PRS → Midsem → 2021PCS1017

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① → let → P → event where someone is (+ve).

and D_i → event that ~~the person~~ the person has 'D_i' disease.

$$Pr(D_1) = \left(\frac{2}{3}\right)$$
$$Pr(D_2) = \left(\frac{1}{3}\right)$$

and given that → $Pr\left(\frac{P}{D_1}\right) = \left(\frac{1}{4}\right)$

$$Pr\left(\frac{P}{D_2}\right) = \left(\frac{1}{2}\right)$$

let E → be the event that out of 3 test, 2 comes to be (+ve) & 1 (-ve). This will follow binomial distributi.

so → $Pr\left(\frac{E}{D_1}\right) = \frac{Pr(E \cap D_1)}{Pr(D_1)}$

$$\Rightarrow \frac{{}^3C_2 * \left\{Pr\left(\frac{P}{D_1}\right)\right\}^2 * \left\{Pr\left(\frac{\bar{P}}{D_1}\right)\right\}}{Pr(D_1)}$$

$$\Rightarrow \frac{3 * \left(\frac{1}{4}\right)^2 * \frac{3}{4}}{2/3}$$

$$\Rightarrow 3 * \frac{1}{16} * \frac{3}{4} * \frac{3}{2} = \left(\frac{27}{128}\right).$$

$$Pr(E/D_1) = \frac{Pr(E \cap D_1)}{Pr(D_1)}$$

$Pr(E \cap D_1) = Pr(D_1) * Pr(E/D_1)$ { which is also equal to pr. of 1st illness. }

$$\Rightarrow \frac{2}{3} * \left\{ {}^3C_2 * Pr\left(\frac{P}{D_1}\right)^2 * Pr\left(\frac{\bar{P}}{D_1}\right) \right\}$$

$$\Rightarrow \frac{\cancel{2}}{\cancel{3}} * \left\{ \cancel{3} * \frac{1}{\cancel{16}\,8} * \frac{3}{4} \right\} \Rightarrow \frac{3}{32}$$

→ Pr(first illness)

and → $$Pr\left(\frac{E}{D_2}\right) = \frac{Pr(E \cap D_2)}{Pr(D_2)}$$

$$Pr(E \cap D_2) = Pr(D_2) \cdot Pr(E/D_2)$$

$$\Rightarrow \frac{1}{3} * \left\{ {}^3C_2 * Pr\left(\frac{P}{D_2}\right)^2 * Pr\left(\frac{\bar{P}}{D_2}\right) \right\}$$

$$\Rightarrow \frac{1}{\cancel{3}} * \left\{ \cancel{3} * \frac{1}{4} * \frac{1}{2} \right\} \Rightarrow \frac{1}{8}$$

→ Pr(2nd illness)

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(2) → since, intially, there were 2 equi-probable boxes having N elements.

....'N' element....   ....'N' elem....

Box (1) → B_1 Box (2) → B_2

→ it is mentioned that when the 1st box is emptied (not found empty) → other contains exactly r matches.

So → we need to remove $(N-1)$ matches from (B_1) & $(N-r)$ matches of B_2 in any order.

and after those $(2N-r-1)$ remove op.^{n},

Next one should be from box ①.

$$Pr(B_1) = \frac{1}{2} \; ; \; Pr(B_2) = \frac{1}{2}$$

& removing $(N-1)$ from B_1 & $(N-r)$ from B_2 out of $(2N-r-1)$ will follow ~~final~~ binomial Distribution.

So → Pr(exactly 'r' matches when 'B' is emptied is) →

$$\Rightarrow \left\{ {}^{2N-r-1}C_{N-1} * \left(\frac{1}{2}\right)^{N-1} * \left(\frac{1}{2}\right)^{N-r} \right\} * \left(\frac{1}{2}\right)$$

$$\boxed{\Rightarrow \binom{2N-r-1}{N-1} * \left(\frac{1}{2}\right)^{2N-r}}$$ (answer)

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③ → $\lambda = 2$ accidents per week.

Let E_i → be the ~~event~~ R.v. which denotes no. of accidents in (i) weeks. (it will follow Poisson Distribution).

(i). Pr(at most 2 acc. during one week) →

$$= Pr(E_1 \leq 2) = Pr(E_1 = 0) + Pr(E_1 = 1) + Pr(E_1 = 2)$$

$$\Rightarrow \frac{e^{-\lambda} \cdot \lambda^0}{0!} + \frac{e^{-\lambda} \cdot \lambda^1}{1!} + \frac{e^{-\lambda} \cdot \lambda^2}{2!}$$

$$\Rightarrow e^{-\lambda} \cdot \left\{ 1 + \lambda + \frac{\lambda^2}{2} \right\}$$

$e^{-2} * \{1 + 2 + \frac{4}{2}\} \Rightarrow \left(\frac{5}{e^2}\right)$.

