FACULDADE DE
CIÊNCIAS E TECNOLOGIA
DEPARTAMENTO DE INFORMÁTICA

Foundations of Shared Memory

lectures 09 & 10 (2025-04-07)

Master in Computer Science and Engineering

— Concurrency and Parallelism / 2024-25 —

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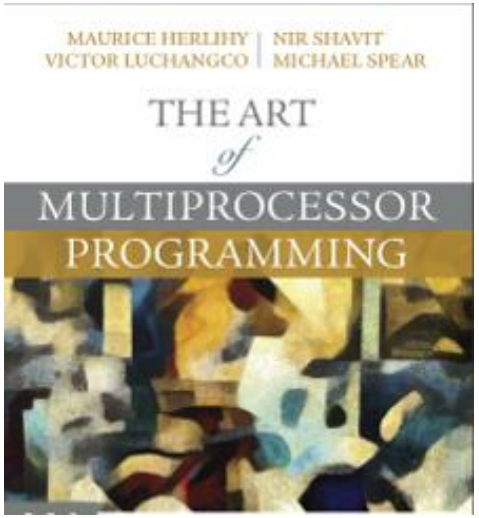
Based in the companion slides from "The Art of Multiprocessor Programming"

Outline

- Concurrent Objects
  - Correctness
  - Sequential Objects
  - Quiescent and Sequential Consistency
  - Linearizability
  - Progress Conditions



  - Bibliography:
    - **Chapters 4** of book

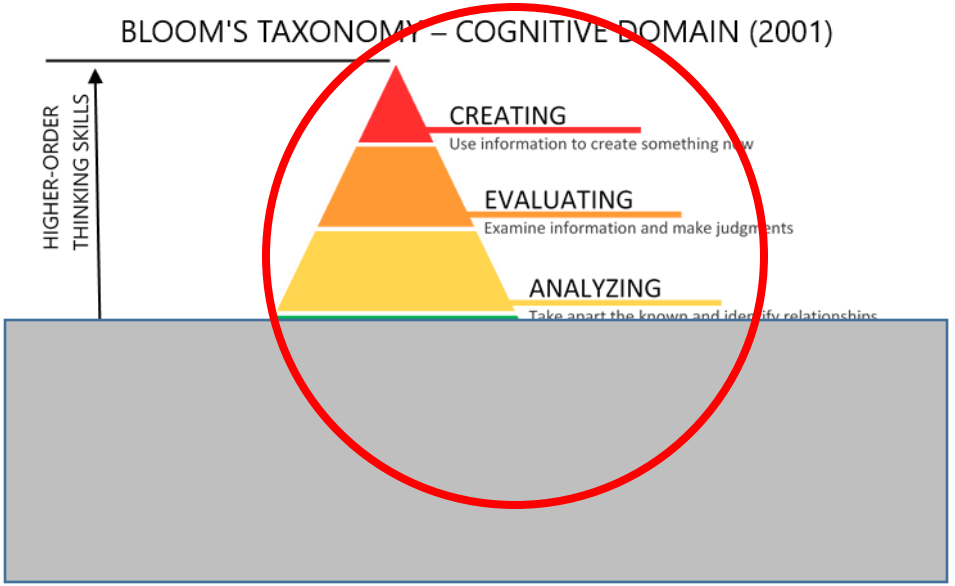
      Herlihy M., Shavit N., Luchangco V., Spear M.;
      The Art of Multiprocessor Programming;
      Morgan Kaufmann (2020); ISBN: 978-0-12-415950-1

Last Lecture

- Defined concurrent objects using linearizability and sequential consistency
- Fact: implemented linearizable objects (two thread FIFO Queue) in read-write memory without mutual exclusion
- Fact: hardware does not provide linearizable read-write memory

Fundamentals

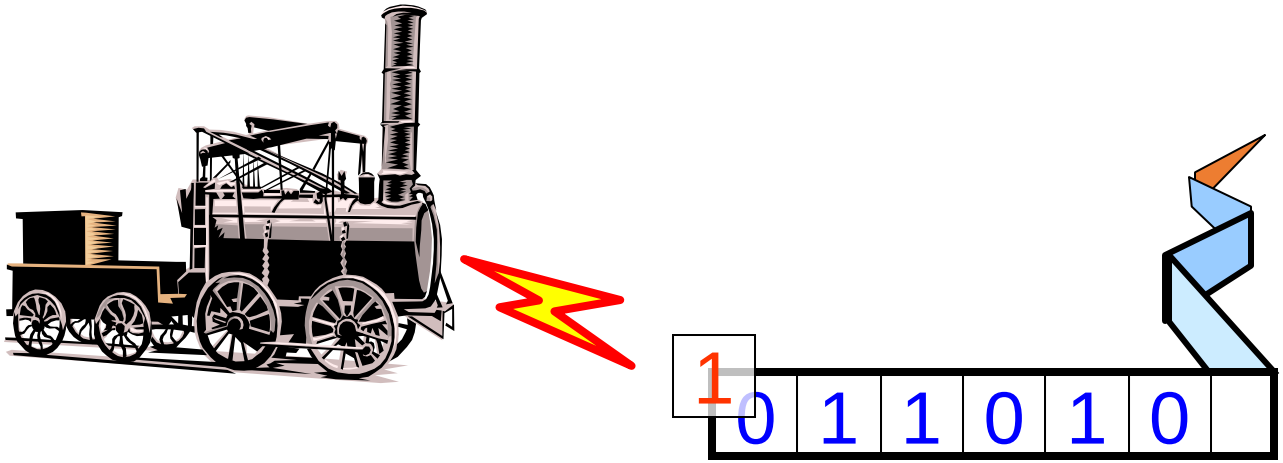
- What is the weakest form of communication that supports mutual exclusion?
- What is the weakest shared object that allows shared-memory computation?



Alan Turing

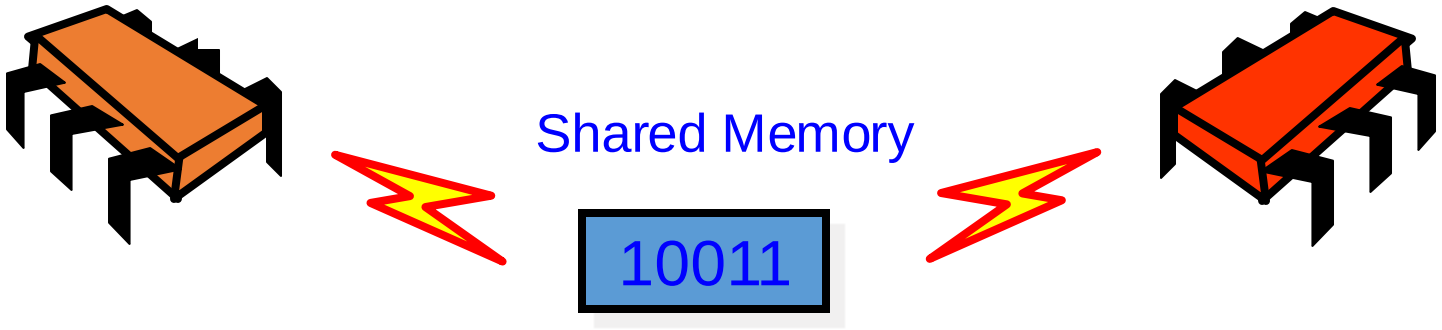
- Showed what is and is not computable on a sequential machine
- Still best model there is

Turing Computability



- Mathematical model of computation
- What is (and is not) computable
- Efficiency (mostly) irrelevant

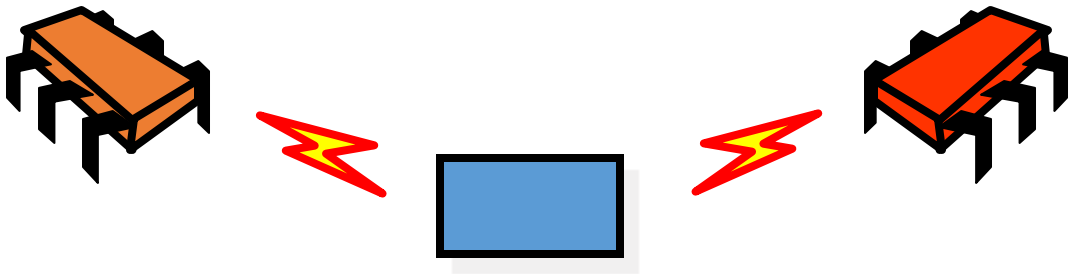
Shared-Memory Computability?



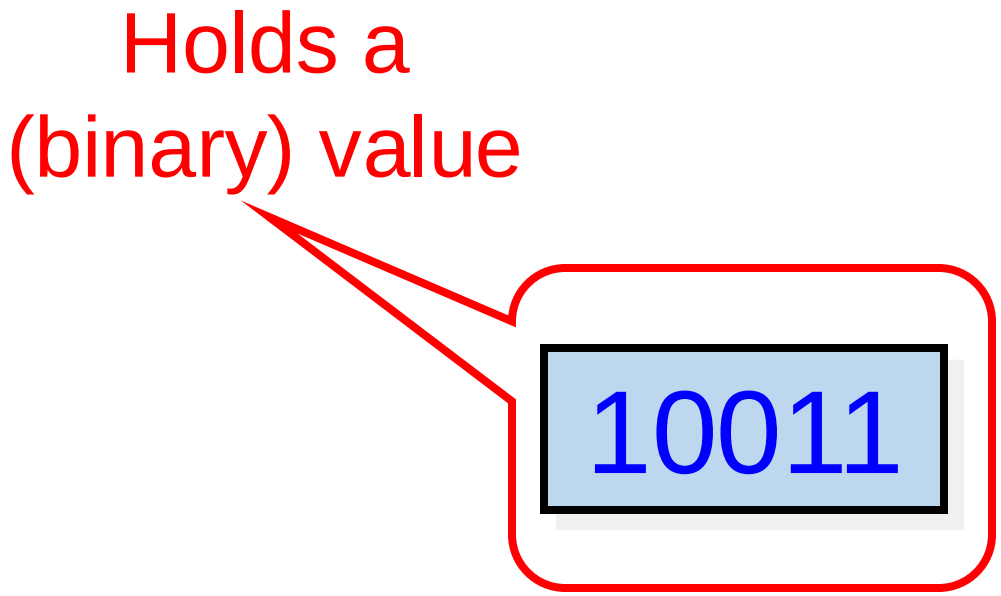
- Mathematical model of concurrent computation
- What is (and is not) concurrently computable
- Efficiency (mostly) irrelevant

Foundations of Shared Memory

- To understand modern multiprocessors, we need to ask some basic questions …
  - What is the weakest useful form of shared memory?
  - What can it do? What can we do with it?

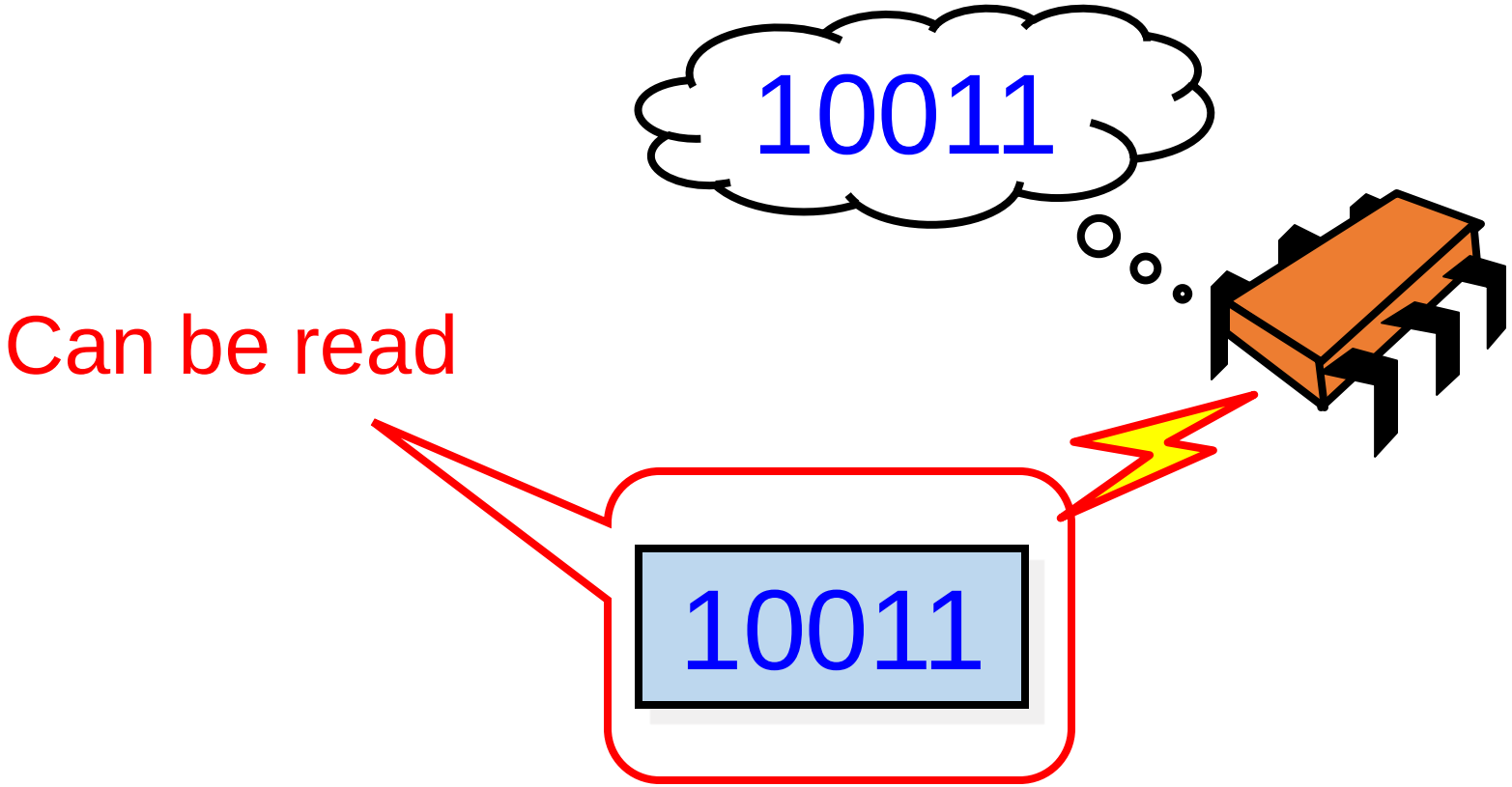


Register*

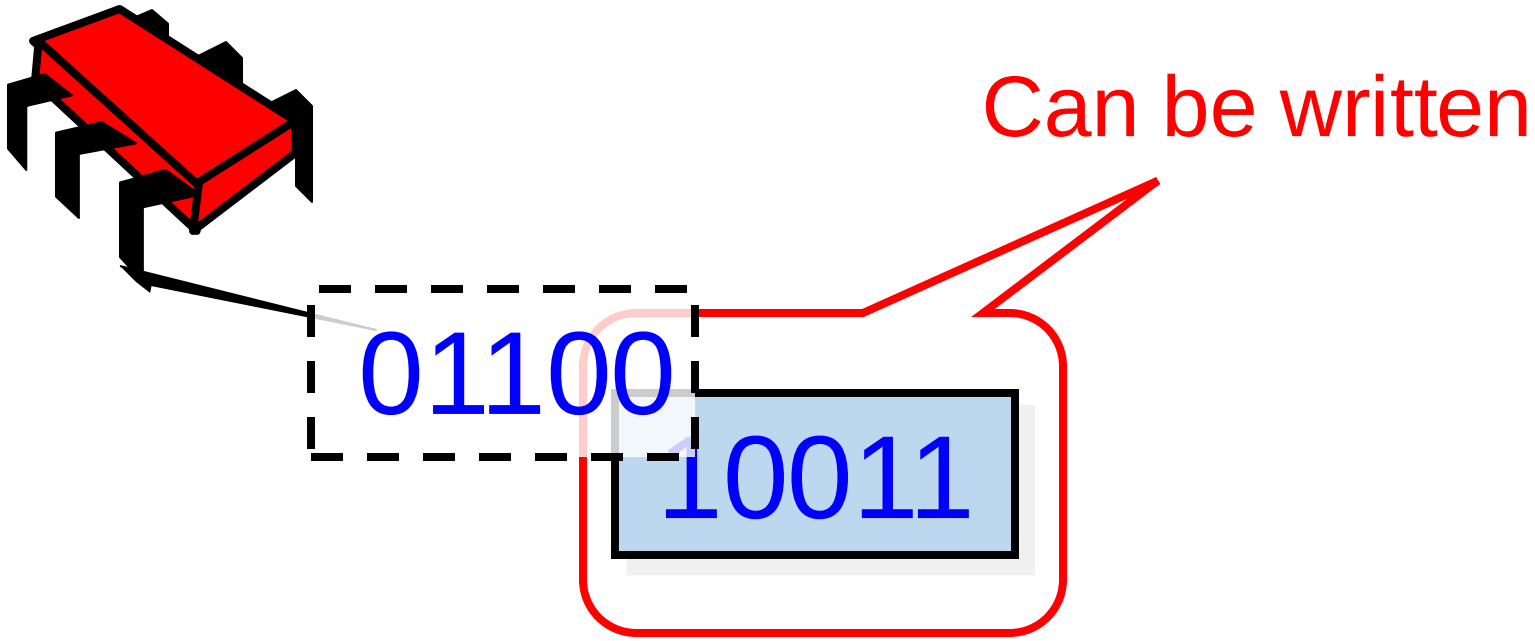


*** A memory location: name is historical**

Register



Register

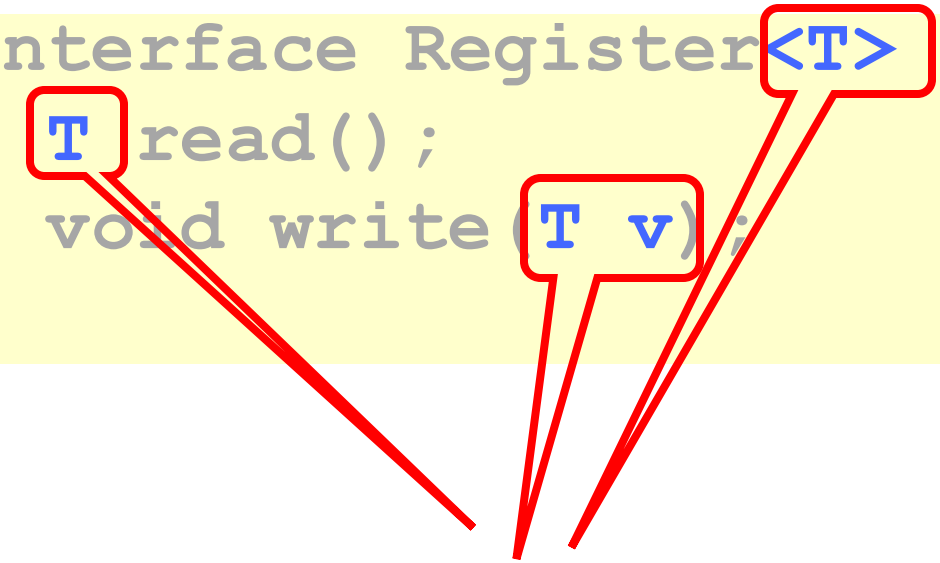


Registers

```
public interface Register<T> {
   public T read();
   public void write(T v);
}
```

