Lecture 19

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Eigen Values and Eigen Vectors

Change of basis matrix

Let V be a vector space with ordered bases $B = \{\mathbf{v}_1, \dots, \mathbf{v}_n\}$ and $B' = \{\mathbf{w}_1, \dots, \mathbf{w}_n\}$. If $T = I : V \to V$ is an identity linear transformation, then $[\mathbf{v}]_{B'} = [I]_B^{B'}[\mathbf{v}]_B$ and the matrix $[T]_B^{B'} = [I]_B^{B'}$ is called *transition matrix* from B to B'.

Exercise

$\checkmark$ *Show that transition matrix $[I]_B^{B'}$ is invertible and $\left([I]_B^{B'}\right)^{-1} = [I]_{B'}^{B}$.*

Similar Matrices

Exercise

Let U, V, W be finite dimensional vector spaces with ordered bases B, B' and B''. If $T : U \to V$ and $S : V \to W$ are linear transformations, then $[S \circ T]_B^{B''} = [S]_{B'}^{B''} [T]_B^{B'}$.

Definition

Let A, B be $n \times n$ matrices. Then A is said to be similar to B, if there exists an invertible matrix P such that $B = P^{-1}AP$.

Change of basis

Let V be a finite dimensional vector space, B_1 and B_2 two ordered bases for V, and $T : V \to V$ a linear operator. Let $P = [I]_{B_2}^{B_1}$ be the transition matrix from B_2 to B_1. Then $[T]_{B_2} = P^{-1}[T]_{B_1}P$.

Hint: Use $T = I \circ T \circ I$. Then $[T]_{B_2} = [I \circ T \circ I]_{B_2} = [I]_{B_1}^{B_2}[T]_{B_1}[I]_{B_2}^{B_1}$.

Eigenvalues and Eigenvectors

Definition

- Let $T : V \to V$ be a linear transformation. Then an *eigenvector* of T is a nonzero vector $\mathbf{v}$ such that $T(\mathbf{v}) = \lambda\mathbf{v}$ for some scalar λ. The scalar $\lambda \in \mathbb{F}$ is called *eigenvalue* of T.
- For any $n \times n$ matrix A over $\mathbb{F}$, a nonzero vector $X \in \mathbb{F}^n$ is called a eigenvector of A, if $AX = \lambda X$ for some scalar $\lambda \in \mathbb{F}$. The subspace $A_\lambda = \{X \in \mathbb{F}^n : AX = \lambda X\}$ is called the eigenspace of A corresponding to λ. Clearly, $A_\lambda = N(A - \lambda I)$.