CSL003P1M : Probability and Statistics
QuestionSet - 05: Expectation and Variance of Random Variables

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1. An urn contains r distinct balls labeled $1, 2, \ldots, r$. Balls are drawn with replacement until exactly $k \leq r$ distinct objects have been obtained. Let S_k be the random variable which denotes the size of the sample required. Compute $E[S_k]$.

2. Suppose that a sample of size n is to be chosen randomly (without replacement) from an urn containing N balls, of which m are white and $N - m$ are black. If we let X denote the number of white balls selected, then

$$p(i) = P\{X = i\} = \frac{\binom{m}{i}\binom{N-m}{n-i}}{\binom{N}{n}}, \quad i = 0, 1, \ldots, n$$

   A random variable whose probability mass function is given by the above equation for some values of n, N, m is said to be a **Hypergeometric random variable**.

   Compute $E[X]$ and $Var[X]$.

3. Consider the previous question. Find $E[X]$ and $Var[X]$ when the sampling is done with the replacement.

4. Let n numbers be selected from the N numbers $1, 2, \ldots, N$. Let S_n be the random variable which denotes the sum of selected numbers. Find $E[S_n]$.

5. Let there be N_1 white balls and N_2 black balls; n balls are drawn at random, (a) with replacement, (b) without replacement. What is the expected number of white balls in the sample?

6. An exam consists of multiple-choice test consisting of two problems. The first problem has 3 possible answers and the second one has 5. The students chooses, at random, one answer as the right one from each of the two problems. Let X be the random variable which denotes the number of right answers given by the student. Find $E[X]$ and $Var(X)$.

7. Suppose you have INR 15 and you are participating in a game. Your strategy is as follows:

   - You bet INR 1 for the toss. If a head appears on the first toss, you receive INR 1 and then you withdraw the game.
   - Otherwise, if a tail appears, you bet INR 2 for the next toss. If a head appears on the next toss, you win and withdraw the game.
   - Otherwise, if a tail appears, you bet INR 4 for the next toss. If a head appears on the next toss, you win and withdraw the game.
   - Otherwise, if a tail appears, you bet remaining INR 8 for the next toss. If a head appears on the next toss, you win and withdraw the game.

   What is your expected gain?

8. (a) There are two identical coins. The property of these coins is such that if one coin appears head, then the second one also appears head. Suppose these two coins are tossed. Let $P(\text{Head}) = p$ and X be the random variable which denotes the number of heads that appear. Find $E[X]$ and $Var[X]$.

   (b) Suppose two different coins are thrown independently and the probability that head appears are p_1 and p_2. Let X be the random variable which denotes the number of heads that appear. Find $E[X]$ and $Var[X]$.

9. A die is thrown until "6" or "an odd number" appears three times. Find the expected number of throws:

   (a) when this experiment is done only once.

   (b) when this experiment is repeated ten times.

10. A player pays INR 1 for playing the following game: three dice are thrown; the player gets INR 1 if one 6 appears, the player gets INR 2 if two 6 appear and the player gets INR 8 if three 6 appear. Find the expected gain of the player. How much reward should be given to the player if three 6 appear so that the expected gain should be zero?