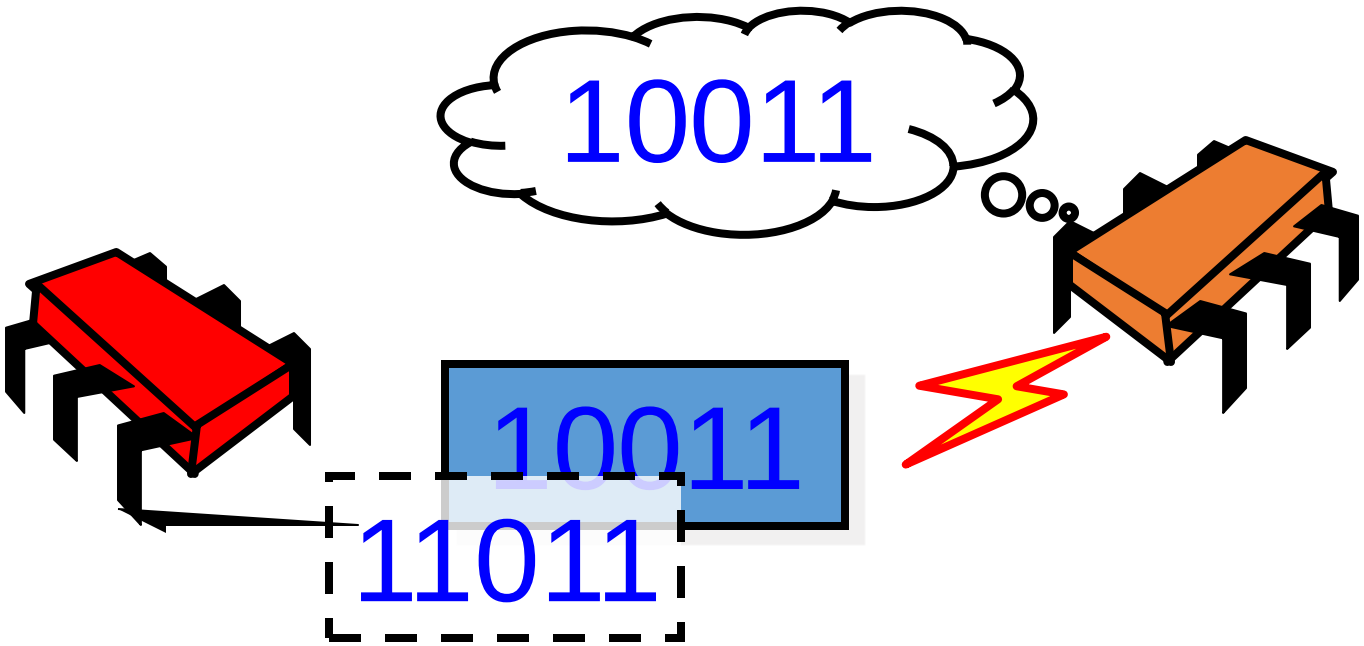
Registers

```
public interface Register<T> {
  public T read();
  public void write(T v);
}
```

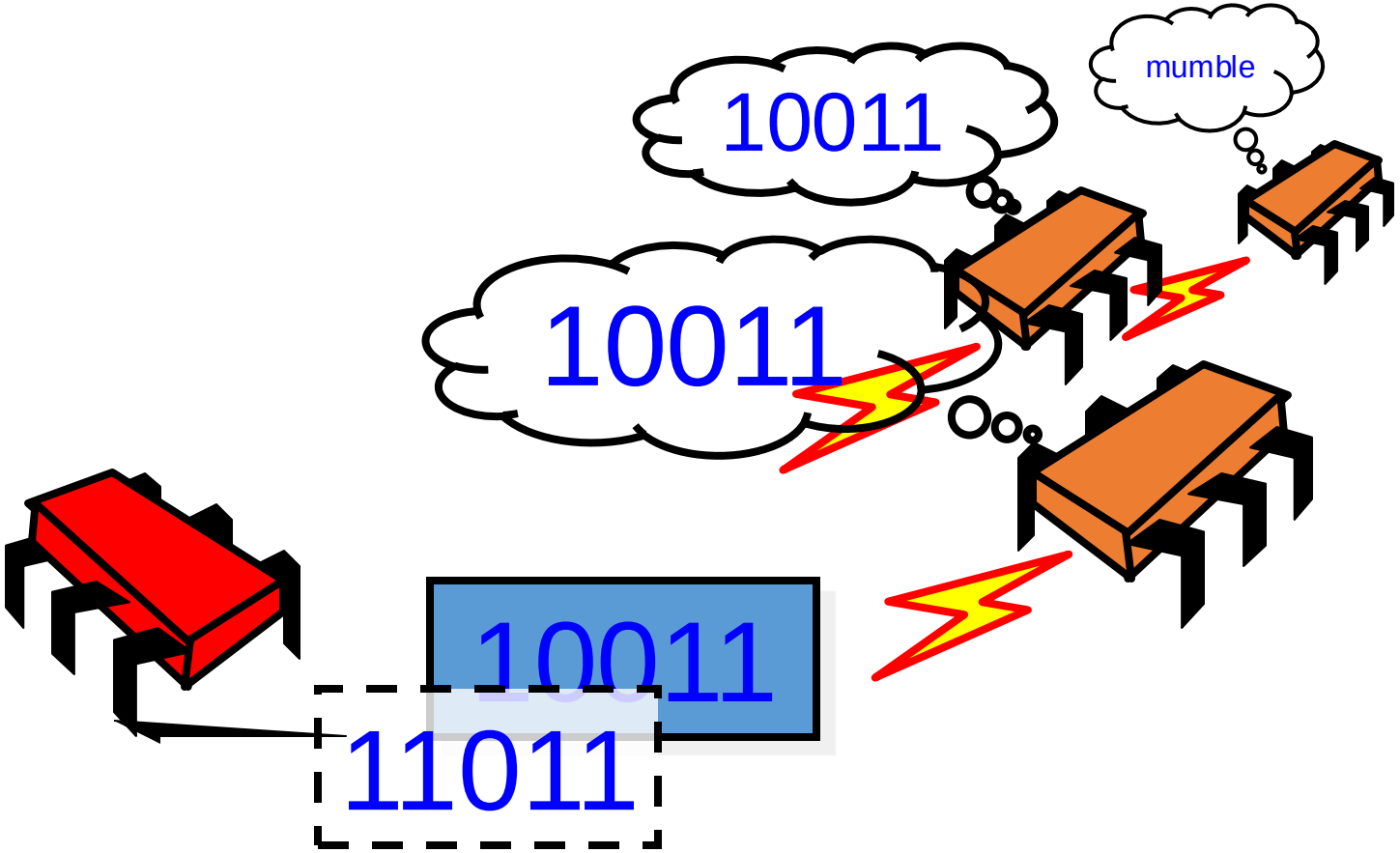


Type of register
(usually Boolean or *m*-bit Integer)

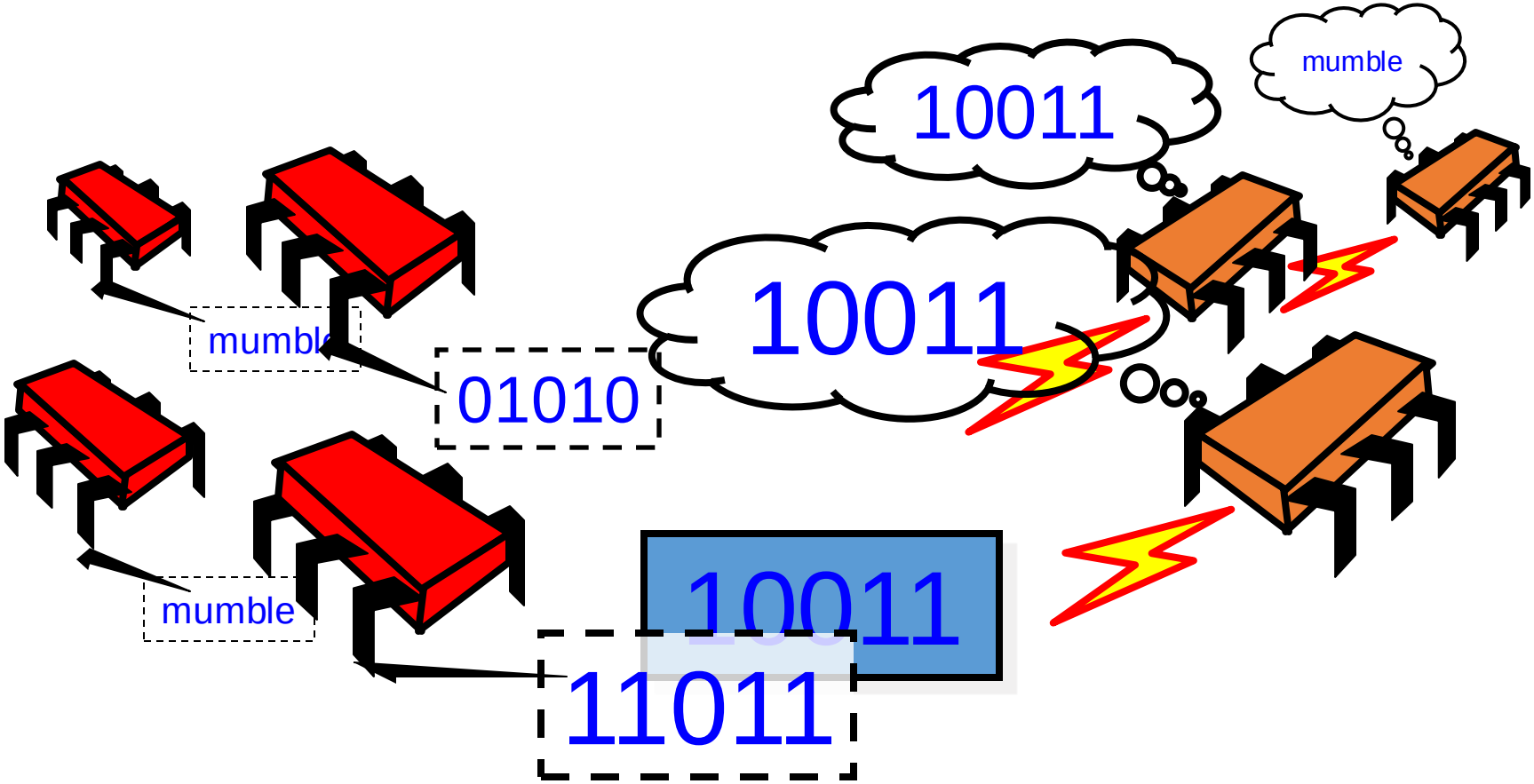
Single-Reader/Single-Writer Register (SRSW)



Multi-Reader /Single-Writer Register (MRSW)

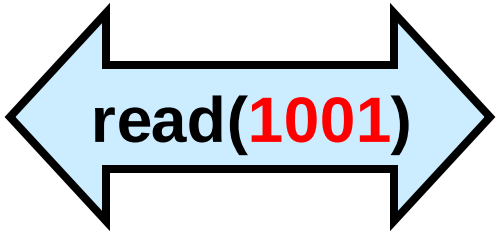


Multi-Reader/Multi-Writer Register (MRMW)



