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Abstract

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Chapter 1

Introduction

1.1 Motivation

Fully autonomously driving cars have the potential to rule out human driving error which is at least a contributing factor to most accidents today. Many social and technical obstacles have yet to be overcome until fully autonomous cars become market-ready (Maurer et al., 2016). However, many Advanced driver assistance systems (ADAS) such as adaptive cruise control, lane keeping and changing assistance, and automated collision mitigation are already deployed in modern cars.

The extend to which an ADAS takes control varies. While the potential prevention of human-error caused accidents increases with the elaborateness of intervention by an assistant system, excessive intervention drastically limits the driver's autonomy. A loss of driver autonomy can turn driving into a monotonous and tedious supervisory task. Drivers easily become inattentive and are more prone to distract themselves, for example by looking on their phone. However, as long as assistance systems are not sufficient to handle all situations, a concentrated human will remain necessary to take actions in situations the assistance system fails. Leaving the driver with a pure supervision task can lead to a long transition time for the driver when it is required to retake control of the vehicle (Wang et al., 2020). Being in control means having to concentrate. Therefore, the goal should be to keep the human driver in control as much as possible but to assist when help is really needed. As a result, driving pleasure is enhanced and drivers are prevented from relying too heavily on the assistance systems.

15% of injury crashes in the US were associated with driver distraction in 2018 (NHTSA, 2020). Therefore, it seems reasonable to make the extend of the ADAS's activation dependent on the driver's level of attention. Whenever a driver is inattentive or distracted, an ADAS needs to be particularly sensitive. Yet in what way can an assistance system detect that a driver is distracted?

1.2 Problem overview

There have bee attempts to develop systems that determining the psychological state of a driver in real time while driving. The application of eye tracking technology or analysis of camera footage using machine learning models is conceivable and has led to promising results. However, as promising as promising as these methods are, they are not readily available yet. Furthermore, they are quite intrusive and could be seen as an encroachment on privacy. Thus, the driver's level of attention is essentially unknown. Nevertheless, one can assume

that distracted drivers act differently. Among other things, deviations such as increased reaction times and altered steering behavior are likely.

A two-fold problem arises: On the one hand, a lane keeping assistance system has to be able to identify when drivers are distracted by observing their behavior. On the other hand, the system must have the capability to provide meaningful assistance.

Intuitively, it seems reasonable to solve both problems individually; using a model that takes the available data, such as the driver's steering behavior, as input to classify whether the driver is distracted, and another model that assists a distracted driver in steering the car. Both could be trained using example data. However, this supervised approach entails two challenges: First, driving is a sequential decision process. An action influences future actions and driving situations in which decisions have to be made are essentially unique. Second, an activation of the assistance system can affect on how drivers behave. Drivers may adjust to the system. It is not possible to create a dataset that covers these dynamics entirely.

Reinforcement learning (RL) allows a system to learn and represent its behavior by interacting with it rather than learning from past experience. Therefore, RL constitutes a promising method to develop an ADAS or even a fully autonomous driving agent and its application in this area is a very active research area with many successful results (Kiran et al., 2021). Because learning is achieved by exploration rather than from examples, RL is able to perform well in sequential decision making tasks. Moreover, reinforcement learning algorithms can be extended to support learning with a partially observable state (Sutton & Barto, 2018, p. 466). While the agent can perceive the car's environment with sensors, the attention level of the driver is hidden. Nevertheless, only one RL agent is needed to both learn how to assist in driving and to classify when this is desired due to a distracted driver.

The result is a shared control scenario where both the human driver and the agent can actively control (e.g. steer, brake, accelerate) the car simultaneously. Each can indirectly perceive the actions of the other by observing the state of the car. Thereby, on the one hand, the agent is able to analyze the driving behavior of the human and, on the other hand, the human can notice the assistance of the agent and may adapt to it.

Learning in a real-world situation is not feasible in the context of this thesis. Despite the inevitable high safety risk, it would also require an enormous investment of resources, and the complexity of a real-world driving scenario represents an insurmountable obstacle. Instead, the agent learns in a simulation environment with a simulated human driver. The Open Racing Car Simulator (TORCS), a racing car simulator that allows to model various driving situations (Espié et al., 2005) is used as simulation environment. It offers a good balance between realism and resource efficiency and has been utilized in many papers regarding RL-based driving before. An Adaptive Control of Thought-Rational (ACT-R) cognitive model is employed to simulate the human's actions. The model is able to keep the car in its lane, perform lane changes, and avoid collision with other road users. It captures behavioral differences between attentive

and inattentive human drivers. Furthermore, a human-subject experiment is performed in the TORCS simulation environment to identify if the agent is able to generalize well enough to be useful for actual human drivers.

1.3 Contributions

The main goal and differentiator of this thesis is to utilize reinforcement learning for a shared-control driving task with unknown attention of the human driver. One of the main challenges is that near real-time decisions of the agent are necessary. This drastically limits the time available for online planning. Accordingly, the implementation needs to be very efficient. Solving the problem using an algorithm that requires discretized states (e.g. steering angle categories) is contrasted with a solution using an algorithm directly supporting continuous states.

1.4 Outline

The rest of the proposal is organised as follows:

Chapter 4 describes the problem in a formal manner using a Partially observable Markov decision process (POMDP).

Chapter 2 summarizes and reviews important literature that serves as the foundation of the thesis.

?? presents the initial research plan for the rest of the thesis, including important milestones and deadlines.

Chapter 2

Literature

This thesis focuses on efficient online POMDP planning. The two most notable fast online POMDP algorithms are DESPOT (Ye et al., 2017) and POMCP (Silver & Veness, 2010). Both apply Monte Carlo tree search to evaluate the quality of candidate policies. At each time point, a simulator of the environment is used to form a search tree from multiple simulations in order to evaluate the resulting hypothetical histories by their mean return, leading to a real action of the agent in the environment and thus to a new real observation (Silver & Veness, 2010). DESPOT addresses and improves upon POMCP's problem of a poor worst-case performance bound (Ye et al., 2017).

Both POMCP and DESPOT can handle continuous state spaces but would have to be modified in order to support continuous action or observation spaces (Sunberg & Kochenderfer, 2018). Sunberg and Kochenderfer provide two online algorithms for POMDP with continuous state, action, and observation spaces: POMCPOW for simulating approximate state trajectories, and PFT-DPW for simulating approximate belief trajectories.

Offline and online approaches can be combined by using an approximate policy computed offline as a default policy (Gelly & Silver, 2007), or by considering a sequence of macro-actions to reduce the size of the search horizon (He et al., 2011). Especially when there is only very little time for online planning, incorporating an offline approximation into an online approach can be useful (Ross et al., 2008).

Lam and Sastry, 2014 provide a framework for using a POMDP to model a Human-in-the-loop (HITL) control system. Their framework serves as foundation for this thesis and is further described in Chapter 4. The framework is used in a case study where an agent assists a potentially drowsy human driver in keeping the car centered in its lane. Whether or not the driver is drowsy remains unknown to the agent. The agent's estimation of the driver's drowsiness is based on the humans actions such as turning the steering wheel and opening or closing its eyes. Intervention by the agent is possible both by alerting the driver with a warning, and by actively steering the car. Any intervention is penalized with the aim to interfere with the driver as little as possible but as much as required in order to keep the car centered when the driver is drowsy. They approximately solve their POMDP problem with an offline randomized point-based value iteration approach. The policy is computed by iteratively sampling a finite set of random points from the agent's belief space. The agent thus interacts randomly with the environment in order to find an approximation of the optimal policy. The employed model of the human's internal state is rather simplistic and based on handcrafted transition probabilities. The state and action spaces are discrete.

