(20) → Till (3).
(30) → Till (7).
last → 15 Night

259-107 | Week 38

SEPTEMBE 202

Tuesday →

linear Algebra

lec (1) → (1). Symmetric Matrix ⇒ $\boxed{A = A^T}$

lec (2) → Matrices & system of linear eq.^{n} →

product of matrices as linear combination.

squar matrix Inverse → $\boxed{AA^{-1} = A^{-1}A = I}$

lec (3) → RREF →

lec (4) → Sol.^{n} of linear system of eq.^{n}

⇒ lec (8) → (i) if W_1 & W_2 are subspaces of a vector space V. then $W_1 \cup W_2$ is a subspace of V if and only if $W_1 \subseteq W_2$ (or) $W_2 \subseteq W_1$.

(ii). intersection of any 2 subspaces of a vector space V is always a subspace in V.

lecture (21) → $\boxed{\text{Geometric} \leq \text{A.M. Multiplicity}}$

→ 2 similar matrices have same determinant, same characteristic eq.^{n} and same eigen values.

→ check whether matrix is diagonalizable?

lecture (22) → Suppose 'A' is diagonalizable, then A^n will also be diagonalizable not vice-versa.

$\boxed{\text{Cayley-Hamilton Theorem}}$

lecture (23) → * Constant term in charact. poly. $\chi_A(x)$ is det.(A).

→ $\boxed{\text{finding minimal polynomial}}$

{ → A matrix is diagonalizable iff $m_A(x)$ has distinct roots & splits into linear factor. }

lecture (24) → Eigen vector corresponding to diff. eigen values are L.I.

Inner product space → cond.(1) → $\langle x,x \rangle \geq 0$ & if $=0$, $x=0$

cond.(2) → $\langle \alpha x+\beta y, z \rangle = \alpha \langle x,z \rangle + \beta \langle y,z \rangle$

cond.(3) → $\langle x,y \rangle = \overline{\langle y,x \rangle}$

and property → $\boxed{\langle x, \alpha y + \beta z \rangle = \bar{\alpha}\langle x,y \rangle + \bar{\beta}\langle y,z \rangle}$

9 ⇒ Lecture (25) → ① if $u, v \in V$, then if $\langle u, v \rangle = 0$, then $u \perp v$.

10 ② mutually orthogonal set of vectors are L.I. as well.

11 ③ $\{0\} \perp V$

12 ⇒ Lecture (26) → ① Orthogonal decomposition.

1 ② Cauchy-schwartz inequality.

2 $|\langle u, v \rangle| \leq \|u\| \cdot \|v\|$ & equality holds if $\{u, v\}$ are L.D.

3

③ Triangle inequality → $\|u+v\| \leq \|u\| + \|v\|$

4 ④ parallelogram " → $\|u+v\|^2 + \|u-v\|^2 = 2\|u\|^2 + 2\|v\|^2$

5

⑤ Orthonormal set →

→ 7 Lecture (27) ⇒ Gram-Schmidt procedure.

⇒ Lecture (29) ⇒ for 'A' to be orthogonal matrix.

$$\boxed{AA^T = I = A^TA}$$

→ can convert as orthonormal as well, row ~~for~~ vectors are ⊥ to each other.

⇒ lecture (30) → Orthogonal complements.

(1). V.S = V & $U \subseteq W$, then $\boxed{W^{\perp} \subseteq U^{\perp}}$

(2). for $A_{m \times n}$ → $\boxed{R(A) \perp N(A)}$

so → $\boxed{\begin{array}{l} \{R(A)\}^{\perp} = N(A) \\ \& \ \{C(A)\}^{\perp} = N(A^{\perp}) \end{array}}$ ⇒ Only if inner prod. standard def.^n is used.

⇒ To find $U^{\perp}$ ⇒ cal.^c basis of U, form a matrix with basis as rows & cal.^c Null space. That'll be result.

& $\boxed{V = U \oplus U^{\perp}}$; {Total sum},

⇒ $\boxed{Dim(U^{\perp}) = Dim(V) - Dim(U)}$

⇒ lecture (31) → for $A_{m \times n}$ → $\boxed{R^n = R(A) \oplus N(A)}$

$\boxed{R^m = C(A) \oplus N(A^T)}$

Minimization prob.

