CSL003P1M : Probability and Statistics
QuestionSet - 04: Discrete Random Variables and Some Discrete Probability Distributions

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1. Let
$$p(k, \lambda) = e^{-\lambda}\frac{\lambda^k}{k!}.$$
Express $p(k, \lambda)$ in terms of k, λ and $p(k-1, \lambda)$.

2. Let
$$q(k, n, p) = \binom{n}{k} p^k (1-p)^{n-k}.$$
Express $q(k, n, p)$ in terms of k, n, p and $q(k-1, n, p)$.

3. A box contains seven black balls numbered $1, 2, \ldots, 7$ and three white balls numbered $8, 9, 10$. Five balls are randomly selected, (a) without replacement, (b) with replacement.
For each of the cases (a) and (b) give the distribution:
   (a) of the number of black balls in the sample;
   (b) of the minimum number in the sample;
   (c) of the maximum number in the sample;
   (d) of the minimum number of balls needed for selecting a black ball.

4. Do the previous problem but with a small change - now white balls are numbered $1, 2$ and 3.

5. A machine makes items of which 10% are defective. Every hour the producer draws a sample of size 10 for inspection. If the sample contains no defective item, he does not stop the machine. What is the probability that the machine runs for 12 hours without being stopped? (You can make suitable assumptions).

6. It has been observed that one person per thousand persons is subjected to airline accident in an year. Suppose that an insurance company has insured 5000 persons in a town. What is the probability that at most 2 insured persons will have an airline accident this year?

7. Tata Airlines observes that on an average 5% of the persons who do flight reservations do not show up for the flight. The airlines sells 100 seats for a plane which has only 95 seats. What is the probability that there will be a seat available for every person who shows up for the flight?

8. It has been observed that two accidents happen in a factory per week. What is the probability that there will be at most two accidents, (i) during one week, (b) during two weeks and (c) in each of two weeks.

9. Suppose that the population of Jalpaiguri is 5 lakhs. It is observed that in Jalpaiguri, the suicide rate is 4 suicides per 10 lakhs inhabitants per month. Find the probability that in Jalpaiguri, there will be at most 4 suicides in the month of October. Find the probability that during one year, there were at least two months in which more than 4 suicides occurred.

10. How many times a fair coin must be tossed so that with the probability 0.95, there will be at least one head and at least one tail?