CSL003P1M: Probability and Statistics
Mid-Sem

November 01, 2021

Total Marks : 32 | Duration: 2 hours | Maximum Marks : 30

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1. There are two kinds of diseases D_1, D_2 and it is suspected that a patient has one of the diseases. Assume that the population who are suffering from these diseases are in the ratio $2:1$. If a patient gives a test then

   - 25% of the cases of D_1 turn out to be positive.
   - 50% of the cases of D_2 turn out to be positive..

   A patient who is suffering from a particular disease gives three same tests out of which two come out to be positive. Find the probability for each of the two illness. $[2+2]$

2. A box contains seven black balls numbered $1, 2, \ldots, 7$ and three white balls numbered $8, 9, 10$. Five balls are randomly selected, (a) without replacement, (b) with replacement. For each of the cases (a) and (b), give the distribution of the minimum number in the sample. $[2+2]$

3. It has been observed that two accidents happen in a factory per week. What is the probability that there will be at most two accidents, (i) during one week, (b) during two weeks and (c) in each of two weeks. $[1+2+1]$

4. An urn contains r distinct balls labeled $1, 2, \ldots, r$. Balls are drawn with replacement until exactly $k \leq r$ distinct objects have been obtained. Let S_k be the random variable which denotes the size of the sample required. Compute $E[S_k]$. (You may use the fact that $E[\sum_{i=1}^{k} X_i] = \sum_{i=1}^{k} E[X_i]$) $[4]$

5. Suppose you have INR 15 and you are participating in a game. Your strategy is as follows:

   - You bet INR 1 for the toss. If a head appears on the first toss, you receive INR 1 and then you withdraw the game. **Otherwise**, if a tail appears, you bet INR 2 for the next toss. If a head appears on the next toss, you win and withdraw the game. **Otherwise**, if a tail appears, you bet INR 4 for the next toss. If a head appears on the next toss, you win and withdraw the game. **Otherwise**, if a tail appears, you bet remaining INR 8 for the next toss. If a head appears on the next toss, you win and withdraw the game.

   What is your expected gain? $[4]$

6. Two ideal dice are thrown. Let X be the score on the first die and Y be the larger of two scores. Write down the joint distribution of X and Y. $[4]$

7. Suppose there is a family with two children. We define three random variables X_1, X_2 and X_3. For $i = 1, 2$, $X_i = 1$ if ith child is a boy otherwise 0. The r.v. $X_3 = 1$ if there is only one boy in the family, otherwise 0. Are X_i's pairwise independent? Are X_i's mutually independent? Justify your answer in each cases. $[2+2]$

8. In Banach's match box problem, find the probability that at the moment when the first box is emptied (not found empty) the other contains exactly r matches (where $r = 1, 2, \ldots, N$). $[4]$