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1.

a. $a_k = \frac{0.8}{2^{k-1}}$

a1 = 0.8
a2 = 0.4
a3 = 0.2
a4 = 0.1
a5 = $0.8 / 2^4 = 0.8 / 16 = 0.05$

b. a3 = 2.5a2 - a1
0.2 = 2.5 * 0.4 - 0.8
0.2 = 1 - 0.8
0.2 = 0.2

$a_{k+1} = 2.5\, a_k - a_{k-1}$

$a_k = 2.5 * 0.8/2^{k-1} - 0.8/2^{k-2}$

$a_k = 2/2^{k-1} - 0.8/2^{k-2}$

```
function [a, a_hat] = no1()

f = @(k) 0.8/2^(k-1)
a_hat = 1:50
a_hat(1) = 0.8
a_hat(2) = 0.5
a = 1:50

for i=1:50
  if i > 2
    a_hat(i) = 2.5 * a_hat(i-1) - a_hat(i-2)
  endif
  a(i) = f(i)
  #disp(sprintf('a_hat: %.308f', f_hat(i)));
  #disp(sprintf('a: %.308f', f(i)));
endfor

endfunction
```

c.

```
>> a_hat'
ans =

   8.0000e-01
   4.0000e-01
   2.0000e-01
   1.0000e-01
   5.0000e-02
   2.5000e-02
   1.2500e-02
   6.2500e-03
   3.1250e-03
   1.5625e-03
   7.8125e-04
   3.9062e-04
   1.9531e-04
   9.7656e-05
   4.8828e-05
   2.4414e-05
   1.2207e-05
   6.1035e-06
   3.0518e-06
   1.5259e-06
   7.6292e-07
   3.8143e-07
   1.9066e-07
   9.5212e-08
   4.7373e-08
   2.3221e-08
   1.0679e-08
   3.4769e-09
  -1.9868e-09
  -8.4440e-09
  -1.9123e-08
  -3.9364e-08
  -7.9287e-08
  -1.5885e-07
  -3.1784e-07
  -6.3576e-07
  -1.2716e-06
  -2.5431e-06
  -5.0863e-06
  -1.0173e-05
  -2.0345e-05
  -4.0690e-05
```

```
>> a'
ans =

   8.0000e-01
   4.0000e-01
   2.0000e-01
   1.0000e-01
   5.0000e-02
   2.5000e-02
   1.2500e-02
   6.2500e-03
   3.1250e-03
   1.5625e-03
   7.8125e-04
   3.9063e-04
   1.9531e-04
   9.7656e-05
   4.8828e-05
   2.4414e-05
   1.2207e-05
   6.1035e-06
   3.0518e-06
   1.5259e-06
   7.6294e-07
   3.8147e-07
   1.9073e-07
   9.5367e-08
   4.7684e-08
   2.3842e-08
   1.1921e-08
   5.9605e-09
   2.9802e-09
   1.4901e-09
   7.4506e-10
   3.7253e-10
   1.8626e-10
   9.3132e-11
   4.6566e-11
   2.3283e-11
   1.1642e-11
   5.8208e-12
   2.9104e-12
   1.4552e-12
   7.2760e-13
```

d. Forward error =
$|ahat_{50} - a_{50}| / |a_{50}| = |-0.010417 - 1.4211e-15| / 1.4211e-15 = 7.3301e+12$
Forward error yang sangat besar ni disebabkan oleh akumulasi error dari ahat dimana pada suatu ketika ahat mulai bernilai negatif

```
>> abs(a_hat(50) - a(50)) / a(50)
ans = 7.3301e+12
```

Semenjak a_hat 30, tepatnya pada a_hat 29, aproksimasi menjadi nilai minus dikarenakan akurasi a_hat yang perlahan berkurang, sehingga ketika dilakukan pengurangan terjadi akumulasi error dimana $ahat_{k-2} > 2.5 * ahat_{i-1}$

```
>> 2.5 * a_hat(28) - a_hat(27)
ans = -1.9868e-09
>> a(28)
ans = 5.9605e-09
>> a_hat(28)
ans = 3.4769e-09
```

2. TBD
   a. Pertama saya coba denan q = 1, c = 2000000, s = 0.9

```
>> x
x =

   1.0718e+06   7.4408e+05   5.7435e+05   4.6985e+05   3.9874e+05
```

Model saya ternyata sekitar ½ dari X_n aslinya, maka hanya perlu mengalikan c dengan 2.

   b.