Safe Register

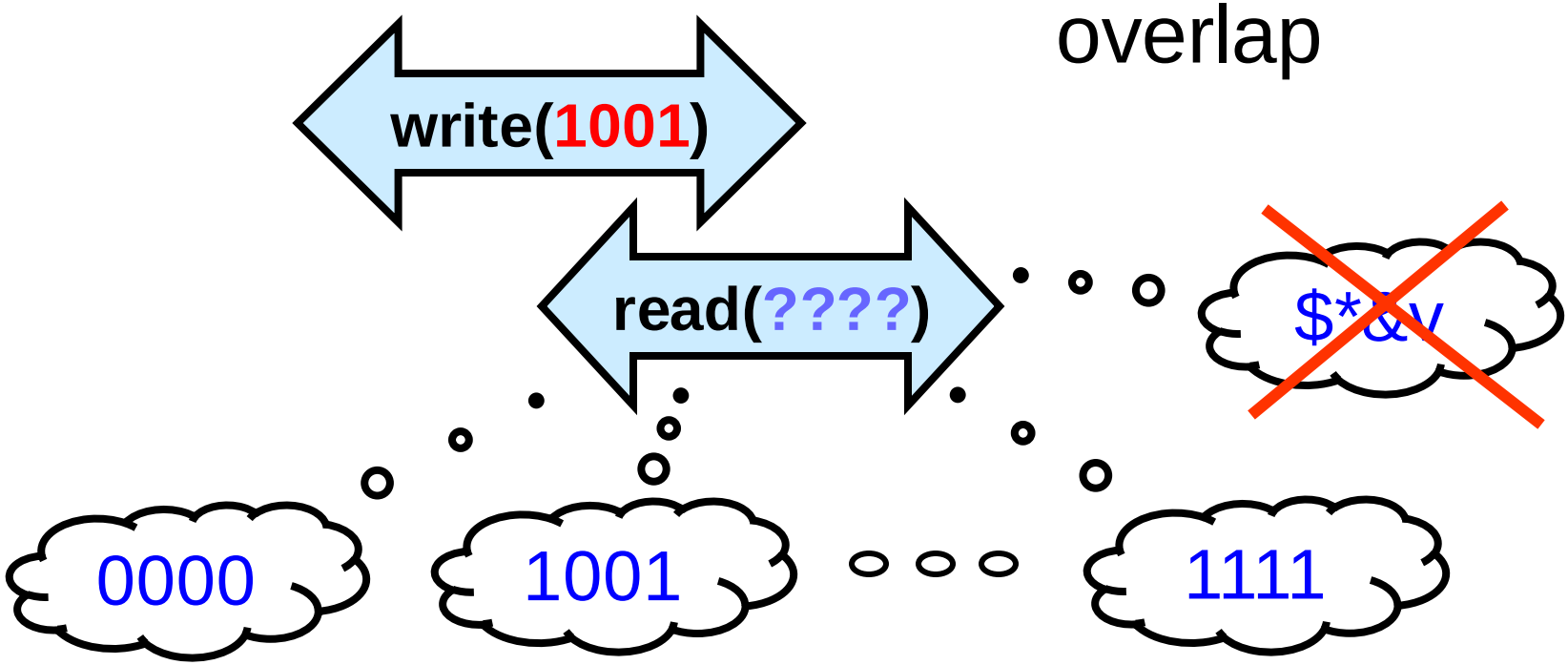
OK if reads and writes don't overlap



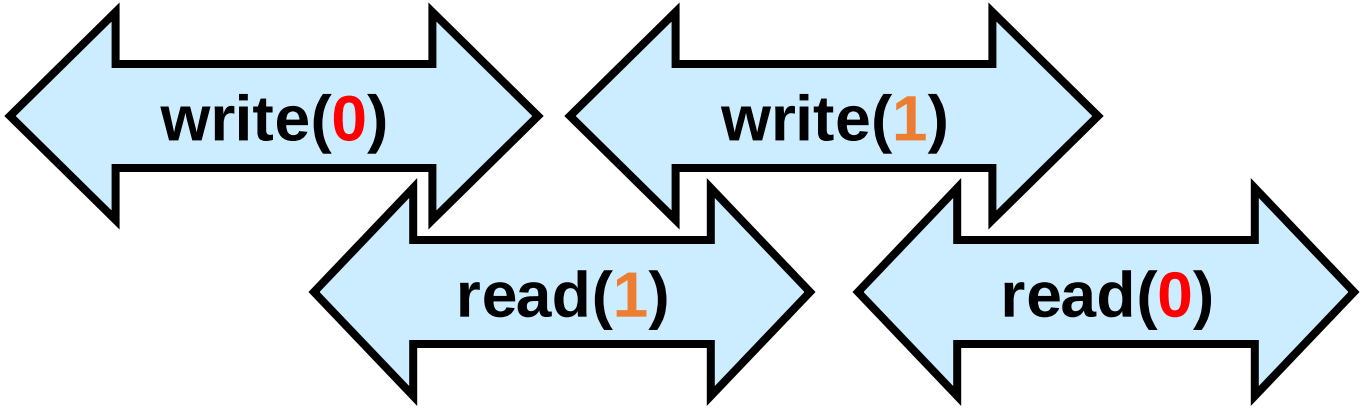
Safe Register

valid ≠ correct/meaningful

Some valid value if reads and writes do overlap

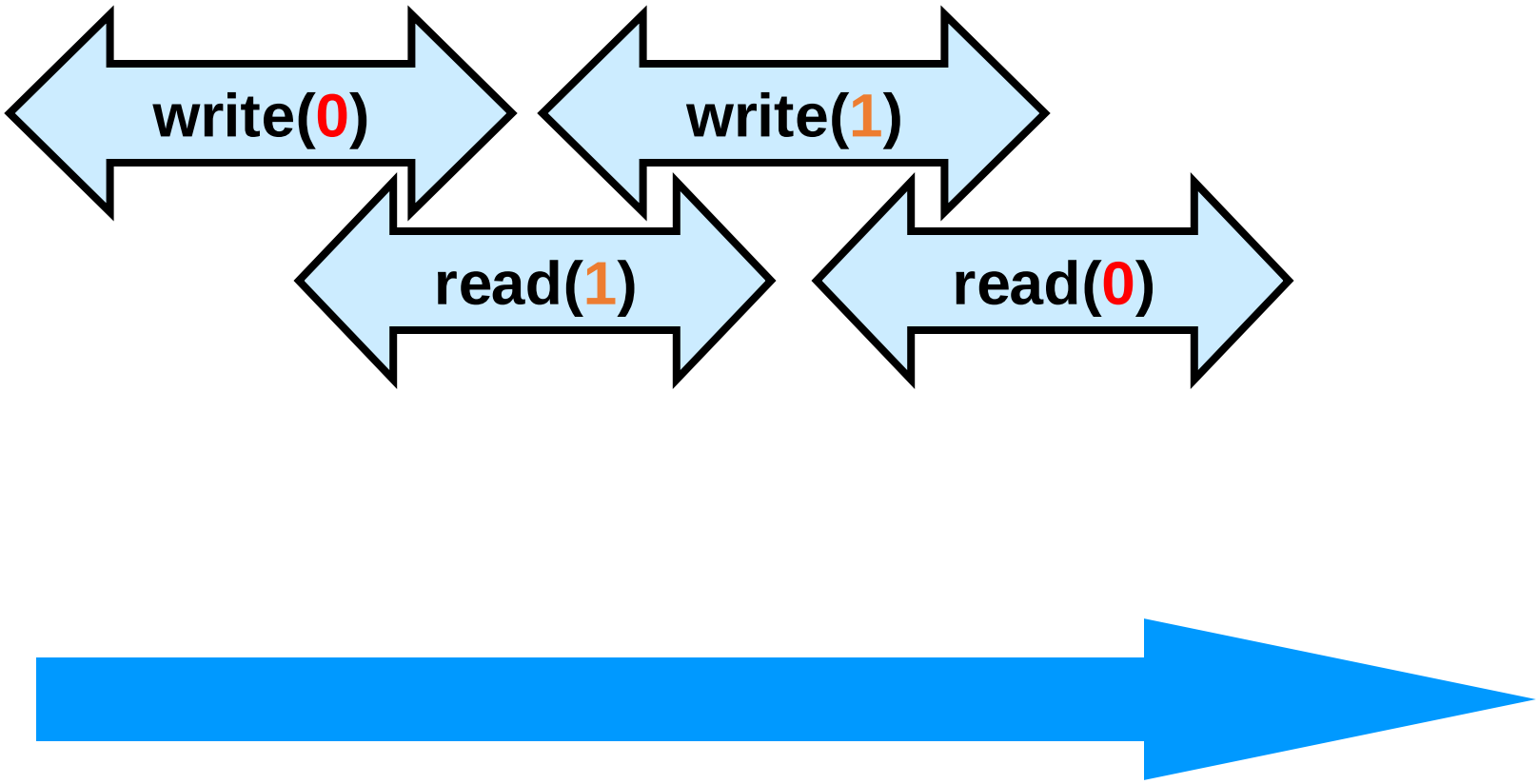


Regular Register

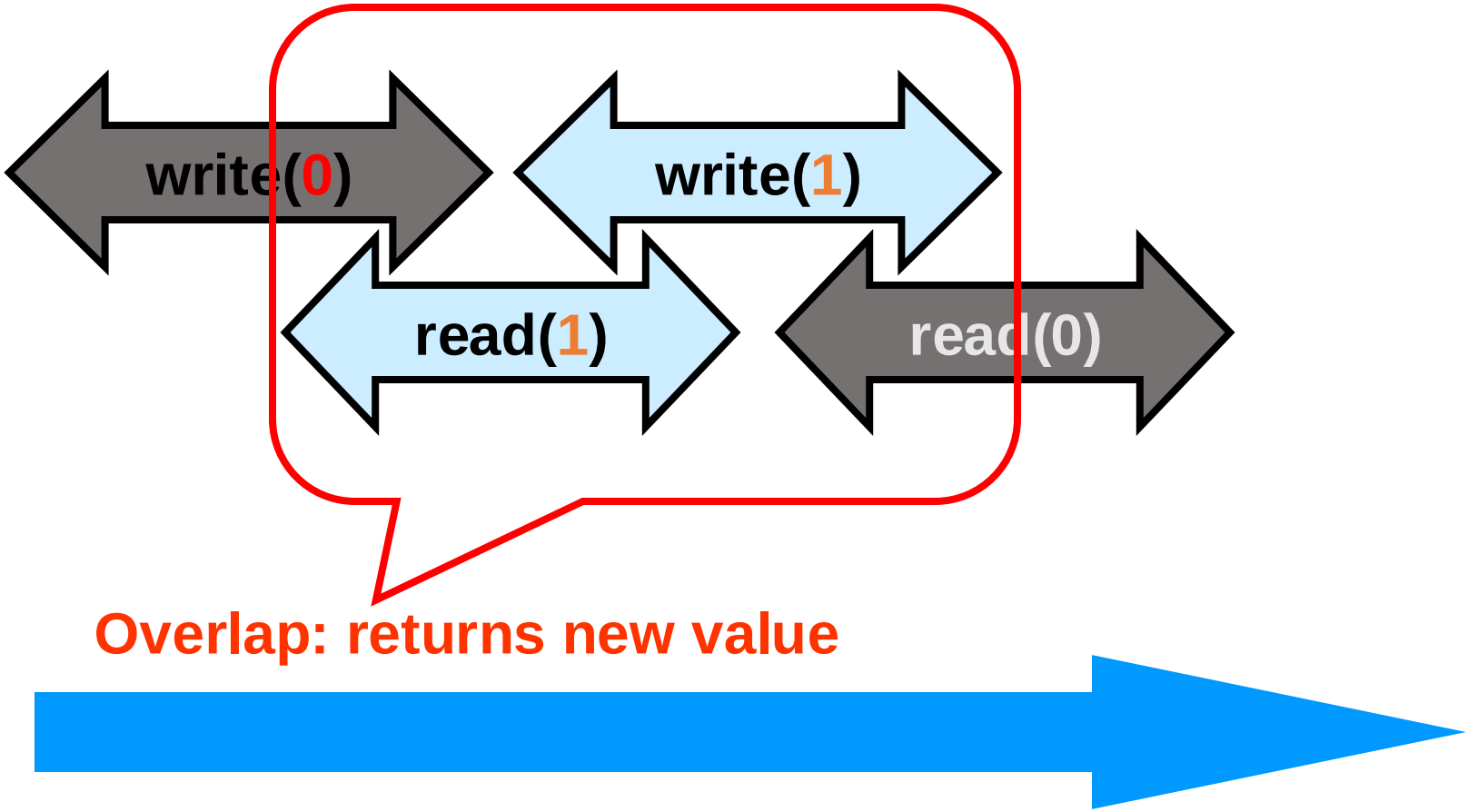


- Single Writer
- Readers return:
  - Old value if no overlap (safe)
  - Old or one of new values if overlap

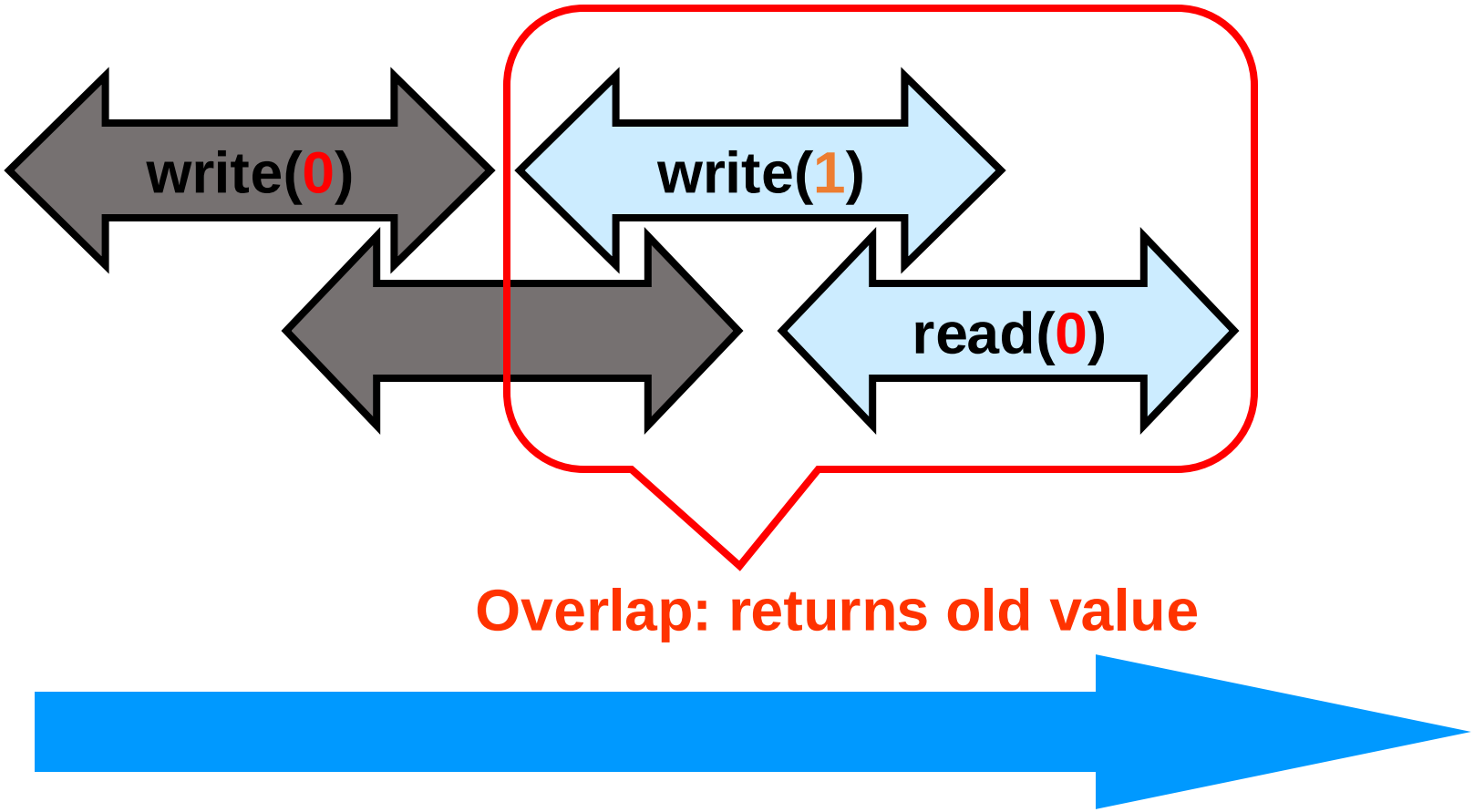
Regular or Not?



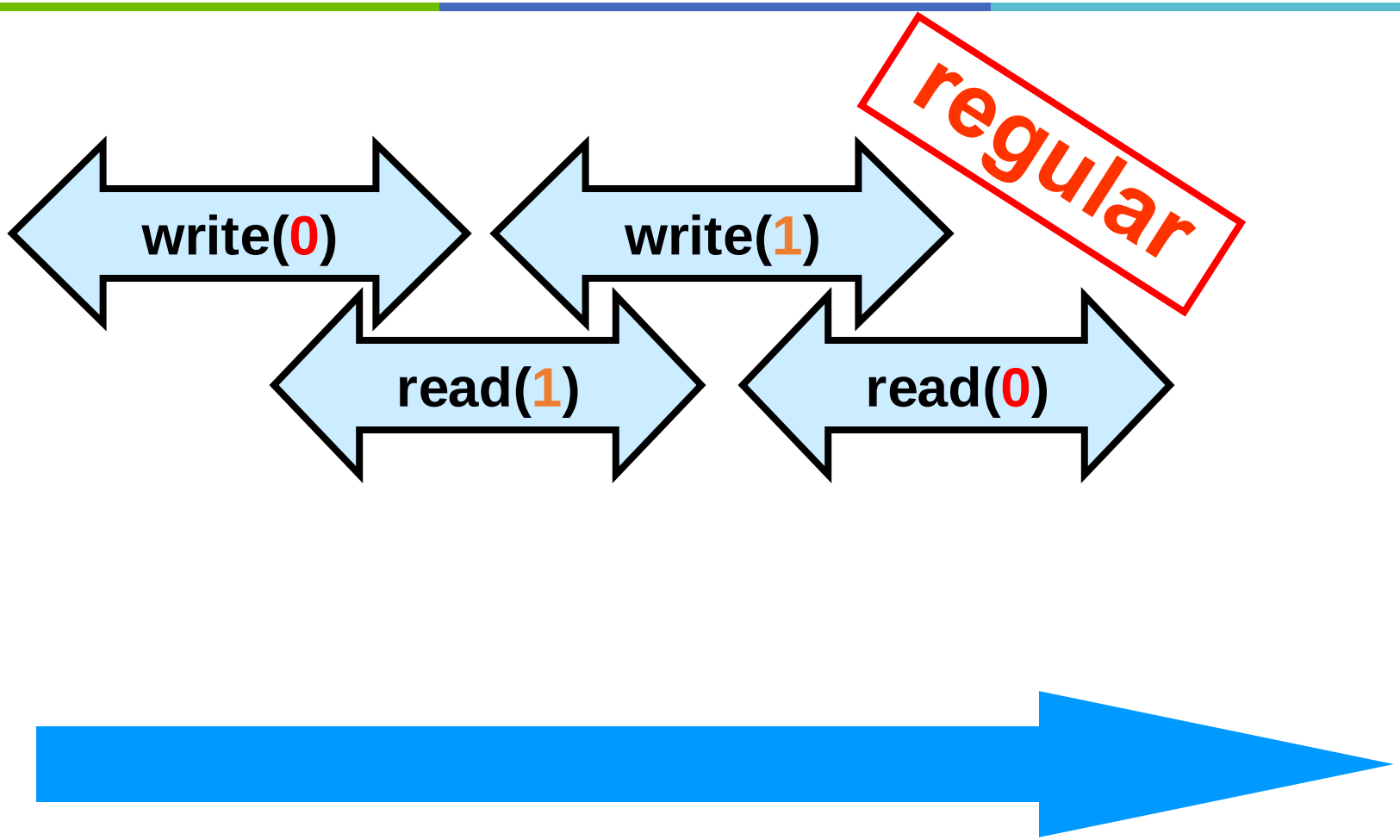
Regular or Not?



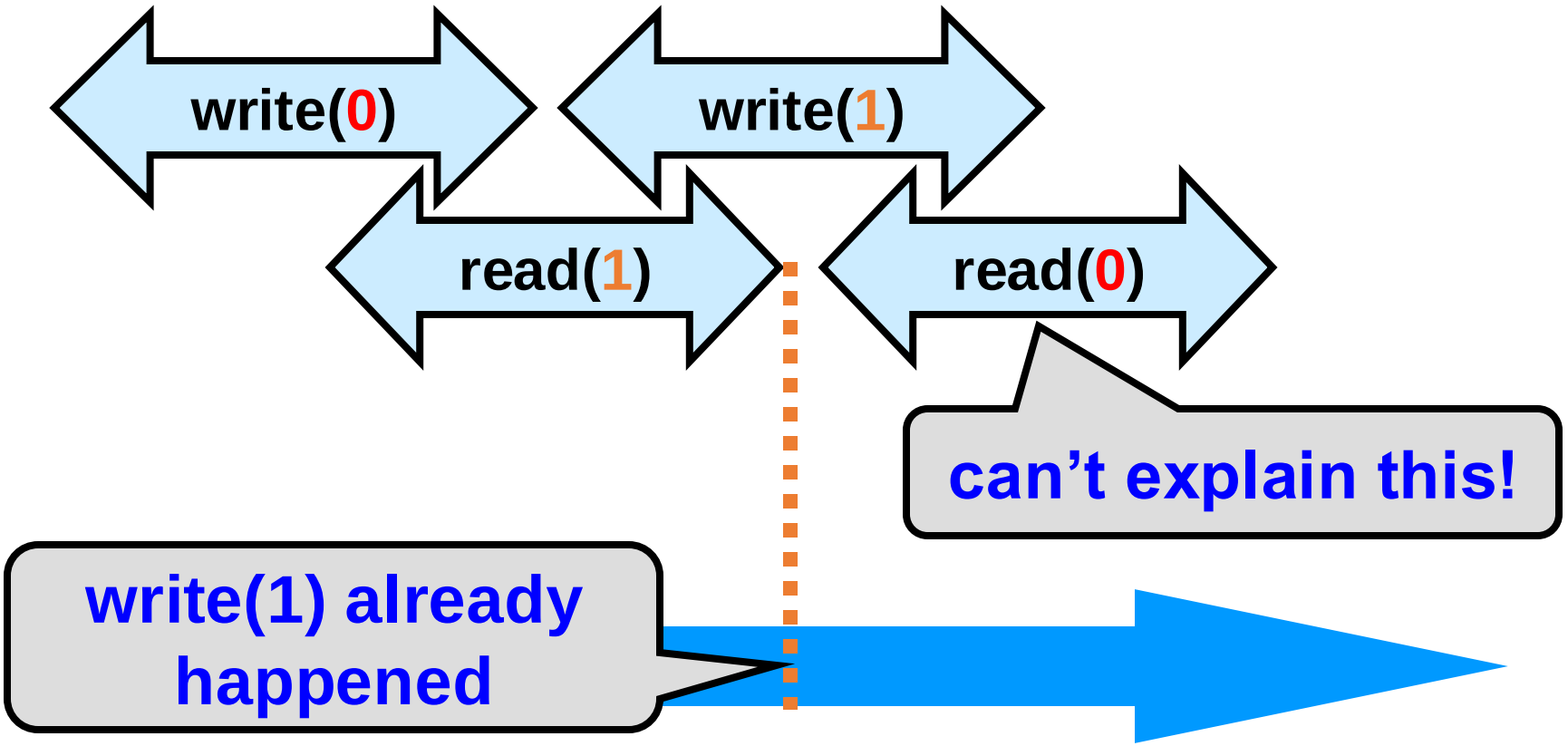
Regular or Not?



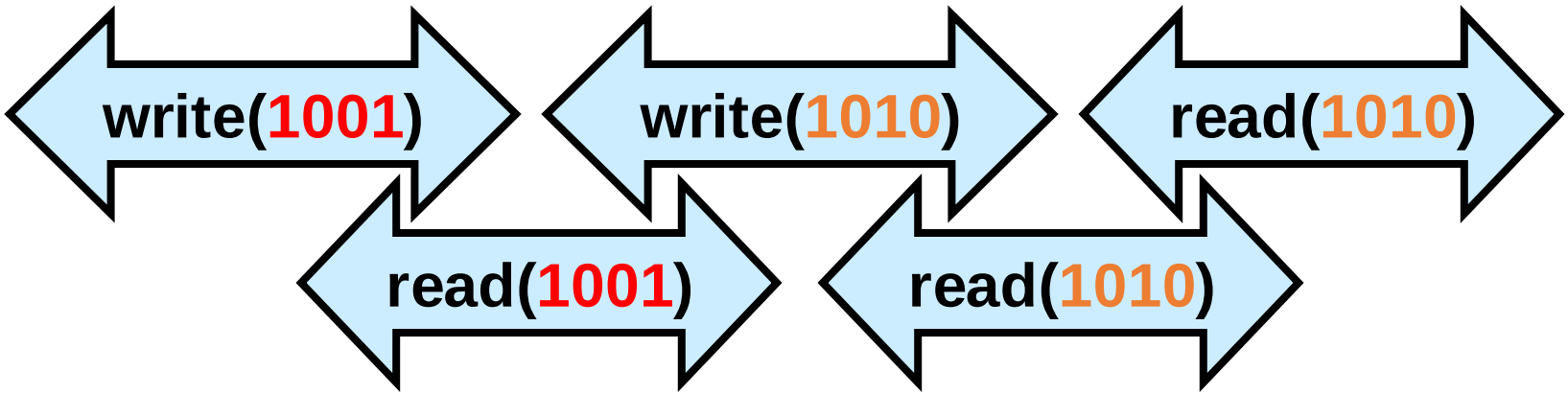
Regular or Not?



