Linear Algebra → Tutorial (I)

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(1). →

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APRIL 2020

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(2) → A, B are invertible matrix with same sizes.

Since, we know that →

$$AA^{-1} = I \quad \text{—(1)}$$
$$BB^{-1} = I \quad \text{—(2)}$$

AB is multiplicable & invertible.

So →

$$(AB) \cdot (AB)^{-1} = I$$

$$\underbrace{A^{-1} \cdot A}_{I} \cdot B \cdot (AB)^{-1} = A^{-1} \cdot I = A^{-1}$$

$$B \cdot (AB)^{-1} = A^{-1}$$

$$\underbrace{B^{-1} \cdot B}_{I} \cdot (AB)^{-1} = B^{-1} \cdot A^{-1}$$

$$\boxed{(AB)^{-1} = B^{-1} A^{-1}}$$

(3) → $(AA^{-1} = I) \longleftrightarrow \{(A^T) \cdot (A^T)^{-1} = I\}$

L.H.S. → $AA^{-1} = I$

take transpose →

$$(AA^{-1})^T = (I)^T$$

$$(A^{-1})^T \cdot A^T = I$$

$$\boxed{(A^T)^{-1} \cdot A^T = I} = \boxed{\text{R.H.S.}}$$

Similarily we can prove backwards as well.

(i). (4) → if 'A' is a square matrix of size (m×m). So →

$$A = \frac{1}{2}(A + A)$$

$$A = \underbrace{\frac{(A+A^T)}{2}}_{\text{symmetric Matrix}} + \underbrace{\frac{(A-A^T)}{2}}_{\text{skew-symmetric Matrix.}}$$

for Proof →

L.H.S. → $(A+A^T)^T$

$\Rightarrow A^T + (A^T)^T$

$\Rightarrow (A + A^T)$

$\Rightarrow$ so, symmetric Matrix.

L.H.S. → $(A - A^T)^T$

$\Rightarrow A^T - (A^T)^T$

$\Rightarrow A^T - A$

$\Rightarrow -(A - A^T)$

$\Rightarrow$ so, skew-symmetric Matrix.

(ii). A, B are symmetric ⇒ $A = A^T$
$B = B^T$

so, for (AB) →

$$(AB)^T = B^T \cdot A^T$$

$$\Rightarrow \boxed{B \cdot A \neq AB}$$

So, it is only possible iff $\boxed{AB = BA}$.

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(5) → A, B are 2 U.T. matrices.

So →

$$A = \begin{bmatrix} a_{ij} \end{bmatrix}_{m \times m} \; ; \quad \underline{\text{where}} \rightarrow a_{ij} = 0; \text{ if } i > j.$$

$$B = \begin{bmatrix} b_{ij} \end{bmatrix}_{n \times p} \; ; \quad \underline{\text{where}} \rightarrow b_{ij} = 0; \text{ if } i > j$$

$$\underset{(c_{ij})}{C = AB} \Rightarrow \left[\sum_{k=1}^{n} a_{ik} \cdot b_{kj} \right]_{m \times p} \Rightarrow$$ we only need to check elements where $\boxed{i > j}$,

$$\Downarrow$$

$$\Rightarrow \left[\sum_{k=1}^{i-1} \cancelto{0}{a_{ik}} \cdot b_{kj} \right] + \left[\sum_{k=i}^{n} a_{ik} \cdot \cancelto{0}{b_{kj}} \right]$$

$\Rightarrow 0 + 0 \Rightarrow 0$; {So, C is also a U.T. matrix}.

(6) → AB are such that AB is defined.

So, suppose →

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}_{2 \times 2} \; ; \quad B = \begin{bmatrix} x & y \\ z & w \end{bmatrix}_{2 \times 2}$$

So → $AB = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} x & y \\ z & w \end{bmatrix}$.

$$\Rightarrow \begin{bmatrix} ax + bz & ay + bw \\ cx + dz & cy + dw \end{bmatrix}_{2 \times 2}$$ ~~tot~~ (R)

So → (i) Rows of AB as linear combination of Rows of B →
we can write that as →

$$\Rightarrow \begin{bmatrix} ax & ay \\ cx & cy \end{bmatrix} + \begin{bmatrix} bz & bw \\ dz & dw \end{bmatrix} \quad \text{— (1).}$$

$$\Rightarrow \begin{matrix} a\cdot \\ c\cdot \end{matrix}\begin{bmatrix} x & y \\ x & y \end{bmatrix} + \begin{matrix} b\cdot \\ d\cdot \end{matrix}\begin{bmatrix} z & w \\ z & w \end{bmatrix}$$

$$= \begin{bmatrix} a\cdot[x \ y] + b\cdot[z \ w] \\ c\cdot[x \ y] + d\cdot[z \ w] \end{bmatrix} \Rightarrow \text{Req.}$$

(ii). Coloumns of AB can be written as L.C. of coloumns of A. from eq.^n (1). →

$$\Rightarrow \begin{matrix} x\cdot \\ \cancel{y}\cdot \end{matrix}\begin{bmatrix} a & a \\ c & c \end{bmatrix} + \begin{matrix} z\cdot \\ w\cdot \end{matrix}\begin{bmatrix} b & b \\ d & d \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x\cdot\begin{bmatrix} a \\ c \end{bmatrix} + z\cdot\begin{bmatrix} b \\ d \end{bmatrix} & y\cdot\begin{bmatrix} a \\ c \end{bmatrix} + w\cdot\begin{bmatrix} b \\ d \end{bmatrix} \end{bmatrix}$$

⇒ Req.

