Linear Algebra → Tutorial ②

① → Both parts are easy. Do yourself.

~~②~~ → for 2×2 matrix → $\begin{bmatrix}1&0\\0&1\end{bmatrix}$; $\begin{bmatrix}1&0\\0&0\end{bmatrix}$; $\begin{bmatrix}0&1\\0&0\end{bmatrix}$; $\cancel{\begin{bmatrix}0&0\\1&0\end{bmatrix}}$; $\cancel{\begin{bmatrix}0&0\\0&1\end{bmatrix}}$

Use α, β for values which can take any values.

for 2×3 matrix →

$\begin{bmatrix}1&0&0\\0&1&0\end{bmatrix}$; $\begin{bmatrix}1&0&0\\0&0&1\end{bmatrix}$; $\begin{bmatrix}0&1&0\\0&0&1\end{bmatrix}$;

$\begin{bmatrix}1&0&0\\0&0&0\end{bmatrix}$; $\begin{bmatrix}0&1&0\\0&0&0\end{bmatrix}$; $\begin{bmatrix}0&0&1\\0&0&0\end{bmatrix}$;

$\cancel{\begin{bmatrix}0&0&0\\1&0&0\end{bmatrix}}$; $\cancel{\begin{bmatrix}0&0&0\\0&1&0\end{bmatrix}}$; $\cancel{\begin{bmatrix}0&0&0\\0&0&1\end{bmatrix}}$; {Non-zero rows should be below}

Do yourself for 3×3 →

③. ⓐ.

$$\left[\begin{array}{cccc|c}3&2&7&9&7\\6&14&22&15&13\\1&4&5&2&2\end{array}\right]$$

10 Sunday

$$\Big\downarrow \; R_2 \to R_2 - 2R_1, \; R_3 = 3R_3 - R_1$$

$$\left[\begin{array}{cccc|c}3&2&7&9&7\\0&10&8&-3&-1\\0&10&8&-3&-1\end{array}\right] \xrightarrow{R_3 \to R_3 - R_2}$$

$$\left[\begin{array}{cccc|c} 3 & 2 & 7 & 9 & 7 \\ 0 & 10 & 8 & -3 & -1 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$$\Big\downarrow R_1 \to 5R_1 - R_2$$

$$\left[\begin{array}{cccc|c} 15 & 0 & 27 & 48 & 36 \\ 0 & 10 & 8 & -3 & -1 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$$\Big\downarrow$$

$$\left[\begin{array}{cccc|c} \boxed{1} & 0 & 27/15 & 48/15 & 36/15 \\ 0 & \boxed{1} & 8/10 & -3/10 & -1/10 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

free variable

so→ let $z = 'P'$

$w = 'Q'$

so→

$$y + \frac{8}{10}\cdot P - \frac{3}{10}\cdot Q = \frac{-1}{10}$$

$$10\,y = 3Q - 1 - 8P$$

$$\boxed{y = \left(\frac{3Q - 1 - 8P}{10}\right)}$$

and→

$$x + \frac{27}{15}P + \frac{48Q}{15} = \frac{36}{15}$$

$$x = \frac{36 - 27P - 48Q}{15} \Rightarrow \boxed{x = \left(\frac{12 - 9P - 16Q}{5}\right)}$$

So, sol^n vector →

$$\begin{bmatrix} \left(\frac{12-9P-16Q}{5}\right) & \left(\frac{3Q-1-8P}{10}\right) & P & Q \end{bmatrix}_{4\times1}$$

check if something else is asked regarding Homogeneous eq.^{n}.

(5) → finding inverse →

$[A \mid I]$

$$\Rightarrow \left[\begin{array}{ccc|ccc} 2 & 1 & -1 & 1 & 0 & 0 \\ 0 & 2 & 1 & 0 & 1 & 0 \\ 5 & 2 & -3 & 0 & 0 & 1 \end{array}\right] \quad R_3 \to 2R_3 - 5R_1$$

$$\Rightarrow \left[\begin{array}{ccc|ccc} 2 & 1 & -1 & 1 & 0 & 0 \\ 0 & 2 & 1 & 0 & 1 & 0 \\ 0 & -1 & -1 & -5 & 0 & 2 \end{array}\right]$$

