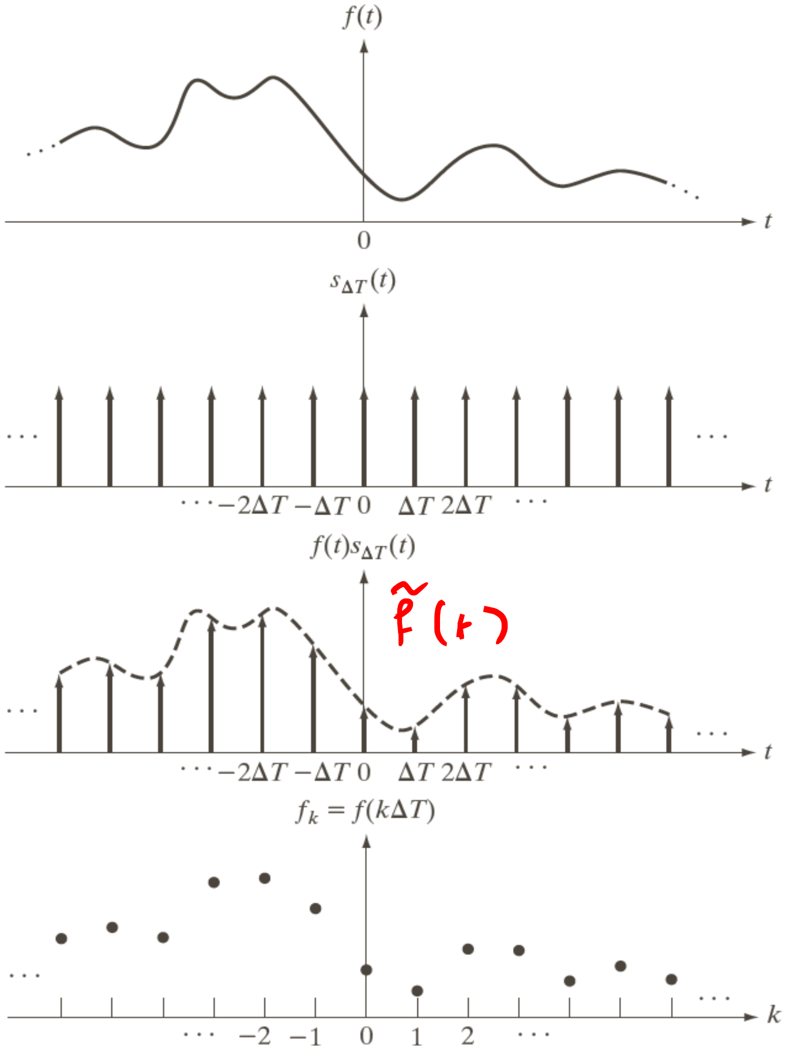
Frequency Domain Filtering

Topics

- Fourier series
- Impulses and their sifting property
- FT of functions of one continuous variable
- FT of convolution
- Sampling (Fourier transform of sampled functions)

