Department of Mathematics
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CSD001P5M **Linear Algebra** **Tutorial: 08**

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1. What happens if we apply Gram-Schmidt procedure on a linearly dependent set?

2. Let $<,>$ be any inner product on $\mathbb{R}^n$. Show that $< x, y >= x^t Ay$ for all vectors $x, y \in \mathbb{R}^n$ where A is the symmetric $n \times n$ matrix whose (i, j) th entry is $< e_i, e_j >$, the vector e_i being the standard basis vectors of $\mathbb{R}^n$.

3. Use Gram-Schmidt process to transform each of the following into an orthonormal basis; (a) $\{(1,1,1),(1,0,1),(0,1,2)\}$ for $\mathbb{R}^3$ with dot product. (b) Same set as in (i) but using the inner product defined by $< (x,y,z),(x',y',z') >= xx' + 2yy' + 3zz'$.

4. Describe all 2×2 orthogonal matrices. Prove that action of any orthogonal matrix on a vector $v \in \mathbb{R}^2$, is either a rotation or a reflection about a line.

5. Let $\mathbb{R}_3(x)$ be a vector space of all polynomials of degree at most 3. Then check that $\langle p(x), q(x) \rangle = \int_{-1}^{1} p(x)q(x)dx$ is an inner product on $\mathbb{R}_3(x)$. By considering the basis $\{1, x, x^2, x^3\}$ of $\mathbb{R}_3(x)$, find an orthonormal basis for $\mathbb{R}_3(x)$.

6. Let A be an orthogonal matrices. Then show that $||Av|| = ||v||$ for all $v \in \mathbb{R}^n$.

7. Let A and B be two $n \times n$ orthogonal matrices. Prove that AB and BA are both orthogonal matrices.

8. Prove that a upper triangular matrices is orthogonal if and only if it is identity matrices.

9. The rows of an orthogonal matrices of size $n \times n$ forms a basis for $\mathbb{R}^n$.

10. Determine an orthonormal basis of $\mathbb{R}^4$ containing the vectors $\frac{1}{2}(1,1,1,1)$ and $\frac{1}{2}(1,-1,-1,1)$.

11. Let V be a real inner product space and $\{v_1, \dots, v_m\}$ be a basis of V. Prove that there exist exactly 2^m orthonormal basis $\{e_1, \dots, e_m\}$ of V such that $\text{span}\{v_1, \dots, v_j\} = \text{span}\{e_1, \dots, e_j\}$.

12. Let V be a vector space with orthonormal basis $\{e_1, dos, e_n\}$. Prove that

$$||v|| = |\langle v, e_1 \rangle| + \cdots + |\langle v, e_n \rangle|.$$