Sadigh et al., 2016 evaluate how active probing can be utilized by autonomous vehicles in driving scenarios to reduce their uncertainty about a hidden psychological state of human drivers on the road. Three different scenarios are modeled: First, the agent wants to cross an intersections with other cars driving on the crossed road. The autonomous car can cautiously approach in order to probe the other cars for attentiveness; if they react by reducing their speed, they are likely attentive. Second, the agent drives on a highway with human drivers approaching from behind. The goal is to avoid rear-end collisions, which are especially likely in the case of inattentive human drivers. Active probing can be performed by the agent through braking. If an approaching driver does not slow down, the driver is most likely not paying attention. Third, the agent actively probes for an aggressive or timid driving style of other drivers by nudging into their lane. A human driver is expected to either slow down to allow the agent to switch lanes (timid driving style), or speed up to discourage the agent to switch lanes (agressive). This approach of actively provoking human responses rather than just passively observing leads to a significant improvement in classifying the human drivers' hidden attentiveness. It can potentially also be utilized in a shared control setting. The agent could utilize minor interventions to reduce uncertainty about the human's internal state.

Furthermore, the work of Sadigh et al., 2016 is relevant with regard to how they represent their problem as a POMDP with a continuous state and action space and plan online using Model Predictive Control (MPC). At every time step, the agent uses an embedded human model to make predictions over a finite horizon about the actions a human would take in response to its own actions. The agent knows how a human would act in different psychological states, the state itself, however, is hidden. It is assumed that the human always tries to maximize its reward. The agent chooses a policy that maximizes its reward while accounting for the human's (potential) actions depending on the hidden psychological state the agent believes the human to be in. The real human actions that are observed after the agent executes an action are used to update the agent's belief about the human's internal state. The human model is learned a priori using inverse reinforcement learning (IRL) using demonstrations of human behavior for which the human's internal state is known.

Wang et al., 2020 provide an overview of papers regarding decision making and human driver modeling for driver-vehicle shared control scenarios. Insight about recent developments, different architectures, and remaining challenges is provided. Of particular relevance are the different modes for the communication of authority between human and agent and the cognitive modeling approaches that are discussed.

Chapter 3

Theoretical foundation

3.1 Sequential decision making

Lane keeping of a car is a sequential decision making task. Every steering action that is performed directly influences the choice of the best succeeding steering actions. Markov decision processes (MDP) are well suited and widely used to model sequential decision making tasks. An MDP is a discrete time framework for a decision maker, the agent, to interact with an environment. At every time step, the environment is in a certain state, fully observable by the agent. The agent interacts with the environment by performing an action that determines the next state of the environment. The underlying assumption, the Markov property, is that the next state of the environment only depends on its current state and the agent's action. The transition to a succeeding state after an action has been performed does not need to be deterministic but can be probabilistic, accounting for randomness in the environment. After performing an action, the agent receives a numerical reward (also called return). The agent's goal is to maximize the cumulative reward it receives over time. An action that leads to a high immediate return is not optimal if another action leads to a higher cumulative reward in the long run. Thus, the agent needs to find an optimal policy that decides the best action to take in every state. In case the state transition probabilities are known to the agent, the optimal policy can be found using model-based techniques such as value or policy iteration. If the transition model is unknown, model-free reinforcement learning can be applied to learn an optimal policy.

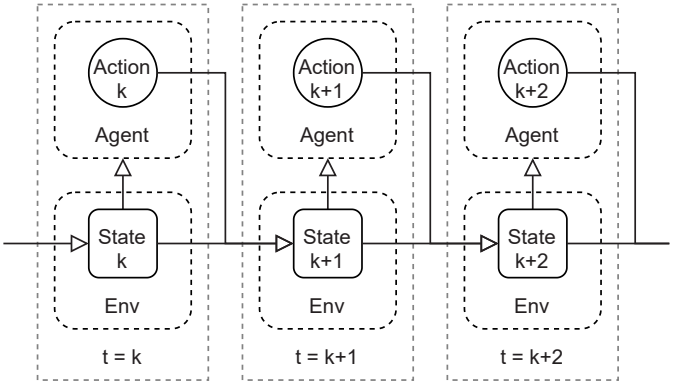


Figure 3.1: Markov decision process (MDP)

Assisting a human driver in the lane keeping task is essentially a sequential

decision making task as well. However, the agent that is assisting the human driver does not know about the driver's internal psychological state, and therefore her attention. A distracted driver may steer poorly and needs assistance. But how can the agent tell whether the driver is distracted? Reading the driver's mind is not feasible and even if it were, it would be too invasive for this task. Instead, the agent needs to estimate the driver's internal state in order to act adequately. A POMDP is a generalization of an MDP that allows to plan under uncertainty. Even without observing the full state of the agent's environment, of which the driver is part of, a POMDP allows the agent to estimate the environment's true state using the partial information it observes. A POMDP serves as the foundation of this thesis. The lane keeping assistance problem this thesis aims to solve can be defined as a POMDP. First, a formal definition is needed.

3.2 Partially observable Markov decision process (POMDP)

The POMDP generalizes the MDP for planning under uncertainty. The environment's true state is unknown to the agent. It has to rely on observations with partial information about the environment's true state to choose its actions. Kaelbling et al., 1998 define a POMDP as a tuple (S, A, T, R, O, Z), where:

- S is the set of all possible states $s \in S$ of the environment. A state describes the environment at a time point. It must not be an all-encompassing description but must include all relevant information to make decisions. The state is hidden from the agent. This is the main difference to an MDP.
- A is the set of all possible actions $a \in A$ the agent can perform in the environment.
- $T : S \times A \times S \to [0, 1]$ defines the conditional state transition probabilities. $T(s, a, s') = Pr(s'|s, a)$ constitutes the probability of transitioning to state s' after performing action a in state s.
- $R : S \times A \to \mathbb{R}$ is the reward function providing the agent with a reward of $R(s, a)$ after performing action a in state s.
- O is the set of all possible observations $o \in O$. Observations are the agent's source of information about the environment, enabling the agent to estimate the environment's state.
- $Z : S \times A \times O \to [0, 1]$ defines the conditional observation probabilities. $Z(s', a, o) = Pr(o|s', a)$ represents the probability of receiving observation o at state s' after performing action a in the previous state.

At any time, the environment is in some state s. Unlike in the case of an MDP, the agent cannot directly observe the environment's state. Instead, the agent receives an observation o that provides partial information about the current state. The agent uses the observations it perceives over time to estimate

the true state of the environment in order to choose adequate actions. At any time step t, it has to take into account the complete history h_t of actions and observations until t:

$$h_t = \{a_0, o_1, ..., o_{t-1}, a_{t-1}, o_t\} \tag{3.1}$$

Keeping a collection of all past observations and actions is very memory expensive. A less memory demanding alternative is to only keep a probability distribution over the states at every step, called a belief b. $b(s, h)$ denotes the probability of being in state s given history h.

