Statistical Inference: Peer Assessment Part 1

Requirements

- Show the sample mean and compare it to the theoretical mean of the distribution.
- Show how variable the sample is (via variance) and compare it to the theoretical variance of the distribution.
- Show that the distribution is approximately normal.

Including Libraries

```
library("data.table")
library(ggplot2)
```

Instructions

```
# set seed for reproducability
set.seed(31)

# Set lambda = 0.2
lambda <- 0.2

# 40 samples
n <- 40

# 1000 simulations
simulations <- 1000

# Simulate
simulated_exponentials <- replicate(simulations, rexp(n, lambda))

# Calculate mean of exponentials
means_exponentials <- apply(simulated_exponentials, 2, mean)
```

Question 1

Show where the distribution is centered at and compare it to the theoretical center of the distribution.

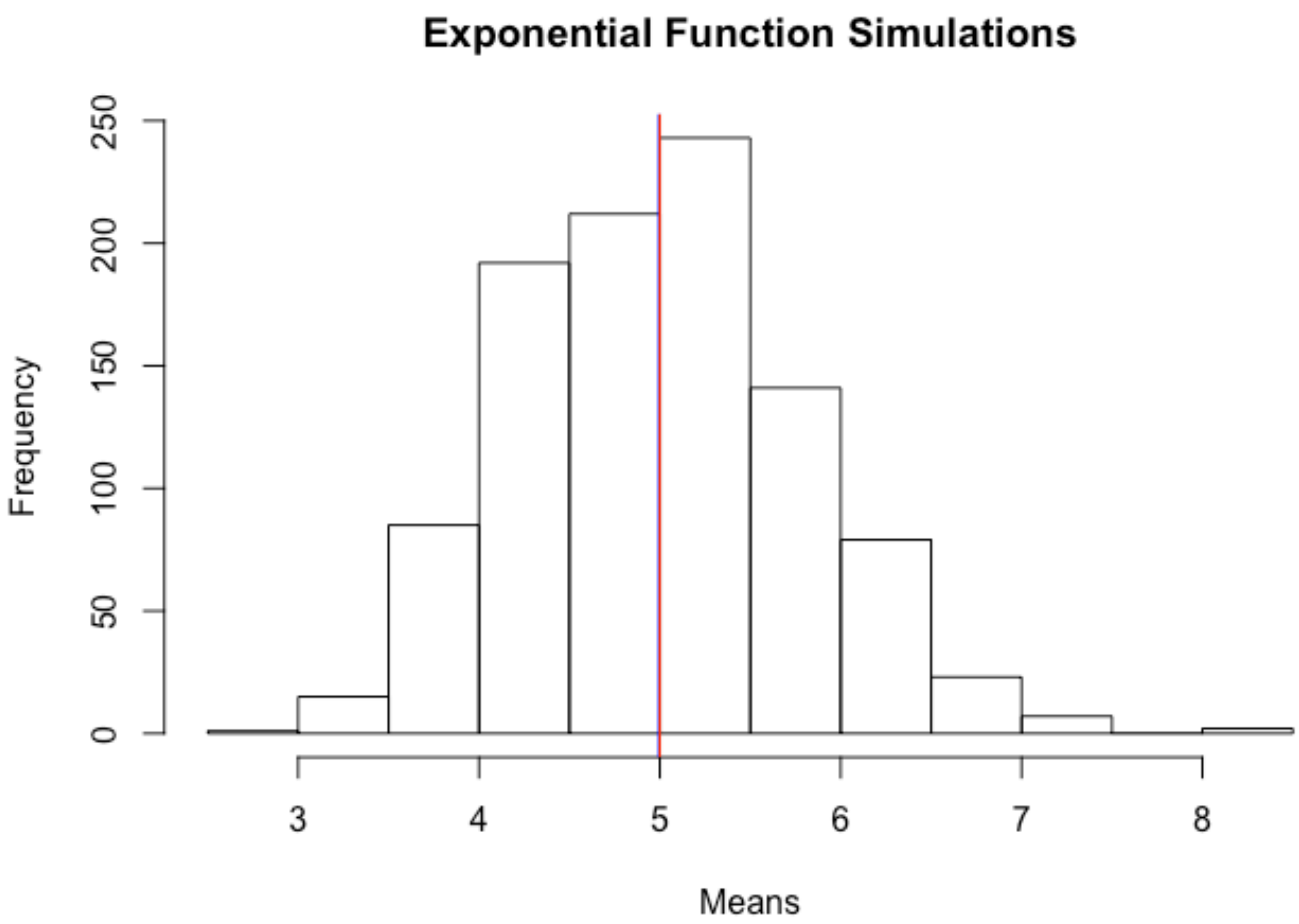
```
analytical_mean <- mean(means_exponentials)
analytical_mean
```

```
## [1] 4.993867
```

```
# analytical mean
theory_mean <- 1/lambda
theory_mean
```

```
## [1] 5
```

```
# visualization
hist(means_exponentials, xlab = "Means", main = "Exponential Function Simulations")
abline(v = analytical_mean, col = "blue")
abline(v = theory_mean, col = "red")
```



The analytics mean is 4.993867 while the theoretical mean is 5. The center of distribution of averages of 40 exponentials is very close to the theoretical center of the distribution.

