Tutorial → (8) ⇒ Linear Algebra

(1) → $\langle v_1, v_2, v_3, \ldots, v_k \rangle$; linearly dependent vectors.

→ u_k can be written as sum of (L.C. of all L.I. vectors.) and (another ⊥ vector to it).

→ $u_1 = v_1$
$u_2 = v_2 - \langle v_2, u_1 \rangle . u_1$ {supposing all are orthonormal vectors}.

$\vdots$

$$u_k = v_k - \langle v_k, u_1 \rangle u_1 - \langle v_k, u_2 \rangle . u_2 - \langle v_k, u_3 \rangle . u_3 - \ldots\ldots - \langle v_k, u_{k-1} \rangle . u_{k-1} \quad \text{—— (1)}$$

Since v_k is L.C. of remaining. so, →

$$v_k = c_1 v_1 + c_2 v_2 + c_3 v_3 + \ldots\ldots + c_{k-1} . v_{k-1}$$
$$\Rightarrow \lambda_1 u_1 + \lambda_2 u_2 + \lambda_3 u_3 + \ldots\ldots + \lambda_{k-1} . u_{k-1}$$

from eqn (1) →

$$u_k = v_k - \lambda_1 u_1 - \lambda_2 u_2 - \ldots\ldots - \lambda_{k-1} . u_{k-1}$$
$$\Rightarrow v_k - (v_k) \Rightarrow \vec{0}$$

This canbe proved using pictorial repr.n in one line.

$x, y \in \mathbb{R}^n$

(2) → (P.T) → $\langle x, y \rangle = x^t_{1\times n} \cdot A_{n\times n} \cdot Y_{n\times 1}$

$\boxed{A = A^T} \Rightarrow A_{(i,j)} = \langle e_i, e_j \rangle$

⇓ that means

$$A_{i,j} = \begin{cases} 0, & i \neq j \\ T, & i = j \end{cases}$$

$A_{n\times n} \rightarrow$ {Diagonal matrix}

$$A_{n\times n} = \begin{bmatrix} \|e_1\|^2 & 0 & 0 & 0 & \cdots \\ 0 & \|e_2\|^2 & 0 & 0 & \cdots \\ 0 & 0 & \|e_3\|^2 & 0 & \cdots \\ 0 & 0 & 0 & \|e_4\|^2 & \cdots \\ \cdots & & & & \end{bmatrix}$$

So, if $x^t = \langle x_1, x_2, x_3, \cdots, x_n \rangle$

$Y = \langle Y_1, Y_2, Y_3, \vdots, Y_n \rangle$

$= (x^t \cdot A) \cdot Y$

$\Rightarrow [x_1\|e_1\|^2 \quad x_2\|e_2\|^2 \quad x_3\|e_3\|^2 \cdots \cdots x_n\|e_n\|^2] \cdot \begin{bmatrix} Y_1 \\ Y_2 \\ Y_3 \\ \vdots \\ Y_n \end{bmatrix}$

Is this true?

$\Rightarrow (x_1 Y_1 + x_2 Y_2 + x_3 Y_3 + \cdots + x_n Y_n)$

Que. → Will norm of v changes with changing def. of I.P.S.?

③. → ⓐ. using dot product →

$$V = \{v_1, v_2, v_3\}.$$

$$u_1 = v_1 = (1,1,1)$$

$$u_2 = v_2 - \frac{\langle v_2, u_1 \rangle}{\langle u_1, u_1 \rangle} \cdot u_1 \Rightarrow (1,0,1) - \langle (1,0,1),(1,1,1) \rangle \cdot (1,1,1)$$

$$\Rightarrow (1,0,1) - \frac{2}{3}(1,1,1) \Rightarrow \left(\frac{1}{3}, -\frac{2}{3}, \frac{1}{3}\right)$$

$$\Rightarrow (1,-2,1)$$

$$u_3 = v_3 - \frac{\langle v_3, u_2 \rangle}{\langle u_2, u_2 \rangle} \cdot u_2 - \frac{\langle v_3, u_1 \rangle}{\langle u_1, u_1 \rangle} \cdot u_1$$

$$\Rightarrow (0,1,2) - \{0 - \frac{3}{5} \cdot (1,1,1)$$

$$\Rightarrow \left(-\frac{3}{5}, \frac{2}{5}, \frac{7}{5}\right) \Rightarrow (-3,2,7)$$

ⓑ. $\langle (x,y,z),(x',y',z') \rangle = (xx' + 2yy' + 3zz')$

$$u_1 = (1,1,1)$$

$$u_2 = (1,0,1) - \frac{\langle (1,0,1),(1,1,1) \rangle}{\langle (1,1,1),(1,1,1) \rangle} (1,1,1)$$

$$\Rightarrow (1,0,1) - \frac{\cancel{4}\,2}{\cancel{6}\,3}(1,1,1) \Rightarrow \left(\frac{1}{3}, -\frac{2}{3}, \frac{1}{3}\right)$$

$$\Rightarrow (1,-2,1)$$

$\parallel^{ly} \rightarrow$ u_3.

