(2) → $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 4 \\ 3 & 0 & -3 \end{bmatrix}$

To find the characterstic eq.^{n} →

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} 1-\lambda & 2 & 3 \\ 0 & 2-\lambda & 4 \\ 3 & 0 & -3-\lambda \end{vmatrix} = 0$$

$$(1-\lambda)\cdot\{(2-\lambda)\cdot-(\lambda+3)\} - 2\cdot\{-12\}$$
$$+3\cdot\{-3\cdot(2-\lambda)\} = 0$$

$$(1-\lambda)\cdot\{(\lambda+3)\cdot(\lambda-2)\} + 24 + 9\cdot(\lambda-2) = 0$$

$$(1-\lambda)\cdot\{\lambda^2+\lambda-6\} + 24 + 9\lambda - 18 = 0$$

$$\cancel{\lambda^2} + \lambda - \cancel{6} - \lambda^3 - \cancel{\lambda^2} + 6\lambda + 9\lambda + \cancel{6} = 0$$

$$-\lambda^3 + 16\lambda = 0$$

$$\boxed{\lambda^3 - 16\lambda = 0} \Rightarrow \lambda(\lambda^2 - 16) = 0$$

$$\boxed{\lambda\cdot(\lambda+4)(\lambda-4) = 0}$$

so, $\boxed{\lambda = 0, 4, -4}$

so, A.M. of all eigen values 0, 4, (-4)

$\Rightarrow$ 1.

$\rightarrow$ $\boxed{G.M. \leq A.M.}$

$\hookrightarrow$ so, G.M. of all E.V. = 1.

Bcoz G.M. $\neq 0$

$\longrightarrow$ Since AM. = GM. for all eigenvalues,

so, $\longrightarrow$

A is Diagonalizable

Cal.^{c} E.V. for $\lambda = 0$ $\longrightarrow$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 4 \\ 3 & 0 & -3 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \left[\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 0 & 2 & 4 & 0 \\ 3 & 0 & -3 & 0 \end{array}\right]$$

$\downarrow R_1 \rightarrow R_1 - R_2$

$\downarrow R_3 \rightarrow \frac{1}{3} R_3$

$$\left[\begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ 0 & 2 & 4 & 0 \\ 1 & 0 & -1 & 0 \end{array}\right]$$

$\downarrow R_3 \rightarrow R_3 - R_1$; $R_2 \rightarrow \frac{1}{2} R_2$

$$\left[\begin{array}{ccc|c} \boxed{1} & 0 & -1 & 0 \\ 0 & \boxed{1} & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$\downarrow$

free valiable $\longrightarrow$ $z = 1$;

$y + 2z = 0$

$\boxed{y = (-2)}$

& $x - z = 0$

$\boxed{x = 1}$.

E.V. $\Rightarrow$ $\begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$

cal^c E.V. for $(\lambda = 4)$ $\longrightarrow$

$$\begin{bmatrix} -3 & 2 & 3 \\ 0 & -2 & 4 \\ 3 & 0 & -7 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \left[\begin{array}{ccc|c} -3 & 2 & 3 & 0 \\ 0 & -2 & 4 & 0 \\ 3 & 0 & -7 & 0 \end{array}\right] \xrightarrow[R_2 \rightarrow -\frac{1}{2} R_2]{R_3 \rightarrow R_3 + R_1} \left[\begin{array}{ccc|c} -3 & 2 & 3 & 0 \\ 0 & 1 & -2 & 0 \\ 0 & 2 & -4 & 0 \end{array}\right]$$

$\downarrow R_3 \rightarrow R_3 - 2R_2$

$$\left[\begin{array}{ccc|c} -3 & 2 & 3 & 0 \\ 0 & 1 & -2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

So $\rightarrow$ $z = 1$;

$y = 2$;

$-3x + 2y + 3z = 0$

$+3x = 7$

$x = 7/3$

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26	27	28	29	30	31	

So → E.V. ⇒ $\begin{bmatrix} 7/3 \\ 2 \\ 1 \end{bmatrix}$

Calᶜ: E.V. for $\lambda = (-4)$ →

$$\left[\begin{array}{ccc|c} 5 & 2 & 3 & 0 \\ 0 & 6 & 4 & 0 \\ 3 & 0 & 1 & 0 \end{array}\right]$$