Regular ≠ Linearizable

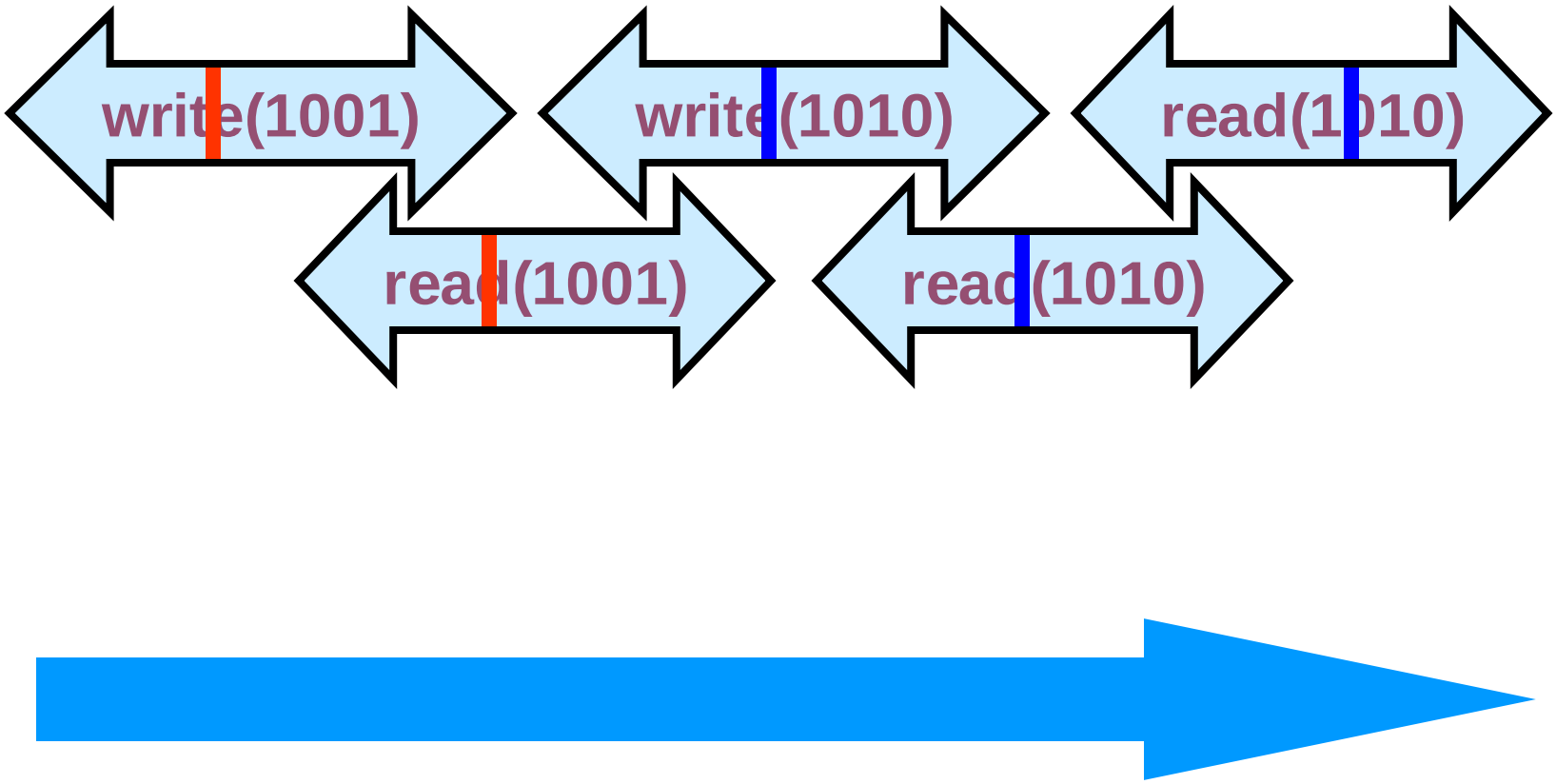


Atomic Register

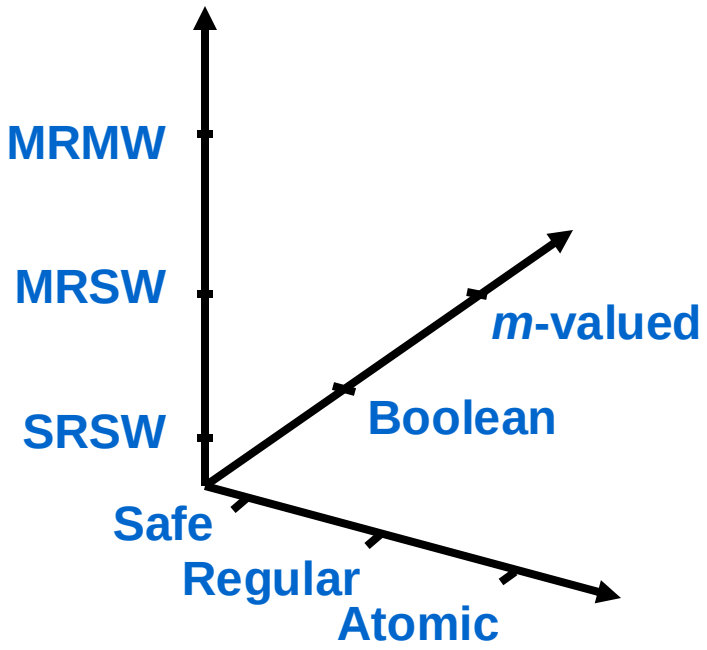


Linearizable to sequential safe register

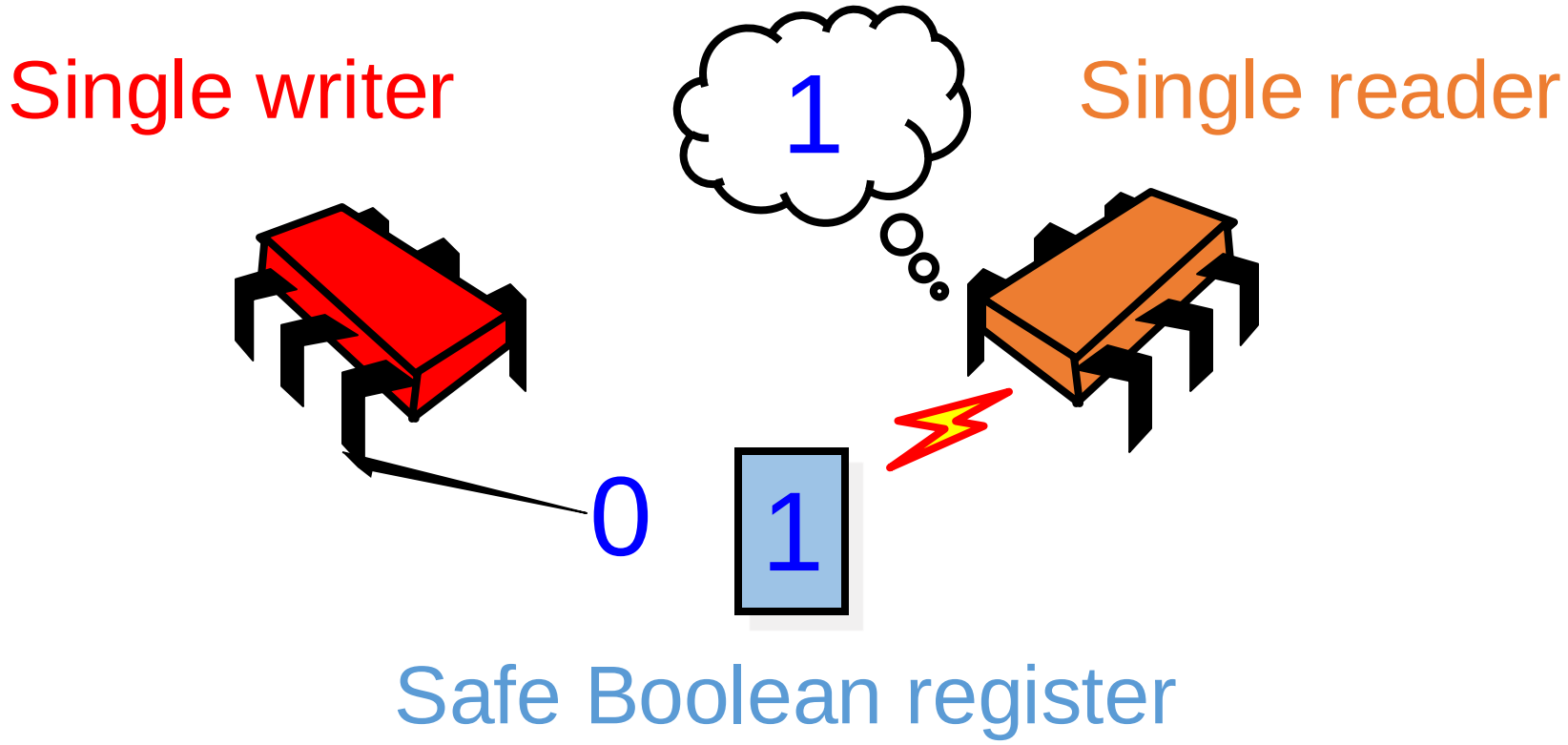
Atomic Register



Register Space



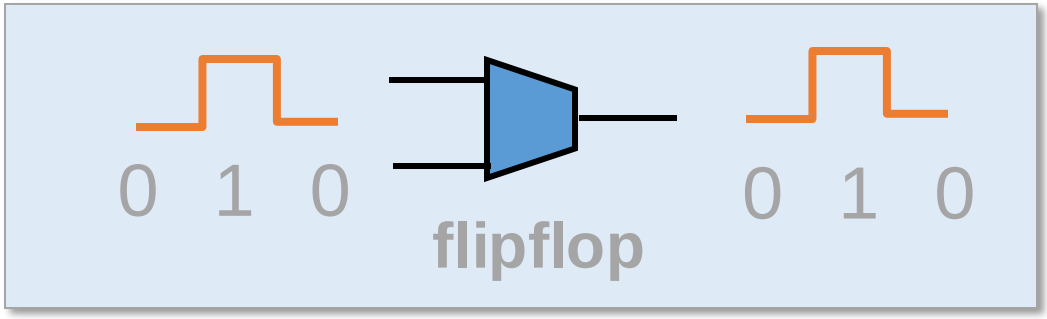
Weakest Register



Weakest Register

Single writer

Single reader



Get correct reading
if not during state transition

Results

- From SRSW safe Boolean register
  - All the other registers
  - Mutual exclusion
- But not everything!
  - Consensus hierarchy

Foundations of the field

The really cool stuff …

Locking within Registers

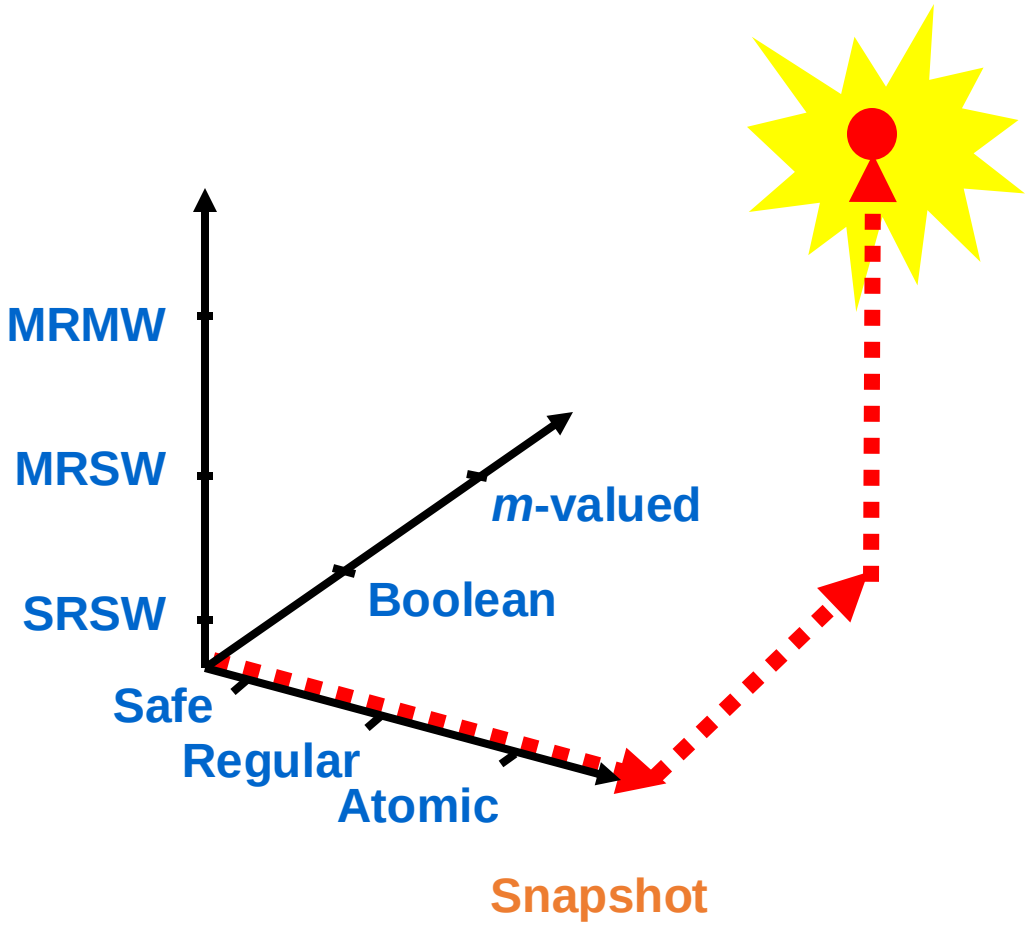
- Not interesting to rely on mutual exclusion in register constructions
- We want to use registers to implement mutual exclusion!
- It's cheating to use mutual exclusion to implement itself!

Definition

An object implementation is ***wait-free***
if every method call completes
in a finite number of steps

- No mutual exclusion
  - Thread could halt in critical section
  - Build mutual exclusion from registers

From Safe SRSW Boolean to Atomic Snapshots



Road Map

- SRSW safe Boolean
- MRSW safe Boolean
- MRSW regular Boolean
- MRSW regular
- MRSW atomic
- MRMW atomic
- Atomic snapshot

