Department of Mathematics
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CSD001P5M **Linear Algebra** **Tutorial: 02**

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1. Reduce the following matrices to an RREF matrix using elementary row operations:

   (a) $A = \begin{bmatrix} 1 & 2 & -3 & 0 \\ 2 & 4 & -2 & 2 \\ 3 & 6 & -4 & 1 \end{bmatrix}$

   (b) $B = \begin{bmatrix} 1 & -2 & 3 & -1 \\ 2 & -1 & 2 & 2 \\ 3 & 1 & 2 & 3 \end{bmatrix}$

2. Explicitly describe all non-zero 2×2 RREF matrices. You may also try to do this for 2×3 and 3×3 RREF matrices.

3. (a) Row reduce the augmented matrix of the system given below to an RREF matrix:

$$\begin{aligned} 3x + 2y + 7z + 9w &= 7 \\ 6x + 14y + 22z + 15w &= 13 \\ x + 4y + 5z + 2w &= 2 \end{aligned}$$

   (b) Express the solution (if the system is consistent) in the form of a vector $\mathbf{u}$ which is a particular solution plus scalar multiples of vector(s) which are solutions of the associated homogeneous system.

4. Is it possible for a non-homogeneous system $AX = b, b \neq 0$, to be inconsistent when the associated homogeneous system $AX = 0$ has a unique solution (i.e. only the trivial solution) ? Answer YES or NO, and justify your answer. If YES, construct an example and verify. If NO, explain with reference to suitable propositions and theorems.

5. Determine the inverse of the given matrix A using row reduction.

$$A = \begin{bmatrix} 2 & 1 & -1 \\ 0 & 2 & 1 \\ 5 & 2 & -3 \end{bmatrix}$$

6. Recall that if e is an elementary row-operation and E is the corresponding elementary matrix, then $e(A) = E(A)$. Illustrate with one example each for scaling, interchange and replacement operations (the minimum size of the matrices in your examples should be 3×3).

7. Prove the fact stated in Problem 6 in the general case, i.e. for any row operation e and any matrix A. (Note: the three cases of scaling, replacement and interchange require separate proofs.)

8. Let A be an $n \times n$ matrix. Show that the system $AX = b$ has a solution for every column vector $b \in \mathbb{R}^n$ if and only if A is invertible.

9. Let
$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ 1 & 0 & 8 \end{bmatrix}$$
be an invertible matrix. Find the second column of A^{-1} without actually computing A^{-1}.

10. Suppose $AB = AC$, where B and C are $n \times k$ matrices and A is invertible. Show that $B = C$. Is this true, in general, when A is not invertible ? Justify your answer (give proof if true, give counter-example if false).

11. Let
$$A = \begin{bmatrix} 1 & 2 & 1 & 0 \\ -1 & 0 & 3 & 5 \\ 1 & -2 & 1 & 1 \end{bmatrix}$$
Find a row-reduced echelon matrix R which is row-equivalent to A and an invertible 3×3 matrix P such that $R = PA$.