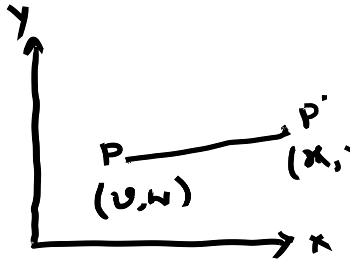
Lecture - 6

Translation:-



Translation vector

$\rightarrow x = v + x_0$

$\rightarrow y = w + y_0$

$$\begin{bmatrix} x_0 \\ y_0 \end{bmatrix}$$

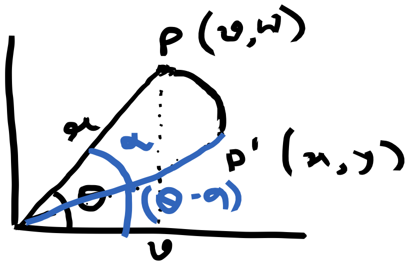
$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v \\ w \end{bmatrix} + \begin{bmatrix} x_0 \\ y_0 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 & 0 & x_0 \\ 0 & 1 & y_0 \end{bmatrix} \begin{bmatrix} v \\ w \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 0 & x_0 \\ 0 & 1 & y_0 \\ 0 & 0 & 1 \end{bmatrix}}_{T} \begin{bmatrix} v \\ w \\ 1 \end{bmatrix}$$

$$\boxed{\begin{bmatrix} x & y & 1 \end{bmatrix} = \begin{bmatrix} v & w & 1 \end{bmatrix} \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ x_0 & y_0 & 1 \end{bmatrix}}_{T^T}}$$

* Rotation:-



$v = r \cos\theta$

$w = r \sin\theta$

$x = r \cos(\theta - \alpha)$

$y = r \sin(\theta - \alpha)$

$$x = r\cos\theta\cos\alpha + r\sin\theta\sin\alpha = v\cos\alpha + w\sin\alpha$$

$$y = r\sin\theta\cos\alpha - r\sin\alpha\cos\theta = w\cos\alpha - v\sin\alpha$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \underbrace{\begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}}_{T} \begin{bmatrix} v \\ w \end{bmatrix}$$

Scalling :- S_x , S_y

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} S_x & 0 \\ 0 & S_y \end{bmatrix} \begin{bmatrix} v \\ w \end{bmatrix}$$

* Vector & matrix operations :-

$\bar{F}_{n\times 1} \leftarrow F_{\sqrt{n}\times\sqrt{n}}$

$$D_e(\bar{f},\bar{g}) = \left[(\bar{f}-\bar{g})^T(\bar{f}-\bar{g})\right]^{\frac{1}{2}}.$$

$$= \|\bar{f}-\bar{g}\|_2 \quad [L_2\text{ norm}]$$

$$= \left[(f_1-g_1)^2 + (f_2-g_2)^2 + \cdots + (f_n-g_n)^2\right]^{1/2}$$

$$\bar{f} = \begin{bmatrix} f_1 \\ f_2 \\ \vdots \\ f_n \end{bmatrix}_{n\times 1} \qquad \bar{g} = \begin{bmatrix} g_1 \\ g_2 \\ \vdots \\ g_n \end{bmatrix}_{n\times 1}$$