$$b_t(s, h) = Pr(s_t = s | h_t = h) \tag{3.2}$$

The belief is a sufficient statistic for the agent to form a decision about its next action (Smallwood & Sondik, 1973). Thus, only the belief needs to be kept and can be recursively updated whenever an action is performed and a new observation arises. The agent starts with an initial belief b_0 about the initial state of the environment. At every subsequent time step, the new belief b' can be recursively calculated based on the previous belief b, the last action a and the current observation o. The previous belief can then be discarded as the history it represents is no longer up-to-date. For an exact update of the belief one can apply the Bayes theorem:

$$\begin{aligned} b'(s') &= Pr(s'|o, a, b) \\ &= \frac{Pr(o|s', a, b) Pr(s'|a, b)}{Pr(o|a, b)} \\ &= \frac{Pr(o|s', a) \sum_{s \in S} Pr(s'|a, b, s) Pr(s|a, b)}{Pr(o|a, b)} \\ &= \frac{Z(s', a, o) \sum_{s \in S} T(s, a, b) b(s)}{Pr(o|a, b)} \end{aligned} \tag{3.3}$$

The agent chooses its actions based on its belief according to its policy π. The agent's policy defines the action to choose at any given belief state. It describes the strategy for every possible situation the agent can encounter. Solving a POMDP consists in finding an optimal policy π^* that maximizes the the cumulative reward obtained over some time horizon N starting from initial belief b_0 using a discount factor $0 \leq \lambda \leq 1$:

$$\pi^* = argmax_\pi \; E\left[\sum_{t=0}^{N} \sum_{s \in S} b_t(s) \sum_{a \in A} \lambda^t R(s, a) \pi(b_t, a) | b_0\right] \tag{3.4}$$

The return that is gained by following a policy π from a certain belief b can be obtained with the value function $V^\pi(b)$:

$$V^\pi(b) = \sum_{a \in A} \pi(b, a) \left[\sum_{s \in S} b(s) R(s, a) + \lambda \sum_{o \in O} Pr(o|b, a) V^\pi(b')\right] \tag{3.5}$$

The optimal policy π^* maximizes $V^\pi(b_0)$. For any POMDP there exists at least one optimal policy.

3.3 Key challenges

3.3.1 Curse of dimensionality and curse of history

Computing an optimal policy for a POMDP is challenging for two distinct but interdependent reasons (Pineau et al., 2006). On the one hand, there is the so-called curse of history: Finding an optimal policy is like searching through the space of possible action-observation histories. The number of distinct histories grows exponentially with the size of the time horizon. Therefore, planning further into the future increases the computation complexity exponentially. While finding an optimal policy can be relatively easy for short histories, it becomes computationally infeasible for larger time horizons. On the other hand, there is the curse of dimensionality: The belief space is a $|S|$-dimensional. Therefore, the size of the belief space, representing the number of states in a POMDP, grows exponentially with $|S|$.

The task of finding an optimal policy for a finite POMDP is PSPACE-complete (Papadimitriou & Tsitsiklis, 1987). Therefore, solving POMDP to optimality is computationally infeasible with a large state space or time horizon. For this reason, approximate algorithms are often applied.

3.3.2 Unknown transition and observation probabilities

For many problems, it is difficult or impossible to know the probability distributions T or Z explicitly. This is also the case for the shared control lane keeping scenario assessed in this thesis. Neither the transition probabilities, nor the observation probabilities are known a priori. The belief update method using Bayes' theorem presented in Equation 3.3 is not computable without knowing the probabily distributions explicitly. However, exact updates are too complex for problems with a large state space in any case (Silver & Veness, 2010). Some solution approaches circumvent the problem of unknown transition and observation probability distributions by only requiring a generative model that can sample state and observation transitions. A generative model can stochastically generate a successor state, reward, and observation, given the current state and action. Thereby, it implicitly defines the transition and observation probabilities, even if they are not explicitly known. The generative model used in this thesis is described in detail in Section 4.2.5.

3.4 Algorithms to aproximately solve large POMDP

3.4.1 Offline and online solvers

There are two general approaches to solve POMDP: offline and online. Online solvers compute the optimal policy prior to execution for all possible future

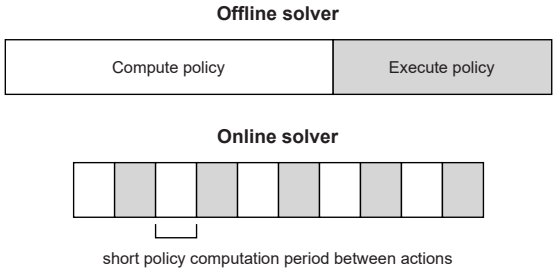


Figure 3.2: Comparison of offline and online solving procedure

scenarios. Their advantage is that once the policy is found, its the application does only have a very minimal, negligible time overhead. However, offline planning is hard to scale to complex problems as the number of possible future scenarios grows exponentially with the size of the time horizon (curse of history). Furthermore, while the performance for small to medium sized POMDP can be quite good, computing the policy may take a very long time, and even small changes in the dynamics of the environment require a full recomputation (Ross et al., 2008). Online solvers interleave planning and plan execution. At every time step, only the current belief is considered to compute the next optimal action by searching ahead until a certain depth is reached. On the one hand, the scalability is greatly increased. On the other hand, sufficiently more online computation than with offline planning is required. The amount of available online planning time at each time step limits the performance.

Because the state space

3.4.2 Monte Carlo tree search solvers

Other solvers have build upon the principle from POMCP of using Monte Carlo tree search for POMDP planning. Notably, there are DESPOT, POMCPOW, ... TODO ... They all have an important requirement in common: The observation probabilities need to be known to the solver. Thereby, particles that are added to the belief can be weighed by how likely their associated observation is at the current belief state after performing a certain action in the prior state. However, the observation probabilities are essentially unknown in our assisted driving scenario.

Chapter 4

Methodology

4.1 Lane keeping with a human in the loop as a POMDP

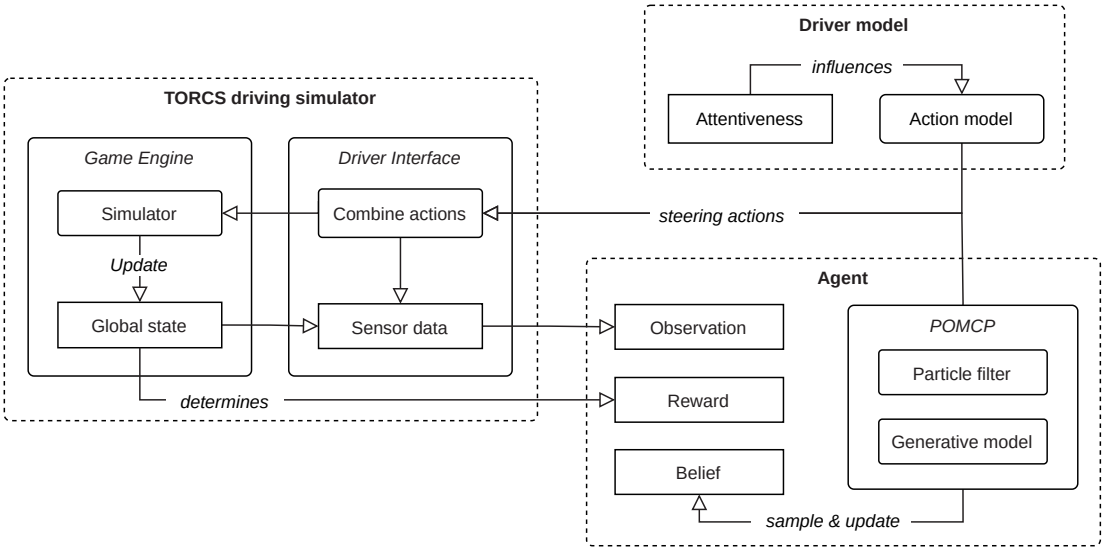


Figure 4.1: Solution approach overview

4.1.1 Driving simulator TORCS

4.1.1.1 State

The tracks will be round courses. Thus, there is no terminal state if everything goes well. If the car reaches an off-track position, however, the car is reset to be in the initial starting position again.

