# Interest Rate Shocks

September 21, 2025

## 1 Impact of Interest Rate Shocks on Bank Stock VaR

### 1.1 Part 1: Data Gathering

#### 1.1.1 Dara Source: Yahoo Finance & pandas_datareader

[1]:
```
import yfinance as yfin
import pandas_datareader as pdr
tickers = ['JPM', 'BAC', 'C', 'WFC']
start_date = '1980-01-01'
end_date = '2025-09-12'
ticker = yfin.Tickers(tickers)
stock = ticker.history(interval = '1d', start = start_date, end = end_date,␣
 ↪auto_adjust=False, actions=False, period=None)
stock.head()
```

```
[*********************100%***********************]  4 of 4 completed
```

[1]:
```
Price       Adj Close                              Close                      \
Ticker            BAC         C JPM       WFC        BAC          C JPM
Date
1980-01-02   0.552944  5.996466 NaN  0.224495   1.687500  14.473125 NaN
1980-01-03   0.547824  6.125422 NaN  0.221210   1.671875  14.784375 NaN
1980-01-04   0.547824  6.060940 NaN  0.223400   1.671875  14.628750 NaN
1980-01-07   0.552944  6.125422 NaN  0.223400   1.687500  14.784375 NaN
1980-01-08   0.547824  6.211388 NaN  0.224495   1.671875  14.991875 NaN

Price                       High              … Low                 Open  \
Ticker            WFC        BAC          C  … JPM       WFC        BAC
Date                                         …
1980-01-02   1.067708   1.718750  14.888125  … NaN  1.062500   1.703125
1980-01-03   1.052083   1.687500  14.940000  … NaN  1.052083   1.687500
1980-01-04   1.062500   1.671875  14.836250  … NaN  1.052083   1.671875
1980-01-07   1.062500   1.703125  14.888125  … NaN  1.062500   1.671875
1980-01-08   1.067708   1.703125  15.043750  … NaN  1.052083   1.687500

Price                              Volume
Ticker                C JPM       WFC      BAC         C JPM       WFC
Date
```

```
1980-01-02  14.784375 NaN  1.062500   36000  102892 NaN   398400
1980-01-03  14.473125 NaN  1.062500   31200  100000 NaN   132000
1980-01-04  14.784375 NaN  1.052083   35200   32771 NaN    93600
1980-01-07  14.628750 NaN  1.062500  260000   19277 NaN  2124000
1980-01-08  14.836250 NaN  1.062500  160800   27711 NaN  4312800

[5 rows x 24 columns]
```

[2]:
```python
dgs10 = pdr.DataReader('DGS10', 'fred', start_date, end_date)
dgs10.head()
```

[2]:
```
            DGS10
DATE
1980-01-01    NaN
1980-01-02  10.50
1980-01-03  10.60
1980-01-04  10.66
1980-01-07  10.63
```

[3]:
```python
stock.to_csv('D:/Finance/risk/stock.csv')
dgs10.to_csv('D:/Finance/risk/dgs10.csv')
```

## 1.2 Part 2: Data Processing

### 1.2.1 The file imported comes from SQL after filtering and check for null values

[4]:
```python
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
data = pd.read_csv('D:/Finance/risk/int rate shock.csv',parse_dates = ['date'])
data.head()
data.info()
data.describe()
```

```
<class 'pandas.core.frame.DataFrame'>
RangeIndex: 2498 entries, 0 to 2497
Data columns (total 6 columns):
 #   Column         Non-Null Count  Dtype
---  ------         --------------  -----
 0   date           2498 non-null   datetime64[ns]
 1   BAC_adj_close  2498 non-null   float64
 2   C_adj_close    2498 non-null   float64
 3   JPM_adj_close  2498 non-null   float64
 4   WFC_adj_close  2498 non-null   float64
 5   DGS10_rate     2498 non-null   float64
dtypes: datetime64[ns](1), float64(5)
memory usage: 117.2 KB
```

[4]:
```
                               date  BAC_adj_close  C_adj_close  JPM_adj_close  \
count                          2498    2498.000000  2498.000000    2498.000000
mean   2019-12-31 02:40:49.959968      25.627776    48.431900     104.663244
min           2015-01-02 00:00:00       9.010223    26.317577      40.855118
25%           2017-06-30 18:00:00      19.606886    40.916053      72.642828
50%           2020-01-01 00:00:00      25.315926    48.013577      93.941360
75%           2022-06-29 18:00:00      32.005223    56.268813     134.033249
max           2024-12-31 00:00:00      46.671661    70.963501     246.293076
std                             NaN       8.898195     9.379918      44.877595

       WFC_adj_close   DGS10_rate
count    2498.000000  2498.000000
mean       41.487561     2.479788
min        18.853188     0.520000
25%        38.417059     1.730000
50%        41.487934     2.320000
75%        44.443168     3.107500
max        76.102943     4.980000
std         8.419010     1.052637
```

[5]:
```python
data['date'] = pd.to_datetime(data['date'], format='%d/%m/%Y')
data.head()
```

[5]:
```
        date  BAC_adj_close  C_adj_close  JPM_adj_close  WFC_adj_close  \
0 2015-01-02      14.277457    40.653790      46.948074      40.370869
1 2015-01-05      13.862693    39.372589      45.490562      39.263794
2 2015-01-06      13.447928    37.986496      44.311058      38.444576
3 2015-01-07      13.511739    38.338638      44.378670      38.673370
4 2015-01-08      13.790908    38.915539      45.370377      39.529495

   DGS10_rate
0        2.12
1        2.04
2        1.97
3        1.96
4        2.03
```

### 1.3 This is an example of python connecting sql data directly - warnings can be ignored

[6]:
```python
import pyodbc
import pandas as pd

conn_str = (
    "DRIVER={ODBC Driver 17 for SQL Server};"
    "SERVER=XX;"                # replace XX with the server name from SSMS
    "DATABASE=finance;"
    "Trusted_Connection=yes;"
    "Encrypt=yes;"
    "TrustServerCertificate=yes;"
)

conn = pyodbc.connect(conn_str)

df = pd.read_sql_query("""
    SELECT
        s.date,
        s.BAC_adj_close,
        s.C_adj_close,
        s.JPM_adj_close,
        s.WFC_adj_close,
        r.dgs10_rate AS DGS10_rate
    FROM stocks AS s
    INNER JOIN dgs10 AS r
        ON s.date = r.date
    WHERE s.date >= '2015-01-01'
      AND s.date <= '2024-12-31'
      AND NOT(
        s.BAC_adj_close IS NULL
        OR s.C_adj_close IS NULL
        OR s.JPM_adj_close IS NULL
        OR s.WFC_adj_close IS NULL
        OR r.dgs10_rate IS NULL
      )
    ORDER BY s.date;
""", conn)

conn.close()
df.head()
```

```
C:\Users\amusi\AppData\Local\Temp\ipykernel_41212\4177353140.py:15: UserWarning:
pandas only supports SQLAlchemy connectable (engine/connection) or database
string URI or sqlite3 DBAPI2 connection. Other DBAPI2 objects are not tested.
Please consider using SQLAlchemy.
  df = pd.read_sql_query("""
```

[6]:

```
         date  BAC_adj_close  C_adj_close  JPM_adj_close  WFC_adj_close  \
0  2015-01-02      14.277457    40.653790      46.948074      40.370869
1  2015-01-05      13.862693    39.372589      45.490562      39.263794
2  2015-01-06      13.447928    37.986496      44.311058      38.444576
3  2015-01-07      13.511739    38.338638      44.378670      38.673370
4  2015-01-08      13.790908    38.915539      45.370377      39.529495

   DGS10_rate
```

```
0        2.12
1        2.04
2        1.97
3        1.96
4        2.03
```

[7]:
```
from scipy.stats import norm
df = df.set_index('date')
```

### 1.3.1 Returns & Rate changes

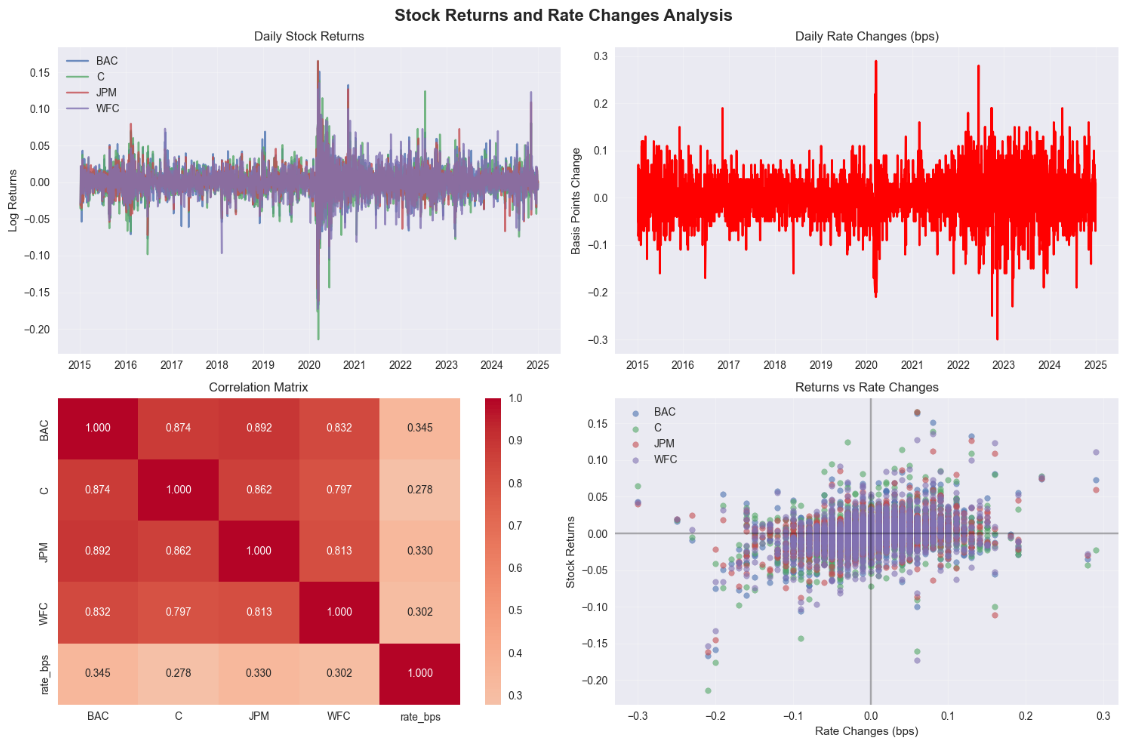
[9]:
```
stock_list = ['BAC_adj_close','C_adj_close','JPM_adj_close','WFC_adj_close']
log_ret = np.log(df[stock_list]).diff().dropna()
log_ret = log_ret.rename(columns={
    'BAC_adj_close': 'BAC',
    'C_adj_close': 'C',
    'JPM_adj_close': 'JPM',
    'WFC_adj_close': 'WFC'
})
rate_bps = df['DGS10_rate'].diff().dropna()
df_change = pd.concat([log_ret, rate_bps.rename('rate_bps')], axis=1)
print(df_change.head())
```

```
                 BAC         C       JPM       WFC  rate_bps
date
2015-01-05 -0.029481 -0.032022 -0.031537 -0.027806     -0.08
2015-01-06 -0.030376 -0.035839 -0.026271 -0.021085     -0.07
2015-01-07  0.004734  0.009227  0.001525  0.005934     -0.01
2015-01-08  0.020451  0.014935  0.022100  0.021896      0.07
2015-01-09 -0.018092 -0.022586 -0.017540 -0.016567     -0.05
```

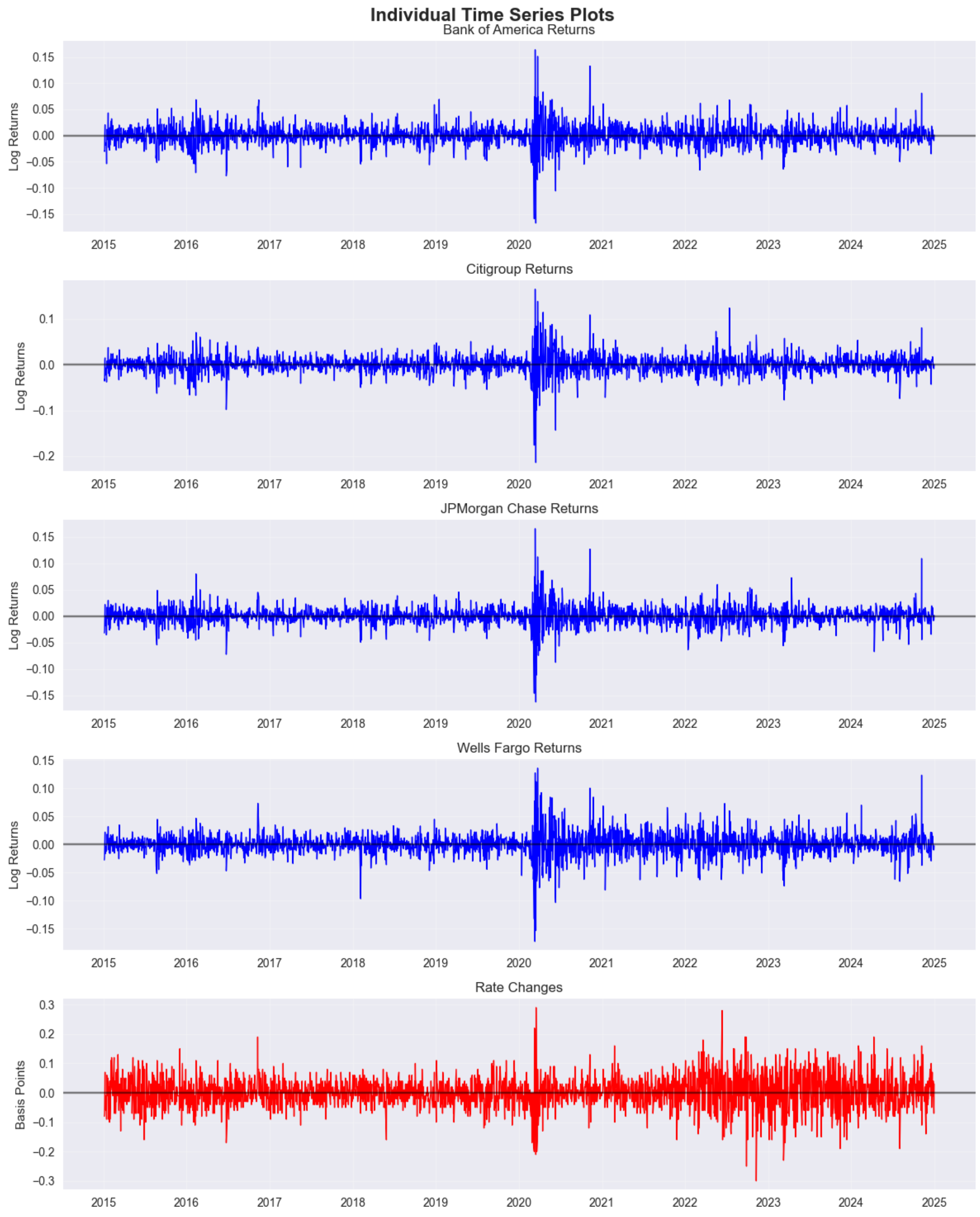
[10]:
```
import matplotlib.pyplot as plt
import seaborn as sns

# Set up the plotting style
plt.style.use('seaborn-v0_8')
fig, axes = plt.subplots(2, 2, figsize=(15, 10))
fig.suptitle('Stock Returns and Rate Changes Analysis', fontsize=16,␣
 ↪fontweight='bold')

# Plot 1: Individual stock returns over time
for stock in ['BAC', 'C', 'JPM', 'WFC']:
    axes[0, 0].plot(df_change.index, df_change[stock], label=stock, alpha=0.8)

axes[0, 0].set_title('Daily Stock Returns')
axes[0, 0].set_ylabel('Log Returns')
axes[0, 0].legend()
```

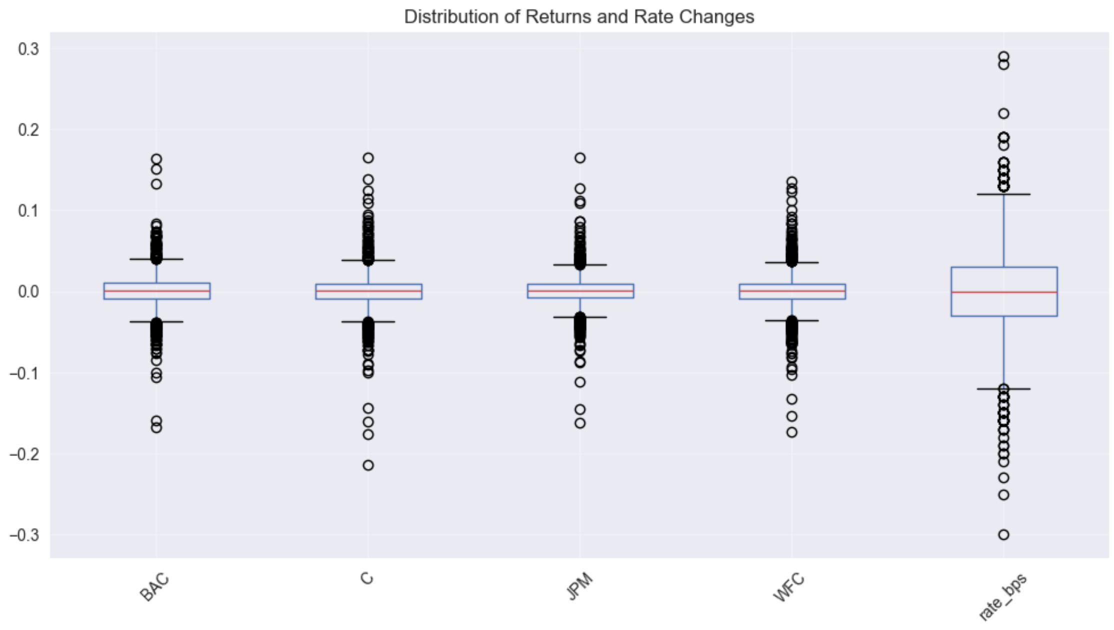
```
axes[0, 0].grid(True, alpha=0.3)

# Plot 2: Rate changes over time
axes[0, 1].plot(df_change.index, df_change['rate_bps'], color='red',␣
 ↪linewidth=2)
axes[0, 1].set_title('Daily Rate Changes (bps)')
axes[0, 1].set_ylabel('Basis Points Change')
axes[0, 1].grid(True, alpha=0.3)

# Plot 3: Correlation heatmap
correlation_matrix = df_change.corr()
sns.heatmap(correlation_matrix, annot=True, cmap='coolwarm', center=0,
            ax=axes[1, 0], fmt='.3f')
axes[1, 0].set_title('Correlation Matrix')

# Plot 4: Scatter plot of returns vs rate changes
for stock in ['BAC', 'C', 'JPM', 'WFC']:
    axes[1, 1].scatter(df_change['rate_bps'], df_change[stock],
                      alpha=0.6, label=stock, s=30)

axes[1, 1].set_title('Returns vs Rate Changes')
axes[1, 1].set_xlabel('Rate Changes (bps)')
axes[1, 1].set_ylabel('Stock Returns')
axes[1, 1].axhline(y=0, color='black', linestyle='-', alpha=0.3)
axes[1, 1].axvline(x=0, color='black', linestyle='-', alpha=0.3)
axes[1, 1].legend()
axes[1, 1].grid(True, alpha=0.3)

plt.tight_layout()
plt.show()

# Additional individual plots for better visualization
fig2, axes2 = plt.subplots(5, 1, figsize=(12, 15))
fig2.suptitle('Individual Time Series Plots', fontsize=16, fontweight='bold')

stocks = ['BAC', 'C', 'JPM', 'WFC', 'rate_bps']
titles = ['Bank of America Returns', 'Citigroup Returns',
          'JPMorgan Chase Returns', 'Wells Fargo Returns', 'Rate Changes']

for i, (stock, title) in enumerate(zip(stocks, titles)):
    color = 'blue' if stock != 'rate_bps' else 'red'
    axes2[i].plot(df_change.index, df_change[stock], color=color, linewidth=1)
    axes2[i].set_title(title)
    axes2[i].set_ylabel('Log Returns' if stock != 'rate_bps' else 'Basis␣
 ↪Points')
    axes2[i].grid(True, alpha=0.3)
    axes2[i].axhline(y=0, color='black', linestyle='-', alpha=0.5)
```

```
plt.tight_layout()
plt.show()

# Box plot to show distribution of returns and rate changes
plt.figure(figsize=(12, 6))
df_change[['BAC', 'C', 'JPM', 'WFC', 'rate_bps']].boxplot()
plt.title('Distribution of Returns and Rate Changes')
plt.xticks(rotation=45)
plt.grid(True, alpha=0.3)
plt.show()
```



| | BAC | C | JPM | WFC | rate_bps |
|---|---|---|---|---|---|
| BAC | 1.000 | 0.874 | 0.892 | 0.832 | 0.345 |
| C | 0.874 | 1.000 | 0.862 | 0.797 | 0.278 |
| JPM | 0.892 | 0.862 | 1.000 | 0.813 | 0.330 |
| WFC | 0.832 | 0.797 | 0.813 | 1.000 | 0.302 |
| rate_bps | 0.345 | 0.278 | 0.330 | 0.302 | 1.000 |

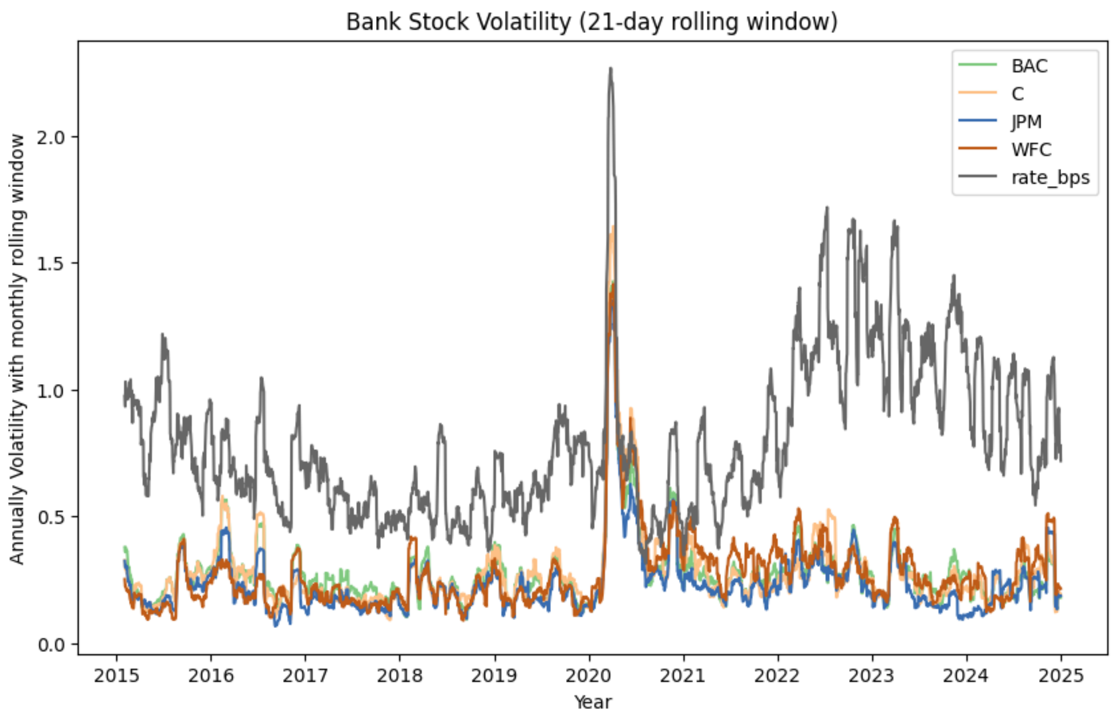

Individual Time Series Plots
Bank of America Returns
Citigroup Returns
JPMorgan Chase Returns
Wells Fargo Returns
Rate Changes
Log Returns
Basis Points
2015
2016
2017
2018
2019
2020
2021
2022
2023
2024
2025



### 1.3.2 Volatility analysis

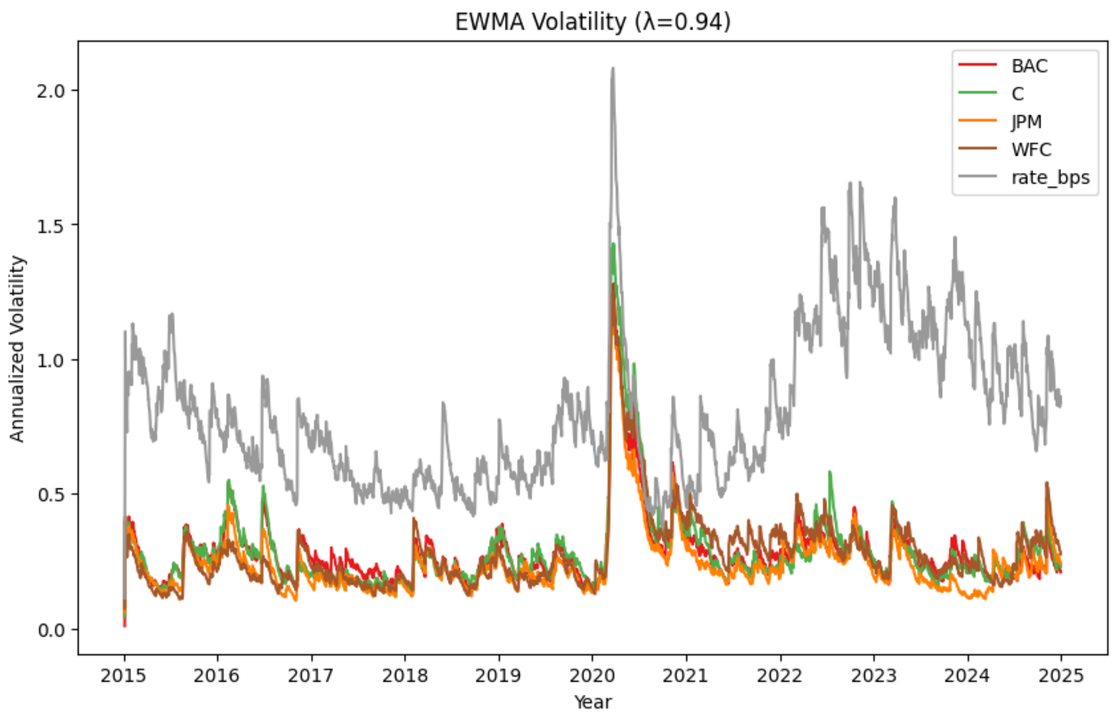
[10]:
```
# SMA volatility
window = 21 # 1 month window
print(df_change.columns[0:5])
vol_daily = df_change.rolling(window).std()
vol_annual = vol_daily * np.sqrt(252)
vol_annual.plot(figsize=(10,6),label=df_change.columns[0:5],colormap='Accent')
plt.xlabel('Year')
plt.ylabel('Annually Volatility with monthly rolling window')
plt.title('Bank Stock Volatility (21-day rolling window)')
plt.legend()
plt.show()
```

```
Index(['BAC', 'C', 'JPM', 'WFC', 'rate_bps'], dtype='object')
```