Question 2

Show how variable it is (via variance) and compare it to the theoretical variance of the distribution.

```
# Standard deviation of distribution
standard_deviation_dist <- sd(means_exponentials)
standard_deviation_dist
```

```
## [1] 0.7931608
```

```
# Standard deviation from analytical expression
standard_deviation_theory <- (1/lambda)/sqrt(n)
standard_deviation_theory
```

```
## [1] 0.7905694
```

```
# Variance of distribution
variance_dist <- standard_deviation_dist^2
variance_dist
```

```
## [1] 0.6291041
```

```
# Variance from analytical expression
variance_theory <- ((1/lambda)*(1/sqrt(n)))^2
variance_theory
```

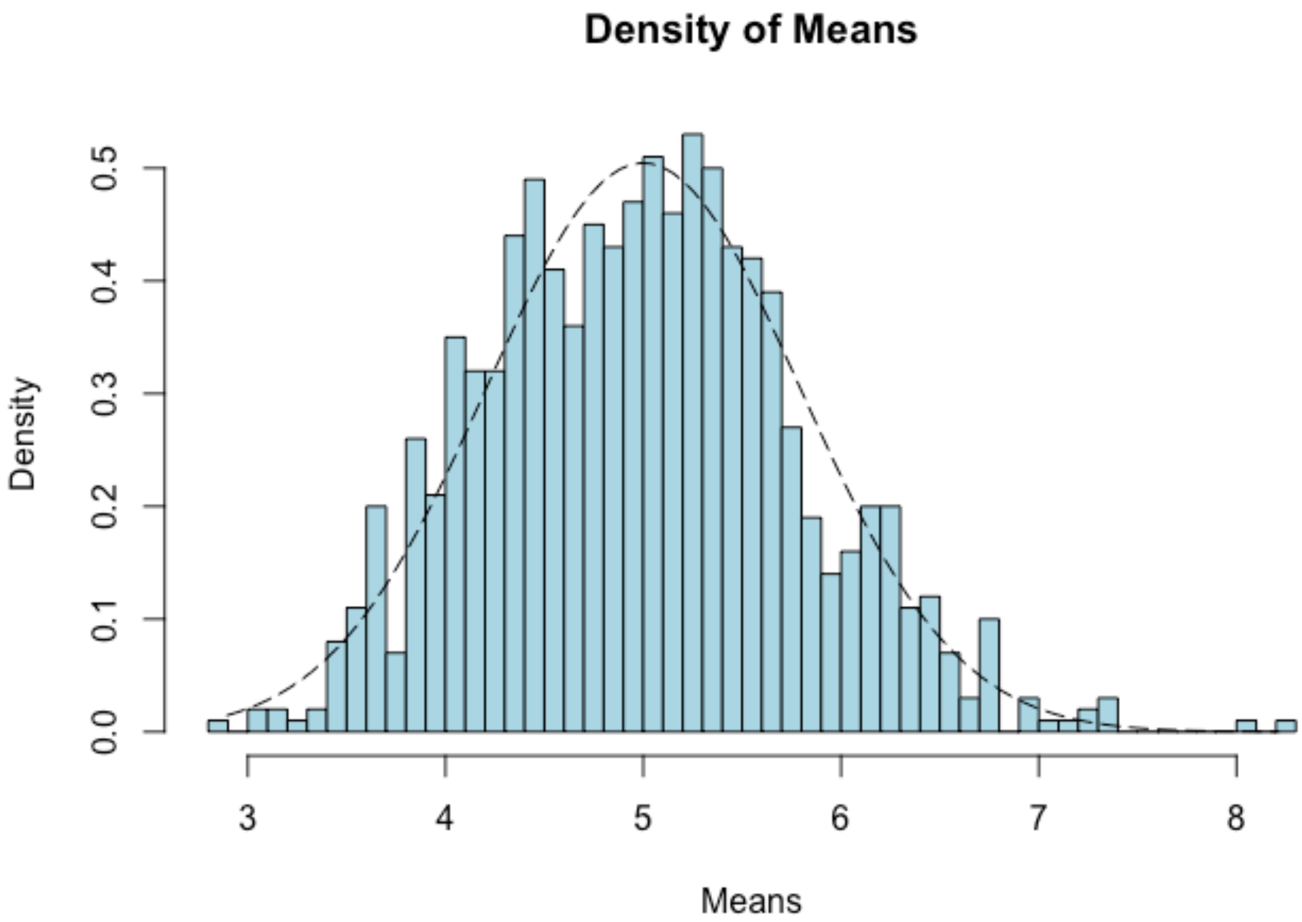
```
## [1] 0.625
```

Standard Deviation of the distribution is 0.7931608 with the theoretical SD calculated as 0.7905694. The theoretical variance is calculated as $((1/\text{lambda}) * (1/n^{0.5}))^2 = 0.625$. The actual variance of the distribution is 0.6291041.

