Image Restoration (contd.)

Basic approaches to remove degradation

- Inverse filtering

- Wiener filtering

- Least square error filtering

- many more…

$G(u,v) = F(u,v)H(u,v) + N(u,v)$

Inverse filtering

- Once we have the degradation function H(u,v), we can restore the image by

$$\hat{F}(u,v) = \frac{G(u,v)}{H(u,v)}$$

Freq. domain representation of degraded image

$$\hat{F}(u,v) = F(u,v) + \frac{N(u,v)}{H(u,v)}$$

if $H(u,v) \to 0$

$\frac{N(u,v)}{H(u,v)} \to \infty$

- It will be a problem when H(u,v) is very small.

Results

40

70

85

a b
c d

FIGURE Restoring Fig. 5.25(b) with Eq. (5.7-1). (a) Result of using the full filter. (b) Result with *H* cut off outside a radius of 40; (c) outside a radius of 70; and (d) outside a radius of 85.

Weiner filtering

- Here we try to minimize the square error

$$e^2 = E\{(f - \hat{f})^2\}$$

- The solution is

$$\hat{F}(u,v) = \left[\frac{H^*(u,v)S_f(u,v)}{S_f(u,v)|H(u,v)|^2 + S_\eta(u,v)}\right]G(u,v)$$

$$= \left[\frac{H^*(u,v)}{|H(u,v)|^2 + S_\eta(u,v)/S_f(u,v)}\right]G(u,v)$$

$$= \left[\frac{1}{H(u,v)}\frac{|H(u,v)|^2}{|H(u,v)|^2 + S_\eta(u,v)/S_f(u,v)}\right]G(u,v)$$

Inverse vs Wiener Filtering

a b c

FIGURE 5.28 Comparison of inverse and Wiener filtering. (a) Result of full inverse filtering of Fig. 5.25(b). (b) Radially limited inverse filter result. (c) Wiener filter result.

a	b	c
d	e	f
g	h	i

FIGURE 5.29 (a) 8-bit image corrupted by motion blur and additive noise. (b) Result of inverse filtering. (c) Result of Wiener filtering. (d)–(f) Same sequence, but with noise variance one order of magnitude less. (g)–(i) Same sequence, but noise variance reduced by five orders of magnitude from (a). Note in (h) how the deblurred image is quite visible through a "curtain" of noise.

Constrained Least Square Filtering:-

$$g = Hf + n.$$

$$\sum_{x=0}^{M-1} \sum_{y=0}^{N-1} \left[\nabla^2 f(x,y) \right]^2$$

$$Pf$$

$$\hat{f} = \arg\min_{\hat{f}} \left\{ \underbrace{\| g - H\hat{f} \|_2^2} + \gamma \cdot \underbrace{\| Pf \|_2^2} \right\}$$

