CSL003P1M: Probability and Statistics
Quiz I

October 25, 2021

Total Marks : 16 | Duration: 1 hour | Maximum Marks : 15

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1. In a classroom there are n students.

   (a) What is the probability that at least two students have the same birthday?

   (b) What is the minimum value of n which secures probability $1/2$ that at least two have a common birthday.

   Note: You may make suitable assumptions. [2 + 2]

2. Among the permutations of $\{1, 2, \cdots, n\}$, there are some called derangements, in which none of the n integers appears in its natural place. Thus, $(i_1, i_2, \cdots, i_n)$ is a derangement if $i_1 \neq 1$, $i_2 \neq 2$, $\cdots$, and $i_n \neq n$. Let D_n be the number of derangements of $\{1, 2, \cdots, n\}$. Prove that

$$D_n = n! \left[1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \cdots + \frac{(-1)^n}{n!}\right].$$

   [4]

3. Suppose that the probability of an insect laying r eggs follow Poisson distribution with parameter λ. Assume that the probability of an egg developing is p. Assuming mutual independence of the eggs, find out the probability distribution of a total of k survivors. [4]

4. N players $P_1, P_2, \ldots, P_N$ throw a biased coin whose probability of heads equals p. P_1 starts the game, P_2 plays second, P_3 plays third etc. in cyclic order. The player who first throws a head wins. Find the probability that $P_i (i = 1, 2, \ldots, N)$ will be the winner. [4]