CSL003P1M : Probability and Statistics
QuestionSet - 07: Conditional Distribution and Joint Distribution of Some Functions of Random Variables

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1. Let X, Y and Z be independent geometric random variables with the same parameter p. Find
   - $P\{X = Y\}$.
   - $P\{X \geq 2Y\}$.
   - $P\{X + Y \leq Z\}$.

2. Let X and Y be independent geometric random variables with the same parameter p. Let $U = \min(X, Y)$ and $V = X - Y$. Are U and V independent? Justify your answer.

3. Let $X_1, X_2, \ldots, X_r$ be mutually independent random variables, each having the uniform distribution $P\{X_i = k\} = 1/N$ for $k = 1, 2, \ldots, N$. Let $U_n = \min(X_1, X_2, \ldots, X_n)$ and $V_n = \max(X_1, X_2, \ldots, X_n)$. Find the distributions of U_n and V_n.

4. If $X_1, X_2, \ldots, X_k$ are distributed according to the multinomial distribution, the conditional distribution of X_1, given $X_2 = n_2, \ldots, X_{k-1} = n_{k-1}$, is binomial with parameters $n - (n_2 + \cdots + n_{k-1})$ and $p_1/(p_1 + p_k)$.

5. An urn contains balls numbered 1 to N. Let X and Y be the largest and smallest number drawn in n drawings when random sampling with replacement is used.
   (a) Find the joint distribution of X and Y.
   (b) Find the conditional probability that the first two drawings are j and k, given that $X = r$.

6. **Simulating a perfect coin:** Given a biased coin such that the probability of heads is p, we simulate a perfect coin as follows. Throw the biased coin twice. Interpret HT as success and TH as failure; if neither event occurs repeat the throws until a decision is reached.
   (a) Show that this model leads to Bernoulli trials with $p = 1/2$.
   (b) Find the distribution of the number of throws required to reach a decision.

7. **Sampling inspection:** Suppose that items with a probability p of being acceptable are subjected to inspection in such a way that the probability of an item being inspected is p'. We have four classes, namely, "acceptable and inspected", "acceptable but not inspected", etc. with corresponding probabilities pp', pq', $p'q$, qq' where $q = 1 - p$ and $q' = 1 - p'$.

   Let N be the number of items passing the inspection desk (both inspected and uninspected) before the first defective is found, and let K be the (undiscovered) number of defectives among them. Find the joint distribution of N and K and the marginal distributions.