$$-2H^T(g - Hf) + 2\gamma P^T P f = 0$$

$$-H^T g + H^T H f + \gamma P^T P f = 0$$

$$(H^T H + \gamma P^T P) f = H^T g.$$

$$\hat{f} = (H^T H + \gamma P^T P)^{-1} H^T g.$$

$$H^T H = H^*(u,v)\, H(u,v) = |H(u,v)|^2$$

In freq. domain

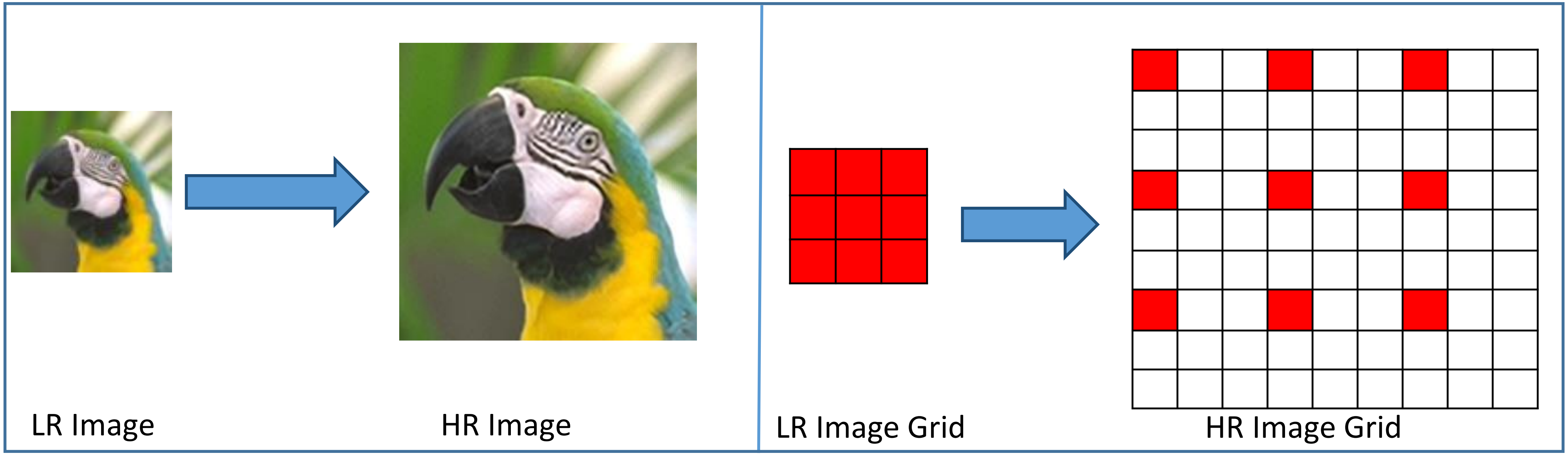
$$\hat{F}(u,v) = \left[\frac{H^*(u,v)}{|H(u,v)|^2 + \gamma |P(u,v)|^2} \right] G(u,v)$$

$$P(x,y) = \begin{bmatrix} 0 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 0 \end{bmatrix}$$

Super Resolution Imaging

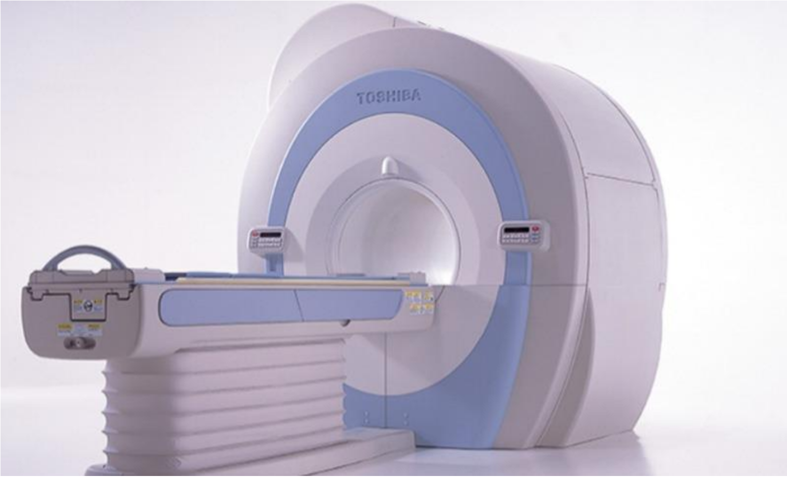
Objective of Super-resolution (SR)

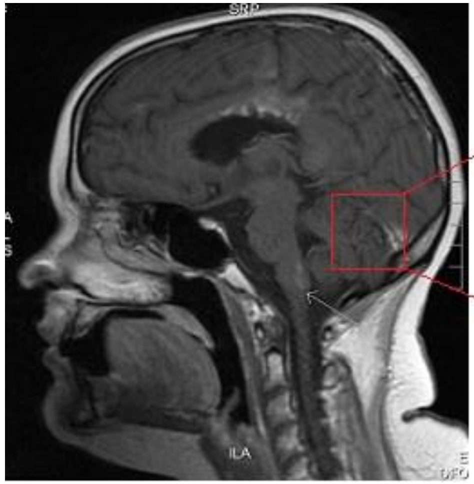
- Obtaining a high resolution (HR) image from the degraded low resolution (LR) image(s) is called SR.



