$$f(t) = \sum_{n=-\infty}^{\infty} \delta(t - n\Delta T)$$

$$F(\mu) = \frac{1}{\Delta T} \sum_{n=-\infty}^{\infty} \delta\left(\mu - \frac{n}{\Delta T}\right)$$

$$f(t) * h(t) \longleftrightarrow F(\mu) H(\mu)$$

<u>Sampling:-</u>

$$\mathcal{F}\{\tilde{f}(t)\} = F(\mu) * S(\mu)$$

$$S(\mu) = \frac{1}{\Delta T} \sum_{n} \delta\left(\mu - \frac{n}{\Delta T}\right)$$

$$F(\mu) * S(\mu) = \int_{-\infty}^{\infty} F(\tau) S(\mu - \tau) \, d\tau$$

$$= \frac{1}{\Delta T} \int_{-\infty}^{\infty} F(\tau) \sum_{n=-\infty}^{\infty} \delta\left(\mu - \tau - \frac{n}{\Delta T}\right) d\tau.$$

$$= \frac{1}{\Delta T} \sum_{n=-\infty}^{\infty} \int_{-\infty}^{\infty} F(\tau) \delta\left(\underbrace{\mu - \tau - \frac{n}{\Delta T}}_{0}\right) d\tau$$

$$\tau = \mu - \frac{n}{\Delta T}$$

$$= \frac{1}{\Delta T} \sum_{n=-\infty}^{\infty} F\left(u - \frac{n}{\Delta T}\right)$$

$f(t) \longleftrightarrow F(u)$

Fourier Transform of a sampled f= is an infinite, periodic sequence of copies of $F(u)$, the transform of the original, continuous f=. $\frac{1}{\Delta T}$.

Sampling Theorem :-

A continuous, band-limited f= can be recovered completely from a set of it's samples if the samples are achieved at a rate exceeding twice the highest freq content of the f=

$$\frac{1}{\Delta T} > 2\, u_{max}$$

$\frac{1}{\Delta T} > 2\, u_{max} \longrightarrow$ Oversampling

$\frac{1}{\Delta T} = 2\, u_{max} \longrightarrow$ Critically-Sampling.

$\frac{1}{\Delta T} < 2\, u_{max} \longrightarrow$ under-Sampling.

DFT of one variable:-

$$\tilde{F}(\mu) = \int_{-\infty}^{\infty} \tilde{f}(t)\, e^{-j2\pi\mu t}\, dt$$ → sampled signal

$$= \int_{-\infty}^{\infty} \sum_{n=-\infty}^{\infty} f(t)\, \delta(t - n\Delta T)\, e^{-j2\pi\mu t}\, dt$$

$$= \sum_{n} \int_{-\infty}^{\infty} f(t)\, \delta(t - n\Delta T)\, e^{-j2\pi\mu t}\, dt$$

$$= \sum_{n} f(n\Delta T)\, e^{-j2\pi\mu n\Delta T}$$

$$\tilde{F}(\mu) = \sum_{n=-\infty}^{\infty} f_n\, e^{-j2\pi\mu n\Delta T} \quad \text{— (1)}$$

$\tilde{F}(\mu)$ $\mu = 0$ to $\frac{1}{\Delta T}$

$$\mu = \frac{m}{M\Delta T} \qquad m = 0, 1, \ldots, M-1$$

$$F_m = \sum_{n=0}^{M-1} f_n\, e^{-j2\pi \frac{m}{M\Delta T} n\Delta T}$$

DFT $\quad F_m = \sum_{n=0}^{M-1} f_n \, e^{-j2\pi mn/M} \qquad m = 0, 1, \cdots, M-1$

IDFT $\quad f_n = \frac{1}{M} \sum_{m=0}^{M-1} F_m \, e^{j2\pi mn/M} \qquad n = 0, 1, \cdots, M-1$

$$F(u) = \sum_{x=0}^{M-1} f(x) \, e^{-j2\pi ux/M} \qquad u = 0, 1, \cdots, M-1$$

$$f(x) = \frac{1}{M} \sum_{u=0}^{M-1} F(u) \, e^{j2\pi ux/M} \qquad x = 0, 1, \cdots, M-1$$

\# $\quad f(x) = \{\underset{\substack{\uparrow \\ 0}}{1}, 2, 4, \underset{\substack{\uparrow \\ 3}}{4}\}$

$F(u) = ?$

$$F(0) = \sum_{x=0}^{3} f(x) \, e^{-j2\pi 0 x/M} = \sum_{x=0}^{3} f(x)$$

$$= 1 + 2 + 4 + 4 = 11$$

$$F(1) = \sum_{x=0}^{3} f(x) \, e^{-j2\pi(1)x/4}$$

$$= 1 \cdot e^{0} + 2 \, e^{-j2\pi \cdot 1/4} + 4 \cdot e^{-j2\pi \cdot 2/4} + 4 \cdot e^{-j\frac{3\pi}{2}}$$

$$= 1 - 2j - 4 + 4j = -3 + 2j$$

$F(2) = ?$ $F(3) = ?$

$F(0) = ✓$, $F(1) = ✓$, $F(2) = ✓$, $F(3) = ✓$

IDFT $f(x) = ?$