CPSC-354 Report

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Abstract

This report is a collection of notes, discussions, and homework solutions for CPSC 354. Homework 1 discusses the MIU puzzle and proves why `MI` cannot become `MU`. Homework 2 introduces abstract rewriting systems (ARS), includes TikZ diagrams, and classifies systems by termination, confluence, and unique normal forms. Homework 3 studies a rewriting system over $\{a, b\}$ and explores its equivalence classes and termination. Homework 4 proves termination of two classic algorithms using measure functions. Homework 5 evaluates a small λ-calculus "workout" using α- and β-rules, with notes connecting these ideas to modern languages. Homework 6 develops fixpoints and recursion: we compute `fact 3` step-by-step with the rules for `fix`, `let`, and `let rec`, and give a Y-combinator encoding of factorial. We also include a Class Notes & Discussion section that addresses questions about evaluation contexts, fixpoints, recursion, confluence vs. termination, and practical trade-offs.

Contents

1 Introduction

This report documents my learning week by week. Each section includes my notes and discussions of the lecture topics, followed by my homework solutions. The purpose is to show understanding, practice writing in LaTeX, and explain the material in my own words.

2 Week 1: The MIU Puzzle

2.1 Notes and Discussion

The first week introduced the MIU puzzle, created by Douglas Hofstadter. It is a formal system that begins with the word `MI` and has four rules. The puzzle asks whether it is possible to reach `MU`. This is important because it shows how formal systems can be analyzed with invariants, a key concept in logic and computer science.

2.2 Homework

Rules:

1. If a string ends with `I`, add a `U`.
2. If a string starts with `M`, double the part after `M`.
3. Replace `III` with `U`.
4. Remove `UU`.

Why `MI` cannot become `MU`: Let $\#I(w)$ be the number of `I`'s in w. Initially $\#I(\texttt{MI}) = 1$. Each rule preserves $\#I(w) \pmod 3$:

- Rule 1: no effect.
- Rule 2: doubles the count, $1 \mapsto 2$, never $0 \pmod 3$.
- Rule 3: subtracts 3, same remainder.
- Rule 4: no effect.

Thus $\#I(w) \pmod 3$ is invariant. Since `MU` has $\#I = 0$, it cannot be reached.

Deeper explanation: The invariant argument proves that all reachable words have I-count $\equiv 1$ or $2 \pmod 3$, never 0. The first letter M never disappears, so all reachable words start with M. Another way: define a homomorphism $h(M) = 0, h(U) = 0, h(I) = 1 \pmod 3$. Every rule preserves h, so $h(\texttt{MI}) = 1$ and $h(\texttt{MU}) = 0$. Contradiction.

3 Week 2: Abstract Rewriting Systems

3.1 Notes and Discussion

This week focused on abstract rewriting systems. An ARS consists of a set A and a relation R. We ask whether rewriting always stops (termination), whether different rewrite paths can rejoin (confluence), and whether elements end up in unique normal forms.

3.2 Homework

Same problem statement as before: draw each ARS and decide termination, confluence, and whether the system has *unique normal forms for all elements* (UN-for-all).

Examples (drawings unchanged)

1: $A = \{\}, R = \{\}$ (empty system)

$$\begin{tikzcd} a \end{tikzcd}$$

2: $A = \{a\}, R = \{\}$

$$\begin{tikzcd} a \arrow[loop above] \end{tikzcd}$$

3: $A = \{a\}, R = \{(a, a)\}$

$$\begin{tikzcd} a \arrow[r] \arrow[dr] & b \\ & c \end{tikzcd}$$

4: $A = \{a, b, c\}, R = \{(a, b), (a, c)\}$

$$\begin{tikzcd} a \arrow[loop above] \arrow[r] & b \end{tikzcd}$$

5: $A = \{a, b\}, R = \{(a, a), (a, b)\}$

$$\begin{tikzcd} a \arrow[r] \arrow[dr] & b \arrow[loop above] \\ & c \end{tikzcd}$$

6: $A = \{a, b, c\}, R = \{(a, b), (b, b), (a, c)\}$

$$\begin{tikzcd} a \arrow[r] \arrow[dr] & b \arrow[loop above] \\ & c \arrow[loop right] \end{tikzcd}$$

7: $A = \{a, b, c\}, R = \{(a, b), (b, b), (a, c), (c, c)\}$

Classification (solutions rewritten)

We now use **UN-for-all**: every element must have a (unique) normal form.

