CSL003P1M : Probability and Statistics
Lecture 01 (Recap)

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Classical Definition of Probability

$$P(E) = \frac{F}{T}$$

$P(E)$ is the probability of event E,
F is the number of times the **favourable** event E could happen,
T is the number of times the **whole** event could happen.

Exercise-1

Three winning tickets are drawn from an urn of 100 tickets (without replacement). What is the probability of winning for a person who buys: (a) 4 tickets? (b) only one ticket?

Solution: (a)

- A persons wins when he/she wins with **at least** one ticket, say it a lucky ticket.
- The probability that there is no lucky ticket is

$$p_l = \frac{\binom{96}{3}}{\binom{100}{3}}$$

- So, the probability of winning is $1 - p_l$.

Three winning tickets are drawn from an urn of 100 tickets (without replacement). What is the probability of winning for a person who buys: (a) 4 tickets? (b) only one ticket?

Solution: (b)

- The winning probability is

$$\frac{\binom{99}{2}}{\binom{100}{3}}$$

Exercise-2

A bakery makes 80 loaves of bread daily. Ten of them are underweight. An inspector weighs 5 loaves at random. What is the probability that (at least) an underweight loaf will be discovered?

Solution:

- The probability that no underweight loaf will be discovered is

$$p_l = \frac{\binom{70}{5}}{\binom{80}{5}}.$$

- So, the probability that at least one underloaf will be discovered is

$$1 - p_l.$$

Find the probability that among seven persons:

1. No two were born on the same day of the week.
2. at least two were born on the same day.
3. two were born on a Sunday and two on a Tuesday.

Solution: (a)

- The probability that no two were born on the same day of the week is **equal to** the probability that all were born on different days of the week. The probability is

$$\frac{7!}{7^7}.$$

Find the probability that among seven persons:

1. No two were born on the same day of the week.
2. at least two were born on the same day.
3. two were born on a Sunday and two on a Tuesday.

Solution: (b)

- The probability that at least two were born on the same day is

$$1 - \frac{7!}{7^7}.$$

Exercise-3

Find the probability that among seven persons:

1. No two were born on the same day of the week.
2. at least two were born on the same day.
3. exactly two were born on a Sunday and exactly two on a Tuesday.

Solution: (c)

- The number of cases that two were born on Sunday is $\binom{7}{2}$.
- The number of cases that two from the remaining five were born on Friday is $\binom{5}{2}$.
- The number of cases that remaining were born on other five days is 5^3.

Exercise-3

Find the probability that among seven persons:

1. No two were born on the same day of the week.
2. at least two were born on the same day.
3. exactly two were born on a Sunday and exactly two on a Tuesday.

Solution: (c)

- Thus the probability is

$$\frac{\binom{7}{2}\binom{5}{2}5^3}{7^7}$$

Exercise-4

A group of $2N$ boys and $2N$ girls is randomly divided into two equal groups. What is the probability that each group has the same number of boys and girls?

Solution:

- There are $4N$ people.
- If $4N$ people are divided into two equal groups, each group will have $2N$ people.
- If each group has the same number of boys and girls in each group, the total number of ways groups can be created is

$$X = \binom{2N}{N}\binom{2N}{N}.$$

Exercise-4

A group of $2N$ boys and $2N$ girls is randomly divided into two equal groups. What is the probability that each group has the same number of boys and girls?

Solution:

- The total number of ways the two groups can be created is

$$T = \binom{4N}{2N}.$$

- So, the probability is

$$\frac{X}{T}.$$

Exercise-4

A group of $2N$ boys and $2N$ girls is randomly divided into two equal groups. What is the probability that each group has the same number of boys and girls?

Solution:

- Let r be the number of boys and girls in each group.
- Then the number of ways in which two groups can be created where the group one has r number of boys and group two has r number of girls is
$$\binom{2N}{r}\binom{2N}{r}.$$
- So, the total number of ways two groups can be created is
$$F = \sum_{r=1}^{2N} \binom{2N}{r}\binom{2N}{r}.$$

Exercise-4

A group of $2N$ boys and $2N$ girls is randomly divided into two equal groups. What is the probability that each group has the same number of boys and girls?

Solution:

- The total number of ways the two groups can be created is

$$T = \sum_{r_1=1}^{2N} \sum_{r_2=1}^{2N} \binom{2N}{r_1} \binom{2N}{r_2}.$$

- So, the probability is

$$\frac{F}{T}.$$

Exercise-5

Consider r indistinguishable balls randomly distributed in n cells. What is the probability that exactly m cells remain empty?

Solution:

- If there are n cells out of which m cells remain empty, there are $n - m$ cells remain non-empty which can be done in $\binom{n}{m}$ ways.
- Now, we need to fill r indistinguishable balls in $n - m$ cells which is same as finding the number of ways of the solution

$$x_1 + x_2 + \cdots + x_{n-m} = r$$

where $x_i \geq 1$ for all $1 \leq i \leq n - m$.

Exercise-5

Consider r indistinguishable balls randomly distributed in n cells. What is the probability that exactly m cells remain empty?

Solution:

- Consider the equation

$$x_1 + x_2 + \cdots + x_{n-m} = r$$

  where $x_i \geq 1$ for all $1 \leq i \leq n - m$.

- The total number of solutions of the above equaiton is

$$\binom{r-1}{n-m-1}$$

Exercise-5

Consider r indistinguishable balls randomly distributed in n cells. What is the probability that exactly m cells remain empty?

Solution:

- So, the number of ways that r indistingushable balls are randomly distributed in n cells out of which m cells remain empty is
$$F = \binom{n}{m}\binom{r-1}{n-m-1}$$

Exercise-5

Consider r indistinguishable balls randomly distributed in n cells. What is the probability that exactly m cells remain empty?

Solution:

- The total number of ways in which r indistinguishable balls are randomly distributed in n cells is the total number of solutions for the equation

$$x_1 + x_2 + \cdots + x_n = r$$

where $x_i \geq 0$ for all $1 \leq i \leq n$.

- The total number of solutions for the above equation is

$$T = \binom{r+n-1}{n-1}$$

- Thus, the probability is

$$\frac{F}{T}.$$

Birthday Problem

In a classroom there are n students.

1. What is the probability that at least two students have the same birthday?
2. What is the minimum value of n which secures probability $1/2$ that at least two have a commom birthday.

Solution: (Do it yourself!!)

Thank You