Applications:

- Medical Imaging





Magnetic Resonance Imaging Scanner

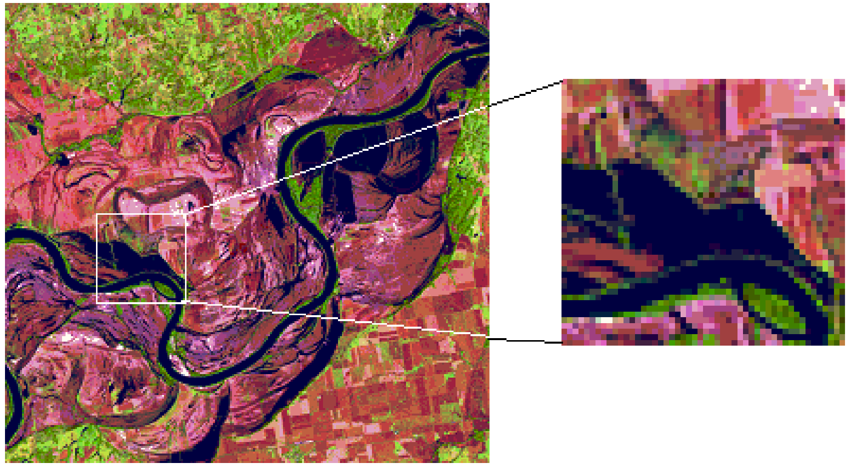
Brain Image

Magnified region of interest (ROI)

Applications: (contd.)

- Remote Sensing

Satellite dedicated for RS



This Landsat image of the Missouri River links to a remote-sensing activity for the Event-Based Science *Flood!* unit. Image shows flood waters as they recede (October 4, 1993)

Magnified ROI part

(From: NASA/Goddard Space Flight Center)

Applications: (contd.)

- Surveillance applications

Surveillance camera

Corridor

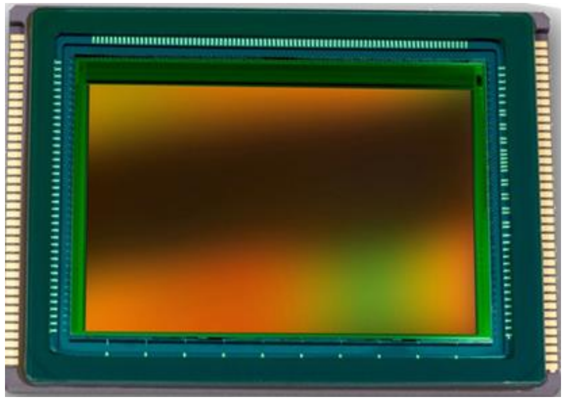
Magnified ROI

(here face region)

and many more...

Other Options to Achieve HR Image

Sensor Modification



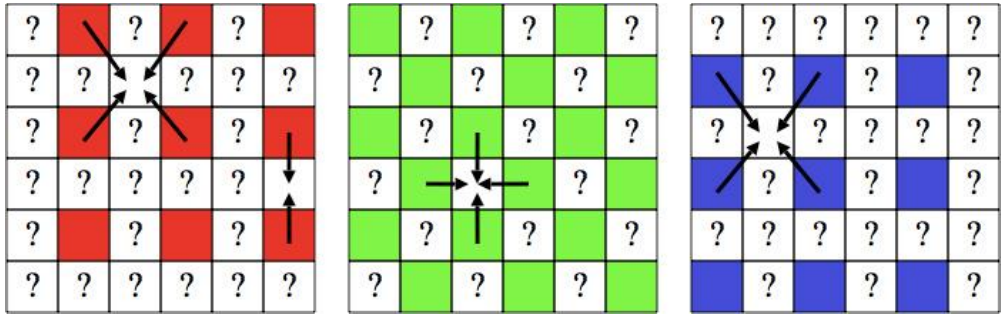
Leica MAX 24MP
CMOS Sensor

- Increase the chip area
- Reduce the size of pixel

Storage requirement
Bandwidth requirement
Distance

Image Processing

- Interpolation Approaches
  - Nearest neighbor, Linear, etc.

