AUTUMN TRIMESTER 2021

GROUP 3

AN ANALYSIS OF INDEX-BASED OPTION PRICING THEORY

FIN42020

DERIVATIVE SECURITIES

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FIN42020 Derivative Securities Assessment Submission Form

INTRODUCTORY BRIEF

By providing a brief insight into the global events surrounding this paper's studied period, this introduction offers the opportunity to fully comprehend the expected volatility and the level of investor confidence in the market at that time. For the days leading up to the studied period (1st March 2011 to 31st March 2011), there were several international events from which one could infer an effect on the pricing of capital market derivatives, the most notable being the emergence of the Libyan Civil War.

Commencing on 15th February 2021, this outbreak sent ripples through commodity markets, as oil supplies to the Western world became uncertain. Albeit more of a ‘white swan’ event than 'black swan', this uncertainty had a similarly-mannered spill-over effect into international capital markets, creating increased uncertainty and negatively affecting security prices. Furthermore, this uncertainty was accentuated by an accompanying soar in oil prices, resulting in a two-year high for the commodity. Thus, this paper holds that this may have created an inflationary dynamic on the pricing of options as investors and institutions became subject to both an increased volatility and an increased demand for instruments of hedging. As illustrated later, this paper proposes a strategy for trading this elevated volatility, and further discusses the theory behind option pricing.

QUESTION 1

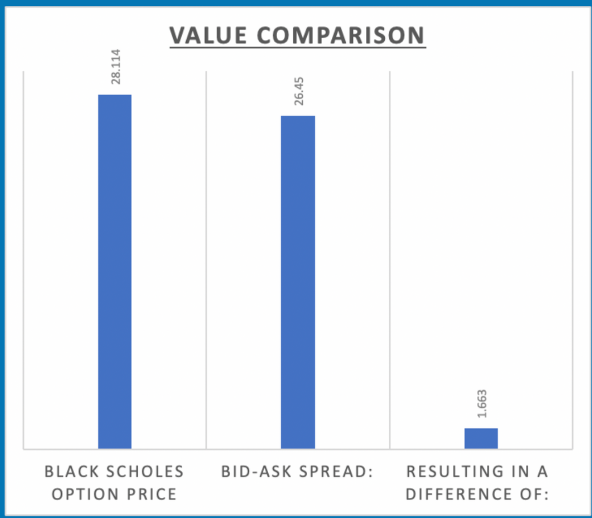
A.

By extracting data from the 'Federal Reserve Economic Data' (FRED) website, one observes that the 'risk-free' rate for the studied period was 0.14%, as seen in the accompanying Excel file (under '*Q1*'). Furthermore, data taken from Bloomberg indicates that the S&P 500 closed at a value of $1306.33 on the date of our option's inception (1st March 2011).

B.

Using a value of 18.203% for Implied Volatility, an Excel-based model was created to implement the Black-Scholes method of option pricing (while assuming a 0% dividend yield). As further indicated by the '*Raw Data*' section of the file, our selected option was the call option with a strike of $1305, as it had the closest Delta to '0.5'.

Running this model gave a value of $28.114 for the option. Comparing this to the bid-ask spread of $26.45, there is a resulting difference of $1.664. In this instance, one can postulate that this is due to the fact that one presumed a 0% dividend yield for this Black Scholes calculation, which inflates the value of the option. Furthermore, this difference could be created by the presence of over-simplified assumptions underpinning the Black Scholes Pricing model, such as no transaction costs or the consistency of interest rates.



C.

By running a '*What-If?*' function on Excel, one can easily enumerate a dividend yield which would cause the value of the Black-Scholes-modelled option to equal that of the bid-ask spread's midpoint value. In this instance, this dividend yield is 3.0027% (*as seen in 'Q1')*.

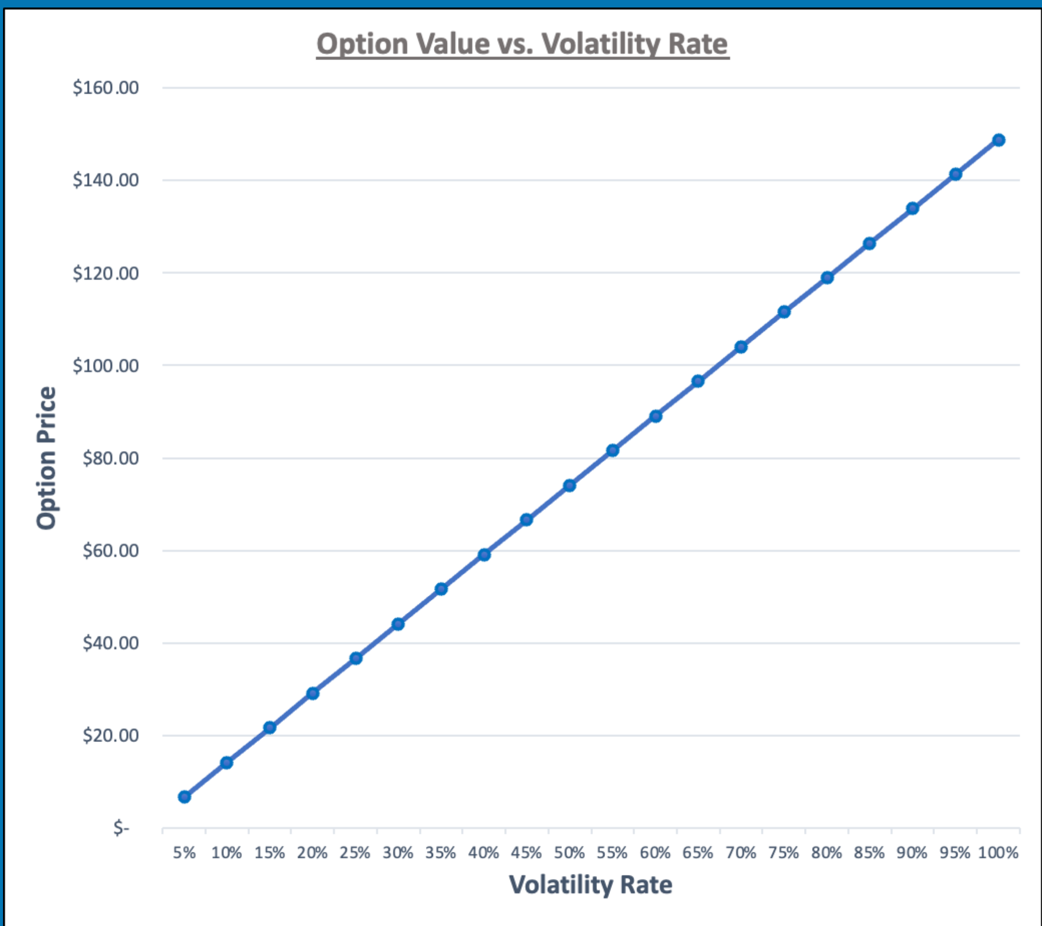
QUESTION 2 (A)

As per the theory underpinning the Black-Scholes model, an increase in the volatility of an option results in an increase in its value, regardless of whether it is a call or a put. The higher the volatility of an option, the higher the likelihood of the option expiring in the money. This is reflected in the accompanying plotted graph (*Exhibit 2(a)*); one observes a linear relation between the value of the option, and the rate of the underlying volatility, *ceteris paribus.*

A Note on Vega

One should not confuse volatility with Vega; volatility is the expected movement of the underlying, while the Vega is the sensitivity of the option price in regard to changes in volatility. As noted in the surface plot, (*Appendix 1),* and as understood from option theory, Vega is affected by both the 'money-ness' and the time to expiry of the option. Thus, Vega is most sensitive when the oscillation of the underlying causes the option to move in and out of the money - a movement which is facilitated by higher volatility. As a final observation, the further in or out of the money an option is, the lower its sensitivity to changes in volatility.