Sampling



$$\tilde{f}(t) = f(t)\, s_{\Delta T}(t)$$

$$= f(t) \sum_{n=-\infty}^{\infty} \delta(t - n\Delta T)$$

$$= \sum_{n=-\infty}^{\infty} f(t)\, \delta(t - n\Delta T)$$

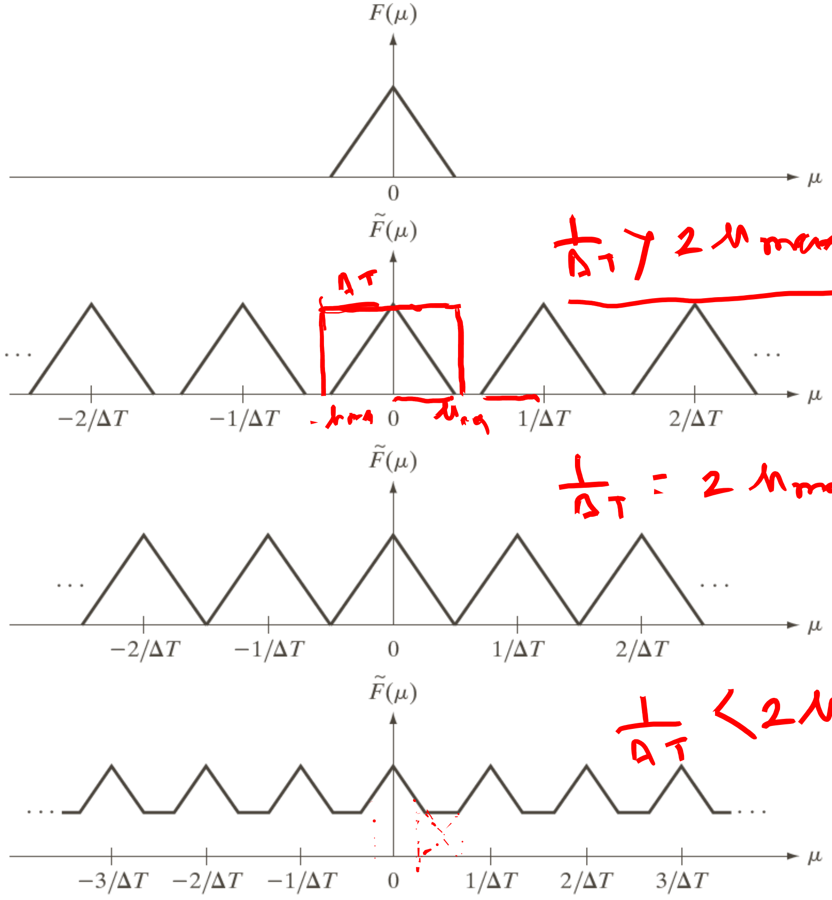
$$\mathcal{F}\{\tilde{f}(t)\}$$

$$= F(\mu) * S(\mu)$$

Sampling Theorem

$$\tilde{F}(u)\,H(u) = F(u)$$

$$H(u) = \begin{cases} \Delta T & -u_{max} \le u \le +u_{max} \\ 0 & \text{otherwise.} \end{cases}$$



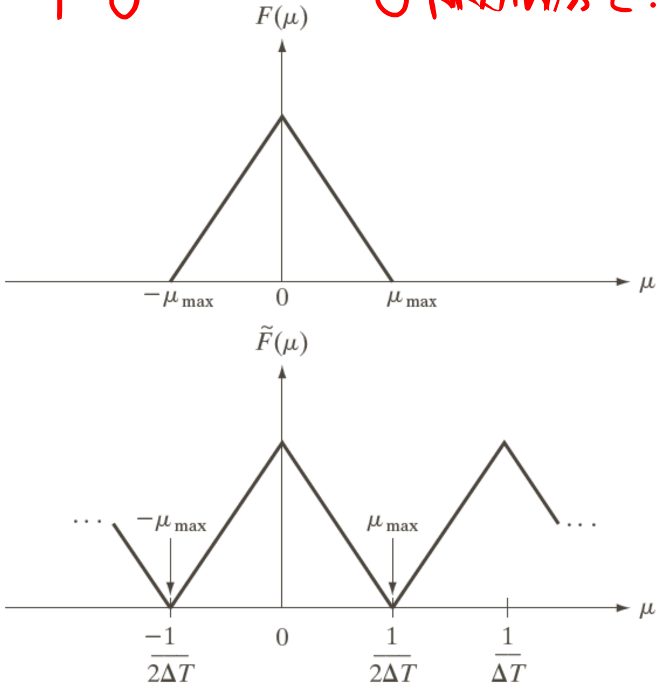
$\frac{1}{\Delta T} > 2u_{max}$

$\frac{1}{\Delta T} = 2u_{max}$

$\frac{1}{\Delta T} < 2u_{max}$

a
b
c
d

FIGURE 4.6 (a) Fourier transform of a band-limited function. (b)–(d) Transforms of the corresponding sampled function under the conditions of over-sampling, critically-sampling, and under-sampling, respectively.