$R_3 \to 2R_3 + R_2$

$$\left[\begin{array}{ccc|ccc} 2 & 1 & -1 & 1 & 0 & 0 \\ 0 & 2 & 1 & 0 & 1 & 0 \\ 0 & 0 & -1 & -10 & 1 & 4 \end{array}\right]$$

$R_1 \to 2R_1 - R_2$
$R_3 \to -R_3$

$$\left[\begin{array}{ccc|ccc} 4 & 0 & -3 & 2 & -1 & 0 \\ 0 & 2 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 10 & -1 & -4 \end{array}\right]$$

$$R_1 \to R_1 + 3R_3$$
$$R_2 \to R_2 - R_3$$

$$\left[\begin{array}{ccc|ccc} 4 & 0 & 0 & 32 & -4 & -12 \\ 0 & 2 & 0 & -10 & 2 & 4 \\ 0 & 0 & 1 & 10 & -1 & -4 \end{array}\right]$$

$$\downarrow$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 8 & -1 & -3 \\ 0 & 1 & 0 & -5 & 1 & 2 \\ 0 & 0 & 1 & 10 & -1 & -4 \end{array}\right]$$

→ Sol.ⁿ.

(6) → $e(A) = E(A)$

let matrix A be → $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 10 & 13 \\ 2 & 5 & 9 \end{bmatrix}$

for scaling:

$e \Rightarrow R_2 \to R_2 * 3$

$$\begin{bmatrix} 1 & 2 & 3 \\ 12 & 30 & 39 \\ 2 & 5 & 9 \end{bmatrix} \text{ --- } (1)$$

So →

$E \Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

elementary matrix

So → $EA \Rightarrow$ same matrix as (1).

⇒ for ~~replacement~~ interchange in A ⟶ $e \Rightarrow R_1 \leftrightarrow R_2$

$$\Downarrow$$

$$eA = \begin{bmatrix} 4 & 10 & 13 \\ 1 & 2 & 3 \\ 2 & 5 & 9 \end{bmatrix} \quad \text{—(2)}$$

corresponding E ⟶

$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

so ⟶ EA will give same as (2).

for replacement in A ⟶ ??

?? (1) ⟶ Great intution. But how to prove?

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Good Quest.

check if it's correct

Q.→ $A_{n \times n}$ →

Prove that →

system (Ax=b) has a sol.^{n} iff A is for every coloumn vec. (⟷) invertible

part ⓐ → Given L.H.S. → it means all coloums has pivot elements & there are no free variables.

Only possibility in that case, is (if we convert A to RREF form) that A's RREF will form an identity matrix which is invertible.

(R.H.S.)

part ⓑ → Given RHS. → A is invertible. ($AA^{-1}=I$)

then RREF of A will be identity. so, sol.^{n} for every vector will exist.

↳ L.H.S.

→ बहुत ज्यादा गलत, इससे ज्यादा गलत कुछ है ही नहीं दुनिया में।

MAY 2020

Su	Mo	Tu	We	Th	Fr	Sa
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3	4	5	6	7	8	9
10	11	12	13	14	15	16
17	18	19	20	21	22	23
24	25	26	27	28	29	30
31						

(9). → $[A \mid I]$

$\Downarrow$

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 2 & 5 & 3 & 0 & 1 & 0 \\ 1 & 0 & 8 & 0 & 0 & 1 \end{array}\right]$$

$\downarrow R_1 \to 5R_1 - 2R_2$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 9 & 5 & -2 & 0 \\ 2 & 5 & 3 & 0 & 1 & 0 \\ 1 & 0 & 8 & 0 & 0 & 1 \end{array}\right]$$

$\downarrow R_2 \to \frac{1}{5} R_2$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 9 & 5 & -2 & 0 \\ 2/5 & 1 & 3/5 & 0 & 1/5 & 0 \\ 1 & 0 & 8 & 0 & 0 & 1 \end{array}\right]$$

2nd coloumn of A^{-1}.

(10) → $\boxed{AB = AC}$

→ if A is invertible → $AA^{-1} = A^{-1}A = I$

$$A(B-C) = 0$$
$$A^{-1}.A\,(B-C) = A^{-1}.0 = 0$$
$$I.\,(B-C) = (B-C) = 0$$
$$\boxed{B = C}$$

→ Now if A is not invertible., then → $\boxed{AA^{-1} \neq I}$

(1)

$AB = AC$

$\boxed{A(B-C) = 0}$

→ Now, since A is non-invertible & (B−C) is (n×K) matrix. So, (B−C) is also non-invertible so, it is not necessary for either of 'A' or (B−C) to be zero matrix.