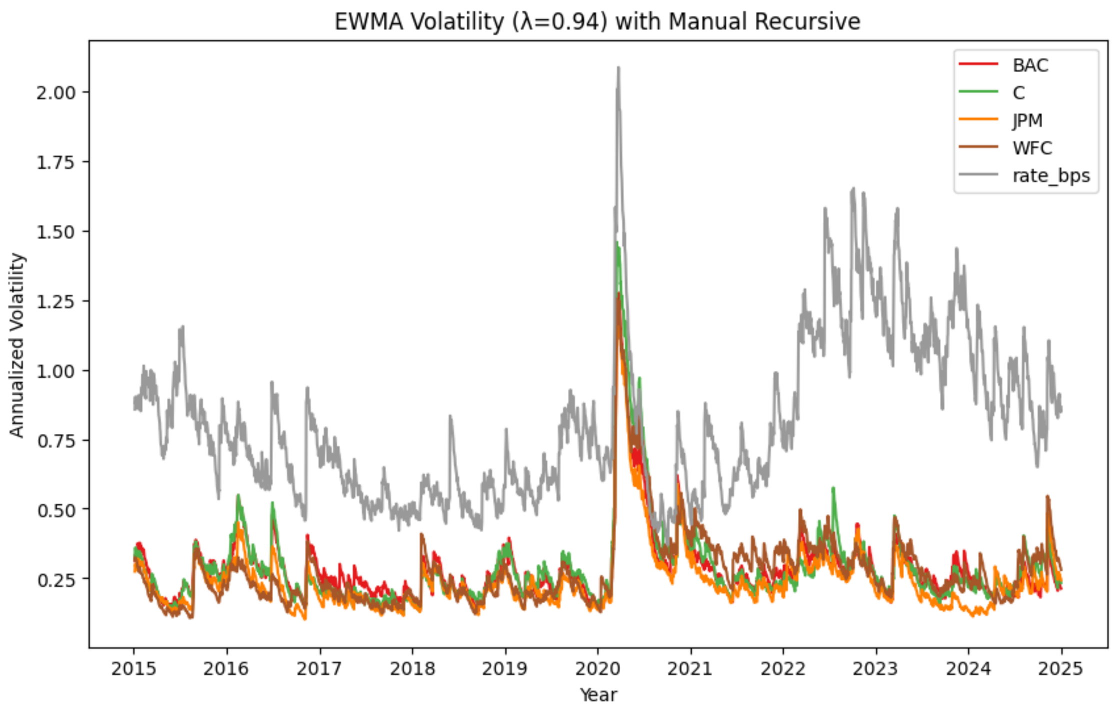


[11]:
```
# EWMA volatility
lambda_ = 0.94
ewma_vol = df_change.ewm(
    span=(2/(1-lambda_)-1),
    adjust=False
).std() * np.sqrt(252)   # annualized

print(ewma_vol.tail())

ewma_vol.plot(figsize=(10,6),label=log_ret.columns,colormap='Set1')
plt.xlabel('Year')
plt.ylabel('Annualized Volatility')
plt.title('EWMA Volatility ( =0.94)')
plt.legend()
plt.show()
```

```
                 BAC         C       JPM       WFC  rate_bps
date
2024-12-24  0.232101  0.254413  0.265593  0.306643  0.862469
2024-12-26  0.225680  0.246993  0.257659  0.297356  0.841949
2024-12-27  0.219438  0.240838  0.252359  0.290869  0.822295
2024-12-30  0.215592  0.236625  0.246751  0.284736  0.862118
2024-12-31  0.209178  0.229436  0.239302  0.276188  0.839173
```



[12]:
```
# manual recursive EWMA
tickers = df_change.columns
lambda_ = 0.94
ewma_vol_manual = pd.DataFrame(index=df_change.index,columns=df_change.columns)
for ticker in tickers:
    r = df_change[ticker]
    vol = np.zeros(len(r))
    vol[0] = np.std(r)
    for i in range(1,len(df_change)):
        vol[i] =  np.sqrt(lambda_ * vol[i-1]**2 + (1 - lambda_) * r.
↪iloc[i-1]**2)
        ewma_vol_manual.loc[df_change.index,ticker] = vol
ewma_vol_annual = ewma_vol_manual * np.sqrt(252)

ewma_vol_annual.plot(figsize=(10,6),label=log_ret.columns,colormap='Set1')
plt.xlabel('Year')
plt.ylabel('Annualized Volatility')
plt.title('EWMA Volatility ( =0.94) with Manual Recursive')
plt.legend()
plt.show()
```

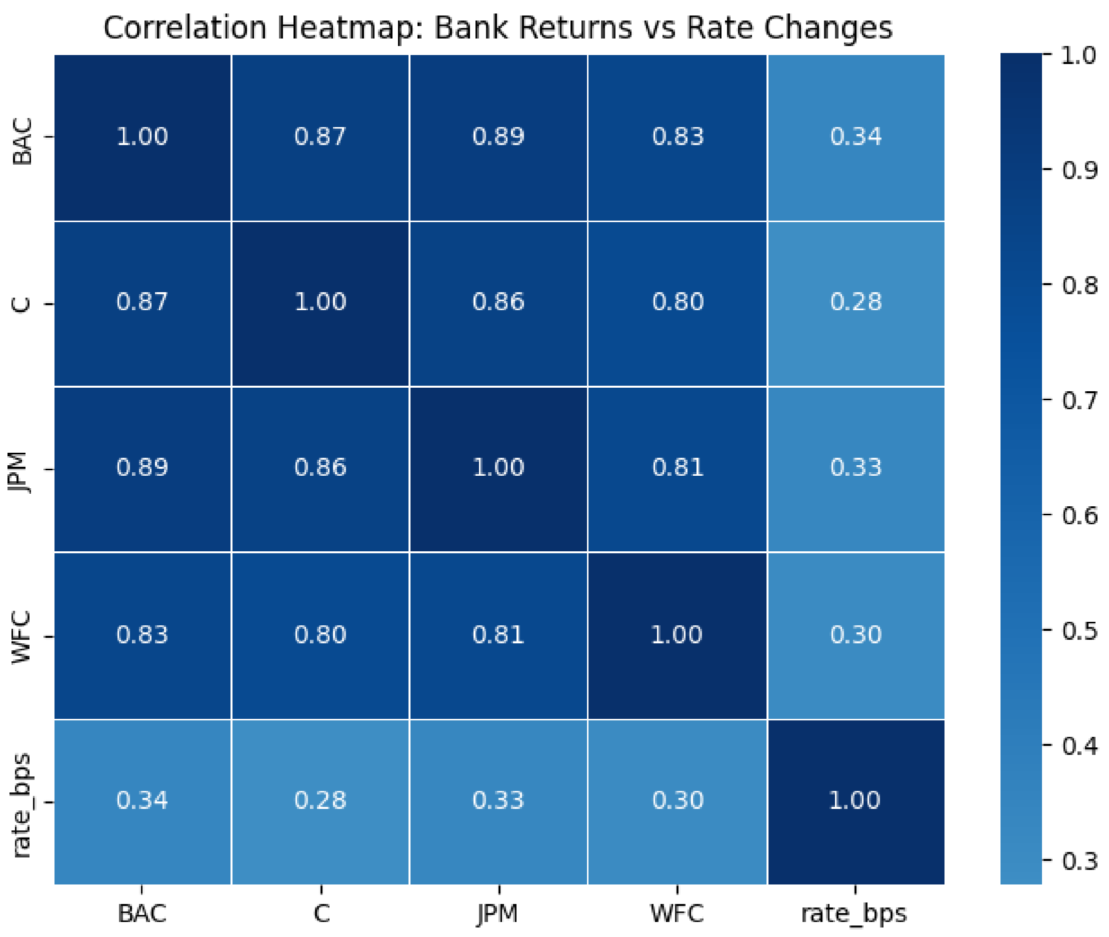


### 1.3.3 Correlation analysis

[13]:
```python
# correlation analysis
import seaborn as sns
corr = df_change.corr()
print(corr)
plt.figure(figsize=(8,6))
sns.heatmap(
    corr,
    annot=True,         # show values
    fmt=".2f",
    cmap="Blues",
    center=0,
    linewidths=0.5
)

plt.title("Correlation Heatmap: Bank Returns vs Rate Changes")
plt.show()
```

```
               BAC         C       JPM       WFC  rate_bps
BAC       1.000000  0.873763  0.892117  0.831937  0.344644
C         0.873763  1.000000  0.862386  0.797335  0.278270
JPM       0.892117  0.862386  1.000000  0.812747  0.329798
WFC       0.831937  0.797335  0.812747  1.000000  0.301539
```

```
rate_bps   0.344644   0.278270   0.329798   0.301539   1.000000
```

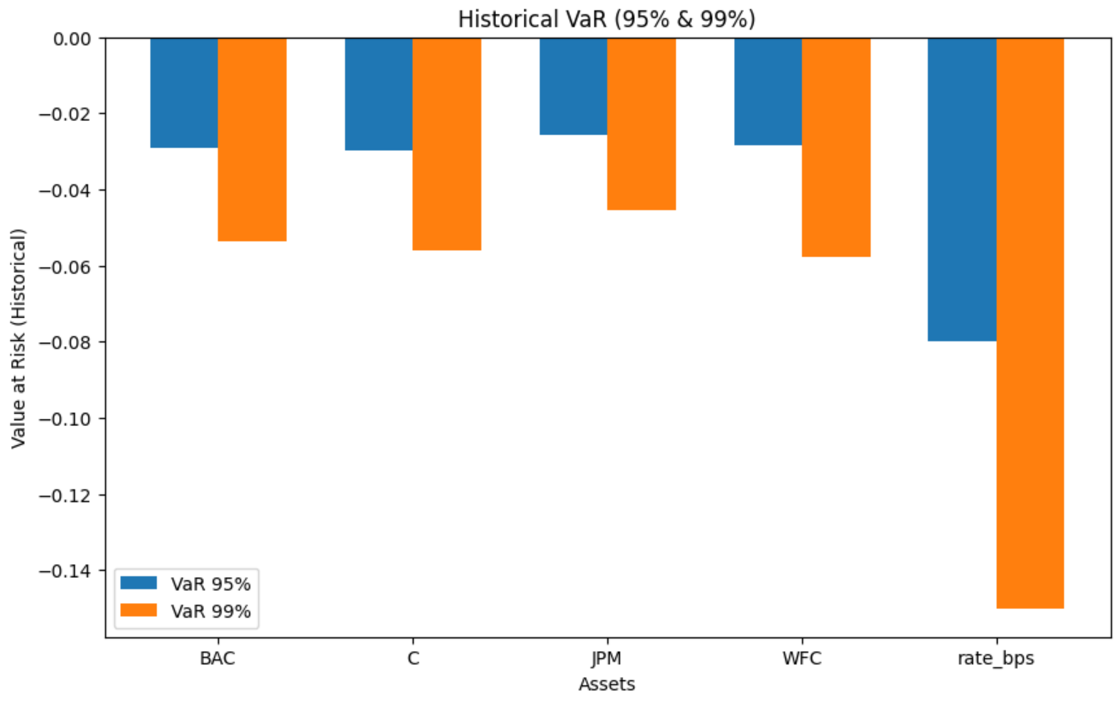


### 1.3.4 VaR analysis

[14]:
```
# historical VaR
var_his = pd.DataFrame(index=tickers,columns=['var95','var99'])
for ticker in tickers:
    var_his.loc[ticker,'var95'] = np.percentile(df_change[ticker],5)
    var_his.loc[ticker,'var99']= np.percentile(df_change[ticker],1)
print(var_his)

plt.figure(figsize=(10,6))
x = np.arange(0,5)  # or it can be written as x = np.arange(len(var_his.index))
bar_width = 0.35

plt.bar(x - bar_width/2, var_his['var95'], width=bar_width, label='VaR 95%')
plt.bar(x + bar_width/2, var_his['var99'], width=bar_width, label='VaR 99%')
```

```python
plt.xticks(x, var_his.index)
plt.xlabel("Assets")
plt.ylabel("Value at Risk (Historical)")
plt.title("Historical VaR (95% & 99%)")
plt.legend()
plt.show()

#portfolio var
weights = np.array([0.25,0.25,0.25,0.25])
port_ret = df_change.iloc[:,0:4].dot(weights)
port_var95 = np.percentile(port_ret, 5)
port_var99 = np.percentile(port_ret, 1)

print("Portfolio Historical VaR 95%:{:.4f}".format(port_var95))
print("Portfolio Historical VaR 99%:{:.4f}".format(port_var99))
```

```
             var95     var99
BAC       -0.029137 -0.053712
C         -0.029749 -0.055975
JPM       -0.025803 -0.045499
WFC        -0.02848 -0.057614
rate_bps      -0.08     -0.15
```

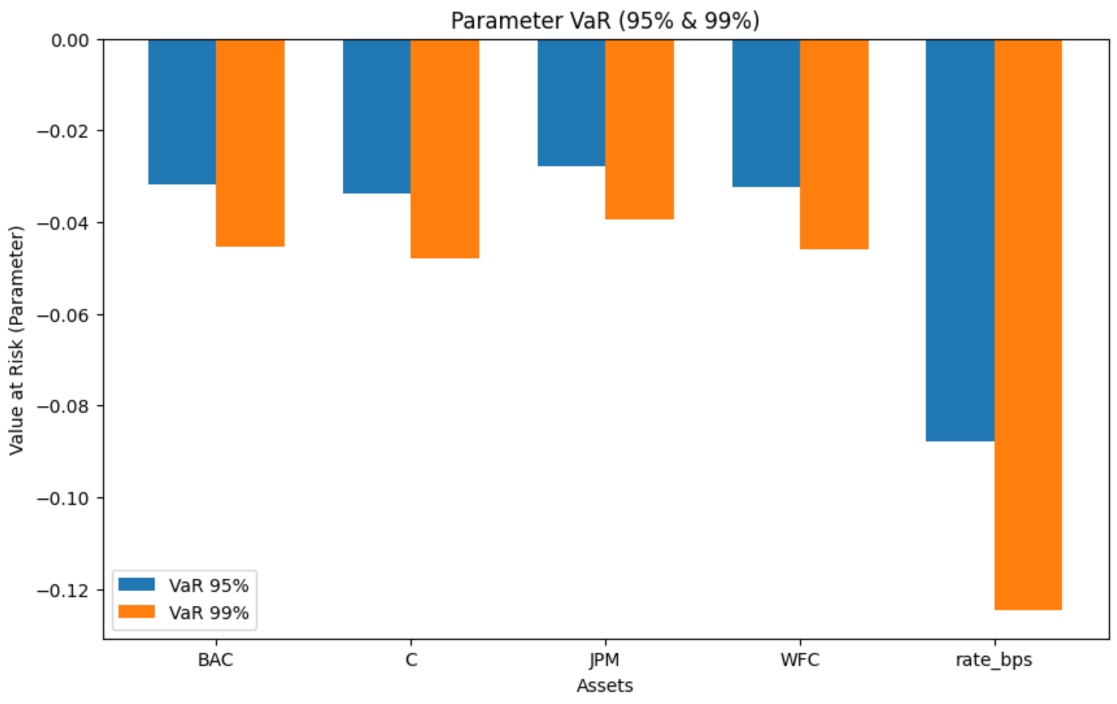


```
Portfolio Historical VaR 95%:-0.0272
Portfolio Historical VaR 99%:-0.0472
```

[15]:
```
# parameter VaR
import scipy.stats as st
para = pd.DataFrame(index=tickers,columns=['mu','sigma'])
var_para = pd.DataFrame(index=tickers,columns=['var95','var99'])
for ticker in tickers:
    para.loc[ticker,'mu'] = df_change[ticker].mean()
    para.loc[ticker,'sigma'] = df_change[ticker].std()
print(para)
for ticker in tickers:
    var_para.loc[ticker,'var95'] = st.norm.ppf(0.05,loc = para.
 ↪loc[ticker,'mu'], scale = para.loc[ticker,'sigma'])
    var_para.loc[ticker,'var99'] = st.norm.ppf(0.01,loc = para.
 ↪loc[ticker,'mu'], scale = para.loc[ticker,'sigma'])
print(var_para)

plt.figure(figsize=(10,6))
x = np.arange(0,5)   # or it can be written as x = np.arange(len(var_his.index))
bar_width = 0.35

plt.bar(x - bar_width/2, var_para['var95'], width=bar_width, label='VaR 95%')
plt.bar(x + bar_width/2, var_para['var99'], width=bar_width, label='VaR 99%')

plt.xticks(x, var_his.index)
plt.xlabel("Assets")
plt.ylabel("Value at Risk (Parameter)")
plt.title("Parameter VaR (95% & 99%)")
plt.legend()
plt.show()

#portfolio var
port_mu = port_ret.mean()
port_sigma = port_ret.std()
port_var95_para = st.norm.ppf(0.05,loc = port_mu, scale = port_sigma)
port_var99_para = st.norm.ppf(0.01,loc = port_mu, scale = port_sigma)

print("Portfolio Parameter VaR 95%:{:.4f}".format(port_var95_para))
print("Portfolio Parameter VaR 99%:{:.4f}".format(port_var99_para))
```

```
                mu     sigma
BAC       0.000443  0.019663
C         0.000211  0.020734
JPM       0.000646   0.01728
WFC       0.000215  0.019873
rate_bps  0.000985  0.053956
             var95     var99
BAC        -0.0319   -0.0453
C        -0.033893 -0.048023
JPM      -0.027777 -0.039553
```

```
WFC      -0.032473 -0.046017
rate_bps -0.087764 -0.124534
```



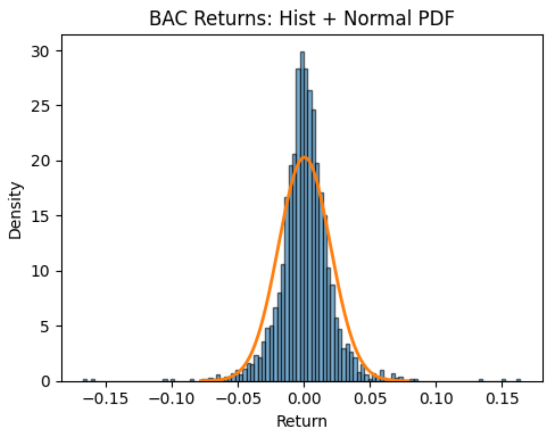
```
Portfolio Parameter VaR 95%:-0.0296
Portfolio Parameter VaR 99%:-0.0420
```

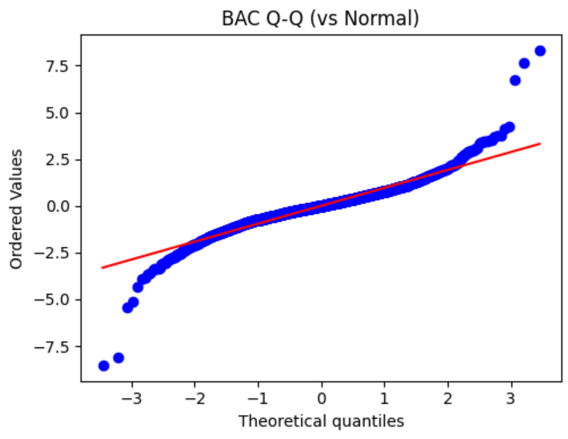
## 2 Part 3 Topic Analysis – Hypothesis 1: Distributional assumption

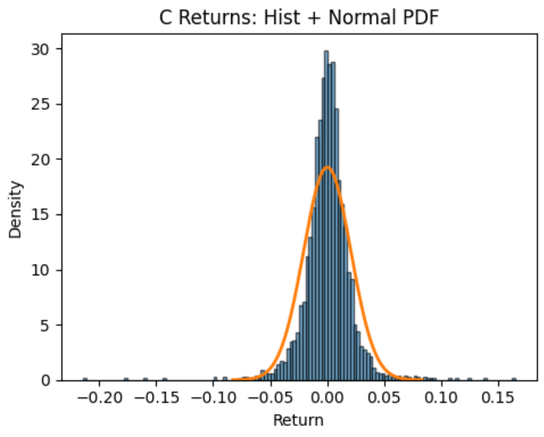
### 2.1 Are returns normally distributed?

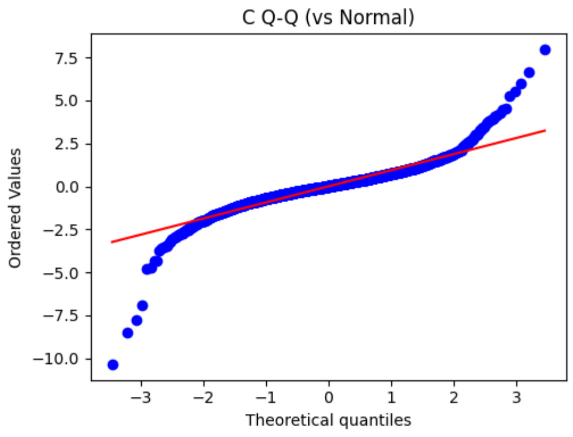
[16]:
```
from scipy import stats
stock_tickers = ['BAC','C','JPM','WFC']
results = []
for ticker in stock_tickers:
    mu = np.mean(log_ret[ticker])
    sigma = np.std(log_ret[ticker],ddof=1) #ddof=1, sample unbiased std; ddof=0␣
 ↪population std
    skew = stats.skew(log_ret[ticker],bias = False) #bias=false, unbiased;␣
 ↪bias=true, biased
    kurtosis = stats.kurtosis(log_ret[ticker],fisher=True,bias=False)␣
 ↪#fisher=True, excess kurtosis; fisher=false, kurtosis
    jb_stat,jb_p = stats.jarque_bera(log_ret[ticker])
    results.append({
        'Ticker':ticker,
        'Mean':mu,
        'Std':sigma,
        'Skewness':skew,
        'Excess Kurtosis':kurtosis,
        'JB statistics': jb_stat,
        'JB p-value': jb_p
    })
summary = pd.DataFrame(results).set_index(log_ret.columns)
print(summary.round(4))

def plot_ret(series,title=None):
    x = series.dropna().values
    mu = np.mean(x)
    sigma = np.std(x,ddof=1)
    fig,axes = plt.subplots(1,2,figsize=(10,4))
    ax = axes[0]
    ax.hist(x,bins='auto',density=True,alpha=0.6,edgecolor='black')
    grid = np.linspace(mu - 4*sigma,mu + 4*sigma,500)
    ax.plot(grid,stats.norm.pdf(grid,loc=mu,scale=sigma),linewidth=2)
    ax.set_title(f'{title} Returns: Hist + Normal PDF')
    ax.set_xlabel('Return')
    ax.set_ylabel('Density')

    z = (x-mu)/sigma
    ax = axes[1]
    stats.probplot(z,dist='norm',plot=ax)
    ax.set_title(f'{title} Q-Q (vs Normal)')

    plt.tight_layout()
    plt.show()

for ticker in stock_tickers:
    plot_ret(log_ret[ticker],title=ticker)
```

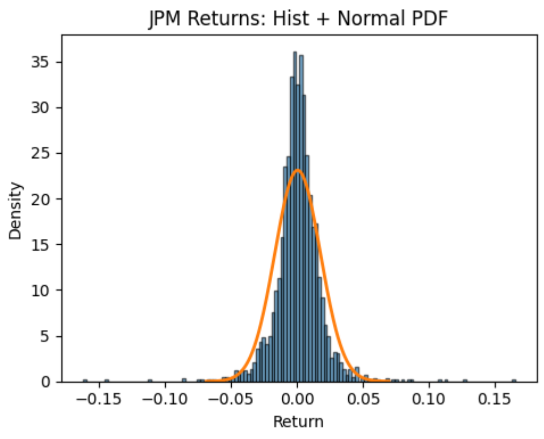
```
    Ticker    Mean     Std  Skewness  Excess Kurtosis  JB statistics  \
BAC    BAC  0.0004  0.0197   -0.0175           9.5353      9417.3108
C        C  0.0002  0.0207   -0.4642          14.0810     20628.7175
JPM    JPM  0.0006  0.0173   -0.0254          13.3782     18540.2616
WFC    WFC  0.0002  0.0199   -0.2198           9.1837      8755.4037

     JB p-value
BAC         0.0
C           0.0
JPM         0.0
WFC         0.0
```