Name	Measurement	Description
Gear **(constant)**	$\{-1, 0, 1, \ldots, 6\}$	Distance of the car from the start line along the track line. **Neither the human driver nor the agent can directly influence this with their actions.**
RPM **(constant)**	$[0, +\inf)$	Number of engine rotations per minute. **Neither the human driver nor the agent can directly influence this with their actions.**
Speed **(constant)**	$(-\inf, +\inf)$ (km/h)	Speed along the longitudinal axis of the car. **Neither the human driver nor the agent can directly influence this with their actions.**
Side force	$(-\inf, +\inf)$ (km/h)	Speed along the transverse axis of the car. This is directly influenced by the steering actions of both human driver and agent.
Distance from start	$[0, +\inf)$ (m)	Distance of the car from the start line along the track line.
Angle	$[-\pi, +\pi]$ (rad)	Angle between car direction and track axis direction.
Lane position	$(-\inf, +\inf)$	Horizontal distance between the car and the track axis. 0 when the car is on the axis, +1 if the car is on the left edge of the track, and −1 if the car is on the right edge of the track. Greater numbers than +1 or smaller numbers than −1 indicate that the car is off-track.
Driver attention	True / False	Whether the human driver is attentive or distracted.

4.1.1.2 Actions

Name	Measurement	Description
In our simplified scenario, both the human driver and the agent can not accelerate, brake or switch gears.		
Steering	$[-2, +2]$	The input to the car is generated by combining the agent's action with the human's steering action (see equation 4.1). For the car, -1 means full right (159 degrees) and $+1$ means full left (21 degrees). A value greater than $+1$ or lower than -1 can effectively reverse an opposite action of the human driver.

The human driver and the agent share control of the steering wheel. The speed of the car is fixed and cannot be altered; neither by human driver nor agent. The steering input of the driver $\mathcal{A}_{steer}^{driver}$ and agent $\mathcal{A}_{steer}^{agent}$ are combined to $A_{steer} \in [-1, +1]$ using equation 4.1.

The agent needs to be able to fully counteract a distracted driver's actions. In the extreme case, while the car is in a curve, a distracted driver could steer into the opposite direction of the trajectory of the lane center. Thus, the car would not only diverge from the lane center but would even get off the road completely. The agent thus needs to reverse the driver's action in order to keep the car centered in the lane and follow the road curve. Therefore, we define the range for the agent's steering action as follows: $\mathcal{A}_{steer}^{agent} \in [-2, +2]$.

$$\mathcal{A}_{\text{steer}} = \min(\,-1,\, \max(\,1,\, (\mathcal{A}_{\text{steer}}^{\text{driver}} + \mathcal{A}_{\text{steer}}^{\text{agent}})\,)\,) \tag{4.1}$$

4.1.1.3 Observations

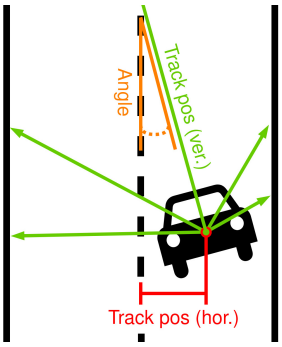


Figure 4.2: Main environment observations

Name	Measurement	Description
Constant state parameters are not observed as they do not influence learning. **The observations are not noisy.**		
Angle	$[-\pi, +\pi]$ (rad)	Angle between car direction and track axis direction
Side force	$(-\inf, +\inf)$ (km/h)	Speed along the transverse axis of the car. This is directly influenced by the steering actions of both human driver and agent.
Track position (horizontal)	$(-\inf, +\inf)$	Horizontal distance between the car and the track axis. 0 when the car is on the axis, +1 if the car is on the left edge of the track, and −1 if the car is on the right edge of the track. Greater numbers than +1 or smaller numbers than −1 indicate that the car is off-track.
Track position (vertical)	[0,200] (m)	Vector of 5 range finder sensors (of 19 available in TORCS). The range finders serve as lookahead by returning the distance between the car and the track edge in a given forward angle between −90 and +90 degrees with respect to the car axis.
Driver steering (last time step)	$[-1, +1]$	The agent perceives the last input of the human. This is not the action of the human in the next but in the last time step. The agent does not know which action the human is going to choose simultaneous to its own action. −1 means full right (159 degrees) and +1 means full left (21 degrees).

4.1.2 Driver model

The driver model is simplistic. If the driver is attentive, its actions are optimal. The driver model returns the action that steers the car as close to the center of the lane as possible. In this case, the agent should not interfere. However, if a distracted driver is modeled, the driver just repeats the last action it performed while being attentive. This can have the effect of the driver's action to overshoot with the car diverging from the center of the lane. Following, the agent has to identify distracted driving and counteract.

When the driver model is initialized, it is randomly set to be attentive or distracted. The driver stays in this state for a randomly chosen discrete time period between ten and 60 seconds for an attentive state and between two and six seconds for a distracted state. After the chosen time period, the state of

attentiveness switches; a previously attentive driver becomes distracted, and a previously distracted driver becomes attentive. The process repeats until the experiment is over.

4.1.2.1 Simple driver model

4.1.2.2 Steering over-correction

4.1.2.3 Steering over-correction and noise

4.1.3 Reward

The overall goal for the agent is to only assist the driver in keeping the car centered in its lane. Therefore, this is the main source of reward for the agent. The more centered the car is at a certain time step, the more reward r is received. However, the agent is supposed to leave the human driver with as much autonomy as possible. Thus, any intervention by the agent is penalized. Minor smooth interventions are generally preferred over large abrupt steering actions. Accordingly, the penalty is (exponentially) dependent on the intensity of the agent's action. The general assumption is that an attentive driver performs better in keeping the car centered than an inattentive driver. The agent has to predict whether a driver is attentive or not in order to choose its actions correctly. An incorrect prediction of the driver's actions will lead to overshooting and thus be negatively reflected in the reward for keeping the car centered. Lastly, the car is never supposed to leave the lane. Consequently, leaving the lane is highly penalized.

$$\mathcal{R} = \mathcal{R}_{\text{center}} - \mathcal{P}_{\text{act intensity}} - \mathcal{P}_{\text{off-lane}}$$

$$\mathcal{R}_{\text{center}} = \begin{cases} r - r * |Pos_{hor}| & \text{if} \quad |Pos_{hor}| \leq 1 \\ 0 & \text{if} \quad \text{off-lane} \end{cases}$$

$$\mathcal{P}_{\text{act intensity}} = |\mathcal{A}_{\text{steer}}|^{p_{\text{int}}}$$

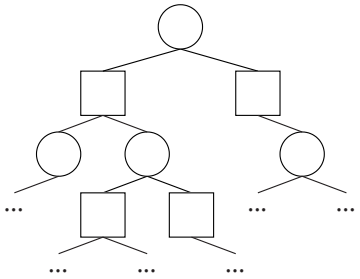
$$\mathcal{P}_{\text{off-lane}} = \begin{cases} p_{\text{off}} & \text{if} \quad |Pos_{hor}| > 1 \\ 0 & \text{if} \quad |Pos_{hor}| \leq 1 \end{cases}$$

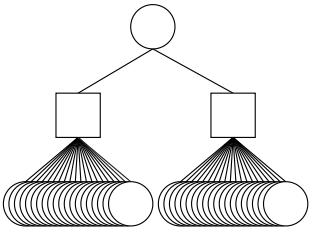
4.2 Solution approach using the POMCP algorithm

