Que (7) →

$$A = \begin{bmatrix} 3 & 2 & 2 \\ 2 & 3 & -2 \end{bmatrix}_{2\times3}$$

$\boxed{AV = U\Sigma}$; for SVD ⇒ $\boxed{A = U\Sigma V^T}$

$\boxed{A^TA \Rightarrow V\Sigma^2V^T}$

Now →

$$A^TA \Rightarrow \begin{bmatrix} 3 & 2 \\ 2 & 3 \\ 2 & -2 \end{bmatrix} \cdot \begin{bmatrix} 3 & 2 & 2 \\ 2 & 3 & -2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 13 & 12 & 2 \\ 12 & 13 & -2 \\ 2 & -2 & 8 \end{bmatrix}$$

Cal^c. E.V. →

$$(A^TA - \lambda I) \Rightarrow \begin{bmatrix} 13-\lambda & 12 & 2 \\ 12 & 13-\lambda & -2 \\ 2 & -2 & 8-\lambda \end{bmatrix}$$

$$\Rightarrow (13-\lambda)\cdot\{(13-\lambda)\cdot(8-\lambda) - 4\}$$
$$-12\{96 - 12 + 4\} + 2\cdot\{-24 - 26 + 2\lambda\}.$$

$\Rightarrow$ ___.

$\Rightarrow \lambda_1, \lambda_2, \lambda_3 \rightarrow$ E.V. of $\boxed{A^TA}$

step (2) → we'll cal^c. their respective eigen vector. if 2 eigen values are same → apply gram-schmidt & make orthonormal.

$$V = \begin{bmatrix} \text{E.V. corresponding to} \\ \text{E.V.} \end{bmatrix} \Rightarrow \begin{bmatrix} v_1 & v_2 & v_3 \end{bmatrix}$$

$$\sigma_1 = \sqrt{\lambda_1}$$
$$\sigma_2 = \sqrt{\lambda_2}$$
$$\sigma_3 = \sqrt{\lambda_3}$$

$$U = \begin{bmatrix} \frac{Av_1}{\sigma_1} & \frac{Av_2}{\sigma_2} & \frac{Av_3}{\sigma_3} \end{bmatrix}$$

→ These are steps. ⇒ so → $\boxed{AV = U\Sigma}$