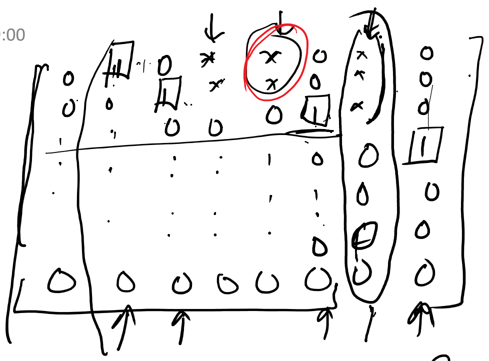
$A_E =$ 

$$\begin{pmatrix} a \\ b \\ c \\ 0 \\ \vdots \\ 0 \end{pmatrix} = ae_1 + be_2 + ce_3$$

Basis for column space A_E.

$(1, 0, 0, \ldots 0)^t = e_1$

$(0, 1, 0, \ldots 0)^t = e_2$

$(0, 0, 1, 0, \ldots 0)^t = e_3$

$(0, 0, 0, 1, 0, \ldots 0)^t = e_4$

$B = \{e_1, e_2, e_3, e_4\}$

B is L.I

$$\mathcal{C}(A_E) = \text{span} \overbrace{\{ \underset{e_1}{c_1}, \underset{e_2}{c_3}, c_4, c_5, \underset{e_3}{c_6}, c_7, \underset{e_4}{c_8} \}}^{S}$$

WTP. $c_7 \in \text{span}\, \underline{(S \setminus \{c_7\})}$

If $v \in \text{span}(S \setminus \{v\})$, then $\text{span}(S) = \text{span}(S \setminus v)$

$S = \left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}, \begin{bmatrix} 3 \\ 5 \\ 7 \end{bmatrix} \right\}$

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 5 \\ 3 & 4 & 7 \end{bmatrix}$$

Basic columns

Basis for column space of A

$$\left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix} \right\}$$

$R_2 - 2R_1, \quad R_3 - 3R_1$

$$\sim \begin{bmatrix} 1 & 2 & 3 \\ 0 & -1 & -1 \\ 0 & -2 & -2 \end{bmatrix}$$

$R_2 \to -R_2$

$$\sim \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 1 \\ 0 & -2 & -2 \end{bmatrix}$$

$R_3 \to R_3 + 2R_2$

$R_1 \to R_1 - 2R_2$

$$\sim \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

Let $S = \{v_1, v_2, \ldots v_n\} \subseteq \mathbb{R}^m$

Find a subset of S which is a basis for $\text{Span}(S)$.

Consider $A = [v_1 \; v_2 \; \ldots \; v_n]$

$$\text{Span}(S) = \mathcal{C}(A)$$

$$ax + by = 0$$
$$cax + cby = 0$$

$$\underbrace{\begin{bmatrix} a & b \\ ca & cb \end{bmatrix}}_{A} \begin{bmatrix} x \\ y \end{bmatrix} = 0$$

$\dim \mathcal{C}(A) = 1$

$\dim(\mathcal{N}(A)) = 1$

$1 + 1 = 2$ (Number of colu

$$x + y + z = 0$$

$$\underbrace{\begin{bmatrix} 1 & 1 & 1 \end{bmatrix}}_{A} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$\dim(\mathcal{N}(A)) = \text{nullity}(A) = 2$

$\text{rank}(A) = 1$

$1 + 2 = 3$ (Number of colu

Rank-Nullity theorem.

$$\text{rank}(A) + \text{nullity}(A) = \#\text{ columns of } A.$$

$x_1 \ldots x_{n-r}, y_1 \ldots y_r$

c_1 $(1, 0, \ldots 0, \alpha_{11} \ldots \alpha_{1r})$

$+ c_2$ $(0, 1, 0 \ldots 0, \alpha_{21}, \ldots \alpha_{2r})$

$\vdots$

$+ c_{n-r}$ $(0, 0, \ldots 1, \alpha_{n-r,1} \ldots, \alpha_{n-r,r})$

Set is L.I

$$+ c_{n-r}\underbrace{(0, 0, \ldots, 1, \alpha_{n-r,1}, \ldots, \alpha_{n-r,r})}_{\text{Basis for } N(A)}$$

$$\dim(N(A)) = n - r = \text{nullity}$$

$$\text{rank} = r$$

$$\text{rank} + \text{nullity} = r + n - r = n = \#\text{ columns}$$

$$W = \{f \in \mathbb{R}_4[x] : f(1) = 0 = f(-1)\}$$

$$\underline{f = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4}$$

$f(1) = 0$, $\quad a_0 + a_1 + a_2 + a_3 + a_4 = 0$

$f(-1) = 0$, $\quad a_0 - a_1 + a_2 - a_3 + a_4 = 0$

$\dim(W) = 3$

$$A \quad \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 & 1 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix} = 0$$

$\text{Rank}(A) = 2$

$\text{Nullity}(A) = 3$

$R_2 \to R_2 - R_1$

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & -2 & 0 & -2 & 0 \end{bmatrix}$$

$R_2 \to -\frac{1}{2} R_2$

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix}$$

$R_1 \to R_1 - R_2$

free variables $- a_2, a_3, a_4$

$$\begin{array}{ccccc} a_0 & a_1 & a_2 & a_3 & a_4 \\ 1 & 0 & 1 & 0 & 1 \end{array}$$

We directly get Basis of N(A) by this method.

$$\begin{array}{cccccc} & a_0 & a_1 & a_2 & a_3 & a_4 \\ & \left[\begin{array}{c} 1 \\ 0 \end{array}\right. & \begin{array}{c} 0 \\ 1 \end{array} & \begin{array}{c} 1 \\ 0 \end{array} & \begin{array}{c} 0 \\ 1 \end{array} & \left.\begin{array}{c} \\ 0 \end{array}\right] \end{array}$$

<u>free variable</u> – $\underset{1}{a_2}, \underset{0}{a_3, a_4}$

$\underset{0}{a_2} = \underset{1}{a_3}, \underset{0}{a_4}$

$a_0 + a_2 + a_4 = 0$

$a_1 + a_3 = 0$

$\Rightarrow a_0 = -(a_2 + a_4)$

$a_1 = -a_3$

$a_2 = 1,\ a_3 = a_4 = 0,\quad a_0 = -1 \quad (-1, 0, 1, 0, 0)$

$a_2 = 0,\ a_3 = 1,\ a_4 = 0,\quad a_1 = -1 \quad (0, -1, 0, 1, 0)$

$a_2 = 0,\ a_3 = 0,\ a_4 = 1,\quad a_0 = -1 \quad (-1, 0, 0, 0, 1)$

$f_1 = -1 + x^2, \quad f_2 = -x + x^3, \quad f_3 = -1 + x^4$

$\{f_1, f_2, f_3\}$ forms a basis for W.