

01

214-152 | Week 31

AUGUST  
2021

Saturday

~~Q1~~ → lecture 23 →

9

Tutorial → ⑦

10

① →  $\alpha$  → scalar

11

$$\boxed{Ax = \lambda X}$$

→  $\lambda$  are eigenvalues.

12

⇒

$$(A - \alpha I) \cdot X = \lambda X,$$

1

$$\boxed{(A - \alpha I - \lambda I) \cdot X = 0}$$

2

3

4

5

② →

$$\boxed{Ax = \lambda X}$$

— ①

where  $\lambda$  is scalar &  
'X' is EV.  $\neq 0$ 

6

7

③ →

$$A^2 = A$$

$$\rightarrow A^2 \cdot X = A \cdot X$$

$$A \cdot (A \cdot X) = (A \cdot X)$$

$$A \cdot (\lambda X) = \lambda \cdot X$$

$$\lambda \cdot (AX) = \lambda \cdot X$$

$$\lambda^2 \cdot X = \lambda \cdot X$$

$$\boxed{(\lambda^2 - \lambda) \cdot X = 0}$$

02 Sunday

Now, since EV. is  
not equal to 0. so

$$(\lambda^2 - \lambda) = 0$$

$$\lambda(\lambda - 1) = 0$$

$$\boxed{\lambda = 0, 1}$$



$$(b) \rightarrow \boxed{A^m = 0} ; A \in \boxed{m \times m}$$

$$(A^m \cdot x) = (0 \cdot x)$$

$$\lambda \cdot (A^{m-1} \cdot x) = (0 \cdot x)$$

$$\lambda^2 \cdot (A^{m-2} \cdot x) = (0 \cdot x)$$

$$\lambda^m \cdot x = (0 \cdot x)$$

$$\boxed{(\lambda^m - 0) \cdot x = 0} \Rightarrow \lambda^m = 0$$

$$\boxed{x = 0} \Leftarrow$$

$$(a) \rightarrow A_{n \times n} \rightarrow \text{all e.v.} \Rightarrow 0$$

$$(b) \rightarrow (\lambda - 0)^n = 0 \rightarrow \begin{bmatrix} 0 & 0 & 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & \dots & 0 \end{bmatrix} M.$$

$$\boxed{GM = AM} \text{ for } \lambda = 0;$$

Diagonalizable  $\Leftarrow$

all  $(n)$  free variables.

$\Rightarrow$  Not necessary that it's diagonalizable.



Tuesday

$$(4) \rightarrow (6) \quad \begin{bmatrix} 1 & 1 \\ 4 & 1 \end{bmatrix} \Rightarrow A$$

$$(A - \lambda I) = \begin{bmatrix} 1-\lambda & 1 \\ 4 & 1-\lambda \end{bmatrix} \begin{matrix} (x \cdot 0) \\ (x \cdot 0) \end{matrix} = (x \cdot A)$$

$$(1-\lambda)^2 - 4 = 0$$

$$(1-\lambda+2) \cdot (1-\lambda-2) = 0 \quad x \cdot A$$

$$(3-\lambda) \cdot (-1-\lambda) = 0 \rightarrow \text{characteristic poly}$$

$$\lambda = (-1, 3) \rightarrow \text{minimal poly}$$

$$X(x) = (3-x) \cdot (-1-x)$$

$$M(x) = (3-x) \cdot (-1-x)$$

$$M_A(x) = \{(3-A) \cdot (-1-A)\}$$

$$M_A(x) \Rightarrow \{(A-3) \cdot (A+1)\}$$

$$\Rightarrow \begin{bmatrix} -2 & 1 \\ 4 & -2 \end{bmatrix} \cdot \begin{bmatrix} 2 & 1 \\ 4 & 2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$$

$$-27 + 45 - 24 + 6$$

$$-51 + 51$$

$$\begin{array}{r|l} 1 & 5-8+6 \\ 8 & 20-16+6 \\ \hline & 26 \\ 24 & 72 \end{array}$$



$$\textcircled{b} \rightarrow \begin{bmatrix} -1-\lambda & 1 & 2 \\ 2 & 2-\lambda & 2 \\ -3 & -6 & -6-\lambda \end{bmatrix}$$