Road Map

- SRSW safe Boolean
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Next

Register Names

```
public class SafeBoolMRSWRegister
 implements Register<Boolean> {
  public boolean read() { … }
  public void write(boolean x) { … }
}
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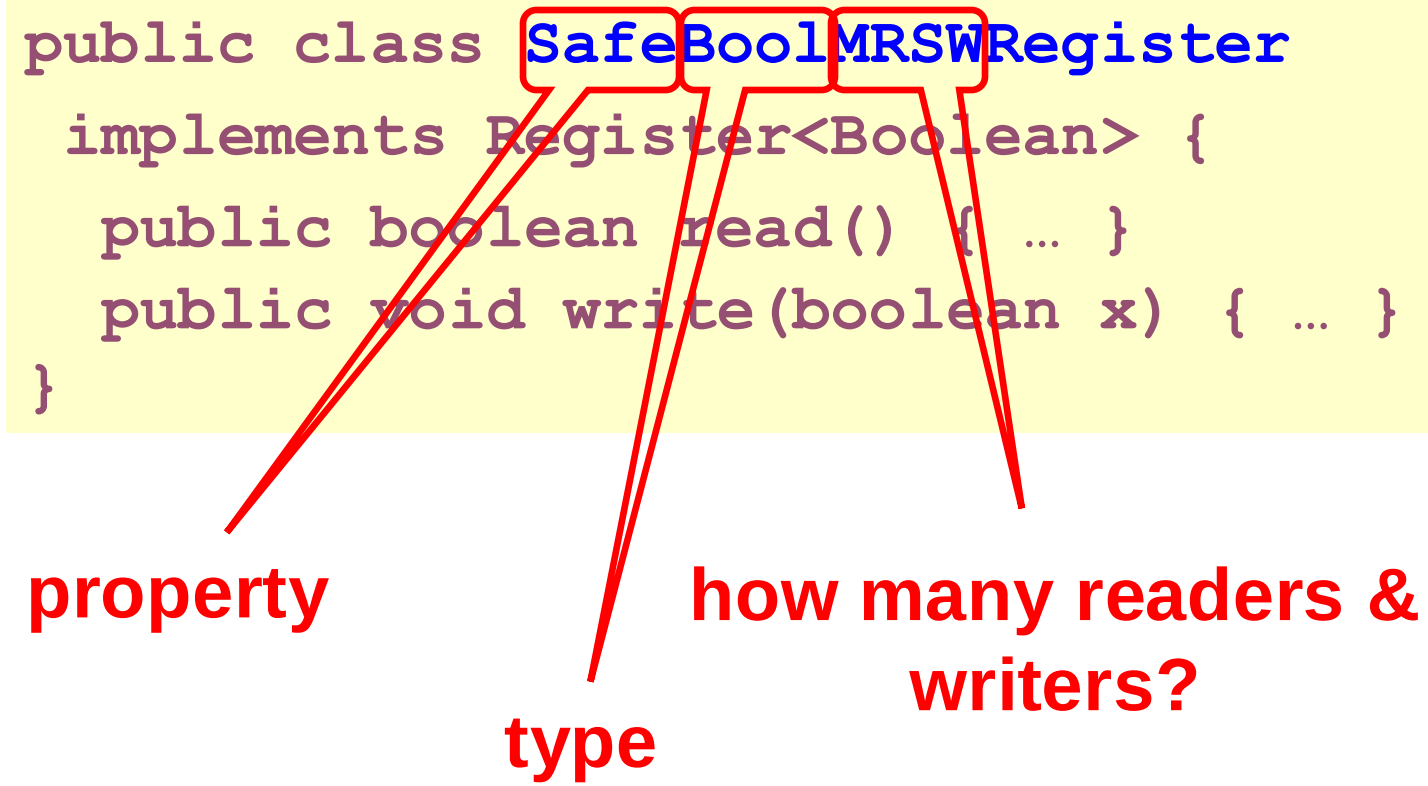
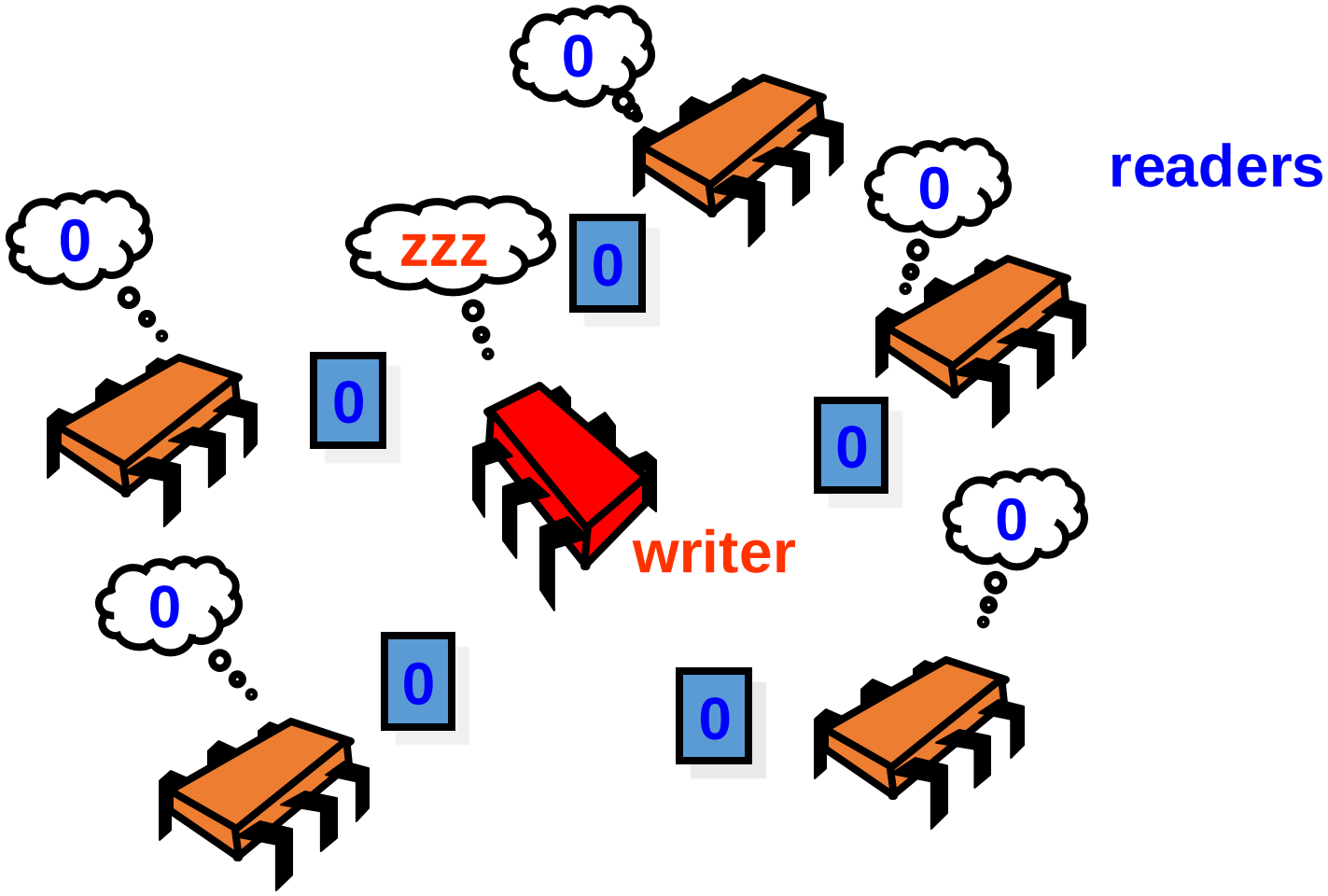
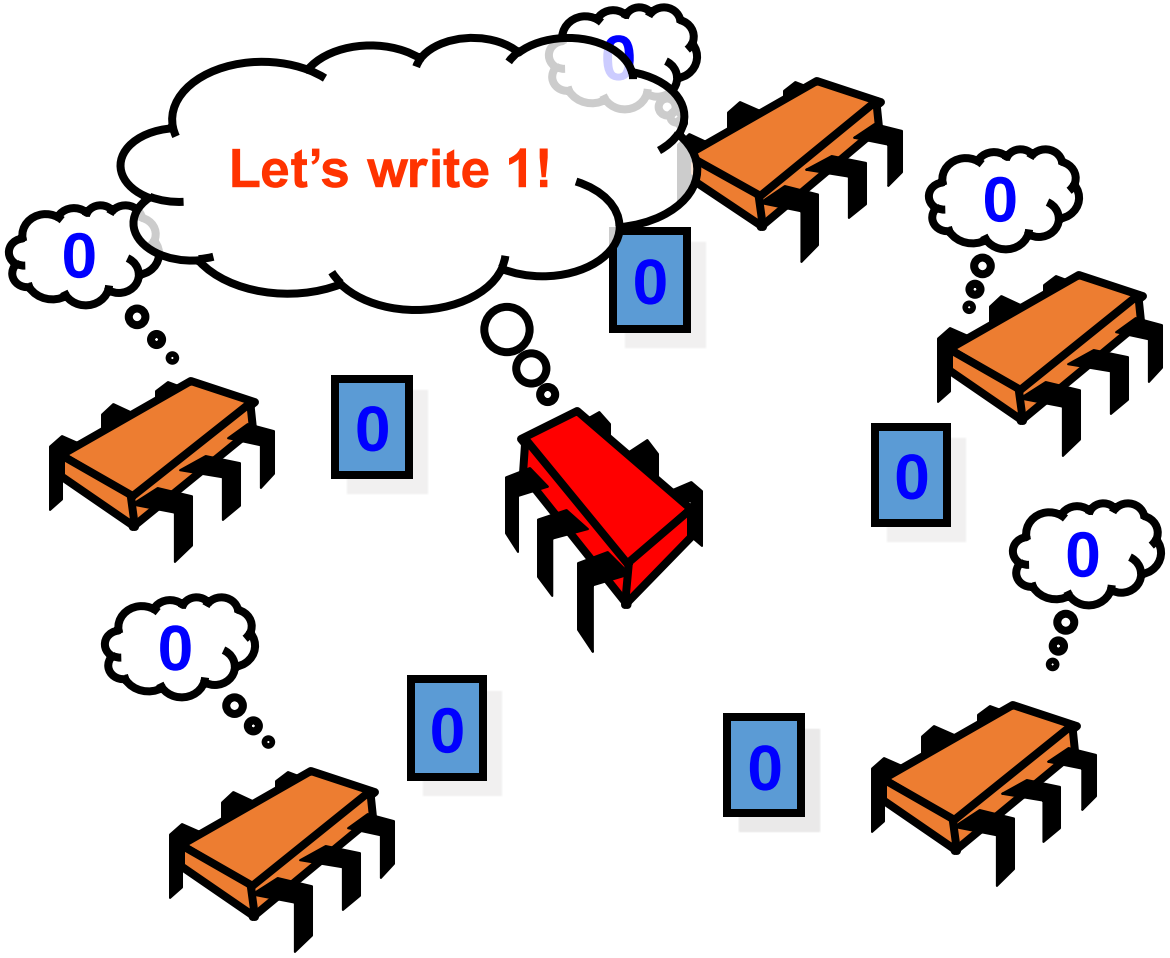


(3)

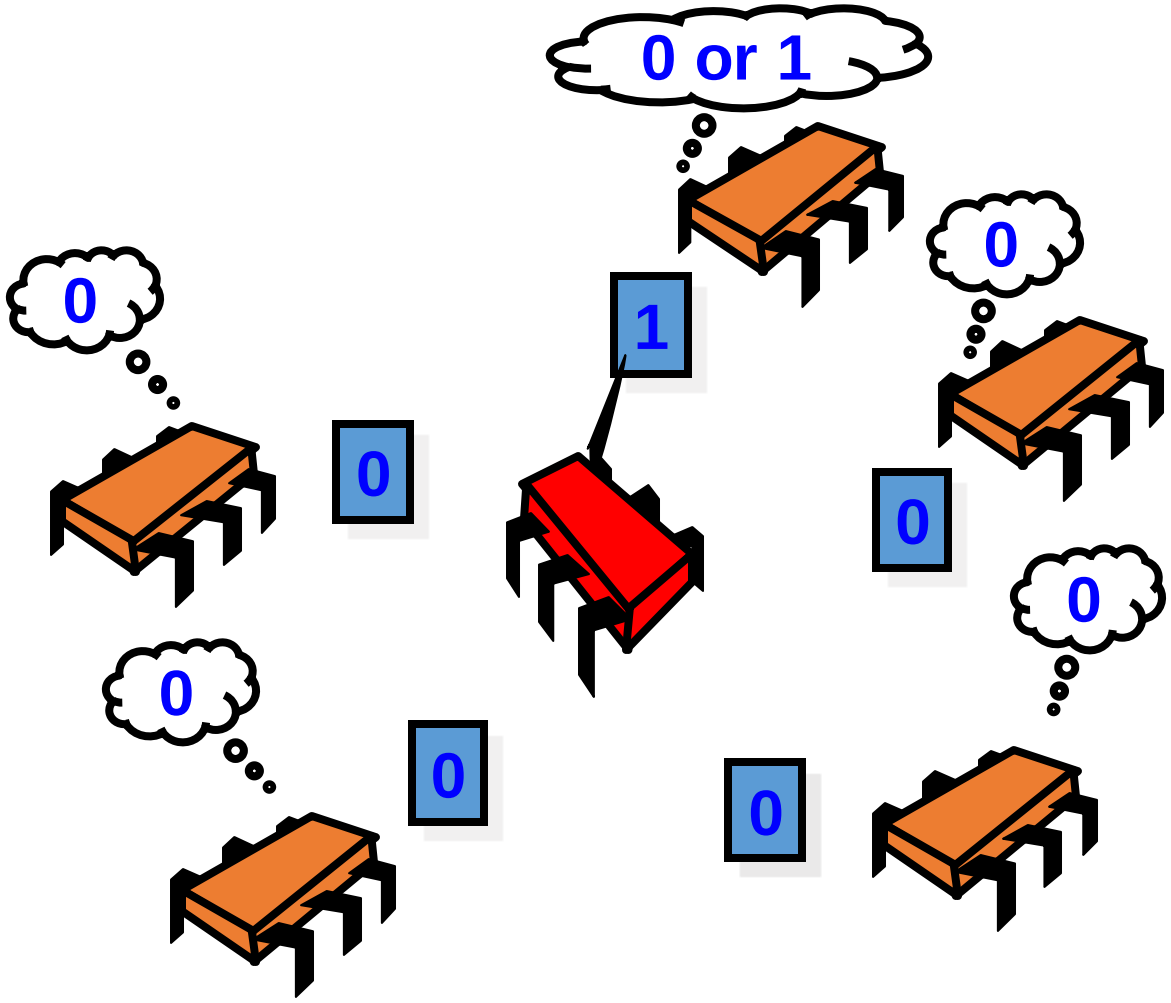
Safe Boolean MRSW from Safe Boolean SRSW



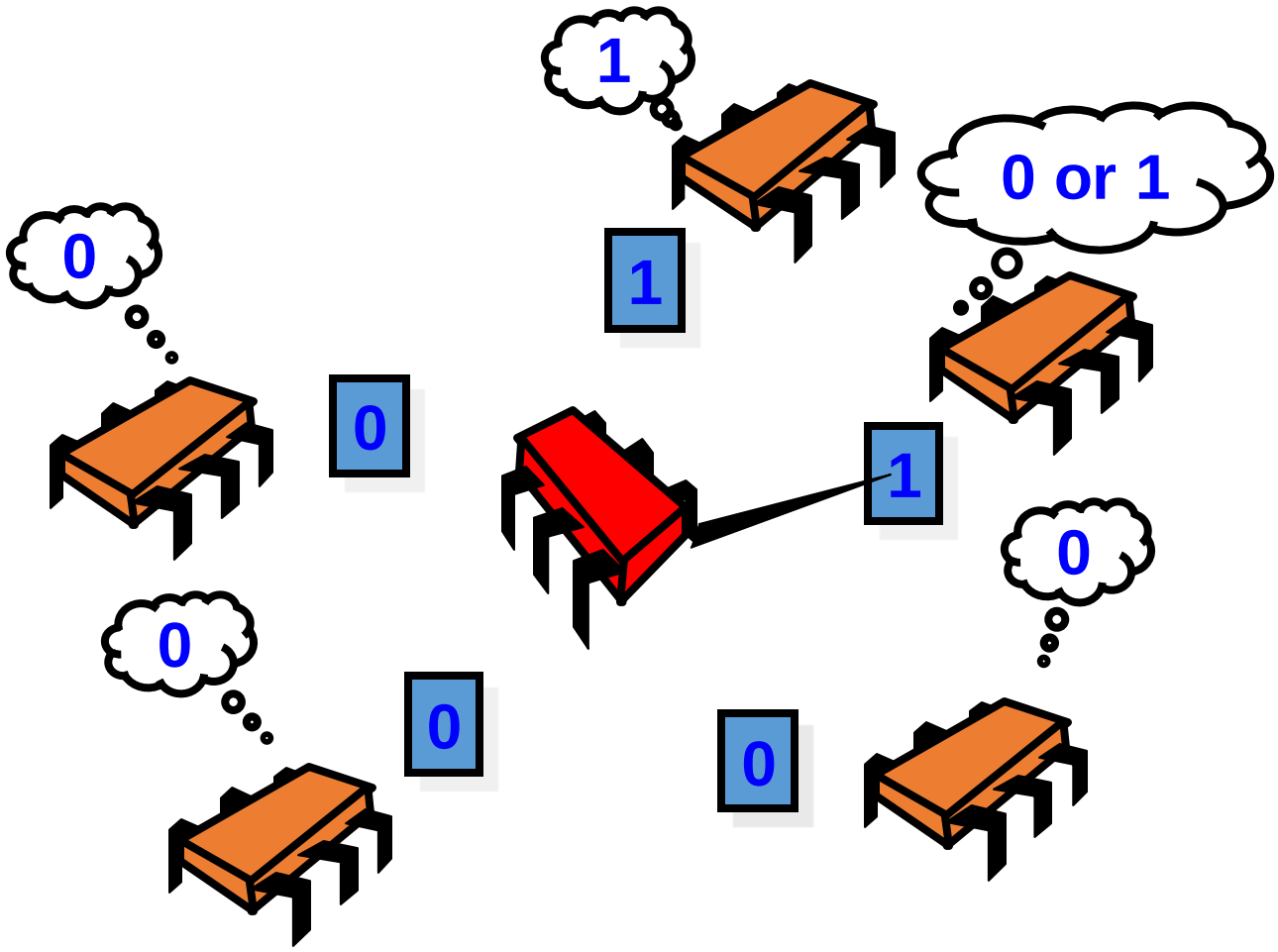
Safe Boolean MRSW from Safe Boolean SRSW



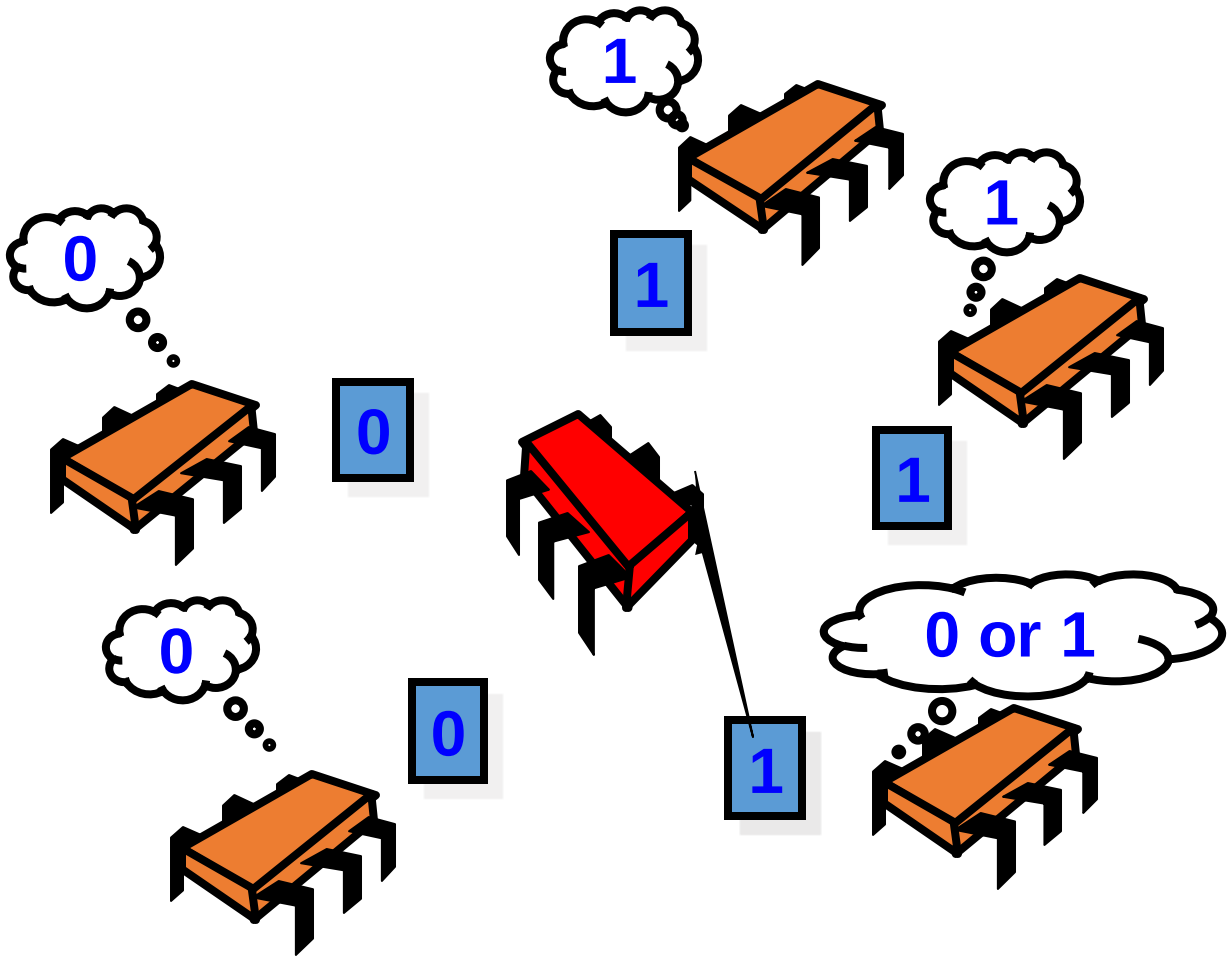
Safe Boolean MRSW from Safe Boolean SRSW



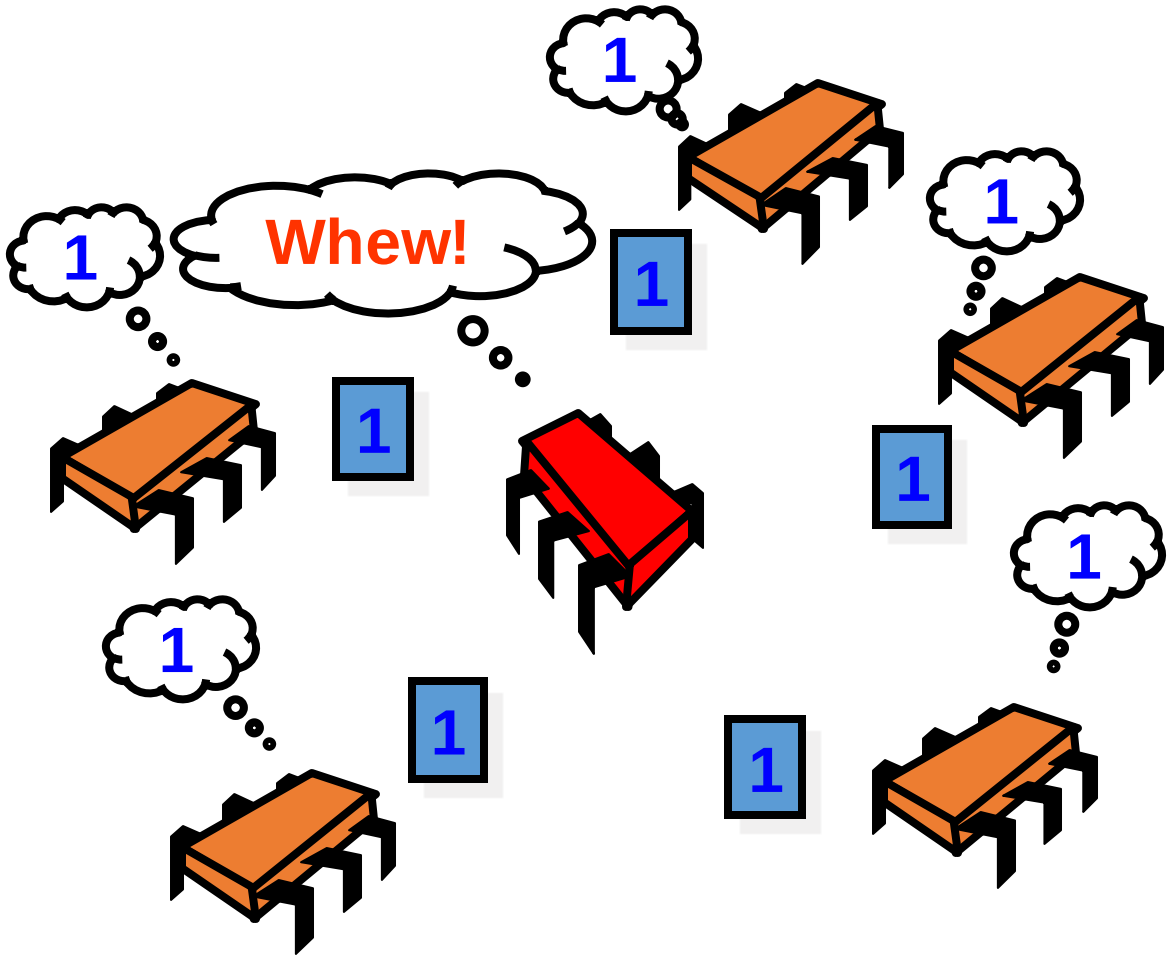
Safe Boolean MRSW from Safe Boolean SRSW



Safe Boolean MRSW from Safe Boolean SRSW



Safe Boolean MRSW from Safe Boolean SRSW



Safe Boolean MRSW from Safe Boolean SRSW

