$\checkmark$ Q. → what is an object & a field?

21 ② → ask if $\boxed{1.x = x}$ → '1' is the multiplicative identity. MAY 2020

142-224 | Week 21

Thursday

Linear Algebra (Tutorial → ⑧)

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① → $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & - & - \\ a_{31} & - & - \end{bmatrix}_{3\times3}$ → is a V.S. over $\mathbb{R}$.

$B = \begin{bmatrix} - & - & - \\ - & - & - \\ - & - & - \end{bmatrix}_{3\times3}$

$(A+B) = \begin{bmatrix} a_{11}+b_{11} & - & - \\ - & - & - \\ - & - & - \end{bmatrix}_{3\times3}$; all these values will exist in $\mathbb{R}$. So → $(A+B)$ also exist in $\mathbb{R}$.

C.A will also exist in $\mathbb{R}$. ⇒ so, it's a real V.S.

→ Simⁿ, we can prove for complex. V.S.

② → $A = \begin{bmatrix} - & - & - \\ 0 & - & - \\ 0 & 0 & - \end{bmatrix}$; ⇒ $a_{ij} = 0,\ \forall (i > j)$

Rest $a_{ij} \in \mathbb{R}$

Performing both addⁿ & scaler multiplication will give an U.T.M.

→ so, it's a Real V.S.

③ → ask, how $I\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} k \\ 0 \end{bmatrix}$;

④ → ask for union subspace quest.

③ → $x \oplus y = (x+y-1)$; over $\mathbb{R}$

since simple addition is closed over $\mathbb{R}$
so, it'll also be closed.

$$a \cdot x = \underbrace{\underbrace{\underbrace{a \cdot \underbrace{(x-1)}_{\text{closed}}}_{\text{closed}} + 1}_{\text{closed}}}$$

closed

Suppose additive identity = 'Q'

$$x \oplus Q = x$$
$$x + Q - 1 = x$$
$$Q = \underbrace{x - (x)} + 1$$
$$Q = 0 + 1$$
$$\boxed{Q = 1} \rightarrow \text{additive identity}$$

R.V.S. in $\mathbb{R}$.

$M \cdot x = x$

$M(x-1)+1 = x$

$\Downarrow$

$\boxed{M = 1}$

$\downarrow$

M.I.

④ → (a). $\mathbb{C}$ over $\mathbb{C}$ → Yes V.S.

(b). $\mathbb{C}$ over $\mathbb{R}$ → Yes V.S. { B'coz values are drawn from $\mathbb{R}$ }.

(c). $\mathbb{R}$ over $\mathbb{C}$ → Not V.S. { B'coz values are drawn from C for scaler mult. }.

(D). $\mathbb{R}$ over $\mathbb{R}$ → Yes V.S. → obviously.

(5) → (a). Subspace in $\mathbb{R}^3$.

(b) → (Not). Bcoz don't follow additive closure

(c) → (Yes). Bcoz only possibility is $\boxed{x=0}$ & $\boxed{y=0}$. So, both are closed.

(d) → (Not). Both op.^{n} not closed.

(e) → (Not). scaler multp.^{n} is not closed.

(f) → (Yes). Both closed.

(g) → (Not). if scaler is negative.

(h) → (a). (Not). if we use (i) in scaler multiplication.

(b) → (Yes). Both closed.

~~(i)~~ → for $\{ax+by+cz=d\}$ →

(i) → if a, b, c all are zero, then 'd' must be 0.

⇒ ~~Only 'd' should be zero~~

(9) → (10) →

$$\begin{bmatrix} -2 & 4 \\ 4 & 0 \end{bmatrix} = x\begin{bmatrix} 1 & 2 \\ 1 & -1 \end{bmatrix} + y\begin{bmatrix} -2 & 3 \\ 1 & 4 \end{bmatrix} + z\begin{bmatrix} -1 & 3 \\ 2 & 1 \end{bmatrix}$$

$$\begin{bmatrix} -2 & 4 \\ 4 & 0 \end{bmatrix} = \begin{bmatrix} x-2y-z & 2x+3y+3z \\ x+y+2z & -x+4y+z \end{bmatrix}$$

$$\Downarrow$$

$$\left[\begin{array}{ccc|c} 1 & -2 & -1 & -2 \\ 2 & 3 & 3 & 4 \\ 1 & 1 & 2 & 4 \\ -1 & 4 & 1 & 0 \end{array}\right]$$

$$\Downarrow$$

$$\left[\begin{array}{ccc|c} 1 & -2 & -1 & -2 \\ 0 & 7 & 5 & 8 \\ 0 & 3 & 3 & 6 \\ 0 & \cancel{2} & 0 & -\cancel{2} \end{array}\right]$$

$$\Downarrow$$

$$\left[\begin{array}{ccc|c} 1 & -2 & -1 & -2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 5 & 15 \\ 0 & 0 & 3 & 9 \end{array}\right]$$

$$\Downarrow$$

$$\left[\begin{array}{ccc|c} 1 & -2 & -1 & -2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 1 & 3 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 1 & 3 \end{array}\right] \Rightarrow \begin{matrix} x = (-1) \\ y = (-1) \\ z = 3 \end{matrix}$$

⟹ Not eq.ⁿ but polynomial addition will be closed ↓ But $\boxed{\text{not}}$ scaler mult.

(11) → (a). W is a subset of V.

for addition →

$x^3 + x^2 + x + 1 = 0$ — (1).

$-x^3 - x^2 - x + 2 = 0$ — (2).

So, Not V.S. But their addition wouldn't exist in W.

(b) → Not V.S. ∵ in case of a(−ve) scaler multp.ⁿ.

(c) → (V.S). Both closed.

(12) → (a). Already proved. Yes.

(b).

(13) → Do yourself.

(14). → Can easily prove that 'U+W' is a subspace of V.

Part (b). → let U+W is not smallest V.S. but X is. So, there exist 'e' ∈ (U+W) but 'e' ∉ (X)

Now, since → e ∈ U+W.
$\boxed{e = (v+w)}$ for some v ∈ U & w ∈ W.

→ But this ~~(v+w)~~ (v) & (w) also exist in X. as it contains all elements from U & W as well. Now V.S. also true for additive close, which means that for (v) & (w) → (w+v) exist in X. which is opposite to assumption. So → (U+W) is smallest V.S.

(9) → (d) → Yes V.S. {Just proved above}.

(e) → Yes but why?

Ask To (B) → explain this