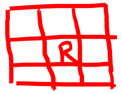
Spatial Filtering (contd..)

N_1 $\quad$ N_2 $\quad$ Result

$x[n] * h[n] = y[n]$ $\quad (N_1 + N_2 - 1)$ without zero padding



Multiply

Addition

Shift

f $\quad$ w

$M_1 \times N_1$ $\quad$ $M_2 \times N_2$

$f * w = g$

$(M_1 + M_2 - 1) \times (N_1 + N_2 - 1)$

Correlation: $w(s,t) \circledast f(x,y)$

$$= \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t)\, f(x+s,\, y+t)$$

$$a = \left(\frac{M_2 - 1}{2}\right); \quad b = \left(\frac{N_2 - 1}{2}\right)$$

$\underline{w}^T \underline{z}$

$= w_1 z_1 + w_2 z_2 + \dots$

$+ w_{mn} z_{mn}$

$= R$

Convolution

$$w(s,t) * f(x,y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t)\, f(x-s,\, y-t)$$

Generating Spatial Filter Masks

- For averaging: $R = \frac{1}{9}\sum_{i=1}^{9} z_i$

$\frac{1}{9} \times$

z_1 1	z_2 1	z_3 1
z_4 1	z_5 1	z_6 1
z_7 1	z_8 1	z_9 1

$\frac{1}{16} \times$

1	2	1
2	4	2
1	2	1

$w_1 = h(-1,-1)$

$(-1,-1)$ w_1	$(-1,0)$ w_2	w_3
w_4	$(0,0)$ w_5	$(0,1)$ w_6
w_7	w_8 $(1,0)$	w_9 $(1,1)$

y

x

- For convolving with Gaussian function: $h(x,y) = e^{-\frac{x^2+y^2}{2\sigma^2}}$

$w_1 \; (\sigma^2 = 1) = e^{-\frac{2}{2}} = e^{-1} = 0.36$

$\boxed{W_5 = \text{Centre}} = 1$

$W_2 = e^{-0.5} = 0.6$

Generalized Filtering Operation

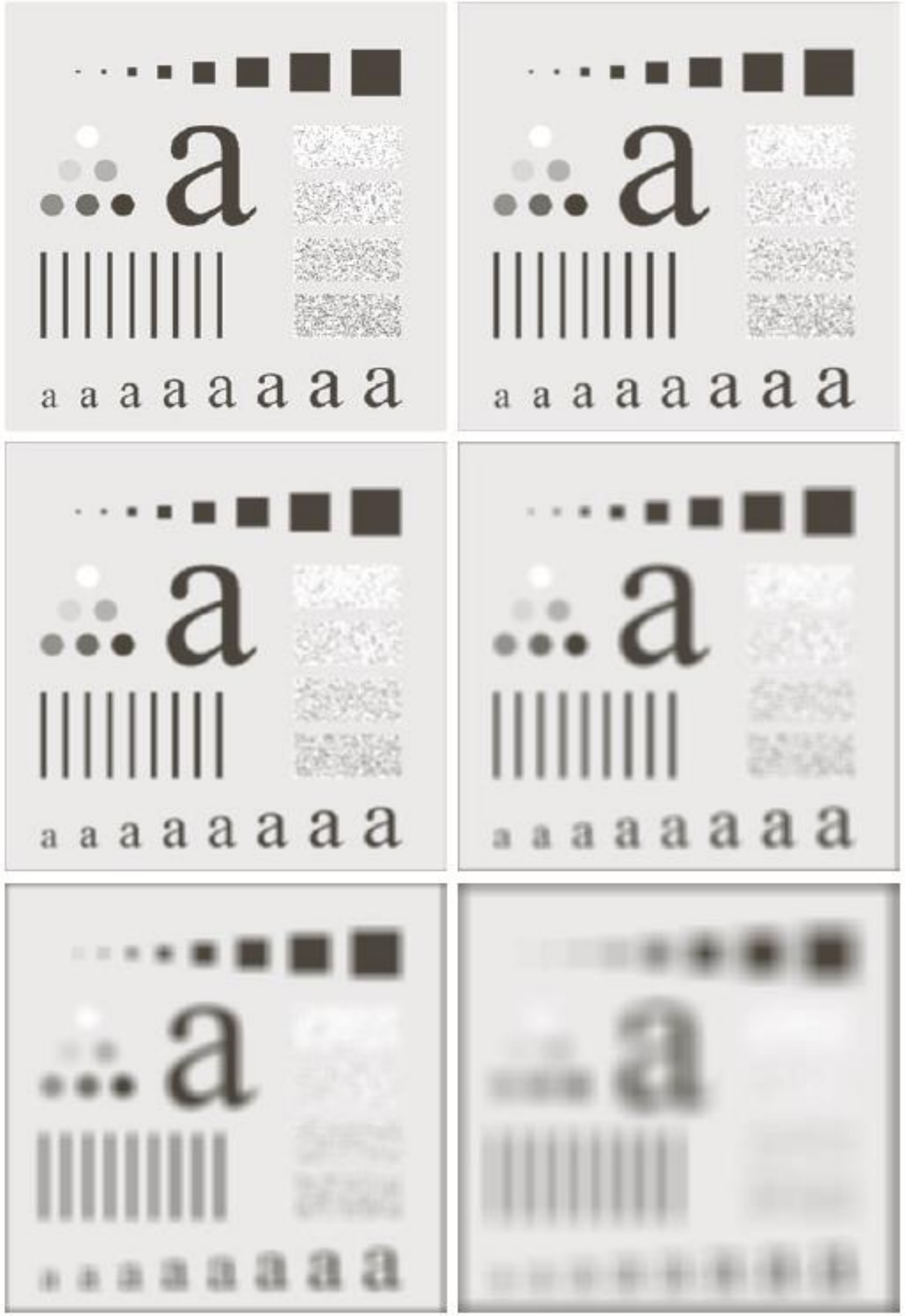
$$g(x, y) = \frac{\sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s, t) f(x + s, y + t))}{\sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s, t)}$$