ARS	Terminating	Confluent	UN-for-all	Why
1	Yes	Yes	Yes	Nothing rewrites (vacuous).
2	Yes	Yes	Yes	a is already normal and unique.
3	No	Yes	No	$a \to a$ forever; a has no NF.
4	Yes	No	No	$a \to b$, $a \to c$ (two distinct NFs).
5	No	Yes	Yes	$a \to b$, b normal; both end at b.
6	No	No	No	b loops; $a \to c$ NF, not joinable with b.
7	No	No	No	b, c loop; no element has an NF.

Remarks.

- Non-termination alone does not rule out UN-for-all (see 5), but any element without an NF breaks it (3,6,7).
- In terminating ARSs, confluence $\Rightarrow$ UN-for-all (every element reaches a unique NF).
- Item 4 is the canonical "diamond failure": two different normal forms reachable from the same source.

4 Week 3: Strings over {a,b}

4.1 Notes and Discussion

We studied rewriting rules over alphabet $\{a, b\}$. This shows how simple systems can classify strings into equivalence classes. We also saw how orienting rules one way can make a system terminating without changing equivalence.

4.2 Homework 3

Exercise 5 rules:

$$ab \to ba, \quad ba \to ab, \quad aa \to \varepsilon, \quad b \to \varepsilon$$

Examples: `abba` $\to$ `aa` $\to \varepsilon$. `bababa` $\to$ `aaa` $\to$ `a`.

Not terminating because swaps loop. Equivalence classes: even number of a's $\to \varepsilon$, odd $\to a$. Normal forms: ε, a. Fix: orient $ba \to ab$ only.

Exercise 5b rules:

$$ab \leftrightarrow ba, \quad aa \to a, \quad b \to \varepsilon$$

Examples: `abba` $\to$ `a`. `bababa` $\to$ `a`.

Equivalence: all b vanish, runs of a collapse to one a. Classes: $\{\varepsilon, a\}$. Fix: orient swaps, keep $aa \to a$ and $b \to \varepsilon$.

5 Week 4: Measure Functions and Termination

5.1 Notes and Discussion

This week we looked at *measure functions* to prove that algorithms terminate. A measure maps the state of a computation to a value in a well-founded set (usually the natural numbers) and must *strictly decrease* with every loop iteration or recursive call. Because there are no infinite descending chains in a well-founded set, the computation must eventually stop.

Scope note: Here we define a measure and show it strictly decreases

Class notes invariant proof that Euclid returns gcd**.**

- **Key fact.** $\gcd(a,b) = \gcd\big(b,\, a - qb\big)$ for any integer q.
- **Update.** Writing $a = qb + r$ with $r = a \bmod b$, the step $(a,b) \mapsto (b,r)$ keeps the gcd unchanged.
- **Invariant.** Before each loop test: $\gcd(a,b) = \gcd(a_0,b_0)$.
- **End.** When $b = 0$, $\gcd(a_0,b_0) = \gcd(a,0) = |a|$, the returned value.

5.2 Homework 4

HW 4.1

Problem. Considering

```
while b != 0:
    temp = b
    b = a % b
    a = temp
return a
```

Under which conditions does this algorithm always terminate? Find a measure function and prove termination.