(ii). ~~No. avrg~~ average No. of accidents per 2 weeks $(\lambda') \Rightarrow$ 4 accidents per 2 week.

Pr(atmost 2 accidents during 2 weeks) $\rightarrow$

$\Rightarrow Pr(E_2 \leq 2) = Pr(E_2 = 0) + Pr(E_2 = 1) + Pr(E_2 = 2)$

$= \frac{e^{-\lambda'} \cdot (\lambda')^0}{0!} + \frac{e^{-\lambda'} \cdot (\lambda')^1}{1!} + \frac{e^{-\lambda'} \cdot (\lambda')^2}{2!}$

$\Rightarrow e^{-\lambda'} \cdot \{1 + \lambda' + \frac{\lambda'^2}{2}\}$

$\Rightarrow e^{-4} \cdot \{1 + 4 + 8\} \Rightarrow \left(\frac{13}{e^4}\right)$

(iii). Pr(at most 2 accidents in each of 2 weeks) $\rightarrow$

~~$Pr(E_1 \leq 2 \cap E$~~

Pr(atmost 2 acc. in week 1 $\cap$ atmost 2 acc. in week 2)

$\Rightarrow$ Both of these are indendent to each other & equal as well. So $\rightarrow$

$\Rightarrow$ Pr(at most 2 acc. in week 1). Pr(at most 2 accidents in week 2).

$\Rightarrow$ ~~2~~ $\{Pr($at most 2 acc. in week 1$)\}^2$

$=$ ~~$2 Pr(E_1 \leq 2) \Rightarrow$~~ $\Rightarrow \{Pr(E_1 \leq 2)\}^2$

$\Rightarrow \left(\frac{5}{e^2}\right)^2 = \left(\frac{25}{e^4}\right)$

② → Balls →

$\{1, 2, 3, 4, 5, 6, 7\}$ & $\{8, 9, 10\}$

Black balls. white balls.

let X_{min} → Denotes R.V. which is min.^m out of all 5 fetched values.

So → $X_{min} = \{1, 2, 3, 4, 5, 6\}$.

ⓐ. without Replacement →

$Pr(X_{min} = k) \Rightarrow$ out of 10 values, one values must be 'k' & rest 4 values must be fetched from $(10-k)$ values.

$$\boxed{Pr(X_{min} = k) \Rightarrow \frac{1 \times {}^{10-k}C_4}{{}^{10}C_5}}$$; for $k = 1, 2, 3, 4, 5, 6$

{ Assumption → Ordering of no. fetched doesn't matter. }

ⓑ. with Replacement →

$Pr(X_{min} = k) =$ All 5 values can be any value among last $(10-k+1)$ values.

So → $$\boxed{Pr(X_{min} = k) = \frac{(10-k+1)^5}{10^5}}$$; for $k = 1, 2, 3, 4, 5, 6$

(5). initially → 15 Rs.; let X_i → R.V. that denotes 'i' Rs. are put on bet & wins.

Case (a). if bets 1 Rs. & wins → gain → $(15-1+1)-15$

$\Rightarrow 15 - 15$

$\Rightarrow 0$

$Pr(X_1) = (\text{head appears})$

$\Rightarrow \left(\frac{1}{2}\right)$

Case (b) → $Pr(X_2) = \left(\frac{1}{2} * \frac{1}{2}\right) \Rightarrow \{\text{Tail} * \text{head}\}$.

$\Rightarrow \left(\frac{1}{4}\right)$; gain $\Rightarrow (15-1+2)-15$

$\Rightarrow 16 - 15$

$\Rightarrow 1$

Case (c) →

$P(X_4) = \frac{1}{2} * \frac{1}{2} * \frac{1}{2} \Rightarrow \{\text{Tail} * \text{Tail} * \text{head}\}$.

$\Rightarrow \frac{1}{8}$; gain $\Rightarrow (15-1-2+4)-15$

$\Rightarrow 16 - 15$

$\Rightarrow 1$

Case (d) →

$P(X_8) = \frac{1}{2} * \frac{1}{2} * \frac{1}{2} * \frac{1}{2} \Rightarrow \{\text{Tail} * \text{Tail} * \text{Tail} * \text{head}\}$.

$\Rightarrow \frac{1}{16}$. gain $\Rightarrow 15-1-2-4+8-15$

$\Rightarrow 16 - 15$

$\Rightarrow 1$.

$Pr(\text{all tails}) \Rightarrow \left(\frac{1}{16}\right)$; gain $\Rightarrow (-15)$

gain = → (1); → (−15)

Expected gain ⇒ $1 \cdot Pr(\text{while gain } 1) - 15 \cdot Pr(\text{gain} = -15)$

$$\Rightarrow 1 * \frac{15}{16} - 15 * \frac{1}{16}$$

$$\Rightarrow \frac{15-15}{16} \Rightarrow \underline{0}$$

so, expected gain = $\underline{0}$