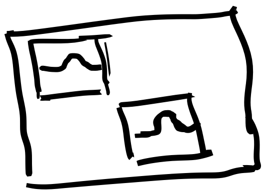
Lecture - 8

1

Interpolation:-



-> Finite region of support

-> Smooth, no-discontinuity

-> Shift invariant.

$$f(x) \longrightarrow \boxed{H} \longrightarrow g(x)$$

$$f(x-x_0) \longrightarrow \boxed{H} \longrightarrow g(x-x_0)$$

B-spline fn:- A piecewise polynomial fn ~~that~~ can be used to provide local approximation of curves using small no of parameters.

$$x(t) = \sum_{i=0}^{n} P_i \, B_{i,k}(t)$$

i = no. of samples we have to use for approximation. $(n+1)$

P_i = Control points

$B_{i,k}$ = normalized B-spline fn. of order k.

Order 1

$$B_{i,1} = \begin{cases} 1 & t_i \le t < t_{i+1} \\ 0 & \text{otherwise.} \end{cases}$$

$$B_{i,k}(t) = \frac{(t-t_i)\,B_{i,k-1}(t)}{t_{i+k-1} - t_i} + \frac{(t_{i+k} - t)\,B_{i+1,k-1}(t)}{t_{i+k} - t_{i+1}}$$

$$B_{i,k}(t) = B_{0,k}(t-i) \Leftarrow \text{Shift invariant.}$$

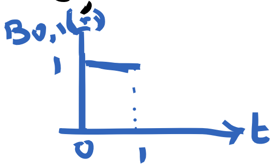
$$B_{0,1}(t) = \begin{cases} 1 & 0 \le t < 1 \\ 0 & \text{otherwise.} \end{cases}$$

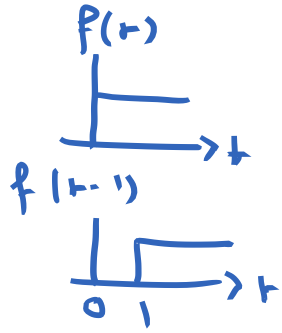
$t_i \le t < t_{i+1}$

$i = 0$

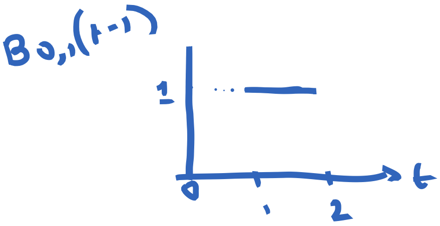
$t_0 = 0$

$t_1 = 1$





$$B_{1,1}(t) = B_{0,1}(t-1) = \begin{cases} 1 & 1 \le t < 2 \\ 0 & \text{else} \end{cases}$$

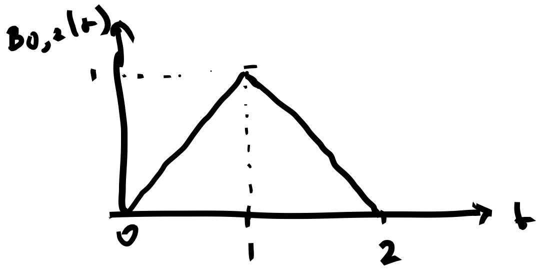


$$B_{2,1}(t) = B_{0,1}(t-2) = \begin{cases} 1 & 2 \le t < 3 \\ 0 & \text{else} \end{cases}$$

$$B_{0,2}(t) = \frac{(t-0)\, B_{0,1}(t)}{1-0} + \frac{(2-t)\, B_{1,1}(t)}{2-1}$$

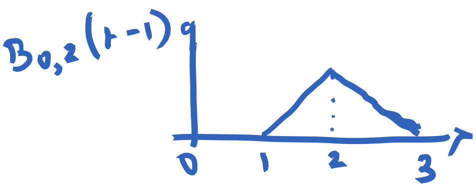
$$= t\, B_{0,1}(t) + (2-t)\, B_{1,1}(t)$$

$$B_{0,2}(t) = \begin{cases} t & 0 \le t < 1 \\ 2-t & 1 \le t < 2 \\ 0 & \text{otherwise} \end{cases}$$