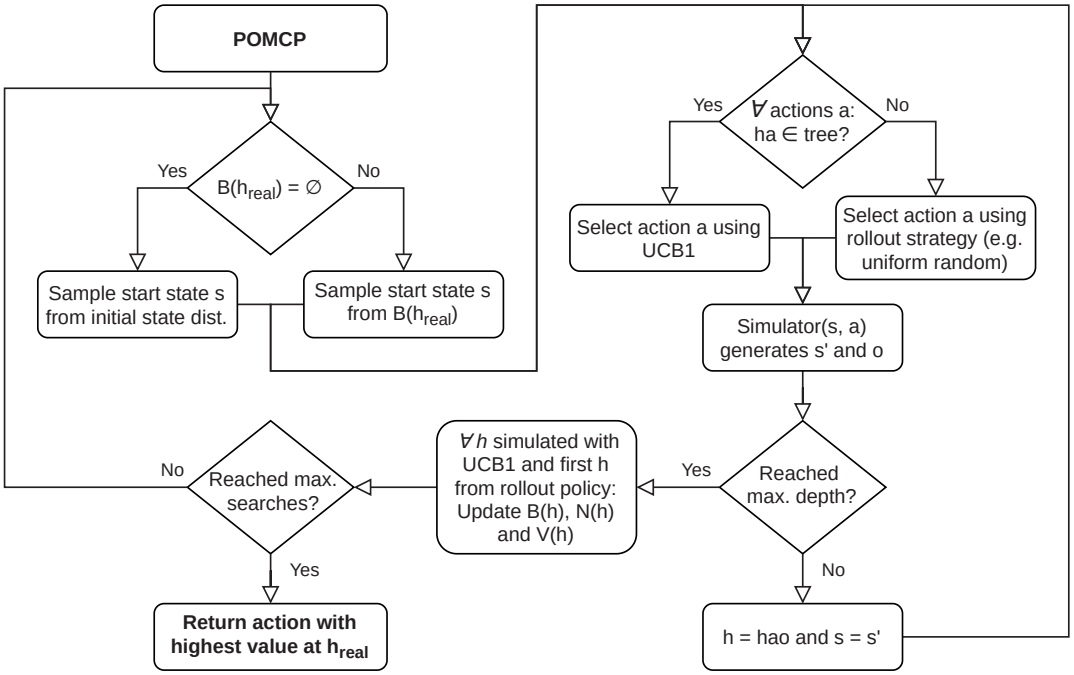
4.2.1 Discretization

4.2.2 General POMCP definition

Partially observable Monte-Carlo Planning (POMCP) constructs a search tree with nodes representing histories h of actions and observations. At each node, $N(h)$ stores the number of times the represented history h has been encountered, $V(h)$ is the node's value that is approximated by the average return of simulations starting at history h, and $B(h)$ represents the node's belief over







the real environment's state. $B(h)$ is a collection of potential states where the likelihood of each state is given by the relative number of times it is included in the collection.

If the belief at the node representing h_{real} is empty, an initial state distribution I is used to sample a start state s for the search. Otherwise, $B(h_{real})$ is utilized. The search tree is then searched in two stages. First, in the case that the search tree already contains child nodes for all actions at the current history, UCB1 is used for the action selection. Exploration is achieved by increasing the value of rarely-tried actions by an exploration bonus. Second, when the tree is missing a node for a potential action at the current history, a rollout policy is used to select actions. In the most simple case, this means choosing uniformly random over the action space. In either case, the selected action is executed on the start

state s, leading to a successive state s', observation o, and reward r. The process is repeated with resulting successive states until a maximum depth of the tree is reached. Afterwards, the beliefs, counts, and values are updated at all nodes for the histories resulting from the UCB1 action selection, and the node for the first history resulting from the rollout policy. The belief is updated by adding the successive states s' from the simulator to the collections $B(ho)$ in the nodes. If the maximum planning time has not yet been reached, another start state is sampled and the whole process repeats. When the time runs out, the action a_{best} with the highest value at the current history h_{real} is returned. After this action is executed in the real environment, with an observation o_{last} the tree can be pruned. Only the nodes from history $h_{real}a_{best}o_{last}$ onward stay relevant as all other histories are rendered impossible.

4.2.3 Particle deprivation and particle injection

Particle filter approaches, POMCP included, can fail due to a phenomenon called particle deprivation. Because of the random nature of the process, the belief can sometimes converge towards a state that is far from the environment's true state. Particles that differ from the converged state have a low probability to be selected while sampling (low relative count). Hence, with each iteration, they become scarcer until they are completely erased from the belief. At this point, the agent is sure to be in an erroreous state and cannot recover anymore. Particle injection (also called particle reinvigoration) is a method to counteract this problem by introducing a number of random particles to the belief at each iteration (Kochenderfer et al., 2021). While this reduces the accuracy of the belief, it prevents its complete convergence towards a wrong state.

Particle injection is used to increase the variance of the belief about the driver model state. Only observable information is used. Concretely, particle injection is implemented by adding driver model states with a random number of remaining actions and the same action as the one that was last observed. The number of remaining actions can be lower than the minimum defined in Section 4.1.2 because this limit is only intended for initial sampling and the true remaining number of driver actions in a particular state might be lower after having performed actions already. Like in the original POMCP paper, the amount of transformed particles that are added before each planning step is 1/16 of the number of searches. The particles can be added during policy execution, and therefore, do not influence planning time.

4.2.4 Action selection and preferred actions

4.2.5 Generative model

Chapter 5

Experimental setup

The task for the agent in the experiment is to keep the car centered in a highway lane. Thus, the track used for the agent's evaluation needs to represent such a scenario. Most tracks readily available in the racing car simulator TORCS are race tracks. These are much wider than common roads and the width often differs in different segments. To ensure a realistic scenario, a one-lane track with a continuous width of 3.5m, which is common for European roads, is used. The track covers a wide array of scenarios. It includes long straight segments, both left and right curves, and multiple curves of alternating directions in a row. By ensuring that all common highway scenarios are covered by the track, a single track is sufficient.

The car used for the experiment does not have a big impact on driving performance, as long as it is consistent during all experiments. To ensure that an action's effect at a particular position are consistent, the speed of the car is constant.

The driver is pulled for an action every 0.1 seconds. The simulation tick rate is 0.002 seconds. When the driver is not pulled, its last action is repeated. It follows, that every action is repeated during 50 simulation ticks. The simulation is not in real time. Therefore, the simulation waits for the agent's planning. If the agent is pulled for an action, the environment does not change until the agent's next action is decided and performed.

5.1 Evaluated scenarios

5.1.1 Driver model

5.1.2 Action space and action selection

5.2 Design decisions

5.3 Hyperparameter optimization

There are a number of hyperparameters. Most importantly, there is the planning time. This is the time the agent is allowed to search in and expand its search tree in order to find the most likely current state and best course of action. More planning time can result in a wider and/or deeper tree. The search horizon is limited by a discount threshold. If this threshold is reached, the search is stopped and no more actions will be performed for the current trajectory and, if there is enough planning time left, a new state is sampled from the current belief and a new planning trajectory is expanded. Moreover, there is an exploration constant.

This value, determined before the start of the experiment, assigns actions that have not been tried before more expected reward and thus favors exploration.

5.3.0.1 Number of searches

5.3.0.2 Exploration constant

5.3.0.3 Discount horizon

5.4 Performance metrics

Due to the randomness involved in the driver model, each experiment run will lead to a different scenario. Therefore, to get credible results, the experiment has to be repeated many times. The average discounted return over all experiment runs serves as performance metric. This result is compared with the average reward of an agent that always performs the optimal reaction to the action of the driver model. The closer the POMCP agent's reward is to the optimal agent's reward, the better was the planning.