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(7) $\rightarrow$ $\boxed{AB=0}$; where $A, B \neq O$

$$\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} = 0.$$

(8) $\rightarrow$ (a).

$$\begin{bmatrix} 1 & 4 & -1 \\ -2 & -8 & 2 \\ -3 & 12 & -3 \\ 2 & 5 & 3 \end{bmatrix}$$

$\downarrow$ $R_2 = R_2 + 2R_1$, $R_3 = R_3 - 3R_1$, $R_4 = R_4 - 2R_1$

$$\Rightarrow \begin{bmatrix} 1 & 4 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & -3 & 5 \end{bmatrix}$$

$\downarrow$ $R_2 \longleftrightarrow R_4$; $R_2 = -\frac{1}{3} R_2$

$$\Rightarrow \begin{bmatrix} \boxed{1} & 4 & 1 \\ 0 & \boxed{1} & -5/3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$ (REF form)

$\downarrow$ $R_1 = R_1 - 4R_2$

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$$\begin{bmatrix} 1 & 0 & 23/3 \\ 0 & 1 & -5/3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$ (RREF form)

(b) →
$$\begin{bmatrix} 5 & 6 & -7 & 2 \\ -1 & -2 & 3 & 0 \\ 0 & 4 & 1 & 3 \end{bmatrix}$$

$\downarrow R_1 \leftrightarrow R_2 ; R_1 = -R_1$

$$\begin{bmatrix} 1 & 2 & -3 & 0 \\ 5 & 6 & -7 & 2 \\ 0 & 4 & 1 & 3 \end{bmatrix}$$

$\downarrow R_2 = R_2 - 5R_1$

$$\begin{bmatrix} \boxed{1} & 2 & -3 & 0 \\ 0 & \boxed{-4} & 8 & 2 \\ 0 & 4 & 1 & 3 \end{bmatrix}$$

$\downarrow R_3 \rightarrow R_3 + R_2$
$R_2 = -\frac{1}{4} R_2$

$$\begin{bmatrix} \boxed{1} & 2 & -3 & 0 \\ 0 & \boxed{1} & -2 & -\frac{1}{2} \\ 0 & 0 & 9 & 5 \end{bmatrix}$$

$\downarrow R_3 \rightarrow \frac{1}{9} R_3$

$$\begin{bmatrix} \boxed{1} & 2 & -3 & 0 \\ 0 & \boxed{1} & -2 & -\frac{1}{2} \\ 0 & 0 & \boxed{1} & \frac{5}{9} \end{bmatrix}$$ (REF form)

↓ can convert it to RREF.

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126-240 | Week 19

Tuesday

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(c) $\rightarrow \begin{bmatrix} 2 & -4 & 1 & 6 \\ -4 & 0 & 3 & -1 \\ 0 & 1 & -1 & 3 \end{bmatrix}$

$\downarrow \begin{matrix} R_2 = R_2 + 2R_1 \\ R_1 = \frac{1}{2}R_1 \end{matrix}$

$\begin{bmatrix} \boxed{1} & -2 & \frac{1}{2} & 3 \\ 0 & -8 & 5 & 11 \\ 0 & +1 & -1 & 3 \end{bmatrix}$

$\downarrow R_2 \longleftrightarrow R_3$

$\begin{bmatrix} \boxed{1} & -2 & \frac{1}{2} & 3 \\ 0 & \boxed{1} & -1 & 3 \\ 0 & -8 & 5 & 11 \end{bmatrix}$

$\downarrow R_3 \rightarrow R_3 + 8R_2$

$\begin{bmatrix} \boxed{1} & -2 & \frac{1}{2} & 3 \\ 0 & \boxed{1} & -1 & 3 \\ 0 & 0 & -3 & 35 \end{bmatrix}$

$\downarrow R_3 = -\frac{1}{3} R_3$

$\begin{bmatrix} \boxed{1} & -2 & \frac{1}{2} & 3 \\ 0 & \boxed{1} & -1 & 3 \\ 0 & 0 & \boxed{1} & -\frac{35}{3} \end{bmatrix}$ (REF form

$\downarrow$ (convert it to RREF)

9. → $\begin{bmatrix} 1 & 4 \\ 3 & 11 \end{bmatrix}$

$R_2 \to R_2 - 3R_1$

$$\begin{bmatrix} 1 & 4 \\ 0 & -1 \end{bmatrix}$$

$\downarrow R_2 \to -1 \cdot R_2$

$$\begin{bmatrix} \boxed{1} & 4 \\ 0 & \boxed{1} \end{bmatrix} \text{ (REF)}$$

$\downarrow$ (RREF)

$$\begin{bmatrix} \boxed{1} & 0 \\ 0 & \boxed{1} \end{bmatrix}$$

→ Yes, a different REF form are possible in this case, but not RREF form.

$R_1 \leftrightarrow R_2$

$$\begin{bmatrix} 3 & 11 \\ 1 & 4 \end{bmatrix}$$

$\downarrow R_1 \to R_1/3$

$$\begin{bmatrix} 1 & 11/3 \\ 1 & 4 \end{bmatrix}$$

$\downarrow R_2 \to R_2 - R_1$

$$\begin{bmatrix} 1 & 11/3 \\ 0 & 1/3 \end{bmatrix}$$

$\downarrow R_2 = 3 \cdot R_2$

$$\begin{bmatrix} \boxed{1} & 11/3 \\ 0 & \boxed{1} \end{bmatrix} \text{ (REF)}$$

$\downarrow$ (RREF)

$$\begin{bmatrix} \boxed{1} & 0 \\ 0 & \boxed{1} \end{bmatrix}$$

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