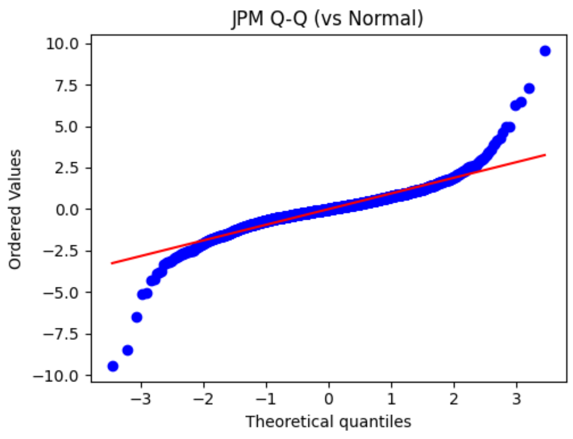

BAC Returns: Hist + Normal PDF
Density
Return

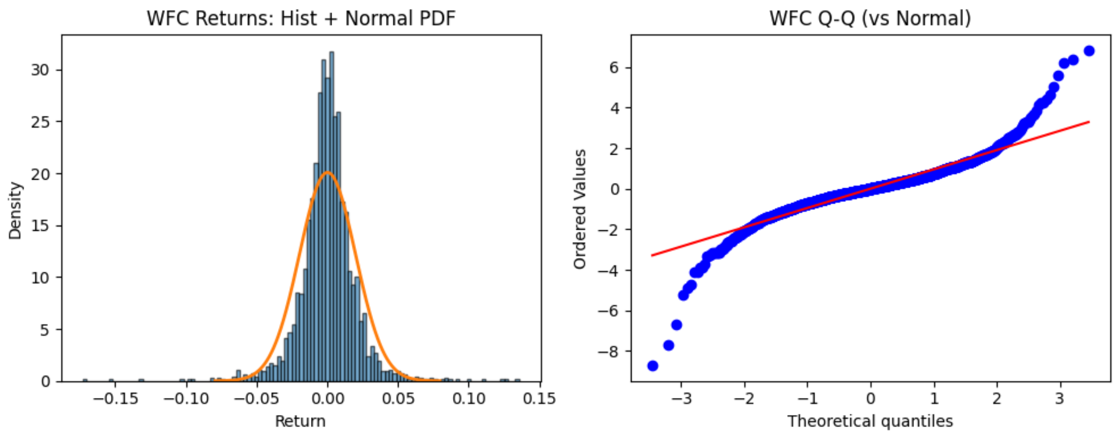

BAC Q-Q (vs Normal)
Ordered Values
Theoretical quantiles


C Returns: Hist + Normal PDF
Density
Return


C Q-Q (vs Normal)
Ordered Values
Theoretical quantiles


JPM Returns: Hist + Normal PDF
Density
Return


JPM Q-Q (vs Normal)
Ordered Values
Theoretical quantiles



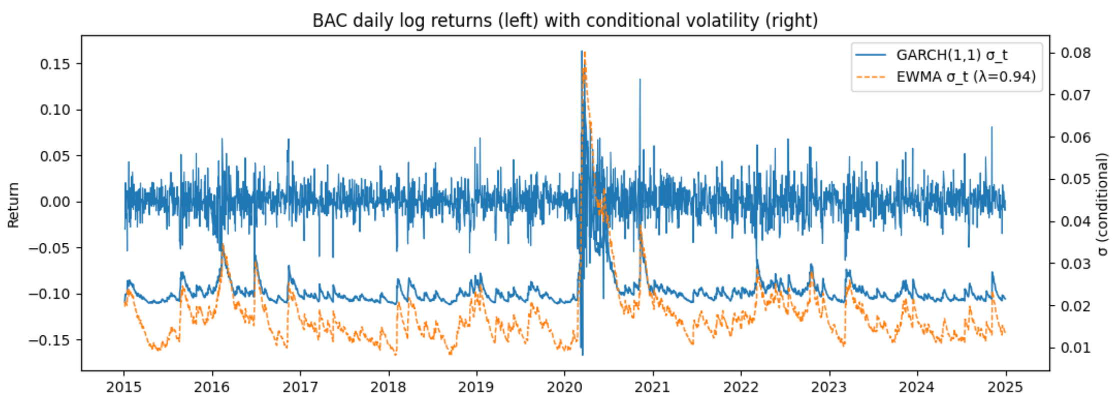
## 3 Part 4 Topic Analysis – Hypothesis 2: Volatility clustering

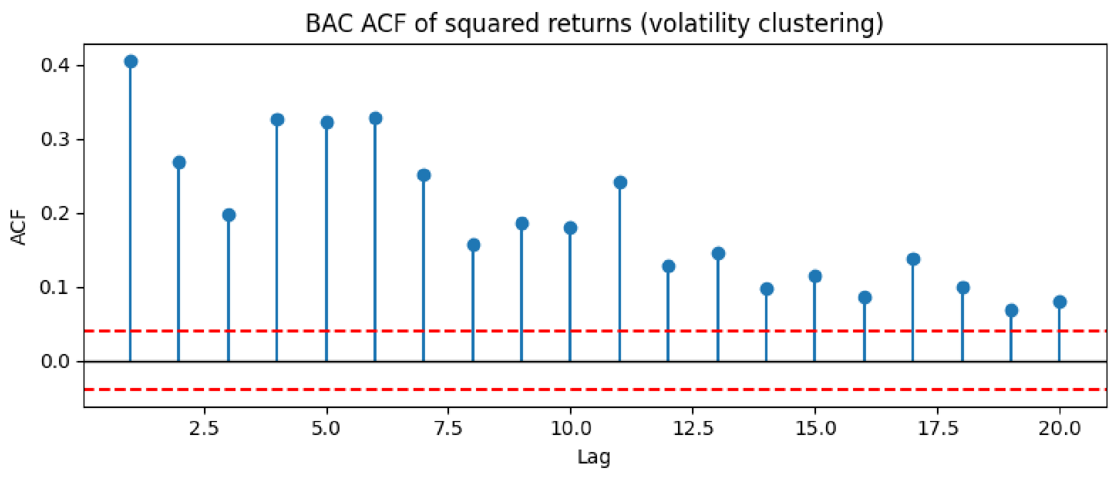
### 3.1 Do returns exhibit autocorrelation in volatility?

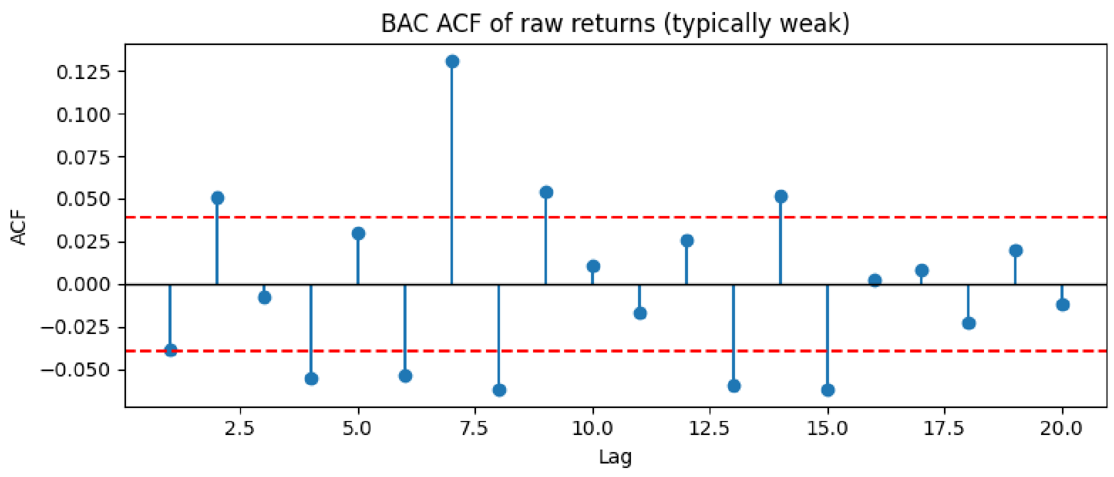
[17]:
```
from scipy.optimize import minimize

# ACF ACF(k) = Σ(x x  ) / Σ(x ²) for t = k+1 to n
def acf(x, nlags=20):  #nlags=20: number of lags to compute (defaults to 20)
    x = np.asarray(x)   # converts the input x to a NumPy array
    x = x[~np.isnan(x)] # removes any nan
    n = x.size
    x = x - x.mean()  # mean-centers the data, crucial for autocorrelation␣
↪calculation, removes any constant bias
    denom = np.dot(x, x) # dot product of x with itself, = sum of squares
    ac = [1.0]  # autocorrelation at lag 0 is always 1.0 (perfect correlation␣
↪with itself)
    for k in range(1, nlags+1):  # loops through each lag from 1 to n
        num = np.dot(x[:-k], x[k:])
        # numerator for lag k
        # x[:-k] selects all elements except the last k elements
        # x[k:] selects all elements starting from position k
        # computes the covariance between the time series and its lagged version
        ac.append(num / denom)
    return np.array(ac)

# Ljung-Box Q test
def ljung_box_q(x, lags=20):
    x = np.asarray(x)
    x = x[~np.isnan(x)]
    n = x.size
    rho = acf(x, nlags=lags)[1:]
    # use acf function to compute autocorrelations
    # slices the array to exclude lag 0
    # rho contains autocorrelations for lags 1 through lags
    k = np.arange(1, lags+1)
    Q = n * (n + 2) * np.sum((rho**2) / (n - k))
    # Ljung-Box Q statistic: Q = n(n + 2) × Σ( ² / (n - k)) for k = 1 to m
    pval = 1 - stats.chi2.cdf(Q, df=lags)  #p-value for the test
    return Q, pval


def arch_lm_test(eps, lags=10):
    """
    arch_lm_test that takes two parameters:
    eps: the residual series from a model
    lags=10: number of lags to include in the test (defaults 10)
    Interpretation of the ARCH-LM test:
    Null hypothesis: No ARCH effects (no conditional heteroscedasticity)
    Alternative hypothesis: ARCH effects exist (volatility clustering)
    If p-value < significance level (e.g., 0.05): Reject null - ARCH effects␣
 ↪detected
    If p-value  significance level: Fail to reject null - no significant ARCH␣
 ↪effects
    """
    y = eps**2  # residual squre
    y = y[~np.isnan(y)]
    T = y.size
    X = np.ones((T - lags, 1))  # T - lags: lose lags when creating lagged␣
 ↪variables
    for j in range(1, lags+1):
        X = np.column_stack([X, y[lags-j:T-j]])
        # y[lags-j:T-j] creates the j-th lag of the squared residuals
        # np.column_stack adds each lag as a new column to the design matrix
        # after this loop, X contains: [constant, lag1, lag2, ..., lag_lags]
        """
        Time:   0  1  2  3  4  5  6  7  8  9  (indices)
        y:      1  2  3  4  5  6  7  8  9  10
        y_dep:  4  5  6  7  8  9  10   (starts at index 3)
        Lag 1:  3  4  5  6  7  8  9    (y[t-1]) y_dep
        Lag 2:  2  3  4  5  6  7  8    (y[t-2]) y_dep
        Lag 3:  1  2  3  4  5  6  7    (y[t-3]) y_dep
        y_dep[0] = 4 (y ²)
        Lag 1[0] = 3 (y ²) ← y  ²
        Lag 2[0] = 2 (y ²) ← y  ²
        Lag 3[0] = 1 (y ²) ← y  ²
        y ² =   +  ·y  ² +  ·y  ² +  ·y  ² +
        """
    y_dep = y[lags:] # squared residuals starting from position lags
    beta = np.linalg.lstsq(X, y_dep, rcond=None)[0]  #linear algebra;␣
↪rcond=None uses the default threshold for determining the matrix rank
    y_hat = X @ beta  # predicted values
    ssr = np.sum((y_dep - y_hat)**2)  # Sum of Squared Residuals (SSR)
    sst = np.sum((y_dep - y_dep.mean())**2)  # Total Sum of Squares (SST)
    R2 = 1 - ssr/sst
    LM = (T - lags) * R2  # LM test statistic: LM = n × R²
    pval = 1 - stats.chi2.cdf(LM, df=lags)
    return LM, pval


def garch11_fit(r):
    """
    return series r
    GARCH(1,1) volatility:  ² =   + ·  ² + ·  ²
    """
    r = np.asarray(r)
    r = r[~np.isnan(r)]
    n = r.size
    def negloglike(params):  # negative log-likelihood function needed␣
↪minimized (no minimize, only maximize)
        mu, omega, alpha, beta = params
        if omega <= 1e-12 or alpha < 0 or beta < 0 or (alpha + beta) >= 0.9999:
            return 1e12  # penalize invalid region
        # returns a large penalty value if constraints are violated
        eps = r - mu  #residual
        var = np.empty(n)
        # initialize variance with sample variance
        var0 = np.var(eps, ddof=1)
        var[0] = max(omega + alpha * eps[0]**2 + beta * var0, 1e-12)
        # max ensures variance > = 0
        for t in range(1, n):
            var[t] = omega + alpha * eps[t-1]**2 + beta * var[t-1]
            #  ² =   + ·  ² + ·  ²
            if var[t] <= 0:
                return 1e12
        # Gaussian log-likelihood
        ll = -0.5 * (np.log(2*np.pi) + np.log(var) + (eps**2)/var)
        # logL = -½[log(2 ) + log( ²) +  ²/ ²]
        return -np.sum(ll)
    # initial guesses
    mu0 = np.mean(r)
    var0 = np.var(r, ddof=1)
    x0 = np.array([mu0, 0.1*var0, 0.05, 0.9])
    """
    0.1*var0:  is typically much smaller than the unconditional variance
    0.05: Common initial value for ARCH coefficient ( )
    0.9: Common initial value for GARCH coefficient ( ) - volatility is␣
↪persistent
    """
    # bounds and constraint (alpha+beta < 1)
    bnds = [(-1, 1), (1e-12, None), (0, 1), (0, 1)]
    """
    mu: between -1 and 1 (reasonable for daily returns)
    omega: positive ( 1e-12 for numerical stability)
    alpha: between 0 and 1 (non-negative, typically < 0.3)
    beta: between 0 and 1 (non-negative, typically 0.7-0.95)
    """
    cons = ({'type': 'ineq', 'fun': lambda p: 0.9999 - (p[2] + p[3])})
    # ineq : the function should return   0

    res = minimize(negloglike, x0, method='SLSQP', bounds=bnds,␣
↪constraints=cons, options={'maxiter': 2000})
    # SLSQP: Sequential Least Squares Programming - good for constrained␣
↪optimization
    # maxiter=2000: up to 2000 iterations for convergence
    mu, omega, alpha, beta = res.x
    # rebuild conditional variance with fitted params
    eps = r - mu
    var = np.empty(n)
    var_init = np.var(eps, ddof=1)
    var[0] = max(omega + alpha * eps[0]**2 + beta * var_init, 1e-12)
    for t in range(1, n):
        var[t] = omega + alpha * eps[t-1]**2 + beta * var[t-1]

    out = {
        'mu': mu, 'omega': omega, 'alpha': alpha, 'beta': beta,
        'alpha+beta': alpha + beta,
        'sigma': np.sqrt(var),
        'success': res.success, 'message': res.message
    }
    return out

def ewma_sigma(eps, lam=0.94):
    eps = np.asarray(eps)
    n = eps.size
    var = np.empty(n)
    var[0] = np.var(eps, ddof=1)
    for t in range(1, n):
        var[t] = lam * var[t-1] + (1 - lam) * eps[t-1]**2
    return np.sqrt(var)
```

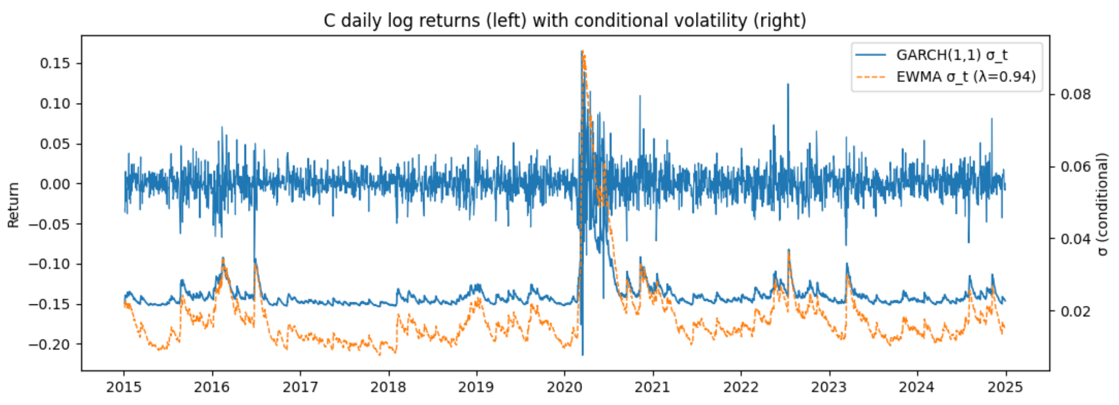
[38]:
```
nlags_show = 20
arch_lags = 10
```

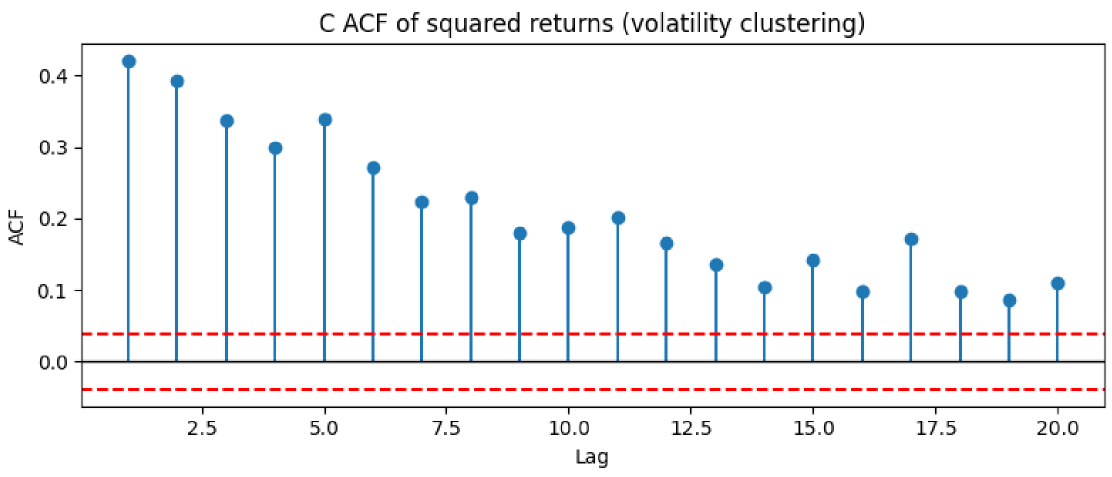
```
ewma_lambda = 0.94


garch_table = []
for ticker in stock_tickers:
    r = log_ret[ticker].dropna()
    t_idx = r.index

    # 1) ACFs (returns vs squared/absolute returns)
    rho_r = acf(r.values, nlags=nlags_show)
    rho_sq = acf((r.values**2), nlags=nlags_show)
    rho_abs = acf(np.abs(r.values), nlags=nlags_show)

    lags_plot_r   = np.arange(1, nlags_show+1)
    rho_r_plot    = rho_r[1:]
    lags_plot_sq  = np.arange(1, nlags_show+1)
    rho_sq_plot   = rho_sq[1:]

    conf = 1.96 / np.sqrt(len(r.values))

    # 2) Tests on squared returns (vol clustering)
    Q, Qp = ljung_box_q(r.values**2, lags=nlags_show)
    LM, LMp = arch_lm_test(r.values - r.mean(), lags=arch_lags)

    # 3) GARCH(1,1) fit
    fit = garch11_fit(r.values)
    mu, omega, alpha, beta, aplusb = fit['mu'], fit['omega'], fit['alpha'],␣
 ↪fit['beta'], fit['alpha+beta']
    sigma_garch = pd.Series(fit['sigma'], index=t_idx)
    eps = r.values - mu
    # JB on standardized residuals (innovations)
    z = eps / sigma_garch.values
    jb_stat, jb_p = stats.jarque_bera(z)
    sigma_ewma = pd.Series(ewma_sigma(eps, lam=ewma_lambda), index=t_idx)

    garch_table.append({
        'Ticker': ticker, 'mu': mu, 'omega': omega, 'alpha': alpha, 'beta':␣
 ↪beta,
        'alpha+beta': aplusb,'JB_stat(z)': jb_stat,'JB_p(z)': jb_p,
        f'LjungBox_Q({nlags_show}) on r^2': Q, 'LB_p': Qp,
        f'ARCH_LM({arch_lags})': LM, 'ARCH_p': LMp,
        'SLSQP_success': fit['success']
    })

    # 4) Plots
    # (a) Returns with fitted conditional vol (GARCH & EWMA)
    fig, ax1 = plt.subplots(figsize=(11, 4))
    ax1.plot(t_idx, r.values, lw=0.8)
    ax1.set_title(f"{ticker} daily log returns (left) with conditional␣
 ↪volatility (right)")
    ax1.set_ylabel("Return")

    ax2 = ax1.twinx()
    ax2.plot(t_idx, sigma_garch.values, lw=1.2, label='GARCH(1,1)  _t')
    ax2.plot(t_idx, sigma_ewma.values, lw=1.0, linestyle='--', label=f"EWMA  _t␣
 ↪( ={ewma_lambda})")
    ax2.set_ylabel("  (conditional)")
    ax2.legend(loc="upper right")
    plt.tight_layout()
    plt.show()


    # (b) ACF of squared returns (no lag 0), with bands + y=0
    plt.figure(figsize=(8, 3.5))
    plt.stem(lags_plot_sq, rho_sq_plot, basefmt=" ")
    plt.axhline(conf,  linestyle='--', color='red')
    plt.axhline(-conf, linestyle='--', color='red')
    plt.axhline(0,     color="black", linewidth=1)
    plt.title(f"{ticker} ACF of squared returns (volatility clustering)")
    plt.xlabel("Lag"); plt.ylabel("ACF")
    plt.tight_layout(); plt.show()

    # (c) ACF of raw returns (no lag 0), with bands + y=0
    plt.figure(figsize=(8, 3.5))
    plt.stem(lags_plot_r, rho_r_plot, basefmt=" ")
    plt.axhline(conf,  linestyle='--', color='red')
    plt.axhline(-conf, linestyle='--', color='red')
    plt.axhline(0,     color="black", linewidth=1)
    plt.title(f"{ticker} ACF of raw returns (typically weak)")
    plt.xlabel("Lag"); plt.ylabel("ACF")
    plt.tight_layout(); plt.show()


# 5) Parameter table
garch_df = pd.DataFrame(garch_table).set_index('Ticker')
display(garch_df.round(6))
```

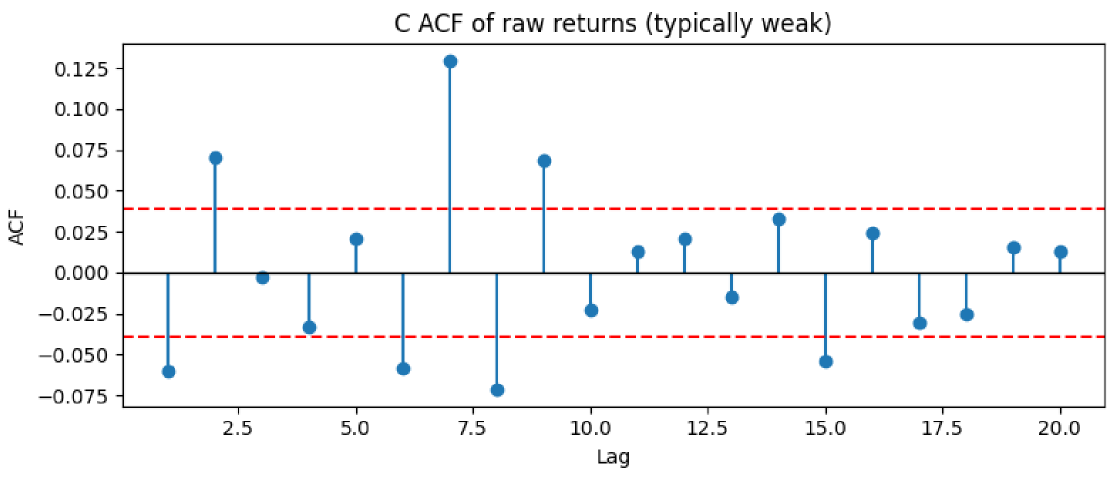

BAC daily log returns (left) with conditional volatility (right)
GARCH(1,1) σ_t
EWMA σ_t (λ=0.94)
Return
σ (conditional)