Chapter 6

Results

6.1 Lower and upper performance bound

The benchmarks for the performance of the agent are the performance of the driver without assistance system as lower bound, and the performance of an agent that always reacts optimally to the driver's actions as upper bound. For both baselines, 50 runs with up to 1000 actions each, if no terminal state is reached earlier, are performed for each of the three driver models.

In the case of the independent driver, no run was completed successfully. At some point in any run the driver becomes distracted and fails to adjust to a change of the course of the road, leading to lane departure. Table 6.1 shows that the more complex the driver model is, the worse is its performance. The simple driver model and the driver model with steering overcorrection lead to a few runs with a relatively high number of successful actions before a lane departure occurs. The reason for this is that the driver only repeats it's last attentive action after becoming distracted. In some cases this means that the distracted driver just continues to steer straight which is less likely to lead to lane departure on the highway track than consecutively steering left or right.

The agent that reacts optimally to the driver's actions has full knowledge about the environment, as well as the driver's next action and is equipped with a continuous optimal steering policy. Therefore, it can easily choose its action by checking which combined action is the closest to the optimal steering. Due to the discretization of the action space, a possible combination that equals the optimal steering is unlikely but the agent performs whatever action leads to the closest result. It finishes every run successfully and leads to average cumulative rewards of 999.3 for all driver models.

	Mean reward	Min #actions	Max #actions
Simple driver	54.39	17	347
Over correction	51.26	17	233
Over correction and noise	31.08	14	74

Table 6.1: Independent driver performance

6.2 Hyperparameter optimization

Hyperparameter optimization is performed for agents with all three action configurations and the driver model with steering overcorrection but without noise. Grid search is used to search for the combination of search horizon and

exploration constant that leads to the highest average cumulative reward after 20 runs with up to 1000 actions each and 1000 searches per planning step.

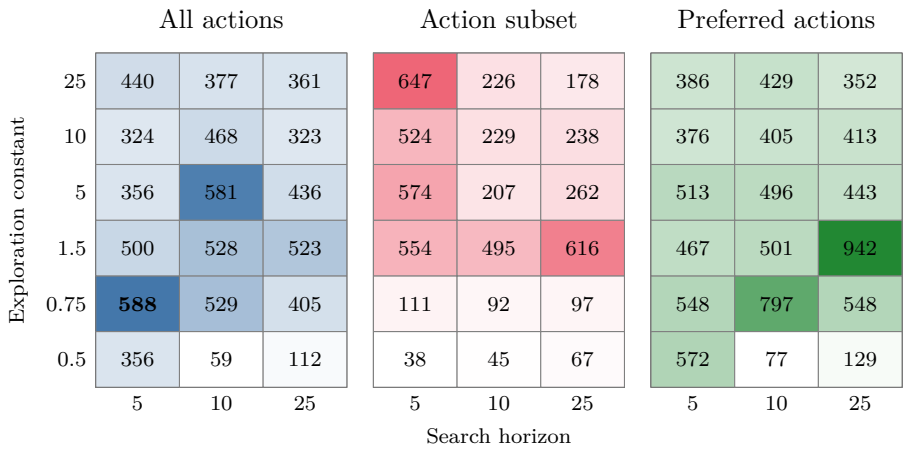


Figure 6.1: Average cumulative rewards for combinations of search horizon and exploration constant for all three action configurations (20 runs with up to 1000 actions each and 1000 searches per planning step; driver model with steering overcorrection but without noise).

For the agent with a full, unweighted action space, two combinations lead to a sufficiently higher average cumulative reward than the others: The combination of a search horizon of 5 actions with an exploration constant of 0.75 leads to a reward of 587.67, and the combination of planning 10 actions ahead with an exploration constant of 5 results in a reward of 581.40. The shallower the search horizon, the less planning time is needed at every planning step as fewer actions need to be simulated. Thus, at the same performance, a lower search horizon is preferable. The combination of a search horizon of 5 actions and an exploration constant of 0.75 is used for the further experiments.

In the case of the agent restricted to using a subset of the action space, the combination of a search horizon of 5 actions and an exploration constant of 25 yields the highest average cumulative reward of 646.83. Only the combination of looking ahead five actions with an exploration constant of 1.5 comes relatively close with a reward of 616.47. As a lower search horizon is preferable, the setup for the further evaluation for this agent is a search horizon of 5 actions with an exploration constant of 25.

The best combination of search horizon and exploration constant for the agent with preferred actions is 25 actions and 1.5 respectively. This combination yields an average cumulative reward of 942.05 which is sufficiently higher than the return of any other combination. Consequently, this is the combination used for this agent in subsequent experiments.

6.3 Reward convergence behavior

The agent's challenge is threefold: First, it must accurately determine where the car is located on the track. Its observations are accurate, however, because of the discretization, multiple actual positions map to the same observation (See Section 4.1.1.3). Second, it needs to estimate whether the driver is attentive or not. Third, it must decide on an appropriate action choice, taking into account future consequences of the chosen action. In order to achieve this, at each planning step, the agent can search ahead by simulating experiences based on its belief of the state of the environment and the driver. Performing more searches means evaluating more possible scenarios. In turn, more evaluated scenarios enable the agent to form a better policy and therefore selecting the better next action. However, after some amount of searches, the information gain from additional searches decreases and performance is expected to converge (Silver & Veness, 2010). The number of searches is directly related to the planning time. Planning time is a limiting factor for the application of a planner. Thus, knowing after how many searches the performance converges, and therefore being able to choose the minimal number of searches to perform for a good result, is critical. Below, the convergence behavior of the three evaluated agents is assessed for the scenarios of the simple driver model, the driver model with steering overcorrection after regaining attentiveness, and the driver with overcorrection and noise.

6.3.1 Simple driver model

For the experiment with the simple driver model, using the full action space, utilizing only the subset of minor steering actions, and employing preferred actions lead to similar convergence behavior. Already with just 200 searches during planning, the average cumulative reward is above 600 for all agents. However, as it can be seen in Table 6.2, the number of runs that lead to a terminal state is high. In the runs resulting in a terminal state, the agents are able to assist the drivers well at first but suffer from belief divergence after some time. Then, their belief deviates noticably from the true states of the environment and the driver. The agents are not able anymore to make accurate assumptions about the state of the environment and whether the driver is attentive or not. Thus, they are rendered unable to decide on the right actions to keep the car centered.

The agent using the full range of actions already causes only four terminal states with 300 searches, whereas the other two agents need more searches to perform well. The best result for all three agents is achieved with 1500 searches. No run results in a terminal state. The the agent with full action space receives an average cumulative reward of 957.83, the agent with a reduced action space yields 981.99, and the agent using preferred actions gains 973.88.

