# CSL003P1M : Probability and Statistics Lecture 01 (Recap)

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## Classical Definition of Probability

### Classical Definition of Probability

$$P(E) = \frac{F}{T}$$

P(E) is the probability of event E,

F is the number of times the **favourable** event E could happen,

T is the number of times the **whole** event could happen.

Three winning tickets are drawn from an urn of 100 tickets (without replacement). What is the probability of winning for a person who buys: (a) 4 tickets? (b) only one ticket?

### Solution: (a)

- A persons wins when he/she wins with at least one ticket, say it a lucky ticket.
- The probability that there is no lucky ticket is

$$p_l = \frac{\binom{96}{3}}{\binom{100}{3}}$$

• So, the probability of winning is  $1 - p_l$ .



Three winning tickets are drawn from an urn of 100 tickets (without replacement). What is the probability of winning for a person who buys: (a) 4 tickets? (b) only one ticket?

Solution: (b)

• The winning probability is

$$\frac{\binom{99}{2}}{\binom{100}{3}}$$

A bakery makes 80 loaves of bread daily. Ten of them are underweight. An inspector weighs 5 loaves at random. What is the probability that (at least) an underweight loaf will be discovered?

#### Solution:

The probability that no underweight loaf will be discovered is

$$p_l = \frac{\binom{70}{5}}{\binom{80}{5}}.$$

 So, the probability that at least one underloaf will be discovered is

$$1-p_I$$



Find the probability that among seven persons:

- 1 No two were born on the same day of the week.
- 2 at least two were born on the same day.
- 3 two were born on a Sunday and two on a Tuesday.

### Solution: (a)

 The probability that no two were born on the same day of the week is equal to the probability that all were born on different days of the week. The probability is

$$\frac{7!}{7^7}$$

Find the probability that among seven persons:

- 1 No two were born on the same day of the week.
- 2 at least two were born on the same day.
- 3 two were born on a Sunday and two on a Tuesday.

### Solution: (b)

• The probability that at least two were born on the same day is

$$1-\frac{7!}{7^7}$$
.



Find the probability that among seven persons:

- No two were born on the same day of the week.
- 2 at least two were born on the same day.
- exactly two were born on a Sunday and exactly two on a Tuesday.

### Solution: (c)

- The number of cases that two were born on Sunday is  $\binom{7}{2}$ .
- The number of cases that two from the remaining five were born on Friday is  $\binom{5}{2}$ .
- The number of cases that remaining were born on other five days is 5<sup>3</sup>.

Find the probability that among seven persons:

- 1 No two were born on the same day of the week.
- 2 at least two were born on the same day.
- exactly two were born on a Sunday and exactly two on a Tuesday.

Solution: (c)

Thus the probability is

$$\frac{\binom{7}{2}\binom{5}{2}5^3}{7^7}$$



A group of 2N boys and 2N girls is randomly divided into two equal groups. What is the probability that each group has the same number of boys and girls?

#### Solution:

- There are 4N people.
- If 4N people are divided into two equal groups, each group will have 2N people.
- If each group has the same number of boys and girls in each group, the total number of ways groups can be created is

$$X = \binom{2N}{N} \binom{2N}{N}.$$



A group of 2N boys and 2N girls is randomly divided into two equal groups. What is the probability that each group has the same number of boys and girls?

#### Solution:

The total number of ways the two groups can be created is

$$T = \binom{4N}{2N}.$$

So, the probability is

$$\frac{X}{T}$$
.

A group of 2N boys and 2N girls is randomly divided into two equal groups. What is the probability that each group has the same number of boys and girls?

#### Solution:

- Let *r* be the number of boys and girls in each group.
- Then the number of ways in which two groups can be created where the group one has r number of boys and group two has r number of girls is

$$\binom{2N}{r}\binom{2N}{r}$$
.

So, the total number of ways two groups can be created is

$$F = \sum_{r=1}^{2N} {2N \choose r} {2N \choose r}.$$

A group of 2N boys and 2N girls is randomly divided into two equal groups. What is the probability that each group has the same number of boys and girls?

#### Solution:

• The total number of ways the two groups can be created is

$$T = \sum_{r_1=1}^{2N} \sum_{r_2=1}^{2N} {2N \choose r_1} {2N \choose r_2}.$$

So, the probability is

$$\frac{F}{T}$$



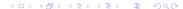
Consider r indistinguishable balls randomly distributed in n cells. What is the probability that exactly m cells remain empty?

#### Solution:

- If there are n cells out of which m cells remain empty, there are n-m cells remain non-empty which can be done in  $\binom{n}{m}$  ways.
- Now, we need to fill r indistinguishable balls in n-m cells which is same as finding the number of ways of the solution

$$x_1 + x_2 + \cdots + x_{n-m} = r$$

where  $x_i \ge 1$  for all  $1 \le i \le n - m$ .



Consider r indistinguishable balls randomly distributed in n cells. What is the probability that exactly m cells remain empty?

#### Solution:

• Consider the equation

$$x_1 + x_2 + \cdots + x_{n-m} = r$$

where  $x_i \ge 1$  for all  $1 \le i \le n - m$ .

The total number of solutions of the above equaiton is

$$\binom{r-1}{n-m-1}$$



Consider r indistinguishable balls randomly distributed in n cells. What is the probability that exactly m cells remain empty?

#### Solution:

 So, the number of ways that r indistingushable balls are randomly distributed in n cells out of which m cells remain empty is

$$F = \binom{n}{m} \binom{r-1}{n-m-1}$$

Consider r indistinguishable balls randomly distributed in n cells. What is the probability that exactly m cells remain empty?

#### Solution:

 The total number of ways in which r indistinguishable balls are randomly distributed in n cells is the total number of solutions for the equation

$$x_1 + x_2 + \cdots + x_n = r$$

where  $x_i \ge 0$  for all  $1 \le i \le n$ .

The total number of solutions for the above equation is

$$T = \binom{r+n-1}{n-1}$$

• Thus, the probability is



### Birthday Problem

In a classroom there are *n* students.

- What is the probability that at least two students have the same birthday?
- ② What is the minimum value of n which secures probability 1/2 that at least two have a commom birthday.

Solution: (Do it yourself!!)

# Thank You