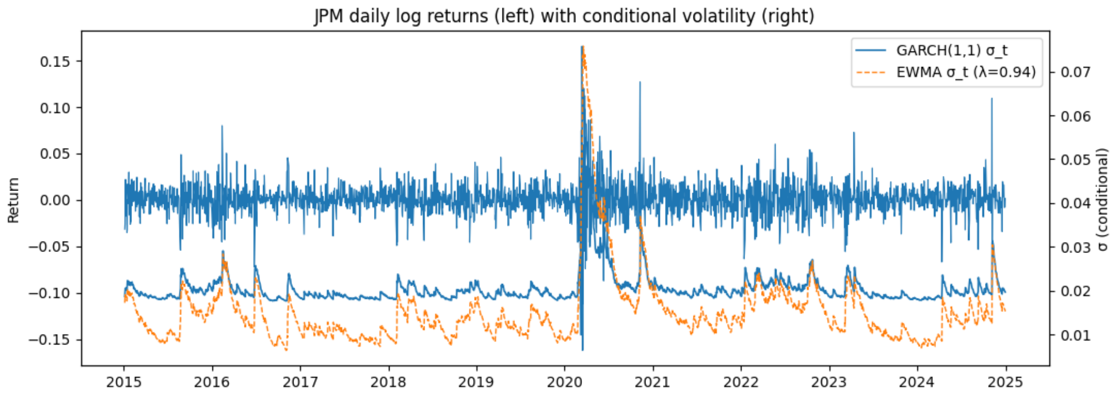

BAC ACF of squared returns (volatility clustering)
ACF
Lag

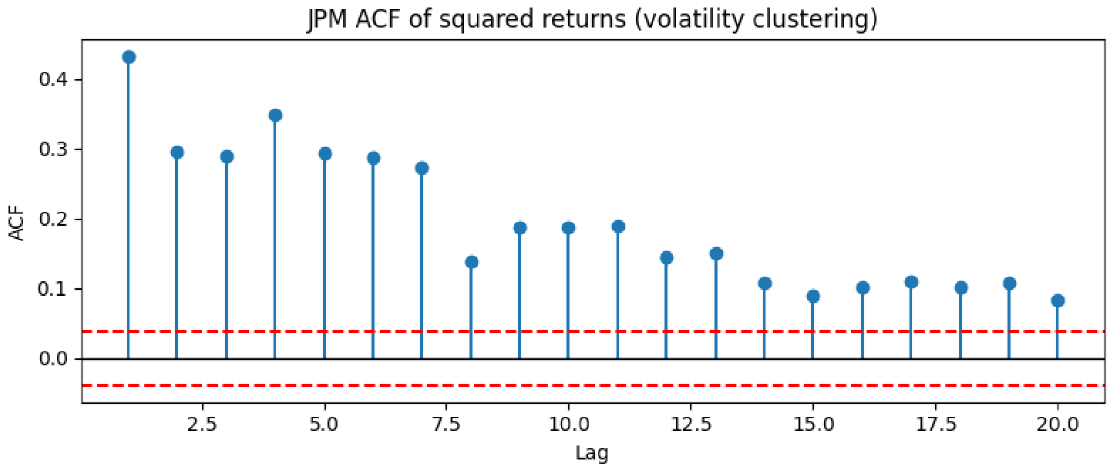

BAC ACF of raw returns (typically weak)
ACF
Lag

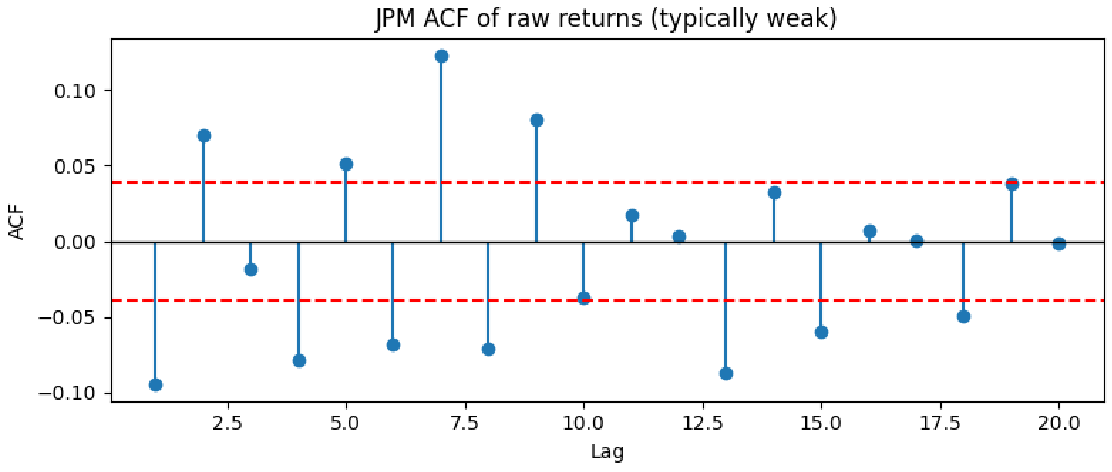

C daily log returns (left) with conditional volatility (right)
Return
σ (conditional)
GARCH(1,1) σ_t
EWMA σ_t (λ=0.94)
0.15
0.10
0.05
0.00
−0.05
−0.10
−0.15
−0.20
0.08
0.06
0.04
0.02
2015
2016
2017
2018
2019
2020
2021
2022
2023
2024
2025

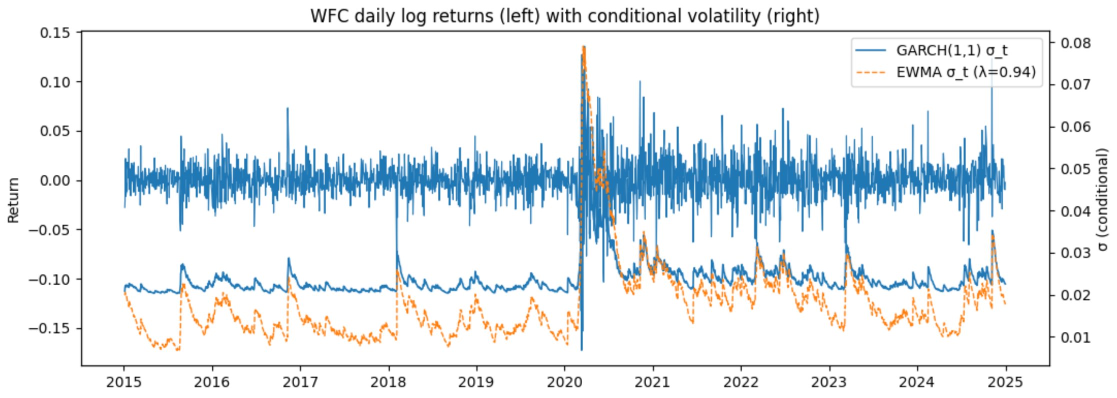

C ACF of squared returns (volatility clustering)
ACF
Lag
0.4
0.3
0.2
0.1
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5
20.0

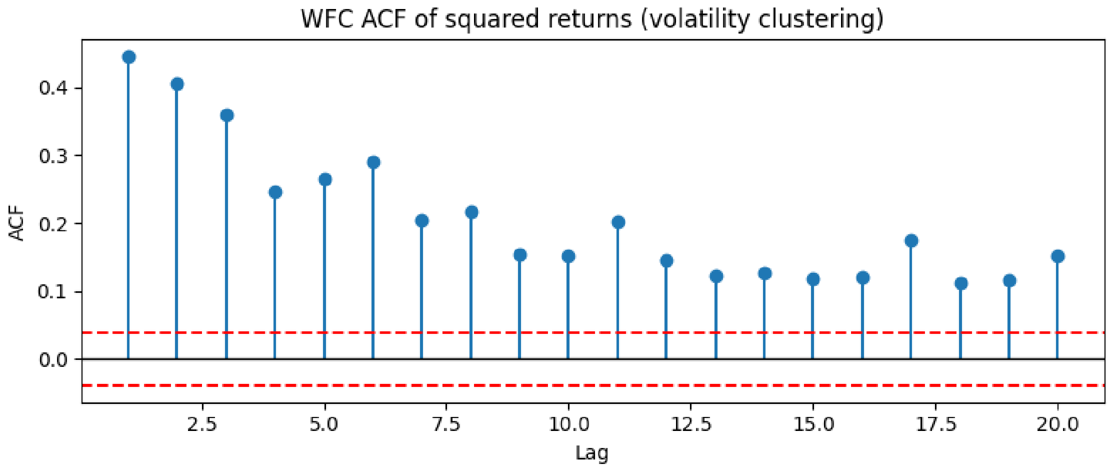

C ACF of raw returns (typically weak)
ACF
Lag
0.125
0.100
0.075
0.050
0.025
0.000
−0.025
−0.050
−0.075
2.5
5.0
7.5
10.0
12.5
15.0
17.5
20.0

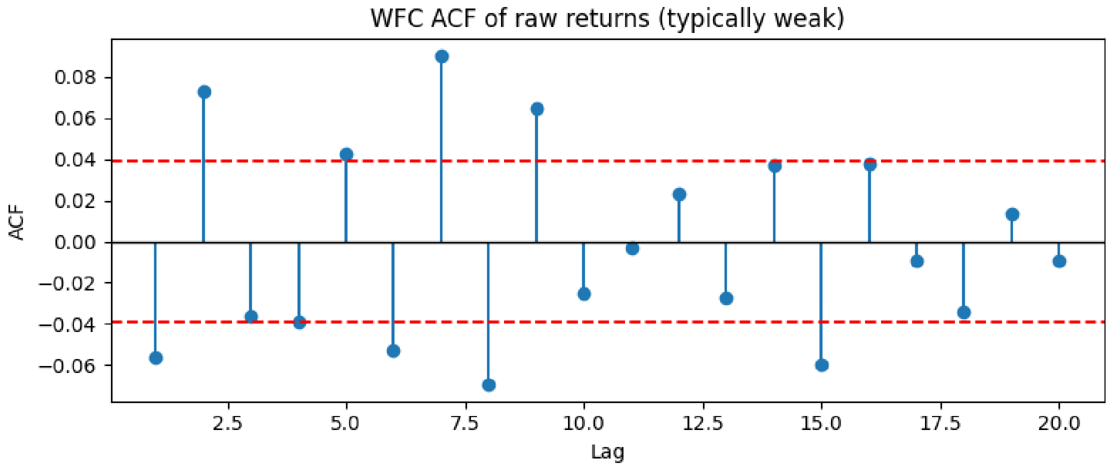

JPM daily log returns (left) with conditional volatility (right)
GARCH(1,1) σ_t
EWMA σ_t (λ=0.94)
Return
σ (conditional)
2015
2016
2017
2018
2019
2020
2021
2022
2023
2024
2025


JPM ACF of squared returns (volatility clustering)
ACF
Lag


JPM ACF of raw returns (typically weak)
ACF
Lag


WFC daily log returns (left) with conditional volatility (right)
GARCH(1,1) σ_t
EWMA σ_t (λ=0.94)
Return
σ (conditional)


WFC ACF of squared returns (volatility clustering)
ACF
Lag


WFC ACF of raw returns (typically weak)
ACF
Lag

```
              mu     omega     alpha      beta  alpha+beta   JB_stat(z)  \
Ticker
BAC     0.000443  0.000039  0.050000  0.900000    0.950000  1022.741103
C       0.000595  0.000043  0.050881  0.898854    0.949735  1756.672208
JPM     0.000646  0.000030  0.050000  0.900000    0.950000  2494.352309
WFC     0.000215  0.000039  0.050000  0.900000    0.950000  1642.096437

        JB_p(z)  LjungBox_Q(20) on r^2  LB_p  ARCH_LM(10)  ARCH_p  \
Ticker
BAC         0.0            2294.990761   0.0   667.479912     0.0
C           0.0            2709.365315   0.0   679.882467     0.0
JPM         0.0            2403.990315   0.0   698.392796     0.0
WFC         0.0            2621.029206   0.0   717.698538     0.0

        SLSQP_success
Ticker
BAC              True
C                True
JPM              True
WFC              True
```

# 4 Part 5 Topic Analysis – Hypothesis 3: Asymmetric volatility

## 4.1 Do negative returns increase volatility more than positive returns of the same magnitude?

[40]:
```python
def _aic_bic(ll, k, n):  # ll: log-likelihood value; k: number of parameters in␣
↪the model; n: number of observations
    aic = 2*k - 2*ll  # AIC = 2k - 2log(L)
    bic = k*np.log(n) - 2*ll  # BIC = k·log(n) - 2log(L)
    return aic, bic

def _num_hessian(f, x0, eps=1e-6):
    # simple central-difference Hessian (symmetric)
    x0 = np.asarray(x0, dtype=float)
    k = x0.size
    H = np.zeros((k, k), dtype=float)
    f0 = f(x0)
    for i in range(k):
        xi_p = x0.copy(); xi_m = x0.copy()
        xi_p[i] += eps; xi_m[i] -= eps
        f_ip = f(xi_p); f_im = f(xi_m)
        H[i, i] = (f_ip - 2*f0 + f_im) / (eps**2)
        # Diagonal elements: Second derivative approximation using central␣
↪difference:  ²f/ x ²   [f(x+ e ) - 2f(x) + f(x- e )] /  ²
        for j in range(i+1, k):
            xpp = x0.copy(); xpm = x0.copy()
            xmp = x0.copy(); xmm = x0.copy()
            xpp[i]+=eps; xpp[j]+=eps
            xpm[i]+=eps; xpm[j]-=eps
            xmp[i]-=eps; xmp[j]+=eps
            xmm[i]-=eps; xmm[j]-=eps
            H_ij = (f(xpp) - f(xpm) - f(xmp) + f(xmm)) / (4*eps**2)
            # Off-diagonal elements: Mixed partial derivative approximation:␣
 ↪ ²f/ x x   [f(x+ e + e ) - f(x+ e - e ) - f(x- e + e ) + f(x- e - e )] / (4 ²)
            H[i, j] = H[j, i] = H_ij
    return H

def _stderr_from_hessian(negloglike, params_hat):
    """
    compute parameter standard errors from Hessian
    negloglike: negative log-likelihood function
    params_hat: estimated parameter values
    """
    # Var(theta)   [H(negloglike)]^{-1}; H is evaluated at the optimum
    # Variance-covariance matrix is approximately the inverse of the Hessian
    H = _num_hessian(negloglike, np.array(params_hat))
    try:
        cov = np.linalg.inv(H)   # invert the Hessian to get␣
 ↪variance-covariance matrix
        se = np.sqrt(np.diag(cov))  # extracts standard errors as square roots␣
 ↪of diagonal elements
    except np.linalg.LinAlgError:
        se = np.full(len(params_hat), np.nan)  # if Hessian inversion fails,␣
 ↪returns nan
    return se

def garch_llf_from_sigma(r, mu, sigma, var_floor=1e-10):
    eps = r - mu
    var = np.maximum(sigma**2, var_floor)
    ll  = -0.5*(np.log(2*np.pi) + np.log(var) + (eps**2)/var)
    return np.sum(ll)
```

[41]:
```
# ---------- Robust SE helpers (drop-in) ----------
import numpy as np

def _is_pos_def(M):
    try:
        # Cholesky is a good PD check
        np.linalg.cholesky(M)
        return True
    except np.linalg.LinAlgError:
        return False
```

```python
def _robust_inverse(H, ridge0=1e-8, max_tries=8):
    """
    Try to invert Hessian H. If not PD, add ridge*I, growing ridge geometrically.
    Fall back to pseudo-inverse if needed. Returns (cov, used_pinv, ridge_used).
    """
    I = np.eye(H.shape[0])
    ridge = ridge0
    for _ in range(max_tries):
        H_r = H + ridge * I
        if _is_pos_def(H_r):
            try:
                cov = np.linalg.inv(H_r)
                return cov, False, ridge
            except np.linalg.LinAlgError:
                pass
        ridge *= 10.0  # increase regularization
    # fallback
    cov = np.linalg.pinv(H)
    return cov, True, 0.0

def _stderr_from_hessian_robust(negloglike, params_hat, hess_eps=1e-5, ridge0=1e-8):
    """
    Safer SEs from numerical Hessian of NEGATIVE log-likelihood.
    Returns SE array and a small diagnostics dict.
    """
    H = _num_hessian(negloglike, np.array(params_hat, dtype=float), eps=hess_eps)
    cov, used_pinv, ridge_used = _robust_inverse(H, ridge0=ridge0)

    # Diagonal can still be slightly negative due to numerical noise: clip to nan
    diag = np.diag(cov)
    se = np.empty_like(diag)
    se[:] = np.nan
    pos = diag > 0
    se[pos] = np.sqrt(diag[pos])

    info = {
        "hess_eps": hess_eps,
        "ridge_used": ridge_used,
        "used_pinv": bool(used_pinv),
        "min_diag_cov": float(np.nanmin(diag)),
        "max_diag_cov": float(np.nanmax(diag))
    }
    return se, info
```

```
[42]: # sigma_t^2 =   +   _{t-1}^2 +   I( _{t-1}<0)  _{t-1}^2 +   _{t-1}^2
      """
      GJR-GARCH(1,1) model: Allows asymmetric volatility response
        I(   <0)    ²: Extra volatility effect for negative shocks (leverage effect)
      I(   <0) is an indicator function (1 if negative, 0 otherwise)
      """
      def fit_gjr_garch(r):
          r = np.asarray(r, float)
          r = r[~np.isnan(r)]
          n = len(r)
          var_floor = 1e-10
          def nll(theta):  # Negative Log-Likelihood Function
              mu, omega, alpha, gamma, beta = theta
              # basic constraints
              if omega <= 1e-12 or alpha < 0 or gamma < 0 or beta < 0:
                  return 1e12
              if (alpha + 0.5*gamma + beta) >= 0.999:   #tighter bound
                  return 1e12
              eps = r - mu
              var = np.empty(n)
              v0 = np.var(eps, ddof=1)
              var[0] = max(omega + (alpha + 0.5*gamma + beta)*v0, var_floor)   #␣
       ↪variance >= 0
              for t in range(1, n):
                  Ineg = 1.0 if eps[t-1] < 0 else 0.0
                  v = omega + (alpha + gamma*Ineg)*eps[t-1]**2 + beta*var[t-1]
                  if not np.isfinite(v):           # CHANGE: keep the finiteness␣
       ↪check on v
                      return 1e12
                  var[t] = max(v, var_floor)       # CHANGE: floor and then assign
              ll = -0.5*(np.log(2*np.pi) + np.log(var) + (eps**2)/var)  #log␣
       ↪likelihood
              S = np.sum(ll)
              if not np.isfinite(S):
                  return 1e12
              return -S

          mu0 = np.mean(r); v0 = np.var(r, ddof=1)
          x0  = np.array([mu0, 0.05*v0, 0.05, 0.05, 0.90])
          bnds = [(-1,1), (1e-12,None), (0,1), (0,1), (0,0.999)]
          cons = ({'type':'ineq','fun': lambda th: 0.999 - (th[2] + 0.5*th[3] +␣
       ↪th[4])})

          res = minimize(nll, x0, method='SLSQP', bounds=bnds, constraints=cons,␣
       ↪options={'maxiter':2000})
          mu, omega, alpha, gamma, beta = res.x
           """
```

```
     : sample mean
     : 5% of sample variance
     : 0.05 (typical ARCH effect)
     : 0.05 (typical leverage effect)
     : 0.9 (typical persistence)
    res.x is the optimized parameter vector that minimizes the negative␣
↪log-likelihood function
    """
    # optimizes using SLSQP with constraints

    # reconstructs conditional variance series with fitted parameters
    eps = r - mu
    var = np.empty(n); v_init = np.var(eps, ddof=1)
    var[0] = max(omega + (alpha + 0.5*gamma + beta)*v_init, var_floor)
    for t in range(1, n):
        Ineg = 1.0 if eps[t-1] < 0 else 0.0
        var[t] = max(omega + (alpha + gamma*Ineg)*eps[t-1]**2 + beta*var[t-1],␣
↪var_floor)
    sigma = np.sqrt(var)

    # computes standard errors(using hessian) and t-statistics for hypothesis␣
↪testing
    # computes standard errors(using robust Hessian) and t-statistics
    se, se_info = _stderr_from_hessian_robust(nll, res.x, hess_eps=1e-5,␣
↪ridge0=1e-8)
    tstats = res.x / se if np.all(np.isfinite(se)) else np.full_like(res.x, np.
↪nan)
    # print("GJR-GARCH SE diagnostics:", se_info)
    # calculates log-likelihood, AIC, and BIC for model comparison
    llf = -nll(res.x); k = len(res.x)
    aic, bic = _aic_bic(llf, k, n)
    out = {'params': {'mu':mu,'omega':omega,'alpha':alpha,'gamma':gamma,'beta':
↪beta},
           'se': {'mu':se[0],'omega':se[1],'alpha':se[2],'gamma':se[3],'beta':
↪se[4]},
           't':  {'mu':tstats[0],'omega':tstats[1],'alpha':tstats[2],'gamma':
↪tstats[3],'beta':tstats[4]},
           'sigma': sigma, 'eps': eps, 'llf': llf, 'aic': aic, 'bic': bic,␣
↪'success': res.success, 'message': res.message}
    return out
```

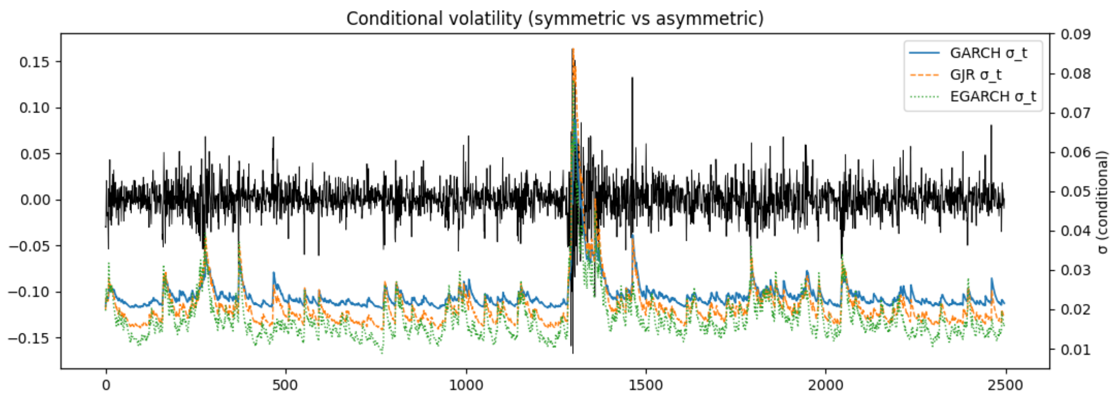
[43]:
```
# log( _t^2) =   +   log( _{t-1}^2) +   (|z_{t-1}| - E|Z|) +   z_{t-1}, z ~ N(0,1)
"""
EGARCH(1,1) model: Exponential GARCH model for asymmetric volatility
Models the log of variance (always positive, no constraints needed)
 (|z  | - E|Z|): Magnitude effect (volatility clustering)
```

```
  z  : Sign effect (leverage/asymmetry)
E|Z| = √(2/ ) for standard normal
"""
def fit_egarch(r):
    r = np.asarray(r, float)
    r = r[~np.isnan(r)]
    n = len(r)
    EabsZ = np.sqrt(2/np.pi)  # calculates E|Z| for standard normal
    var_floor = 1e-10
    sig_floor = 1e-5
    clip_lo, clip_hi = -25.0, 25.0

    def nll(theta):
        mu, omega, alpha, gamma, beta = theta
        if not (-0.999 < beta < 0.999):     # CHANGE: tighter beta
            return 1e12
        if not np.all(np.isfinite(theta)):  # CHANGE
            return 1e12
        eps = r - mu
        logv = np.empty(n)
        logv[0] = np.log(np.var(eps, ddof=1) + var_floor)   # initialize␣
 ↪log-variance = log of sample variance
        for t in range(1, n):
            sig_prev = np.exp(0.5*logv[t-1])
            sig_prev = max(sig_prev, sig_floor)
            z_prev = eps[t-1] / sig_prev   # standardized residuals: z   =   /
 ↪    ~ N(0,1)
            lv = omega + beta*logv[t-1] + alpha*(np.abs(z_prev) - EabsZ) +␣
 ↪gamma*z_prev
            if not np.isfinite(lv): return 1e12
            logv[t] = np.clip(lv, clip_lo, clip_hi)

        var = np.maximum(np.exp(logv), var_floor)
        ll  = -0.5*(np.log(2*np.pi) + np.log(var) + (eps**2)/var)
        S   = np.sum(ll)
        if not np.isfinite(S): return 1e12
        return -S

    mu0 = np.mean(r)
    x0  = np.array([mu0, -0.1, 0.1, -0.05, 0.9])  # reasonable starts
    """
     : -0.1 (typical for log-variance intercept)
     : 0.1 (volatility clustering effect)
     : -0.05 (leverage effect, typically negative)
     : 0.9 (high persistence in log-volatility)
    """
    bnds = [(-1,1), (None,None), (None,None), (None,None), (-0.9999,0.9999)]
    res = minimize(nll, x0, method='SLSQP', bounds=bnds, options={'maxiter':
↪2000})
    mu, omega, alpha, gamma, beta = res.x

    # reconstructs the volatility series with fitted parameters
    eps = r - mu
    logv = np.empty(n); logv[0] = np.log(np.var(eps, ddof=1) + var_floor)
    for t in range(1, n):
        sig_prev = np.exp(0.5*logv[t-1]); sig_prev = max(sig_prev, sig_floor)
        z_prev   = eps[t-1] / sig_prev
        lv = omega + beta*logv[t-1] + alpha*(np.abs(z_prev) - EabsZ) +␣
↪gamma*z_prev
        logv[t] = np.clip(lv, clip_lo, clip_hi)
    var   = np.maximum(np.exp(logv), var_floor)
    sigma = np.sqrt(var)
    # computes standard errors, t-stats, and information criteria
    se, se_info = _stderr_from_hessian_robust(nll, res.x, hess_eps=1e-5,␣
↪ridge0=1e-8)
    tstats = res.x / se if np.all(np.isfinite(se)) else np.full_like(res.x, np.
↪nan)
    # print("EGARCH SE diagnostics:", se_info)

    llf = -nll(res.x); k = len(res.x)
    aic, bic = _aic_bic(llf, k, n)

    out = {'params': {'mu':mu,'omega':omega,'alpha':alpha,'gamma':gamma,'beta':
↪beta},
           'se': {'mu':se[0],'omega':se[1],'alpha':se[2],'gamma':se[3],'beta':
↪se[4]},
           't':  {'mu':tstats[0],'omega':tstats[1],'alpha':tstats[2],'gamma':
↪tstats[3],'beta':tstats[4]},
           'sigma': sigma, 'eps': eps, 'llf': llf, 'aic': aic, 'bic': bic,␣
↪'success': res.success, 'message': res.message}
    return out
```