EXHIBIT 2(a): CHANGE IN OPTION VALUE AGAINST 5% INCREMENTS IN VOLATILITY



QUESTION 2 (B)

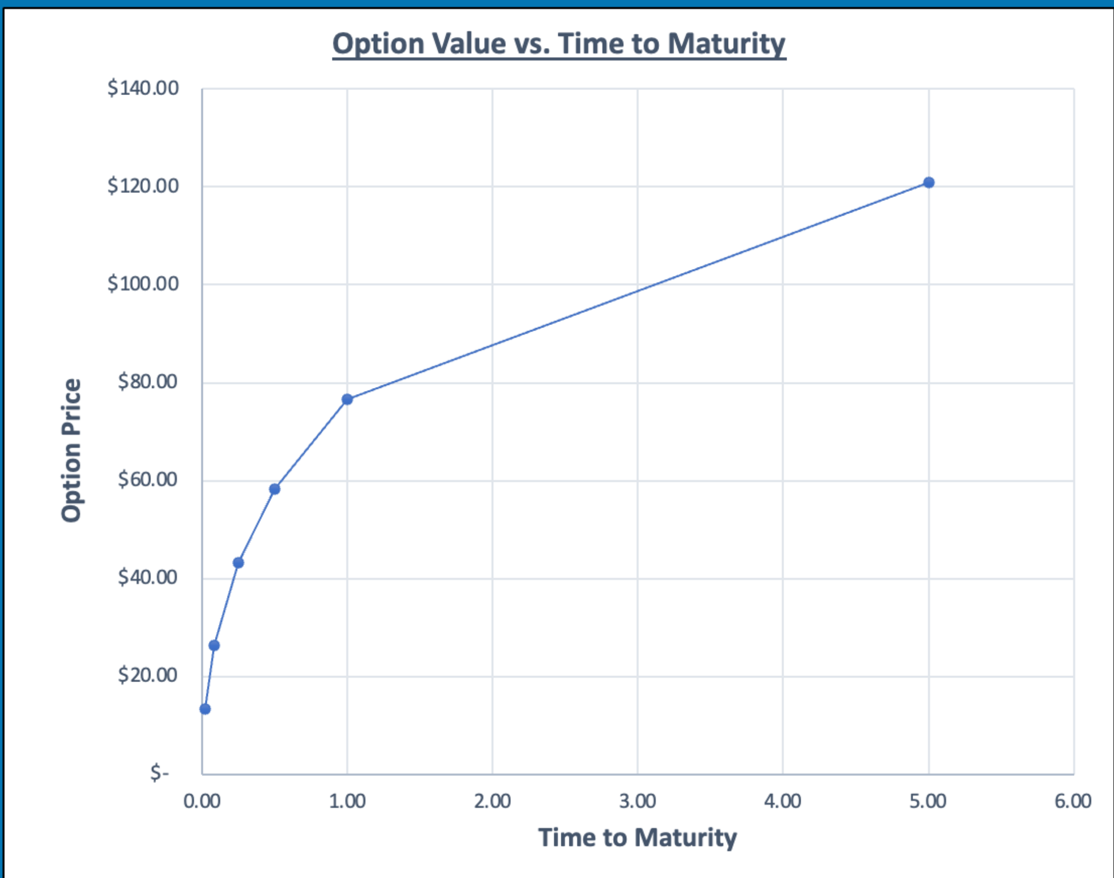
Options are subject to time decay - meaning they lose their extrinsic value with the passage of time. As expressed by both the plotted graph (*Exhibit 2(b))* and the surface plot (*Appendix 2)*, this decay takes the form of a non-linear depreciation. This is reflected in options with a shorter time to expiry, as their daily time decay accelerates due to the increased emphasis on the time value of money. In essence, the value that is provided to the option by an increased time to maturity comes from the fact that the option is more likely to expire in-the-money. However, in certain instances where the option is extremely in-the-money, an increase in maturity can actually cause the option's value to decrease.

A Note on Theta

The Theta of an option is its Greek associated with the passage of time - it is typically always negative as a lesser time to expiration induces a lower valuation. The results of this surface graph affirm the previous statement of non-linear decay - the call option is subject to an exponential 'drop-off' in price the closer it is to expiration.

Furthermore, the plotted Theta appears to be at its most sensitive when options are at-the-money as opposed to those which are either in-the-money or out-the-money, which is in further accordance with option theory.

EXHIBIT 2(b): CHANGE IN OPTION VALUE AGAINST CHANGES IN TIME TO MATURITY



QUESTION 2 (C)

As indicated by the combination of the plotted graph (*Exhibit 2(c))* and surface plot (*Appendix 3*), the influence of changing interest rates is negligible when compared to that of increased volatility or time to expiry. However, the augmented valuation of options arises due to the fact that when there is an increase in interest rates, one can earn a greater return for allocating resources to an interest-bearing security. In this way, options, particularly calls, become more attractive as interest rates rise, as there is now a 'cost savings effect' between buying calls and buying the relevant underlying security.

A Note on Rho

Rho is the least price-significant Greek in this paper's analysis - as illustrated in the surface plot, the impact of interest rate changes on option valuations is relatively small. Furthermore, Rho is larger for options that are further in-the-money, and decreases as the option moves towards its strike price. Thus, Rho is at its maximum value at the point of highest interest rate and longest time to expiry.

EXHIBIT 2(c): CHANGE IN OPTION VALUE AGAINST 0.25% INTEREST RATE INCREMENTALS

In order to discuss the theory behind the payoffs, two approaches will be taken, one which considers this question using strict Black Scholes Pricing Theory assumptions, and one which does not.

