~~(c)~~ → lecture (23) →

Tutorial → (7)

~~(b)~~ →

(1). → $\alpha \rightarrow$ scaler

$\boxed{Ax = \lambda X}$ → λ are eigenValues.

⇒ $(A - \alpha I) \cdot X_1 = \lambda_1 X_1$

$\boxed{(A - \alpha I - \lambda_1 I) \cdot X_1 = 0}$

(2) → $\boxed{Ax = \lambda X}$ — (1). where 'λ' is scaler & 'X' is EV. $\neq 0$

(a) → $A^2 = A$

$A^2 \cdot X = A \cdot X$

$A \cdot (A \cdot X) = (A \cdot X)$

$A \cdot (\lambda X) = \lambda \cdot X$

$\lambda \cdot (AX) = \lambda \cdot X$

$\lambda^2 \cdot X = \lambda \cdot X$

$\boxed{(\lambda^2 - \lambda) \cdot X = 0}$

Now, since EV. is not equal to 0. so—

$(\lambda^2 - \lambda) = 0$

$\lambda(\lambda - 1) = 0$

$\boxed{\lambda = 0, 1}$

02 Sunday

(b) → $\boxed{A^m = 0}$; A E $\boxed{m \geq 1}$

$(A^m \cdot x) = (0 \cdot x)$

$\lambda \cdot (A^{m-1} \cdot x) = (0 \cdot x)$

$\lambda^2 \cdot (A^{m-2} \cdot x) = (0 \cdot x)$

$(\lambda^m \cdot x) = (0 \cdot x)$

$\boxed{(\lambda^m - 0) \cdot x = 0}$ ⇒ $\lambda^m = 0$

$\boxed{\lambda = 0}$

3 → $A_{n \times n}$ → all e.v. ⇒ 0

(b) → $(\lambda - 0)^n = 0$ → $\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & \cdots \\ & 0 & & & & \\ & & 0 & & & \\ & & & & & \end{bmatrix}$

$\boxed{GM = AM}$ for ($\lambda = 0$);

all(n) free variables.

⇓

Diagonalizable.

(a) → Not necessary that it's diagonalizable.

(4) → (a). $\begin{bmatrix} 1 & 1 \\ 4 & 1 \end{bmatrix} \Rightarrow A$

$$(A-\lambda I) = \begin{bmatrix} 1-\lambda & 1 \\ 4 & 1-\lambda \end{bmatrix}$$

$$(1-\lambda)^2 - 4 = 0$$

$$(1-\lambda+2)\cdot(1-\lambda-2) = 0$$

$$\boxed{(3-\lambda)\cdot(-1-\lambda) = 0} \rightarrow \text{characteristic poly}$$

$$\Downarrow$$

$$\boxed{\lambda = (-1), 3} \qquad \text{minimal poly.}$$

$$\chi(x) = (3-x)\cdot(-1-x)$$

$$M(x) = (3-x)\cdot(-1-x)$$

$$M_A(x) = \{(3-A)\cdot(-1-A)\}$$

$$M_A(x) \Rightarrow \{(A-3)\cdot(A+1)\}$$

$$\Rightarrow \begin{bmatrix} -2 & 1 \\ 4 & -2 \end{bmatrix} \cdot \begin{bmatrix} 2 & 1 \\ 4 & 2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = \underline{0}$$

$-27 + 45 - 24 + 6$

$\cancel{-51 + 51}$

(b) $\rightarrow$ $$\begin{bmatrix} -1-\lambda & 1 & 2 \\ 2 & 2-\lambda & 2 \\ -3 & -6 & -6-\lambda \end{bmatrix}$$

$$\Rightarrow (-1-\lambda)\cdot\{(2-\lambda)\cdot(-6-\lambda)+12\} - 1\{(-12-2\lambda)+6\} + 2\cdot\{-12+3\cdot(2-\lambda)\}$$

$$\Rightarrow -(\lambda+1)\cdot\{(\lambda-2)\cdot(\lambda+6)+12\} + 1\cdot\{2\lambda+6\} - 2\{3\lambda+6\}$$

$$\Rightarrow -(\lambda+1)\cdot\{\lambda^2+4\lambda\cancel{-12}\cancel{+12}\} + 2\lambda+6-6\lambda-12$$

$$\Rightarrow -\lambda^3-4\lambda^2-\lambda^2-4\lambda-4\lambda-6$$

$$\Rightarrow -\lambda^3-5\lambda^2-8\lambda-6$$

$$\Rightarrow -(\lambda^3+5\lambda^2+8\lambda+6)$$

$$\Rightarrow -\{\lambda^2(\lambda+3)+2\lambda(\lambda+3)+2(\lambda+3)\}$$

$$\Rightarrow -\{(\lambda+3)\cdot(\lambda^2+2\lambda+2)\}$$

b^2-4ac

$\downarrow$

$4-4\times1\times2$

-4

5 → (a).
$$\begin{bmatrix} 0-x & 0 & 2 \\ 0 & (2-x) & 0 \\ 2 & 0 & 3-x \end{bmatrix}$$

$\Rightarrow -x\{(2-x)\cdot(3-x)\} + 2\cdot\{-2\cdot(2-x)\}$

$\Rightarrow (2-x)\cdot\{x(3-x)+2\cdot 2\}$

$\Rightarrow (x-2)\cdot\{3x-x^2+4\}$

$\Rightarrow (x-2)\cdot -\{x^2-3x-4\}$

$\Rightarrow -(x-2)\cdot(x-4)(x+1) \longrightarrow$ char. eq.^{n}

$\boxed{x = -1, 2, 4}$

Minimal Polynomial → Diagonalizable → Compute {P^{-1} & P} & Diagonal matrix

So → $B = PAP^{-1}$
→ $B^2 = PAP^{-1}\cdot P\cdot AP^{-1}$

$\boxed{B^2 \Rightarrow PAP^{-1}}$

Similarly → $\boxed{B^{100} = P\cdot A^{100}\cdot P^{-1}}$

(b) $\rightarrow$ $\begin{bmatrix} 1-x & 1 & -1 \\ -1 & (1-x) & 1 \\ -1 & 1 & (1-x) \end{bmatrix}$

$\Rightarrow (1-x)\cdot\{(1-x)^2-1\} - 1\cdot\{\cancel{-1}+x+\cancel{1}\}$
$-1\cdot\{\cancel{-1}+\cancel{1}-x\}$

$\Rightarrow (1-x)\cdot\{x^2+\cancel{1}-2x-\cancel{1}\} - \cancel{x}+\cancel{x}$

$\Rightarrow -(x-1)\cdot(x-2)\cdot x \rightarrow$ Char. eq.^{n}.

So $\rightarrow$ $\boxed{x = 0, 1, 2}$

Similar as Ques. 5 (a).

(6) $\rightarrow$ —

$-\{x^2-3x+2\}\cdot x$

(7) $\rightarrow$

$\Rightarrow -\{x^3-3x^2+2x\}$