Roll No. → 2021PCS1017 (AMBUJ MISHRA)

1. → (a). Pr(atleast 2 student have same birthday)

⇒ 1 − Pr(No 2 student have same birthday out of all n students)

$$\Rightarrow 1 - \frac{365}{365} \times \frac{364}{365} \times \cdots \times \frac{365-(n-1)}{365}$$

which is basically → $\left\{1 - \frac{{}^{365}P_n}{(365)^n}\right\}$ ⇒ Considering this year has only 365 days.

(b) → for 2nd part →

Pr(atleast 2 students have same birthday) ⇒

$$= 1 - \frac{\cancel{365}}{\cancel{365}} * \frac{\cancel{365}*\left(1-\frac{1}{365}\right)}{\cancel{365}} * \frac{\cancel{365}*\left(1-\frac{2}{365}\right)}{\cancel{365}} * \cdots$$

$$\cdots\cdots * \frac{\cancel{365}*\left(1-\frac{n-1}{365}\right)}{\cancel{365}}$$

$$\Rightarrow 1 - \left\{\left(1-\frac{1}{365}\right) * \left(1-\frac{2}{365}\right) * \cdots\cdots * \left(1-\frac{n-1}{365}\right)\right\} \quad \text{—— (1).}$$

Now from expansion →

$e^x = 1 + x + \frac{x^2}{2!} + \cdots\cdots$ ⇒ if x is very small, then →

$$\boxed{e^x = 1 + x}$$

$$\Rightarrow 1 - \left\{ e^{-1/365} * e^{-2/365} * \cdots * e^{-(n-1)/365} \right\}$$

$$\Rightarrow 1 - \left\{ e^{-\left(\frac{1+2+\cdots+(n-1)}{365}\right)} \right\}$$

$$\Rightarrow 1 - e^{-\frac{n\cdot(n-1)}{2\cdot 365}} \Rightarrow \left(1 - e^{\frac{n(1-n)}{2\cdot 365}}\right)$$

Now, since given that $\rightarrow$ $1 - e^{\frac{n(1-n)}{2\cdot 365}} = 1/2$

$$e^{\frac{n(1-n)}{2\cdot 365}} = 1/2 \Rightarrow 2\cdot e^{\frac{n(1-n)}{2\cdot 365}} = 1$$

Take $\log_e$ both sides $\Rightarrow$

$$\log_e 2 + \frac{n(1-n)}{730} = 0$$

$$730 \cdot \log_e 2 = n(n-1)$$

$$\boxed{n^2 - n - 730 \log_e 2 = 0}$$

$$n = \frac{+1 \pm \sqrt{1 + 4 \times 1 * 730 \log_e 2}}{2}$$

$$= \frac{1 \pm \sqrt{2024.9898}}{2} = \frac{1 \pm 44.97}{2}$$

$$\Rightarrow \frac{1 + 44.97}{2}$$

$$\Rightarrow 22.98$$

$\boxed{n \approx 23}$ answer

Question (3) →

Let 'X' be a R.V. denoting that how many eggs will the insect lay. Since, it follows poisson →

So → $Pr(X=r) = \left\{ \frac{e^{-\lambda} \cdot \lambda^{r}}{r!} \right\}$

Idea → Now out of these 'r', there will be only 'k' survivors. But, these 'r' values can also vary which we'll add the prob. because those will be independent event. If we are calculating for a random 'r'.

Now let 'Y' be a R.V. denoting how many eggs will survive out of 'r' eggs coming from X.

