CSL003P1M : Probability and Statistics
QuestionSet - 06: Joint Distribution, Independent Random Variables and Sum of Independent Random Variables

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1. The joint distribution of (X, Y) is defined by $P\{X = 0, Y = 0\} = P\{X = 0, Y = 1\} = P\{X = 1, Y = 1\} = 1/3$. Find the marginal distribution functions of X and Y.

2. A die is thrown 12 times. Let X be the number of appearances of 6 and Y the number of appearances of 1. Find the joint distribution of (X, Y).

3. Two ideal dice are thrown. Let X be the score on the first die and Y be the larger of two scores. Write down the joint distribution of X and Y.

4. Suppose there is a family with two children. We define three random variables X_1, X_2 and X_3. For $i = 1, 2$, $X_i = 1$ if ith child is a boy otherwise 0. The r.v. $X_3 = 1$ if there is only one boy in the family, otherwise 0. Are X_i's pairwise independent? Are X_i's mutually independent?

5. Three distinguishable balls are placed randomly into three cells. Let X_i denote the number of balls in the cell number i and N denote the number of occupied cells. Form tables for joint distributions of (a) (N, X_1) and (n) (X_1, X_2).

6. In five tosses of a coin, let X, Y, Z be the number of heads, the number of head runs, and the length of the largest head run. Tabulate the 32 sample points together with the corresponding values of X, Y and Z. By simple counting derive the joint distribution of the pairs (X, Y), (X, Z), (Y, Z) and the distributions of $X + Y$ and XY.

7. Suppose the number of trials n is large and the p_i small so that the $np_i = \lambda_i$ are moderate. Show that the multinomial distribution can be approximated by the so-called multiple Poisson distribution
$$e^{-(\lambda_1+\lambda_2+\cdots+\lambda_k)} \frac{\lambda_1^{n_1} \lambda_2^{n_2} \cdots \lambda_k^{n_k}}{n_1! n_2! \cdots n_k!}$$

8. Consider two discrete random variables X and Y with joint probability function $p_{ij} = P\{X = x_i, Y = y_j\}$. Show that X and Y are independent if and only if the probability matrix $P = (p_{ij})$ has rank 1.

9. There are n balls numbered $1, 2, \cdots, n$. "r" balls are selected at random with replacement. What is the probability that the largest selected number is m?

10. In a sequence of Bernoulli trials, (a) let X be the length of the run (of either successes or failures) started by the first trial. Find the distribution of X. (b) Let Y be the length of the second run. Find the distribution of Y.

    Find the joint distribution of X, Y.