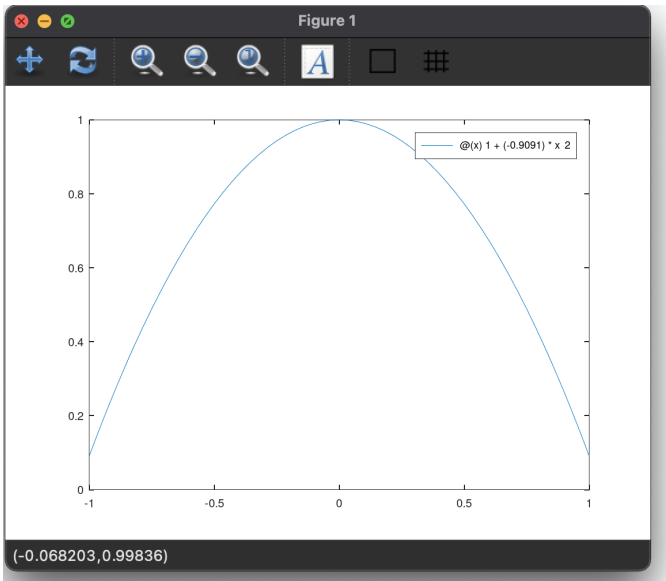
```
>> [x] = no2()
x =

   2.1435e+06   1.4882e+06   1.1487e+06   9.3970e+05   7.9749e+05
```

   c. X_50 = 788589
   rel_error
   $Xhat_{50} - X_{50}| / |X_{50}| = |6.0452e+05 - 788589| / 788589 = 0.2334$

```
>> x_real = 788589;
>> err = abs(f(8) - x_real)/x_real
err = 0.2334
```

3. TBD
   a. X E {-1, 0, 1}
      p(x) = a0 + a1x + a2x^2

      a0 - a1 + a2 = 1/11 (i)
      a0 = 1 (ii)
      a0 + a1 + a2 = 1/11 (iii)




```
>> A = [1 -1 1; 1 0 0; 1 1 1];
>> b = [1/11; 1; 1/11];
>> A\b
ans =

   1.0000
        0
  -0.9091

>> coeff = [A\b];
>> p = @(x) 1 + (-0.9091) * x^2
p =

@(x) 1 + (-0.9091) * x ^ 2

>> fplot(p, [-1, 1])
```

Sangat mirip, karena menggunakan persamaan normal yang mereduce norm(| b - Ax|).

Error menggunakan forward error, error pun cukup baik

```
>> f = @(x) 1/(1 + 10*x^2)
f =

@(x) 1 / (1 + 10 * x ^ 2)

>> err = @(x) abs(p(x))
err =

@(x) abs (p (x))

>> err = @(x) abs(p(x) - f(x))/f(x);
>> err(1)
ans = 1.0000e-04
>> err(-1)
ans = 1.0000e-04
>> err(0)
ans = 0
```

b.

```
function [A, s, b] = no3()
f = @(x) 1/(1 + 10*x^2)

n = 101
s = 1:101
b = 1:101
A = zeros(101, 101);

for i=0:100
  s(i+1) = i/50 - 1;
  A(i+1, :) = getRow(i);
  b(i+1) = f(i);
endfor

endfunction

function [row] = getRow(i)
  row = 1:101;
  cur = 1;
  for k=0:100
    row(k+1) = cur;
    cur = cur * i;
  endfor
endfunction
```