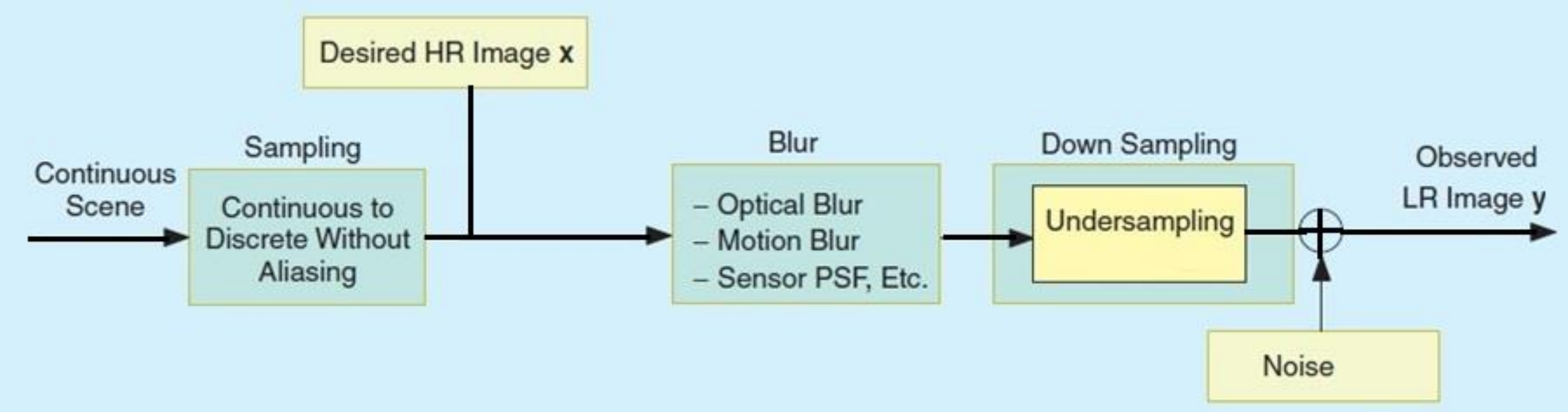


A typical linear interpolation

Blur the edges, corners, textures, etc.

LR Image Formation Model

$3x_1 + 2x_2 = 5$



- Mathematically, formation of LR image can be expressed as:

$$\boldsymbol{y} = \boldsymbol{D}\boldsymbol{H}\boldsymbol{x} + \boldsymbol{n}$$

$\boldsymbol{y} \in \mathbb{R}^{\boldsymbol{b}}$ - LR image,

$\boldsymbol{x} \in \mathbb{R}^{\boldsymbol{a}}$ - HR original image,

$\boldsymbol{D} \in \mathbb{R}^{\boldsymbol{b}\times\boldsymbol{a}}$ - Down-sampling operator,

$\boldsymbol{H} \in \mathbb{R}^{\boldsymbol{a}\times\boldsymbol{a}}$ - Operator responsible for blurring,

$\boldsymbol{n} \in \mathbb{R}^{\boldsymbol{b}}$ - Noise component. $(a > b)$

$a = 2$

$b = 1$

$DH = I$

$y = x + n$

$y = Hx + n$

$y = DHx + n$

- SR aims to recover $\boldsymbol{x}$ from $\boldsymbol{y}$.

