a) $r_{ik} = Pr(h_i = k | x_i, \theta^{(t)}) = \dfrac{Pr(x_i | h_i = k, \theta^{(t)}) \, Pr(h_i = k | \theta^{(t)})}{\sum_{j=1}^{K} Pr(x_i | h_i = j, \theta^{(t)}) \, Pr(h_i = j | \theta^{(t)})}$

$$= \frac{\lambda_k \, Norm_{x_i}[\mu_k, \Sigma_k]}{\sum_{j=1}^{K} \lambda_j \, Norm_{x_i}[\mu_j, \Sigma_j]} \longrightarrow (1)$$

$$Norm_x[\mu, \Sigma] = \frac{1}{(2\pi)^{D/2} |\Sigma|^{1/2}} e^{[-1/2 (x-\mu)^T \Sigma^{-1} (x-\mu)]}$$

Given $\Sigma = I$

So

$$Norm_x[\mu, \Sigma] = \frac{1}{(2\pi)^{I/2}} \exp^{[-1/2 (x-\mu)^T (x-\mu)]}$$

Substituting in (1)

$$= \frac{\lambda_k \frac{1}{\sqrt{2\pi}} e^{(-1/2 (x_i - \mu_k)^2)}}{\sum_{j=1}^{K} \lambda_j \frac{1}{\sqrt{2\pi}} e^{(-1/2 (x_i - \mu_j)^2)}}$$

$$= \frac{\lambda_k e^{(-1/2 (x_i - \mu_k)^2)}}{\sum_{j=1}^{K} \lambda_j e^{(-1/2 (x_i - \mu_j)^2)}}$$

1(c) Defn: $r_{ik} = \begin{cases} 1 & \text{if } k = k' \\ 0 & \text{otherwise} \end{cases}$

$$v_k^{[t+1]} = \frac{\sum_{i=1}^{n} r_{ik} x_i}{\sum_{i=1}^{n} r_{ik}}$$

if $k = k'$ $\quad p_{k'}^{[t+1]} = x_i$

else $\quad v_k^{[t+1]}$ doesn't exist

So GMM is meanshift now.

1(b)