Department of Mathematics
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CSD001P5M **Linear Algebra** **Tutorial: 10**

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1. Using the normal equation, find the least square solution of $AX = b$.

   a) $A = \begin{bmatrix} 2 & 1 \\ -2 & 0 \\ 2 & 3 \end{bmatrix}, \quad b = \begin{bmatrix} -5 \\ 8 \\ 1 \end{bmatrix}$ b) $A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix}, \quad b = \begin{bmatrix} 1 \\ 3 \\ 8 \\ 2 \end{bmatrix}$

2. Find the least possible solution using QR factorization of the following system:

   a) $A = \begin{bmatrix} 2 & 3 \\ 2 & 4 \\ 1 & 1 \end{bmatrix}, \quad b = \begin{bmatrix} 7 \\ 3 \\ 1 \end{bmatrix}$ b) $A = \begin{bmatrix} 1 & -1 \\ 1 & 4 \\ 1 & -1 \\ 1 & 4 \end{bmatrix}, \quad b = \begin{bmatrix} -1 \\ 6 \\ 5 \\ 7 \end{bmatrix}$

3. Find a least square line $y = ax + b$ to the points $(0,1), (1,1), (2,3), (3,3), (4,1)$.

4. Find an orthogonal matrix P and a diagonal matrix D such that $P^t AP = D$, where A is

   a) $\begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & 1 \\ 3 & 1 & 1 \end{bmatrix}$ b) $\begin{bmatrix} 3 & -2 & 4 \\ -2 & 6 & 2 \\ 4 & 2 & 3 \end{bmatrix}$ c) $\begin{bmatrix} 7 & -4 & 4 \\ -4 & 5 & 0 \\ 4 & 0 & 9 \end{bmatrix}$

5. Write the spectral decomposition of the matrix in Question 4

6. Suppose A and B are orthogonally diagonalizable and $AB = BA$. Show that AB is also orthogonally diagonalizable.

7. Find the QR factorization of the following matrix:

   a) $\begin{bmatrix} 1 & 2 & 5 \\ -1 & 1 & -4 \\ -1 & 4 & -3 \\ 1 & -4 & 7 \\ 1 & 2 & 1 \end{bmatrix}$ b) $\begin{bmatrix} 1 & 3 & 5 \\ -1 & -3 & 1 \\ 0 & 2 & 3 \\ 1 & 5 & 2 \\ 1 & 5 & 8 \end{bmatrix}$

8. Find the orthonormal basis for the column space of the following matrix:

a) $\begin{bmatrix} 3 & -5 & 1 \\ 1 & 1 & 1 \\ -1 & 5 & -2 \\ 3 & -7 & 8 \end{bmatrix}$ b) $\begin{bmatrix} -1 & 6 & 6 \\ -3 & -8 & 3 \\ 1 & -2 & 6 \\ 1 & -4 & -3 \end{bmatrix}$

9. Let A be a matrix. Suppose columns of A are linearly and $A = QR$ is a QR factorization of A. Then show that columns of A forms a basis for column space of A.

10. Suppose $A = QR$, where Q is an orthogonal matrix of size $m \times n$ and R is $n \times n$. Show that if columns of A are linearly independent, then R is invertible.

11. Suppose $A = QR$, where R is invertible. Show that if $\mathscr{C}(A) = \mathscr{C}(Q)$.

12. Suppose $A = QR$ is an QR factorization.of an $m \times n$ matrix, where columns of A are linear independent. Let A_1 be the submatrix of A consisting first p columns of A. Then find the QR factorization of A_1.