CSL003P1M : Probability and Statistics
QuestionSet - 02: Axiomatic Approach of Probability

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1. Let S denotes the total students in IIT Jammu and A_1, A_2, A_3, A_4 the sets of first year (both B.Tech and M.Tech), second year (both B.Tech and M.Tech), third year and fourth year students. Let F denote the set of female students and J the set of Jammu students. Express in words each of the following sets:
   (a) $(A_1 \cup A_2)F$.
   (b) $F\bar{J}$.
   (c) $A_1\bar{F}\bar{J}$.
   (d) $A_3F\bar{J}$.
   (e) $(A_1 \cup A_2)JF$.

2. Express each of the events using set-theoretic operations on the events A, B and C,
   (a) at least one of the events A, B, C occurs.
   (b) at most one of the events A, B, C occurs.
   (c) none of the events A, B, C occurs.
   (d) all three events occur.
   (e) exactly one of the events A, B, C occurs.
   (f) A and B occur but not C.
   (g) A occurs, if not then B does not occur either.

3. A family has 4 children. Let the events
   (a) A: the number of boys and girls are same.
   (b) B: the first and fourth child are girls.
   (c) C: three successive children of the same sex.
   (d) D: boys and girls alternate.

   Define explicity the sample space S and the events A, B, C and D.

4. Given $P(A) = 1/3$, $P(B) = 1/4$, $P(AB) = 1/6$, find the following probabilities:
$$P(\bar{A}),\quad P(\bar{A} \cup B),\quad P(A \cup \bar{B}),\quad P(A\bar{B}),\quad P(\bar{A} \cup \bar{B}).$$

5. Given $P(A) = 3/4$ and $P(B) = 3/8$, is it true that
   (a) $P(A \cup B) \geq 3/4$?
   (b) $1/8 \leq P(AB) \leq 3/8$?

Justify your answer in each case.

6. For any two events A and B, is it true that

$$P(AB) - P(A)P(B) = P(\bar{A})P(B) - P(\bar{A}B) = P(A)P(\bar{B}) - P(A\bar{B}).$$

7. Prove inclusion-exclusion identity (see lecture slide 03, slide number 10).

8. Prove or dispprove the following inequalities. For events $E_1, E_2, \ldots, E_n$, we have

   (a) $P(\cup_{i=1}^{n} E_i) \leq \sum_{i=1}^{n} P(E_i)$.

   (b) $P(\cup_{i=1}^{n} E_i) \geq \sum_{i=1}^{n} P(E_i) - \sum_{j<i} P(E_i E_j)$.

   (c) $P(\cup_{i=1}^{n} E_i) \leq \sum_{i=1}^{n} P(E_i) - \sum_{j<i} P(E_i E_j) + \sum_{k<j<i} P(E_i E_j E_k)$.

9. Consider the quadratic equation $ax^2 + bx + c = 0$. The values a, b, c are chosen after throwing a die three times. Find the probability that there are (a) real roots and (b) complex roots assuming all outcomes of the die are equally likely.

10. N couples go to a party. Each male randomly chooses a female partner (without any clash) in a dance event. What is the probability that no male chooses his wife as a female dance partner?

11. N people go to offer their prayers to a prayer-room. The one who enetrs in the prayer-room has to keep one's shoes outside the room; none is allowed to enter with shoes. Assume everyone wears a pair of shoes. Suddenly, the fire breaks out and everone leaves wearing a pair of shoes randomly. Find out the probability that

    (a) no one will wear his own shoes (in a pair of shoes, one may be correct or both may be wrong).

    (b) everyone wears a wrong left shoe and a wrong right shoe.

12. An urn contains nr balls numbered $1, 2, \cdots, n$ in such a way that there are exctly r number of balls bearing the number i for $i = 1, 2, \ldots, n$.

    (a) N balls are drawn at random without replacement. Find the probability that

       i. exactly m of the numbers will appear in the sample,

       ii. each of the n numbers will appear at least once.

    (b) Balls are drawn randomly until each of the numbers $1, 2, \ldots, n$ appears at least once. Find the probability that

       i. exactly m balls will be needed.

13. An urn contains n_1 red and n_2 blue balls. Two balls are randomly drawn (without replacement) from the urn. Find the probability that both drawn balls are red..

14. Birthday Problem: In a classroom there are n students.

    (a) What is the probability that at least two students have the same birthday?

    (b) What is the minimum value of n which secures probability $1/2$ that at least two have a commom birthday.

    Assume there are r days in a year and a student's birthday may lie in any day of the year with equal probability.

15. A man buys a packet of Kurkure for his child. A packet of Kurkure contains exactly one of the photos of the four characters from a TV-series - (a) Motu, (b) Patlu, (c) Dr. Jhatka and (d) Inspector Chingum. A man gets a free packet if he shows the four different characters' photos. Assume that all the four photos have the same probability of appearing in a packet. Find the probability that a man who buys 8 packets will get: (i) one free packet, (ii) two free packets.