```
public class SafeBoolMRSWRegister
 implements Register<Boolean> {
 private SafeBoolSRSWRegister[] r =
   new SafeBoolSRSWRegister[N];
  public void write(boolean x) {
   for (int j = 0; j < N; j++)
    r[j].write(x);
  }
  public boolean read() {
   int i = ThreadID.get();
   return r[i].read();
  }}
```

Safe Boolean MRSW from Safe Boolean SRSW

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    r[j].write(x);
  }
  public boolean read() {
   int i = ThreadID.get();
   return r[i].read();
  }}
```

Each thread has own safe SRSW register

Safe Boolean MRSW from Safe Boolean SRSW

```
public class SafeBoolMRSWRegister
 implements BooleanRegister {
 private SafeBoolSRSWRegister[] r =
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  public void write(boolean x) {
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  }
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   int i = ThreadID.get();
   return r[i].read();
  }}
```

write method

Safe Boolean MRSW from Safe Boolean SRSW

```
public class SafeBoolMRSWRegister
 implements BooleanRegister {
 private SafeBoolSRSWRegister[] r =
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  public void write(boolean x) {
   for (int j = 0; j < N; j++)
    r[j].write(x);
  }
  public boolean read() {
   int i = ThreadID.get();
   return r[i].read();
  }}
```

Write each thread's register one at a time

Safe Boolean MRSW from Safe Boolean SRSW

```
public class SafeBoolMRSWRegister
 implements BooleanRegister {
 private SafeBoolSRSWRegister[] r =
   new SafeBoolSRSWRegister[N];
  public void write(boolean x) {
   for (int j = 0; j < N; j++)
    r[j].write(x);
  }
  public boolean read() {
   int i = ThreadID.get();
   return r[i].read();
  }}
```

read method

Safe Boolean MRSW from Safe Boolean SRSW

```
public class SafeBoolMRSWRegister
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    r[j].write(x);
  }
  public boolean read() {
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   return r[i].read();
  }}
```
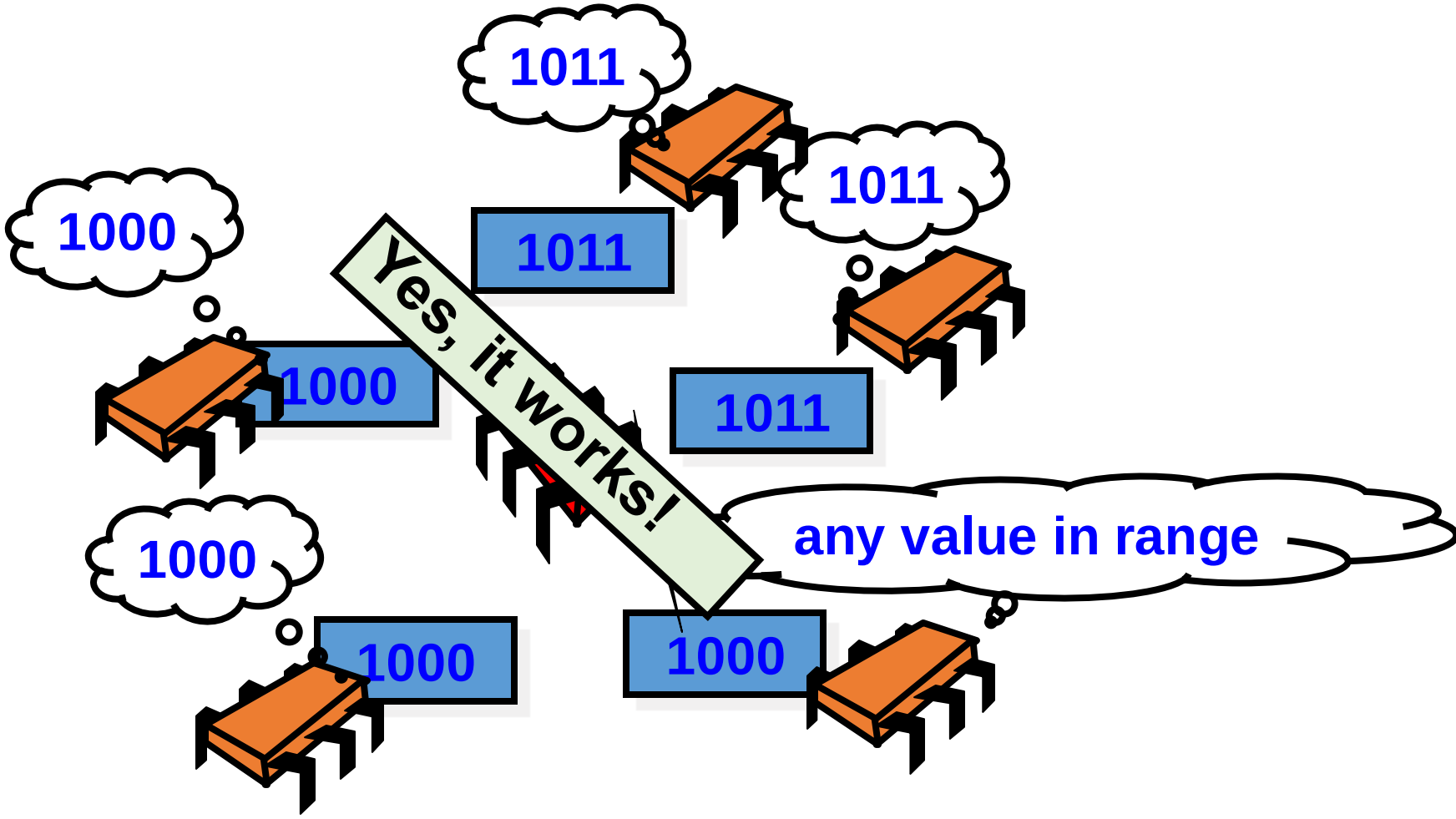

Read my own register

Safe Multi-Valued MRSW from Safe Multi-Valued SRSW?



Road Map

- SRSW safe Boolean
- MRSW safe Boolean
- MRSW regular Boolean
- MRSW regular
- MRSW atomic
- MRMW atomic
- Atomic snapshot

Questions?

Road Map

- SRSW safe Boolean
- MRSW safe Boolean
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- MRSW atomic
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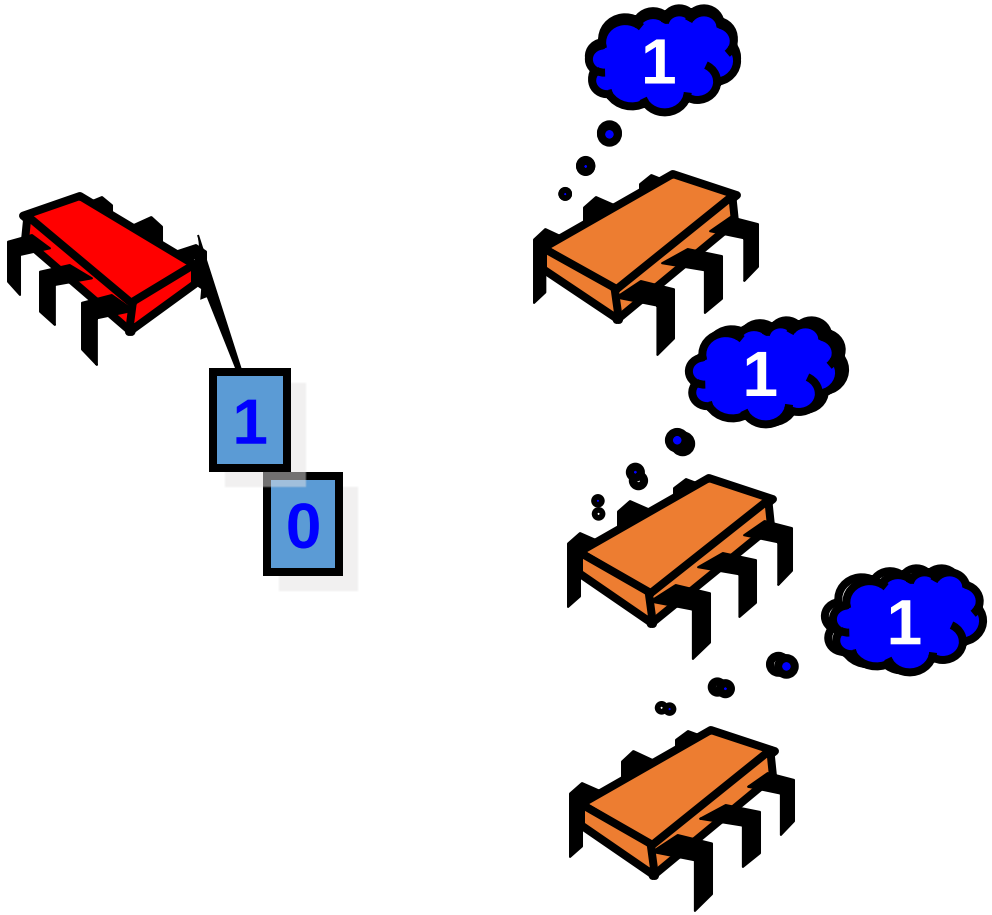
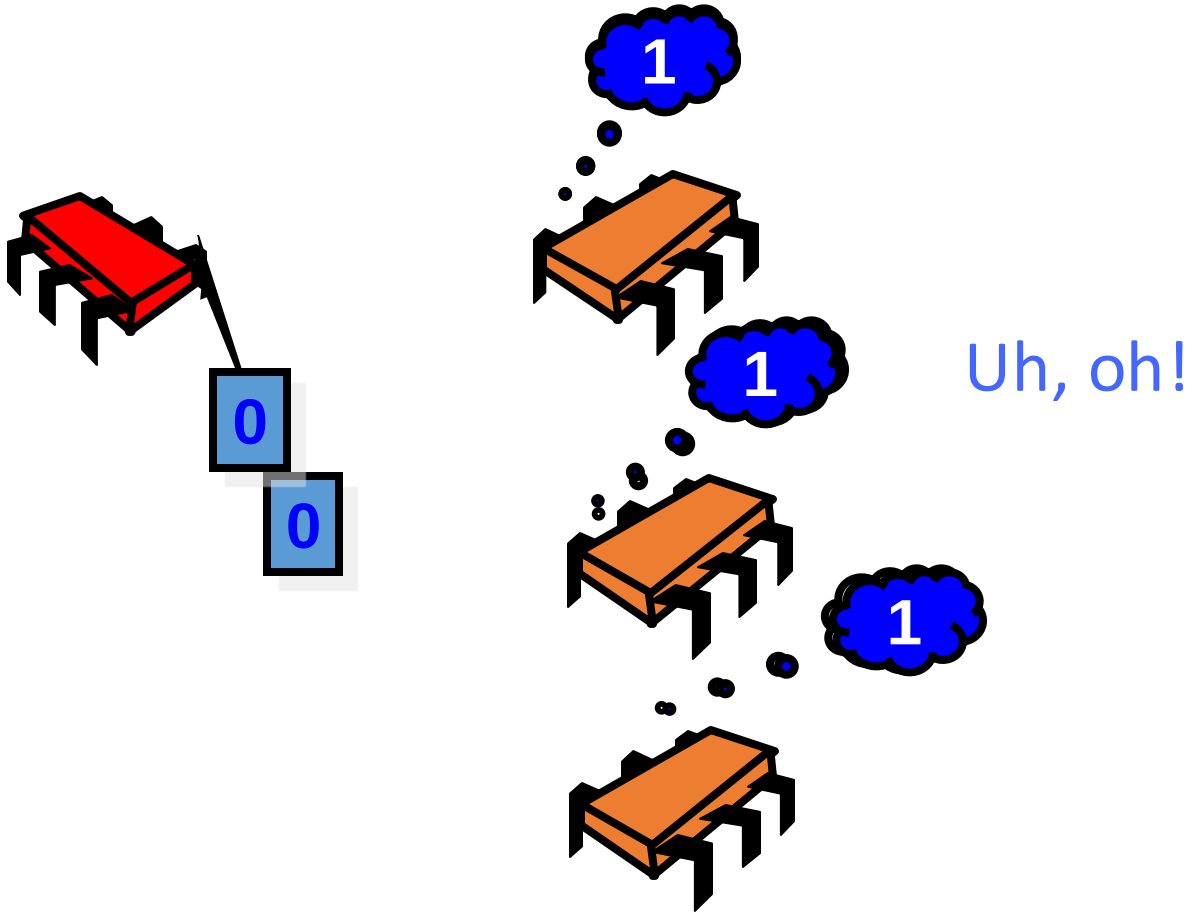
- Atomic snapshot

Regular Boolean MRSW from Safe Boolean MRSW

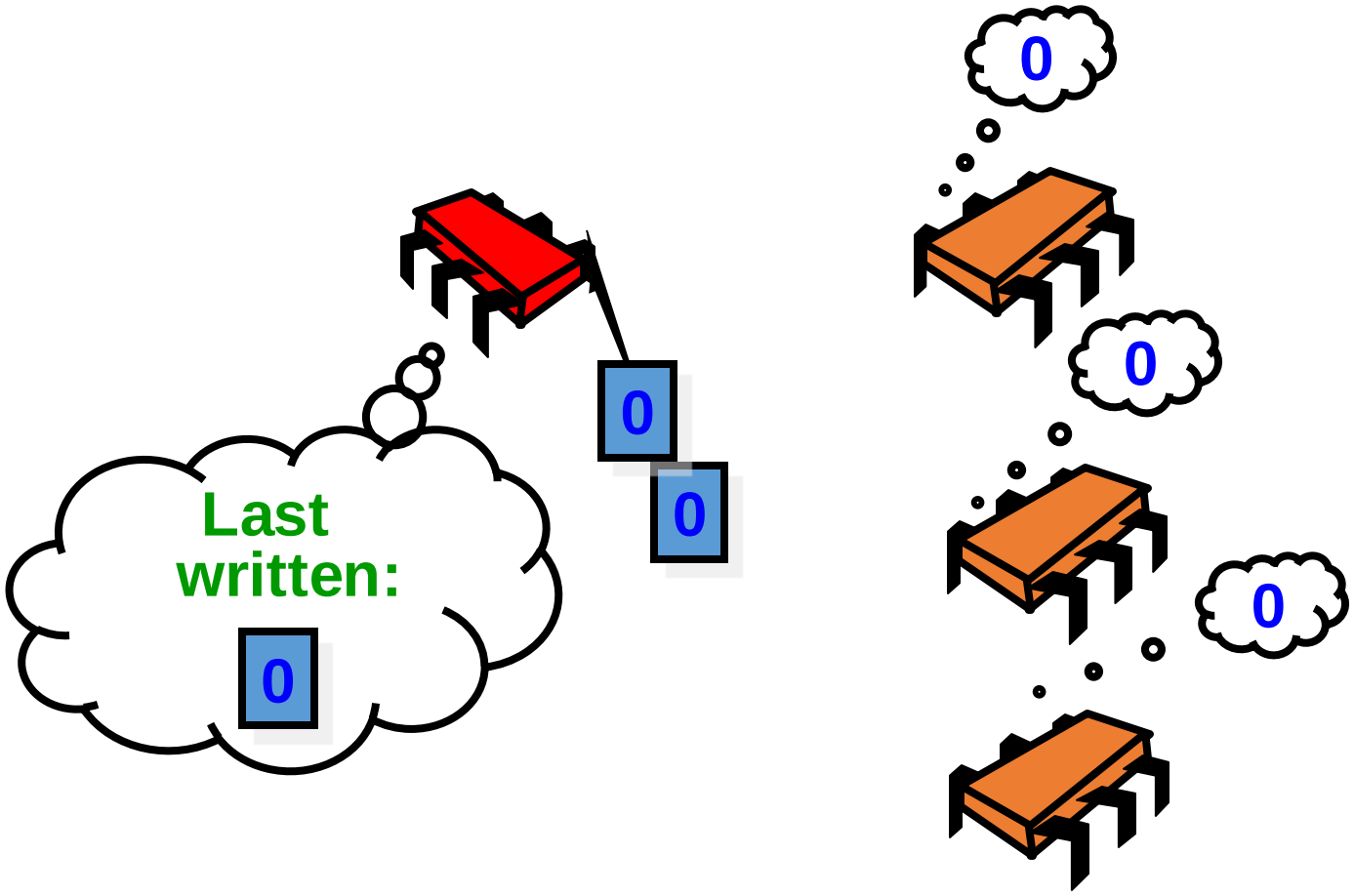


Regular Boolean MRSW from Safe Boolean MRSW



Uh, oh!

Regular Boolean MRSW from Safe Boolean MRSW



Regular Boolean MRSW from Safe Boolean MRSW

```
public class RegBoolMRSWRegister
 implements Register<Boolean> {
  private boolean old;
  private SafeBoolMRSWRegister value;
  public void write(boolean x) {
   if (old != x) {
    value.write(x);
    old = x;
   }}
  public boolean read() {
   return value.read();
  }}
```

Regular Boolean MRSW from Safe Boolean MRSW

```
public class RegBoolMRSWRegister
 implements Register<Boolean> {
  threadLocal boolean old;
  private SafeBoolMRSWRegister value;
  public void write(boolean x) {
   if (old != x) {
    value.write(x);
    old = x;
   }}
  public boolean read() {
   return value.read();
  }}
```

Last bit this thread wrote

(made-up syntax)

Regular Boolean MRSW from Safe Boolean MRSW

```
public class RegBoolMRSWRegister
 implements Register<Boolean> {
  threadLocal boolean old;
  private SafeBoolMRSWRegister value;
  public void write(boolean x) {
   if (old != x) {
    value.write(x);
    old = x;
   }}
  public boolean read() {
   return value.read();
  }}
```

Actual value

Regular Boolean MRSW from Safe Boolean MRSW

```
public class RegBoolMRSWRegister
 implements Register<Boolean> {
  threadLocal boolean old;
  private SafeBoolMRSWRegister value;
  public void write(boolean x) {
    if (old != x) {
     value.write(x);
     old = x;
    }}
  public boolean read() {
   return value.read();
  }}
```

Is new value different from last value I wrote?