Results

FIGURE 3.33 (a) Original image, of size 500×500 pixels. (b)–(f) Results of smoothing with square averaging filter masks of sizes $m = 3, 5, 9, 15$, and 35, respectively. The black squares at the top are of sizes 3, 5, 9, 15, 25, 35, 45, and 55 pixels, respectively; their borders are 25 pixels apart. The letters at the bottom range in size from 10 to 24 points, in increments of 2 points; the large letter at the top is 60 points. The vertical bars are 5 pixels wide and 100 pixels high; their separation is 20 pixels. The diameter of the circles is 25 pixels, and their borders are 15 pixels apart; their intensity levels range from 0% to 100% black in increments of 20%. The background of the image is 10% black. The noisy rectangles are of size 50×120 pixels.

a b
c d
e f

15x15



Order Statistic (Non-linear) Filters

255 0

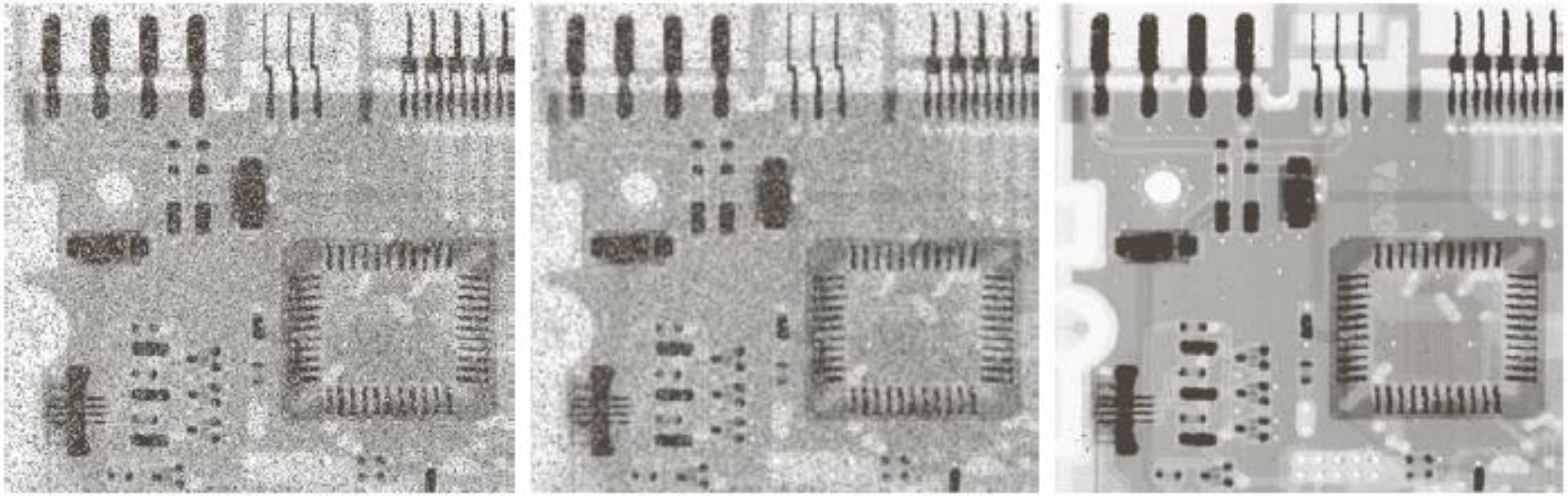
- Median filter
- Max filter
- Min Filter

Median Filtering.