ill-posed nature of SR

$$\boldsymbol{y} = \boldsymbol{DHx} + \boldsymbol{n}$$

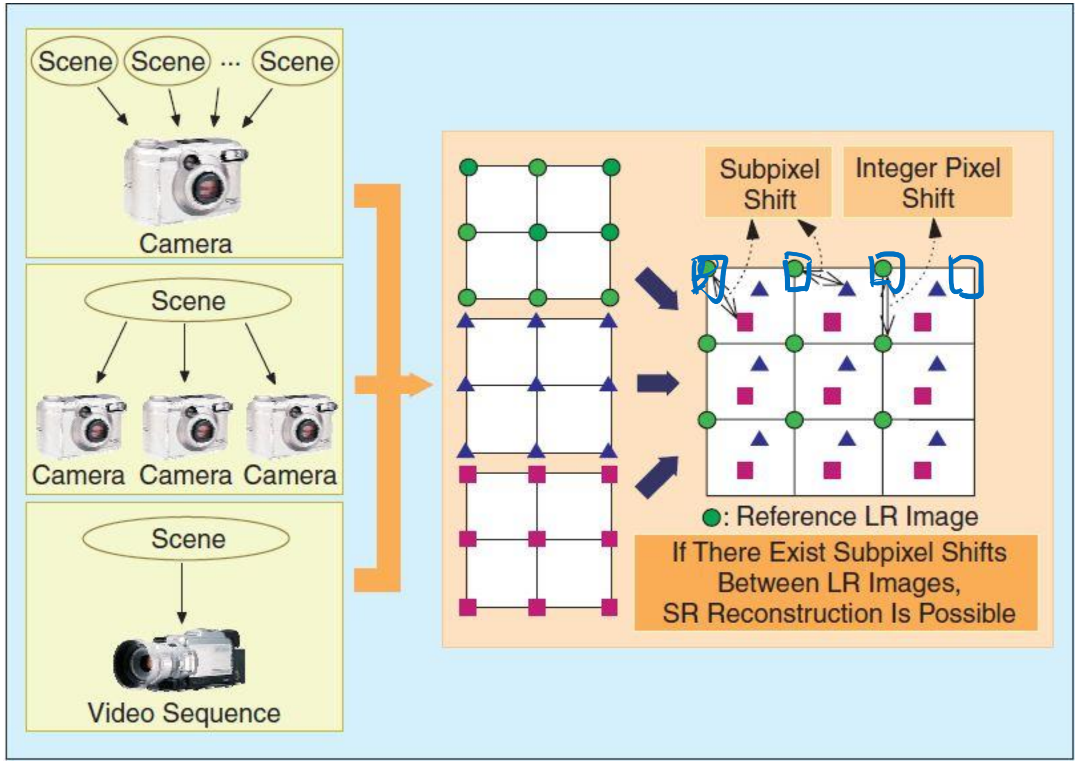
- $\boldsymbol{DH}$ together have more no. of columns than the no. of rows.
- Infinitely many solution are possible.
- Regularization is required

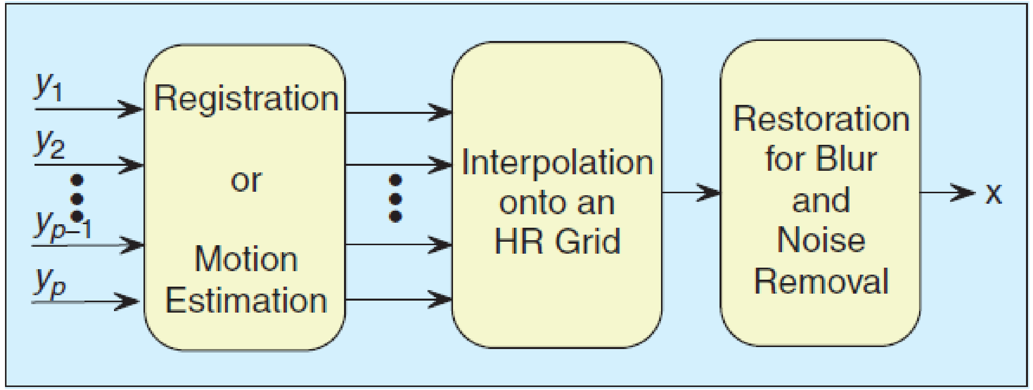
Classification

Based on the number of LR images required to perform SR, it can be classified into three classes:

- Multiple images SR – requires multiple LR images with sub-pixel shift precision.
- Example based single image SR -- requires single LR image without any precision.
- SR from single image – doesn’t require any other image than the test image

