# Lecture - 5

## Distance Measures :-

$p \approx (x, y)$ ; $q \approx (s, t)$ ; $z \approx (u, v)$

D (i) if $D(p, q) \geq 0$

$D(p, q) == 0$ iff $p == q$.

(ii) $D(p, q) == D(q, p)$

(iii) $D(p, z) \leq D(p, q) + D(q, z)$

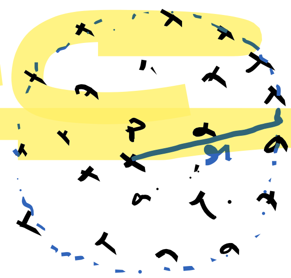
D is a distance $f^n$ or metric.

$p_{(x,y)}$ $\qquad$ $q\ (s, t)$

Euclidean Distance:

$$D_E(p, q) = \left[ (x-s)^2 + (y-t)^2 \right]^{1/2}$$

$$S = \{ q \mid D(p, q) \leq r \}$$



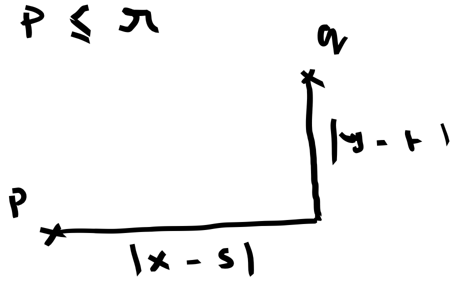

* $D_4$ distance or City block distance or Manhattan dist:-

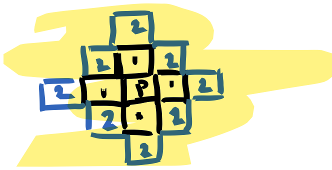
$$D_4(p, q) = |x - s| + |y - t|$$

$p \leq r$

$r = 1$

$r = 2$



## * Chess Board Distance or $D_8$ Distance:-

$$D_8(p, q) = \max\left(|x-s|, |y-t|\right).$$

$$S = \{ q \mid D_8(p, q) \le r \}$$

| | | | | |
|---|---|---|---|---|
| 2 | 2 | 2 | 2 | 2 |
| 2 | 1 | 1 | 1 | 2 |
| 2 | 1 | p | 1 | 2 |
| 2 | 1 | 1 | 1 | 2 |

## * Array Vs. Matrix Operations:-

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \qquad B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$$

✓ **Array Product:-**

$$A \odot B = \begin{bmatrix} a_{11}\,b_{11} & a_{12}\,b_{12} \\ a_{21}\,b_{21} & a_{22}\,b_{22} \end{bmatrix}$$

**Matrix Product:-**

$$AB = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$$

$$= \begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} & a_{11}b_{12} + a_{12}b_{22} \\ a_{21}b_{11} + a_{22}b_{21} & a_{21}b_{12} + a_{22}b_{22} \end{bmatrix}$$

## * Linear Vs. Non-linear :-

$$a_i f_i(x,y) \rightarrow \boxed{H} \rightarrow a_i H[f_i(x,y)] = a_i g_i(x,y)$$

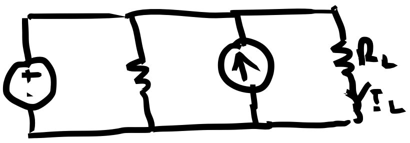
$$+ \qquad\qquad\qquad + \qquad\qquad\qquad +$$

$$a_j f_j(x,y) \longrightarrow a_j H[f_j(x,y)] = a_j g_j(x,y)$$

$$a_i f_i(x,y) + a_j f_j(x,y) \longrightarrow a_i g_i(x,y) + a_j g_j(x,y)$$

(i) Homogeneity :- Scaling the i/p will produce an O/p which will be scaled by the ~~f~~ same factor.

(ii) Superposition :-




$$I_{L_1} + I_{L_2} = I_L$$

$$a_i f_i(x,y) + a_j f_j(x,y) \longrightarrow \boxed{\Sigma} \longrightarrow a_i g_i(x,y) + a_j g_j(x,y)$$

$a_i = 2$

$f_i = [8, 1, 6]$ $\qquad a_i f_i = [16, 2, 12]$

$f_j = [10, 30, 15]$ $\qquad a_j f_j = [10, 30, 15]$

$a_j = 1$

$$[16, 2, 12] + [10, 30, 15] = [26, 32, 27] \rightarrow \boxed{\Sigma} \rightarrow \Sigma\, 26 + 32 + 27$$
$$= 85.$$

$$2 \times [8,1,6] \longrightarrow \boxed{\Sigma} \longrightarrow 16+2+12 = 30$$

$$+ \Big\} \rightarrow 85$$

$$1 \times [10,30,15] \longrightarrow \boxed{\Sigma} \longrightarrow 10+30+15 = 55$$

Hence '$\Sigma$' is linear.

# Non-linear:-

$a_i = 2$

$f_i = [8, 1, 6]$

$$a_i f_i \longrightarrow \boxed{\text{max}} \longrightarrow a_i g_i = 16$$

$$+ \Big\} \; 46$$

$a_j = 1$

$f_j = [10, 30, 15]$

$$a_j f_j \longrightarrow \boxed{\text{max}} \longrightarrow a_j g_j = 30$$

$$a_i f_i + a_j f_j \longrightarrow \boxed{\text{max}} \longrightarrow 32$$

$[26, 32, 27]$

'MAX' is non-linear

* Arithmetic Operations:-

$$s(x,y) = f(x,y) + g(x,y)$$

$$d(x,y) = f(x,y) - g(x,y)$$

$$p(x,y) = f(x,y) \times g(x,y)$$

$$v(x,y) = f(x,y) \div g(x,y)$$

* 0 $\underset{0}{f(x,y)} - \underset{255}{g(s,t)} = -255$

$\underset{255}{f(x,y)} + \underset{255}{g(s,t)} = 510$

510 } 255
255
0 } 0
⋮
255

[0 - 255]

$f_m = f - \min(f)$

$-255 - (-255) = 0$

$\hat{f}_m = \dfrac{f_m}{\max(f_m)}$ $\hat{f}_m$ [0 - 1]

[0 - 255]

$\boxed{\hat{\hat{f}}_m = (L-1) \cdot \hat{f}_m}$ 0-255

$L = 2^k$

$k = 8$