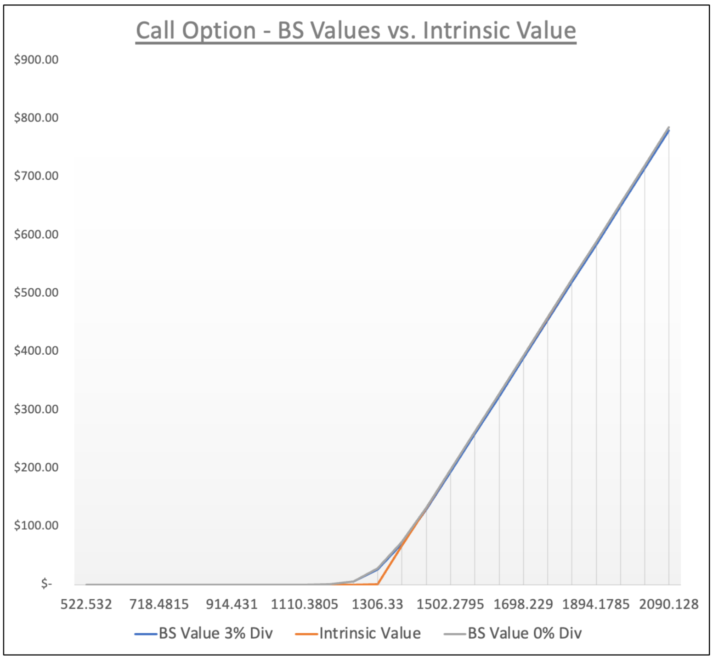
1) In Accordance with BS Pricing Theory

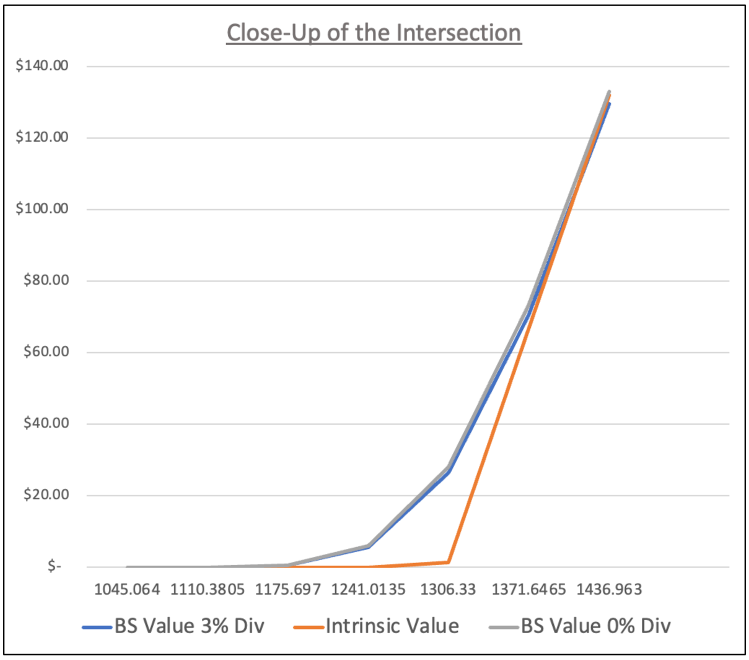
One of the main assumptions of Black Scholes Pricing Theory is the fact that there are no dividends paid out in the lifetime of the stock. Plotting this, one shall see that the ‘BS Value 0% Div’ line and the ‘intrinsic value’ line do not intersect at any point on the curve. As understood from option theory, this is due to the fact that the Black Scholes value of an option is equal to the intrinsic value of said option plus the time value of money - which should always be greater than the intrinsic value alone.

2) The Inclusion of the Dividend Yield

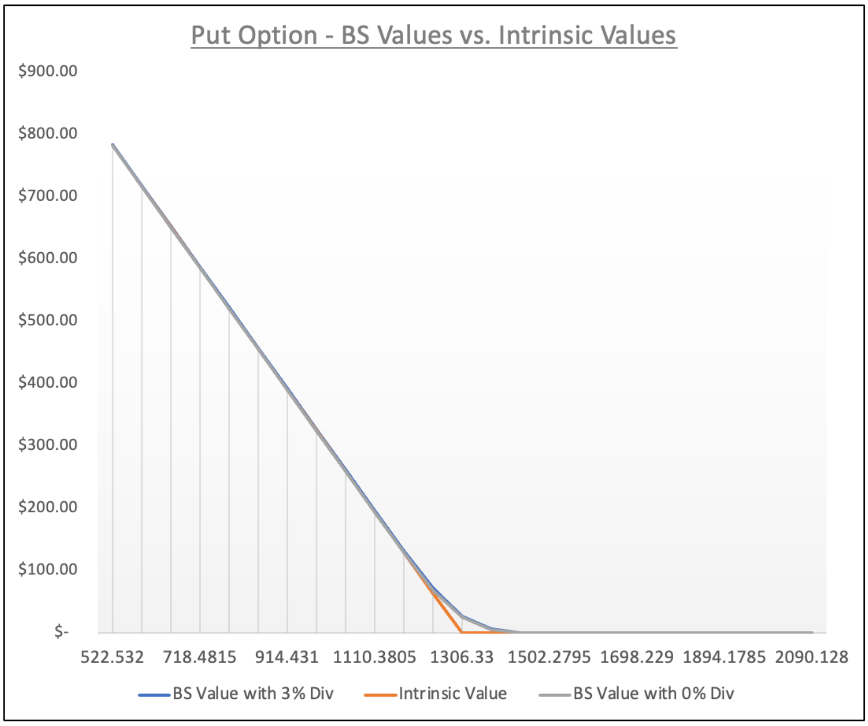
While the inclusion of the dividend yield contradicts BS pricing theory, it helps us to understand why one would exercise an option earlier than its expiration date. If the dividend yield is larger than the risk-free rate, then it is discounting the time value of money of the option at a faster rate than the risk-free rate can improve it. Thus, this creates a dynamic whereby the intrinsic value of the option intersects the Black Scholes pricing model, as seen with the line 'BS Value 3% Div'.

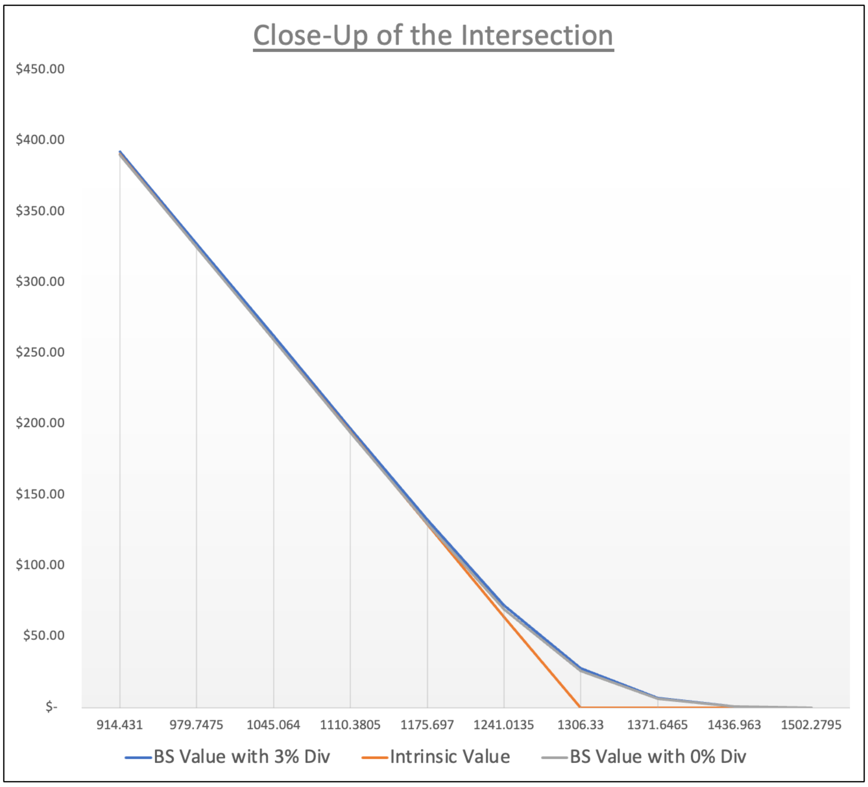
It is important to note that any discussion around early exercising an option should make the assumption that one is dealing with non-dividend American options, as opposed to European options. Nevertheless, if one was to make this assumption, early exercise would still be considered rare, as the power of the Black Scholes Model to calculate the value of American options is undermined as it does not account for the value of early exercise. Furthermore, this paper advises against the early exercising of a non-dividend American call option, as this diminishes the potential for further returns. As the price of the underlying ameliorates, the intrinsic value will cross over the Black Scholes-modelled value and become more precious. It is then at this point that one should exercise their option early - however, as we are dealing with European options, early exercising is not an option. Instead, at this point (approximately $1400) one should sell the option for the Black Scholes price.