(01) least squared method → $\boxed{A^T A \cdot x = A^T \cdot b_{C(A)}}$

SEPTEMBER					2020	
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{ * How to find $U^{\perp}$ using gram-schmidt proc. where standard inner prod. is not used? }

$\Rightarrow (U^{\perp})^{\perp} = U$

→ Orthogonal projection: $P_U(v) = \frac{\langle v, x \rangle}{\|x\|^2} \cdot x$

where $U = \mathrm{span}\{x\}$ & $x \in V$.

→ lecture (32) →

{Try to solve orthogonal proj. questions using $v = u + w$ method, where $u \in U$, & $w \in U^{\perp}$.}

$\boxed{\text{check how to find sol}^n \text{ for } (b = Ax)?}$ → $\begin{bmatrix} \langle b, e_1 \rangle \\ \langle b, e_2 \rangle \\ \vdots \\ \langle b, e_n \rangle \end{bmatrix}$ → rows of (A).

{last कुछ Pages समझो।}

→ lecture (33) → To find least sq. sol.^{n} →

(a). if consistent $Ax = b$, then sol.^{n} x

(b). if inconsistent, ~~& col(A) are L.I.~~ then →

$\boxed{A^T . A . \hat{x} = A^T b}$ → $\boxed{\text{Normal eq.}^n}$

& (i). $\boxed{\mathrm{Rank}(A) = \mathrm{Rank}(A^T A)}$

(ii). ~~If~~ $A^T A$ is invertible iff colloumns of A are L.I.

→ lecture (34) →

(1). Diagonalization of symmetric matrix:

→ if A is a real symmetric matrix, then, A is orthogonally diagonalizable. such that →

$$\boxed{A = \underbrace{P}_{\text{Orthogonal \& invertible}}\underbrace{D}_{\text{diagonal}}P^{-1}}$$

(2). if A is $(n \times n)$ symmetric matrix, then eigen values of A are real & eigen vector, corresponding to different eigen values are L.I. & orthogonal to each other.

→ lecture (35) →

(1) Spectral Decomposition → (of symmetric matrix)

find all eigen values & eigen vectors (orthogonal) corresponding to those → then →

$$\begin{bmatrix} u_1^T \\ u_2^T \\ \vdots \\ u_n^T \end{bmatrix} . A . [u_1\ u_2 \cdots u_n] = \begin{bmatrix} \lambda_1 & \cdots & \cdots \\ \cdots & \lambda_2 & \cdots \\ \cdots & \cdots & \ddots \\ \cdots & \cdots & \lambda_n \end{bmatrix}$$

$$\boxed{A = \_\_\_\_\_\_}$$

How to find least square soln using this?

(2) QR factorization →

$\boxed{\text{Read the procedure} \rightarrow}$

Lecture (37) → Quadratic forms →

for every quadratic form, $\exists$ a unique symmetric matrix: ← $\boxed{Q(x) = x^T . A . x}$ → symmetric.

→ Change of Variables in Quadratic form →

since $Q(x) = x^T . A . x$

& for $\boxed{y = Px}$ → $Q(y) = x^T . \underbrace{(P^T A P)}_{D} . x$

$\boxed{Q(y) = x^T D x}$ (or) $y^T D y$.

→ Principal Axis Theorem →

$A \rightarrow (n \times n)$, sym. matrix.

$\exists \Rightarrow x = Py$; such that $x^T A x \Rightarrow y^T D y$ with no cross prod.

⇒ lect. → (38) → if A is $n \times n$ sym. matrix,
then $Q(x) = x^T A x$ is →

(1). +ve definite → iff all E.V. > 0

(2) −ve " → " " " < 0

(3) +ve semidefinite → " " " " ≥ 0

(4) −ve " → " " " ≤ 0

(5) Indefinite → iff. both

⇒ lect. (39) → $m = \min\{Q(x) : \|x\| = 1\}$.
$M = \max\{\ "\ \ \ "\ \}$

⇒ lect (40) → Singular Value Decomposition →

$AV = U\Sigma \Rightarrow \boxed{A = U\Sigma V^T}$ ✓

$\boxed{AA^T = U\Sigma^2 U^T}$ ✓

$\boxed{\text{Rank}(A) = \text{Dim } C(A) = r}$

⇒ put singular values in ↓ order

⇒ lec. 41 → $\Sigma = \begin{array}{c} \begin{bmatrix} D_{r \times r} & 0 \\ 0 & 0 \end{bmatrix}_{m \times n} \rightarrow m-r \\ \downarrow \\ n-r \end{array}$

Solving SVD →
⇒ calc. σ →
⇒ V is made of E.V. of Σ^2.