③. ⓐ → T: map from $R_3[x] \longrightarrow R_2[x]$.

Suppose → $\boxed{P(x) = ax^3 + bx^2 + cx + d}$; as $P(x) \in R_3[x]$.

$\boxed{P'(x) = (3ax^2 + 2bx + c);}$

$\boxed{P''(x) = (6ax + 2b)}$

Sol → Given →

$$T(P(x)) = -x(3ax^2 + 2bx + c) - 2x^2 \cdot (6ax + 2b);$$

$$\Rightarrow -3ax^3 - 2bx^2 - cx - 12ax^3 - 4bx^2$$

$$\boxed{T(x) = (-15ax^3 - 6bx^2 - cx)}$$

To prove L.T. →

$$T(c_1x_1 + c_2x_2) =$$

$$= -15 \cdot a \cdot (c_1x_1 + c_2x_2)^3 - 6b(c_1x_1 + c_2x_2)^2 - c(c_1x_1 + c_2x_2)$$

$$\Rightarrow c_1 \cdot (-15ax_1^3 - 6bx_1^2 - cx_1) + c_2 \cdot (-15ax_2^3 - 6bx_2^2 - cx_2)$$

ⓑ →