Contrastingly to the call option, the intrinsic value of the put option decreases as the value of the underlying increases and reverts to zero once the underlying is out-of-the-money. However, unlike the former, this paper's put option does not intercept the Black Scholes price at any point. Thus, one would hold this option until expiry. Nevertheless, if one was to hypothesise this interception (typically due to a higher dividend yield), as an alternative to just selling the option for the Black Scholes price one could instead theoretically profit from arbitrage opportunity by taking a long position in the option and shorting the underlying. This would result in a conceptual profit for the investor, which they could then reinvest at the risk-free rate. As a final note, this crossover of the intrinsic line does not occur whether the dividend yield is considered in the calculations or not, as illustrated below.





QUESTION 4

Before any considerations of option strategy implementation, one should acknowledge the aforementioned macroeconomic events. As previously stated, the greater amount of volatility present in the market at the inception of this paper's studied options may have caused these derivatives to be more costly; the savvy investors can capitalise on this volatility via the writing of said derivatives.

One can use a GARCH model to estimate the forecasted volatility of financial markets, in order to decide on an appropriate strategy. In the '*Volatility Values Table*', this forecasted volatility, as well as the value of the VIX index on the trade date, the option's implied volatility, and the actual realised volatility for the month have been documented.

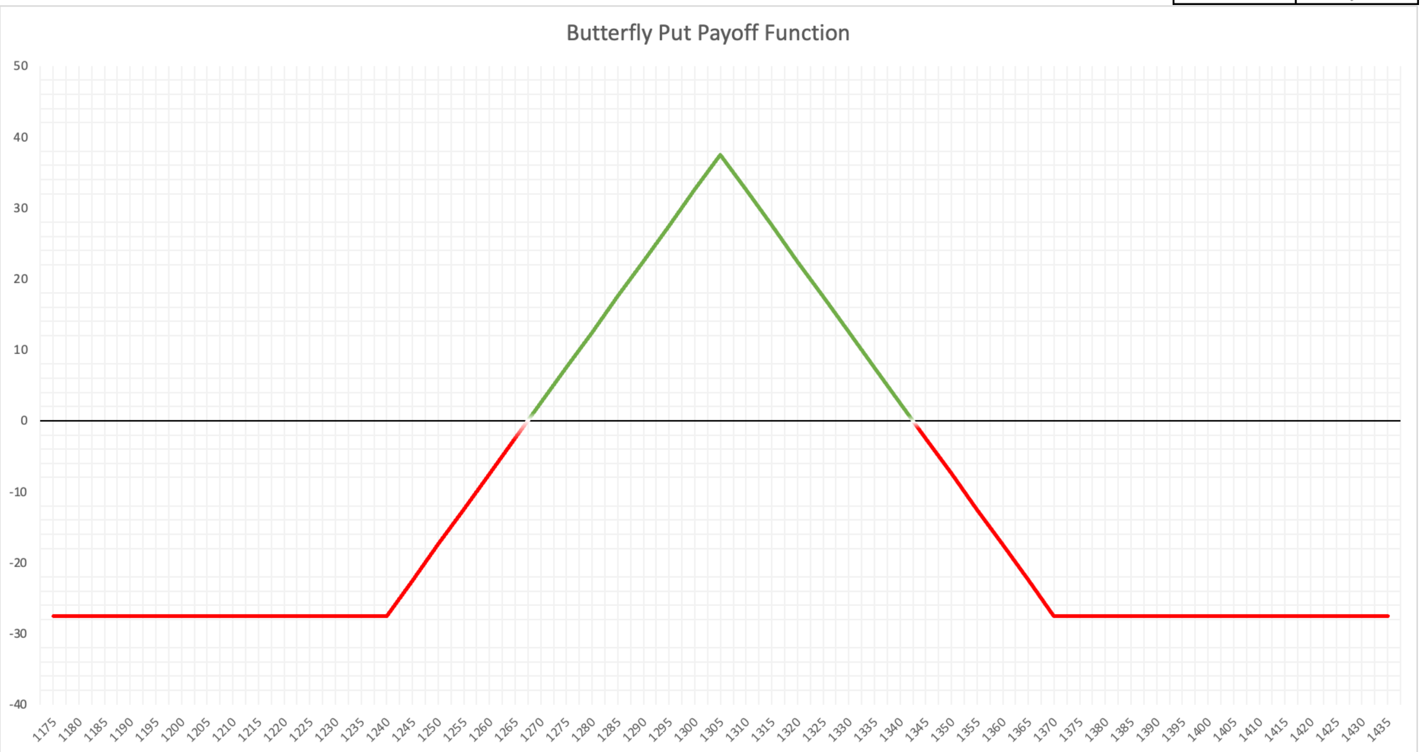
Input	***Value***
Annualised Forecasted Volatility	17.32
Annualised Realised Volatility	15.52
VIX value on trade date	21.01
Implied volatility of your option from option metrics	0.18203

Volatility Values Table

From first observations, one can note that the VIX value on the trade date has been elevated by a relatively large amount in comparison to the forecasted volatility. This divergence in volatilities, due to the increased fear in the market at the time surrounding the Libyan Civil War, would have had an inflationary effect on the prices of our studied options. As the hypothetical investor has stated that they wish to open a *spread* to trade volatility, this paper holds that a butterfly spread is the optimal choice. While a strangle or straddle strategy could also be considered, these techniques are categorised as *combinations,* and thus are less suitable in this instance. The potential payoffs from this strategy have been illustrated on the left hand side of the following page, whereby one can observe that if the price of the underlying stays within the range of $1270 to $1340, then the investor stands to hold a profit. However, once these thresholds are breached, then the investor begins to incur losses.

While 'forecasted volatility' may not be the greatest estimator of the actual realised volatility, the fact that it is lower than our implied volatility means that one could infer an expected decrease in volatility - which further supports the decision to implement a butterfly strategy. As a final note, this paper proposes the implementation of puts instead of calls in the butterfly strategy. When considering strike prices of $1260, $1305 and $1360, one can actually earn more from the use of the former ($37.5), rather than the latter ($37.2).