$$\Rightarrow (-1-\lambda) \cdot \{ (2-\lambda) \cdot (-6-\lambda) + 12 \} - 1 \cdot \{ (-12-2\lambda) + 6 \} + 2 \cdot \{ -12 + 3 \cdot (2-\lambda) \}$$

$$\Rightarrow -(\lambda+1) \cdot \{ (\lambda-2) \cdot (\lambda+6) + 12 \} + 1 \cdot \{ 2\lambda+6 \} - 2 \cdot \{ 3\lambda+6 \}$$

$$\Rightarrow -(\lambda+1) \cdot \{ \lambda^2 + 4\lambda - 12 + 12 \} + 2\lambda + 6 - 6\lambda - 12$$

$$\Rightarrow -\lambda^3 - 4\lambda^2 - \lambda^2 - 4\lambda - 4\lambda - 6$$

$$\Rightarrow -\lambda^3 - 5\lambda^2 - 8\lambda - 6$$

$$\Rightarrow -(\lambda^3 + 5\lambda^2 + 8\lambda + 6)$$

$$\Rightarrow -\{ \lambda^2(\lambda+3) + 2\lambda(\lambda+3) + 2(\lambda+3) \}$$

$$\Rightarrow -\{ (\lambda+3) \cdot (\lambda^2 + 2\lambda + 2) \}$$

$$\Rightarrow -\{ (\lambda+3) \cdot (\lambda^2 + 2\lambda + 2) \}$$

$$\Rightarrow -\{ (\lambda+3) \cdot (\lambda^2 + 2\lambda + 2) \}$$

$$b^2 - 4ac$$

$$4 - 4 \times 1 \times 2$$

$$\textcircled{-4}$$

AUGUST 2020						
Su	Mo	Tu	We	Th	Fr	Sa
31						1
2	3	4	5	6	7	8
9	10	11	12	13	14	15
16	17	18	19	20	21	22
23	24	25	26	27	28	29



06

219-147 | Week 32

AUG  
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Thursday

(5) → (a).

$$\begin{bmatrix} 0-x & 0 & 2 \\ 0 & (2-x) & 0 \\ 2 & 0 & 3-x \end{bmatrix}$$

$$\Rightarrow -x \cdot \{(2-x) \cdot (3-x)\} + 2 \cdot \{-2 \cdot (2-x)\}$$

$$\Rightarrow (x-2) \cdot \{-x(3-x) + 2 \cdot 2\}$$

$$\Rightarrow (x-2) \cdot \{3x - x^2 + 4\}$$

$$\Rightarrow (x-2) \cdot \{-x^2 - 3x - 4\}$$

$$\Rightarrow -(x-2) \cdot (x-4) \cdot (x+1) \rightarrow \text{char. eq.}$$

$$\boxed{x = -1, 2, 4}$$

Minimal Polynomial → Diagonalizable

so  $B = PAP^{-1}$   $\rightarrow$  Compute  $\{P^{-1} \& P\}$   
 $B^2 = PAP^{-1} \cdot PAP^{-1}$   $\downarrow$   
2 Diagonal matrix

$$\boxed{B^2 \Rightarrow PAP^{-1}}$$

Similarly  $\rightarrow \boxed{B^{100} = P \cdot A^{100} \cdot P^{-1}}$

$$(b) \rightarrow \begin{bmatrix} 1-n & 1 & -1 \\ -1 & (1-n) & 1 \\ -1 & 1 & (1-n) \end{bmatrix}$$

$$\Rightarrow (1-n) \cdot \{ (1-n)^2 - 1 \} - 1 \cdot \{ -1 + n + 1 \} \\ - 1 \cdot \{ -1 + 1 - n \}$$

$$\Rightarrow (1-n) \cdot \{ n^2 + 1 - 2n - 1 \} - n + n$$

$$\Rightarrow -(n-1) \cdot (n-2) \cdot 2 \rightarrow \text{Char. eq.}^n$$

$$\text{So } \rightarrow \boxed{n = 0, 1, 2}$$

Similar as Ques. (5)(a).

$$(c) \rightarrow - \{ n^2 - 3n + 2 \} \cdot n$$

$$(d) \rightarrow - \{ n^3 - 3n^2 + 2n \}$$