$$B_{0,3}(t) = \frac{(t-0)\, B_{0,2}(t)}{2-0} + \frac{(3-t)\, B_{1,2}(t)}{3-1}$$

$$= \frac{t}{2} B_{0,2}(t) + \frac{3-t}{2} B_{1,2}(t)$$



$B_{0,3}(t)$: $t^2/2 \; ; \; 0 \le t < 1$

$\frac{t}{2}(2-t) + \frac{3-t}{2}(t-1) \qquad 1 \le t < 2$

$= -t^2 + 3t - 1.5$

$$= \left(\frac{3-t}{2}\right)(3-t) \qquad 2 \leq t < 3$$

$$= \frac{(3-t)^2}{2}.$$

$$B_{0,3}(t) = \begin{cases} t^2/2 & 0 \leq t < 1 \\ -t^2+3t-1.5 & 1 \leq t < 2 \\ \frac{(3-t)^2}{2} & 2 \leq t < 3 \\ 0 & \text{elsewhere} \end{cases}$$

$$B_{0,4}(t) = \frac{t\, B_{0,3}(t)}{3-0} + \frac{(4-t)\, B_{1,3}(t)}{4-1}$$

$$= \frac{t}{3} B_{0,3}(t) + \frac{(4-t)}{3} B_{1,3}(t) \longrightarrow B_{0,3}(t-1)$$

$$B_{0,4}(t) = \frac{t^3}{6} \qquad 0 \leq t < 1$$

$$\left.\begin{aligned} &\frac{t}{3}\left[-t^2+3t-1.5\right] + \frac{4-t}{3}\frac{(t-1)^2}{2} \\ &= \frac{-3t^3+12t^2-12t+4}{6} \end{aligned}\right\} \quad 1 \leq t < 2$$

$$\left.\begin{aligned} &\frac{t}{3}\frac{(3-t)^2}{2} + \frac{(4-t)}{3}\left[-(t-1)^2+3(t-1)-1.5\right] \\ &= \frac{3t^3-24t^2+60t-44}{6} \end{aligned}\right\} \quad 2 \leq t < 3$$

$$\frac{4-t}{3}\frac{\left[3-(t-1)\right]^2}{2} = \frac{(4-t)^3}{6} \qquad 3 \leq t < 4$$

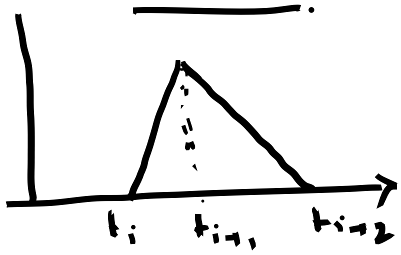
Constant

$B_{i,1}(t)$ $\equiv NN$

Linear

$B_{i,2}(t)$

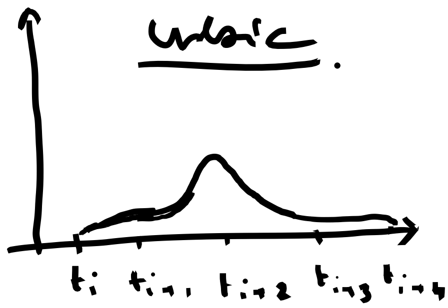
Bilinear



$B_{i,3}(t)$ quadratic

$B_{i,4}(t)$ cubic

Bicubic



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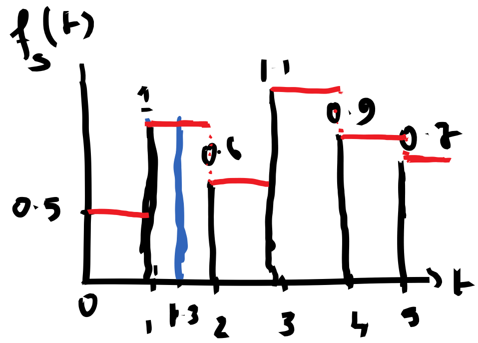


Using Constant interpolation

$t = 1.3$ What is $f_s(t)$?