[29]:
```
r = log_ret['BAC'].dropna().values

gjr = fit_gjr_garch(r)
eg  = fit_egarch(r)
g11 = garch11_fit(r)

def _row(name, res, k):
    return {
        'Model': name,
        'mu': res['params']['mu'],
        'omega': res['params']['omega'],
        'alpha': res['params']['alpha'],
        'gamma': res['params']['gamma'],
        'beta': res['params']['beta'],
        't(gamma)': res['t']['gamma'],
        'LLF': res['llf'],
        'AIC': res['aic'],
        'BIC': res['bic']
    }

import pandas as pd
rows = [
    _row('GARCH(1,1)', {'params':{'mu':g11['mu'],'omega':g11['omega'],'alpha':
 ↪g11['alpha'],'gamma':np.nan,'beta':g11['beta']},
                      't':{'gamma':np.nan}, 'llf':- np.inf, 'aic':np.nan,␣
 ↪'bic':np.nan}, 4),
    _row('GJR-GARCH', gjr, 5),
    _row('EGARCH',    eg,  5),
]
cmp_df = pd.DataFrame(rows).set_index('Model')
# fill AIC/BIC/LLF for symmetric model using its 4 params (mu,omega,alpha,beta)
n = len(r)
llf_g11 = - (lambda: (  # reuse your negloglike inside a quick closure
    (lambda mu,omega,alpha,beta: (
        (lambda eps,var: np.sum(-0.5*(np.log(2*np.pi)+np.log(var)+(eps**2)/
 ↪var)))(
            r-mu,
            (lambda eps: (
                np.array([max(omega + alpha*eps[0]**2 + beta*np.
 ↪var(eps,ddof=1),1e-12)] +
                         [0]*(len(eps)-1), dtype=float)
            ))(r-mu)
        )
    ))(g11['mu'], g11['omega'], g11['alpha'], g11['beta'])
))()
# the above is clunky; simpler: recompute with your garch11_fit's negloglike if␣
 ↪you kept it accessible
# For brevity, set directly using stored path:
llf_g11 = garch_llf_from_sigma(r, g11['mu'], g11['sigma'])
aic_g11, bic_g11 = _aic_bic(llf_g11, 4, len(r))
cmp_df.loc['GARCH(1,1)', ['LLF','AIC','BIC']] = [llf_g11, aic_g11, bic_g11]

print(cmp_df.round(4))
print("\nInterpretation tip: Significant  (>~1.96 in |t|) supports asymmetry␣
 ↪(leverage).")

import pandas as pd
```

```python
idx = pd.Index(np.arange(len(r)), name='t')
plt.figure(figsize=(11,4))
plt.plot(idx, r, lw=0.6, color='black', label='returns')
ax = plt.gca().twinx()
ax.plot(idx, g11['sigma'], lw=1.2, label='GARCH  _t')
ax.plot(idx, gjr['sigma'], lw=1.0, linestyle='--', label='GJR  _t')
ax.plot(idx, eg['sigma'],  lw=1.0, linestyle=':',  label='EGARCH  _t')
ax.set_ylabel('  (conditional)')
ax.legend(loc='upper right')
plt.title('Conditional volatility (symmetric vs asymmetric)')
plt.tight_layout(); plt.show()
```

```
C:\Users\amusi\AppData\Local\Temp\ipykernel_14248\2288054900.py:33:
RuntimeWarning: divide by zero encountered in log
  (lambda eps,var: np.sum(-0.5*(np.log(2*np.pi)+np.log(var)+(eps**2)/var)))(
C:\Users\amusi\AppData\Local\Temp\ipykernel_14248\2288054900.py:33:
RuntimeWarning: divide by zero encountered in divide
  (lambda eps,var: np.sum(-0.5*(np.log(2*np.pi)+np.log(var)+(eps**2)/var)))(
C:\Users\amusi\AppData\Local\Temp\ipykernel_14248\2288054900.py:33:
RuntimeWarning: invalid value encountered in add
  (lambda eps,var: np.sum(-0.5*(np.log(2*np.pi)+np.log(var)+(eps**2)/var)))(

                mu   omega   alpha   gamma  beta  t(gamma)       LLF  \
Model
GARCH(1,1)  0.0004  0.0000  0.0500     NaN  0.90       NaN  6370.3824
GJR-GARCH   0.0004  0.0000  0.0500  0.0500  0.90       NaN  6538.6351
EGARCH      0.0006 -0.3993  0.1694 -0.1094  0.95   -8.1538  6620.4016

                   AIC         BIC
Model
GARCH(1,1) -12732.7649 -12709.4735
GJR-GARCH  -13067.2701 -13038.1559
EGARCH     -13230.8032 -13201.6890

Interpretation tip: Significant  (>~1.96 in |t|) supports asymmetry (leverage).
```



## 5 Part 6 Topic Analysis – Hypothesis 4: Risk model comparison

### 5.1 Do heavy-tailed models yield better risk estimates than normal distribution models?

[ ]:

```

```

## 6 Part 7 Backtesting and Validation

[30]:

```python
import numpy as np
import pandas as pd

def _to_np_array(x):
    if isinstance(x, (list, tuple, pd.Series, pd.Index)):
        return np.asarray(x)
    elif isinstance(x, pd.DataFrame):
        # if DataFrame with one column, squeeze; else raise
        if x.shape[1] == 1:
            return x.iloc[:, 0].to_numpy()
        raise ValueError("Provide a 1-D array/Series for returns/VaR, not a␣
↪multi-column DataFrame.")
    """
    if x is a Python list/tuple, a pandas Series, or a pandas Index, convert it␣
↪to a NumPy array using np.asarray
    np.asarray is preferred over np.array because it avoids unnecessary copying␣
↪when x is already an array-like object
    if x is a pandas DataFrame:
    if it has exactly one column, we "squeeze" it to 1-D by selecting that␣
↪column x.iloc[:, 0] and converting to a NumPy array with .to_numpy()
    VaR backtests need a single series of numbers, not a table
    if it has more than one column, we raise an error. This prevents silent␣
↪mistakes
    """
    return x  # assume numpy array


def detect_violation_rule(var_series):
    """
    Decide the comparison direction based on VaR sign convention
    If VaR is mostly negative (quantile of returns), violation is r < VaR
    If VaR is positive loss threshold, violation is r < -VaR
    """
    v = _to_np_array(var_series)
    neg_share = np.mean(v < 0)
    return "r<var" if neg_share > 0.5 else "r<-var"


def compute_violations(returns, var_series, alpha=0.99, rule=None):
    """
    returns: array-like of realized returns (same freq as VaR forecasts)
    var_series: array-like of VaR forecasts (either negative return quantile,␣
↪or positive loss)
    alpha: VaR confidence level (e.g., 0.99 for 99% VaR)
    rule: 'r<var' or 'r<-var' (auto-detected if None)
    """
    r = _to_np_array(returns)
    v = _to_np_array(var_series)
    if r.shape != v.shape:
        raise ValueError("returns and var_series must have the same shape")

    if rule is None:
        rule = detect_violation_rule(v)

    if rule == "r<var":
        viol = r < v
    elif rule == "r<-var":
        viol = r < -v
    else:
        raise ValueError("rule must be one of {'r<var','r<-var'}")

    n = viol.size
    x = int(viol.sum())
    rate = x / n if n > 0 else np.nan
    exp = (1 - alpha) * n
    """
    n: total number of days
    x: number of violations (days actual loss was worse than predicted)
    rate: proportion of violations (e.g., 12 violations out of 250 days = 4.8%)
    exp: expected number of violations based on the confidence level
    At 99% VaR, we expect about 1% of days to breach → 0.01 × n
    """
    return viol.astype(bool), {"n": n, "violations": x, "violation_rate": rate,␣
↪"expected_violations": exp, "rule": rule, "alpha": alpha}


# Kupiec POF test (unconditional)

def kupiec_pof(alpha, x, n):
    """
    Kupiec (1995) Proportion of Failures test (LR_uc)
    We compare LR to chi-square(1) 95% critical value 3.841
    Returns dict with LR statistic and a 95% reject flag
    """
    if n <= 0:
        return {"LR_uc": np.nan, "pi_hat": np.nan, "reject_95pct": None}
    pi = 1 - alpha
    pi_hat = x / n
    """
    Expected violation probability (pi):
    At 99% VaR → expected probability of violation = 1 - 0.99 = 0.01 = 1%
    Observed violation rate (pi_hat):
    What actually happened: x / n
    """
    # Handle edge cases to avoid log(0)
    eps = 1e-12
    pi_hat = min(max(pi_hat, eps), 1 - eps)
    pi = min(max(pi, eps), 1 - eps)

    ll_uncond = (n - x) * np.log(1 - pi) + x * np.log(pi)
    ll_cond = (n - x) * np.log(1 - pi_hat) + x * np.log(pi_hat)
    LR_uc = -2.0 * (ll_uncond - ll_cond)  # Likelihood ratio test (LR_uc)
    """
    LRT = -2 × (log-likelihood of the simpler model - log-likelihood of the␣
 ↪complex model)
    """
    reject = LR_uc > 3.841  # chi2_0.95, df=1
    return {"LR_uc": float(LR_uc), "pi_hat": float(pi_hat), "reject_95pct":␣
 ↪bool(reject)}


# Christoffersen independence test (clustering)

def christoffersen_independence(violations):
    """
    Tests whether violations are independent (no clustering).
    Builds 2x2 transition matrix:
        N00: 0->0, N01: 0->1
        N10: 1->0, N11: 1->1
    LR_ind ~ chi-square(1). Reject if > 3.841 (95%).
    """
    v = _to_np_array(violations).astype(int)
    if v.size < 2:
        return {"N00":0,"N01":0,"N10":0,"N11":0,"LR_ind":np.nan,"reject_95pct":
 ↪None}
    """
    Input: violations = a sequence of 0/1 by day (1 = VaR breach, 0 = no breach)
    Convert to integers and ensure there's at least 2 days to form transitions
    """
    v_prev = v[:-1]
    v_next = v[1:]
    N00 = int(((v_prev == 0) & (v_next == 0)).sum())
    N01 = int(((v_prev == 0) & (v_next == 1)).sum())
    N10 = int(((v_prev == 1) & (v_next == 0)).sum())
    N11 = int(((v_prev == 1) & (v_next == 1)).sum())
    # A large N11 means once you get a breach, another breach is more likely␣
↪next day → clustering.

    # Transition probabilities
    eps = 1e-12
    N0_ = N00 + N01
    N1_ = N10 + N11
    p01 = N01 / N0_ if N0_ > 0 else 0.0
    p11 = N11 / N1_ if N1_ > 0 else 0.0
    p = (N01 + N11) / (N00 + N01 + N10 + N11 + eps)

    """
    p01: Probability of a violation tomorrow given no violation today
    p01=N01/(N00+N01)
    p11: Probability of a violation tomorrow given a violation today
    p11=N11/(N10+N11)
    p: Overall (unconditional) probability of a violation on any day
    p=(N01+N11)/(N00+N01+N10+N11)
    """

    # Log-likelihoods
    def _log(x): return np.log(max(x, eps))
    ll_indep = (N00 * _log(1 - p) + N01 * _log(p) +
                N10 * _log(1 - p) + N11 * _log(p))
    ll_markov = (N00 * _log(1 - p01) + N01 * _log(p01) +
                 N10 * _log(1 - p11) + N11 * _log(p11))
    """
    Independence model (no memory):
    Tomorrow's breach chance is always p, regardless of today.
    Log-likelihood uses p for both 0→1 and 1→1 transitions.
    Markov (1-step memory) model:
    Tomorrow's chance depends on today:
    If today=0, use p01
    If today=1, use p11
    This model captures clustering if present.
    """
    LR_ind = -2.0 * (ll_indep - ll_markov)  # # Likelihood ratio test (LR_uc)
    reject = LR_ind > 3.841  # chi2_0.95, df=1
    return {"N00":N00,"N01":N01,"N10":N10,"N11":N11,
            "p01":float(p01),"p11":float(p11),"p_hat":float(p),
            "LR_ind":float(LR_ind),"reject_95pct":bool(reject)}
```

```python
# Conditional coverage (Kupiec + Christoffersen combined test)

def conditional_coverage(alpha, violations):
    """
    LR_cc = LR_uc + LR_ind ~ chi-square(2).
    Reject if > 5.991 (95%).
    """
    v = _to_np_array(violations).astype(bool)
    n = v.size
    x = int(v.sum())
    out_uc = kupiec_pof(alpha, x, n)
    """
    compares the observed breach rate (x/n) to the target (1-alpha)
    If there are too many or too few breaches vs. expected, the test statistic␣
 ↪LR_uc will be large
    """
    out_ind = christoffersen_independence(v)
    """
    checks for clustering of breaches (e.g., many around shock weeks)
    If breaches depend on yesterday (e.g., breach today makes breach tomorrow␣
 ↪more likely), LR_ind will be large
    """
    LR_cc = out_uc["LR_uc"] + out_ind["LR_ind"]
    """
    combined statistic LR_cc tests both properties simultaneously
    Under the null ("model has correct frequency and no clustering")
    """
    reject = LR_cc > 5.991  # chi2_0.95, df=2
    return {"LR_cc": float(LR_cc), "reject_95pct": bool(reject),
            "LR_uc": float(out_uc["LR_uc"]), "LR_ind": float(out_ind["LR_ind"])}


# Event-window clustering around known shock announcements

def violation_cluster_around_events(dates, violations, event_dates,␣
 ↪window_days=1):
    """
    Count how many violations occur within +/- window_days of given events.
    Reports enrichment vs. time coverage as a simple reality check.

    dates: array-like of pd.Timestamp (or strings parseable by pd.to_datetime)
    violations: bool array aligned to dates
    event_dates: list/array of dates (FOMC/announcement dates)
    window_days: non-negative integer window size
    """
    dts = pd.to_datetime(_to_np_array(dates))
    v = _to_np_array(violations).astype(bool)
    if dts.size != v.size:
        raise ValueError("dates and violations must align in length")

    ev = pd.to_datetime(pd.Index(event_dates))   # event dates (e.g. covid,␣
↪2008 financial crisis)
    # build mask of days that fall within any event window
    in_window = np.zeros(dts.shape[0], dtype=bool)
    for e in ev:
        lo = e - pd.Timedelta(days=window_days)  # dates +- window_length
        hi = e + pd.Timedelta(days=window_days)
        in_window |= (dts >= lo) & (dts <= hi)  # whole event process

    n_total = v.size
    n_in = int(in_window.sum())  # how many days in window
    n_out = n_total - n_in  # how many days not in window
    viol_in = int((v & in_window).sum())
    viol_out = int((v & (~in_window)).sum())

    share_time_in = n_in / n_total if n_total > 0 else np.nan  #What fraction␣
↪of time is near events?
    share_viol_in = (viol_in / (viol_in + viol_out)) if (viol_in + viol_out) >␣
↪0 else np.nan
    # What fraction of all breaches happened near events?
    enrichment = (share_viol_in / share_time_in) if (share_time_in not in [0,␣
↪np.nan]) else np.nan
    """
    Interpretation:
    > 1: breaches are over-represented near events → clustering around shocks
      1: breaches occur near events in proportion to time → no special␣
↪clustering
    < 1: breaches are under-represented near events
    """

    return {
        "n_total_days": n_total,
        "n_days_in_windows": n_in,
        "n_days_outside": n_out,
        "violations_in_windows": viol_in,
        "violations_outside": viol_out,
        "share_of_time_in_windows": float(share_time_in),
        "share_of_violations_in_windows": float(share_viol_in),
        "enrichment_ratio": float(enrichment)  # >1 suggests clustering near␣
↪events
    }
```

```
# Pretty printer / Summary

def summarize_backtest(returns, var_series, alpha=0.99, dates=None,␣
↪event_dates=None, window_days=1, rule=None):
    viol_flags, base = compute_violations(returns, var_series, alpha=alpha,␣
↪rule=rule)

    # Tests
    pof = kupiec_pof(alpha, base["violations"], base["n"])
    ind = christoffersen_independence(viol_flags)
    cc = conditional_coverage(alpha, viol_flags)

    # Optional event clustering
    cluster = None
    if dates is not None and event_dates is not None and len(event_dates) > 0:
        cluster = violation_cluster_around_events(dates, viol_flags,␣
↪event_dates, window_days=window_days)

    # Print summary (basic formatting)
    print("=== VaR Backtest Summary (H5) ===")
    print(f"Alpha (confidence):          {alpha:.4f}")
    print(f"Observations (n):            {base['n']}")
    print(f"Violations (x):              {base['violations']}")
    print(f"Violation rate (x/n):        {base['violation_rate']:.6f}")
    print(f"Expected violations:         {base['expected_violations']:.2f}")
    print(f"Violation rule used:         {base['rule']}")
    print("\n-- Kupiec POF (Unconditional Coverage) --")
    print(f"LR_uc:                       {pof['LR_uc']:.4f}  | Reject@95%:␣
↪{pof['reject_95pct']}")
    print("-- Christoffersen Independence (Clustering) --")
    print(f"N00={ind['N00']} N01={ind['N01']} N10={ind['N10']}␣
↪N11={ind['N11']}")
    print(f"LR_ind:                      {ind['LR_ind']:.4f}  | Reject@95%:␣
↪{ind['reject_95pct']}")
    print("-- Conditional Coverage (POF + Independence) --")
    print(f"LR_cc:                       {cc['LR_cc']:.4f}  | Reject@95%:␣
↪{cc['reject_95pct']}")
    if cluster is not None:
        print("\n-- Event-Window Check (interest-rate shock announcements) --")
        print(f"Days in ±{window_days}d windows: ␣
↪{cluster['n_days_in_windows']} / {cluster['n_total_days']} "
              f"({cluster['share_of_time_in_windows']:.2%} of time)")
        print(f"Violations in windows:       ␣
↪{cluster['violations_in_windows']} / "
```

```
              f"{cluster['violations_in_windows'] +␣
↪cluster['violations_outside']}")
        print(f"Share of violations in windows:␣
↪{cluster['share_of_violations_in_windows']:.2%}")
        print(f"Enrichment ratio (viol/time): {cluster['enrichment_ratio']:.3f}␣
↪ (>1 suggests clustering)")

    return {"basic": base, "pof": pof, "independence": ind, "conditional": cc,␣
↪"cluster": cluster}
```

[33]:
```
# ==========================================
# H5 VaR Backtest on YOUR data
# Uses: log_ret, stock_tickers, ewma_sigma, and your H5 helper functions
# ==========================================
import numpy as np
import pandas as pd
from scipy import stats as _st

# Interest-rate shock events (Fed target range changes only)
# Window suggestion: +/- 1 trading day (as in your backtest)

event_dates_ir = [
    # 2015 liftoff
    "2015-12-16",

    # 2016
    "2016-12-14",

    # 2017
    "2017-03-15", "2017-06-14", "2017-12-13",

    # 2018
    "2018-03-21", "2018-06-13", "2018-09-26", "2018-12-19",

    # 2019 (cuts)
    "2019-07-31", "2019-09-18", "2019-10-30",

    # 2020 emergency cuts (COVID)
    "2020-03-03",  # -50 bps emergency
    "2020-03-15",  # to 0-0.25% at emergency meeting

    # 2022 (start of hiking cycle)
    "2022-03-16", "2022-05-04", "2022-06-15", "2022-07-27",
    "2022-09-21", "2022-11-02", "2022-12-14",

    # 2023
    "2023-02-01", "2023-03-22", "2023-05-03", "2023-07-26",
    # 2024 (cuts)
    "2024-09-18", "2024-11-07", "2024-12-18",
]

# ----- 1) Settings & inputs -----
alpha = 0.99
window_days = 1                       # event window ±1 day
weights = np.array([0.25, 0.25, 0.25, 0.25])    # equal-weight portfolio
tickers_use = stock_tickers           # ['BAC','C','JPM','WFC']

# Align/clean returns
R = log_ret[tickers_use].dropna().copy()
R.index = pd.to_datetime(R.index)

# ----- 2) Portfolio returns -----
port_ret = R.dot(weights).rename("PORT").dropna()

# ----- 3) VaR A: Rolling Historical (nonparametric) -----
lookback = 250  # ~1y
var_hist = port_ret.rolling(lookback).quantile(1 - alpha).dropna()  # negative␣
  ↪1% quantile of returns
ret_hist = port_ret.loc[var_hist.index]

print("=== Portfolio: Rolling Historical VaR (nonparametric) ===")
_ = summarize_backtest(
    returns=ret_hist.to_numpy(),
    var_series=var_hist.to_numpy(),
    alpha=alpha,
    dates=ret_hist.index.to_numpy(),
    event_dates=event_dates,
    window_days=window_days,
    rule=None,   # auto-detect VaR sign
)

# ----- 4) VaR B: EWMA Parametric (RiskMetrics-style mean ~ 0) -----
# Conditional sigma_t from portfolio returns
def _ewma_sigma(series, lam=0.94):
    # you already have ewma_sigma(eps, lam); using that directly for␣
  ↪consistency:
    return pd.Series(ewma_sigma(series.values - series.values.mean(), lam=lam),␣
  ↪index=series.index)

sigma_ewma_p = _ewma_sigma(port_ret, lam=0.94)
mu_roll = port_ret.rolling(60).mean().fillna(0.0)  # short-term mean; can also␣
  ↪use 0
z = _st.norm.ppf(1 - alpha)                         # ~ -2.326 for alpha=0.99
```

```python
var_ewma = (mu_roll + z * sigma_ewma_p).dropna()
ret_ewma = port_ret.loc[var_ewma.index]

print("\n=== Portfolio: EWMA Parametric VaR ===")
_ = summarize_backtest(
    returns=ret_ewma.to_numpy(),
    var_series=var_ewma.to_numpy(),
    alpha=alpha,
    dates=ret_ewma.index.to_numpy(),
    event_dates=event_dates,
    window_days=window_days,
    rule=None,
)

# ----- 5) (Optional) Per-asset backtests with both VaRs -----
for tkr in tickers_use:
    r = R[tkr].dropna()

    # (A) Rolling historical VaR on the asset
    v_hist = r.rolling(lookback).quantile(1 - alpha).dropna()
    rr = r.loc[v_hist.index]
    print(f"\n=== {tkr}: Rolling Historical VaR ===")
    _ = summarize_backtest(
        returns=rr.to_numpy(),
        var_series=v_hist.to_numpy(),
        alpha=alpha,
        dates=rr.index.to_numpy(),
        event_dates=event_dates,
        window_days=window_days,
        rule=None,
    )

    # (B) EWMA parametric VaR on the asset
    sig = _ewma_sigma(r, lam=0.94)
    mu_r = r.rolling(60).mean().fillna(0.0)
    v_ew = (mu_r + z * sig).dropna()
    rr2 = r.loc[v_ew.index]
    print(f"\n=== {tkr}: EWMA Parametric VaR ===")
    _ = summarize_backtest(
        returns=rr2.to_numpy(),
        var_series=v_ew.to_numpy(),
        alpha=alpha,
        dates=rr2.index.to_numpy(),
        event_dates=event_dates,
        window_days=window_days,
        rule=None,
    )
```

```python
# ----- 6) (Nice to have) List violation dates for the portfolio -----
viol_flags_hist, _base_hist = compute_violations(ret_hist.to_numpy(), var_hist.
 ↪to_numpy(), alpha=alpha, rule=None)
viol_dates_hist = ret_hist.index[np.where(viol_flags_hist)[0]].date.tolist()

viol_flags_ewma, _base_ewma = compute_violations(ret_ewma.to_numpy(), var_ewma.
 ↪to_numpy(), alpha=alpha, rule=None)
viol_dates_ewma = ret_ewma.index[np.where(viol_flags_ewma)[0]].date.tolist()

print("\nFirst 12 portfolio violation dates (Historical VaR):")
for d in viol_dates_hist[:12]:
    print(" -", d)

print("\nFirst 12 portfolio violation dates (EWMA VaR):")
for d in viol_dates_ewma[:12]:
    print(" -", d)
```

```
=== Portfolio: Rolling Historical VaR (nonparametric) ===
=== VaR Backtest Summary (H5) ===
Alpha (confidence):          0.9900
Observations (n):            2248
Violations (x):              33
Violation rate (x/n):        0.014680
Expected violations:         22.48
Violation rule used:         r<var

-- Kupiec POF (Unconditional Coverage) --
LR_uc:                       4.3460  | Reject@95%: True
-- Christoffersen Independence (Clustering) --
N00=2184 N01=30 N10=30 N11=3
LR_ind:                      6.3095  | Reject@95%: True
-- Conditional Coverage (POF + Independence) --
LR_cc:                       10.6555  | Reject@95%: True

-- Event-Window Check (interest-rate shock announcements) --
Days in ±1d windows:  24 / 2248 (1.07% of time)
Violations in windows:       5 / 33
Share of violations in windows: 15.15%
Enrichment ratio (viol/time): 14.192  (>1 suggests clustering)

=== Portfolio: EWMA Parametric VaR ===
=== VaR Backtest Summary (H5) ===
Alpha (confidence):          0.9900
Observations (n):            2497
Violations (x):              57
Violation rate (x/n):        0.022827
```

```
Expected violations:          24.97
Violation rule used:          r<var

-- Kupiec POF (Unconditional Coverage) --
LR_uc:                        30.4497  | Reject@95%: True
-- Christoffersen Independence (Clustering) --
N00=2387 N01=52 N10=52 N11=5
LR_ind:                       6.5693  | Reject@95%: True
-- Conditional Coverage (POF + Independence) --
LR_cc:                        37.0190  | Reject@95%: True

-- Event-Window Check (interest-rate shock announcements) --
Days in ±1d windows:  24 / 2497 (0.96% of time)
Violations in windows:        4 / 57
Share of violations in windows: 7.02%
Enrichment ratio (viol/time): 7.301  (>1 suggests clustering)

=== BAC: Rolling Historical VaR ===
=== VaR Backtest Summary (H5) ===
Alpha (confidence):           0.9900
Observations (n):             2248
Violations (x):               35
Violation rate (x/n):         0.015569
Expected violations:          22.48
Violation rule used:          r<var

-- Kupiec POF (Unconditional Coverage) --
LR_uc:                        6.0211  | Reject@95%: True
-- Christoffersen Independence (Clustering) --
N00=2180 N01=32 N10=32 N11=3
LR_ind:                       5.6831  | Reject@95%: True
-- Conditional Coverage (POF + Independence) --
LR_cc:                        11.7042  | Reject@95%: True

-- Event-Window Check (interest-rate shock announcements) --
Days in ±1d windows:  24 / 2248 (1.07% of time)
Violations in windows:        5 / 35
Share of violations in windows: 14.29%
Enrichment ratio (viol/time): 13.381  (>1 suggests clustering)

=== BAC: EWMA Parametric VaR ===
=== VaR Backtest Summary (H5) ===
Alpha (confidence):           0.9900
Observations (n):             2497
Violations (x):               49
Violation rate (x/n):         0.019624
Expected violations:          24.97
Violation rule used:          r<var
```

```
-- Kupiec POF (Unconditional Coverage) --
LR_uc:                       18.2406  | Reject@95%: True
-- Christoffersen Independence (Clustering) --
N00=2404 N01=43 N10=43 N11=6
LR_ind:                      12.9970  | Reject@95%: True
-- Conditional Coverage (POF + Independence) --
LR_cc:                       31.2376  | Reject@95%: True

-- Event-Window Check (interest-rate shock announcements) --
Days in ±1d windows:  24 / 2497 (0.96% of time)
Violations in windows:       4 / 49
Share of violations in windows: 8.16%
Enrichment ratio (viol/time): 8.493  (>1 suggests clustering)

=== C: Rolling Historical VaR ===
=== VaR Backtest Summary (H5) ===
Alpha (confidence):          0.9900
Observations (n):            2248
Violations (x):              35
Violation rate (x/n):        0.015569
Expected violations:         22.48
Violation rule used:         r<var

-- Kupiec POF (Unconditional Coverage) --
LR_uc:                       6.0211  | Reject@95%: True
-- Christoffersen Independence (Clustering) --
N00=2179 N01=33 N10=33 N11=2
LR_ind:                      2.4151  | Reject@95%: False
-- Conditional Coverage (POF + Independence) --
LR_cc:                       8.4363  | Reject@95%: True

-- Event-Window Check (interest-rate shock announcements) --
Days in ±1d windows:  24 / 2248 (1.07% of time)
Violations in windows:       3 / 35
Share of violations in windows: 8.57%
Enrichment ratio (viol/time): 8.029  (>1 suggests clustering)

=== C: EWMA Parametric VaR ===
=== VaR Backtest Summary (H5) ===
Alpha (confidence):          0.9900
Observations (n):            2497
Violations (x):              56
Violation rate (x/n):        0.022427
Expected violations:         24.97
Violation rule used:         r<var

-- Kupiec POF (Unconditional Coverage) --
```

```
LR_uc:                        28.7909  | Reject@95%: True
-- Christoffersen Independence (Clustering) --
N00=2390 N01=50 N10=50 N11=6
LR_ind:                       10.1312  | Reject@95%: True
-- Conditional Coverage (POF + Independence) --
LR_cc:                        38.9221  | Reject@95%: True

-- Event-Window Check (interest-rate shock announcements) --
Days in ±1d windows:  24 / 2497 (0.96% of time)
Violations in windows:        4 / 56
Share of violations in windows: 7.14%
Enrichment ratio (viol/time): 7.432  (>1 suggests clustering)

=== JPM: Rolling Historical VaR ===
=== VaR Backtest Summary (H5) ===
Alpha (confidence):           0.9900
Observations (n):             2248
Violations (x):               32
Violation rate (x/n):         0.014235
Expected violations:          22.48
Violation rule used:          r<var

-- Kupiec POF (Unconditional Coverage) --
LR_uc:                        3.5998  | Reject@95%: False
-- Christoffersen Independence (Clustering) --
N00=2187 N01=28 N10=28 N11=4
LR_ind:                       11.1227  | Reject@95%: True
-- Conditional Coverage (POF + Independence) --
LR_cc:                        14.7225  | Reject@95%: True

-- Event-Window Check (interest-rate shock announcements) --
Days in ±1d windows:  24 / 2248 (1.07% of time)
Violations in windows:        3 / 32
Share of violations in windows: 9.38%
Enrichment ratio (viol/time): 8.781  (>1 suggests clustering)

=== JPM: EWMA Parametric VaR ===
=== VaR Backtest Summary (H5) ===
Alpha (confidence):           0.9900
Observations (n):             2497
Violations (x):               55
Violation rate (x/n):         0.022026
Expected violations:          24.97
Violation rule used:          r<var

-- Kupiec POF (Unconditional Coverage) --
LR_uc:                        27.1687  | Reject@95%: True
-- Christoffersen Independence (Clustering) --
```

```
N00=2391 N01=50 N10=50 N11=5
LR_ind:                      7.1486  | Reject@95%: True
-- Conditional Coverage (POF + Independence) --
LR_cc:                       34.3173  | Reject@95%: True

-- Event-Window Check (interest-rate shock announcements) --
Days in ±1d windows:  24 / 2497 (0.96% of time)
Violations in windows:       3 / 55
Share of violations in windows: 5.45%
Enrichment ratio (viol/time): 5.675  (>1 suggests clustering)

=== WFC: Rolling Historical VaR ===
=== VaR Backtest Summary (H5) ===
Alpha (confidence):          0.9900
Observations (n):            2248
Violations (x):              31
Violation rate (x/n):        0.013790
Expected violations:         22.48
Violation rule used:         r<var

-- Kupiec POF (Unconditional Coverage) --
LR_uc:                       2.9171  | Reject@95%: False
-- Christoffersen Independence (Clustering) --
N00=2190 N01=26 N10=26 N11=5
LR_ind:                      16.8945  | Reject@95%: True
-- Conditional Coverage (POF + Independence) --
LR_cc:                       19.8116  | Reject@95%: True

-- Event-Window Check (interest-rate shock announcements) --
Days in ±1d windows:  24 / 2248 (1.07% of time)
Violations in windows:       5 / 31
Share of violations in windows: 16.13%
Enrichment ratio (viol/time): 15.108  (>1 suggests clustering)

=== WFC: EWMA Parametric VaR ===
=== VaR Backtest Summary (H5) ===
Alpha (confidence):          0.9900
Observations (n):            2497
Violations (x):              49
Violation rate (x/n):        0.019624
Expected violations:         24.97
Violation rule used:         r<var

-- Kupiec POF (Unconditional Coverage) --
LR_uc:                       18.2406  | Reject@95%: True
-- Christoffersen Independence (Clustering) --
N00=2402 N01=45 N10=45 N11=4
LR_ind:                      5.7211  | Reject@95%: True
```

```
-- Conditional Coverage (POF + Independence) --
LR_cc:                        23.9617  | Reject@95%: True

-- Event-Window Check (interest-rate shock announcements) --
Days in ±1d windows:  24 / 2497 (0.96% of time)
Violations in windows:        3 / 49
Share of violations in windows: 6.12%
Enrichment ratio (viol/time): 6.370  (>1 suggests clustering)

First 12 portfolio violation dates (Historical VaR):
 - 2016-01-07
 - 2016-02-11
 - 2016-06-24
 - 2017-03-21
 - 2017-05-17
 - 2018-02-05
 - 2018-02-08
 - 2018-03-22
 - 2018-12-04
 - 2019-08-14
 - 2020-02-24
 - 2020-02-25

First 12 portfolio violation dates (EWMA VaR):
 - 2015-03-10
 - 2015-06-29
 - 2015-07-08
 - 2015-08-20
 - 2015-08-21
 - 2015-08-24
 - 2016-01-07
 - 2016-06-24
 - 2016-09-26
 - 2017-01-17
 - 2017-03-21
 - 2017-05-17
```