For more searches, the average cumulative rewards are similar for the agents with a subset of actions and preferred actions. In contrast, the agent with a complete action space encounters four terminal states in the trial with 5000 searches and six in the experiment with 7500 seaches. These terminal states are

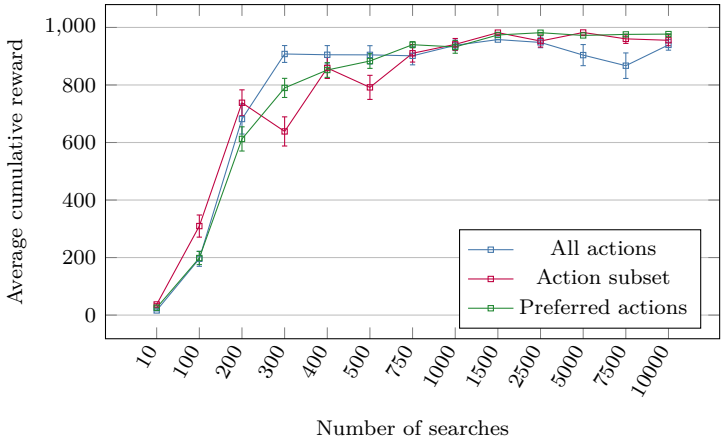


Figure 6.2: Performance comparison of POMCP when utilizing all actions, an action subset, or preferred actions with a simple driver model. Each point shows the mean cumulative reward from 50 runs with 1000 actions each, if no terminal state is reached earlier.

reached early on during the run - executing less than 50 actions before. They are caused by an extreme action of the agent that leads to the opposite of optimal steering behavior. The actions completely overrule the driver's actions who is even concentrated in three of the ten occasions. The reason for choosing these extreme actions is a poor belief that strongly deviates from the true states of the environment and the driver. The agent with preferred actions is less likely to perform extreme actions, and the agent with a subset of minor actions cannot do so.

The agent restricted to use only a subset of minor actions reaches terminal states in two states in the experiments with 2500, 7500, and 10000 searches. These are not caused by belief divergence. In all cases, the car is in a road bend and the driver is distracted, steering into the wrong direction. The agent performs its best possible action by steering as much as possible in the opposite direction than the driver. However, because the agent's range of actions is severely limited, the combination of the agent's and driver's actions is not enough to keep the car in the lane.

Using preferred actions results in only one terminal state for experiments with 1500 searches or more. The reason, like for the terminal states reached by the agent without weighted actions, is belief divergence in combination with an extreme action. Nevertheless, extreme actions are much less likely for the agent with preferred actions. There are multiple occasions where a distracted driver steers into the wrong direction in a road bend and the agent is able to correct the steering by effectively reversing the driver's actions.

	10	100	200	300	400	500	750	1000	1500	2500	5000	7500	10000
All	50	50	22	4	4	3	3	1	0	1	4	6	1
Subset	50	48	27	26	11	18	6	5	0	2	0	2	2
Preferred	50	50	32	13	8	4	1	3	0	0	1	0	0

Table 6.2: Number of terminal runs by the number of performed searches at each planning step in the experiment with a simple driver model for agents with all three action space types.

6.3.2 Steering over-correction

A driver that overcorrects after regaining attentiveness by steering too strongly in her first attentive action presents a greater challenge for the agents. When choosing its next action, an agent also need to account for the driver's possible overcorrection. The amount of overcorrection is stochastic (see Section 4.1.2). Attentive drivers are less predictable when they overcorrect. Rather than just performing the optimal steering action, different overcorrection intensities can lead to a variety of actions at the same position. The complexity of the planning problem is higher. Generally, more searches are needed in order to evaluate a greater variety of possible states from the belief while planning.

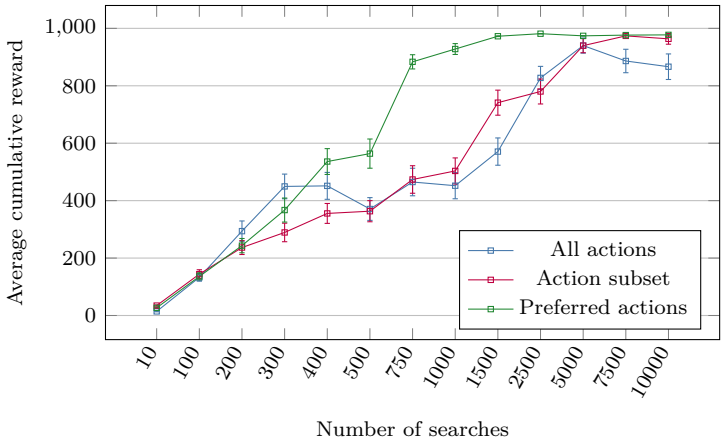


Figure 6.3: Performance comparison of POMCP when utilizing all actions, an action subset, or preferred actions with a driver model that over-corrects when it regains attention. Each point shows the mean cumulative reward from 50 runs with 1000 actions each, if no terminal state is reached earlier.

As it can be seen in Figure 6.3, the agent using preferred actions converges sufficiently earlier towards a good policy than the other two agents. This is mainly caused by a high number of terminal states reached during evaluation runs of the agents with full and reduced action spaces (see Table 6.3). For example, with 750 searches, the agent using preferred actions receives an average

	10	100	200	300	400	500	750	1000	1500	2500	5000	7500	10000
All	50	50	49	45	42	46	42	41	35	13	4	5	6
Subset	50	50	49	49	48	49	42	39	23	17	3	1	1
Preferred	50	50	49	42	35	29	10	3	0	0	0	0	0

Table 6.3: Number of terminal runs by the number of performed searches at each planning step in the experiment with a driver model with steering over correction for agents with all three action space types.

cumulative reward of 883.43 with only 10 runs ending in a terminal state, while the other agents each reach terminal states in 42 runs. The agent with an unrestricted action space yields a return of 464.89, and the one with a restricted action space gains 473.67. In contrast to the experiment with the simple driver model, the terminal states are caused almost exclusively by particle deprivation after a formerly distracted driver becomes attentive again and overcorrects. If the agent did not account for this possibility properly, no matching node for the observation resulting from the overcorrection can be found in the agent's planning tree. In this case, the agent cannot recover and has to fall back to uniformly random action selection (see Section 4.2.4), which usually leads to a terminal state after just a few actions.

The agent with preferred actions is able to form a more accurate belief of the environment's true state with fewer searches than the other two agents. Due to the use of preferred actions, the agent does not start with equal initial values at new nodes during rollouts. Instead, the actions are weighted with domain knowledge (see Section 4.2.4). The likelihood of selecting an action for a rollout is bound to its intensity, with less severe steering actions being preferred as the need for strong steering is scarce on a highway track. Assuming the underlying assumption of the introduced domain knowledge is valid, and the return is confirmed to be higher for a preferred action during initial searches, then exploration is kept to a minimum. If the reward does not drop sufficiently, the agent is allowed to exploit on the preferred action. It is like lowering the threshold of trustworthyness for preferred actions; they need less initial confirmation than others. Thereby, a preferred action, if successful initially, is evaluated relatively often, even with fewer searches. At each evaluation, the simulated next state will be added to the belief of the node representing the chosen action and a resulting observation. Consequently, nodes connected with the preferred actions hold a more comprehensive belief. This results in a lower chance of particle deprivation. The advantage of the agent using preferred actions becomes clearer than before during this experiment.

6.3.3 Steering over-correction and noise

The noise added to the driver's actions is added with the goal to make the driving more realistic, and thereby also more unpredictable and difficult to plan with. However, the performance of the agents in the experiment with over correction

and noise is very similar to the experiment with overcorrection but without noise. Surprisingly, all agents even receive slightly higher average cumulative rewards and show a similar convergence behavior. The agent using preferred actions converges to a reward of roughly 960 with 1000 searches or more. The agent with a full action range reaches peak performance at 5000 searches with a return of about 860, staying at roughly the same level subsequently. The agent with the small action space converges at around 960, with 7500 searches and more. The actual convergence probably occurs with somewhere between 5000 and 7500 searches but no experiment was conducted with a number of searches in between.