$$\bar{g} = \bar{H}\bar{f} + \bar{n}$$

$\bar{g}$ = Blurred and noisy image

$\bar{H}$ = Blurring matrix

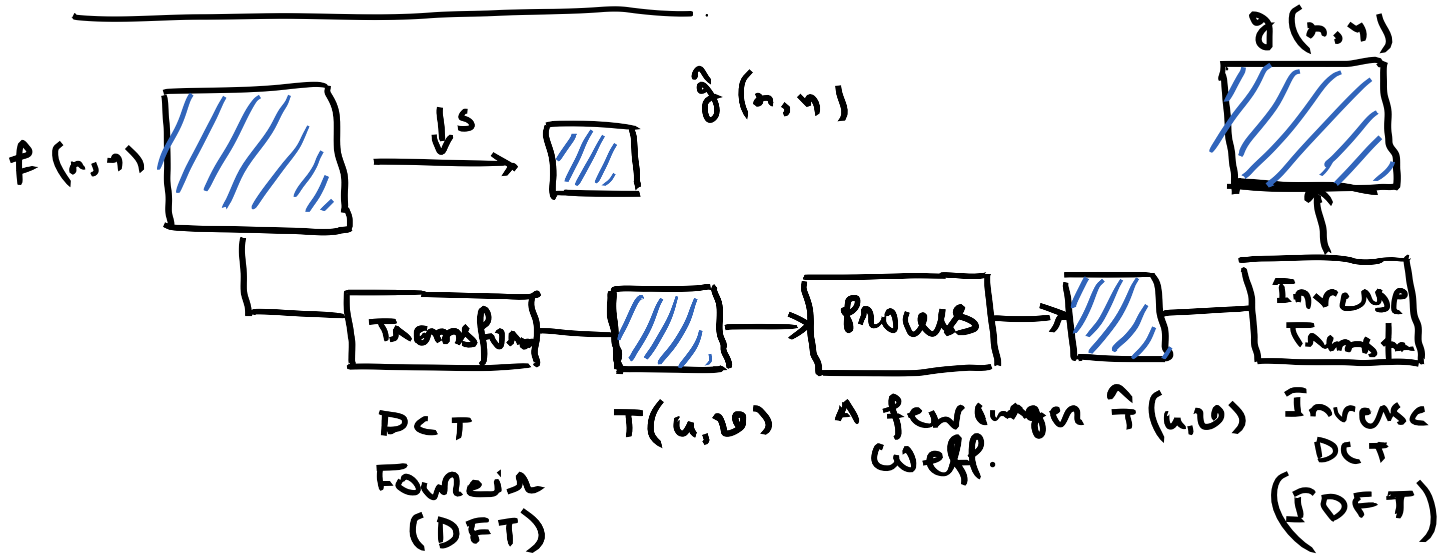
$\bar{f}$ = Original / GT image.

$\bar{n}$ = noise (AWGN)

vector → a (lower case Bold form)

Matrix → $\mathbf{A}$ (upper case Bold form)

* Image Transformation :-



$$\left[f(x,y)\right]_{M\times N}$$

$$T(u,v) = \sum_{x=0}^{M-1}\sum_{y=0}^{N-1} f(x,y)\, \underbrace{r(x,y,u,v)}_{\text{Forward Transformation Kernel}}$$

$u = 0, 1, \dots, M-1$

$v = 0, 1, \dots, N-1$

Process

$$g(x,y) = \hat{f}(x,y) = \sum_{u=0}^{M-1}\sum_{v=0}^{N-1} \hat{T}(u,v)\, \underbrace{s(x,y,u,v)}_{\text{Inverse Transformation kernel}}$$

If the kernel is separable

$$r(x,y,u,v) = r_1(x,u)\, r_2(y,v)$$

If the kernel is symmetric

$$r(x,y,u,v) = r_1(x,u)\, r_1(y,v)$$

1 $r(x,y,u,v) = e^{-j2\pi\left(\frac{ux}{M} + \frac{vy}{N}\right)}$

$$s(x,y,u,v) = \frac{1}{MN} e^{+j2\pi\left(\frac{ux}{M} + \frac{vy}{N}\right)}$$

$$\bar{T} = \bar{A}\bar{F}\bar{A}$$

$$a_{ij} = r_1(i,j)$$

$$\bar{B}\bar{T}\bar{B} = \bar{B}\bar{A}\bar{F}\bar{A}\bar{B} \qquad [\bar{B} = \bar{A}^{-1}]$$

$$\bar{B}\bar{T}\bar{B} = \bar{F}$$

$$\hat{\bar{F}} = \bar{B}\bar{A}\bar{F}\bar{A}\bar{B}$$

* Probabilistic Methods:-

R.V. Z_i $i = 0, 1, \ldots, L-1$

$p(Z_k)$ = probability of kth intensity value.

$$= \frac{n_k}{MN}$$ [n_k = no. of pixels who have Z_k intensity. MN = Dimension of the image]

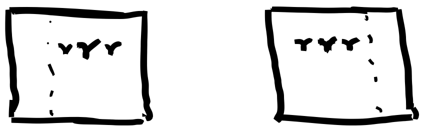
$$\sum_k p(z_k) = 1$$

$$m = \sum_{k=0}^{L-1} z_k p(z_k)$$

$$\sigma^2 = \sum_{k=0}^{L-1} (z_k - m)^2 p(z_k)$$

Image Registration:-

Registration is a process which makes the pixels in two images precisely coincide to the same points in the scene.



→ Disparity map.

↓

Depth map

(i) Image stitching

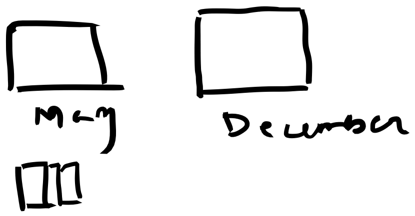
(ii) Stereo imaging

(iii) Remote Sensing.

(iv) Medical imaging.

MRI PET

(v) Template Matching.



$L = 2^k$ $\quad$ k = bit

$L = 2$ $\quad$ L = quantization level.