$\downarrow R_3 \to 5R_3 - 3R_1$

$$\left[\begin{array}{ccc|c} 5 & 2 & 3 & 0 \\ 0 & 6 & 4 & 0 \\ 0 & -6 & -4 & 0 \end{array}\right]$$

$\downarrow R_3 \to R_3 + R_2$

$$\left[\begin{array}{ccc|c} 5 & 2 & 3 & 0 \\ 0 & 6 & 4 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

Now $2 \Rightarrow 1 \Rightarrow$ $\left\{ \begin{array}{l} 6Y + 4Z = 0. \\ Y = -4/6 \\ Y = -2/3 ; \end{array} \right.$

$5X = -2Y - 3Z$

$= \frac{4}{3} - 3 \Rightarrow -\frac{5}{3} \Rightarrow X = -1/3$

5-3

E.V. ⇒ $\begin{bmatrix} -1/3 \\ -2/3 \\ 1 \end{bmatrix}$

So → $P = \begin{bmatrix} 1 & 7/3 & -1/3 \\ -2 & 2 & -2/3 \\ 1 & 1 & 1 \end{bmatrix}$

and → $D = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & -4 \end{bmatrix}$

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Que. (1) $\rightarrow B = [b_1, b_2, b_3, b_4]$

$B' = [b'_1, b'_2, b'_3]$

(i).

$[T]_B^{B'} \Rightarrow [T(b_1)]_{B'}\ [T(b_2)]_{B'}\ [T(b_3)]_{B'}\ [T(b_4$

$T(b_1) \Rightarrow \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix} = c_1 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} + c_2 \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} + c_3 \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

$c_3 = 2$; $c_2 + c_3 = 1$; $c_1 + c_2 + c_3 = 2$

$\boxed{c_2 = (-1)}$; $c_1 = 1$;

$T(b_2) = \begin{bmatrix} 3 \\ 6 \\ 6 \end{bmatrix} = c_1 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} + c_2 \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} + c_3 \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

$c_3 = 6$; $c_2 + c_3 = 3$ | $c_1 + c_2 + c_3 = 6$

$c_2 = -3$; $c_1 = 3$;

$T(b_3) = \begin{bmatrix} 1 \\ 3 \\ 8 \end{bmatrix} = c_1 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} + c_2 \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} + c_3 \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

$c_3 = 3$; $c_2 + c_3 = 1$; $c_1 + 1 = 8$

$c_2 = -2$; $c_1 = 7$

$T(b_4) = \begin{bmatrix} -4 \\ -5 \\ 10 \end{bmatrix} = c_1 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} + c_2 \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} + c_3 \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$

$c_3 = -5$; $c_2 + c_3 = -4$; $c_1 + (-4) = 10$

$c_2 = 1$; $c_1 = 14$

$$[T]_{B}^{B'} = \begin{bmatrix} 1 & 3 & 7 & 14 \\ -1 & -3 & -2 & 1 \\ 2 & 6 & 3 & -5 \end{bmatrix}$$

(ii). Com. RREF $\longrightarrow$

$$\downarrow \; R_2 \to R_2 + R_1, \quad R_3 \to R_3 - 2R_1$$

$$\Rightarrow \begin{bmatrix} 1 & 3 & 7 & 14 \\ 0 & 0 & 5 & 15 \\ 0 & 0 & -11 & -33 \end{bmatrix}$$

$$\downarrow \; R_3 \to -\frac{1}{11} R_3 \; ; \; R_2 \to \frac{1}{5} R_2$$

$$\Rightarrow \begin{bmatrix} 1 & 3 & 7 & 14 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 1 & 3 \end{bmatrix}$$

$$\downarrow \; R_3 \to R_3 - R_2$$

$$\Rightarrow \begin{bmatrix} 1 & 3 & 7 & 14 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\downarrow \; R_1 \to R_1 - 7R_2$$

$$\Rightarrow \begin{bmatrix} \boxed{1} & 3 & 0 & -7 \\ 0 & 0 & \boxed{1} & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \{\text{RREF form}\}$$

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