$$A_\lambda = \{x : Ax = \lambda x\}$$
$$= \{x : Ax = \lambda I x\}$$
$$= \{x : Ax - \lambda I x = 0\}$$
$$= \{x : (A - \lambda I)x = 0\}$$
$$= N(A - \lambda I)$$

Q. 1 How to find the eigenvalues of a matrix

Q. 2 How many distinct eigen value can be occured

Q. 3 Number of Eigen value is infinite or finite?

Ex. (1) $A = I_{n \times n}$ $x \in \mathbb{R}^n$

$$A x = \underbrace{\lambda}_{\text{eigen value}} \underbrace{x}_{\text{eigen vector}}$$

$$Ax = Ix = x$$

$\Rightarrow$ x is an eigenvector of I correspond to eigen value 1.

(2) $A = O_{n \times n}$

$$Ax = O \cdot x = 0 = 0 \cdot x$$

eigenvalue 0, $\forall x \in \mathbb{R}^n \setminus \{0\}$ is an eigen vecto.

(3) $\begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}$, $\lambda_1 \neq \lambda_2$

$$\begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} \lambda_1 \\ 0 \end{bmatrix} = \lambda_1 \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ is an e.v corr. to e.v. λ_1

$\begin{bmatrix}1\\0\end{bmatrix}$ is an e.v corr. to e.v. λ_1

Illy $\begin{bmatrix}0\\1\end{bmatrix}$ ______ λ_2.

(4) $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$A - \lambda I = 0$

$\begin{bmatrix} a-\lambda & b \\ c & d-\lambda \end{bmatrix} = 0$

$\Rightarrow b = 0, \ c = 0$

$a - \lambda = 0$

$d - \lambda = 0$

$a = d$

To find the eigenvalue: $\det(A - \lambda I) = 0$

$AX = \lambda x$

$(A - \lambda I)x = 0$

$(A - \lambda I)x = 0$ has a non-trivial sol.

$\Rightarrow \det(A - \lambda I) = 0$

$\boxed{\text{Hence } \lambda \text{ is an eigen value of } A \text{ iff } \det(A - \lambda I) = 0}$

Ex(4) $T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} 2x_1 - 12x_2 \\ x_1 - 5x_2 \end{bmatrix}$ $T(x) = Ax$

Let $B = \{e_1, e_2\}$ be a basis of $\mathbb{R}^2$. Then

$A = [T]_B = \begin{bmatrix} 2 & -12 \\ 1 & -5 \end{bmatrix}$ is matrix of L.T w.r.t Basis B.

$T(e_1) = \begin{bmatrix} 2 \\ 1 \end{bmatrix} = 2e_1 + 1e_2$

$T(e_2) = \begin{bmatrix} -12 \\ -5 \end{bmatrix} = -12e_1 - 5e_2$

Char. eq $\det(A - \lambda I) = 0$

$$\begin{vmatrix} 2-\lambda & -12 \\ 1 & -5-\lambda \end{vmatrix} = 0$$

$\Rightarrow \quad -(2-\lambda)(5+\lambda) + 12 = 0$

$\Rightarrow \quad \lambda^2 + 3\lambda - 10 + 12 = 0$

$\Rightarrow \quad (\lambda+2)(\lambda+1) = 0$

$\Rightarrow \quad \lambda = -2, -1$ — eigenvalue of A and T.

To find the eigenvector of A corresponding to $\lambda = -1$

$$A + I = \begin{bmatrix} 3 & -12 \\ 1 & -4 \end{bmatrix} \longrightarrow$$ calculate the eigenvector of A for $\lambda=-1$

Ans

$x = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$ is an eigenvector of A corresponding to $\lambda = -1$

we need to find v, such that $[v]_B = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$

$$v = 4e_1 + 1 \cdot e_2 = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$$

Illy find the eigenvector for $\lambda = -2$

$$T : \mathbb{R}_2[x] \longrightarrow \mathbb{R}_2[x]$$

$$a_0 + a_1 x + a_2 x^2 \longrightarrow a_2 + a_0 x + a_1 x^2$$

Find the eigen values and eigenvectors.

- Fix a basis
- find the images of basis
- → Find the matrix of L.T, say A
- Char. eq
- → eigenvalue of A
- → Eigenvectors of A

→ Eigenvector of T

→ Try the example (*) by considering the basis $\{(1,0), (1,1)\}$

→ basis $\{(1,2), (2,1)\}$