X-Axis	Stock Price
Y-Axis	Profit/Loss



Price of Underlying	Butterfly Spread Payoff
1230	-27.5
1235	-27.5
1240	-27.5
1245	-22.5
1250	-17.5
1255	-12.5
1260	-7.5
1265	-2.5
1270	2.5
1275	7.5
1280	12.5
1285	17.5
1290	22.5
1295	27.5
1300	32.5
1305	37.5
1310	32.5
1315	27.5
1320	22.5
1325	17.5
1330	12.5
1335	7.5
1340	2.5
1345	-2.5
1350	-7.5
1355	-12.5
1360	-17.5
1365	-22.5
1370	-27.5
1375	-27.5
1380	-27.5

Strike	Premium In/Out	Index Price	Profit/Payout	Total
1240	-10.3	1325.83	0	-10.3
1305	51.4	1325.83	0	51.4
1370	-68.6	1325.83	44.17	-24.43
			Total Income	16.67

Thus, as seen in the table above, the hypothetical investor could have earned $16.67 for the implementation of this strategy. For further information regarding the payoffs from each of the individual put options, please refer to *Appendix 4.*

QUESTION 5

In option theory, Delta represents the change in the option price versus the change in the price of the underlying asset. Graphically, it is the slope of the line, and mathematically, it is the derivative of the Black Scholes option price in relation to the underlying's price. In trading, it is common practice to cover an option's position using the underlying asset or futures contracts. By doing this, one reduces the risk involved in the investment given changes in price, volatility or time. In this paper, one hedges an at-the-money call option selling on March 1st 2011. Being an at-the-money call, one considers a spot price equal to the strike price of $1306,33. Furthermore, as it is a call option, one calculates deltas in the range of 0 to +1, beginning with the options initial delta of 0.5191, as per the following calculations (assuming a non-dividend yield).

$$\frac{\partial c}{\partial S} = N(d_1) = 0.5191$$

This delta value indicates we need to buy a total of $678.08 of S&P500. Lastly, in order to calculate the profit/loss of the position in the day, we subtract the buying/selling spot ($678.08+TC on the first period) to the closing position on the previous day (in our initial case: we use the ask price of the call).

$$27.2 - 678.08 \cdot (1 + 0.01\%) = -650.95$$

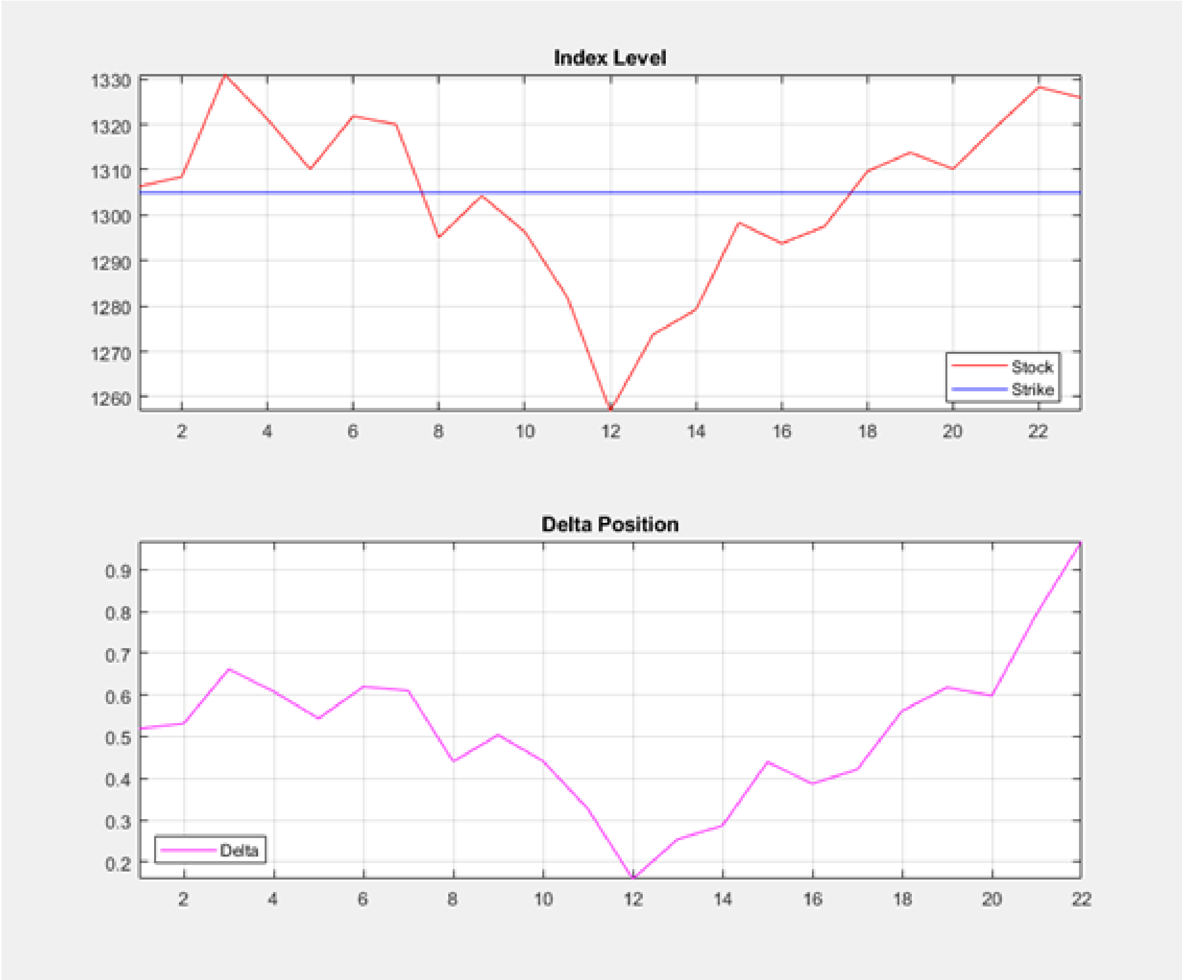
For the next period, we start of the previous delta and bank (P/L) values and calculate the following:

1. Delta for the period, updating the value for the stock, the days to expiry and the price.
2. Delta difference with the previous period.
3. Purchase/Sale of the index in terms of delta.
4. Profit/Loss of the position.