[34]:
```python
def _garch_var_from_fit(fit_out, index, z_alpha):
    """Build VaR series from a fitted model output dict."""
    mu = fit_out['mu'] if 'mu' in fit_out else fit_out['params']['mu']
    sigma = fit_out['sigma']
    var = pd.Series(mu + z_alpha * sigma, index=index).dropna()
    return var

# ---------- 1) Portfolio: fit all three models on portfolio returns ----------
rP = port_ret.dropna()
idxP = rP.index
```

```
print("\n=== Portfolio: GARCH(1,1) VaR ===")
g11P = garch11_fit(rP.values)             # {'mu','sigma',...}
var_g11P = _garch_var_from_fit(g11P, idxP, z)
# align returns with VaR timestamps (drop first obs if any model warmup)
ret_g11P = rP.loc[var_g11P.index]
_ = summarize_backtest(
    returns=ret_g11P.to_numpy(),
    var_series=var_g11P.to_numpy(),
    alpha=alpha,
    dates=ret_g11P.index.to_numpy(),
    event_dates=event_dates,
    window_days=window_days,
    rule=None,
)

print("\n=== Portfolio: GJR-GARCH(1,1) VaR ===")
gjrP = fit_gjr_garch(rP.values)           # has 'params' and 'sigma'
var_gjrP = _garch_var_from_fit(gjrP, idxP, z)
ret_gjrP = rP.loc[var_gjrP.index]
_ = summarize_backtest(
    returns=ret_gjrP.to_numpy(),
    var_series=var_gjrP.to_numpy(),
    alpha=alpha,
    dates=ret_gjrP.index.to_numpy(),
    event_dates=event_dates,
    window_days=window_days,
    rule=None,
)

print("\n=== Portfolio: EGARCH(1,1) VaR ===")
egP = fit_egarch(rP.values)
var_egP = _garch_var_from_fit(egP, idxP, z)
ret_egP = rP.loc[var_egP.index]
_ = summarize_backtest(
    returns=ret_egP.to_numpy(),
    var_series=var_egP.to_numpy(),
    alpha=alpha,
    dates=ret_egP.index.to_numpy(),
    event_dates=event_dates,
    window_days=window_days,
    rule=None,
)

# ---------- 2) Per-asset: fit all three models on each asset ----------
for tkr in tickers_use:
    r = R[tkr].dropna()
```

```python
    idx = r.index

    # GARCH(1,1)
    print(f"\n=== {tkr}: GARCH(1,1) VaR ===")
    g11 = garch11_fit(r.values)
    var_g11 = _garch_var_from_fit(g11, idx, z)
    rr = r.loc[var_g11.index]
    _ = summarize_backtest(
        returns=rr.to_numpy(),
        var_series=var_g11.to_numpy(),
        alpha=alpha,
        dates=rr.index.to_numpy(),
        event_dates=event_dates,
        window_days=window_days,
        rule=None,
    )

    # GJR-GARCH(1,1)
    print(f"\n=== {tkr}: GJR-GARCH(1,1) VaR ===")
    gjr = fit_gjr_garch(r.values)
    var_gjr = _garch_var_from_fit(gjr, idx, z)
    rr = r.loc[var_gjr.index]
    _ = summarize_backtest(
        returns=rr.to_numpy(),
        var_series=var_gjr.to_numpy(),
        alpha=alpha,
        dates=rr.index.to_numpy(),
        event_dates=event_dates,
        window_days=window_days,
        rule=None,
    )

    # EGARCH(1,1)
    print(f"\n=== {tkr}: EGARCH(1,1) VaR ===")
    eg = fit_egarch(r.values)
    var_eg = _garch_var_from_fit(eg, idx, z)
    rr = r.loc[var_eg.index]
    _ = summarize_backtest(
        returns=rr.to_numpy(),
        var_series=var_eg.to_numpy(),
        alpha=alpha,
        dates=rr.index.to_numpy(),
        event_dates=event_dates,
        window_days=window_days,
        rule=None,
    )
```

```
# ---------- 3) (Optional) quick comparison of how many portfolio violations␣
 ↪per model ----------
def _count_viol(returns_s, var_s):
    flags, base = compute_violations(returns_s.to_numpy(), var_s.to_numpy(),␣
 ↪alpha=alpha, rule=None)
    return base['violations'], base['n']

viol_counts = {
    'Hist (250d)': _count_viol(ret_hist, var_hist),
    'EWMA ( =0.94)': _count_viol(ret_ewma, var_ewma),
    'GARCH(1,1)': _count_viol(ret_g11P, var_g11P),
    'GJR-GARCH(1,1)': _count_viol(ret_gjrP, var_gjrP),
    'EGARCH(1,1)': _count_viol(ret_egP, var_egP),
}
print("\n=== Portfolio violation counts (x / n) ===")
for k, (x, n) in viol_counts.items():
    print(f"{k:16s}: {x} / {n}  (expected @99%  {int((1-alpha)*n):.0f})")
```

```
=== Portfolio: GARCH(1,1) VaR ===
=== VaR Backtest Summary (H5) ===
Alpha (confidence):          0.9900
Observations (n):            2497
Violations (x):              11
Violation rate (x/n):        0.004405
Expected violations:         24.97
Violation rule used:         r<var

-- Kupiec POF (Unconditional Coverage) --
LR_uc:                       9.9836   | Reject@95%: True
-- Christoffersen Independence (Clustering) --
N00=2474 N01=11 N10=11 N11=0
LR_ind:                      0.0974   | Reject@95%: False
-- Conditional Coverage (POF + Independence) --
LR_cc:                       10.0810   | Reject@95%: True

-- Event-Window Check (interest-rate shock announcements) --
Days in ±1d windows:  24 / 2497 (0.96% of time)
Violations in windows:       4 / 11
Share of violations in windows: 36.36%
Enrichment ratio (viol/time): 37.833  (>1 suggests clustering)

=== Portfolio: GJR-GARCH(1,1) VaR ===
=== VaR Backtest Summary (H5) ===
Alpha (confidence):          0.9900
Observations (n):            2497
Violations (x):              21
```

```
Violation rate (x/n):         0.008410
Expected violations:          24.97
Violation rule used:          r<var

-- Kupiec POF (Unconditional Coverage) --
LR_uc:                        0.6740  | Reject@95%: False
-- Christoffersen Independence (Clustering) --
N00=2454 N01=21 N10=21 N11=0
LR_ind:                       0.3564  | Reject@95%: False
-- Conditional Coverage (POF + Independence) --
LR_cc:                        1.0303  | Reject@95%: False

-- Event-Window Check (interest-rate shock announcements) --
Days in ±1d windows:  24 / 2497 (0.96% of time)
Violations in windows:        4 / 21
Share of violations in windows: 19.05%
Enrichment ratio (viol/time): 19.817  (>1 suggests clustering)

=== Portfolio: EGARCH(1,1) VaR ===
=== VaR Backtest Summary (H5) ===
Alpha (confidence):           0.9900
Observations (n):             2497
Violations (x):               40
Violation rate (x/n):         0.016019
Expected violations:          24.97
Violation rule used:          r<var

-- Kupiec POF (Unconditional Coverage) --
LR_uc:                        7.7279  | Reject@95%: True
-- Christoffersen Independence (Clustering) --
N00=2418 N01=38 N10=38 N11=2
LR_ind:                       1.9291  | Reject@95%: False
-- Conditional Coverage (POF + Independence) --
LR_cc:                        9.6570  | Reject@95%: True

-- Event-Window Check (interest-rate shock announcements) --
Days in ±1d windows:  24 / 2497 (0.96% of time)
Violations in windows:        3 / 40
Share of violations in windows: 7.50%
Enrichment ratio (viol/time): 7.803  (>1 suggests clustering)

=== BAC: GARCH(1,1) VaR ===
=== VaR Backtest Summary (H5) ===
Alpha (confidence):           0.9900
Observations (n):             2497
Violations (x):               14
Violation rate (x/n):         0.005607
Expected violations:          24.97
```

```
Violation rule used:          r<var

-- Kupiec POF (Unconditional Coverage) --
LR_uc:                        5.7873  | Reject@95%: True
-- Christoffersen Independence (Clustering) --
N00=2468 N01=14 N10=14 N11=0
LR_ind:                       0.1579  | Reject@95%: False
-- Conditional Coverage (POF + Independence) --
LR_cc:                        5.9452  | Reject@95%: False

-- Event-Window Check (interest-rate shock announcements) --
Days in ±1d windows:  24 / 2497 (0.96% of time)
Violations in windows:        4 / 14
Share of violations in windows: 28.57%
Enrichment ratio (viol/time): 29.726  (>1 suggests clustering)

=== BAC: GJR-GARCH(1,1) VaR ===
=== VaR Backtest Summary (H5) ===
Alpha (confidence):           0.9900
Observations (n):             2497
Violations (x):               22
Violation rate (x/n):         0.008811
Expected violations:          24.97
Violation rule used:          r<var

-- Kupiec POF (Unconditional Coverage) --
LR_uc:                        0.3717  | Reject@95%: False
-- Christoffersen Independence (Clustering) --
N00=2452 N01=22 N10=22 N11=0
LR_ind:                       0.3913  | Reject@95%: False
-- Conditional Coverage (POF + Independence) --
LR_cc:                        0.7630  | Reject@95%: False

-- Event-Window Check (interest-rate shock announcements) --
Days in ±1d windows:  24 / 2497 (0.96% of time)
Violations in windows:        3 / 22
Share of violations in windows: 13.64%
Enrichment ratio (viol/time): 14.187  (>1 suggests clustering)

=== BAC: EGARCH(1,1) VaR ===
=== VaR Backtest Summary (H5) ===
Alpha (confidence):           0.9900
Observations (n):             2497
Violations (x):               36
Violation rate (x/n):         0.014417
Expected violations:          24.97
Violation rule used:          r<var
```

```
-- Kupiec POF (Unconditional Coverage) --
LR_uc:                        4.3300  | Reject@95%: True
-- Christoffersen Independence (Clustering) --
N00=2427 N01=33 N10=33 N11=3
LR_ind:                       5.9204  | Reject@95%: True
-- Conditional Coverage (POF + Independence) --
LR_cc:                        10.2504  | Reject@95%: True

-- Event-Window Check (interest-rate shock announcements) --
Days in ±1d windows:  24 / 2497 (0.96% of time)
Violations in windows:        3 / 36
Share of violations in windows: 8.33%
Enrichment ratio (viol/time): 8.670  (>1 suggests clustering)

=== C: GARCH(1,1) VaR ===
=== VaR Backtest Summary (H5) ===
Alpha (confidence):           0.9900
Observations (n):             2497
Violations (x):               15
Violation rate (x/n):         0.006007
Expected violations:          24.97
Violation rule used:          r<var

-- Kupiec POF (Unconditional Coverage) --
LR_uc:                        4.6914  | Reject@95%: True
-- Christoffersen Independence (Clustering) --
N00=2467 N01=14 N10=14 N11=1
LR_ind:                       3.1067  | Reject@95%: False
-- Conditional Coverage (POF + Independence) --
LR_cc:                        7.7981  | Reject@95%: True

-- Event-Window Check (interest-rate shock announcements) --
Days in ±1d windows:  24 / 2497 (0.96% of time)
Violations in windows:        3 / 15
Share of violations in windows: 20.00%
Enrichment ratio (viol/time): 20.808  (>1 suggests clustering)

=== C: GJR-GARCH(1,1) VaR ===
=== VaR Backtest Summary (H5) ===
Alpha (confidence):           0.9900
Observations (n):             2497
Violations (x):               22
Violation rate (x/n):         0.008811
Expected violations:          24.97
Violation rule used:          r<var

-- Kupiec POF (Unconditional Coverage) --
LR_uc:                        0.3717  | Reject@95%: False
```

```
-- Christoffersen Independence (Clustering) --
N00=2453 N01=21 N10=21 N11=1
LR_ind:                        1.7291  | Reject@95%: False
-- Conditional Coverage (POF + Independence) --
LR_cc:                         2.1009  | Reject@95%: False

-- Event-Window Check (interest-rate shock announcements) --
Days in ±1d windows:  24 / 2497 (0.96% of time)
Violations in windows:        3 / 22
Share of violations in windows: 13.64%
Enrichment ratio (viol/time): 14.187  (>1 suggests clustering)

=== C: EGARCH(1,1) VaR ===
```

```
C:\Users\amusi\AppData\Local\Programs\Python\Python313\Lib\site-
packages\scipy\optimize\_slsqp_py.py:435: RuntimeWarning: Values in x were
outside bounds during a minimize step, clipping to bounds
  fx = wrapped_fun(x)
```

```
=== VaR Backtest Summary (H5) ===
Alpha (confidence):            0.9900
Observations (n):              2497
Violations (x):                45
Violation rate (x/n):          0.018022
Expected violations:           24.97
Violation rule used:           r<var

-- Kupiec POF (Unconditional Coverage) --
LR_uc:                         13.1116  | Reject@95%: True
-- Christoffersen Independence (Clustering) --
N00=2408 N01=43 N10=43 N11=2
LR_ind:                        1.2968  | Reject@95%: False
-- Conditional Coverage (POF + Independence) --
LR_cc:                         14.4084  | Reject@95%: True

-- Event-Window Check (interest-rate shock announcements) --
Days in ±1d windows:  24 / 2497 (0.96% of time)
Violations in windows:        4 / 45
Share of violations in windows: 8.89%
Enrichment ratio (viol/time): 9.248  (>1 suggests clustering)

=== JPM: GARCH(1,1) VaR ===
=== VaR Backtest Summary (H5) ===
Alpha (confidence):            0.9900
Observations (n):              2497
Violations (x):                14
Violation rate (x/n):          0.005607
Expected violations:           24.97
Violation rule used:           r<var
```

```
-- Kupiec POF (Unconditional Coverage) --
LR_uc:                        5.7873  | Reject@95%: True
-- Christoffersen Independence (Clustering) --
N00=2468 N01=14 N10=14 N11=0
LR_ind:                       0.1579  | Reject@95%: False
-- Conditional Coverage (POF + Independence) --
LR_cc:                        5.9452  | Reject@95%: False

-- Event-Window Check (interest-rate shock announcements) --
Days in ±1d windows:  24 / 2497 (0.96% of time)
Violations in windows:        3 / 14
Share of violations in windows: 21.43%
Enrichment ratio (viol/time): 22.295  (>1 suggests clustering)

=== JPM: GJR-GARCH(1,1) VaR ===
=== VaR Backtest Summary (H5) ===
Alpha (confidence):           0.9900
Observations (n):             2497
Violations (x):               21
Violation rate (x/n):         0.008410
Expected violations:          24.97
Violation rule used:          r<var

-- Kupiec POF (Unconditional Coverage) --
LR_uc:                        0.6740  | Reject@95%: False
-- Christoffersen Independence (Clustering) --
N00=2454 N01=21 N10=21 N11=0
LR_ind:                       0.3564  | Reject@95%: False
-- Conditional Coverage (POF + Independence) --
LR_cc:                        1.0303  | Reject@95%: False

-- Event-Window Check (interest-rate shock announcements) --
Days in ±1d windows:  24 / 2497 (0.96% of time)
Violations in windows:        3 / 21
Share of violations in windows: 14.29%
Enrichment ratio (viol/time): 14.863  (>1 suggests clustering)

=== JPM: EGARCH(1,1) VaR ===
=== VaR Backtest Summary (H5) ===
Alpha (confidence):           0.9900
Observations (n):             2497
Violations (x):               38
Violation rate (x/n):         0.015218
Expected violations:          24.97
Violation rule used:          r<var

-- Kupiec POF (Unconditional Coverage) --
```

```
LR_uc:                        5.9220  | Reject@95%: True
-- Christoffersen Independence (Clustering) --
N00=2422 N01=36 N10=36 N11=2
LR_ind:                       2.2289  | Reject@95%: False
-- Conditional Coverage (POF + Independence) --
LR_cc:                        8.1510  | Reject@95%: True

-- Event-Window Check (interest-rate shock announcements) --
Days in ±1d windows:  24 / 2497 (0.96% of time)
Violations in windows:        3 / 38
Share of violations in windows: 7.89%
Enrichment ratio (viol/time): 8.214  (>1 suggests clustering)

=== WFC: GARCH(1,1) VaR ===
=== VaR Backtest Summary (H5) ===
Alpha (confidence):           0.9900
Observations (n):             2497
Violations (x):               18
Violation rate (x/n):         0.007209
Expected violations:          24.97
Violation rule used:          r<var

-- Kupiec POF (Unconditional Coverage) --
LR_uc:                        2.1767  | Reject@95%: False
-- Christoffersen Independence (Clustering) --
N00=2461 N01=17 N10=17 N11=1
LR_ind:                       2.4295  | Reject@95%: False
-- Conditional Coverage (POF + Independence) --
LR_cc:                        4.6062  | Reject@95%: False

-- Event-Window Check (interest-rate shock announcements) --
Days in ±1d windows:  24 / 2497 (0.96% of time)
Violations in windows:        3 / 18
Share of violations in windows: 16.67%
Enrichment ratio (viol/time): 17.340  (>1 suggests clustering)

=== WFC: GJR-GARCH(1,1) VaR ===
=== VaR Backtest Summary (H5) ===
Alpha (confidence):           0.9900
Observations (n):             2497
Violations (x):               24
Violation rate (x/n):         0.009612
Expected violations:          24.97
Violation rule used:          r<var

-- Kupiec POF (Unconditional Coverage) --
LR_uc:                        0.0386  | Reject@95%: False
-- Christoffersen Independence (Clustering) --
```

```
N00=2449 N01=23 N10=23 N11=1
LR_ind:                      1.4448  | Reject@95%: False
-- Conditional Coverage (POF + Independence) --
LR_cc:                       1.4833  | Reject@95%: False

-- Event-Window Check (interest-rate shock announcements) --
Days in ±1d windows:  24 / 2497 (0.96% of time)
Violations in windows:       3 / 24
Share of violations in windows: 12.50%
Enrichment ratio (viol/time): 13.005  (>1 suggests clustering)

=== WFC: EGARCH(1,1) VaR ===
=== VaR Backtest Summary (H5) ===
Alpha (confidence):          0.9900
Observations (n):            2497
Violations (x):              41
Violation rate (x/n):        0.016420
Expected violations:         24.97
Violation rule used:         r<var

-- Kupiec POF (Unconditional Coverage) --
LR_uc:                       8.7077  | Reject@95%: True
-- Christoffersen Independence (Clustering) --
N00=2418 N01=37 N10=37 N11=4
LR_ind:                      8.1688  | Reject@95%: True
-- Conditional Coverage (POF + Independence) --
LR_cc:                       16.8766  | Reject@95%: True

-- Event-Window Check (interest-rate shock announcements) --
Days in ±1d windows:  24 / 2497 (0.96% of time)
Violations in windows:       3 / 41
Share of violations in windows: 7.32%
Enrichment ratio (viol/time): 7.613  (>1 suggests clustering)

=== Portfolio violation counts (x / n) ===
Hist (250d)      : 33 / 2248  (expected @99%  22)
EWMA ( =0.94)   : 57 / 2497  (expected @99%  24)
GARCH(1,1)       : 11 / 2497  (expected @99%  24)
GJR-GARCH(1,1)  : 21 / 2497  (expected @99%  24)
EGARCH(1,1)      : 40 / 2497  (expected @99%  24)
```