Multiple Image SR - Framework

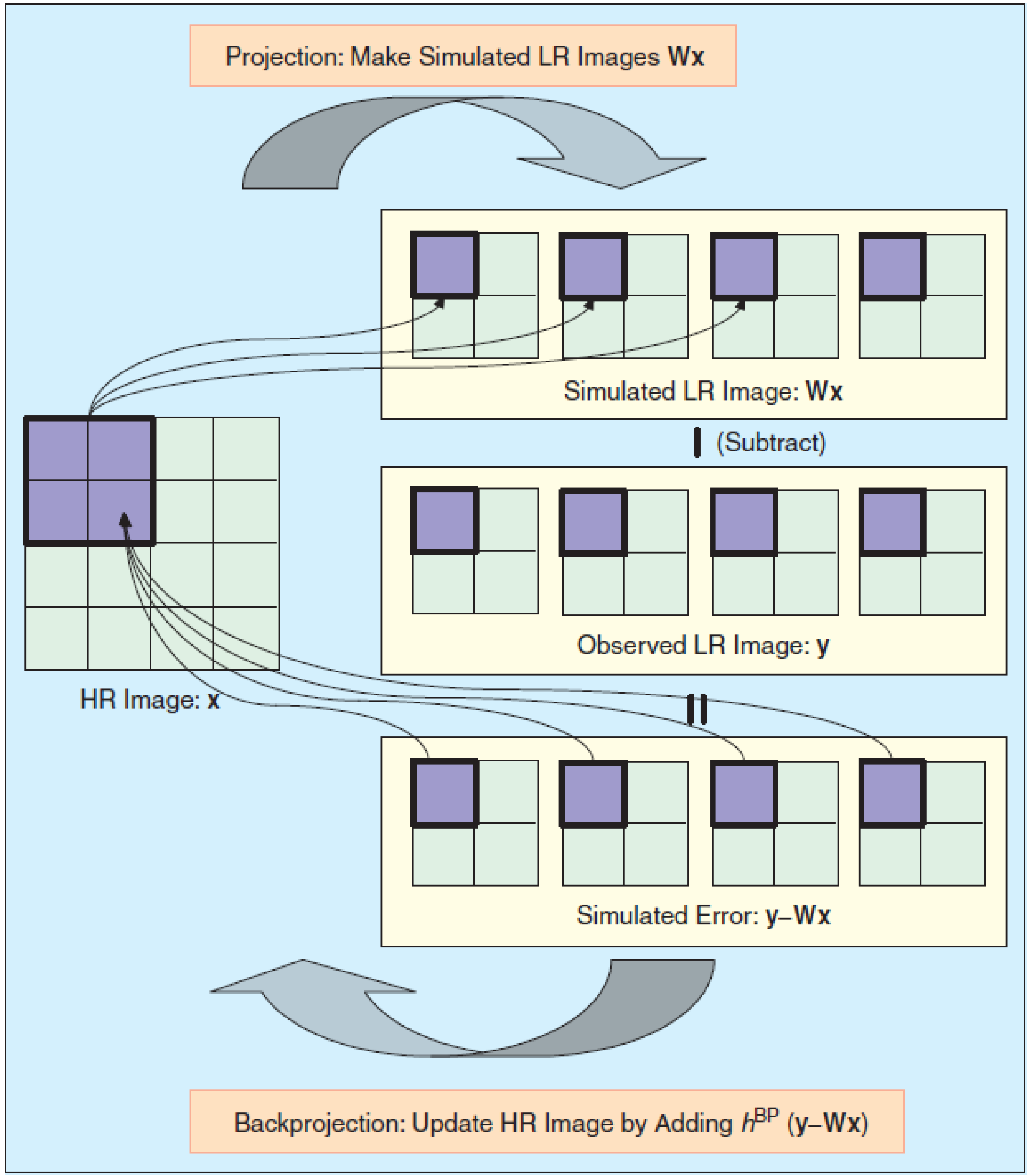




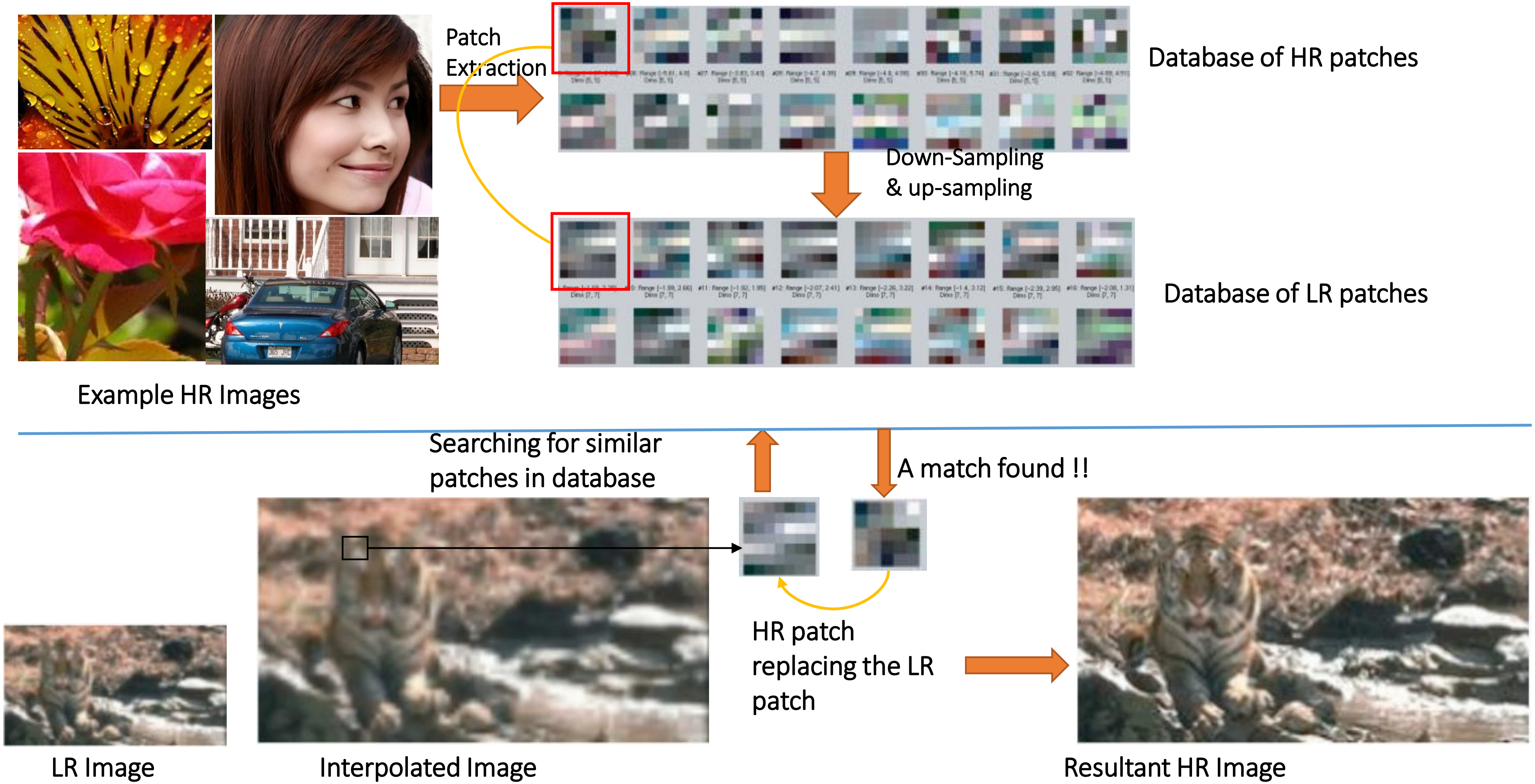
Ref. - Sung Cheol Park; Min Kyu Park; Moon Gi Kang, "Super-resolution image reconstruction: a technical overview," Signal Processing Magazine, IEEE , vol.20, no.3, pp.21-36, May 2003

Iterative Back-Projection

$$\hat{x}^{n+1}[n_1,n_2] = \hat{x}^{n}[n_1,n_2] + \sum_{m_1,m_2 \in \Upsilon_k^{m_1,n_1}} \left(y_k[m_1,m_2] - \hat{y}_k^{n}[m_1,m_2]\right) \times h^{\mathrm{BP}}[m_1,m_2;n_1,n_2]$$



Single Image SR: A Generalized Framework



Results

LR Image
(Interpolated)

HR Image
(Super-resolved)