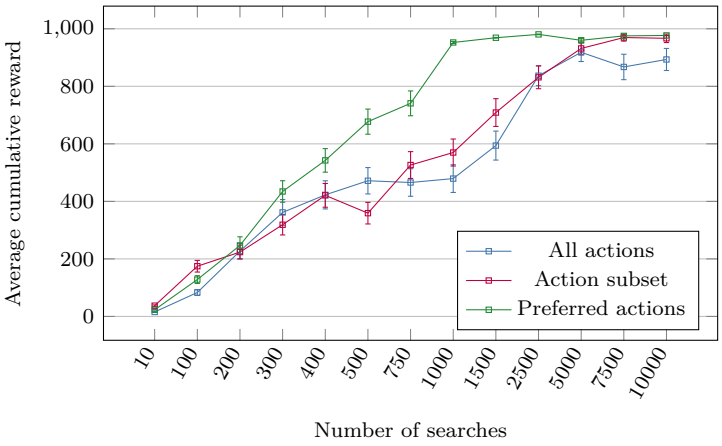


Figure 6.4: Performance comparison of POMCP when utilizing all actions, an action subset, or preferred actions with a driver model that over-corrects when it regains attention and performs noisy actions. Each point shows the mean cumulative reward from 50 runs with 1000 actions each, if no terminal state is reached earlier.

	10	100	200	300	400	500	750	1000	1500	2500	5000	7500	10000
All	50	50	50	44	42	40	40	40	30	15	4	6	5
Subset	50	50	49	47	46	47	37	35	25	14	5	1	1
Preferred	50	50	48	43	37	26	20	1	0	0	3	0	0

Table 6.4: Number of terminal runs by the number of performed searches at each planning step in the experiment with a driver model with steering over correction and noise for agents with all three action space types.

Table 6.4 shows that the number of terminal states reached during experiment runs are comparable for the two agents with unweighted actions. For the agent using preferred actions, the number of terminal states reached at 750 searches is twice as high. In most of these cases, the cause for this is a combination of a

strong overcorrection with a high driver action noise. This combination is unlikely but possible with the random nature of the process. Despite this deviation, the data is very similar to the experiment without noise. The most likely reason for this is the discretization applied to the driver's actions (see Section 4.2.1). Low noise that is added to an action is often lost during discretization. Resulting is the same action as it would have been before adding the noise.

6.4 Mean lane centeredness

The reward reflects how well the agents manage to keep the car centered in lane and angled to the road trajectory. From the last section, it is clear that the number of simulations has a strong impact on the agents' performance. However, the last section only showcased this based on the average cumulative rewards. Figure 6.5 shows the average absolute lane centeredness (distance to the lane center, no matter if left or right of lane center) at every time step in the last experiment with driver action noise and a driver that oversteers after regaining attentiveness. The runs ending in terminal states are taken into account until the terminal state occurs. For the remaining actions, they are ignored in the graphs. It is therefore important to consider them (see Table 6.4).

With just 200 searches, most runs end in terminal states. What is striking is that the average lane centeredness is quite volatile and often drifts off into the extreme. Neither of the three agents is consistently capable of keeping the car centered in the lane. The graph for 500 searches already suggests an improvement. There are less extreme values and the standard errors are lower. The agents with unweighted actions appear to perform better than the one with preferred actions at first glance. However, still almost all runs of the two agents with unweighted actions, and only about half of the runs of the agent with preferred actions lead to terminal states. The performance of the agent with preferred actions is arguably better. Using 1000 searches marks the start of convergence for the agent with preferred actions. The average lane centeredness is consistently lower than with 500 searches throughout the experiment. The agent using a reduced set of actions performs better than with 500 searches, leading to less variation in the lane centeredness and fewer terminal states reached during the experiment. The lane centering of the agent with the full action range is virtually unchanged. When 10000 searches are performed during planning, all agents have converged to their peak performance. The different agents lead to similar results in this case. The agent with preferred actions appears to have a higher variance in its lane centeredness; sometimes, it is comparatively high, followed by periods where the agent achieves lower average values than the other agents. A possible explanation is that this agent accounts better for the times it has to purposefully deviate from the center of the lane, in order to achieve better values subsequently. However, this cannot definitely be concluded.

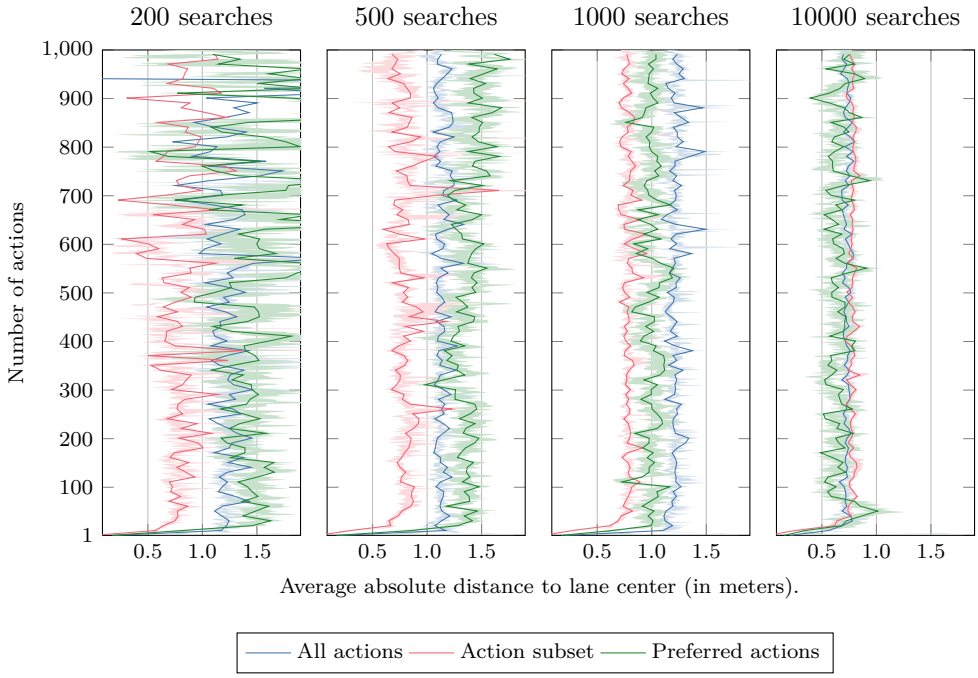


Figure 6.5: Mean lane centeredness for the different agents during experiments with 1000 actions, with oversteering and driver action noise, using 200, 500, 1000, or 10000 searches while planning. In principle, a lower distance is favorable. Nevetheless, it can be better to deviate from the lane center at times in order to reach an overall better performance. The shaded area shows the standard error. *Note: This figure is intended to showcase the difference in lane centeredness between experiments with different numbers of searches for a particular agent. It is not suited to compare the agents with each other without taking into account the different number of terminal runs during the experiments (see Table 6.4). The two agents without preferred actions appear to have lower mean distances than the agent using preferred actions in the first graphs. However, many of their runs end in terminal states. After the terminal state is reached, these runs do not produce additional data and are therefore not reflected anymore in these graphs.*

Chapter 7

Discussion

7.1 Analysis of the results

7.2 Limitations

7.2.1 Driver does not learn or adapt

7.2.2 Long planning time

7.2.3 Action and observation space discretization

7.2.4 Dependency on reliable driver and environment models

7.2.5 Edge cases

Chapter 8

Conclusion and future outlook

8.1 Conclusion

8.2 Road toward application with human drivers

8.2.1 Performance optimization

8.2.2 Integrating realistic driver and environment models

8.2.3 Continuous action and observation space

8.2.4 Using other POMDP solvers

Appendices

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