SIFTING Property

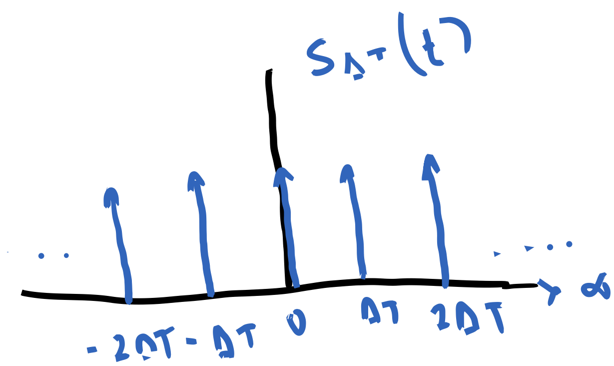
$$\int_{-\infty}^{\infty} f(t)\,\delta(t)\,dt = f(0) \qquad \int_{-\infty}^{\infty} \delta(t)\,dt = 1$$

$$\int_{-\infty}^{\infty} f(t)\,\delta(t-t_0)\,dt = f(t_0)$$

$$\delta(x) = \begin{cases} 1 & x = 0 \\ 0 & x \neq 0 \end{cases}$$

$$\sum_{x=-\infty}^{\infty} \delta(x) = 1$$

$$\sum_{x=-\infty}^{\infty} f(x)\,\delta(x-x_0) = f(x_0)$$



$$S_{\Delta T}(t) = \sum_{n=-\infty}^{\infty} \delta(t - n\Delta T)$$

Periodic Impulses ΔT units apart.

Periodic Signal $\to$ $f(t) = \displaystyle\sum_{n=-\infty}^{\infty} c_n\, e^{+j\frac{2\pi n}{T}t}$ — ①

$$= \sum_{n=-\infty}^{\infty} c_n\, e^{j\omega_0 n t} \qquad \left[\omega_0 = \frac{2\pi}{T}\right]$$

$$\left[u = \frac{1}{T}\right]$$

For ① & ② Fourier Series

$$c_n = \frac{1}{T}\int_{-T/2}^{T/2} f(t)\, e^{-j\frac{2\pi n}{T}t}\, dt \quad \text{for } n = 0, \pm 1, \pm 2 \dots \quad — (2)$$

Inverse Fourier Transform:

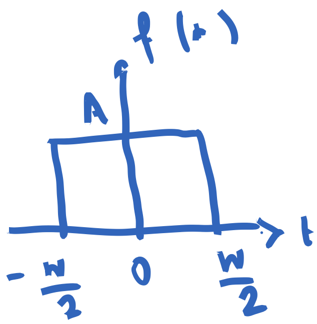
F.T.

* $$f(t) = \int_{-\infty}^{\infty} F(u)\, e^{j2\pi u t}\, du \rightarrow \text{Synthesis f:n.} \quad (3)$$

Fourier Transform

$$F(u) = \int_{-\infty}^{\infty} f(t)\, e^{-j2\pi u t}\, dt. \rightarrow \text{Analysis f:n} \quad — (4)$$

#.



$$F(u) = \int_{-\infty}^{\infty} f(t)\, e^{-j2\pi u t}\, dt$$

$$= \int_{-W/2}^{W/2} A\, e^{-j2\pi u t}\, dt$$

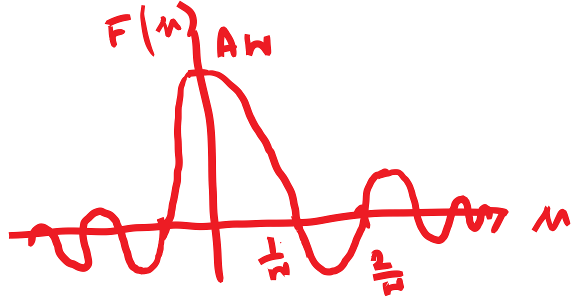
$$e^{j\theta} = \cos\theta + j\sin\theta$$

$$= A \left. \frac{e^{-j2\pi u t}}{-j2\pi u} \right|_{-W/2}^{W/2}$$

$$= \frac{A}{j2\pi u}\left[e^{+j\pi u W} - e^{-j\pi u W}\right]$$

$$= \frac{A}{\pi u} \sin \pi u W = AW \frac{\sin(\pi u W)}{(\pi u W)}$$

$$\frac{\sin(\pi m)}{\pi m} = \text{Sinc}(m)$$



$\pi u W = \pi$

$u = \frac{1}{W}$.

$$\#1.\ \mathfrak{F}[\delta(t)] = \int_{-\infty}^{\infty} \delta(t)\, e^{-j2\pi u t}\, dt$$

$$= 1$$

$$\#\ \mathfrak{F}[\delta(t-t_0)] = \int_{-\infty}^{\infty} \delta(t-t_0)\, e^{-j2\pi u t}\, dt$$

$$= \underline{e^{-j2\pi u t_0}}$$

$e^{j\theta} = \cos\theta + j\sin\theta$ $\qquad x + jy$

$r = |e^{j\theta}| = \sqrt{\cos^2\theta + \sin^2\theta} = 1$ $\qquad r = \sqrt{x^2 + y^2}$

$\theta = \tan^{-1}\left(\frac{y}{x}\right)$

$$\#\ \mathfrak{F}\left[f(t)\, e^{j2\pi u_0 t}\right]$$

$$= \int_{-\infty}^{\infty} f(t)\, e^{j2\pi u_0 t}\, e^{-j2\pi u t}\, dt$$

$$= \int_{-\infty}^{\infty} f(t)\, e^{-j2\pi (u-u_0)t}\, dt$$

$u - u_0 = \nu$

$$= \int_{-\infty}^{\infty} f(t)\, e^{-j2\pi\nu t}\, dt$$

$$= F(\nu) = F(u - u_0)$$

\# $f(t) = 1 \longrightarrow \mathcal{F}[f(t)] = 2\pi\,\delta(\omega)$

$$\int_{-\infty}^{\infty} 1 \cdot e^{-j2\pi\nu t}\, dt \qquad 2\pi a\,\delta(\omega)$$

\# General Periodic Signal:

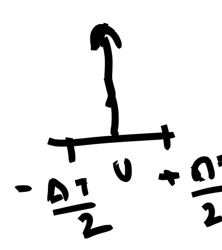
$$f(t) = \sum_{n=-\infty}^{\infty} c_n e^{j\frac{2\pi n}{T}t}$$

$$F(j\omega) = \sum_{n} 2\pi c_n\, \delta\left(\omega - \frac{2\pi n}{T}\right)$$

$$c_n = \frac{1}{T}\int_{-T/2}^{T/2} f(t)\, e^{-j\frac{2\pi n}{T}t}\, dt$$

Consider $f(t) = S_{\Delta T}(t) = \sum_{n=-\infty}^{\infty} \delta(t - n\Delta T)$

$$c_n = \frac{1}{\Delta T}\int_{-\Delta T/2}^{\Delta T/2} \sum_{n=-\infty}^{\infty} \delta(t - n\Delta T)\, e^{-j\frac{2\pi n}{\Delta T}t}\, dt$$



$$= \frac{1}{\Delta T} \int_{-\Delta T/2}^{\Delta T/2} \delta(t) e^{-j\frac{2\pi n}{\Delta T} t} \, dt$$

$$= \frac{1}{\Delta T} e^{0} = \frac{1}{\Delta T}$$

$$F(j\omega) = \frac{2\pi}{\Delta T} \sum_{n=-\infty}^{\infty} \delta\left(\omega - \frac{2\pi n}{\Delta T}\right)$$

$$F(u) = \frac{1}{\Delta T} \sum_{n=-\infty}^{\infty} \delta\left(u - \frac{n}{\Delta T}\right)$$

* <u>Convolution:</u>

$$y(t) = f(t) * h(t) = \int_{-\infty}^{\infty} f(\tau) h(t-\tau) \, d\tau$$

$$F[y(t)]$$

$$= \int_{-\infty}^{\infty} y(t) e^{-j2\pi ut} \, dt$$

$$= \int_{-\infty}^{\infty} \left[\int_{-\infty}^{\infty} f(\tau) h(t-\tau) \, d\tau \right] e^{-j2\pi ut} \, dt$$

$$= \int_{-\infty}^{\infty} f(\tau) \Big[\underbrace{\int_{-\infty}^{\infty} h(t-\tau) e^{-j2\pi u t} dt}_{} \Big] d\tau$$

$h(t) \longleftrightarrow H(u)$

$h(t-\tau) \longleftrightarrow H(u) e^{-j2\pi u\tau}$

$$= \int_{-\infty}^{\infty} f(\tau) H(u) e^{-j2\pi u\tau} d\tau$$

$$= H(u) \int_{-\infty}^{\infty} f(\tau) e^{-j2\pi u\tau} d\tau$$

$$= H(u) F(u)$$

$$\boxed{f(t) * h(t) \longleftrightarrow F(u) H(u)}$$