Answer. This is the classical Euclidean algorithm for $\gcd(a,b)$. It always terminates provided that

$$a, b \in \mathbb{Z} \quad \text{and} \quad b \geq 0,$$

and we interpret % as the mathematical remainder with $0 \leq a \bmod b < b$ for $b > 0$. If the inputs are negative, replace them once by $|a|, |b|$; this preserves gcd and satisfies the condition.)

Measure function. Let the program state be the pair (a,b) with $b \geq 0$. Define

$$\mu(a,b) \;=\; b \in \mathbb{N}.$$

Strict decrease. In an iteration with $b > 0$ we set $b' = a \bmod b$. By definition of remainder,

$$0 \leq b' < b.$$

Therefore $\mu(a',b') = b' < b = \mu(a,b)$. Hence μ strictly decreases on every loop step.

Well-foundedness and termination. μ maps states to the natural numbers with the usual $<$, which is well founded because μ strictly decreases whenever the loop body executes, the loop can execute only finitely many times; eventually $b = 0$ and the algorithm returns. □

Remark 5.1. The algorithm not only terminates; it returns $\gcd(a,b)$. This follows from the class note invariant above and $\gcd(a,0) = |a|$.

HW 4.2

Problem. Consider the fragment of merge sort:

```
def merge_sort(arr, left, right):
    if left >= right:
        return
    mid = (left + right) // 2
    merge_sort(arr, left, mid)
    merge_sort(arr, mid + 1, right)
    merge(arr, left, mid, right)
```

Prove that

$$\varphi(left, right) \;=\; right - left + 1$$

is a measure function for `merge_sort`.

Answer. We show that every recursive call is made with a *strictly smaller* measure and that the measure is bounded below by 0.

Well-founded codomain. $\varphi(left, right)$ is a nonnegative integer whenever $left \le right$, so the codomain is $\mathbb{N}$ with the usual $<$ (well-founded).

Base case. If $left \ge right$, the function returns immediately. In this case $\varphi \le 1$, and there are no recursive calls.

Decrease for recursive calls. Assume $left < right$ and let $n = \varphi(left, right) = right - left + 1 \ge 2$. With

$$mid = \left\lfloor \frac{left + right}{2} \right\rfloor,$$

the first recursive call is on $[left,\, mid]$ and the second on $[mid + 1,\, right]$. Their measures are

$$\varphi(left, mid) = mid - left + 1, \qquad \varphi(mid + 1, right) = right - mid.$$

Because $left \le mid < right$, we have

$$1 \le \varphi(left, mid) \le \left\lceil \frac{n}{2} \right\rceil < n, \qquad 1 \le \varphi(mid + 1, right) \le \left\lfloor \frac{n}{2} \right\rfloor < n.$$

Thus each recursive call receives strictly smaller measure than n.

Conclusion. Every chain of recursive calls strictly decreases φ and cannot be infinite in $\mathbb{N}$. Therefore `merge_sort` terminates. □

6 Week 5: Lambda Calculus Foundations

6.1 Notes and Discussion

- **Self-application and computation.** Terms like $(\lambda x.\, xx)(\lambda x.\, xx)$ show that functions can take *code as data*. This enables iteration/recursion encodings (fixed points) and explains why untyped λ can express non-termination.
- **Where λ shines.** Useful for building *minimal cores*: interpreters, proof assistants, type checkers, compiler IRs, and reasoning about higher-order functions. It models substitution, scope, closures, and evaluation precisely.