$$\Delta = 0.5315;\ (\Delta_i - \Delta_{i-1}) \cdot S_i = \$\,16.31$$

$$P/L = -650.95 - 16.31 \cdot (1 + 0.01\%) = \$\, -667.26$$

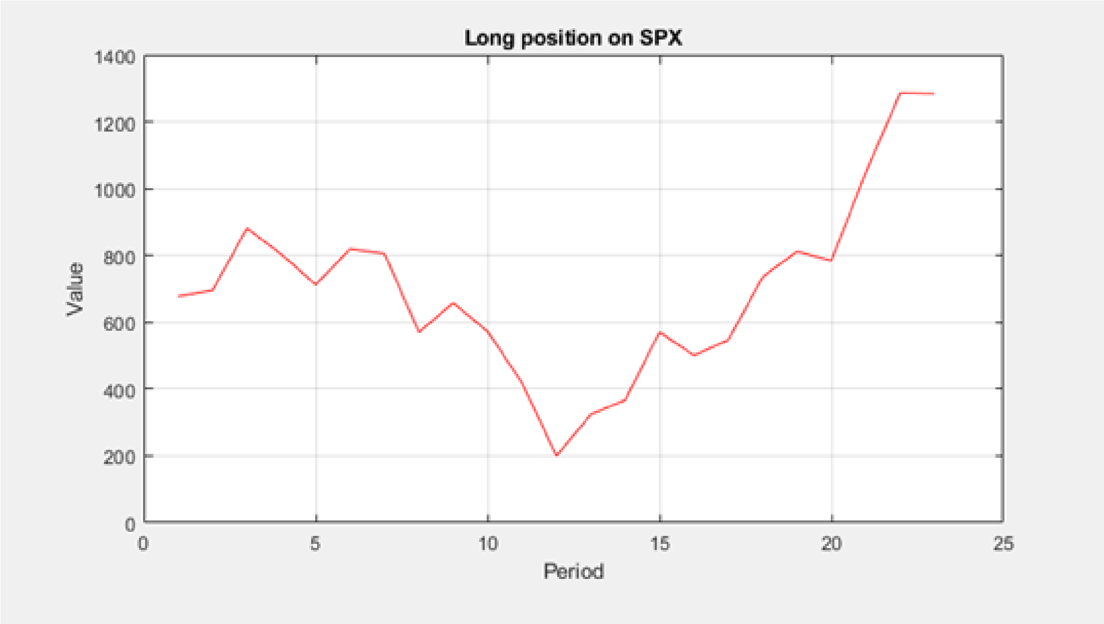
Plotting our calculations over all the periods we obtain the next index and delta results:



As one can see in both graphs, the results of Delta vary over time, and the behaviour is proportional to the index level (except for the last period, where on the day of expiration the call has a 1:1 ratio to the index change). If one looks closer at the index level graph, at the date of expiry the price of the SPX is above the strike price of our call, meaning that if one held until expiry, one would end with a benefit of $20.83. Instead, with a Delta-hedged position and a shorted call, a benefit of $26.62 was achieved, which translates into $5.79 of profit.

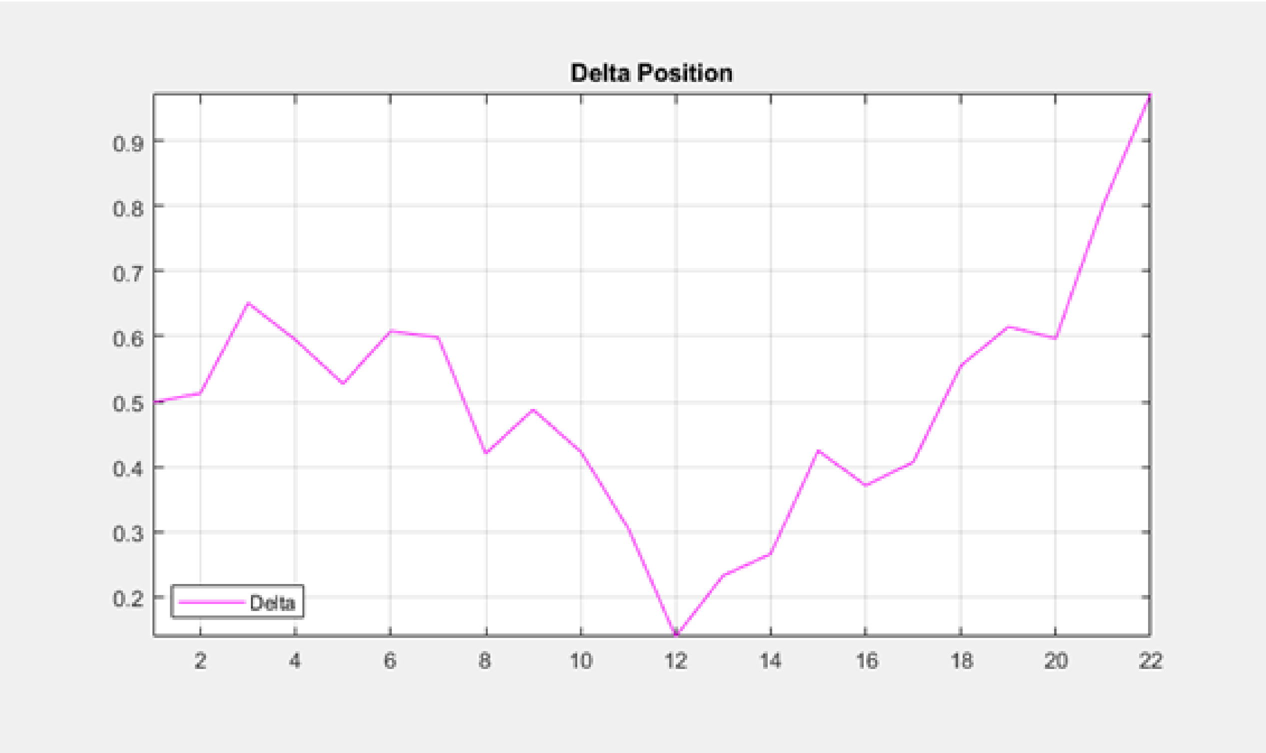
Before moving onto the comparison for volatility, one can also take a look at the bank balance (liabilities) of the delta-hedged position or the total value of the portfolio with the index one holds, as seen in the following graph:

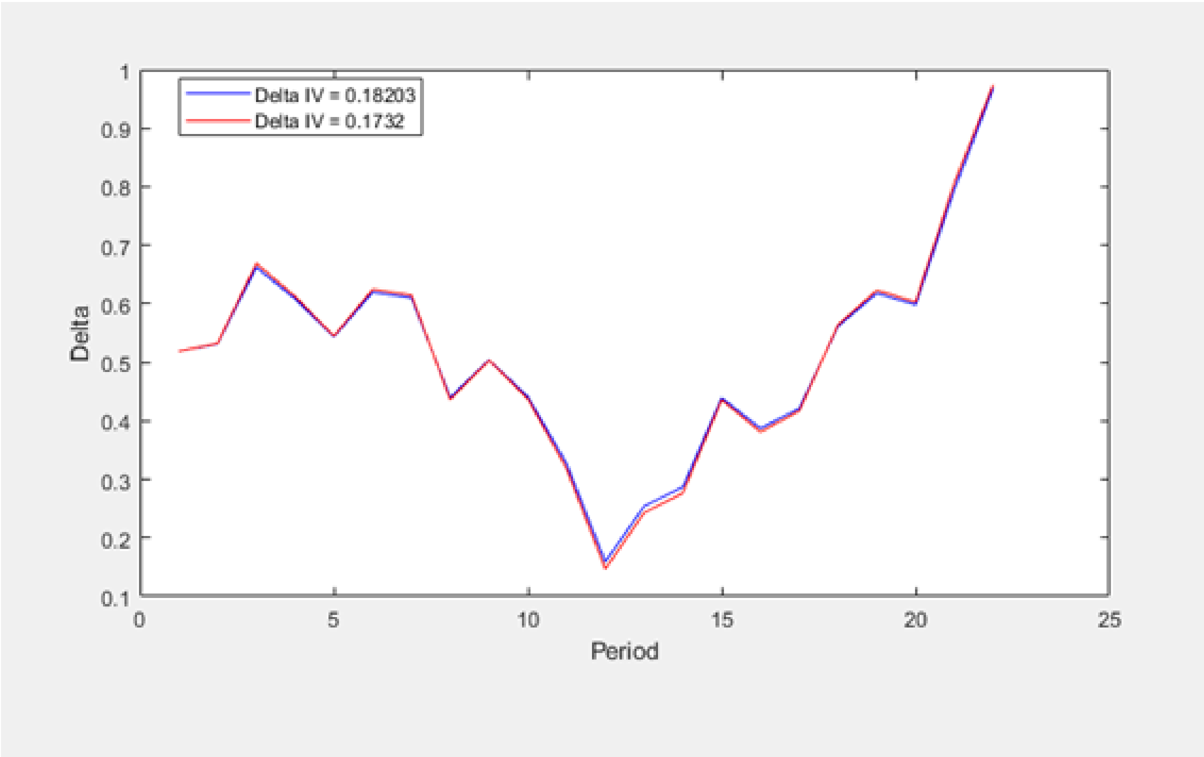
QUESTION 5



FORECASTED VOLATILITY

In the previous question this paper forecasted an annual volatility of 17.32%; this affects directly our Delta values for each period and the final profit of our Delta-hedge. Re-running the calculations one obtains a total profit of $5.44, which is 6.04% less than the previous result. When comparing both Deltas side-by-side, there is less spread in the lower implied volatility forecast.

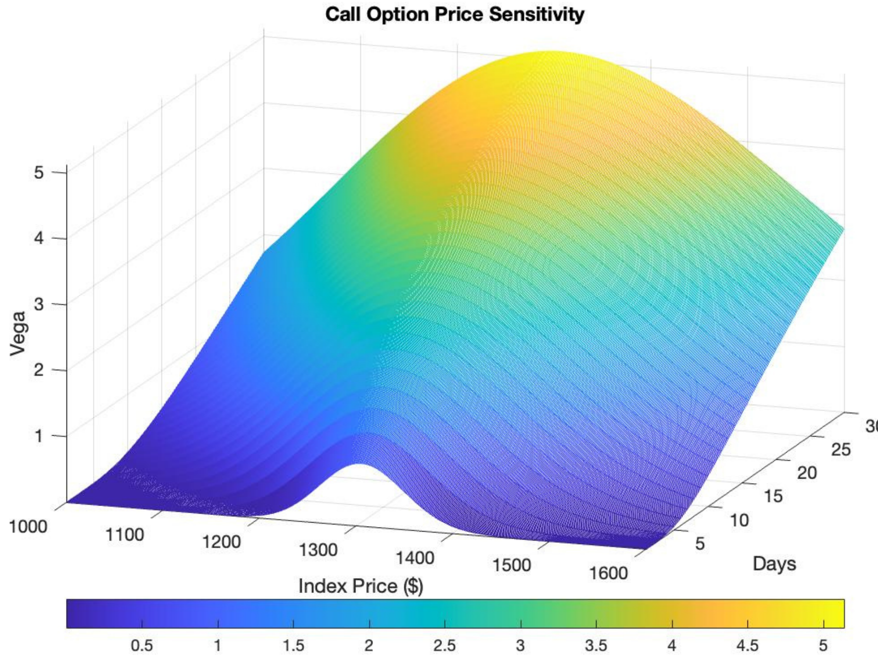




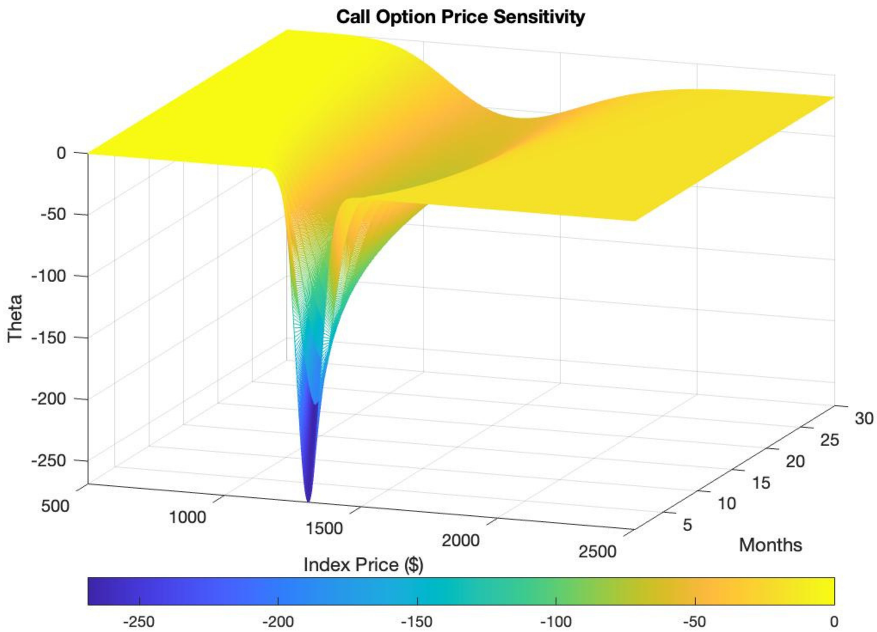
Implied volatility has a direct impact on the Delta: for out-of-the-money options, a higher volatility increases the value of Delta, while decreasing it for in-the-money options. On the other hand, a lower volatility logically causes the opposite effect. In this paper's case, an at-the-money, delta-hedge option, a higher volatility actually translates into a higher profit, as the implemented strategy benefits from movements in both the prices of the underlying and the Delta value. However, in this scenario, the forecasted volatility is lower and probably less accurate, as it is estimated through a model (GARCH) and historical data, in contrast with the implied volatility provided by the Option Metrics data.

To summarise, this paper proposes a profit by shorting the call option and Delta-hedging it, and, the forecasted volatility has resulted in a lesser benefit in comparison to the implied volatility of the option.