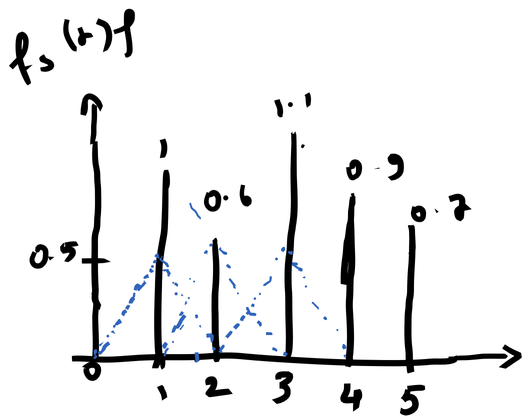
$$f_s(t) = \sum_{i=0}^{n} P_i B_{i,1}(t)$$

$$B_{0,1}(t) = \begin{cases} 1 & 0 \le t < 1 \\ 0 & \text{else} \end{cases}$$

$$B_{1,1}(t) = \begin{cases} 1 & 1 \le t < 2 \\ 0 & \text{else} \end{cases}$$

$$f_s(1.3) = \underbrace{P_0 B_{0,1}(1.3)}_{0} + P_1 B_{1,1}(1.3) + \underbrace{P_2 B_{2,1}(1.3)}_{0} + \cdots$$

$$= P_1 \times 1 = P_1 = 1$$



$f_s(1.7)$ using linear interpolation

$$f_s(1.7) = \sum_{i=0}^{n} P_i \, B_{i,2}(1.7)$$

$$= \underbrace{P_0 B_{0,2}(1.7)} + \underbrace{P_1 B_{1,2}(1.7)}$$

$$+ \underbrace{P_2 B_{2,2}(1.7)}_{0} + \cdots \underbrace{}_{0}$$

$$= P_0 B_{0,2}(1.7) + P_1 B_{0,2}(0.7)$$

$$= 0.5 \times 0.3 + 1 \times 0.7$$

$$= 0.15 + 0.7 = 0.85$$

$$B_{0,2}(t) = \begin{cases} t & 0 \le t < 1 \\ 2-t & 1 \le t < 2 \end{cases}$$

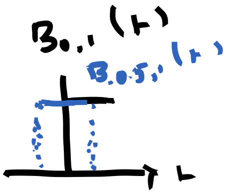
$$B_{1,2}(t) = B_{0,2}(t-1)$$

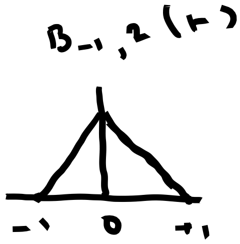
$$\hat{f}(t) = \sum_{i=0}^{n} P_i \, B_{i-s,k}(t)$$

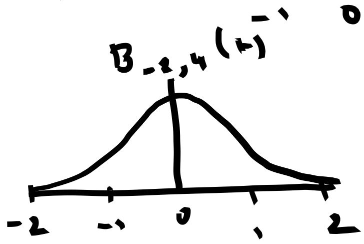
$k = 1 \longrightarrow s = 0.5$

$k = 2 \longrightarrow s = 1$

$k = 4 \longrightarrow s = 2$







$B_{i-1,2}$

$s = 1$

$$f_s(1.7) = P_1 B_{1,2}(1.7) + P_2 B_{2,2}(1.7)$$

$$= 1 \times B_{0,2}(0.7) + 0.6 \times B_{2,2}(1.7)$$

$$f_s(1.7) = \underbrace{P_0 B_{-1,2}(1.7)}_{0} + P_1 B_{0,2}(1.7) + P_2 B_{1,2}(1.7)$$

$$+ \underbrace{P_3 B_{2,2}(1.7)} + \cdots$$

$B_{0,2}(0.7)$

$$= 1 \times 0.3 + 0.6 \times 0.7$$

$$= 0.3 + 0.42 = 0.72$$

* $f_s(2.3) = ?$ using linear interpolation.

* Cubic Interpolation:

$$f(t) = \sum_{i=0}^{n} P_i \, B_{i-2,4}(t)$$

shift

Nearest Neighbour interpolation (constant interpolation)

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