08 October 2021 11:59

$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \end{bmatrix}$

$\mathcal{R}(A) = \underbrace{\text{span}\{(1,2,3), (2,4,6)\}} = \text{Span}\{(1,2,3)\}$

$\mathcal{C}(A) = \text{Span}\left\{\begin{bmatrix}1\\2\end{bmatrix}, \begin{bmatrix}2\\4\end{bmatrix}, \begin{bmatrix}3\\6\end{bmatrix}\right\} = \underline{\text{Span}\left\{\begin{bmatrix}1\\2\end{bmatrix}\right\}}$

$\mathcal{N}^0(A) = \{x : Ax = 0\}$

$= \left\{\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix} : A\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix} = 0\right\}$

$= \left\{\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix} : x_1 + 2x_2 + 3x_3 = 0 = 2x_1 + 4x_2 + 3x_3\right\}$

$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \end{bmatrix}$ $R_2 \to R_2 - 2R_1$ $B = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 0 \end{bmatrix}$

$\mathcal{R}(A) = \mathcal{R}(B)$ $\underline{\mathcal{R}(B)}$ | $C(B) = \text{Span}\left\{\begin{bmatrix}1\\0\end{bmatrix}, \begin{bmatrix}2\\0\end{bmatrix}, \begin{bmatrix}3\\0\end{bmatrix}\right\}$

$C(A) = \text{Span}\left\{\begin{bmatrix}1\\2\end{bmatrix}\right\}$ $= \text{Span}\left\{\begin{bmatrix}1\\0\end{bmatrix}\right\}$

$A = \begin{bmatrix} 1 & 3 \\ 5 & 7 \end{bmatrix}$ $\underrightarrow{R_2 \to R_2 - 5R_1}$ $\underline{\begin{bmatrix} 1 & 3 \\ 0 & -8 \end{bmatrix}}$

B

$\mathcal{R}(A) = \text{Span}\{(1,3), (5,7)\} \subseteq \mathbb{R}^2$

$= \mathbb{R}^2$

$\mathcal{R}(B) = \text{Span}\{(1,3), (0,-8)\} \subseteq \mathbb{R}^2$

$= \mathbb{R}^2$

$\Rightarrow \mathcal{R}(A) = \mathcal{R}(B)$

$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 0 \end{bmatrix}$

$\mathcal{N}(A) = \left\{\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix} : \begin{matrix} x_1 + 2x_2 + 3x_3 = 0 \\ 2x_1 + 4x_2 + 6x_3 = 0 \end{matrix}\right\}$

$= \left\{\begin{bmatrix}x_1\\ \vdots\end{bmatrix} : x_1 + 2x_2 + 3x_3 = 0\right\} = \mathcal{N}(B)$

$$= \left\{ \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} : x_1 + 2x_2 + 3x_3 = 0 \right\} = N(B)$$

Thm1. Proof

$Ax = b$ ① suppose $x = \begin{bmatrix} c_1 \\ \vdots \\ c_n \end{bmatrix}$ be solution of ①

$$A \begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_n \end{bmatrix} = b$$

$$A = \begin{bmatrix} A_1 & \cdots & A_n \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_n \end{bmatrix} = b \Rightarrow c_1 A + c_2 A_2 + \cdots + c_n A_n = b$$

1st col ... nth

$\Rightarrow b \in \text{span}(A_1, \ldots, A_n)$

$\Rightarrow b \in \mathcal{C}(A)$.

Thm 2. Proof

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix}$$

$Ax = 0$ has only trivial sol.

Thm 3. $A = \begin{bmatrix} A_1 & \cdots & A_n \end{bmatrix}$ $B = \begin{bmatrix} B_1 & \cdots & B_n \end{bmatrix}$

$A_{i_1} \ldots A_{i_r} \to$ L.D iff $B_{i_1} \ldots B_{i_r}$ LD

$$A = \begin{bmatrix} A_1 & \cdots & A_n \end{bmatrix} \xrightarrow{R_1, R_2 \ldots R_r} \begin{bmatrix} B_1 & \cdots & B_n \end{bmatrix}$$

$\{A_{i_1} \ldots A_{i_k}\}$ forms a basis for $\mathcal{C}(A)$ iff $\{B_{i_1}, \ldots, B_{i_k}\}$ forms a basis for $\mathcal{C}(B)$

$S = \{(1,2,3), (3,5,7), (1,-2,1), (-1,0,1)\} \subseteq \mathbb{R}^3$

$V = \text{Span}\, S. \subseteq \mathbb{R}^3$

To find Basis V.

$$A = \begin{bmatrix} 1 & 3 & 1 & -1 \\ 2 & 5 & -2 & 0 \\ 3 & 7 & 1 & 1 \end{bmatrix}$$

$\mathcal{C}(A) = V$? X

$V = \{x^t : x \in \mathcal{C}(A)\}$

$$\begin{bmatrix} 3 & 7 & 1 & 1 \end{bmatrix} \qquad V = \{ x^t : x \in \mathcal{C}(A) \}$$

$R_2 \to R_2 - 2R_1, \quad R_3 \to R_3 - 3R_1$

$$= \begin{bmatrix} 1 & 3 & 1 & -1 \\ 0 & -1 & -4 & 2 \\ 0 & -2 & -2 & 4 \end{bmatrix} \qquad R_2 \to -R_2$$

$$= \begin{bmatrix} 1 & 3 & 1 & -1 \\ 0 & 1 & 4 & -2 \\ 0 & -2 & -2 & 4 \end{bmatrix}$$

$R_3 \to R_3 + 2R_2$

$$= \begin{bmatrix} 1 & 3 & 1 & -1 \\ 0 & 1 & 4 & -2 \\ 0 & 0 & 6 & 0 \end{bmatrix} \qquad R_3 \to \frac{1}{6} R_3$$

$$= \begin{bmatrix} \boxed{1} & 3 & 1 & -1 \\ 0 & \boxed{1} & 4 & -2 \\ 0 & 0 & \boxed{1} & 0 \end{bmatrix} = B$$

$\{B_1, B_2, B_3\}$ are L.I

$\Rightarrow \mathcal{C}(B) \subseteq \mathbb{R}^3$

$\Rightarrow \dim \mathcal{C}(B) \leq 3$ and $\{B_1, B_2, B_3\}$ are L.I

Hence $\{B_1, B_2, B_3\}$ forms a basis for $\mathcal{C}(B)$

Since A and B are row equivalent,

$\{A_1, A_2, A_3\}$ forms a basis for $\mathcal{C}(A)$

$\Rightarrow \{(1,2,3), (3,5,7), (1,-2,1)\}$ forms a basis for V.

$P = \{ (x, y, 0) : x, y \in \mathbb{R} \}$

$\text{Basis} = \{ (1,0,0), (0,1,0) \}$

$V = \{ (x_1, x_2, x_3) : a_1 x_1 + a_2 x_2 + a_3 x_3 = 0 \}$.

$\text{Basis} = \{ (a_2, -\underline{a_1}, 0), (a_3, 0, -\underline{a_1}) \}$

Linearly Independent
This is eqn. of plane

Elements of basis ...satisfy our eqn.