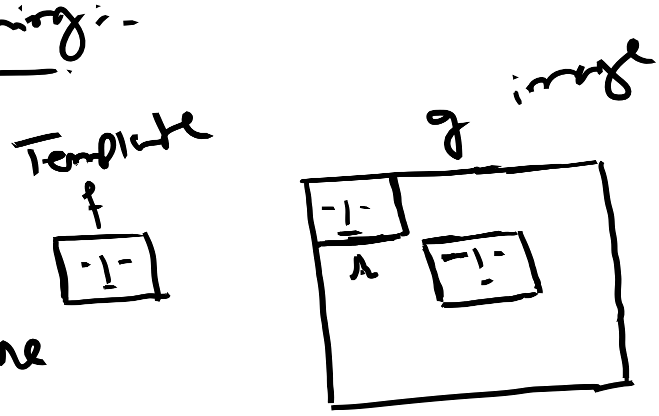
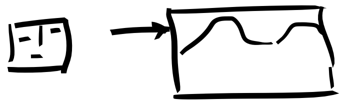
Lecture-7

Template Matching:-



Similarity measure



$\rightarrow \max_{A} |f-g|$

$\rightarrow \iint_A |f-g| \Rightarrow \sum_{i,j \in A}\sum |f(i,j)-g(i,j)|$

$\rightarrow \iint_A |f-g|^2 \Rightarrow \sum\sum_{i,j \in A} [f(i,j)-g(i,j)]^2$

$$\iint_A (f-g)^2 = \underbrace{\iint_A f^2}_{\text{For a Template } f^2 \text{ is fixed}} + \underbrace{\iint_A g^2}_{\text{fixed}} - 2\iint_A fg$$

Distance = mismatch measure

minimum Distance $\longrightarrow$ best match

$\iint f \cdot g \longrightarrow$ match measure / Similarity measure.

* Cauchy – Schwartz inequality :-

$$\iint f \cdot g \le \sqrt{\iint f^2 \cdot \iint g^2}$$

$g = cf$ condition for equality.

$$\sum_{i,j \in A}\sum f(i,j)\, g(i,j) \le \sqrt{\sum_{i,j \in A}\sum f^2(i,j) \sum_{i,j \in A}\sum g^2(i,j)}$$

$$g(i,j) = c\, f(i,j)$$

$$\iint_A f(x,y) \cdot g(\overset{\text{shift}}{x+u, y+v})\, dx\, dy$$

$$\le \Big[\underbrace{\iint_A f^2(x,y)\, dx\, dy}_{R_1} \cdot \underbrace{\iint_A g^2(x+u, y+v)\, dx\, dy}_{R_2}\Big]^{1/2}$$

$$\iint_{-\infty}^{\infty} f(x,y)\, g(x+u, y+v)\, dx\, dy \Rightarrow \text{cross correlation between } f \,\&\, g$$

$$= C_{fg}$$

$\boxed{A}$

$$C_{fg} = \sqrt{R_1 R_2}$$

$$C_{fg} \le (R_1 R_2)^{1/2}$$

Cross- correlation can not be directly used for similarity measure.

$f(x) \longrightarrow \boxed{H} \longrightarrow g(x)$

$f(x-x_0) \longrightarrow \boxed{H} \longrightarrow g(x-x_0)$

$$\frac{C_{fg}}{\sqrt{R_2}} \le \sqrt{R_1}$$

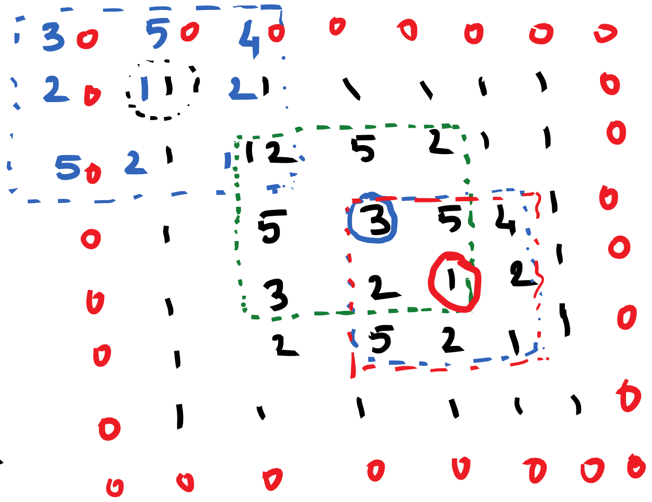
$$C'_{fg} \le \sqrt{R_1} \qquad g = cf.$$

Normalized cross correlation.