So → $Pr(Y=k)$ ~~$Y(k)$~~ $= \left\{ {}^{r}C_{k} \cdot p^{k} \cdot (1-p)^{r-k} \right\}$

so, overall probability, Since these are independent events ⇒

$Pr(\text{for only } r \text{ eggs}) \Rightarrow Pr(X=r) \cdot Pr(Y=k)$

$$\Rightarrow \frac{e^{-\lambda} \cdot \lambda^{r}}{r!} * {}^{n}C_{k} \cdot p^{k} \cdot (1-p)^{r-k}$$

Now for all (r) →

Pr(for overall possible 'r') ⇒

$$Pr = \sum_{r=k}^{\infty} \left(\frac{e^{-\lambda} \cdot \lambda^{r}}{r!} \right) \cdot \left({}^{n}C_{k} \cdot p^{k} \cdot (1-p)^{r-k} \right)$$

$$P_r = \sum_{r=k}^{\infty} \left(e^{-\lambda} \cdot {}^{n}C_k \, p^k\right) \cdot \left(\frac{\lambda^r}{r!} (1-p)^{r-k}\right)$$

$$= e^{-\lambda} \, {}^{n}C_k \cdot p^k \cdot \sum_{r=k}^{\infty} \left(\frac{\lambda^r}{r!} \cdot (1-p)^{r-k}\right)$$

$$\Rightarrow \frac{e^{-\lambda p} \cdot (\lambda p)^k}{k!}$$

which is a poisson distribution with the parameter 'λp'.

(4):→ Let P_i be a player & 'X_i' be a R.V. denoting that ~~that~~ player (X_i) wins in which cycle.

so → $P_r(X_i=1), P_r(X_i=2), P_r(X_i=3), \ldots\ldots$ will all be independent events.

so → P_r(player P_i winning) ⟶

$$P_r(X_i=1) + P_r(X_i=2) + \ldots\ldots \infty$$

$$= (1-p)^{i-1} \cdot p + (1-p)^N \cdot (1-p)^{i-1} \cdot p + (1-p)^{2N} \cdot (1-p)^{i-1} \cdot p + \ldots\ldots \infty \quad \text{—(1)}$$

⇒ eqⁿ (1) is written because R.V. 'X_i' is following Geometric Distribution.

$$\Rightarrow p \cdot (1-p)^{i-1} \cdot \left\{ 1 + (1-p)^N + (1-p)^{2N} + \ldots\ldots \infty \right\}$$

$$= \left\{ \frac{p \cdot (1-p)^{i-1}}{1-(1-p)^N} \right\}$$

(2) → D_n = Total no. of derangements.

D_n = ~~Derangement of~~ $n! - A(i_1 \cup i_2 \cup i_3 \cup \cdots i_n)$

$$D_n = n! - \left(\sum_{K=1}^{n} i_k - \sum i_j \cdot i_k + \sum i_j i_k i_l - \cdots\cdots \right)$$

⇓

it is basically
Derangement for single element. Sim. ll. →

So → for choosing single elements.

choosing combination of '2'

$$D_n = n! - \left[{}^nC_1 * (n-1)! - {}^nC_2 \cdot (n-2)! + {}^nC_3 \cdot (n-3)! - \cdots\cdots \right]$$

for Rest values.

~~Start~~ ⇒ we have taken this element by element by taking combination.

So →

$$D_n = n! - \left[\frac{n!}{\cancel{(n-1)!} \cdot 1!} * \cancel{(n-1)!} - \frac{n!}{\cancel{(n-2)!} \cdot 2!} * \cancel{(n-2)!} + \frac{n!}{\cancel{(n-3)!} \cdot 3!} * \cancel{(n-3)!} - \cdots\cdots \right]$$

$$D_n = n! - \left[\frac{n!}{1!} - \frac{n!}{2!} + \frac{n!}{3!} \cdots\cdots + \frac{(-1)^{n+1}}{n!} \right]$$

$$D_n = n! - \frac{n!}{1!} + \frac{n!}{2!} - \cdots\cdots + \frac{(-1)^n}{n!}$$

$$\boxed{D_n = n! \left[1 - \frac{1}{1!} + \frac{1}{2!} - \cdots\cdots + \frac{(-1)^n}{n!} \right]}$$