Regular Boolean MRSW from Safe Boolean MRSW

```
public class RegBoolMRSWRegister
 implements Register<Boolean> {
  threadLocal boolean old;
  private SafeBoolMRSWRegister value;
  public void write(boolean x) {
   if (old != x) {
    value.write(x);
    old = x;
   }}
  public boolean read() {
   return value.read();
  }}
```

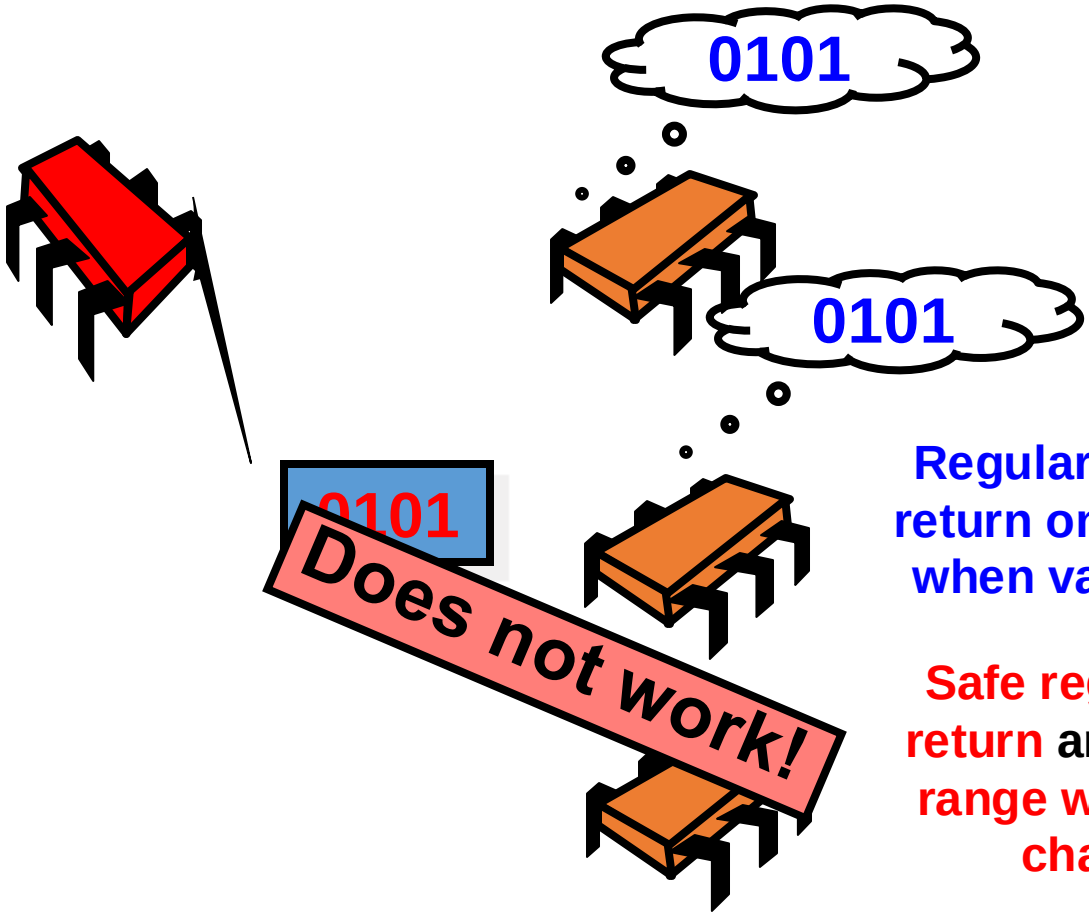
If so, change it (otherwise don't!)

Regular Boolean MRSW from Safe Boolean MRSW

```
public class RegBoolMRSWRegister
 implements Register<Boolean> {
  threadLocal boolean old;
  private SafeBoolMRSWRegister value;
  public void write(boolean x) {
   if (old != x) {
    value.write(x);
    old = x;
   }}
  public boolean read() {
   return value.read();
  }}
```

No more problems with overlap

Regular Multi-Valued MRSW from Safe Multi-Valued MRSW?



Regular register can return only old or new when value changes

Safe register can return any value in range when value changes

Road Map

- SRSW safe Boolean
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Questions?

Road Map

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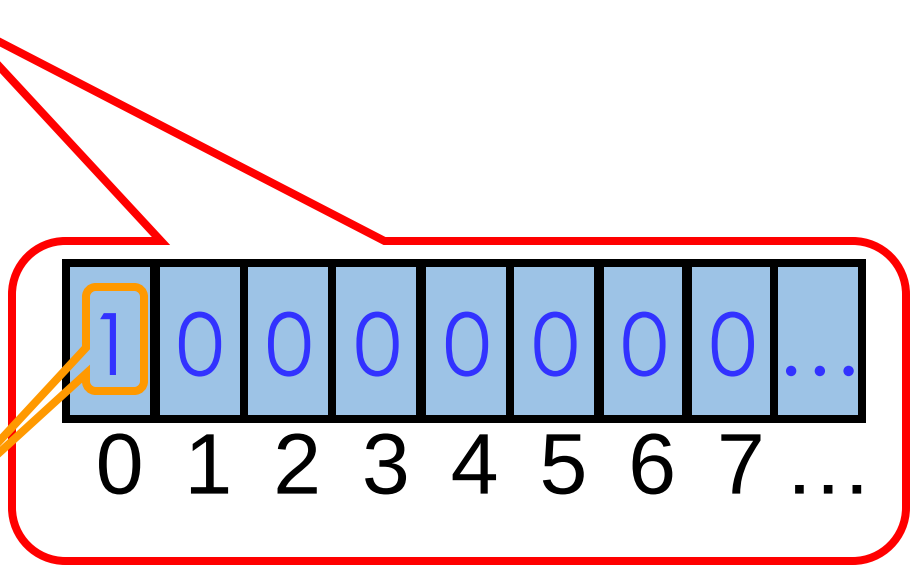
Representing *m* Values

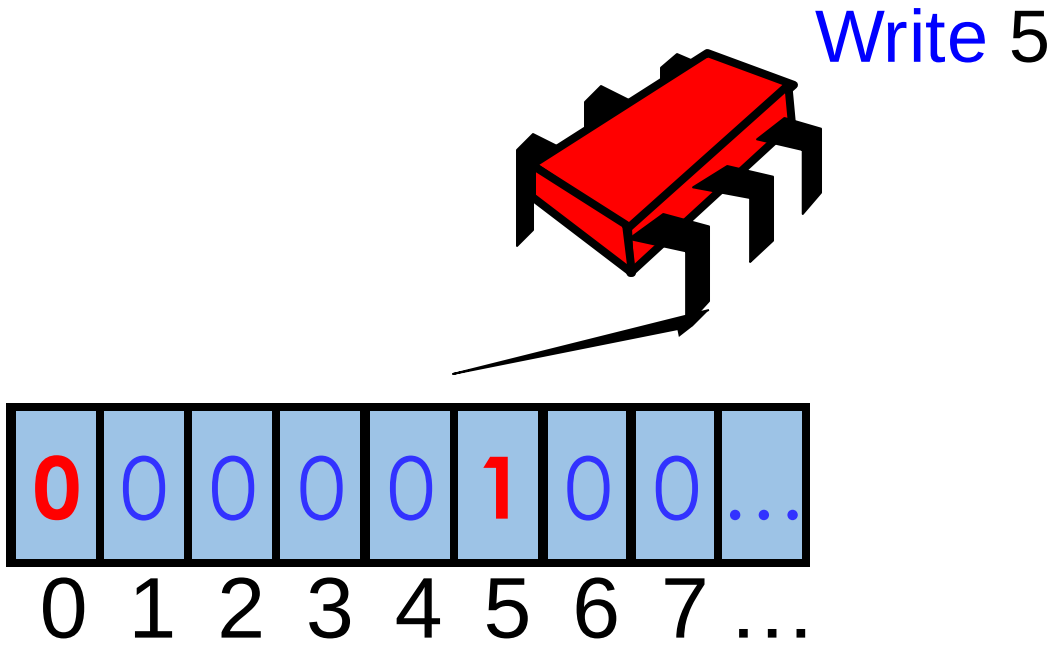
Unary representation:

bit[i]=1 means value=i

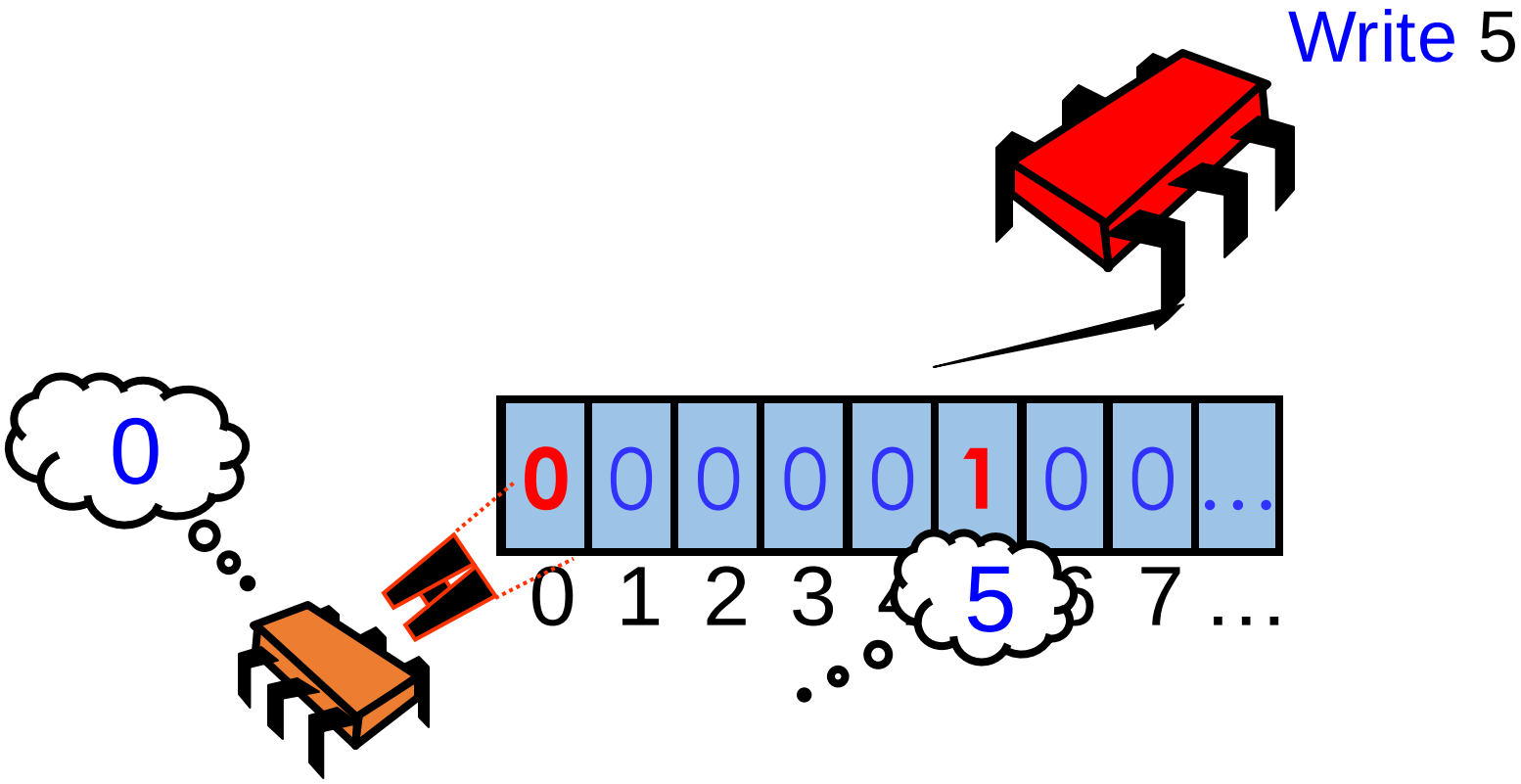


Initially 0

Writing *m*-Valued Register



Writing *m*-Valued Register



MRSW Regular *m*-valued from MRSW Regular Boolean

```
public class RegMRSWRegister implements Register{
  RegBoolMRSWRegister[M] bit;

  public void write(int x) {
    bit[x].write(true);
    for (int i=x-1; i>=0; i--)
      bit[i].write(false);
  }

  public int read() {
    for (int i=0; i < M; i++)
      if (bit[i].read())
        return i;
   }}
```

MRSW Regular *m*-valued from MRSW Regular Boolean

```
public class RegMRSWRegister implements Register{
  RegBoolMRSWRegister[M] bit;

  public void write(int x) {
    bit[x].write(true);
    for (int i=x-1; i>=0; i--)
      bit[i].write(false);
  }

  public int read() {
    for (int i=0; i < M; i++)
      if (bit[i].read())
        return i;
   }}
```

Unary representation: bit[i] means value i

MRSW Regular *m*-valued from MRSW Regular Boolean

```
public class RegMRSWRegisterimplements Register {
  RegBoolMRSWRegister[m] bit;

  public void write(int x) {
    bit[x].write(true);
    for (int i=x-1; i>=0; i--)
      bit[i].write(false);
  }

  public int read() {
    for (int i=0; i < M; i++)
      if (bit[i].read())
        return i;
   }}
```

set bit x

MRSW Regular *m*-valued from MRSW Regular Boolean

```
public class RegMRSWRegisterimplements Register {
  RegBoolMRSWRegister[m] bit;

  public void write(int x) {
    bit[x].write(true);
    for (int i=x-1; i>=0; i--)
      bit[i].write(false);
  }

  public int read() {
    for (int i=0; i < M; i++)
      if (bit[i].read())
        return i;
   }}
```

Clear bits from higher to lower

MRSW Regular *m*-valued from MRSW Regular Boolean

```
public class RegMRSWRegisterimplements Register {
  RegBoolMRSWRegister[m] bit;

  public void write(int x) {
    bit[x].write(true);
    for (int i=x-1; i>=0; i--)
      bit[i].write(false);
  }

  public int read() {
    for (int i=0; i < M; i++)
      if (bit[i].read())
        return i;
  }}
```

Scan from lower to higher & return first bit set

Road Map

- SRSW safe Boolean
- MRSW safe Boolean
- MRSW regular Boolean
- MRSW regular
- MRSW atomic
- MRMW atomic
- Atomic snapshot

Questions?

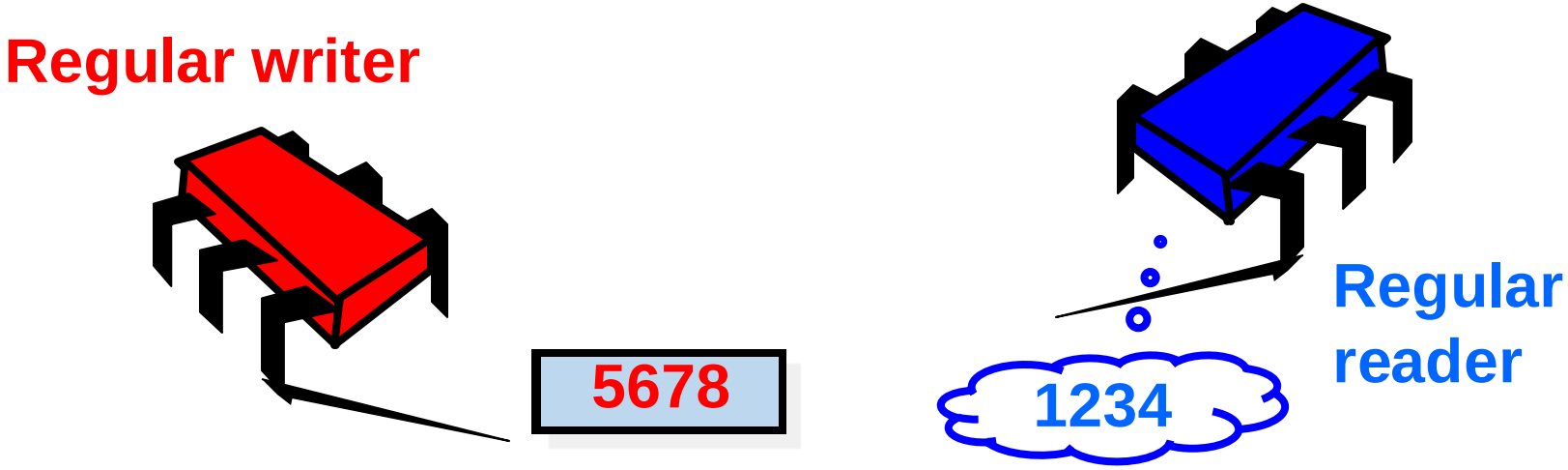
Road Map

- SRSW safe Boolean
- MRSW safe Boolean
- MRSW regular Boolean
- MRSW regular
- MRSW atomic
- MRMW atomic
- Atomic snapshot

Road Map (Slight Detour)

- SRSW safe Boolean
- MRSW safe Boolean
- MRSW regular Boolean
- MRSW regular
- MRSW atomic
- MRMW atomic
- Atomic snapshot

