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~~Q.27~~

(1.) $\rightarrow$ Basis $\Rightarrow \{(1,2,2), (2,1,2), (2,2,1)\}$ for $\mathbb{R}^3$.
$\quad\quad v_1 \quad\quad v_2 \quad\quad v_3$

Now $\rightarrow$ let $\boxed{u_1 = \dfrac{v_1}{\|v_1\|} \Rightarrow \dfrac{1}{3}\cdot(1,2,2)}$

& $w_2 = v_2 - \dfrac{\langle v_2, u_1\rangle}{\langle u_1, u_1\rangle}\cdot u_1 \Rightarrow v_2 - \langle v_2, u_1\rangle \cdot u_1$

$\Rightarrow (2,1,2) - \left\langle (2,1,2), \left(\frac{1}{3}, \frac{2}{3}, \frac{2}{3}\right)\right\rangle \left(\frac{1}{3}, \frac{2}{3}, \frac{2}{3}\right)$

$\Rightarrow (2,1,2) - \left\{\frac{2}{3} + \frac{2}{3} + \frac{4}{3}\right\}\cdot\left(\frac{1}{3}, \frac{2}{3}, \frac{2}{3}\right)$

$\Rightarrow (2,1,2) - \left(\frac{8}{9}, \frac{16}{9}, \frac{16}{9}\right)$

$w_2 \Rightarrow \left(\frac{10}{9}, \frac{-7}{9}, \frac{2}{9}\right)$

thus $\rightarrow$ $u_2 = \dfrac{w_2}{\|w_2\|} \Rightarrow \dfrac{\left(\frac{10}{9}, \frac{-7}{9}, \frac{2}{9}\right)}{\sqrt{\frac{100+49+4}{81}}}$

$\boxed{u_2 \Rightarrow \dfrac{1}{\sqrt{153}}\cdot(10, -7, 2)}$

$$u_3 = \frac{w_3}{\|w_3\|}$$

So $\rightarrow$ $w_3 = v_3 - \langle v_3, u_1 \rangle \cdot u_1 - \langle v_3, u_2 \rangle \cdot u_2$

$$\Rightarrow (2,2,1) - \left\langle (2,2,1), \left(\tfrac{1}{3}, \tfrac{2}{3}, \tfrac{2}{3}\right) \right\rangle \left(\tfrac{1}{3}, \tfrac{2}{3}, \tfrac{2}{3}\right)$$

$$- \left\langle (2,2,1), \frac{1}{\sqrt{153}}(10,-7,2) \right\rangle \frac{1}{\sqrt{153}}(10,-7,$$

$$\Rightarrow (2,2,1) - \left[\frac{2}{3} + \frac{4}{3} + \frac{2}{3}\right] \frac{1}{3}(1,2,2)$$

$$- \frac{1}{\sqrt{153}}(20-14+2) \cdot \frac{1}{\sqrt{153}}(10,-7,2)$$

$$\Rightarrow (2,2,1) - \frac{8}{9} \cdot (1,2,2) - \frac{8}{153}(10,-7,2)$$

$$\Rightarrow \frac{1}{153}(306,306,153) - \frac{1}{153}(136,272,272)$$

$$- \frac{1}{153}(80,-56,16)$$

$$\Rightarrow \frac{1}{153}(90,90,-135) \Rightarrow \frac{1}{17}(10,10,-15)$$

$$\Rightarrow \frac{5}{17} \cdot (2,2,-3)$$

$$\boxed{u_3 = \frac{5}{17} \cdot \frac{(2,2,-3)}{\sqrt{17}}}$$

(3) → To find char. eq.^{n} →

$$|A - xI| = 0$$

$$\begin{vmatrix} 2-x & -4 & 3 \\ 2 & -3-x & 5 \\ 1 & -1 & 4-x \end{vmatrix} = 0$$

$$(2-x)\cdot\{+(x+3)(x-4)+5\} + 4\cdot\{8-2x-5\}$$
$$+3\cdot\{-2+(x+3)\} = 0$$

$$(2-x)\cdot\{x^2 - x - 12 + 5\} - 8x + 12 + 3x + 3 = 0$$

$$2x^2 - \cancel{2x} - 14 - x^3 + x^2 + \cancel{7x} - \cancel{5}x + 15 = 0$$

$$-x^3 + 3x^2 + 1 = 0 \rightarrow (1).$$

By using Cayley-Hamilton →

$$-A^3 + 3A^2 + I = 0$$
$$I = (A^3 - 3A^2)$$

Multyplying (A^{-1}) both sides →

$$\boxed{A^{-1} = A^2 - 3A}$$

$$A^2 = \begin{bmatrix} 2 & -4 & 3 \\ 2 & -3 & 5 \\ 1 & -1 & 4 \end{bmatrix} \cdot \begin{bmatrix} 2 & -4 & 3 \\ 2 & -3 & 5 \\ 1 & -1 & 4 \end{bmatrix}$$

$$A^2 \Rightarrow \begin{bmatrix} -1 & 1 & -2 \\ 3 & -4 & 11 \\ 4 & -5 & 14 \end{bmatrix}$$

So →

$$A^{-1} = A^2 - 3A$$

$$\Rightarrow \begin{bmatrix} -1 & 1 & -2 \\ 3 & -4 & 11 \\ 4 & -5 & 14 \end{bmatrix} - \begin{bmatrix} 6 & -12 & 9 \\ 6 & -9 & 15 \\ 3 & -3 & 12 \end{bmatrix}$$

$$\boxed{A^{-1} \Rightarrow \begin{bmatrix} -7 & 13 & -11 \\ -3 & 5 & -4 \\ 1 & -2 & 2 \end{bmatrix}}$$

$4-6+3$
$-6+12-3$
$6-20+12$

$4-6+5$
$-6+9-5$
9
$6-15+20$
$\Rightarrow 11$

$2-2+4$
$-4+3-4$
-3

$3-5+16$
14.

2. → V. is V.S. in $\mathbb{R}^{3\times3}$.

Basis of V ⇒ B

So → $B = \left\{ \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \right\}$.

Dim.(B) = 9.

~~Generate~~ General observation → A^n will give 'A' every single time. → $\boxed{A^n = A}$

which will not be equal to any diff. 'c'.

let P, & Q are 2 matrix of $(n \times n)$.
such that → P, Q ∈ B. ⟹ such that
$p+l = 1$
$q+l = 1$

$$\underset{(\text{for all } i,j)}{P \times Q} = \left(\sum_{K=1}^{n} p_{iK} \cdot q_{Kj}\right)$$

⇓

this will ~~w~~ be (1) only if both matrix have (1) value at same position,,
in case of other combination ⟹ ~~0 not~~ zero matrix
⇓
which is not part of (B).

so → $A^n = A$ which is $\neq C$
⇓ for all n