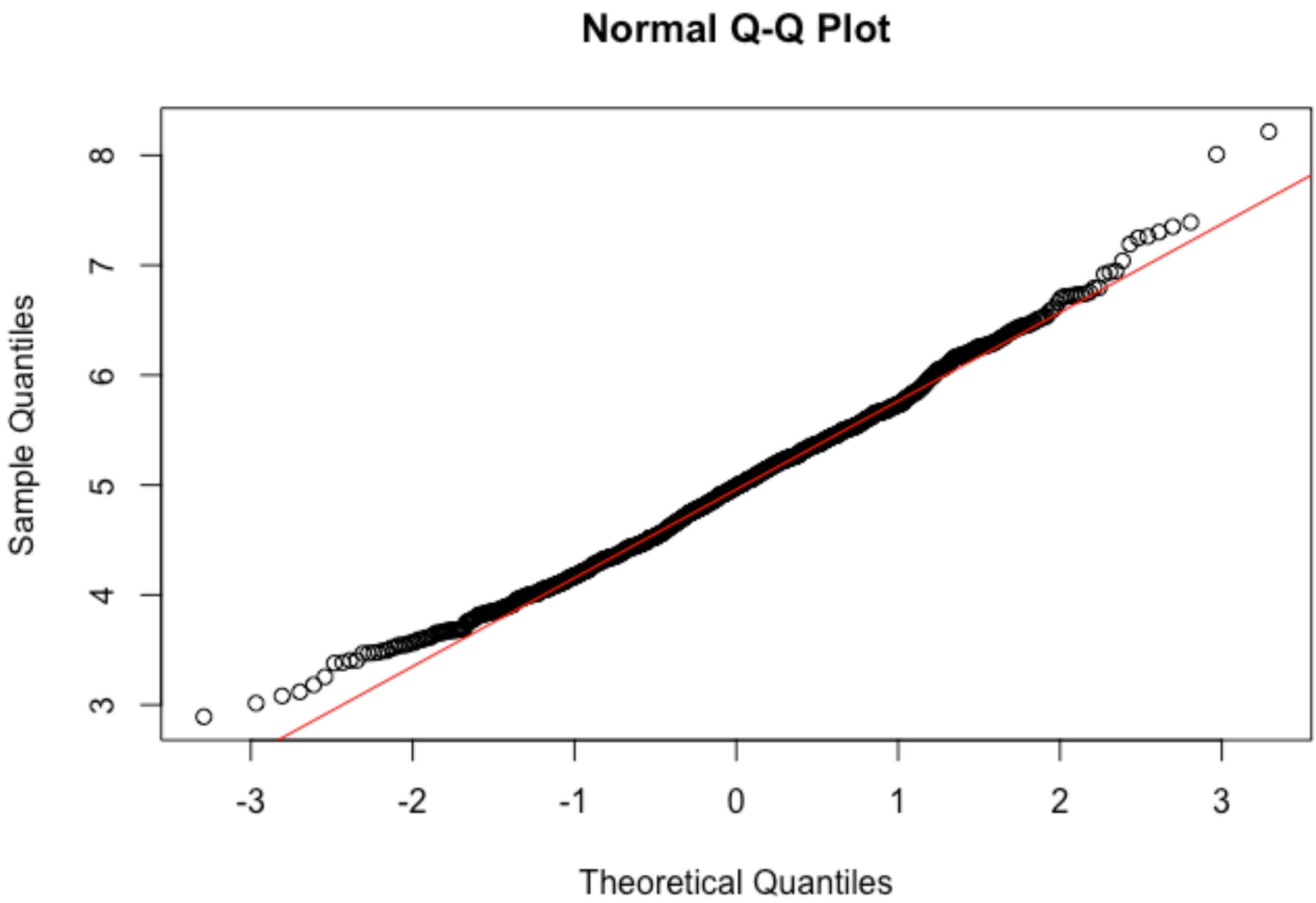
Question 3

Show that the distribution is approximately normal.

```
xfit <- seq(min(means_exponentials), max(means_exponentials), length = 100)
yfit <- dnorm(xfit, mean = 1/lambda, sd = 1/lambda/sqrt(n))
hist(means_exponentials, breaks = n, prob = T, col = "lightblue", main = "Density o
f Means", xlab = "Means", ylab = "Density")
lines(xfit, yfit, pch = 22, col = "black", lty = 5)
```




```
# Compare the distribution of averages of 40 exponentials to a normal distribution
qqnorm(means_exponentials)
qqline(means_exponentials, col = 2)
```



Because of the central limit theorem (CLT), the distribution of averages of 40 exponentials is very close to a normal distribution.