[35]:
```
# ==========================================
# H5 VaR Backtest on YOUR data
# Uses: log_ret, stock_tickers, ewma_sigma, and your H5 helper functions
# ==========================================
import numpy as np
import pandas as pd
```

```python
from scipy import stats as _st

# ----- 0) Event dates (edit/extend as needed) -----
event_dates = [
    # 2008-2009 GFC (included for completeness even if out of sample)
    "2008-03-16",  # Bear Stearns sale to JPM
    "2008-09-15",  # Lehman bankruptcy
    "2008-09-16",  # AIG rescue
    "2008-10-03",  # TARP signed
    "2008-10-08",  # Coordinated emergency rate cuts

    # US downgrade
    "2011-08-05",  # S&P downgrades US

    # Taper Tantrum
    "2013-05-22",  # Bernanke taper remarks

    # Brexit
    "2016-06-23",

    # US Tax Cuts and Jobs Act
    "2017-12-22",

    # COVID shock
    "2020-03-03",  # Fed emergency 50bp cut
    "2020-03-15",  # Fed cut to 0-0.25% + QE restart
    "2020-03-23",  # 'Unlimited' QE / facilities

    # 2022 hike cycle (sample marquee FOMC hikes)
    "2022-06-15",
    "2022-09-21",
    "2022-11-02",

    # 2023 banking stress
    "2023-03-10",  # SVB closed
    "2023-03-12",  # Signature Bank closed
    "2023-03-19",  # Credit Suisse takeover announced
]

# ----- 1) Settings & inputs -----
alpha = 0.99
window_days = 1                    # event window ±1 day
weights = np.array([0.25, 0.25, 0.25, 0.25])   # equal-weight portfolio
tickers_use = stock_tickers        # ['BAC','C','JPM','WFC']

# Align/clean returns
R = log_ret[tickers_use].dropna().copy()
```

```
R.index = pd.to_datetime(R.index)

# ----- 2) Portfolio returns -----
port_ret = R.dot(weights).rename("PORT").dropna()

# ----- 3) VaR A: Rolling Historical (nonparametric) -----
lookback = 250   # ~1y
var_hist = port_ret.rolling(lookback).quantile(1 - alpha).dropna()   # negative␣
  ↪1% quantile of returns
ret_hist = port_ret.loc[var_hist.index]

print("=== Portfolio: Rolling Historical VaR (nonparametric) ===")
_ = summarize_backtest(
    returns=ret_hist.to_numpy(),
    var_series=var_hist.to_numpy(),
    alpha=alpha,
    dates=ret_hist.index.to_numpy(),
    event_dates=event_dates,
    window_days=window_days,
    rule=None,    # auto-detect VaR sign
)

# ----- 4) VaR B: EWMA Parametric (RiskMetrics-style mean ~ 0) -----
# Conditional sigma_t from portfolio returns
def _ewma_sigma(series, lam=0.94):
    # you already have ewma_sigma(eps, lam); using that directly for␣
  ↪consistency:
    return pd.Series(ewma_sigma(series.values - series.values.mean(), lam=lam),␣
  ↪index=series.index)

sigma_ewma_p = _ewma_sigma(port_ret, lam=0.94)
mu_roll = port_ret.rolling(60).mean().fillna(0.0)  # short-term mean; can also␣
  ↪use 0
z = _st.norm.ppf(1 - alpha)                        # ~ -2.326 for alpha=0.99
var_ewma = (mu_roll + z * sigma_ewma_p).dropna()
ret_ewma = port_ret.loc[var_ewma.index]

print("\n=== Portfolio: EWMA Parametric VaR ===")
_ = summarize_backtest(
    returns=ret_ewma.to_numpy(),
    var_series=var_ewma.to_numpy(),
    alpha=alpha,
    dates=ret_ewma.index.to_numpy(),
    event_dates=event_dates,
    window_days=window_days,
    rule=None,
)
```

```
# ----- 5) (Optional) Per-asset backtests with both VaRs -----
for tkr in tickers_use:
    r = R[tkr].dropna()

    # (A) Rolling historical VaR on the asset
    v_hist = r.rolling(lookback).quantile(1 - alpha).dropna()
    rr = r.loc[v_hist.index]
    print(f"\n=== {tkr}: Rolling Historical VaR ===")
    _ = summarize_backtest(
        returns=rr.to_numpy(),
        var_series=v_hist.to_numpy(),
        alpha=alpha,
        dates=rr.index.to_numpy(),
        event_dates=event_dates,
        window_days=window_days,
        rule=None,
    )

    # (B) EWMA parametric VaR on the asset
    sig = _ewma_sigma(r, lam=0.94)
    mu_r = r.rolling(60).mean().fillna(0.0)
    v_ew = (mu_r + z * sig).dropna()
    rr2 = r.loc[v_ew.index]
    print(f"\n=== {tkr}: EWMA Parametric VaR ===")
    _ = summarize_backtest(
        returns=rr2.to_numpy(),
        var_series=v_ew.to_numpy(),
        alpha=alpha,
        dates=rr2.index.to_numpy(),
        event_dates=event_dates,
        window_days=window_days,
        rule=None,
    )

# ----- 6) (Nice to have) List violation dates for the portfolio -----
viol_flags_hist, _base_hist = compute_violations(ret_hist.to_numpy(), var_hist.
 ↪to_numpy(), alpha=alpha, rule=None)
viol_dates_hist = ret_hist.index[np.where(viol_flags_hist)[0]].date.tolist()

viol_flags_ewma, _base_ewma = compute_violations(ret_ewma.to_numpy(), var_ewma.
 ↪to_numpy(), alpha=alpha, rule=None)
viol_dates_ewma = ret_ewma.index[np.where(viol_flags_ewma)[0]].date.tolist()

print("\nFirst 12 portfolio violation dates (Historical VaR):")
for d in viol_dates_hist[:12]:
    print(" -", d)
print("\nFirst 12 portfolio violation dates (EWMA VaR):")
for d in viol_dates_ewma[:12]:
    print(" -", d)
```

```
=== Portfolio: Rolling Historical VaR (nonparametric) ===
=== VaR Backtest Summary (H5) ===
Alpha (confidence):          0.9900
Observations (n):            2248
Violations (x):              33
Violation rate (x/n):        0.014680
Expected violations:         22.48
Violation rule used:         r<var

-- Kupiec POF (Unconditional Coverage) --
LR_uc:                       4.3460   | Reject@95%: True
-- Christoffersen Independence (Clustering) --
N00=2184 N01=30 N10=30 N11=3
LR_ind:                      6.3095   | Reject@95%: True
-- Conditional Coverage (POF + Independence) --
LR_cc:                       10.6555   | Reject@95%: True

-- Event-Window Check (interest-rate shock announcements) --
Days in ±1d windows:  24 / 2248 (1.07% of time)
Violations in windows:       5 / 33
Share of violations in windows: 15.15%
Enrichment ratio (viol/time): 14.192  (>1 suggests clustering)

=== Portfolio: EWMA Parametric VaR ===
=== VaR Backtest Summary (H5) ===
Alpha (confidence):          0.9900
Observations (n):            2497
Violations (x):              57
Violation rate (x/n):        0.022827
Expected violations:         24.97
Violation rule used:         r<var

-- Kupiec POF (Unconditional Coverage) --
LR_uc:                       30.4497   | Reject@95%: True
-- Christoffersen Independence (Clustering) --
N00=2387 N01=52 N10=52 N11=5
LR_ind:                      6.5693   | Reject@95%: True
-- Conditional Coverage (POF + Independence) --
LR_cc:                       37.0190   | Reject@95%: True

-- Event-Window Check (interest-rate shock announcements) --
Days in ±1d windows:  24 / 2497 (0.96% of time)
Violations in windows:       4 / 57
```

```
Share of violations in windows: 7.02%
Enrichment ratio (viol/time): 7.301  (>1 suggests clustering)

=== BAC: Rolling Historical VaR ===
=== VaR Backtest Summary (H5) ===
Alpha (confidence):          0.9900
Observations (n):            2248
Violations (x):              35
Violation rate (x/n):        0.015569
Expected violations:         22.48
Violation rule used:         r<var

-- Kupiec POF (Unconditional Coverage) --
LR_uc:                       6.0211  | Reject@95%: True
-- Christoffersen Independence (Clustering) --
N00=2180 N01=32 N10=32 N11=3
LR_ind:                      5.6831  | Reject@95%: True
-- Conditional Coverage (POF + Independence) --
LR_cc:                       11.7042  | Reject@95%: True

-- Event-Window Check (interest-rate shock announcements) --
Days in ±1d windows:  24 / 2248 (1.07% of time)
Violations in windows:       5 / 35
Share of violations in windows: 14.29%
Enrichment ratio (viol/time): 13.381  (>1 suggests clustering)

=== BAC: EWMA Parametric VaR ===
=== VaR Backtest Summary (H5) ===
Alpha (confidence):          0.9900
Observations (n):            2497
Violations (x):              49
Violation rate (x/n):        0.019624
Expected violations:         24.97
Violation rule used:         r<var

-- Kupiec POF (Unconditional Coverage) --
LR_uc:                       18.2406  | Reject@95%: True
-- Christoffersen Independence (Clustering) --
N00=2404 N01=43 N10=43 N11=6
LR_ind:                      12.9970  | Reject@95%: True
-- Conditional Coverage (POF + Independence) --
LR_cc:                       31.2376  | Reject@95%: True

-- Event-Window Check (interest-rate shock announcements) --
Days in ±1d windows:  24 / 2497 (0.96% of time)
Violations in windows:       4 / 49
Share of violations in windows: 8.16%
Enrichment ratio (viol/time): 8.493  (>1 suggests clustering)
```

```
=== C: Rolling Historical VaR ===
=== VaR Backtest Summary (H5) ===
Alpha (confidence):          0.9900
Observations (n):            2248
Violations (x):              35
Violation rate (x/n):        0.015569
Expected violations:         22.48
Violation rule used:         r<var

-- Kupiec POF (Unconditional Coverage) --
LR_uc:                       6.0211  | Reject@95%: True
-- Christoffersen Independence (Clustering) --
N00=2179 N01=33 N10=33 N11=2
LR_ind:                      2.4151  | Reject@95%: False
-- Conditional Coverage (POF + Independence) --
LR_cc:                       8.4363  | Reject@95%: True

-- Event-Window Check (interest-rate shock announcements) --
Days in ±1d windows:  24 / 2248 (1.07% of time)
Violations in windows:       3 / 35
Share of violations in windows: 8.57%
Enrichment ratio (viol/time): 8.029  (>1 suggests clustering)

=== C: EWMA Parametric VaR ===
=== VaR Backtest Summary (H5) ===
Alpha (confidence):          0.9900
Observations (n):            2497
Violations (x):              56
Violation rate (x/n):        0.022427
Expected violations:         24.97
Violation rule used:         r<var

-- Kupiec POF (Unconditional Coverage) --
LR_uc:                       28.7909  | Reject@95%: True
-- Christoffersen Independence (Clustering) --
N00=2390 N01=50 N10=50 N11=6
LR_ind:                      10.1312  | Reject@95%: True
-- Conditional Coverage (POF + Independence) --
LR_cc:                       38.9221  | Reject@95%: True

-- Event-Window Check (interest-rate shock announcements) --
Days in ±1d windows:  24 / 2497 (0.96% of time)
Violations in windows:       4 / 56
Share of violations in windows: 7.14%
Enrichment ratio (viol/time): 7.432  (>1 suggests clustering)

=== JPM: Rolling Historical VaR ===
=== VaR Backtest Summary (H5) ===
Alpha (confidence):          0.9900
Observations (n):            2248
Violations (x):              32
Violation rate (x/n):        0.014235
Expected violations:         22.48
Violation rule used:         r<var

-- Kupiec POF (Unconditional Coverage) --
LR_uc:                       3.5998  | Reject@95%: False
-- Christoffersen Independence (Clustering) --
N00=2187 N01=28 N10=28 N11=4
LR_ind:                      11.1227  | Reject@95%: True
-- Conditional Coverage (POF + Independence) --
LR_cc:                       14.7225  | Reject@95%: True

-- Event-Window Check (interest-rate shock announcements) --
Days in ±1d windows:  24 / 2248 (1.07% of time)
Violations in windows:       3 / 32
Share of violations in windows: 9.38%
Enrichment ratio (viol/time): 8.781  (>1 suggests clustering)

=== JPM: EWMA Parametric VaR ===
=== VaR Backtest Summary (H5) ===
Alpha (confidence):          0.9900
Observations (n):            2497
Violations (x):              55
Violation rate (x/n):        0.022026
Expected violations:         24.97
Violation rule used:         r<var

-- Kupiec POF (Unconditional Coverage) --
LR_uc:                       27.1687  | Reject@95%: True
-- Christoffersen Independence (Clustering) --
N00=2391 N01=50 N10=50 N11=5
LR_ind:                      7.1486  | Reject@95%: True
-- Conditional Coverage (POF + Independence) --
LR_cc:                       34.3173  | Reject@95%: True

-- Event-Window Check (interest-rate shock announcements) --
Days in ±1d windows:  24 / 2497 (0.96% of time)
Violations in windows:       3 / 55
Share of violations in windows: 5.45%
Enrichment ratio (viol/time): 5.675  (>1 suggests clustering)

=== WFC: Rolling Historical VaR ===
=== VaR Backtest Summary (H5) ===
Alpha (confidence):          0.9900
```

```
Observations (n):             2248
Violations (x):               31
Violation rate (x/n):         0.013790
Expected violations:          22.48
Violation rule used:          r<var

-- Kupiec POF (Unconditional Coverage) --
LR_uc:                        2.9171  | Reject@95%: False
-- Christoffersen Independence (Clustering) --
N00=2190 N01=26 N10=26 N11=5
LR_ind:                       16.8945  | Reject@95%: True
-- Conditional Coverage (POF + Independence) --
LR_cc:                        19.8116  | Reject@95%: True

-- Event-Window Check (interest-rate shock announcements) --
Days in ±1d windows:  24 / 2248 (1.07% of time)
Violations in windows:        5 / 31
Share of violations in windows: 16.13%
Enrichment ratio (viol/time): 15.108  (>1 suggests clustering)

=== WFC: EWMA Parametric VaR ===
=== VaR Backtest Summary (H5) ===
Alpha (confidence):           0.9900
Observations (n):             2497
Violations (x):               49
Violation rate (x/n):         0.019624
Expected violations:          24.97
Violation rule used:          r<var

-- Kupiec POF (Unconditional Coverage) --
LR_uc:                        18.2406  | Reject@95%: True
-- Christoffersen Independence (Clustering) --
N00=2402 N01=45 N10=45 N11=4
LR_ind:                       5.7211  | Reject@95%: True
-- Conditional Coverage (POF + Independence) --
LR_cc:                        23.9617  | Reject@95%: True

-- Event-Window Check (interest-rate shock announcements) --
Days in ±1d windows:  24 / 2497 (0.96% of time)
Violations in windows:        3 / 49
Share of violations in windows: 6.12%
Enrichment ratio (viol/time): 6.370  (>1 suggests clustering)

First 12 portfolio violation dates (Historical VaR):
 - 2016-01-07
 - 2016-02-11
 - 2016-06-24
 - 2017-03-21
 - 2017-05-17
 - 2018-02-05
 - 2018-02-08
 - 2018-03-22
 - 2018-12-04
 - 2019-08-14
 - 2020-02-24
 - 2020-02-25

First 12 portfolio violation dates (EWMA VaR):
 - 2015-03-10
 - 2015-06-29
 - 2015-07-08
 - 2015-08-20
 - 2015-08-21
 - 2015-08-24
 - 2016-01-07
 - 2016-06-24
 - 2016-09-26
 - 2017-01-17
 - 2017-03-21
 - 2017-05-17
```

[36]:
```
def _garch_var_from_fit(fit_out, index, z_alpha):
    """Build VaR series from a fitted model output dict."""
    mu = fit_out['mu'] if 'mu' in fit_out else fit_out['params']['mu']
    sigma = fit_out['sigma']
    var = pd.Series(mu + z_alpha * sigma, index=index).dropna()
    return var

# ---------- 1) Portfolio: fit all three models on portfolio returns ----------
rP = port_ret.dropna()
idxP = rP.index

print("\n=== Portfolio: GARCH(1,1) VaR ===")
g11P = garch11_fit(rP.values)             # {'mu','sigma',...}
var_g11P = _garch_var_from_fit(g11P, idxP, z)
# align returns with VaR timestamps (drop first obs if any model warmup)
ret_g11P = rP.loc[var_g11P.index]
_ = summarize_backtest(
    returns=ret_g11P.to_numpy(),
    var_series=var_g11P.to_numpy(),
    alpha=alpha,
    dates=ret_g11P.index.to_numpy(),
    event_dates=event_dates,
    window_days=window_days,
    rule=None,
)

print("\n=== Portfolio: GJR-GARCH(1,1) VaR ===")
gjrP = fit_gjr_garch(rP.values)          # has 'params' and 'sigma'
var_gjrP = _garch_var_from_fit(gjrP, idxP, z)
ret_gjrP = rP.loc[var_gjrP.index]
_ = summarize_backtest(
    returns=ret_gjrP.to_numpy(),
    var_series=var_gjrP.to_numpy(),
    alpha=alpha,
    dates=ret_gjrP.index.to_numpy(),
    event_dates=event_dates,
    window_days=window_days,
    rule=None,
)

print("\n=== Portfolio: EGARCH(1,1) VaR ===")
egP = fit_egarch(rP.values)
var_egP = _garch_var_from_fit(egP, idxP, z)
ret_egP = rP.loc[var_egP.index]
_ = summarize_backtest(
    returns=ret_egP.to_numpy(),
    var_series=var_egP.to_numpy(),
    alpha=alpha,
    dates=ret_egP.index.to_numpy(),
    event_dates=event_dates,
    window_days=window_days,
    rule=None,
)

# ---------- 2) Per-asset: fit all three models on each asset ----------
for tkr in tickers_use:
    r = R[tkr].dropna()
    idx = r.index

    # GARCH(1,1)
    print(f"\n=== {tkr}: GARCH(1,1) VaR ===")
    g11 = garch11_fit(r.values)
    var_g11 = _garch_var_from_fit(g11, idx, z)
    rr = r.loc[var_g11.index]
    _ = summarize_backtest(
        returns=rr.to_numpy(),
        var_series=var_g11.to_numpy(),
        alpha=alpha,
        dates=rr.index.to_numpy(),
        event_dates=event_dates,
        window_days=window_days,
        rule=None,
    )

    # GJR-GARCH(1,1)
    print(f"\n=== {tkr}: GJR-GARCH(1,1) VaR ===")
    gjr = fit_gjr_garch(r.values)
    var_gjr = _garch_var_from_fit(gjr, idx, z)
    rr = r.loc[var_gjr.index]
    _ = summarize_backtest(
        returns=rr.to_numpy(),
        var_series=var_gjr.to_numpy(),
        alpha=alpha,
        dates=rr.index.to_numpy(),
        event_dates=event_dates,
        window_days=window_days,
        rule=None,
    )

    # EGARCH(1,1)
    print(f"\n=== {tkr}: EGARCH(1,1) VaR ===")
    eg = fit_egarch(r.values)
    var_eg = _garch_var_from_fit(eg, idx, z)
    rr = r.loc[var_eg.index]
    _ = summarize_backtest(
        returns=rr.to_numpy(),
        var_series=var_eg.to_numpy(),
        alpha=alpha,
        dates=rr.index.to_numpy(),
        event_dates=event_dates,
        window_days=window_days,
        rule=None,
    )

# ---------- 3) (Optional) quick comparison of how many portfolio violations␣
 ↪per model ----------
def _count_viol(returns_s, var_s):
    flags, base = compute_violations(returns_s.to_numpy(), var_s.to_numpy(),␣
 ↪alpha=alpha, rule=None)
    return base['violations'], base['n']

viol_counts = {
    'Hist (250d)': _count_viol(ret_hist, var_hist),
    'EWMA ( =0.94)': _count_viol(ret_ewma, var_ewma),
    'GARCH(1,1)': _count_viol(ret_g11P, var_g11P),
    'GJR-GARCH(1,1)': _count_viol(ret_gjrP, var_gjrP),
    'EGARCH(1,1)': _count_viol(ret_egP, var_egP),
}
```

```
print("\n=== Portfolio violation counts (x / n) ===")
for k, (x, n) in viol_counts.items():
    print(f"{k:16s}: {x} / {n}  (expected @99%  {int((1-alpha)*n):.0f})")
```

```
=== Portfolio: GARCH(1,1) VaR ===
=== VaR Backtest Summary (H5) ===
Alpha (confidence):          0.9900
Observations (n):            2497
Violations (x):              11
Violation rate (x/n):        0.004405
Expected violations:         24.97
Violation rule used:         r<var

-- Kupiec POF (Unconditional Coverage) --
LR_uc:                       9.9836  | Reject@95%: True
-- Christoffersen Independence (Clustering) --
N00=2474 N01=11 N10=11 N11=0
LR_ind:                      0.0974  | Reject@95%: False
-- Conditional Coverage (POF + Independence) --
LR_cc:                       10.0810  | Reject@95%: True

-- Event-Window Check (interest-rate shock announcements) --
Days in ±1d windows:  24 / 2497 (0.96% of time)
Violations in windows:       4 / 11
Share of violations in windows: 36.36%
Enrichment ratio (viol/time): 37.833  (>1 suggests clustering)

=== Portfolio: GJR-GARCH(1,1) VaR ===
=== VaR Backtest Summary (H5) ===
Alpha (confidence):          0.9900
Observations (n):            2497
Violations (x):              21
Violation rate (x/n):        0.008410
Expected violations:         24.97
Violation rule used:         r<var

-- Kupiec POF (Unconditional Coverage) --
LR_uc:                       0.6740  | Reject@95%: False
-- Christoffersen Independence (Clustering) --
N00=2454 N01=21 N10=21 N11=0
LR_ind:                      0.3564  | Reject@95%: False
-- Conditional Coverage (POF + Independence) --
LR_cc:                       1.0303  | Reject@95%: False

-- Event-Window Check (interest-rate shock announcements) --
Days in ±1d windows:  24 / 2497 (0.96% of time)
Violations in windows:       4 / 21
```

```
Share of violations in windows: 19.05%
Enrichment ratio (viol/time): 19.817  (>1 suggests clustering)

=== Portfolio: EGARCH(1,1) VaR ===
=== VaR Backtest Summary (H5) ===
Alpha (confidence):           0.9900
Observations (n):             2497
Violations (x):               40
Violation rate (x/n):         0.016019
Expected violations:          24.97
Violation rule used:          r<var

-- Kupiec POF (Unconditional Coverage) --
LR_uc:                        7.7279  | Reject@95%: True
-- Christoffersen Independence (Clustering) --
N00=2418 N01=38 N10=38 N11=2
LR_ind:                       1.9291  | Reject@95%: False
-- Conditional Coverage (POF + Independence) --
LR_cc:                        9.6570  | Reject@95%: True

-- Event-Window Check (interest-rate shock announcements) --
Days in ±1d windows:  24 / 2497 (0.96% of time)
Violations in windows:        3 / 40
Share of violations in windows: 7.50%
Enrichment ratio (viol/time): 7.803  (>1 suggests clustering)

=== BAC: GARCH(1,1) VaR ===
=== VaR Backtest Summary (H5) ===
Alpha (confidence):           0.9900
Observations (n):             2497
Violations (x):               14
Violation rate (x/n):         0.005607
Expected violations:          24.97
Violation rule used:          r<var

-- Kupiec POF (Unconditional Coverage) --
LR_uc:                        5.7873  | Reject@95%: True
-- Christoffersen Independence (Clustering) --
N00=2468 N01=14 N10=14 N11=0
LR_ind:                       0.1579  | Reject@95%: False
-- Conditional Coverage (POF + Independence) --
LR_cc:                        5.9452  | Reject@95%: False

-- Event-Window Check (interest-rate shock announcements) --
Days in ±1d windows:  24 / 2497 (0.96% of time)
Violations in windows:        4 / 14
Share of violations in windows: 28.57%
Enrichment ratio (viol/time): 29.726  (>1 suggests clustering)
```

```
=== BAC: GJR-GARCH(1,1) VaR ===
=== VaR Backtest Summary (H5) ===
Alpha (confidence):          0.9900
Observations (n):            2497
Violations (x):              22
Violation rate (x/n):        0.008811
Expected violations:         24.97
Violation rule used:         r<var

-- Kupiec POF (Unconditional Coverage) --
LR_uc:                       0.3717  | Reject@95%: False
-- Christoffersen Independence (Clustering) --
N00=2452 N01=22 N10=22 N11=0
LR_ind:                      0.3913  | Reject@95%: False
-- Conditional Coverage (POF + Independence) --
LR_cc:                       0.7630  | Reject@95%: False

-- Event-Window Check (interest-rate shock announcements) --
Days in ±1d windows:  24 / 2497 (0.96% of time)
Violations in windows:       3 / 22
Share of violations in windows: 13.64%
Enrichment ratio (viol/time): 14.187  (>1 suggests clustering)

=== BAC: EGARCH(1,1) VaR ===
=== VaR Backtest Summary (H5) ===
Alpha (confidence):          0.9900
Observations (n):            2497
Violations (x):              36
Violation rate (x/n):        0.014417
Expected violations:         24.97
Violation rule used:         r<var

-- Kupiec POF (Unconditional Coverage) --
LR_uc:                       4.3300  | Reject@95%: True
-- Christoffersen Independence (Clustering) --
N00=2427 N01=33 N10=33 N11=3
LR_ind:                      5.9204  | Reject@95%: True
-- Conditional Coverage (POF + Independence) --
LR_cc:                       10.2504  | Reject@95%: True

-- Event-Window Check (interest-rate shock announcements) --
Days in ±1d windows:  24 / 2497 (0.96% of time)
Violations in windows:       3 / 36
Share of violations in windows: 8.33%
Enrichment ratio (viol/time): 8.670  (>1 suggests clustering)

=== C: GARCH(1,1) VaR ===
=== VaR Backtest Summary (H5) ===
Alpha (confidence):          0.9900
Observations (n):            2497
Violations (x):              15
Violation rate (x/n):        0.006007
Expected violations:         24.97
Violation rule used:         r<var

-- Kupiec POF (Unconditional Coverage) --
LR_uc:                       4.6914  | Reject@95%: True
-- Christoffersen Independence (Clustering) --
N00=2467 N01=14 N10=14 N11=1
LR_ind:                      3.1067  | Reject@95%: False
-- Conditional Coverage (POF + Independence) --
LR_cc:                       7.7981  | Reject@95%: True

-- Event-Window Check (interest-rate shock announcements) --
Days in ±1d windows:  24 / 2497 (0.96% of time)
Violations in windows:       3 / 15
Share of violations in windows: 20.00%
Enrichment ratio (viol/time): 20.808  (>1 suggests clustering)

=== C: GJR-GARCH(1,1) VaR ===
=== VaR Backtest Summary (H5) ===
Alpha (confidence):          0.9900
Observations (n):            2497
Violations (x):              22
Violation rate (x/n):        0.008811
Expected violations:         24.97
Violation rule used:         r<var

-- Kupiec POF (Unconditional Coverage) --
LR_uc:                       0.3717  | Reject@95%: False
-- Christoffersen Independence (Clustering) --
N00=2453 N01=21 N10=21 N11=1
LR_ind:                      1.7291  | Reject@95%: False
-- Conditional Coverage (POF + Independence) --
LR_cc:                       2.1009  | Reject@95%: False

-- Event-Window Check (interest-rate shock announcements) --
Days in ±1d windows:  24 / 2497 (0.96% of time)
Violations in windows:       3 / 22
Share of violations in windows: 13.64%
Enrichment ratio (viol/time): 14.187  (>1 suggests clustering)

=== C: EGARCH(1,1) VaR ===

C:\Users\amusi\AppData\Local\Programs\Python\Python313\Lib\site-
```

```
packages\scipy\optimize\_slsqp_py.py:435: RuntimeWarning: Values in x were
outside bounds during a minimize step, clipping to bounds
  fx = wrapped_fun(x)

=== VaR Backtest Summary (H5) ===
Alpha (confidence):          0.9900
Observations (n):            2497
Violations (x):              45
Violation rate (x/n):        0.018022
Expected violations:         24.97
Violation rule used:         r<var

-- Kupiec POF (Unconditional Coverage) --
LR_uc:                       13.1116  | Reject@95%: True
-- Christoffersen Independence (Clustering) --
N00=2408 N01=43 N10=43 N11=2
LR_ind:                      1.2968  | Reject@95%: False
-- Conditional Coverage (POF + Independence) --
LR_cc:                       14.4084  | Reject@95%: True

-- Event-Window Check (interest-rate shock announcements) --
Days in ±1d windows:  24 / 2497 (0.96% of time)
Violations in windows:       4 / 45
Share of violations in windows: 8.89%
Enrichment ratio (viol/time): 9.248  (>1 suggests clustering)

=== JPM: GARCH(1,1) VaR ===
=== VaR Backtest Summary (H5) ===
Alpha (confidence):          0.9900
Observations (n):            2497
Violations (x):              14
Violation rate (x/n):        0.005607
Expected violations:         24.97
Violation rule used:         r<var

-- Kupiec POF (Unconditional Coverage) --
LR_uc:                       5.7873  | Reject@95%: True
-- Christoffersen Independence (Clustering) --
N00=2468 N01=14 N10=14 N11=0
LR_ind:                      0.1579  | Reject@95%: False
-- Conditional Coverage (POF + Independence) --
LR_cc:                       5.9452  | Reject@95%: False

-- Event-Window Check (interest-rate shock announcements) --
Days in ±1d windows:  24 / 2497 (0.96% of time)
Violations in windows:       3 / 14
Share of violations in windows: 21.43%
Enrichment ratio (viol/time): 22.295  (>1 suggests clustering)
```

```
=== JPM: GJR-GARCH(1,1) VaR ===
=== VaR Backtest Summary (H5) ===
Alpha (confidence):          0.9900
Observations (n):            2497
Violations (x):              21
Violation rate (x/n):        0.008410
Expected violations:         24.97
Violation rule used:         r<var

-- Kupiec POF (Unconditional Coverage) --
LR_uc:                       0.6740  | Reject@95%: False
-- Christoffersen Independence (Clustering) --
N00=2454 N01=21 N10=21 N11=0
LR_ind:                      0.3564  | Reject@95%: False
-- Conditional Coverage (POF + Independence) --
LR_cc:                       1.0303  | Reject@95%: False

-- Event-Window Check (interest-rate shock announcements) --
Days in ±1d windows:  24 / 2497 (0.96% of time)
Violations in windows:       3 / 21
Share of violations in windows: 14.29%
Enrichment ratio (viol/time): 14.863  (>1 suggests clustering)

=== JPM: EGARCH(1,1) VaR ===
=== VaR Backtest Summary (H5) ===
Alpha (confidence):          0.9900
Observations (n):            2497
Violations (x):              38
Violation rate (x/n):        0.015218
Expected violations:         24.97
Violation rule used:         r<var

-- Kupiec POF (Unconditional Coverage) --
LR_uc:                       5.9220  | Reject@95%: True
-- Christoffersen Independence (Clustering) --
N00=2422 N01=36 N10=36 N11=2
LR_ind:                      2.2289  | Reject@95%: False
-- Conditional Coverage (POF + Independence) --
LR_cc:                       8.1510  | Reject@95%: True

-- Event-Window Check (interest-rate shock announcements) --
Days in ±1d windows:  24 / 2497 (0.96% of time)
Violations in windows:       3 / 38
Share of violations in windows: 7.89%
Enrichment ratio (viol/time): 8.214  (>1 suggests clustering)

=== WFC: GARCH(1,1) VaR ===
=== VaR Backtest Summary (H5) ===
Alpha (confidence):          0.9900
Observations (n):            2497
Violations (x):              18
Violation rate (x/n):        0.007209
Expected violations:         24.97
Violation rule used:         r<var

-- Kupiec POF (Unconditional Coverage) --
LR_uc:                       2.1767  | Reject@95%: False
-- Christoffersen Independence (Clustering) --
N00=2461 N01=17 N10=17 N11=1
LR_ind:                      2.4295  | Reject@95%: False
-- Conditional Coverage (POF + Independence) --
LR_cc:                       4.6062  | Reject@95%: False

-- Event-Window Check (interest-rate shock announcements) --
Days in ±1d windows:  24 / 2497 (0.96% of time)
Violations in windows:       3 / 18
Share of violations in windows: 16.67%
Enrichment ratio (viol/time): 17.340  (>1 suggests clustering)

=== WFC: GJR-GARCH(1,1) VaR ===
=== VaR Backtest Summary (H5) ===
Alpha (confidence):          0.9900
Observations (n):            2497
Violations (x):              24
Violation rate (x/n):        0.009612
Expected violations:         24.97
Violation rule used:         r<var

-- Kupiec POF (Unconditional Coverage) --
LR_uc:                       0.0386  | Reject@95%: False
-- Christoffersen Independence (Clustering) --
N00=2449 N01=23 N10=23 N11=1
LR_ind:                      1.4448  | Reject@95%: False
-- Conditional Coverage (POF + Independence) --
LR_cc:                       1.4833  | Reject@95%: False

-- Event-Window Check (interest-rate shock announcements) --
Days in ±1d windows:  24 / 2497 (0.96% of time)
Violations in windows:       3 / 24
Share of violations in windows: 12.50%
Enrichment ratio (viol/time): 13.005  (>1 suggests clustering)

=== WFC: EGARCH(1,1) VaR ===
=== VaR Backtest Summary (H5) ===
Alpha (confidence):          0.9900
```

```
Observations (n):             2497
Violations (x):               41
Violation rate (x/n):         0.016420
Expected violations:          24.97
Violation rule used:          r<var

-- Kupiec POF (Unconditional Coverage) --
LR_uc:                        8.7077  | Reject@95%: True
-- Christoffersen Independence (Clustering) --
N00=2418 N01=37 N10=37 N11=4
LR_ind:                       8.1688  | Reject@95%: True
-- Conditional Coverage (POF + Independence) --
LR_cc:                        16.8766  | Reject@95%: True

-- Event-Window Check (interest-rate shock announcements) --
Days in ±1d windows:  24 / 2497 (0.96% of time)
Violations in windows:        3 / 41
Share of violations in windows: 7.32%
Enrichment ratio (viol/time): 7.613  (>1 suggests clustering)

=== Portfolio violation counts (x / n) ===
Hist (250d)     : 33 / 2248  (expected @99%   22)
EWMA ( =0.94)   : 57 / 2497  (expected @99%   24)
GARCH(1,1)      : 11 / 2497  (expected @99%   24)
GJR-GARCH(1,1)  : 21 / 2497  (expected @99%   24)
EGARCH(1,1)     : 40 / 2497  (expected @99%   24)
```