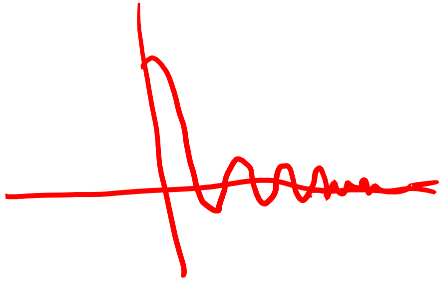


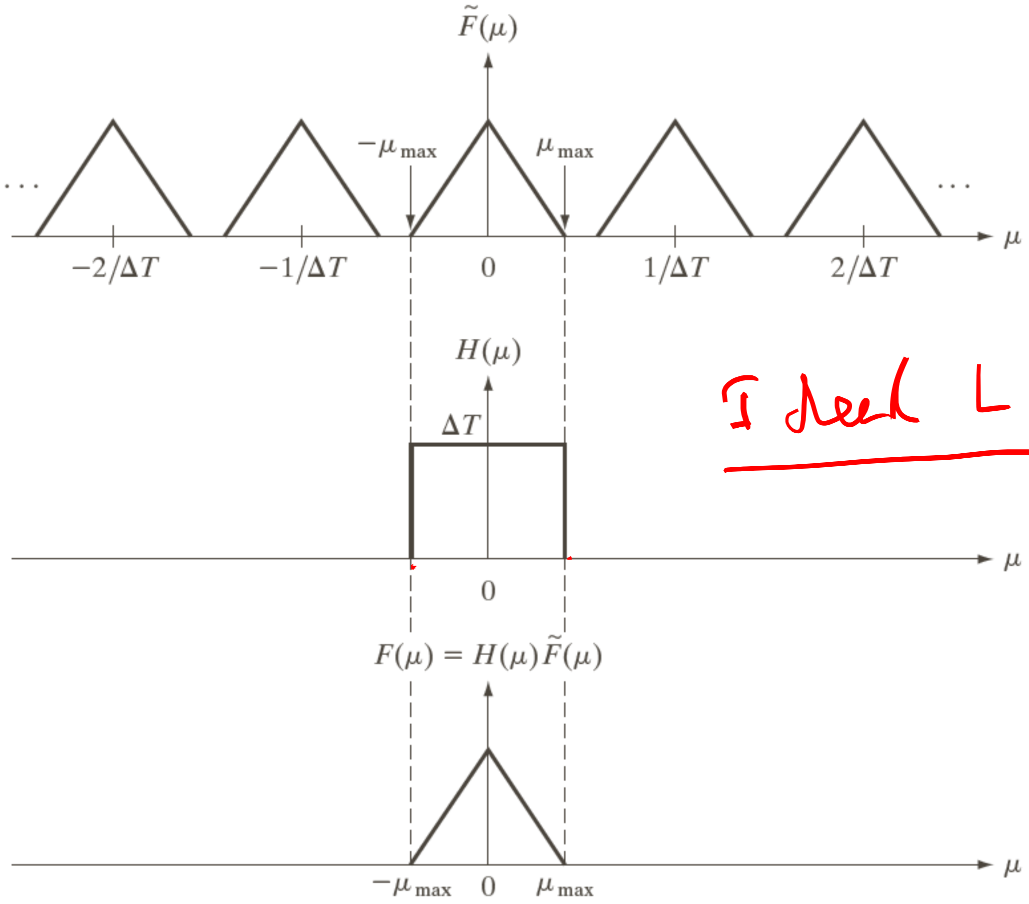
a
b

FIGURE 4.7 (a) Transform of a band-limited function. (b) Transform resulting from critically sampling the same function.

$$f(t) = \int_{-\infty}^{\infty} F(u)\, e^{j2\pi ut}\, du$$

Signal Recovery



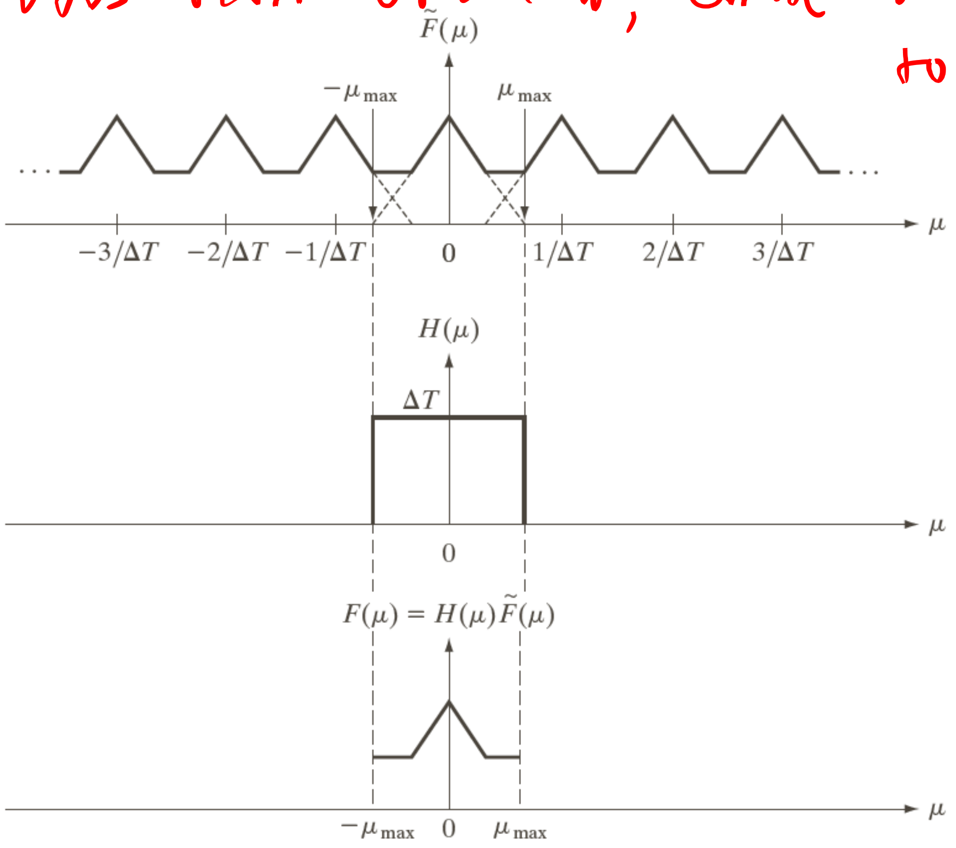


Ideal LPF

Aliasing

The net effect of lowering the sampling rate below the Nyquist rate (Sampling theorem) is that the periods now overlap, and it becomes impossible to isolate a single period of the transform.

This effect is known as freq. aliasing or aliasing.



a
b
c

FIGURE 4.9 (a) Fourier transform of an under-sampled, band-limited function. (Interference from adjacent periods is shown dashed in this figure). (b) The same ideal lowpass filter used in Fig. 4.8(b). (c) The product of (a) and (b). The interference from adjacent periods results in aliasing that prevents perfect recovery of $F(\mu)$ and, therefore, of the original, band-limited continuous function. Compare with Fig. 4.8.

DFT (Example)