5 70 90

25 100 2

15 128 200



a b c

FIGURE 3.35 (a) X-ray image of circuit board corrupted by salt-and-pepper noise. (b) Noise reduction with a 3 × 3 averaging mask. (c) Noise reduction with a 3 × 3 median filter. (Original image courtesy of Mr. Joseph E. Pascente, Lixi, Inc.)

5, 70, 90, 25, 100, 2, 15, 128, 200

2, 5, 15, 25, 70, 90, 100, 128, 200

Sharpening Spatial Filters

- The principal objective of sharpening is to highlight transitions in intensity.
- Sharpening can be accomplished by spatial differentiation.
- Image differentiation enhances edges and other discontinuities.
- It deemphasizes areas with slowly varying intensities.

Integration → Smoothen image

Differentiation → Sharpening "

$$\frac{\partial f}{\partial x} = f(x+1) - f(x)$$

$$\frac{\partial^2 f}{\partial x^2} = f(x+1) + f(x-1) - 2f(x)$$

$f(x-1)$ $\quad f(x)$ $\quad f(x+1)$

Ramp

$$x(t) = t$$

$$\frac{\partial x(t)}{\partial t} = 1$$

$$\frac{\partial^2 x(t)}{\partial t} = 0$$

$$\frac{df}{dx} = \frac{f(x+h) - f(x)}{h}$$

$$= f(x+1) - f(x)$$

Properties of 1st derivative

2nd derivative

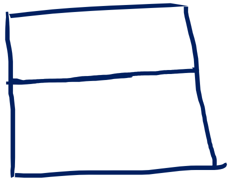
(i) (i) must be '0' in areas of constant intensity.

(ii) (ii) must be non-zero at the onset of an intensity ramp or step.

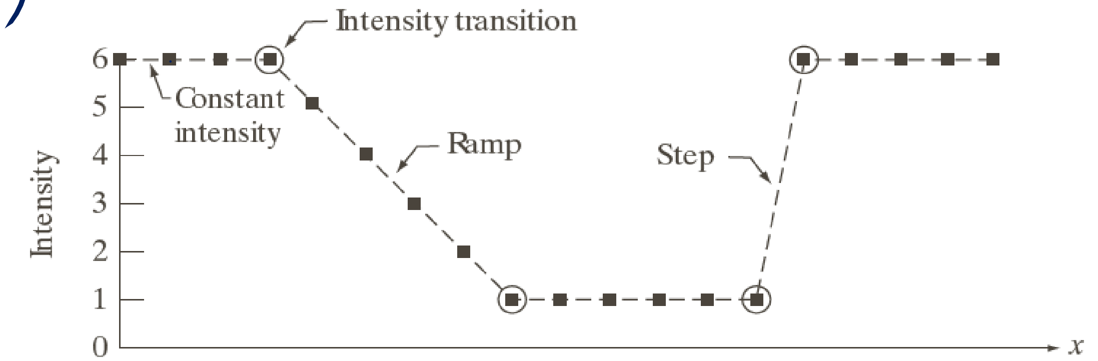
(iii) must be non-zero along the ramp.

(iii) must be zero along the ramp.

