Roll No. → 2021PCS1017

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1. → $V = \{(x, y, z, w) : x+y+2z+2w = 0\} \subset \mathbb{R}^4$

(a). → so, suppose any $\vec{v_1} \in V$ & $\vec{v_2} \in V$, such that:

$$\vec{v_1} = (x_1, y_1, z_1, w_1) : x_1 + y_1 + 2z_1 + 2w_1 = 0 \quad \text{—— (1)}$$

$$\& \quad \vec{v_2} = (x_2, y_2, z_2, w_2) : x_2 + y_2 + 2z_2 + 2w_2 = 0 \quad \text{—— (2)}$$

To check closure under addiⁿ →

for $\vec{v_1} + \vec{v_2} = (x_1 + x_2, y_1 + y_2, z_1 + z_2, w_1 + w_2)$

so →

$= (x_1 + x_2) + (y_1 + y_2) + 2(z_1 + z_2) + 2 \cdot (w_1 + w_2)$

$\Rightarrow \underbrace{(x_1 + y_1 + 2z_1 + 2w_1)}_{0} + \underbrace{(x_2 + y_2 + 2z_2 + 2w_2)}_{0}$

$\Rightarrow 0.$ closed under addⁿ.

for scaler multⁿ →

suppose → $\vec{v_3} = k \cdot \vec{v_1} \Rightarrow (kx_1, ky_1, kz_1, kw_1)$

so → $= k \cdot (x_1 + y_1 + 2z_1 + 2w_1) = 0$

⇒ closed under scaler multⁿ.

⇒ V is a V.S. over R.

(b) → Given set (S) ⇒ $\{(2, -2, 1, -1)\}$

since we know that → $\boxed{\text{Dim}^n(V) = 4}$

~~& Dimⁿ of S → Dim(S) = 1~~

we need to apply plus-minus theorem.

so → I propose that remaining 3 elements (vector) required to make 'S' a basis are →

~~[illegible]~~ $(0,-2,0,0)$, $(0,0,1,0)$ and $(0,0,0,-1)$

So, Now, checking whether S is L.I. →

$$= \begin{bmatrix} 2 & -2 & 1 & -1 \\ 0 & -2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \rightarrow \text{(Newly made S)}$$

$\downarrow \; R_1 \rightarrow R_1 - (R_2 + R_3 + R_4);$

$$\begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & -2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \xrightarrow[R_3 = -R_3;]{R_1 = \frac{1}{2}R_1; \; R_2 = -\frac{1}{2}R_2;} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

ie., they are LI.

and dim$^n$ of $S = 4$, so, it'll be a basis.

so → $S = \{(2,-2,1,-1), (0,-2,0,0), (0,0,1,0), (0,0,0,-1)\}$.