- **Composition and numerals.** Church numerals encode iteration: $\mathbf{n}\, f\, x = f^n x$. Function composition $(f \circ g)$ corresponds to multiplying numerals: $(\mathbf{m} \circ \mathbf{n})\, f\, x = f^{mn} x$. Plain application behaves like *exponentiation in reverse order*: $\mathbf{m}\,\mathbf{n} = \mathbf{n}^{\mathbf{m}}$ (apply $\mathbf{m}$ times), so $\mathbf{2}\,\mathbf{3} = \mathbf{9}$ and $\mathbf{3}\,\mathbf{2} = \mathbf{8}$.
- **Confluence (Church–Rosser).** If a term reduces to two results, there exists a common reduct. Meaning: evaluation order doesn't affect the final normal form (when it exists). Languages borrow this idea to justify equational reasoning and optimizations.
- **From λ to languages.** Add types (safety), effects (state, I/O), data types, and evaluation strategy to get modern languages. Typed λ-calculi (System F, Hindley–Milner) underlie ML/Haskell; effect systems/monads model side effects.
- **Haskell links.** Purity, first-class functions, laziness (normal-order–like), algebraic data types, and type inference all line up with typed λ-calculus. Monads/Applicatives are structured ways to compose computations.
- **Names and α-conversion.** Variables need not be single letters; names are irrelevant up to α-equivalence (consistent renaming). Modern languages reflect this via lexical scope, hygienic macros, and compiler renaming to avoid capture.
- **The role of α in practice.** Compilers implement renaming and fresh-name generation to maintain scope hygiene (e.g., SSA form, macro expansion).
- **Termination and type systems.** Untyped λ allows non-termination (Y-combinator). Total languages (Coq/Agda) *forbid* general recursion by checking termination via structural/lexicographic and multivariate measures; complex patterns (exponential, multi-dimension decrease) are handled with well-founded orders and sized types. Downsides: proofs/measures can cost compile time (sometimes exponential).
- **Original intent.** Church invented λ for the foundations of mathematics (functions as rules of calculation) and to study effective computability—leading to Church–Turing thesis.
- **Recursion vs loops.** Prefer recursion when structure is naturally recursive (trees, syntax, divide-and-conquer) or when immutability/purity matters. Prefer loops when iterating in-place over arrays or when tail recursion isn't optimized.
- **What can/can't be expressed.** Untyped λ can encode any computable function (Turing complete). Exact analysis over reals (symbolic integrals) can be encoded with representations, but decidability and performance are not guaranteed; many problems are undecidable.
- **Meaning vs representation.** α-equivalence shows semantics can be invariant under renaming. Suggests programs' meaning depends on structure and binding, not surface spelling.
- **Infinitely long calls?** Reductions can be infinite (non-terminating), but any single step is finite. Typed total calculi restrict to well-founded recursion to rule these out.

6.2 Homework 5: Lambda Calculus Workout

Problem

Evaluate (practice α and β rules; match parentheses):

$$\Big(\lambda f.\, \lambda x.\, f(f\, x)\Big)\ \Big(\lambda f.\, \lambda x.\, f(f(f\, x))\Big).$$

Solution

Let

$$\mathbf{two} \equiv \lambda f.\lambda x.\, f(fx), \qquad \mathbf{three} \equiv \lambda f.\lambda x.\, f(f(fx)).$$

Then

$$\mathbf{two}\;\mathbf{three} \xrightarrow{\beta} \lambda x.\,\mathbf{three}\,(\mathbf{three}\,x).$$

Compute:

$$\mathbf{three}\,x \xrightarrow{\beta} \lambda y.\,x(x(x\,y)), \qquad \mathbf{three}\,(\lambda y.\,x(x(x\,y))) \xrightarrow{\beta} \lambda z.\,x^9 z.$$

Therefore

$$\boxed{\lambda x.\lambda z.\,x^9 z}$$

which is the Church numeral **nine**.

Swap note. $\mathbf{3}\,\mathbf{2} = \mathbf{8}$, not $\mathbf{9}$, because application acts like exponentiation with reversed order: $\mathbf{m}\,\mathbf{n} = \mathbf{n}^{\mathbf{m}}$.

7 Week 6: Fixpoints and Recursion

7.1 Class Notes & Discussion

This section responds concisely to discussion questions from class.

1. **Reducing inside larger expressions and step rules for `fix`, `let`, `if`.** We use *evaluation contexts*: reduce inside the syntactic position allowed by the strategy (e.g., call-by-value reduces the scrutinee of `if`, the bound expression of `let`, and the argument to `fix` until it is a λ). Rules:

   $$\text{fix } F \;\to\; F(\text{fix } F), \qquad \text{let } x = e_1 \text{ in } e_2 \;\to\; (\lambda x.\,e_2)\,e_1, \qquad \text{if true then } e_1 \text{ else } e_2 \to e_1.$$

   Contexts ensure we can justify each inner step when `fix`, `let`, or `if` occurs inside a bigger term.

2. **Fixpoint combinator and recursion.** A combinator Y with $YF = F(YF)$ provides a value YF that is a fixed point of F. If F encodes one step of a recursive definition, YF is its recursive solution. Languages often bake this in as a primitive `fix`.

3. **Factorial with Y instead of `fix`.**

   $$\text{fact} \;\equiv\; Y\,(\lambda f.\,\lambda n.\text{ if } n = 0 \text{ then } 1 \text{ else } n \cdot f(n-1)).$$

4. **Trade-offs of using Y in practice.** Y relies on non-termination to exist; in typed, strongly normalizing calculi, it breaks termination. In practical languages, explicit `rec`/`let rec` or named functions yield clearer code, better typing, and easier optimization/debugging.

5. **Can Y-style recursion act like loops?** Yes. Encode state as parameters. E.g., a while loop over (n, acc) uses a recursive function that tests a condition and tail-calls with updated state.

6. **Why no definable `fix` in strongly normalizing typed λ?** If `fix` were definable, we could define a non-terminating term Ω, contradicting strong normalization. Hence total calculi exclude general `fix`.

7. **Making a non-confluent ARS confluent by adding rules—will termination break?** Possibly. Adding join rules may introduce cycles. Confluence and termination are orthogonal; the *Knuth–Bendix completion* process may fail or loop.

8. **Why do we need a fixed-point combinator? What limitation does it solve?** Pure λ without recursion lacks self-reference. Fixed points supply self-reference (*solutions to equations* $x = F(x)$), enabling definitions of recursive functions.

9. **A "better" way to compute a fixed point than raw Y?** In typed settings: *typed* fix with guarded/structural recursion, or domain-theoretic least fixed points over CPOs; in practice: language-level recursion, loops, or higher-order iterators (map/fold).

10. **'fact' vs 'factorial'.** Abbreviation only; names are α-variant placeholders.
11. **Does reduction order matter for termination when using `fix`?** Yes. Confluence (Church–Rosser) says normal forms are unique when they exist, not that evaluation terminates. Some orders diverge while others terminate.
12. **What if `fix` is applied to a non-function?** The unfolding step $\text{fix } F \to F(\text{fix } F)$ expects F to be a function. Otherwise the program is stuck (or a runtime type error in typed languages).

7.2 Homework 6

We use these computation rules:

$$\text{fix } F \to F(\text{fix } F), \quad \text{let } x = e_1 \text{ in } e_2 \to (\lambda x.\, e_2)\, e_1, \quad \text{let rec } f = e_1 \text{ in } e_2 \to \text{let } f = (\text{fix } (\lambda f.\, e_1)) \text{ in } e_2.$$

Problem A: Compute `fact 3`

Start with

$$\text{let rec fact} = \lambda n.\ \text{if } n = 0 \text{ then } 1 \text{ else } n * \text{fact}(n-1) \text{ in fact } 3.$$

Derivation

$$
\begin{aligned}
& \text{let rec fact} = \lambda n.\ \text{if } n = 0 \text{ then } 1 \text{ else } n * \text{fact}(n-1) \text{ in fact } 3 \\
\to\ & \textbf{(def of let rec)} \\
& \text{let fact} = \text{fix } (\lambda f.\ \lambda n.\ \text{if } n = 0 \text{ then } 1 \text{ else } n * f(n-1)) \text{ in fact } 3 \\
\to\ & \textbf{(def of let)} \\
& (\lambda\text{fact. fact } 3)\ \big(\text{fix } F\big) \quad \text{where } F \equiv \lambda f.\ \lambda n.\ \text{if } n = 0 \text{ then } 1 \text{ else } n * f(n-1) \\
\to\ & \textbf{(}\beta\textbf{; substitute } \text{fix } F \textbf{ for fact)} \\
& (\text{fix } F)\ 3 \\
\to\ & \textbf{(def of fix)} \\
& \big(F(\text{fix } F)\big)\ 3 \\
\to\ & (\beta) \\
& (\lambda n.\ \text{if } n = 0 \text{ then } 1 \text{ else } n * (\text{fix } F)(n-1))\ 3 \\
\to\ & \textbf{(}\beta\textbf{; substitute } 3 \textbf{ for } n\textbf{)} \\
& \text{if } 3 = 0 \text{ then } 1 \text{ else } 3 * (\text{fix } F)(2) \\
\to\ & \textbf{(def of if)} \\
& 3 * (\text{fix } F)(2) \\
\to\ & \textbf{(def of fix)} \\
& 3 * \big(F(\text{fix } F)\big)\, 2 \\
\to\ & (\beta) \\
& 3 * (\lambda n.\ \text{if } n = 0 \text{ then } 1 \text{ else } n * (\text{fix } F)(n-1))\ 2 \\
\to\ & \textbf{(}\beta\textbf{; substitute } 2 \textbf{ for } n\textbf{)} \\
& 3 * \big(\text{if } 2 = 0 \text{ then } 1 \text{ else } 2 * (\text{fix } F)(1)\big) \\
\to\ & \textbf{(def of if)} \\
& 3 * \big(2 * (\text{fix } F)(1)\big) \\
\to\ & \textbf{(def of fix)} \\
& 3 * \big(2 * \big(F(\text{fix } F)\big)\, 1\big) \\
\to\ & (\beta) \\
& 3 * \big(2 * (\lambda n.\ \text{if } n = 0 \text{ then } 1 \text{ else } n * (\text{fix } F)(n-1))\ 1\big) \\
\to\ & \textbf{(}\beta\textbf{; substitute } 1 \textbf{ for } n\textbf{)} \\
& 3 * \big(2 * \big(\text{if } 1 = 0 \text{ then } 1 \text{ else } 1 * (\text{fix } F)(0)\big)\big) \\
\to\ & \textbf{(def of if)} \\
& 3 * \big(2 * \big(1 * (\text{fix } F)(0)\big)\big) \\
\to\ & \textbf{(def of fix)} \\
& 3 * \big(2 * \big(1 * \big(F(\text{fix } F)\big)\, 0\big)\big) \\
\to\ & (\beta) \\
& 3 * \big(2 * \big(1 * (\lambda n.\ \text{if } n = 0 \text{ then } 1 \text{ else } n * (\text{fix } F)(n-1))\ 0\big)\big) \\
\to\ & \textbf{(}\beta\textbf{; substitute } 0 \textbf{ for } n\textbf{)} \\
& 3 * \big(2 * \big(1 * (\text{if } 0 = 0 \text{ then } 1 \text{ else } 0 * (\text{fix } F)(-1))\big)\big) \\
\to\ & \textbf{(def of if)} \\
& 3 * \big(2 * (1 * 1)\big) \ = \ 3 * 2 * 1 \ = \ \boxed{6}.
\end{aligned}
$$

Problem B: Y-combinator factorial and one unfold

Let

$$Y \equiv \lambda g.\ (\lambda x.\ g(x\,x))(\lambda x.\ g(x\,x)), \quad G \equiv \lambda f.\ \lambda n.\ \text{if } n = 0 \text{ then } 1 \text{ else } n * f(n-1).$$

Define fact $\equiv YG$. Then

$$\text{fact} \;=\; YG \;\to\; G(YG) \;=\; \lambda n.\ \text{if } n = 0 \text{ then } 1 \text{ else } n * (YG)(n-1),$$

which is exactly one recursive "unfold".

Remark 7.1. In total/strongly normalizing typed calculi, Y cannot be defined without sacrificing termination. Practical languages provide explicit recursion or `fix`, often with typing restrictions.

8 Conclusion

Week 1: Learned about invariants and why MU is impossible. **Week 2:** Revisited ARS pictures and re-answered termination/confluence/UN with the UN-for-all convention. **Week 3:** Explored rewriting with $\{a, b\}$, equivalence classes, and how termination can be fixed.
Week 4: Used measure functions to prove termination of Euclid's algorithm and merge sort.
Week 5: Connected λ-calculus ideas (self-application, confluence, Church numerals) to modern language design and evaluated the workout.
Week 6: Explained evaluation contexts and fixpoints, computed `fact 3` step-by-step with labeled rules, and encoded factorial via the Y-combinator.

9 Week 7: Grammars, Parse Trees, and ASTs

9.1 Class Notes & Discussion

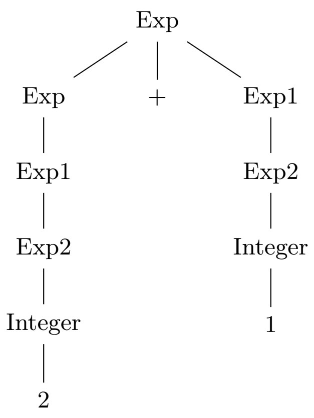
- **Fixed-point combinators and performance.** Using Y/`fix` introduces an extra unfolding step ($YF \to F(YF)$). Named recursion or `let rec` generally performs better since the compiler can optimize it directly without creating closures on each call.
- **LLMs as metaprograms.** Large language models can be viewed as metaprograms that translate semantics (intent or meaning) into code. However, they do not replace the traditional compiler pipeline—semantic interpretation, parsing, optimization, and execution still occur afterward.
- **Lisp and syntax readability.** Lisp's simple S-expression syntax gives programmers direct access to abstract syntax, making metaprogramming and macro systems easier to implement. The trade-off is reduced human readability but increased ease of parsing for machines.
- **Ambiguous grammars.** Ambiguity in grammars can make parsing inconsistent. Some languages resolve this through operator precedence and associativity rules. Others use ambiguity intentionally to allow flexible interpretation, though this is rare and risky in language design.
- **Lambda calculus and grammars.** Lambda calculus defines the semantics of computation, while grammars define syntax. Parse trees represent structure that can later be reduced or evaluated according to lambda calculus rules.
- **Parse trees vs. ASTs.** Parse trees (CSTs) include every symbol from the grammar, while ASTs simplify structure by removing redundant nodes like parentheses or punctuation. ASTs capture the program's logical structure, making them easier for compilers to analyze.
- **ASTs and compilation to assembly.** Parse trees and ASTs form the foundation for converting source code into lower-level representations. Compilers traverse ASTs to generate intermediate code (IR), perform optimizations, and finally emit assembly instructions or machine code.

9.2 Homework 7: Context-Free Grammar and Derivation Trees

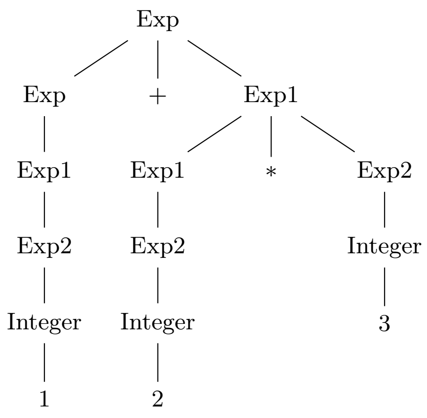
Grammar:

$$\begin{array}{rcl}
\text{Exp} & \to & \text{Exp} + \text{Exp1} \mid \text{Exp1} \\
\text{Exp1} & \to & \text{Exp1} * \text{Exp2} \mid \text{Exp2} \\
\text{Exp2} & \to & \text{Integer} \mid (\,\text{Exp}\,)
\end{array}$$

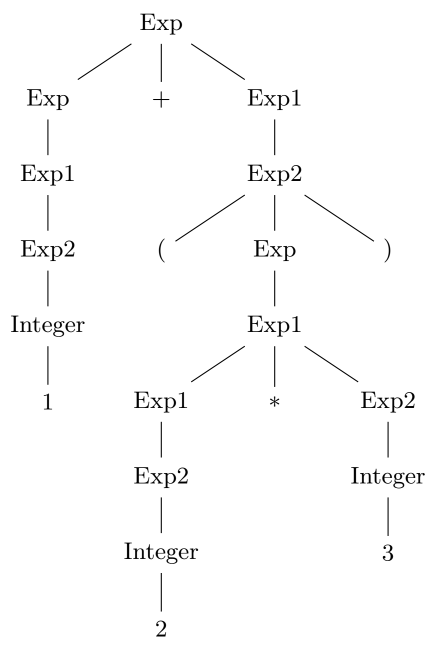
(a) `2+1`



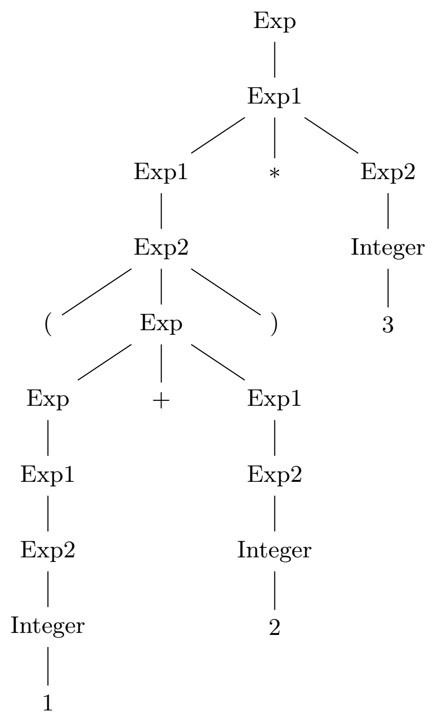
(b) `1+2*3`



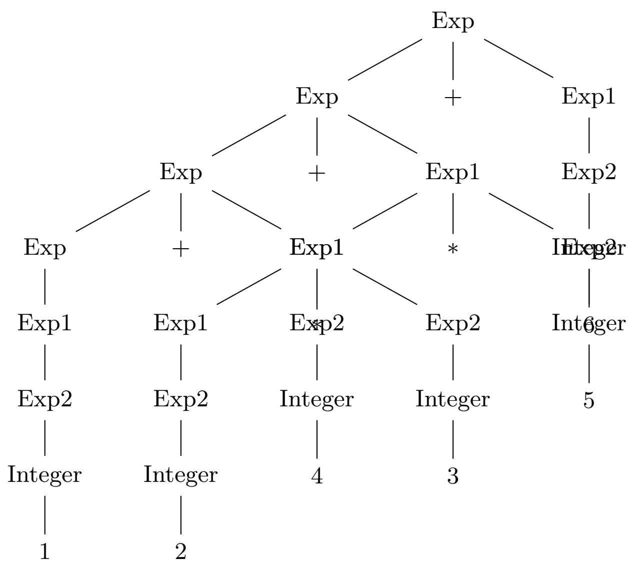
(c) `1+(2*3)`



(d) `(1+2)*3`



(e) `1+2*3+4*5+6`



References

[BLA] Author, LaTeX Overview, Publisher, Year.