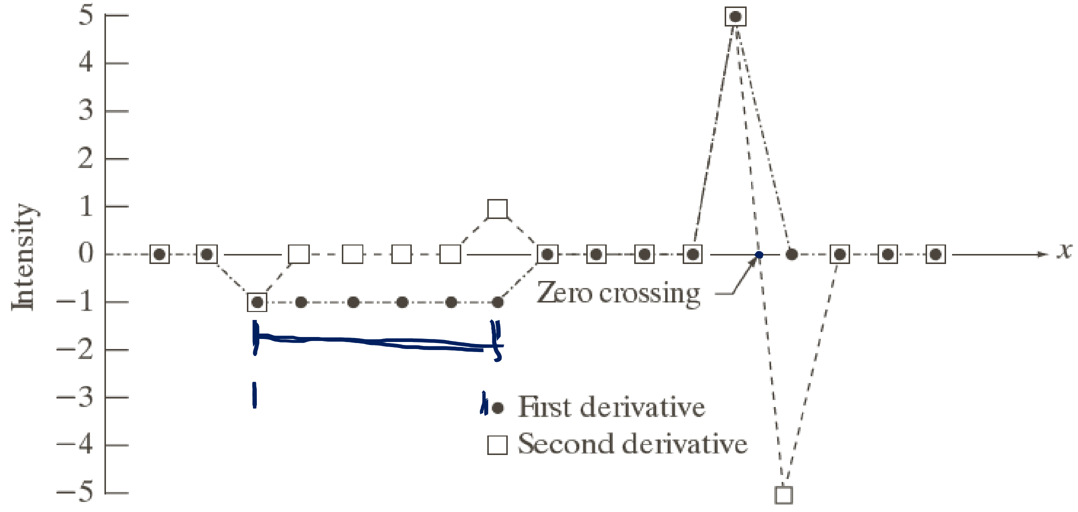
First and Second Derivatives

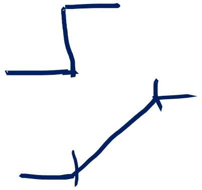


Intensity profile



Scan line	6	6	6	6	5	4	3	2	1	1	1	1	1	1	6	6	6	6	6
1st derivative		0	0	−1	−1	−1	−1	−1	0	0	0	0	0	5	0	0	0	0	
2nd derivative		0	0	−1	0	0	0	0	1	0	0	0	0	5	−5	0	0	0	





Using Second Derivative (The Laplacian)

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

$$\frac{\partial^2 f}{\partial x^2} = f(x+1, y) + f(x-1, y) - 2f(x,y)$$

$$\frac{\partial^2 f}{\partial y^2} = f(x, y+1) + f(x, y-1) - 2f(x,y)$$

$$\nabla^2 f = f(x+1, y) + f(x-1, y) + f(x, y+1) + f(x, y-1) - 4f(x,y) + f(x-1, y+1) + f(x+1, y-1) - 2f(x,y) + f(x+1, y+1) + f(x-1, y-1) - 2f(x,y)$$

0	1	0
1	−4	1
0	1	0

1	1	1
1	−8	1
1	1	1

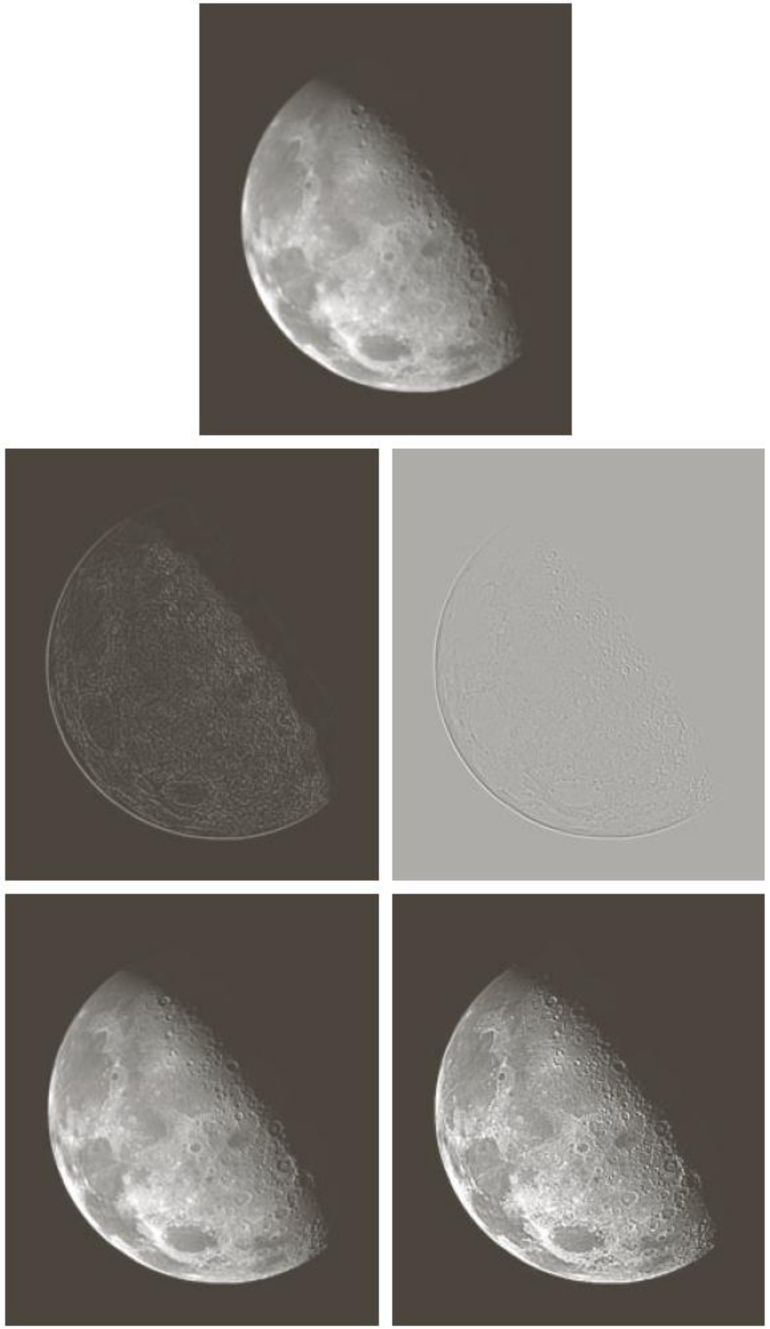
0	−1	0
−1	4	−1
0	−1	0

−1	−1	−1
−1	8	−1
−1	−1	−1

$$g(x,y) = f(x,y) + c\left[\nabla^2 f(x,y)\right]$$

$c = -1$ or $+1$

Results



a
b c
d e

FIGURE 3.38 (a) Blurred image of the North Pole of the moon. (b) Laplacian without scaling. (c) Laplacian with scaling. (d) Image sharpened using the mask in Fig. 3.37(a). (e) Result of using the mask in Fig. 3.37(b). (Original image courtesy of NASA.)

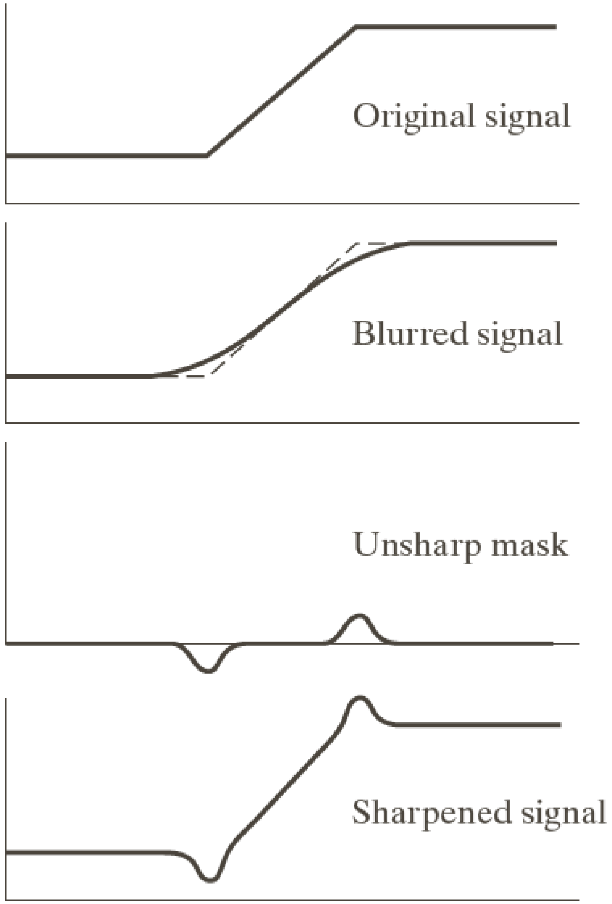
$$\begin{matrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{matrix}$$

$$\begin{matrix} 1 & 1 & 1 \\ 1 & -8 & 1 \\ 1 & 1 & 1 \end{matrix}$$

Unsharp Masking and Highboost Filtering

- $g_{mask}(x, y) = f(x, y) - \bar{f}(x, y)$
- $g(x, y) = f(x, y) + k * g_{mask}(x, y)$
- $k = 1$: unsharp masking
- $k > 1$: highboost filtering

Results



a
b
c
d
e

FIGURE 3.40
(a) Original image.
(b) Result of blurring with a Gaussian filter.
(c) Unsharp mask. (d) Result of using unsharp masking.
(e) Result of using highboost filtering.