So, 'B' is not necessarily equal to 'C' in general if A is non-invertible. But can be in few special cases.

So, for example →

Suppose → $A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$; $(B-C) = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$,

So → $A.(B-C) = 0$; But $\boxed{B \neq C}$

(2) → if homogeneous eq.n $AX = 0$ has trivial sol.n, that means $|A| \neq 0$ or all coloumns contains pivot element for those values of 'A'.

Now for same 'A', eq.n $AX = b$; $b \neq 0$; can only be inconsistent if m eq.n are given but only 'n' variables. &

So, it means that if 'm' rows are given (eq.n) & 'n' variables. So →

$\boxed{m > n}$

(ii) → R can be found easily. To find P →

$$[I \mid A] \longrightarrow (i).$$

$$[PI \mid PA]$$

$$\downarrow$$

$$[P \mid R]$$

So, from eq.^{n} (i) → first cal. RREF of A →

$$A = \left[\begin{array}{cccc|ccc} 1 & 2 & 1 & 0 & 1 & 0 & 0 \\ -1 & 0 & 3 & 5 & 0 & 1 & 0 \\ 1 & -2 & 1 & 1 & 0 & 0 & 1 \end{array}\right]$$

$$\downarrow R_2 \to R_2 + R_1 \qquad R_3 \to R_1 - R_3$$

$$\left[\begin{array}{cccc|ccc} 1 & 2 & 1 & 0 & 1 & 0 & 0 \\ 0 & 2 & 4 & 5 & 1 & 1 & 0 \\ 0 & 4 & 0 & -1 & 1 & 0 & -1 \end{array}\right]$$

$$\downarrow R_3 \to R_3 - 2R_2$$

$$\left[\begin{array}{cccc|ccc} 1 & 2 & 1 & 0 & 1 & 0 & 0 \\ 0 & 2 & 4 & 5 & 1 & 1 & 0 \\ 0 & 0 & -8 & -11 & -1 & -2 & -1 \end{array}\right]$$

$$\left[\begin{array}{cccc} 1 & 2 & 1 & 0 \\ 0 & 2 & 4 & 5 \\ 0 & 0 & -8 & -11 \end{array}\right] \Bigg| \left[\begin{array}{ccc} 1 & 0 & 0 \\ 1 & 1 & 0 \\ -1 & -2 & -1 \end{array}\right]$$

$$\downarrow R_1 \to R_1 - R_2 \;:\; R_3 \to -\tfrac{1}{8} R_3$$

$$\left[\begin{array}{cccc} 1 & 0 & -3 & -5 \\ 0 & 2 & 4 & 5 \\ 0 & 0 & 1 & 11/8 \end{array}\right] \Bigg| \left[\begin{array}{ccc} 0 & -1 & 0 \\ 1 & 1 & 0 \\ 1/8 & 2/8 & 1/8 \end{array}\right]$$

$$\downarrow R_2 \to R_2/2$$

$$\left[\begin{array}{cccc} 1 & 0 & -3 & -5 \\ 0 & 1 & 2 & 5/2 \\ 0 & 0 & 1 & 11/8 \end{array}\right] \Bigg| \left[\begin{array}{ccc} 0 & -1 & 0 \\ 1/2 & 1/2 & 0 \\ 1/8 & 2/8 & 1/8 \end{array}\right]$$

$$\downarrow \begin{array}{l} R_1 \to R_1 + 3R_3 \\ R_2 \to R_2 - 2R_3 \end{array}$$

$$\underbrace{\left[\begin{array}{cccc} \boxed{1} & 0 & 0 & -7/8 \\ 0 & \boxed{1} & 0 & -1/4 \\ 0 & 0 & \boxed{1} & 11/8 \end{array}\right]}_{\text{it is } R} \Bigg| \underbrace{\left[\begin{array}{ccc} 3/8 & -1/4 & 3/8 \\ 1/4 & 0 & -1/4 \\ 1/8 & 1/4 & 1/8 \end{array}\right]}_{\text{it is } P}$$

it is R.   it is P

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