(5) → Dim.(V) = Dim.(W).

$\boxed{T : V \to W}$

So, suppose, we have a L.I. set → $\{u_1, u_2, \ldots, u_r\}$.

So → $c_1 u_1 + c_2 u_2 + \ldots + c_r u_r = 0$, if & only if $\boxed{c_1 = c_2 = \ldots = c_r = 0}$

So → it means that all the linear comb.$^n$ are unique, since it belongs to a V.S. & L.I. →

So; from → $\boxed{T : V \to W.}$

$$\boxed{T(c_1 u_1 + c_2 u_2 + \ldots + c_r u_r) = c_1 . T(u_1) + c_2 . T(u_2) + \ldots + c_r . T(u_r)}$$

↳ Given

So, → L.H.S. will map to diff. images each time, if & only if, in R.H.S., $T(u_1), T(u_2)$ & ... $T(u_r)$ are giving diff$^n$. value each time as linear comb.$^n$.

∴ for this $T(u_1), T(u_2) \ldots T(u_r)$ should be L.I.

→ Bcoz, if they are L.D., 2 comb.$^n$ will give the same value (ie. 0 value).