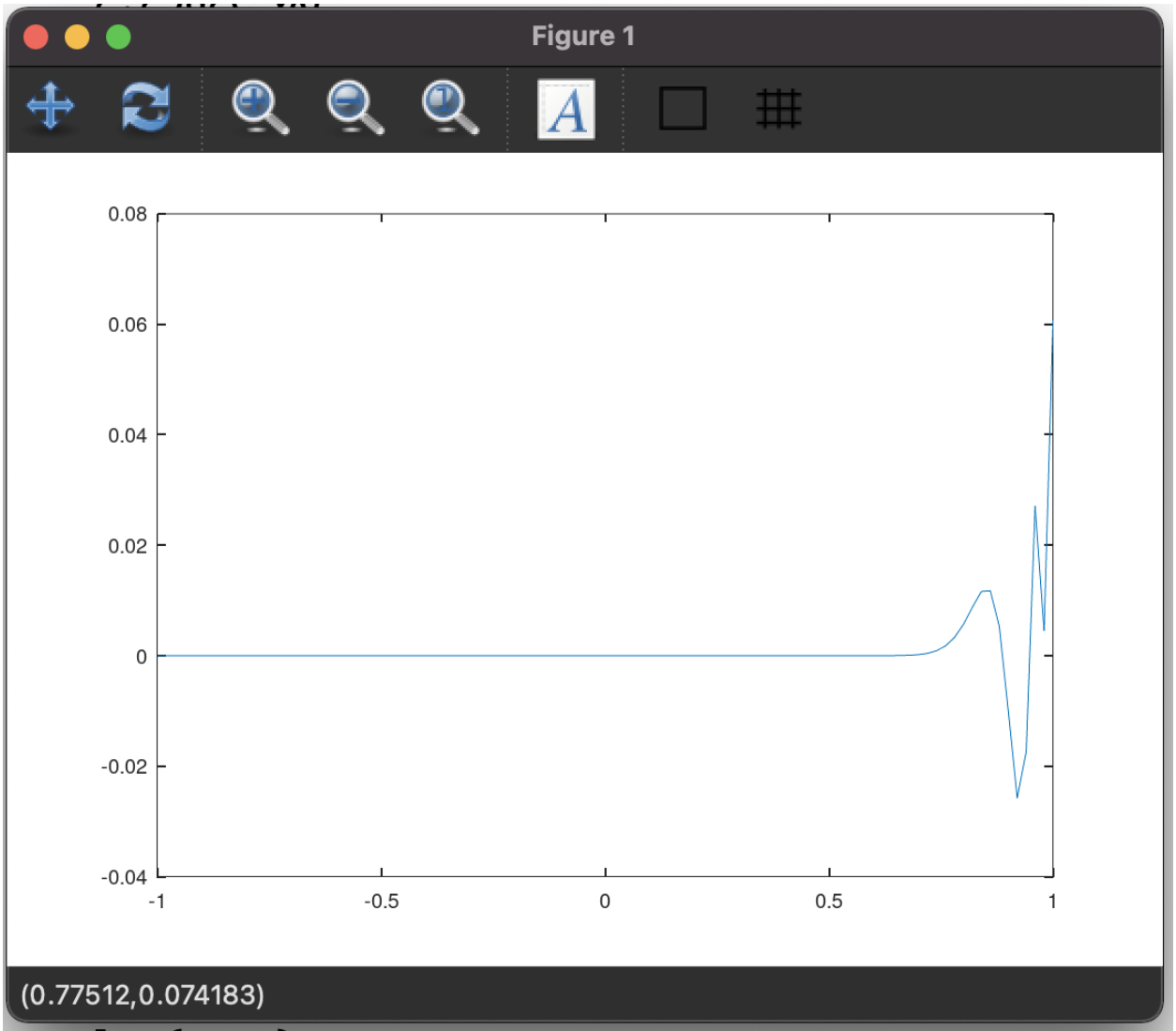
```
>> A\b'
ans =

   5.9647e-186
  -6.8441e-186
   1.3442e-190
  -4.0459e-194
   1.1584e-198
             0
             0
             0
             0
             0
             0
             0
             0
             0
             0
             0
             0
             0
             0
             0
             0
             0
             0
   1.5316e-322
   1.3700e-320
   1.2076e-318
   1.0653e-316
   9.4040e-315
   8.3067e-313
   7.3412e-311
   6.4907e-309
   5.7406e-307
   5.0784e-305
   4.4933e-303
   3.9758e-301
```

c.

```
function [y] = mypol(x)

[A, s, b] = no3();
coeff = A\b';

y = getRow(x) * coeff;

endfunction

function [row] = getRow(i)
  row = 1:101;
  cur = 1;
  for k=0:100
    row(k+1) = cur;
    cur = cur * i;
  endfor
endfunction
```

```
>> A * x
ans =

   5.9647e-186
   3.5034e-186
   3.2452e-157
   2.4774e-140
   2.3323e-128
   4.4809e-119
   1.7107e-111
   4.3698e-105
    1.5418e-99
    1.1991e-94
    2.8290e-90
    2.5421e-86
    1.0293e-82
    2.1339e-79
    2.5011e-76
    1.7931e-73
    8.3775e-71
    2.6866e-68
    6.1733e-66
    1.0536e-63
    1.3770e-61
    1.4143e-59
    1.1673e-57
    7.8945e-56
    4.4490e-54
    2.1205e-52
    8.6599e-51
    3.0658e-49
    9.5059e-48
    2.6054e-46
    6.3648e-45
    1.3963e-43
    2.7692e-42
    4.9957e-41
    8.2438e-40
    1.2507e-38
    1.7526e-37
```

d.



e. Sepertinya terjadi kesalahan
f. D
g. D
h. D
i. p(x) butuh n! Flops, maka harusnya q(x) lebih efisien

4.

a. Saya akan menggunakan bisection method / newton
b. Sepertinya sulit mencari interval [-, +], sehingga gunakan newton, x_0 = 1.5
c. Kelemahan: harus menspecify TOL atau dapat terjadi lama, bisa saja tidak convergen
   Kelebihan: penyelesaian kuadratik
d. Diulang untuk setiap koefisien
e. Asal tebakan tidak jauh dari x*, maka newton memiliki akurasi tinggi untuk konvergen

A dan B terlihat saling berhubungan (A naik, b naik)