21 October 2021 14:54

$f: X \longrightarrow Y$

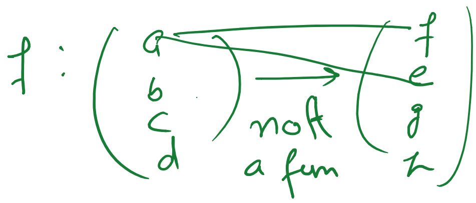
One-one (injective)

$f(x_1) = f(x_2) \Rightarrow x_1 = x_2$

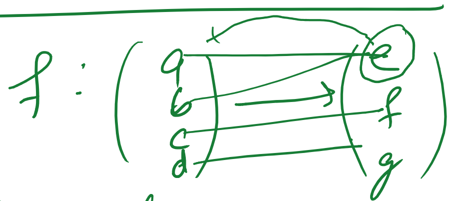
onto (surjective)

$\forall y \in Y, \ \exists x \in X$

st $f(x) = y$




Function :- every element in Domain is associated with a unique element of Range.




f is a function

f is not one-one

f is not onto

$T(u) = T(v)$

$T(u-v) =$

$1 \cdot T(u) + (-1)\, T(v)$

$= T(u) - T(v) = 0$

$T(ax_1 + by_1)$

$= aT(x_1) + bT(y_1)$

T is one-one. we know that $T(0) = 0$

let $x \in N(T)$, $T(x) = 0 \Rightarrow T(x) = T(0) \Rightarrow x = 0$ (AS T is 1-1)

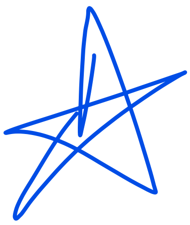
$V = \mathbb{R}_2[x] \longrightarrow W = \mathbb{R}_1[x]$

$T: V \longrightarrow W$ by $T(p(x)) = p'(x) \ \forall \ p(x) \in V$.

1. is T Linear Transformation? Yes
2. is T one-one? $\longrightarrow$ No
3. is T onto? Yes.

We want to check that T is one-one or not.

T is one-one $\Longleftrightarrow N(T) = \{0\}$



Since $T(a) = 0 \ \forall \ a \in \mathbb{R}$

$\Rightarrow N(T) \neq \{0\}$

$\Rightarrow$ T is not one - one.

Let $f(x) \in \mathbb{R}_1[x]$

$f(x) = a_0 + a_1 x$, $a_0, a_1 \in \mathbb{R}$

WTF $g(x) \in V$, s.t $T(g(x)) = f(x)$

$$g(x) = a_0 x + \frac{a_1 x^2}{2} \in V$$

$$T(g(x)) = a_0 + a_1 x = f(x)$$

T is onto

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$V = \mathbb{R}^\infty = W$

$\mathbb{R}^\infty = \{(a_1, a_2, a_3, \ldots) : a_i \in \mathbb{R}\}$

Define $T : V \longrightarrow W$ by

$$T(a_1, a_2, \ldots) = (0, a_1, a_2, a_3, \ldots)$$

1. L.T — Yes
2. 1-1 — Yes
3. onto — No

Let $(a_1, a_2, \ldots) \in N(T)$,

$T(a_1, a_2, \ldots) = (0, 0, 0, \ldots)$

$(0, a_1, a_2, \ldots) = (0, 0, 0, \ldots)$

$\Rightarrow a_1 = a_2 = a_3 = \ldots = 0$

$\Rightarrow$ T is 1-1.

T is not onto, since $(1, 0, 0, \ldots)$ does not have a preimage.

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$V = W = \mathbb{R}^\infty$

Define! $T : V \longrightarrow V$

$T(a_1, a_2, \ldots) = (a_2, a_3, a_4, \ldots)$

<u>Exercise</u> $T$ is linear, onto ~~but~~ not one-one.

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$V = \mathbb{R}_2[x]$, $B = \{1, x-1, (x-1)^2\}$, $p(x) = 2x^2 - 2x - 1$.

want to find the $[p(x)]_B$

$$2x^2 - 2x - 1 = c_1 \cdot 1 + c_2(x-1) + c_3(x-1)^2$$

$$= c_1 + c_2(x-1) + c_3(x^2 - 2x + 1)$$

$$= -1 \cdot 1 + 2(x-1) + 2(x^2 - 2x + 1)$$

$$[p(x)]_B = \begin{bmatrix} -1 \\ 2 \\ 2 \end{bmatrix}$$

$B' = \{1, x, x^2\}$ $\qquad$ $B'' = \{x^2, 1, x\}$

$$[p(x)]_{B'} = \begin{bmatrix} -1 \\ -2 \\ 2 \end{bmatrix} \qquad [p(x)]_{B''} = \begin{bmatrix} 2 \\ -1 \\ -2 \end{bmatrix}$$