Department of Mathematics
Indian Institute of Technology Jammu

CSD001P5M | **Linear Algebra** | **Tutorial: 01**

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1) Given $A = (a_{ij})$ we define the transpose matrix $A^T := (b_{ij})$, where $b_{ij} = a_{ji}$. Show that $(AB)^T = B^T A^T$ if AB is defined.

2) Let A and B be invertible matrices with same size, then show that $(AB)^{-1} = B^{-1}A^{-1}$.

3) Let A be a square matrix, then show that A is invertible if and only if A^T is.

4) Show that every square matrix can be written as a sum of a symmetric and a skew symmetric matrices. Further, show that if A and B are symmetric, then AB is also symmetric if and only if $AB = BA$.

5) Show that product of two upper triangular matrices is upper triangular.

6) Let $\mathbf{A}_1, \ldots, \mathbf{A}_r$ be matrices and $c_1, \ldots, c_r \in \mathbb{R}$. Then an expression of the form $c_1\mathbf{A}_1 + \cdots + c_r\mathbf{A}_r$ is called a $\mathbb{R}$ *-linear combination* of $\mathbf{A}_1, \ldots, \mathbf{A}_r$. Let $\mathbf{A}$ and $\mathbf{B}$ be matrices such that $\mathbf{AB}$ is defined.

   (a) Show that rows of $\mathbf{AB}$ can be written as linear combination of rows of $\mathbf{B}$.

   (b) Show that columns of $\mathbf{AB}$ can be written as linear combination of columns of $\mathbf{A}$.

7) Find nonzero matrices $\mathbf{A}$ and $\mathbf{B}$ such that $\mathbf{AB} = \mathbf{0}$, where $\mathbf{0}$ is a zero matrix.

8) Convert the following matrices into REF and RREF.

a) $\begin{bmatrix} 1 & 4 & -1 \\ -2 & -8 & 2 \\ 3 & 12 & -3 \\ 2 & 5 & 3 \end{bmatrix}$

b) $\begin{bmatrix} 5 & 6 & -7 & 2 \\ -1 & -2 & 3 & 0 \\ 0 & 4 & 1 & 3 \end{bmatrix}$

c) $\begin{bmatrix} 2 & -4 & 1 & 6 \\ -4 & 0 & 3 & -1 \\ 0 & 1 & -1 & 3 \end{bmatrix}$

9) Find two different row echelon forms of $\begin{bmatrix} 1 & 4 \\ 3 & 11 \end{bmatrix}$. Is it possible to find reduced row echelon forms of the given matrix?