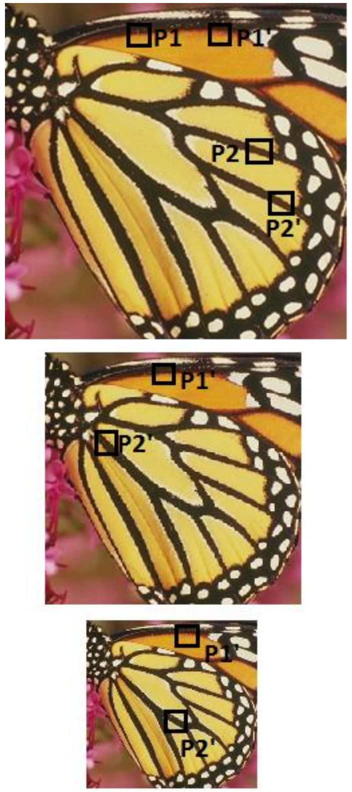
Video SR

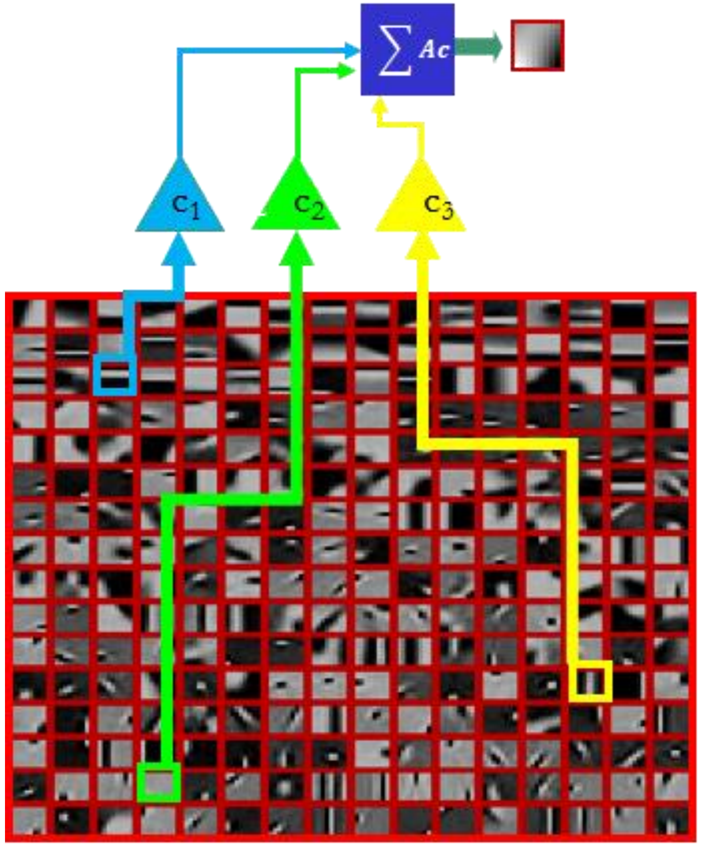
Input LR video (180×256) – Resized

Output HR video (540×768) – Resized

Absence of Example Images

- Form Image pyramid by up/down-sampling the LR image.





Results

LR Image
(Interpolated)

HR Image
(Super-resolved)

Image Denoising based on Anisotropic Diffusion

Anisotropic Diffusion

- Consider the anisotropic diffusion equation:

$$I_t = div(c(x,y,t)\nabla I) = c(x,y,t)\Delta I + \nabla c \cdot \nabla I$$

where c is diffusion coefficient.

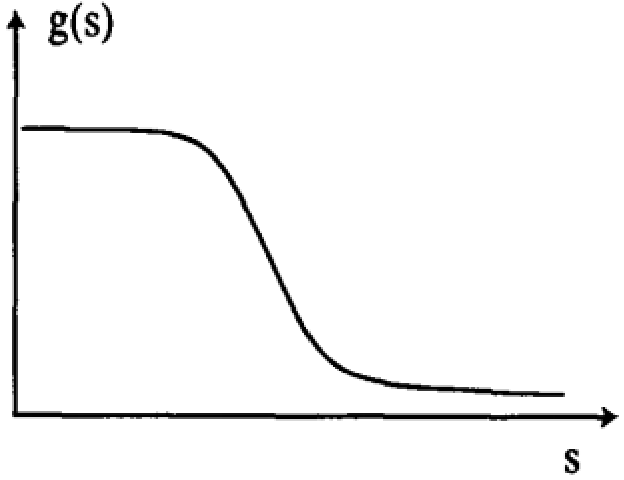
- A common practice is to take

$$c = g(\|\mathbf{E}\|)$$

1. $\mathbf{E}(x,y,t) = \mathbf{0}$ in the interior of each region.
2. $\mathbf{E}(x,y,t) = K\mathbf{e}(x,y,t)$ at each edge point, where $\mathbf{e}$ is a unit vector normal to the edge at the point, and K is the local contrast (difference in the image intensities on the left and right) of the edge.

Anisotropic Diffusion (contd.)

- The qualitative shape of g(.) appears like:



- It has to be non-negative monotonically decreasing function with

  $g(0) = 1$

Anisotropic Diffusion (contd.)

- So, the diffusion will take place in the interior of regions, and it will not affect the region boundaries, where the magnitude of **E** is large.

- A simple estimate of **E** is gradient of the brightness function:

$$\boldsymbol{E}(x, y, t) = \nabla I(x, y, t)$$