Q → what is orthogonal matrix? → intuition

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(4) →

$$A \cdot A^T = I$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} a & c \\ b & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} a^2+b^2 & ac+bd \\ ac+bd & c^2+d^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$\boxed{a^2+b^2 = 1 = c^2+d^2}$; $ac+bd = 0$

$$\left(-\frac{bd}{c}\right)^2 + b^2 = 1$$

$$b^2d^2 + b^2c^2 = c^2$$

$$b^2(c^2+d^2) = c^2$$

$$b^2 = c^2$$

$b = \pm c$; ⟹ $\boxed{a = \pm d}$

$\boxed{ac+bd=0}$

(1). $\begin{bmatrix} +a & +b \\ -b & +a \end{bmatrix}$

(2) $\begin{bmatrix} +a & -b \\ +b & +a \end{bmatrix}$

(3). $\begin{bmatrix} +a & +b \\ +b & -a \end{bmatrix}$

(4). $\begin{bmatrix} +a & -b \\ -b & -a \end{bmatrix}$

	+	d
a +	✓	✓
−		

16 Sunday

sim ⟶ Total (B

$\exists_A$; $\boxed{AA^T = I}$ $\quad v \in \mathbb{R}^2$

$$\underset{2\times 2}{A}\cdot \underset{2\times 1}{v} = \underset{2\times 1}{v'}$$

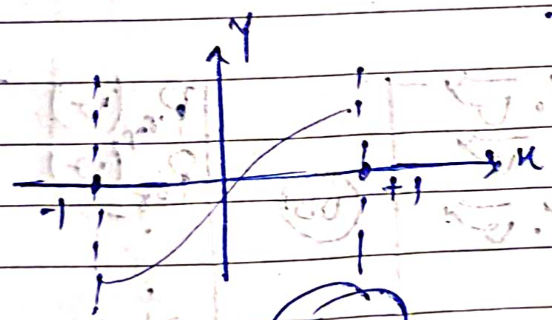
$$A^T \cdot A \cdot v = A^T v'$$

$$\boxed{v = A^T \cdot v'}$$

→ can be written as Linear Transformation.

check,

5 → $\langle P(x), Q(x)\rangle = \int_{-1}^{1} P(x)\cdot Q(x)\cdot dx$



P → I True

if $\langle P(x), P(x)\rangle = 0$

$$\int_{-1}^{+1} \{P(x)\}^2 \cdot dx = 0$$

$\Downarrow$

$$\langle \alpha P_1(x) + \beta P_2(x), Q(x)\rangle = \int_{-1} (\text{———}) \cdot Q(x) \cdot dx$$

$$\Rightarrow \alpha \int_{-1}^{1} \text{———} + \beta \cdot \int_{-1}^{1} \text{———}$$

$\Rightarrow \alpha\text{—} + \beta\text{—} \Rightarrow$ P → II True

So, Does P → III.

ordered basis.

Basis of $\mathbb{R}_3(x) \Rightarrow \{1, x, x^2, x^3\}$.

orthonormal basis of $\mathbb{R}_3 \rightarrow$

$\{(1,0,0,0), (0,1,0,0), (0,0,1,0), (0,0,0,1)\}$.

(6). $\rightarrow$ $\boxed{A \cdot A^T = I}$

$A \rightarrow n \times n$

$$A = \begin{bmatrix} r_1 \\ r_2 \\ \vdots \\ r_n \end{bmatrix}_{n \times n}$$

$\|A \cdot v\| = \|v\|$

$\Downarrow$

$\cancel{\langle A \cdot v, A \cdot v \rangle = \langle v, v \rangle}$

$v = [x_1, x_2, x_3, \ldots, x_n]^T_{n \times 1}$

$$A \cdot \vec{v} = \begin{bmatrix} \vec{r_1} \cdot \vec{v} \\ \vec{r_2} \cdot \vec{v} \\ \vec{r_3} \cdot \vec{v} \\ \vdots \\ \vec{r_n} \cdot \vec{v} \end{bmatrix}_{n \times 1} \text{ (or) } \begin{bmatrix} P_{r_{r_1}}(v) \\ P_{r_{r_2}}(v) \\ \vdots \\ P_{r_{r_n}}(v) \end{bmatrix}$$

~~(or)~~

(7) → $\boxed{A \cdot A^T = I}$; $\boxed{B \cdot B^T = I}$

$(AB) \cdot (AB)^T \Rightarrow (AB \cdot B^T A^T) = A \cdot A^T = (I)$ ✓

$(BA) \cdot (BA)^T = (BA \cdot A^T \cdot B^T) \Rightarrow (I)$ ✓

(8) → $\begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ 0 & a_{22} & a_{23} & \cdots & a_{2n} \\ 0 & 0 & a_{33} & \cdots & a_{3n} \\ \cdots & & & & \\ 0 & 0 & 0 & \cdots & a_{nn} \end{bmatrix} \cdot$ (Transpose)

$\Rightarrow \begin{bmatrix} \|r_1\|^2 & \cdots\cdots & a_{1n} \cdot a_{nn} \\ & & \vdots \\ & & \|r_n\|^2 \end{bmatrix}$

$a_{nn}^2 = 1$

$\boxed{a_{nn} = \pm 1}$; $\boxed{a_{1n} = 0}$ $a_{2n} = 0$, $a_{3n} = 0 \cdots = a_{n-1,n} = 0$

$\|^{ly}$, we can prove for all non-digonal elements = (0)

(Note) → is it necessary to be identity. bcoz any comb.^{n} of sign can occur.

Que:→ Is it necessary that if $A \cdot A^T$ is I, then $A^T A = I$ will be I as well?, would that be called orthogonal?

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Thursday

(9) → Since, Rows are orthogonal vectors & therefore L.I., And since cardinality = n.

so → Basis.

→ $\langle r_1, r_2, \dots r_n \rangle$

$\langle r_1, r_2 \rangle = 0$

$u_1 = r_1 \Rightarrow u_2 = r_2 - \langle r_2, u_1 \rangle \cdot u_1$

$u_2 = r_2 - 0 \Rightarrow r_2$

Sim'ly →