CSL003P1M : Probability and Statistics
Lecture 42 (Testing of Hypothesis-II)

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Exercise-1

A public health official claims that the mean home water use is 350 gallons a day. To verify this claim, a study of 20 randomly selected homes was instigated with the result that the average daily water uses of these 20 homes were as follows:

340	344	362	375
356	386	354	364
332	402	340	355
362	322	372	324
318	360	338	370

Do the data contradict the official's claim?

Assumption: The data have been obtained from a normal population.

Exercise-1

Solution: To determine the data contradict the official's claim, we need to test

$$H_0 : \mu = 350 \quad \text{versus} \quad H_1 : \mu \neq 350$$

We get

$$\bar{X} = 353.8, \quad S = 21.8478$$

Thus the value of the test statistic is

$$T = \frac{\sqrt{20}(3.8)}{21.8478} = 0.7778$$

Because this is less than $t_{0.05,19} = 1.730$, the null hypothesis is accepted at the 10 percent level of significance.

Exercise-1

Indeed the *p*-value of the test data is

$$p\text{-value } = P\{|T_{19}| > 0.7778|\} = 2P\{T_{19} > 0.7778\} = 0.4462$$

indicating the null hypothesis would be accepted at any reasonable significance level, and thus that the data are not inconsistent with the claim of the health official.

Exercise-2

The manufacturer of a new fiberglass tire claims that its average life will be at least $40,000$ miles. To verify this claim, a sample of 12 tires is tested, with their lifetimes (in $10,000$s of miles) being as follows:

Tire	Life	Tire	Life
1	36.1	7	37
2	40.2	8	41
3	33.8	9	36.8
4	38.5	10	37.2
5	37.2	11	33
6	35.8	12	36

Test the manufacturer's claim at the 5 percent level of significance.

Assumption: The data were obtained from a normal population.

Exercise-2

To determine whether the foregoing data are consistent with the hypothesis that the mean life is at least $40,000$ miles, we will test

$$H_0 : \mu \geq 40,000 \quad \text{versus} \quad H_1 : \mu < 40,000$$

A computation gives that

$$\bar{X} = 37.2833, \quad S = 2.7319$$

and so the value of the test statistic is

$$T = \frac{\sqrt{12}(37.2833 - 40)}{2.7319} = -3.4448$$

Since this is less than $-t_{0.05,11} = -1.796$, the null hypothesis is rejected at the 5 percent level of significance.

Exercise-2

Indeed, the *p*-value of the test data is

$$p\text{-value} = P\{T_{11} < -3.4448\} = P\{T_{11} > 3.4448\} = 0.0028$$

indicating that the manufacturer's claim would be rejected at any significance level greater than 0.003.

Exercise-3

Two new methods for producing a tire have been proposed. To ascertain which is superior, a tire manufacturer produces a sample of 10 tires using the first method and a sample of 8 using the second. The first set is to be road tested at location A and the second at location B. It is known from past experience that the lifetime of a tire that is road tested at one of these locations is normally distributed with a mean life due to the tire but with a variance due (for the most part) to the location. Specifically, it is known that the lifetimes of tires tested at location A are normal with standard deviation equal to $4,000$ kilometers, whereas those tested at location B are normal with $\sigma = 6,000$ kilometers. If the manufacturer is interested in testing hypothesis that there is no applicable difference in the mean life of tires produced by either method, what conclusion should be drawn at the 5 percent level of significance if the resulting data are given as

Exercise-3

Tire Lives in Units of 100 Kilometers	
Tires Tested at A	Tires Tested at B
61.1	62.2
58.2	56.6
62.3	66.4
64	56.2
59.7	57.4
66.2	58.4
57.8	57.6
61.4	65.4
62.2	
63.6	

To determine whether there is no applicable difference in the mean life of tires produced by either method, we will test

$$H_0 : \mu_A = \mu_B \quad \text{versus} \quad H_1 : \mu_A \neq \mu_B$$

A computation gives that

$$\bar{X}_A = 61.65 \times 100, \quad \bar{X}_B = 60.025 \times 100$$

and so the value of the test statistic is

$$Z = \frac{(\bar{X}_A - \bar{X}_B)}{\sqrt{\frac{\sigma_A^2}{n_A} + \frac{\sigma_B^2}{n_B}}} \approx 0.066$$

For such a small value of the test statistic (which has a standard normal distribution when H_0 is true), it is clear that the null hypothesis is accepted.

Exercise-3

A test of the hypothesis $H_0 : \mu_A = \mu_B$ (or $H_0 : \mu_A \leq \mu_B$) against the one-sided alternative $H_1 : \mu_A > \mu_B$ would be to

$$\text{accept } H_0 \text{ if } \bar{X}_A - \bar{X}_B \leq z_\alpha \sqrt{\frac{\sigma_A^2}{n} + \frac{\sigma_B^2}{m}}$$

$$\text{reject } H_0 \text{ if } \bar{X}_A - \bar{X}_B > z_\alpha \sqrt{\frac{\sigma_A^2}{n} + \frac{\sigma_B^2}{m}}$$

Exercise-4

Twenty-two volunteers at a cold research institute caught a cold after having been exposed to various cold viruses. A random selection of 10 of these volunteers was given tablets containing 1 gram of vitamin C. These tablets were taken four times a day. The control group consisting of the other 12 volunteers was given placebo tablets that looked and tasted exactly the same as the vitamin C tablets. This was continued for each volunteer until a doctor, who did not know if the volunteer was receiving the vitamin C or the placebo tablets, decided that the volunteer was no longer suffering from the cold. The length of time the cold lasted was then recorded. At the end of this experiment, the following data resulted. $\cdots$
Do the data listed prove that taking 4 grams daily of vitamin C reduces the mean length of time a cold lasts? At what level of significance?

Exercise-4

Treated with Vitamin C	Treated with Placebo
5.5	6.5
6.0	6.0
7.0	8.5
6.0	7.0
7.5	6.5
6.0	8.0
7.5	7.5
5.5	6.5
7.0	7.5
6.5	6.0
	8.5
	7.0

Assumption: The data have been obtained from a normal population and $\sigma^2 = \sigma_x^2 = \sigma_y^2$.
Solution: To prove the above hypothesis, we would need to reject the null hypothesis in a test of

$$H_0 : \mu_p \leq \mu_c \quad \text{versus } H_1 : \mu_p > \mu_c$$

where μ_c is the mean time a cold lasts when the vitamin C tablets are taken and μ_p is the mean time when the placebo is taken.
Let X sample corresponds to those receiving vitamin C and the Y sample to those receiving a placebo. A computation yields that

$$\begin{aligned} \bar{X} &= 6.450, \qquad & \bar{Y} &= 7.125 \\ S_x^2 &= 0.581, \qquad & S_y^2 &= 0.778 \end{aligned}$$

Therefore,

$$S_p^2 = \frac{9}{20} S_x^2 + \frac{11}{20} S_y^2 = 0.689$$

and the value of the test statistic is

$$TS = \frac{-0.675}{\sqrt{0.689\left(\frac{1}{10}+\frac{1}{12}\right)}} = -1.90$$

Since $t_{0.05,20} = 1.725$, the null hypothesis is rejected at the 5 percent level of significance. That is, at the 5 percent level of significance, the evidence is significant in establishing that vitamin C reduces the mean time that a cold persists.

Thank You