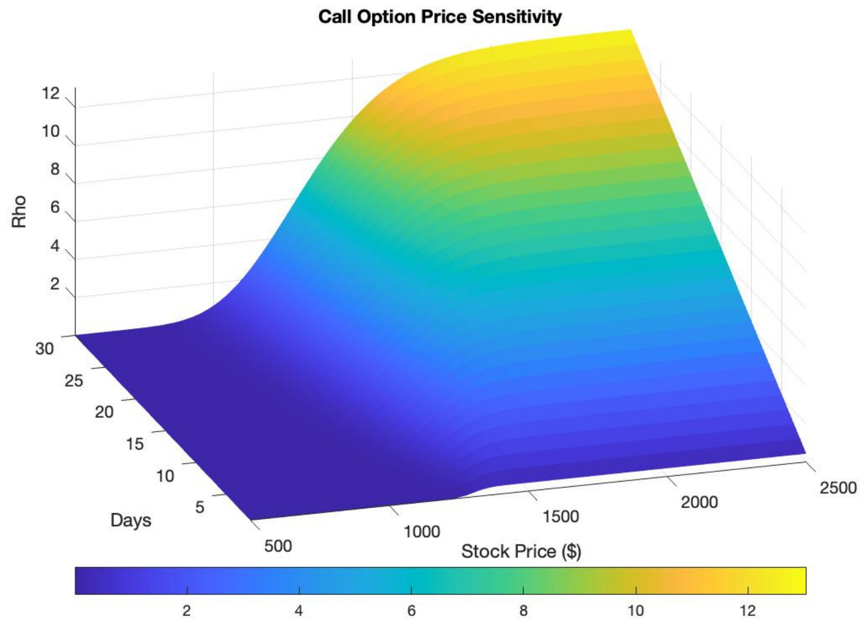
Appendix 1 - Plotting Vega



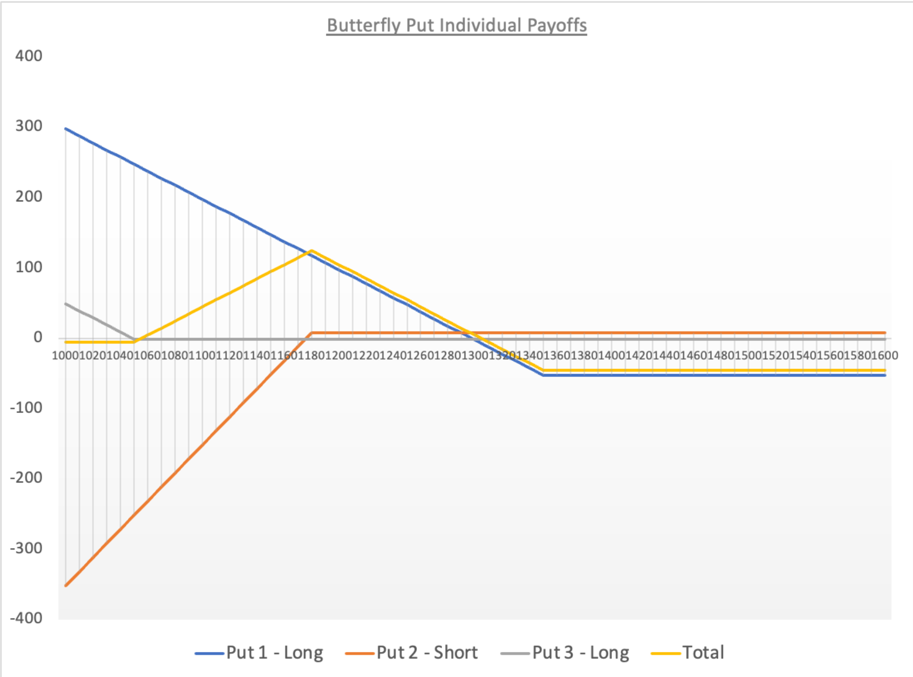
Appendix 2 - Plotting Theta



Appendix 3 - Plotting Rho



Appendix 4 - Butterfly Individual Payoffs



Appendix 5 - Delta hedging results, IV = 0.18203

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Delta Hedging
-----------------------------------------------------------------
 t= 1; delta = 0.5191; bought $ 678.08 of the index; Bank $ -650.95
 t= 2; delta = 0.5315; bought $ 16.31 of the index; Bank $ -667.26
 t= 3; delta = 0.6621; bought $ 173.77 of the index; Bank $ -841.05
 t= 4; delta = 0.6084; bought $ -70.87 of the index; Bank $ -770.19
 t= 5; delta = 0.5435; bought $ -85.04 of the index; Bank $ -685.16
 t= 6; delta = 0.6198; bought $ 100.84 of the index; Bank $ -786.02
 t= 7; delta = 0.6104; bought $ -12.48 of the index; Bank $ -773.54
 t= 8; delta = 0.4402; bought $ -220.45 of the index; Bank $ -553.12
 t= 9; delta = 0.5041; bought $ 83.33 of the index; Bank $ -636.46
 t= 10; delta = 0.4415; bought $ -81.11 of the index; Bank $ -555.36
 t= 11; delta = 0.3269; bought $ -146.92 of the index; Bank $ -408.46
 t= 12; delta = 0.1591; bought $ -210.86 of the index; Bank $ -197.62
 t= 13; delta = 0.2542; bought $ 121.17 of the index; Bank $ -318.80
 t= 14; delta = 0.2869; bought $ 41.81 of the index; Bank $ -360.62
 t= 15; delta = 0.4394; bought $ 197.99 of the index; Bank $ -558.63
 t= 16; delta = 0.3871; bought $ -67.67 of the index; Bank $ -490.97
 t= 17; delta = 0.4215; bought $ 44.59 of the index; Bank $ -535.57
 t= 18; delta = 0.5616; bought $ 183.54 of the index; Bank $ -719.13
 t= 19; delta = 0.6181; bought $ 74.27 of the index; Bank $ -793.41
 t= 20; delta = 0.5985; bought $ -25.75 of the index; Bank $ -767.67
 t= 21; delta = 0.7950; bought $ 259.31 of the index; Bank $ -1027.00
 t= 22; delta = 0.9685; bought $ 230.44 of the index; Bank $ -1257.47
-----------------------------------------------------------------
Total Hedge Profit: $ 5.79
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Appendix 6- Delta hedging results, IV = 0.1732

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-----------------------------------------------------------------
Delta Hedging
-----------------------------------------------------------------
 t= 1; delta = 0.5190; bought $ 677.99 of the index; Bank $ -650.86
 t= 2; delta = 0.5321; bought $ 17.16 of the index; Bank $ -668.03
 t= 3; delta = 0.6689; bought $ 182.10 of the index; Bank $ -850.15
 t= 4; delta = 0.6129; bought $ -74.06 of the index; Bank $ -776.11
 t= 5; delta = 0.5448; bought $ -89.16 of the index; Bank $ -686.96
 t= 6; delta = 0.6249; bought $ 105.81 of the index; Bank $ -792.78
 t= 7; delta = 0.6150; bought $ -13.04 of the index; Bank $ -779.74
 t= 8; delta = 0.4363; bought $ -231.46 of the index; Bank $ -548.31
 t= 9; delta = 0.5034; bought $ 87.56 of the index; Bank $ -635.88
 t= 10; delta = 0.4378; bought $ -85.11 of the index; Bank $ -550.77
 t= 11; delta = 0.3180; bought $ -153.53 of the index; Bank $ -397.26
 t= 12; delta = 0.1467; bought $ -215.34 of the index; Bank $ -181.95
 t= 13; delta = 0.2430; bought $ 122.70 of the index; Bank $ -304.67
 t= 14; delta = 0.2767; bought $ 43.06 of the index; Bank $ -347.74
 t= 15; delta = 0.4358; bought $ 206.57 of the index; Bank $ -554.33
 t= 16; delta = 0.3810; bought $ -70.87 of the index; Bank $ -483.46
 t= 17; delta = 0.4170; bought $ 46.75 of the index; Bank $ -530.22
 t= 18; delta = 0.5642; bought $ 192.82 of the index; Bank $ -723.06
 t= 19; delta = 0.6235; bought $ 77.91 of the index; Bank $ -800.98
 t= 20; delta = 0.6031; bought $ -26.79 of the index; Bank $ -774.20
 t= 21; delta = 0.8066; bought $ 268.48 of the index; Bank $ -1042.71
 t= 22; delta = 0.9746; bought $ 223.23 of the index; Bank $ -1265.96
-----------------------------------------------------------------
Total Hedge Profit: $ 5.41
-----------------------------------------------------------------
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