[50]:
```python
# ==============================================================
# Backtesting → DataFrames for 4 buckets + atomic CSV exports
# ==============================================================

import os, math, numpy as np, pandas as pd, shutil, tempfile
from scipy.stats import norm

# ---------- OUTPUT FOLDER & ATOMIC SAVE ----------
OUT_DIR = r"D:\Finance\risk\exports_bi"
os.makedirs(OUT_DIR, exist_ok=True)

def atomic_save_csv(df: pd.DataFrame, path: str):
    fd, tmp = tempfile.mkstemp(dir=os.path.dirname(path), suffix=".tmp")
    os.close(fd)
    df.to_csv(tmp, index=False)
    if os.path.exists(path):
        os.remove(path)
    shutil.move(tmp, path)

# ---------- REQUIRED PREREQS CHECKS ----------
# Needs: log_ret (DataFrame of daily log returns by ticker), and the Part-7␣
 ↪test functions:
req_funcs =␣
 ↪['compute_violations','kupiec_pof','christoffersen_independence','conditional_coverage','vi
missing = [f for f in req_funcs if f not in globals()]
if missing:
    raise RuntimeError(f"Missing helper(s) from Part 7: {missing}. Run those␣
 ↪cells first.")

# Optionally will use your GARCH functions from Part 4 (if present):
has_garch11 = 'garch11_fit' in globals()
has_gjr     = 'fit_gjr_garch' in globals()
has_egarch  = 'fit_egarch' in globals()

# ---------- CONFIG ----------
ALPHAS          = [0.95, 0.99]      # VaR confidence levels
ROLL_VAR_WINDOW = 250               # rolling window for Hist/Param VaR␣
 ↪forecasts
ANN_SQRT        = math.sqrt(252)

# Define event calendars (EDIT THESE LISTS)
# 1) Interest-rate shock scope (FOMC or key rate decisions)
interest_rate_events = [
    "2020-03-16","2022-06-15","2023-03-22","2023-11-01"  # sample; replace with␣
 ↪your list
]
# 2) Big events scope (crisis days, bank-specific events, geopolitical shocks,␣
 ↪etc.)
big_events = [
    "2020-03-09","2020-03-12","2020-03-16","2023-03-10","2023-03-13"  # sample;␣
 ↪replace with your list
]

# ---------- PORTFOLIO (equal-weight unless you already defined one) ----------
tickers = list(log_ret.columns)
w = np.repeat(1.0/len(tickers), len(tickers))
port_ret = (log_ret.dot(w)).rename("PORT")

# Helper: build rolling Hist & Param VaR series
def rolling_hist_var(series: pd.Series, alpha: float, window: int) -> pd.Series:
    q = 1 - alpha
    return series.rolling(window).quantile(q)

def rolling_param_var(series: pd.Series, alpha: float, window: int) -> pd.
 ↪Series:
    """Rolling parametric VaR using normal assumption; returns a pandas Series.
↪"""
    mu = series.rolling(window).mean()
    sd = series.rolling(window).std(ddof=1)
    vals = norm.ppf(1 - alpha, loc=mu, scale=sd)      # this is a NumPy array
    return pd.Series(vals, index=series.index)        # wrap with the same index

# Helper: build GARCH-family VaR series from fitted sigma_t (full-sample fit)
def garch_var_series(series: pd.Series, model_name: str, alpha: float):
    """
    Fit the corresponding model once; produce VaR_t = mu + z_{1-alpha} *␣
↪sigma_t.
    Uses full-sample conditional sigma_t from your Part-4 estimators.
    """
    if model_name == "garch11" and has_garch11:
        fit = garch11_fit(series.dropna().values)    # returns dict with 'mu'␣
↪and 'sigma' array
        mu = fit['mu']; sigma = pd.Series(fit['sigma'], index=series.dropna().
↪index)
    elif model_name == "gjr" and has_gjr:
        fit = fit_gjr_garch(series.dropna().values)  # returns dict with␣
↪'params' and 'sigma'
        mu = fit['params']['mu']; sigma = pd.Series(fit['sigma'], index=series.
↪dropna().index)
    elif model_name == "egarch" and has_egarch:
        fit = fit_egarch(series.dropna().values)
        mu = fit['params']['mu']; sigma = pd.Series(fit['sigma'], index=series.
↪dropna().index)
    else:
        return None  # model not available

    z = norm.ppf(1 - alpha)   # NOTE: VaR as lower quantile of return → negative
    var_series = (mu + z * sigma).rename(series.name)
    return var_series

# ---------- CORE BACKTEST RUNNER ----------
def run_backtests_for_scope(scope_tag: str, event_dates: list):
    """
    For a given scope (interest-rate shocks vs big events):
      A) Hist & Param VaR (rolling)
      B) GARCH-family VaR (full-sample conditional)
    Returns: (summary_df, flags_df) for the scope.
    """
    rows = []
    flags = []
    # --- A) Hist & Param rolling VaR ---
    for tic in tickers + ['PORT']:
        s = port_ret if tic == 'PORT' else log_ret[tic]
        for a in ALPHAS:
            # HIST
            v_hist = rolling_hist_var(s, a, ROLL_VAR_WINDOW).dropna()
            # align returns to VaR dates
            idx = s.index.intersection(v_hist.index)
            r_use = s.loc[idx].values
            v_use = v_hist.loc[idx].values
            viol, base = compute_violations(r_use, v_use, alpha=a, rule=None)
            out_uc  = kupiec_pof(a, base['violations'], base['n'])
            out_ind = christoffersen_independence(viol)
            out_cc  = conditional_coverage(a, viol)

            # optional event-window enrichment for this scope
            cluster = violation_cluster_around_events(idx, viol, event_dates,␣
↪window_days=1) if event_dates else None

            rows.append({
                'scope': scope_tag,
                'bucket': 'hist_param',       # (1) or (3)
                'model': 'hist',
                'ticker': tic, 'alpha': a,
                'n': base['n'], 'violations': base['violations'],
                'violation_rate': base['violation_rate'],
                'expected_violations': base['expected_violations'],
                'LR_uc': out_uc['LR_uc'], 'reject_uc_95':␣
↪out_uc['reject_95pct'],
                'LR_ind': out_ind['LR_ind'], 'reject_ind_95':␣
↪out_ind['reject_95pct'],
                'LR_cc': out_cc['LR_cc'], 'reject_cc_95':␣
↪out_cc['reject_95pct'],
                'rule': base['rule'],
                'enrichment_ratio': (cluster or {}).get('enrichment_ratio', np.
↪nan)
            })
            flags.append(pd.DataFrame({
                'scope': scope_tag, 'bucket': 'hist_param', 'model': 'hist',
                'ticker': tic, 'alpha': a,
                'date': idx, 'ret': r_use, 'var': v_use,
                'breach_flag': viol.astype(int)
            }))

            # PARAM
            v_para = rolling_param_var(s, a, ROLL_VAR_WINDOW).dropna()
            idx = s.index.intersection(v_para.index)
            r_use = s.loc[idx].values
            v_use = v_para.loc[idx].values
            viol, base = compute_violations(r_use, v_use, alpha=a, rule=None)
            out_uc  = kupiec_pof(a, base['violations'], base['n'])
            out_ind = christoffersen_independence(viol)
            out_cc  = conditional_coverage(a, viol)
            cluster = violation_cluster_around_events(idx, viol, event_dates,␣
↪window_days=1) if event_dates else None

            rows.append({
                'scope': scope_tag,
                'bucket': 'hist_param',
                'model': 'param',
                'ticker': tic, 'alpha': a,
                'n': base['n'], 'violations': base['violations'],
                'violation_rate': base['violation_rate'],
                'expected_violations': base['expected_violations'],
                'LR_uc': out_uc['LR_uc'], 'reject_uc_95':␣
↪out_uc['reject_95pct'],
                'LR_ind': out_ind['LR_ind'], 'reject_ind_95':␣
↪out_ind['reject_95pct'],
                'LR_cc': out_cc['LR_cc'], 'reject_cc_95':␣
↪out_cc['reject_95pct'],
                'rule': base['rule'],
                'enrichment_ratio': (cluster or {}).get('enrichment_ratio', np.
↪nan)
            })
            flags.append(pd.DataFrame({
                'scope': scope_tag, 'bucket': 'hist_param', 'model': 'param',
                'ticker': tic, 'alpha': a,
                'date': idx, 'ret': r_use, 'var': v_use,
                'breach_flag': viol.astype(int)
            }))

    # --- B) GARCH-related VaR (full-sample sigma_t) ---
    garch_models = []
    if has_garch11: garch_models.append('garch11')
    if has_gjr:     garch_models.append('gjr')
    if has_egarch:  garch_models.append('egarch')

    for tic in tickers + ['PORT']:
        s = port_ret if tic == 'PORT' else log_ret[tic]
        for m in garch_models:
            # compute model VaR series for each alpha
            for a in ALPHAS:
                v_m = garch_var_series(s, m, a)
                if v_m is None or v_m.empty:
                    continue
                idx = s.index.intersection(v_m.index)
                r_use = s.loc[idx].values
                v_use = v_m.loc[idx].values
                viol, base = compute_violations(r_use, v_use, alpha=a,␣
↪rule=None)
                out_uc  = kupiec_pof(a, base['violations'], base['n'])
                out_ind = christoffersen_independence(viol)
                out_cc  = conditional_coverage(a, viol)
                cluster = violation_cluster_around_events(idx, viol,␣
↪event_dates, window_days=1) if event_dates else None

                rows.append({
                    'scope': scope_tag,
                    'bucket': 'garch_family',    # (2) or (4)
                    'model': m,
                    'ticker': tic, 'alpha': a,
                    'n': base['n'], 'violations': base['violations'],
                    'violation_rate': base['violation_rate'],
                    'expected_violations': base['expected_violations'],
                    'LR_uc': out_uc['LR_uc'], 'reject_uc_95':␣
↪out_uc['reject_95pct'],
                    'LR_ind': out_ind['LR_ind'], 'reject_ind_95':␣
↪out_ind['reject_95pct'],
                    'LR_cc': out_cc['LR_cc'], 'reject_cc_95':␣
↪out_cc['reject_95pct'],
                    'rule': base['rule'],
                    'enrichment_ratio': (cluster or {}).get('enrichment_ratio',␣
↪np.nan)
                })
                flags.append(pd.DataFrame({
                    'scope': scope_tag, 'bucket': 'garch_family', 'model': m,
                    'ticker': tic, 'alpha': a,
                    'date': idx, 'ret': r_use, 'var': v_use,
                    'breach_flag': viol.astype(int)
                }))

    summary_df = pd.DataFrame(rows)
    flags_df   = pd.concat(flags, axis=0).reset_index(drop=True) if flags else␣
↪pd.DataFrame()
    return summary_df, flags_df

# ---------- RUN FOR BOTH SCOPES (4 BUCKETS TOTAL) ----------
sum_ir,  flg_ir  = run_backtests_for_scope(scope_tag="interest_rate_shock",␣
↪event_dates=interest_rate_events)
```

```python
sum_big, flg_big = run_backtests_for_scope(scope_tag="big_events",␣
    ↪event_dates=big_events)

# Combine for export
backtest_summary_4buckets = pd.concat([sum_ir, sum_big], axis=0).
    ↪reset_index(drop=True)
backtest_flags_4buckets   = pd.concat([flg_ir, flg_big], axis=0).
    ↪reset_index(drop=True)

# ---------- EXPORT ----------
atomic_save_csv(backtest_summary_4buckets, os.path.join(OUT_DIR,␣
    ↪"backtest_summary_4buckets.csv"))
atomic_save_csv(backtest_flags_4buckets,   os.path.join(OUT_DIR,␣
    ↪"backtest_flags_4buckets.csv"))

print("  Backtest exports saved:",
      os.path.join(OUT_DIR, "backtest_summary_4buckets.csv"),
      os.path.join(OUT_DIR, "backtest_flags_4buckets.csv"), sep="\n")
```

```
C:\Users\amusi\AppData\Local\Programs\Python\Python313\Lib\site-
packages\scipy\optimize\_slsqp_py.py:435: RuntimeWarning: Values in x were
outside bounds during a minimize step, clipping to bounds
  fx = wrapped_fun(x)
C:\Users\amusi\AppData\Local\Programs\Python\Python313\Lib\site-
packages\scipy\optimize\_slsqp_py.py:435: RuntimeWarning: Values in x were
outside bounds during a minimize step, clipping to bounds
  fx = wrapped_fun(x)
C:\Users\amusi\AppData\Local\Programs\Python\Python313\Lib\site-
packages\scipy\optimize\_slsqp_py.py:435: RuntimeWarning: Values in x were
outside bounds during a minimize step, clipping to bounds
  fx = wrapped_fun(x)
C:\Users\amusi\AppData\Local\Programs\Python\Python313\Lib\site-
packages\scipy\optimize\_slsqp_py.py:435: RuntimeWarning: Values in x were
outside bounds during a minimize step, clipping to bounds
  fx = wrapped_fun(x)

  Backtest exports saved:
D:\Finance\risk\exports_bi\backtest_summary_4buckets.csv
D:\Finance\risk\exports_bi\backtest_flags_4buckets.csv
```

[51]:
```python
import os, pandas as pd, tempfile, shutil

OUT_DIR = r"D:\Finance\risk\exports_bi"   # or OneDrive/SharePoint folder
os.makedirs(OUT_DIR, exist_ok=True)

def atomic_save_csv(df: pd.DataFrame, path: str):
    """Safely write df → CSV with temp file replace (avoids partial files␣
    ↪during refresh)."""
    tmp_fd, tmp_path = tempfile.mkstemp(dir=os.path.dirname(path), suffix=".
↪tmp")
    os.close(tmp_fd)
    df.to_csv(tmp_path, index=False)
    if os.path.exists(path):
        os.remove(path)
    shutil.move(tmp_path, path)

def _export_if_exists(var_name: str, file_name: str, wide_ok: bool = True):
    """Export a DataFrame/Series if it exists in globals(). Handles␣
↪Series→DataFrame conversion."""
    if var_name in globals():
        obj = globals()[var_name]
        if isinstance(obj, pd.Series):
            df = obj.reset_index()
            # call the non-index column something sensible
            val_col = var_name if wide_ok else "value"
            df.columns = ['date', val_col] if 'date' in df.columns else df.
↪columns
            atomic_save_csv(df, os.path.join(OUT_DIR, file_name))
            print(f"  Saved {var_name} → {file_name}")
        elif isinstance(obj, pd.DataFrame):
            atomic_save_csv(obj, os.path.join(OUT_DIR, file_name))
            print(f"  Saved {var_name} → {file_name}")
        else:
            print(f"… Skipped {var_name}: not a pandas DataFrame/Series")
    else:
        print(f"… Skipped {var_name}: not found")

print("="*12, "EXPORTS", "="*12)

# 1) Correlations (use whichever name you actually have)
_export_if_exists("corr",    "correlations.csv")
_export_if_exists("corr_df", "correlations.csv")   # fallback alias

# 2) VaR (historical / parametric)
_export_if_exists("var_his",  "var_historical.csv")
_export_if_exists("var_para", "var_parametric.csv")

# 3) JB / normality summary
_export_if_exists("summary", "jb_summary.csv")

# 4) GARCH parameter estimates
_export_if_exists("garch_df", "garch_params.csv")

# 5) Asymmetry comparison (GARCH vs GJR vs EGARCH)
_export_if_exists("cmp_df", "asymmetry_models.csv")
```

```python
# 6) Returns (use your existing tidy if present; otherwise save your wide returns)
# Preferred tidy table from your code (if you built it)
_export_if_exists("ret_tidy", "returns_tidy.csv")
# If you never made a tidy table, at least save the wide log returns
_export_if_exists("log_ret", "returns_wide.csv")
# If your df_change holds Δ10Y bps, save it too
if "df_change" in globals() and isinstance(df_change, pd.DataFrame):
    if "rate_bps" in df_change.columns:
        atomic_save_csv(df_change[["rate_bps"]].reset_index(), os.path.join(OUT_DIR, "rate_bps.csv"))
        print("  Saved df_change['rate_bps'] → rate_bps.csv")

# 7) Volatility
# If you already created a tidy volatility table:
_export_if_exists("vol_tidy", "volatility.csv")
# Otherwise export whatever you have (common names in your notebook/PDF)
_export_if_exists("vol_annual",       "volatility_sma_annual.csv")
_export_if_exists("ewma_vol",         "volatility_ewma_annual.csv")
_export_if_exists("ewma_vol_annual",  "volatility_ewma_annual.csv")

# 8) Rolling VaR forecasts (if you built them)
_export_if_exists("var_rolling", "var_rolling.csv")

# 9) Portfolio time series (returns / vol / VaR) if you built a combined table
_export_if_exists("portfolio_ts", "portfolio_timeseries.csv")
# If no combined table, at least export portfolio return series if present
_export_if_exists("port_ret", "portfolio_return_series.csv", wide_ok=False)

# 10) Backtesting exports (only if you already created them)
# (a) Single-scope versions (if you made them separately)
_export_if_exists("backtest_summary", "backtest_summary.csv")
_export_if_exists("backtest_flags",   "backtest_flags.csv")
# (b) Four-bucket versions from the later code we added (if you ran it)
_export_if_exists("backtest_summary_4buckets", "backtest_summary_4buckets.csv")
_export_if_exists("backtest_flags_4buckets",   "backtest_flags_4buckets.csv")

print("  All possible exports attempted. Saved files are in:", OUT_DIR)
```

```
============ EXPORTS ============
  Saved corr → correlations.csv
… Skipped corr_df: not found
  Saved var_his → var_historical.csv
  Saved var_para → var_parametric.csv
  Saved summary → jb_summary.csv
  Saved garch_df → garch_params.csv
```

```
 Saved cmp_df → asymmetry_models.csv
… Skipped ret_tidy: not found
 Saved log_ret → returns_wide.csv
 Saved df_change['rate_bps'] → rate_bps.csv
… Skipped vol_tidy: not found
 Saved vol_annual → volatility_sma_annual.csv
 Saved ewma_vol → volatility_ewma_annual.csv
 Saved ewma_vol_annual → volatility_ewma_annual.csv
… Skipped var_rolling: not found
… Skipped portfolio_ts: not found
 Saved port_ret → portfolio_return_series.csv
… Skipped backtest_summary: not found
… Skipped backtest_flags: not found
 Saved backtest_summary_4buckets → backtest_summary_4buckets.csv
 Saved backtest_flags_4buckets → backtest_flags_4buckets.csv
 All possible exports attempted. Saved files are in: D:\Finance\risk\exports_bi
```

[ ]: