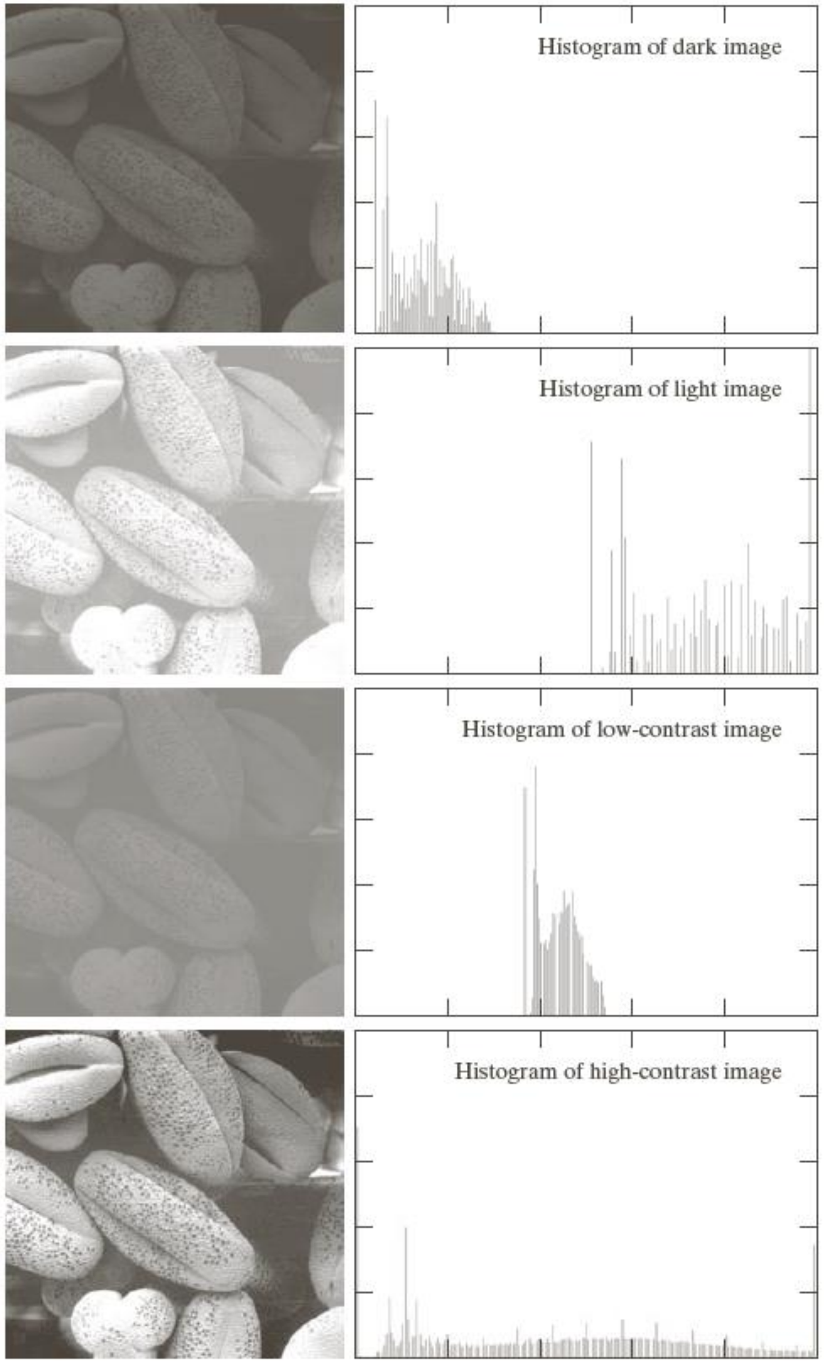
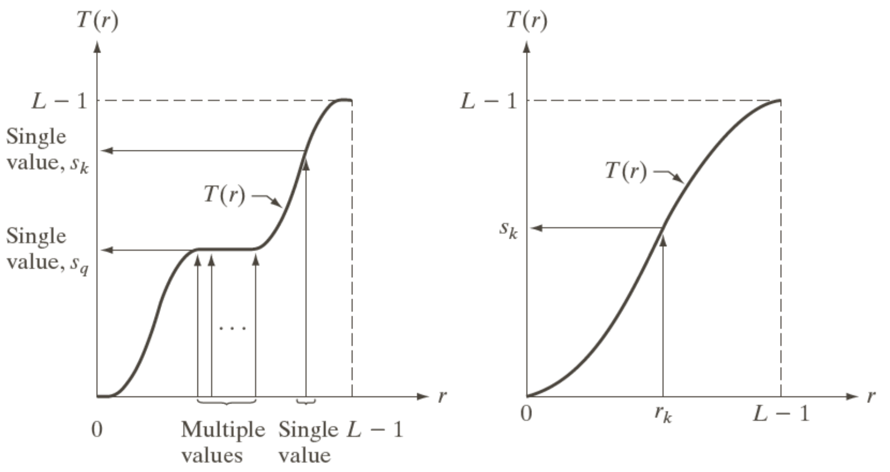
Histogram Processing

- The histogram of a digital image with intensity levels in the range [0,L-1] is a discrete function $h(r_k) = n_k$, where r_k is the kth intensity value and n_k is the no. of pixels in the image with intensity r_k.
- A normalized histogram is given by $p(r_k) = n_k/MN$, for $k = 0, 1,2, \ldots L-1$
- $p(r_k)$ is an estimate of the probability of occurrence of intensity level r_k in an image.
- Sum of all components of a normalized histogram is equal to 1.

Histograms: Examples



Histogram Equalization



- $s = T(r) \qquad 0 \leq r \leq L - 1$
- Assumption about $T(r)$
  - $T(r)$ is monotonically increasing function in the interval $0 \leq r \leq L - 1$
  - $0 \leq T(r) \leq L - 1$ for $0 \leq r \leq L - 1$
- The intensity levels in an image may be viewed as random variables in the interval $[0, L - 1]$.
- R.V. can be described by its PDF.
- A fundamental result from basic probability theory is that if $p_R(r)$ and $T(r)$ are known, and $T(r)$ is continuous and differentiable over the range of values of interest $p_S(s) = p_R(r) \left|\frac{dr}{ds}\right|$

Histogram Equalization (contd.)

- A transformation function of particular importance in image processing has the form $s = T(r) = (L-1)\int_0^r p_R(w)dw$
  - PDFs are always positive
  - Integral of a function is the area under the function (satisfying first condition)
  - Upper limit $r = (L-1)$, the integral evaluates to 1 (the area under a PDF curve always is 1)
  - Maximum value of s is $(L-1)$ (satisfying second condition)
- Find $p_S(s)$. (Tips: Use Leibniz's rule)

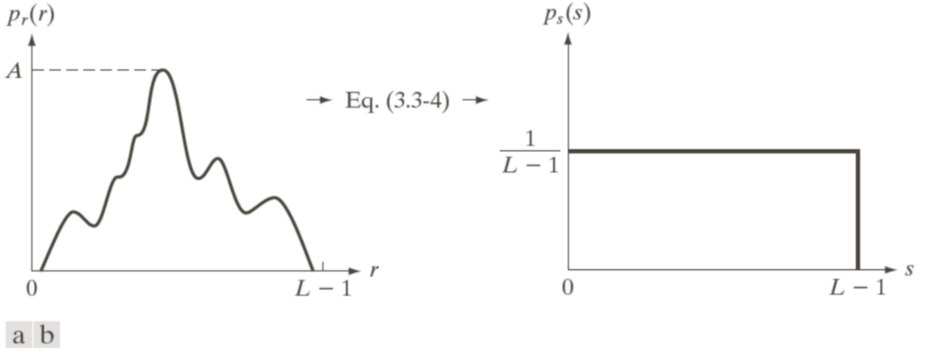


FIGURE 3.18 (a) An arbitrary PDF. (b) Result of applying the transformation in Eq. (3.3-4) to all intensity levels, r. The resulting intensities, s, have a uniform PDF, independently of the form of the PDF of the r's.

Discrete Case: Example

$s_k = (L-1)\sum_{j=0}^{k} p_R(r_j)$

$64 \times 64 = 4096$

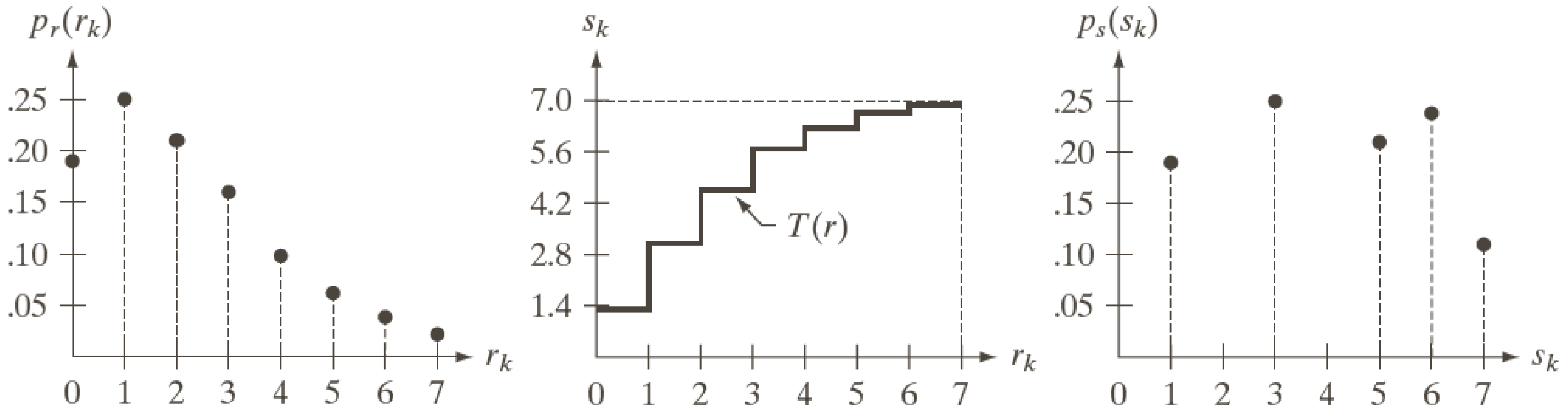
$s_k = \frac{(L-1)}{MN}\sum_{j=0}^{k} n_j$

$L = 8$

$[0, 7]$

r_k	n_k	$p_r(r_k) = n_k/MN$
$r_0 = 0$	790	0.19
$r_1 = 1$	1023	0.25
$r_2 = 2$	850	0.21
$r_3 = 3$	656	0.16
$r_4 = 4$	329	0.08
$r_5 = 5$	245	0.06
$r_6 = 6$	122	0.03
$r_7 = 7$	81	0.02

TABLE 3.1 Intensity distribution and histogram values for a 3-bit, 64×64 digital image.



a b c

FIGURE 3.19 Illustration of histogram equalization of a 3-bit (8 intensity levels) image. (a) Original histogram. (b) Transformation function. (c) Equalized histogram.

$s_0 = (L-1)\,0.19 = 7 \times 0.19 = 1.33 \rightarrow 1$

$s_1 = 7 \times (0.19 + 0.25) = 3.08 \rightarrow 3$

$s_2 = 4.55 \longrightarrow 5$

$s_3 = 5.67 \longrightarrow 6$

$s_4 = 6.23 \longrightarrow 6$

$s_5 = 6.65 \longrightarrow \cancel{7}$

$s_6 = 6.86 \longrightarrow 7$

$s_7 = \cancel{7} \longrightarrow 7$

$$P_s(s_0) = \frac{790}{4096} = 0.19$$

$$P_s(s_1) = \frac{1023}{4096} = 0.24$$

$$P_s(s_2) = \frac{850}{4096} = 0.20$$

$$P_s(s_3) = \frac{656+329}{4096} = 0.24$$

$$P_s(s_5) = \frac{245+122+81}{4096}$$

$$= 0.11$$

Example

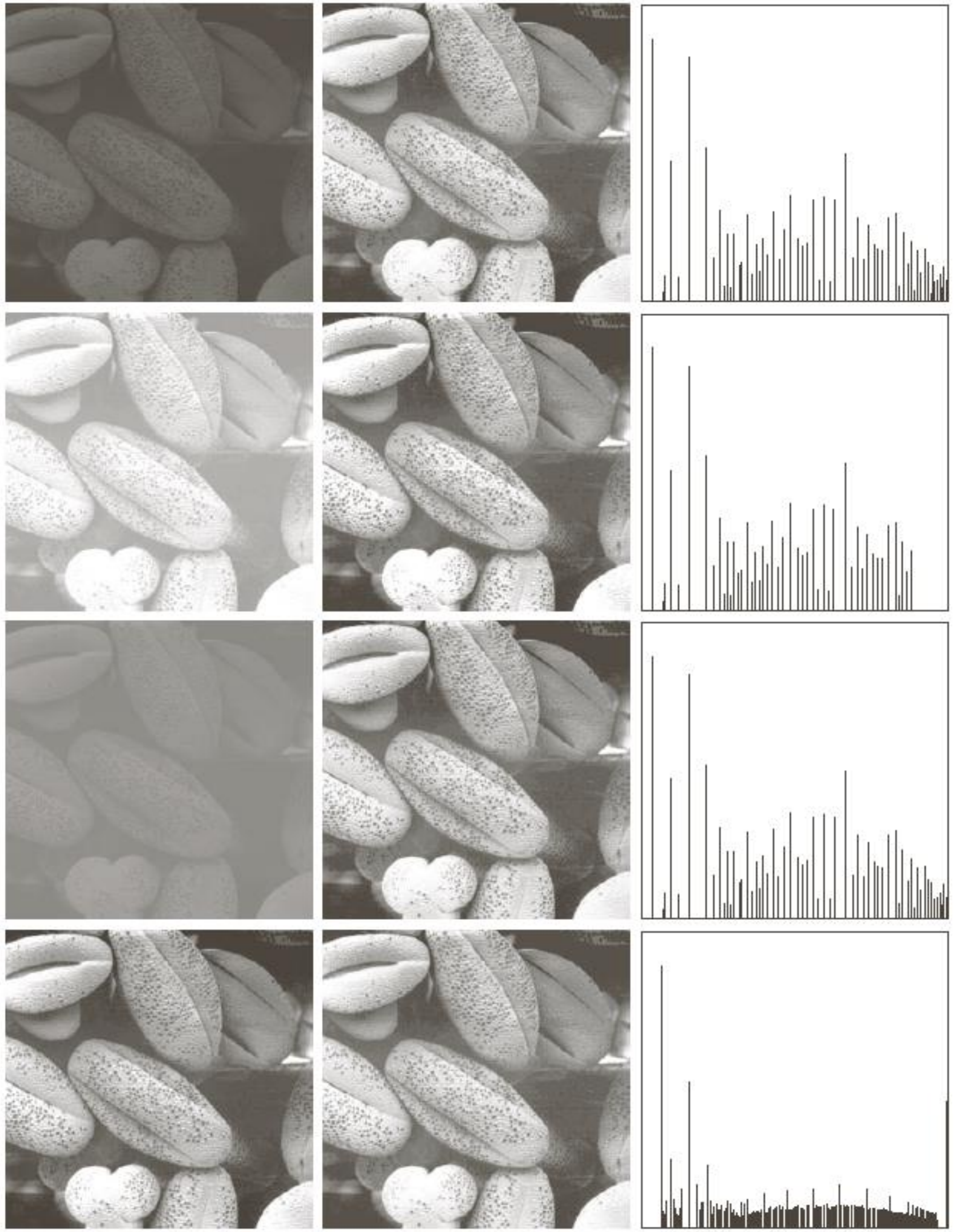


FIGURE 3.20 Left column: images from Fig. 3.16. Center column: corresponding histogram-equalized images. Right column: histograms of the images in the center column.

Histogram Matching (Specification)

$p_z(z)$

- Sometimes it is useful to specify the shape of the histogram that we wish the processed image to have.

  $s = T(r) = (L-1)\int_0^r p_R(w)\,dw$

  - Obtain $p_R(r)$ from the input image, and compute the values of s.
  - Use the specified PDF to obtain the transformation function $G(z)$. $= (L-1)\int_0^z p_z(w)\,dw$
  - Obtain the inverse transformation $z = G^{-1}(s)$
  - Obtain the output image by first equalizing the input image. For each pixel with value s in the equalized image, perform the inverse mapping $z = G^{-1}(s)$ to obtain the corresponding pixel in the output image. When all the pixels have been processed, the PDF of the output image will be equal to the specified PDF.

$p_R(r) = \frac{2r}{(L-1)^2}$ $[0, L-1]$; $p_Z(z) = \frac{3z^2}{(L-1)^3}$

(i) $s = T(r) = (L-1)\int_0^r p_R(w)\,dw = \frac{r^2}{L-1}$

(ii) $G(z) = (L-1)\int_0^z p_Z(v)\,dv = \frac{z^3}{(L-1)^2}$

(iii) $G(z) = s \Rightarrow z = G^{-1}(s)$

$$\frac{z^3}{(L-1)^2} = s \Rightarrow z = \left[(L-1)^2 s\right]^{1/3} = \left[(L-1)\cdot r^2\right]^{1/3}$$

Discrete case:-

(i) $s_k = (L-1) \sum_{j=0}^{k} p_r(r_j)$

(ii) $G(z_q) = (L-1) \sum_{i=0}^{q} p_z(z_i)$

(iii) $G(z_q) = s_k$.

Discrete Case

- Compute the histogram $p_R(r)$ of the given image.
- Compute s_k by histogram equalization and round the resulting values to the integer range $[0, L-1]$.
- Compute all values of the transformation function G for $q = 0,1,2,..,L-1$, where $p_Z(z_i)$ are the values of the specified histogram. Round the values of G to the integers in the range $[0, L-1]$. Store the values of G in a table.
- For every value of s_k, $k = 0,1,2,..,L-1$, use the stored values of G to find the corresponding value of z_q such that $G(z_q)$ is closest to s_k and store these mappings from s to z.
- When more than one value of z_q satisfies the given s_k, choose the smallest value.
- Form the histogram-specified image by first histogram-equalizing the input image and then mapping every equalized pixel value s_k, of this image to the corresponding value z_q in the histogram specified image.

Histogram Matching (contd.)

$s_0 = 1$ $\quad s_5 = 7$

$s_1 = 3$ $\quad s_6 = 7$

$s_2 = 5$ $\quad s_7 = 7$

$s_3 = 6$

$s_4 = 6$

r_k	n_k	$p_r(r_k) = n_k/MN$
$r_0 = 0$	790	0.19
$r_1 = 1$	1023	0.25
$r_2 = 2$	850	0.21
$r_3 = 3$	656	0.16
$r_4 = 4$	329	0.08
$r_5 = 5$	245	0.06
$r_6 = 6$	122	0.03
$r_7 = 7$	81	0.02

$$s_k = (L-1)\sum_{j=0}^{k} p_R(r_j)$$

$$G(z_q) = (L-1)\sum_{i=0}^{q} p_z(z_i)$$

z_q	Specified $p_z(z_q)$	Actual $p_z(z_k)$
$z_0 = 0$	0.00	0.00
$z_1 = 1$	0.00	0.00
$z_2 = 2$	0.00	0.00
$z_3 = 3$	0.15	0.19
$z_4 = 4$	0.20	0.25
$z_5 = 5$	0.30	0.21
$z_6 = 6$	0.20	0.24
$z_7 = 7$	0.15	0.11

TABLE 3.2 Specified and actual histograms (the values in the third column are from the computations performed in the body of Example 3.8).

z_q	$G(z_q)$
$z_0 = 0$	0
$z_1 = 1$	0
$z_2 = 2$	0
$z_3 = 3$	1
$z_4 = 4$	2
$z_5 = 5$	5
$z_6 = 6$	6
$z_7 = 7$	7

TABLE 3.3 All possible values of the transformation function G scaled, rounded, and ordered with respect to z.

s_k	$\rightarrow$	z_q
1	$\rightarrow$	3
3	$\rightarrow$	4
5	$\rightarrow$	5
6	$\rightarrow$	6
7	$\rightarrow$	7

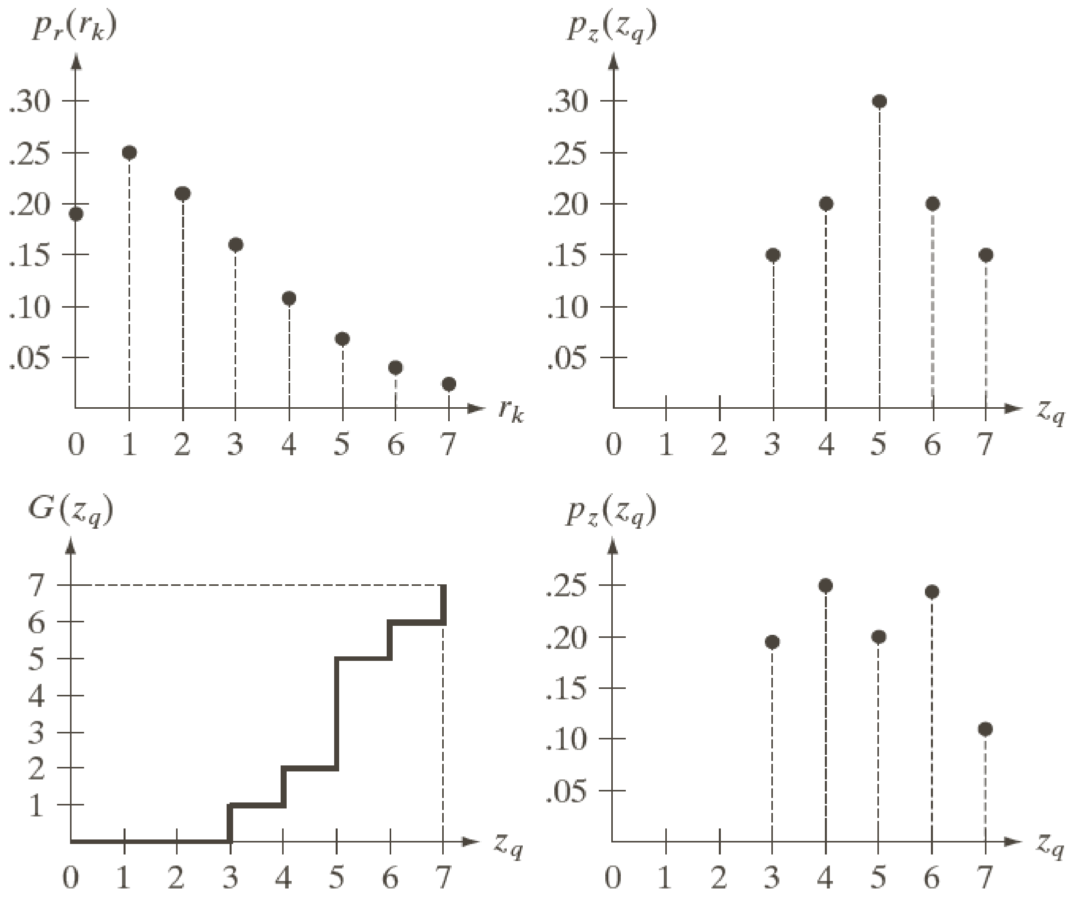
TABLE 3.4 Mappings of all the values of s_k into corresponding values of z_q.

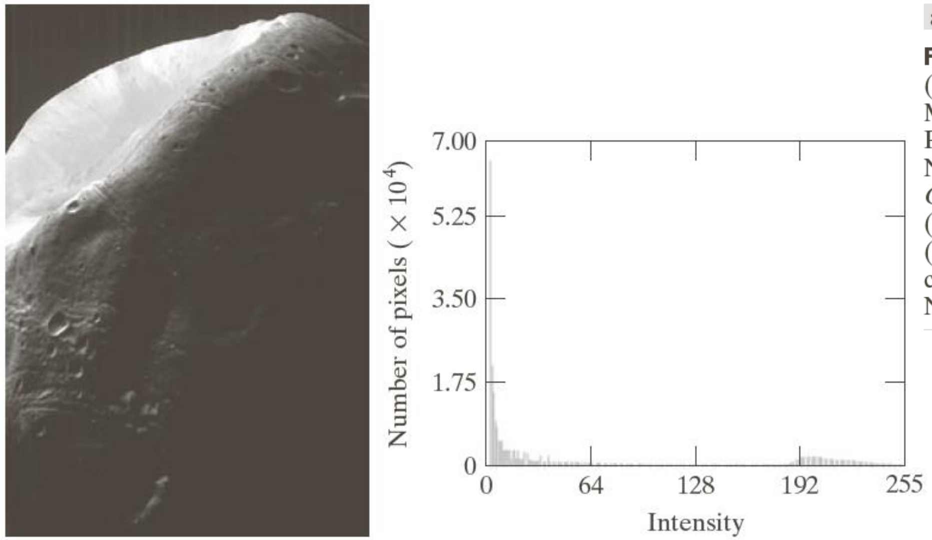
$$z_q = G^{-1}(s_k)$$

$$G(z_q) = s_k$$

$$z_q = 3$$

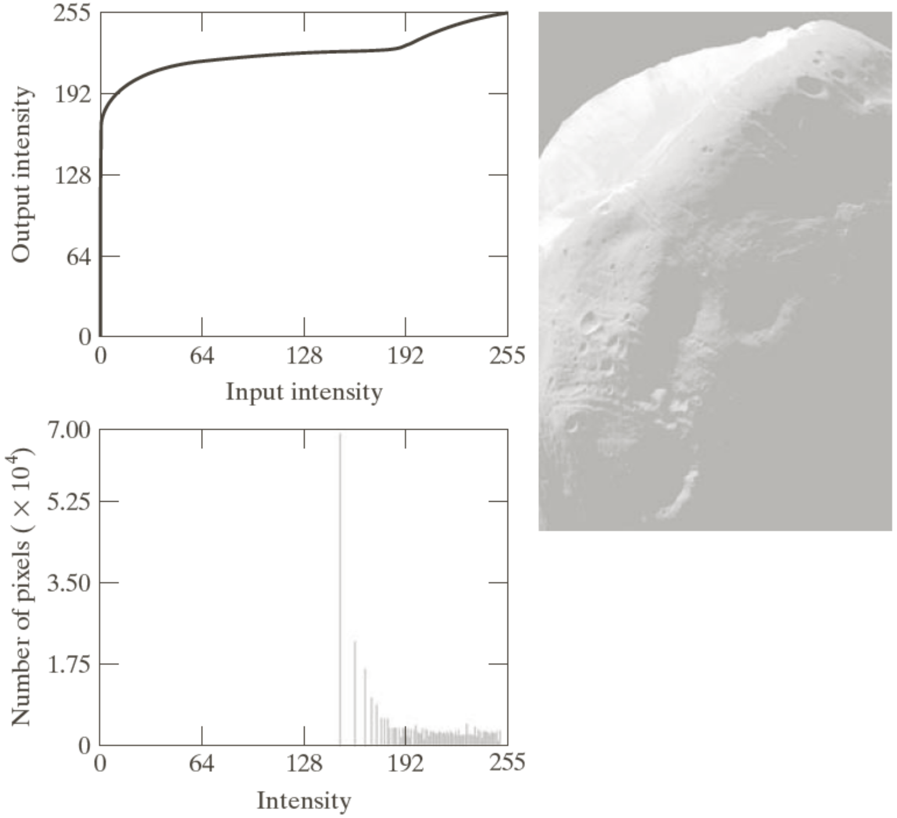
Histogram Matching (contd.)





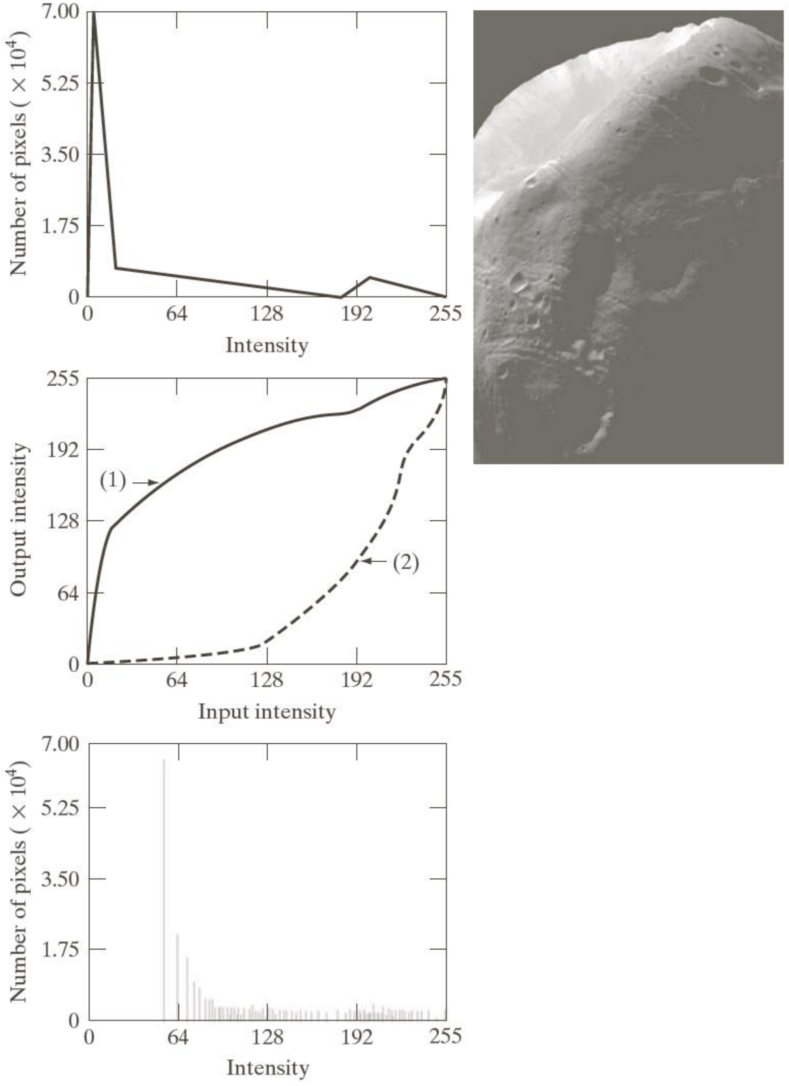
a b

FIGURE 3.23 (a) Image of the Mars moon Phobos taken by NASA's *Mars Global Surveyor.* (b) Histogram. (Original image courtesy of NASA.)



a b
c

FIGURE 3.24 (a) Transformation function for histogram equalization. (b) Histogram-equalized image (note the washed-out appearance). (c) Histogram of (b).



a c
b
d

FIGURE 3.25 (a) Specified histogram. (b) Transformations. (c) Enhanced image using mappings from curve (2). (d) Histogram of (c).

Local Histogram Processing

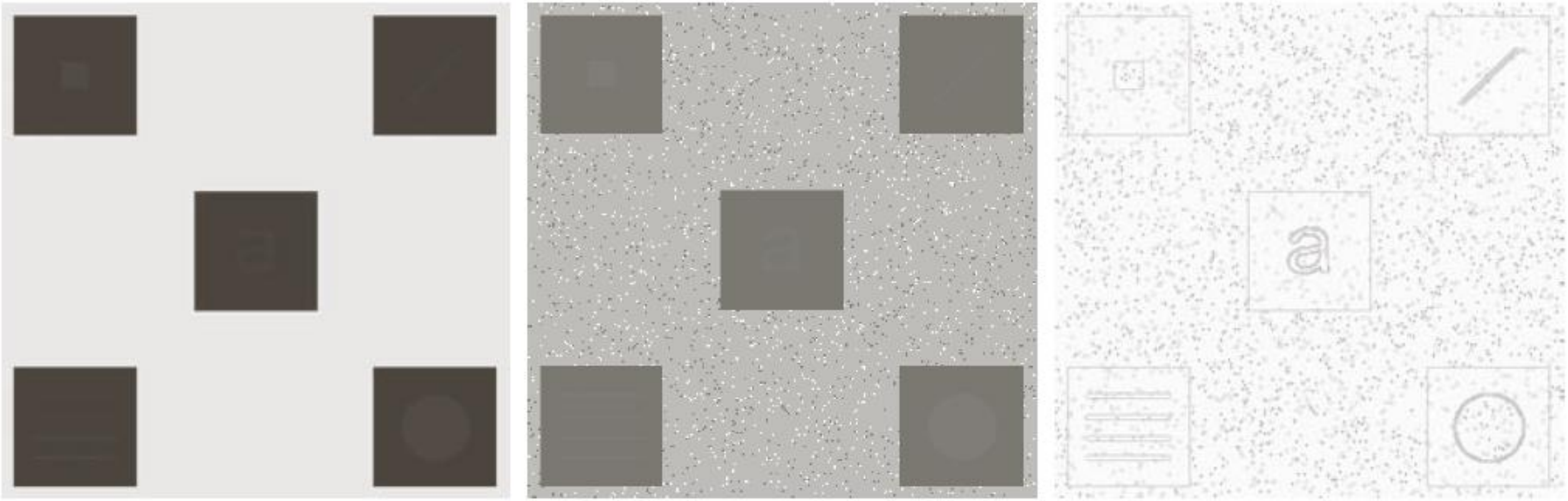


FIGURE 3.26 (a) Original image. (b) Result of global histogram equalization. (c) Result of local histogram equalization applied to (a), using a neighborhood of size 3×3.