$D(a_0 + a_1 x + a_2 x^2) = a_1 + 2a_2 x$

$\text{Ker}(D) = \{ a_0 \in \mathbb{R}_2[x] : a_0 \in \mathbb{R} \}$

$= \{ a_0 : a_0 \in \mathbb{R} \}$

$\mathcal{R}(D) = \mathbb{R}_1[x]$

$\mathcal{R}(D) \subseteq \mathbb{R}_1[x]$ — ①

Let $a_0 + a_1 x \in \mathbb{R}_1[x]$

Let $f = a_0 x + \frac{a_1}{2} x^2 \in \mathbb{R}_2[x]$

$D(f) = D(a_0 x + \frac{a_1}{2} x^2) = a_0 + a_1 x \in \mathcal{R}(D)$

$\Rightarrow \mathbb{R}_1[x] \subseteq \mathcal{R}(D)$ — ②

from ① and ② we get $\mathcal{R}(D) = \mathbb{R}_1[x]$

Let X and Y be sets and $f : X \to Y$ which is one-one.

$g : S \to X$ s.t $g \circ f : X \to X$ is an identity map, where $S \subseteq Y$

Take $S = f(X) = \text{Range}(f)$

Want $g : R(f) \to X$, s.t $g \circ f : X \to X$ is an identity map

Let $y \in R(f)$, $\exists\, x \in X$ s.t $f(x) = y$

$(g \circ f)(x) = x$ main Idea

$g(f(x)) = x$

Define $g(y) = x$, where $f(x) = y$

Then g is a function and $(g \circ f)(x) = x \ \forall x \in X$.

Hence g is inverse map of f.

Ex. $f : \mathbb{R}^2 \longrightarrow \mathbb{R}^3$

$e_1 \longrightarrow e_1 + e_2$

$e_2 \longrightarrow e_2 + e_3$

Verify that f is one-one Linear Transformation.

$R(f) = \{ c_1(e_1+e_2) + c_2(e_2+e_3) : c_1, c_2 \in \mathbb{R} \}$

$f(c_1 e_1 + c_2 e_2) = c_1(e_1+e_2) + c_2(e_2+e_3)$

$f^{-1} : R(f) \longrightarrow \mathbb{R}^2$

$$\boxed{f^{-1}(\underbrace{c_1(e_1+e_2) + c_2(e_2+e_3)}_{y}) = c_1 e_1 + c_2 e_2 \quad \forall\, y \in R(f)}$$

Check that f^{-1} is a L.T.

$T : \underset{B}{V} \longrightarrow \underset{B'}{W}$ $\qquad [T(v)]_{B'} = [T]_B^{B'} [v]_B$

$I : \underset{B}{V} \longrightarrow \underset{B'}{V}$ $\qquad [I(v)]_{B'} = [I]_B^{B'} [v]_B$

$A \sim B$, If $\exists$ invertible matrix P s.t

$P^{-1} A P = B$

$B^2 = (P^{-1} A P)^2 = (P^{-1} A P)(P^{-1} A P)$

$= P^{-1} A^2 P$

$B^k = (P^{-1} A P)(P^{-1} A P)(P^{-1} A P) \cdots (P^{-1} A P) = P^{-1} A^k P$

Observation: If A and B are similar, then $A^k \sim B^k \ \forall k$.

$\begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix}^{1000} \longrightarrow P^{-1} D P$

D: Diagonal mat

$A = P^{-1} D P$

$A^{1000} = P^{-1} D^{1000} P$

Question Let V be a V.S with ordered basis.

$B = \{v_1, \dots, v_n\}$. $T : V \longrightarrow V$

When $[T]_B$ be diagonal?

$$[T]_B = \left[[T(v_1)]_B \ [T(v_2)]_B \ \dots \ T[v_n]_B \right]$$

Suppose $[T]_B$ is diagonal.

$$[T(v_1)]_B = \begin{pmatrix} a_1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix}, \quad T(v_1) = a_1 v_1, \ a_1 \in \mathbb{R}$$

$$[T(v_2)]_B = \begin{pmatrix} 0 \\ a_2 \\ 0 \\ \vdots \\ 0 \end{pmatrix} \quad T(v_2) = a_2 v_2, \ a_2 \in \mathbb{R}$$

$$\vdots$$

$$T(v_n) = a_n v_n, \ a_n \in \mathbb{R}$$

non zero

eigen value. eigen vector

$Ax = \lambda x$. x is a non zero vector.

eigen value eigen vector

Question How to find eigenvector and eigenvalue for a matrix A and linear transformation T