Pattern Recognition and Machine Learning
Mid-Term Exam 2022.
Time - 40 Minutes
Total Marks = 40.

March 30, 2022

(i) Try to explain your answers as much as possible for obtaining full marks.

(ii) Marks shall be also granted for right steps of solution process.

1. Let us consider a Training set $T = \{(x_i, y_i), i = 1, 2, ...l\}$, where l is a very large number and f_0 is the optimal function estimated using the T. Further, let us define a ranodm vraibale $X_i = y_i - f_0(x_i)$ then consider the following statments.

   (a) $X \hookrightarrow \mathcal{N}(0, 1)$

   (b) $E(X_i) = 0$.

   (c) $Var(X_i) = 1$.

   (d) $X \hookrightarrow \mathcal{N}(\mu, \sigma)$, where μ and σ are finite numbers.

   (d) None of above can be said about the distribution of X_i.

   Write the correct statment/statements and breifly describe the reasons behind your answer. 5 $Marks$.

2. Consider the following dataset.

x_1	x_2	Y
0	0	0
0	1	1
1	0	1
1	1	0

Table 1: Dataset

For the given dataset, one has estimated the Least Square linear regression model $f(\mathbf{x}) = w_1x_1 + w_2x_2 + b$ and obtained $w_1 = w_2 = 0$ and $b = 0.5$. The training RMSE is 0.5. The obtained training error, in this case, is due to

(a) high variance.

(b) high bais.

(c) data are identically and independently distirbuted.

(d) the irreducible error is too much.

(d) None of above are true.

Write the correct statment/statements and breifly describe the reasons behind your answer.

5 $Marks.$

3. Consider the basis function $\{2^{||x_i-\mu_1||^2}, 2^{||x_i-\mu_2||^2}\}$, For $\mu_1 = \begin{bmatrix}0\\0\end{bmatrix}$ and $\mu_2 = \begin{bmatrix}1\\0\end{bmatrix}$, Let us say that Least Square estimate (without considering the regularization) for the dataset in the Table -1 is given by $f(x) = 2^{||x_i-\mu_1||^2}w_1 + 2^{||x_i-\mu_1||^2}w_2$. Then obtain the values of w_1 and w_2. 5 $Marks$

4. Let us consider the training set $T = \{(x_i, y_i), i = 1, 2, ...l\}$, briefly describe the steps involved in estimating the function $f(x) = w^Tx + b, w \in \mathbf{R}^n, b \in \mathbf{R}$, using regulrized L_1-norm regression model by computing the gradient. 5 $Marks.$

5. Let us consider a function $f : \mathbf{R}^n \to \mathbf{R}$. Argue why the negative of the gradient direction is the direction of steepest descent. 5 $Marks.$

6. How does the bias-variance decomposition of a Regularized Least Square (RLS) estimator compare with that of ordinary least squares regression estimation?

   (a) RLS estimator has larger bias but, larger variance

   (b) RLS estimator has smaller bias but, larger variance.

   (c) RLS estimator larger bias but, smaller variance.

   (d) RLS estimator has smaller bias but, smaller variance.

   (d) None of above are true.

   Justify your answer with reasons. 5$Marks.$

7. Consider the dataset at Table 2. Find the direction along which the variance of the data is maximum.

   (a) (0.5,0.5)

x_1	x_2
1	0
0	1
1	0
1	1

Table 2: Dataset

(b) (0.12,0)

(c) (0,0.4635)

(d) (0.25, 0.14)

(d) None of above.

Justify your answer. 5 $Marks.$

8. As we, increase the number of training points, which of them are true about our learnt model.

(a) the bais of model decreases.

(b) the variance of model increases.

(c) the variance of model decreases.

(d) bais of the model increases.

(e) Nothing can be said.

Justify your answers with proper reasoning. 5 $Marks.$