#

f Template

$$\begin{matrix} 3 & 5 & 4 \\ 2 & 1 & 2 \\ 5 & 2 & 1 \end{matrix}$$



$C_{fg_1} = 1\times1 + 2\times1 + 2\times1 + 1\times12$

$= 1 + 2 + 2 + 12 = 17$

$C_{fg_2} = 12\times3 + 5\times5 + 2\times4 + 5\times2 + 3\times1 + 5\times2 + 3\times5$
$+ 2\times2 + 1\times1$

$= 112.$

$C_{fg_3} = 3^2 + 5^2 + 4^2 + 2^2 + 1^2 + 2^2 + 5^2 + 2^2 + 1^2$

$= 89.$

Normalization factor :- $\left[\sum_A \sum \left[g^2(x+u, y+v)\right]\right]^{1/2}$

$$c'_{fg_1} = \frac{17}{(1^2+1^2+1^2+12^2)^{1/2}} = 1.40$$

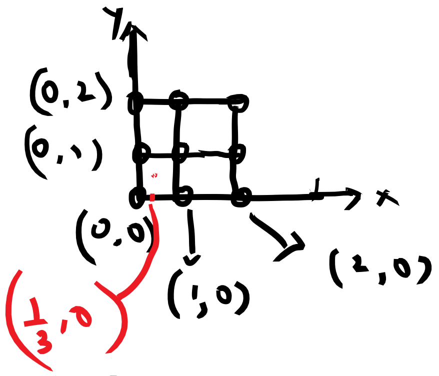
$$c'_{fg_2} = \frac{112}{(12^2+5^2+2^2+5^2+3^2+5^2+3^2+2^2+1^2)^{1/2}} = 7.14$$

$$c'_{fg_3} = \frac{89}{(3^2+5^2+4^2+2^2+1^2+2^2+5^2+2^2+1^2)^{1/2}} = \underline{9.43}$$

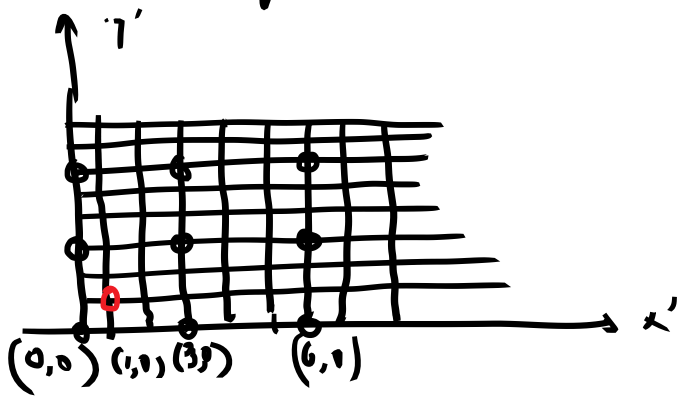
wherever we get the normalized cross-correlation maximum at that location the template best matches the area of the given image.

Interpolation :-

Scaling, ~~Translating~~ Rotate.



$S_x = 3,\ S_y = 3$



1. NN
2. Bi-linear
3. Bi-cubic.

$\theta = 45°$ $\quad \begin{bmatrix} \hat{x} \\ \hat{y} \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$

$(0,0) \rightarrow (0,0)$

$(0,1) \rightarrow \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

$(0.707,\ 0.707) \rightarrow (1,1)$

$(0,2) \rightarrow (1.41,\ 1.41) \rightarrow (1,1)$

$(1,0) \rightarrow \left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right) \rightarrow (1,-1)$

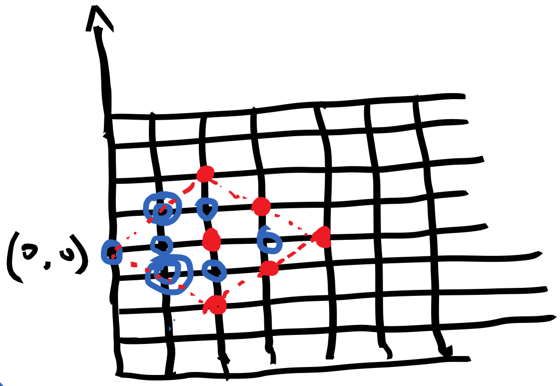
$(1,1) \rightarrow (1.41,\ 0) \rightarrow (1,0)$

$(1,2) \rightarrow \left(\frac{3}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right) \rightarrow (2,1)$

$(2,0) \rightarrow (1.41,\ -1.41) \rightarrow (1,-1)$

$(2,1) \rightarrow \left(\frac{3}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right) \rightarrow (2,-1)$

$(2,2) \rightarrow (2.82,\ 0) \rightarrow (3,0)$



Rounding off - New coordinates