14 October 2021 14:56

$A = [v_1 \;\; \cdots \;\; v_n]$

$A = \begin{bmatrix} w_1 \\ \vdots \\ w_m \end{bmatrix}$

Basis for $\mathcal{C}(A) =$

1. Convert A into RREF A_E
2. Suppose $\{c_1, \ldots, c_r\}$ are the pivotal column of A_E
3. Basis for $\mathcal{C}(A) = \{v_{c_1}, \ldots, v_{c_r}\}$

→ Basis for $\mathcal{R}(A)$

1. Convert A into RREF (A_E)
2. Basis for $\mathcal{R}(A)$ = non zero rows of A_E

→ Rank(A) = $\dim(\mathcal{C}(A)) = \dim(\mathcal{R}(A))$

$\dim(\mathcal{C}(A))$ = # pivotal columns

= # non zero rows in RREF of A

= $\dim(\mathcal{R}(A))$

→ rank(A) = number of L.I rows A. (Exercise.

= # of L.I columns of A

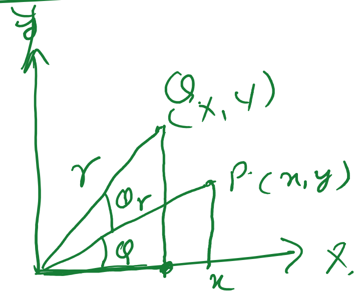
$X = r\cos(\theta + \varphi)$

$Y = r\sin(\theta + \varphi)$

$x = r\cos\varphi$ —— 1

$y = r\sin\varphi$

$X = r\cos(\theta + \varphi)$



$= r(\cos\psi\cos\varphi - \sin\psi\sin\varphi)$

$= x\cos\varphi - y\sin\varphi$

$Y =$

$T\begin{pmatrix} x \\ y \end{pmatrix} = \begin{bmatrix} x+y \\ x-y \end{bmatrix} \quad \forall \begin{bmatrix} x \\ y \end{bmatrix} \in \mathbb{R}^2$

let $\begin{bmatrix} x_1 \\ y_1 \end{bmatrix}, \begin{bmatrix} x_2 \\ y_2 \end{bmatrix} \in \mathbb{R}^2, \quad \alpha, \beta \in \mathbb{R}$

$$T\left(\alpha \begin{bmatrix} x_1 \\ y_1 \end{bmatrix} + \beta \begin{bmatrix} x_2 \\ y_2 \end{bmatrix}\right) = T\begin{bmatrix} \alpha x_1 + \beta x_2 \\ \alpha y_1 + \beta y_2 \end{bmatrix}$$

$$= \begin{bmatrix} \alpha x_1 + \beta x_2 + \alpha y_1 + \beta y_2 \\ \alpha x_1 + \beta x_2 - \alpha y_1 - \beta y_2 \end{bmatrix}$$

$$= \begin{bmatrix} \alpha(x_1 + y_1) + \beta(x_2 + y_2) \\ \alpha(x_1 - y_1) + \beta(x_2 - y_2) \end{bmatrix}$$

$$= \alpha \begin{bmatrix} x_1 + y_1 \\ x_1 - y_1 \end{bmatrix} + \beta \begin{bmatrix} x_2 + y_2 \\ x_2 - y_2 \end{bmatrix}$$

$$= \alpha T\left(\begin{bmatrix} x_1 \\ y_1 \end{bmatrix}\right) + \beta T\left(\begin{bmatrix} x_2 \\ y_2 \end{bmatrix}\right)$$

$\Rightarrow$ T is a linear Transformation from $\mathbb{R}^2 \to \mathbb{R}^2$

$f(x) = Ax$. let $x_1, x_2 \in \mathbb{R}^n, \quad \alpha, \beta \in \mathbb{R}$

$$f(\alpha x_1 + \beta x_2) = A(\alpha x_1 + \beta x_2)$$

$$= \alpha A x_1 + \beta A x_2$$

$$= \alpha f(x_1) + \beta f(x_2)$$

$\rightarrow f$ is L.T.

(3) $f(x_1, x_2, x_3) = (x_1, x_2, 4)$

Scaler multiplication not followed

$f(0,0,0) = (0,0,4)$

$2 \cdot f(0,0,0) = 2(0,0,4) = (0,0,8)$

but $f(2(0,0,0)) = f(0,0,0) = (0,0,4) \neq (0,0,8)$

$\Rightarrow$ f is not L.T.

(4) $f(x_1, x_2, x_3) = (x_1, x_2, 0) \quad \forall (x_1, x_2, x_3) \in \mathbb{R}^3$

Let $(x_1, x_2, x_3), (y_1, y_2, y_3) \in \mathbb{R}^3$, $\alpha, \beta \in \mathbb{R}$

$$\begin{aligned} f(\alpha(x_1, x_2, x_3) + \beta(y_1, y_2, y_3)) &= f(\alpha x_1 + \beta y_1, \alpha x_2 + \beta y_2, \alpha x_3 + \beta y_3) \\ &= (\alpha x_1 + \beta y_1, \alpha x_2 + \beta y_2, 0) \\ &= \alpha(x_1, x_2, 0) + \beta(y_1, y_2, 0) \\ &= \alpha f(x_1, x_2, x_3) + \beta f(y_1, y_2, y_3) \end{aligned}$$

$\Rightarrow$ f is a L.T.

$T: \mathbb{R}^4 \to \mathbb{R}^3$ find $R(T)$

$$T\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} \longrightarrow \begin{bmatrix} a+b \\ b-c \\ a+d \end{bmatrix}$$

Let $B = \{e_1, e_2, e_3, e_4\}$ be a basis for $\mathbb{R}^4$

$$T(e_1) = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \quad T(e_2) = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \quad T(e_3) = \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix}, \quad T(e_4) = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$R(T) = \text{Span}\left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\}$$

$$= \mathbb{R}^3$$

$T : x \to Ax.$

$A e_i =$ i th column of A

$R(T) = \mathcal{C}(A)$

Null space(T) $= \{ x : T(x) = 0 \}$

$= \{ x : Ax = 0 \}$

$=$ null space (A).

$$\begin{bmatrix} 0 & 0 & \boxed{2} \\ 0 & \boxed{1} & 3 \\ 0 & 0 & 0 \end{bmatrix}$$ not in RREF