CSL003P1M: Probability and Statistics
End-Sem

December 29, 2021

Total Marks : 43 $\qquad$ Duration: $2\frac{1}{2}$ hours $\qquad$ Maximum Marks : 40

1. A deck of n numbered cards $(1, 2, \ldots, n)$ is arranged randomly. Let X be the random variable which denotes the number of matches (cards in their natural place). Find $E[X]$ and $Var(X)$.   [2+3]

2. Let X be a continuous random variable having the density $f_X(x) = (1/2)e^{-|x|}$, $-\infty < x < \infty$.

   (a) Find $M_X(t)$.
   (b) Use $M_X(t)$ to find a formula for $E[X^{2n}]$ and $E[X^{2n+1}]$.

   [2+3]

3. Let X have a gamma distribution with parameters α and λ. Show that
$$P\left\{X \geq \frac{2\alpha}{\lambda}\right\} \leq \left(\frac{2}{e}\right)^{\alpha}.$$

   [5]

4. Suppose X follows the exponential distribtuion. Find the density of Y when

   (a) $Y = X^2$
   (b) $Y = |X|$

   [2+2]

5. Given that the density of X and Y is
$$f(x, y) = \frac{2}{(1 + x + y)^3}, \quad x > 0, y > 0,$$

   find (a) $F(x, y)$, (b) $f_X(x)$, (c) $f_Y(y|X = x)$.   [2+2+2]

6. Suppose that X and Y are independent random variables with the same exponential density
$$f(t) = \theta e^{-\theta t}, \quad t > 0.$$

   Find the joint density function of $X + Y$ and X/Y.   [5]

7. Given $f(x_1, \ldots, x_n|\theta) = \frac{e^{-x^2/2\theta}}{\sqrt{2\pi\theta}}$, let the estimator for the parameter θ is
$$\frac{\sum\limits_{i=1}^{n} X_i^2}{n}$$

   where $X_1, \ldots, X_n$ is a random sample from $f(x_1, \ldots, x_n|\theta)$. Is the estimator be an unbiased estimator of θ? Justify your answer.   [6]

8. There are two different techniques a given manufacturer can employ to produce batteries. A random selection of 12 batteries produced by technique I and of 14 produced by technique II resulted in the following capacities (in ampere hours):

Technique I		Technique II	
140	132	144	134
136	142	132	130
138	150	136	146
150	154	140	128
152	136	128	131
144	142	150	137
		130	135

Assumption: The sample data have been obtained from two different normal populations with a common variance.

(a) Determine a 90 percent level two-sided confidence interval for the difference in means.

(b) If the manufacturer is interested in testing hypothesis that there is no applicable difference in the mean capacities of batteries produced by either method, what conclusion should be drawn at the 5 percent level of significance?

[3+2+2]