CSL003P1M : Probability and Statistics
QuestionSet - 03: Conditional Probability, Independence and Bayes Theorem

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1. There are n balls numbered $1, 2, \cdots, n$. "r" balls are selected at random (a) with replacement, (b) without replacement. What is the probability that the largest selected number is m?

2. There are three kinds of diseases D_1, D_2, D_3 and it is suspected that a patient has one of the diseases. Assume that the population who are suffering from these diseases are in the ratio $2 : 1 : 1$. If a patient gives a test then

   - 25% of the cases of D_1 turn out to be positive.
   - 50% of the cases of D_2 turn out to be positive.
   - 90% of the cases of D_3 turn out to be positive.

   A patient who is suffering from a particular disease gives three same tests out of which two come out to be positive. Find the probability for each of the three illness.

3. Suppose you're on a game show, and you're given the choice of three doors: Behind one door is a car; behind the others, goats. You pick a door, say No. 1, and the host, who knows what's behind the doors, opens another door, say No. 3, which has a goat. He then says to you, "Do you want to pick door No. 2?" Is it to your advantage to switch your choice? (Monty Hall Problem: Source - Wikipedia)

4. Three prisoners, A, B and C are in separate cells and sentenced to death. The governor has selected one of them at random to be pardoned. The warden knows which one is pardoned, but is not allowed to tell. Prisoner A begs the warden to let him know the identity of one of the two who are going to be executed. "If B is to be pardoned, give me C's name. If C is to be pardoned, give me B's name. And if I'm to be pardoned, secretly flip a coin to decide whether to name B or C."

   The warden tells A that B is to be executed. Prisoner A is pleased because he believes that his probability of surviving has gone up from 1/3 to 1/2, as it is now between him and C. Prisoner A secretly tells C the news, who reasons that A's chance of being pardoned is unchanged at 1/3, but he is pleased because his own chance has gone up to 2/3. Which prisoner is correct? (Three Prisoner's Problem: Source - Wikipedia)

5. N players $P_1, P_2, \ldots, P_N$ throw a biased coin whose probability of heads equals p. P_1 starts the game, P_2 plays second, P_3 plays third etc. in cyclic order. The player who first throws a head wins. Find the probability that $P_i (i = 1, 2, \ldots, N)$ will be the winner.

6. A lady goes to a work following one of the three routes R_1, R_2, R_3. She chooses one of the routes independent of the weather.

   If it rains, the probability that the lady arrives late is the following:

   - 0.06 if the rote followed is R_1,

- 0.15 if the route followed is R_2,
- 0.12 if the route followed is R_3.

If it does not rain, the lady arrives late is the following:

- 0.05 if the rote followed is R_1,
- 0.10 if the route followed is R_2,
- 0.15 if the route followed is R_3.

(a) Assume that one in every four day is rainy on an average. Then, if she arrives late on a sunny day, what is the probability that she took the route R_3?

(b) If she arrives late on a day, what is the probability that it is a rainy day?

7. 10% of a certain population suffer from a disease. Two independent tests are conducted for any suspected person of this disease. The correct diagnosis rate is 90%. Find the probability that the person is suffering from this disease given

   (a) that both tests are positive;

   (b) that only one test is positive.

8. The probability that a family has k children is given by

$$p_k = \frac{(1-2a)}{2^{(k-1)}}, \text{ for } k \geq 2 \text{ and } p_0 = p_1 = a.$$

   If a family has two girls, what is the probability that

   (a) the family has only two children?

   (b) the family has two girls as well?

9. A high value electric current may burn one or both of the fans in a house if one or both of the electric capacitors are defective. Let the events:

   - C_i : only one capacitor i is defective ($i = 1, 2$);
   - C_3 : both capacitors are defective;
   - F_i : fan j burns out ($j = 1, 2$);
   - F_3 : both fans burn out.

   The conditional probabilities $P(F_j|C_i)$ are provided in the table below:

i/j	F_1	F_2	F_3
C_1	0.7	0.2	0.1
C_2	0.3	0.6	0.1
C_3	0.2	0.2	0.6

   Find the probabilities that

   (a) both capacitors are defective given that both fans were burnt out.

   (b) only capacitor 2 is defective given that both fans were burnt out;

   (c) only capacitor 2 is defective given that at least one fan was burnt out.

   The events C_1, C_2, C_3 have probabilities $0.3, 0.2, 0.1$, respectively.