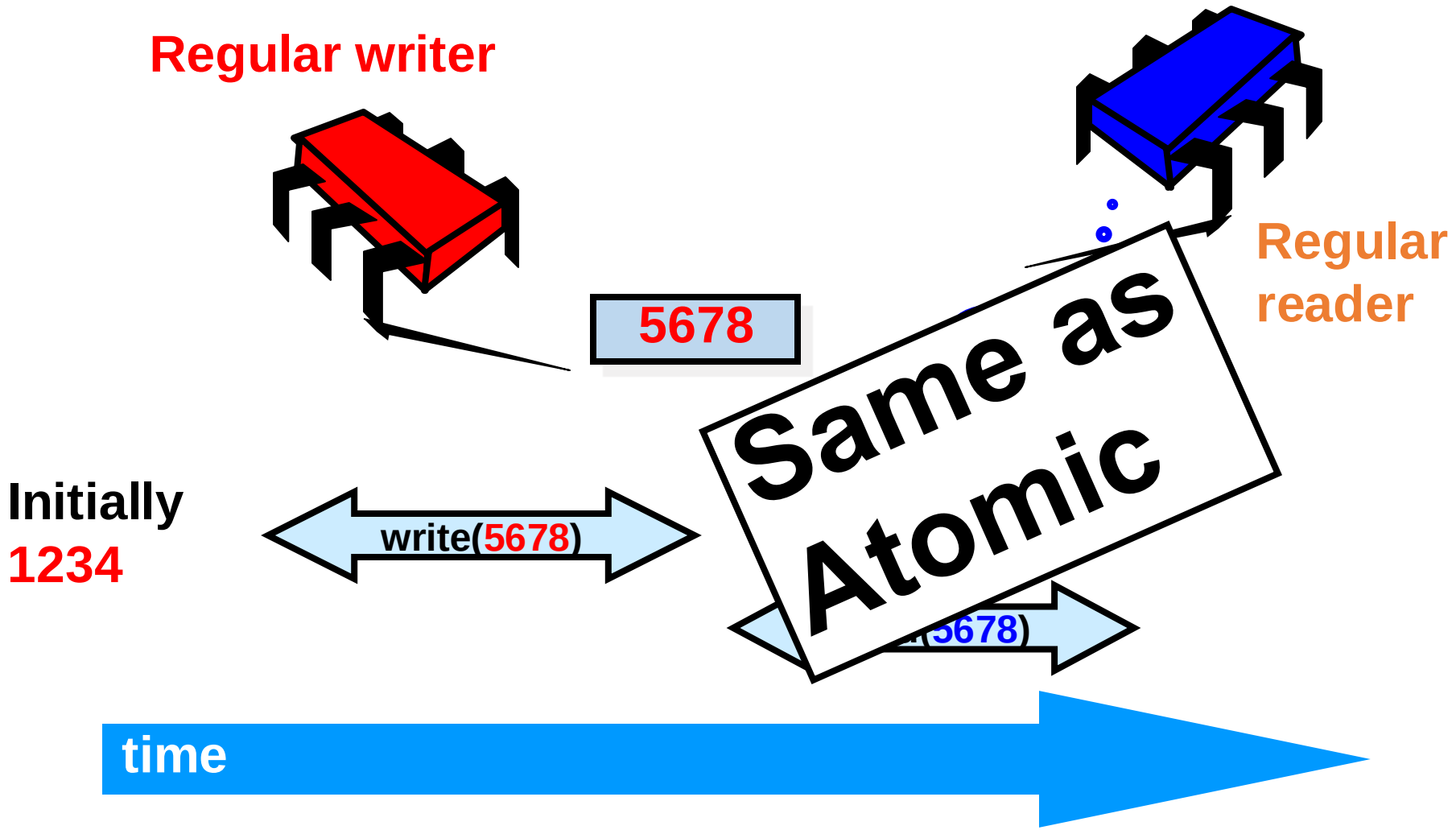
SRSW Atomic from SRSW Regular



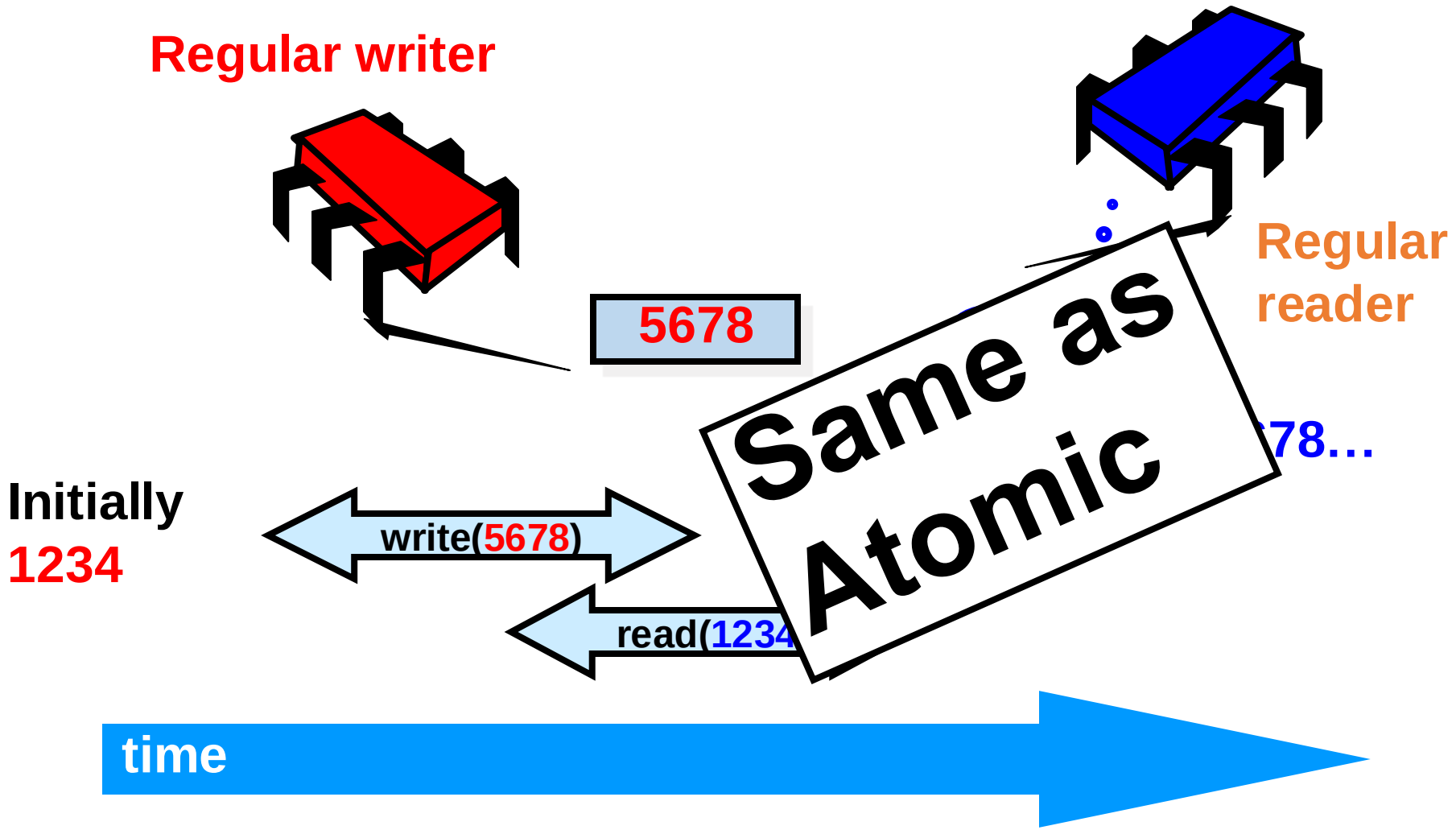
Concurrent Reading

When is this a problem?

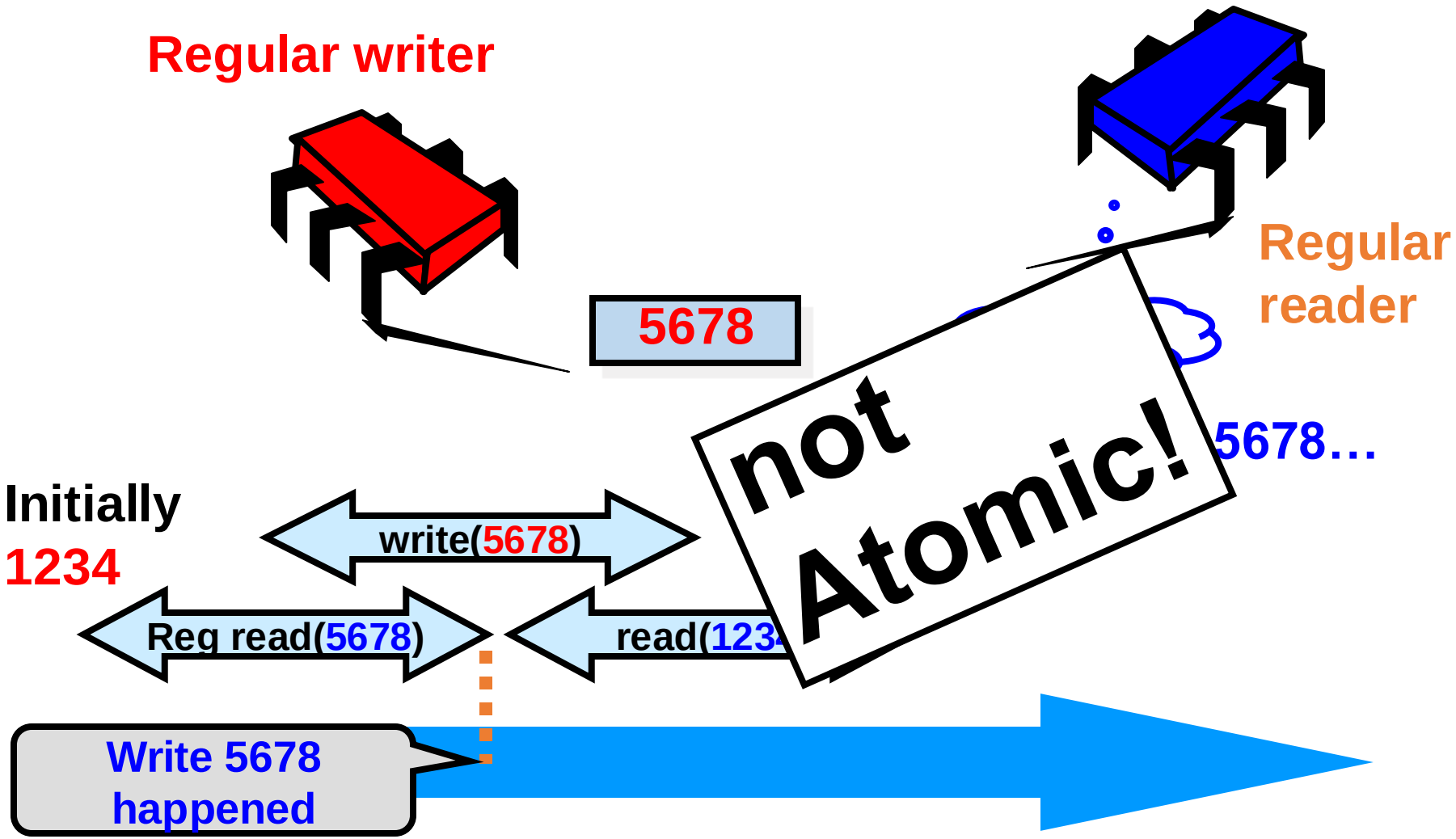
SRSW Atomic From SRSW Regular



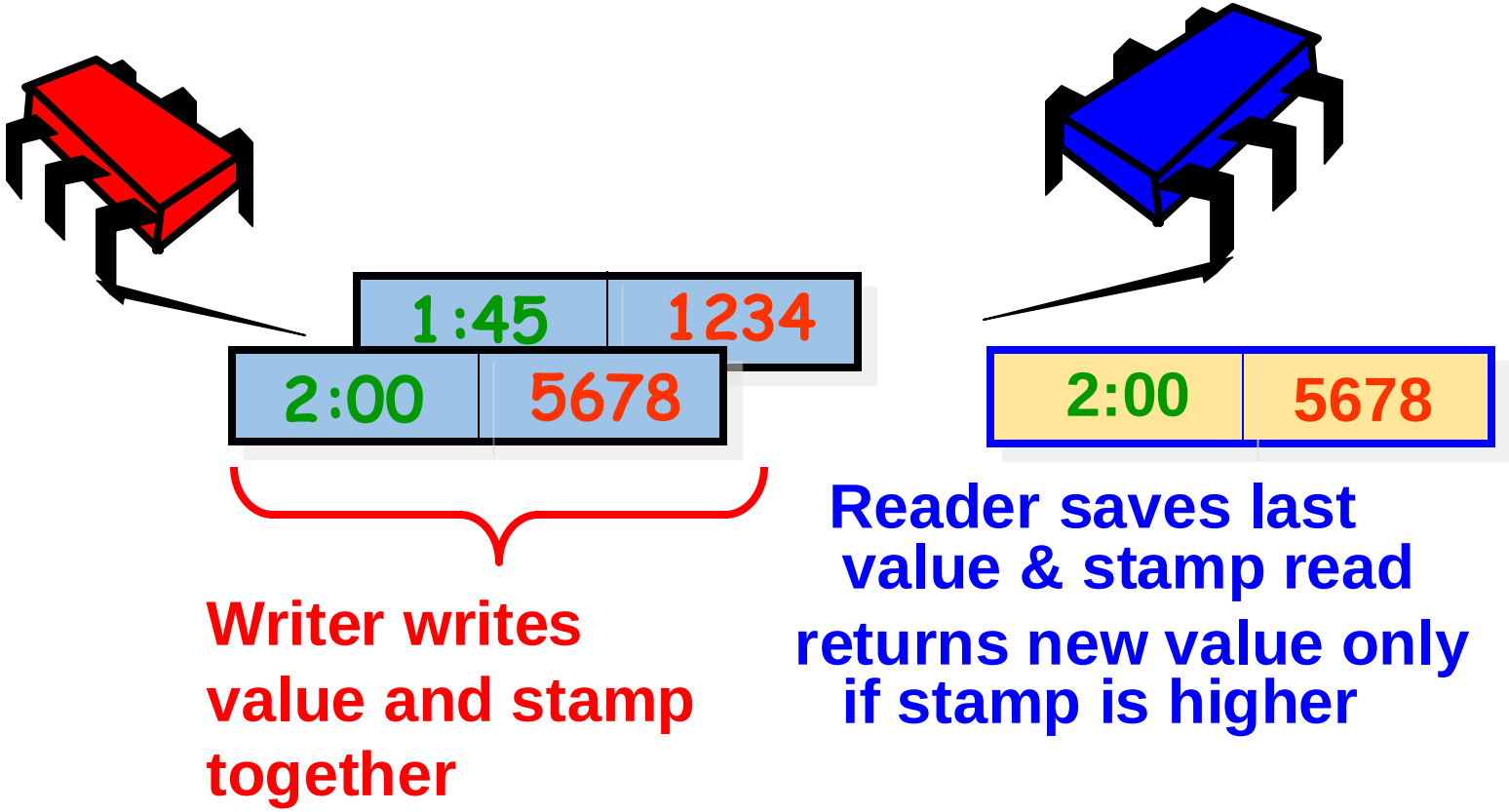
SRSW Atomic From SRSW Regular



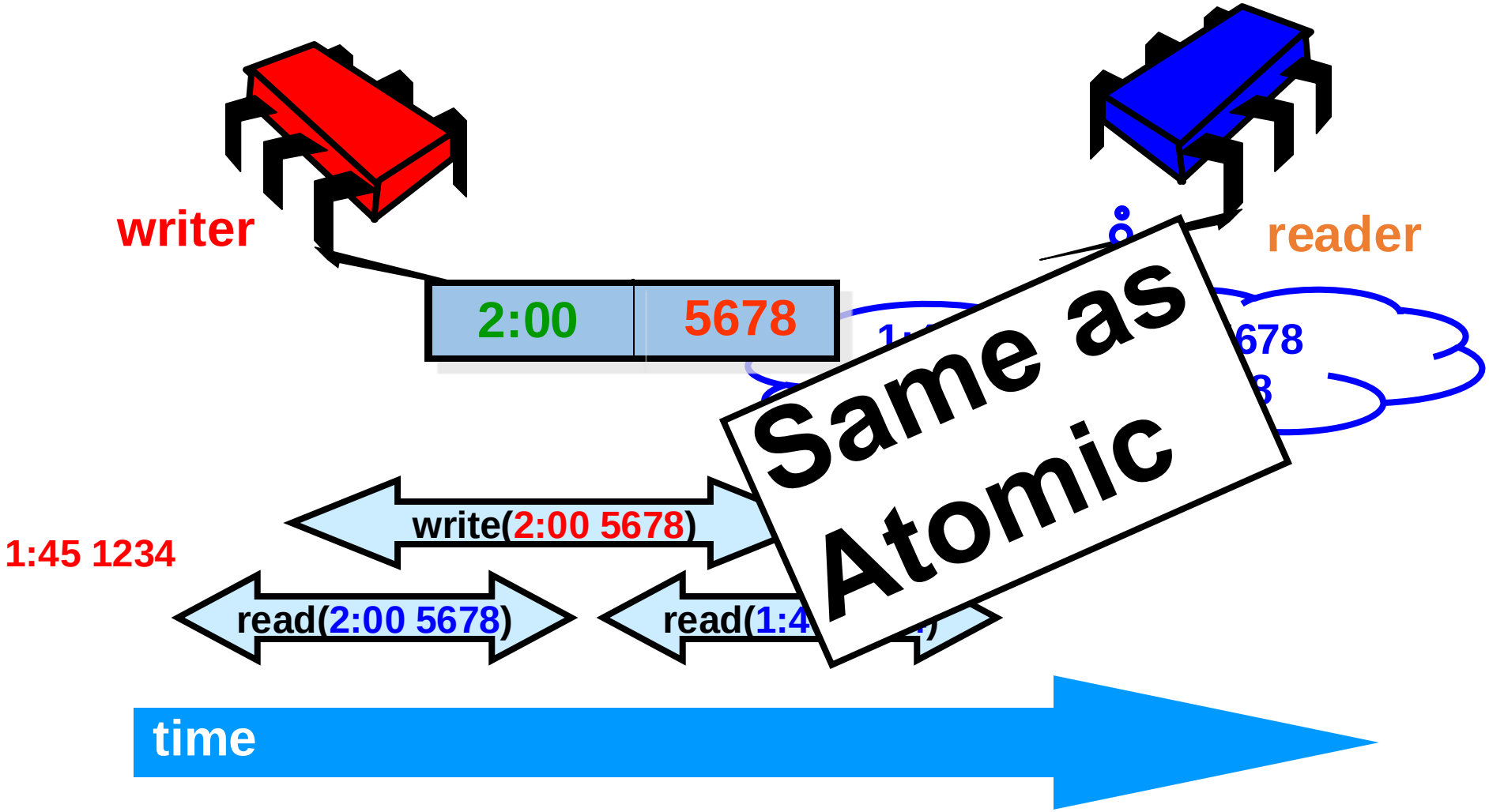
SRSW Atomic From SRSW Regular



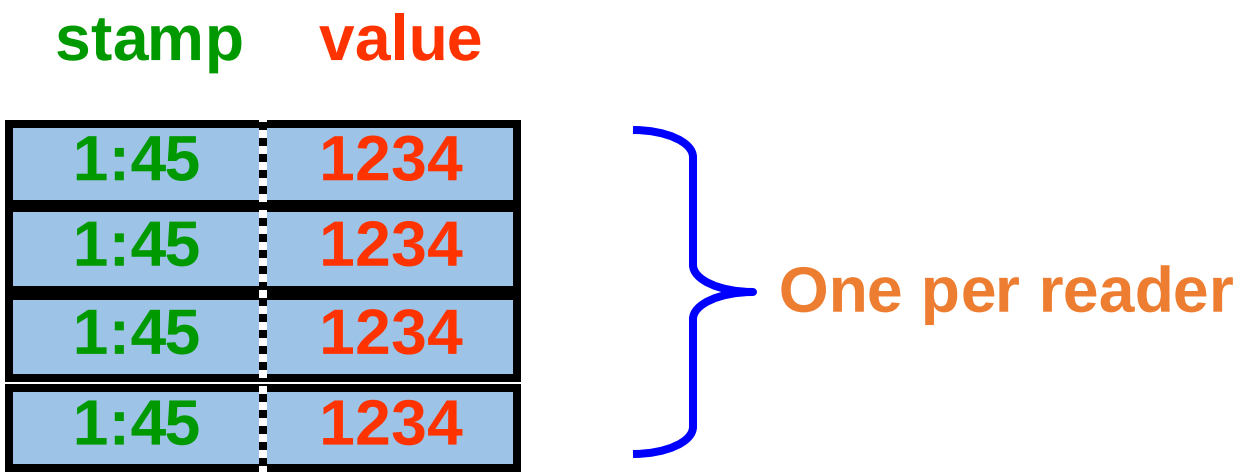
Timestamped Values



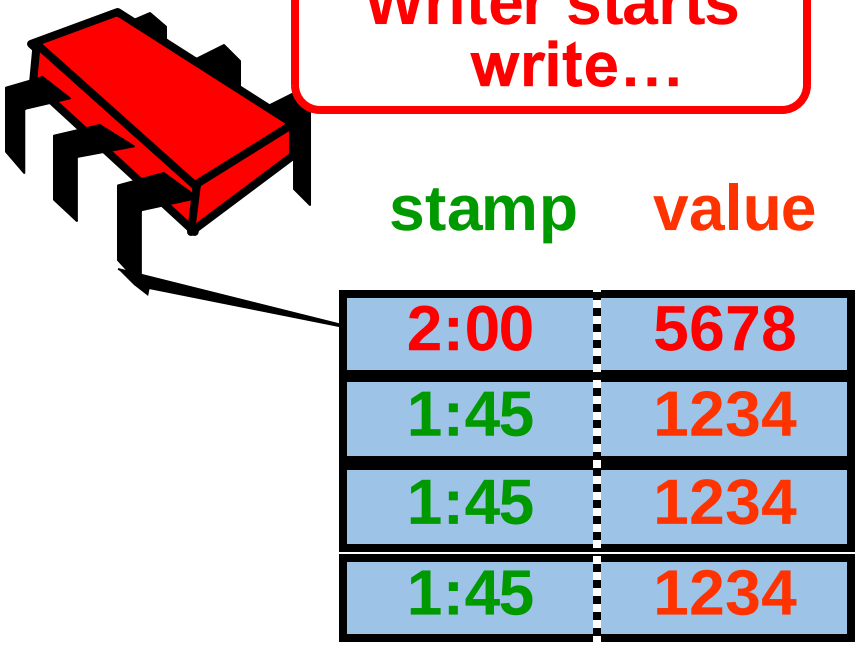
SRSW Atomic From SRSW Regular



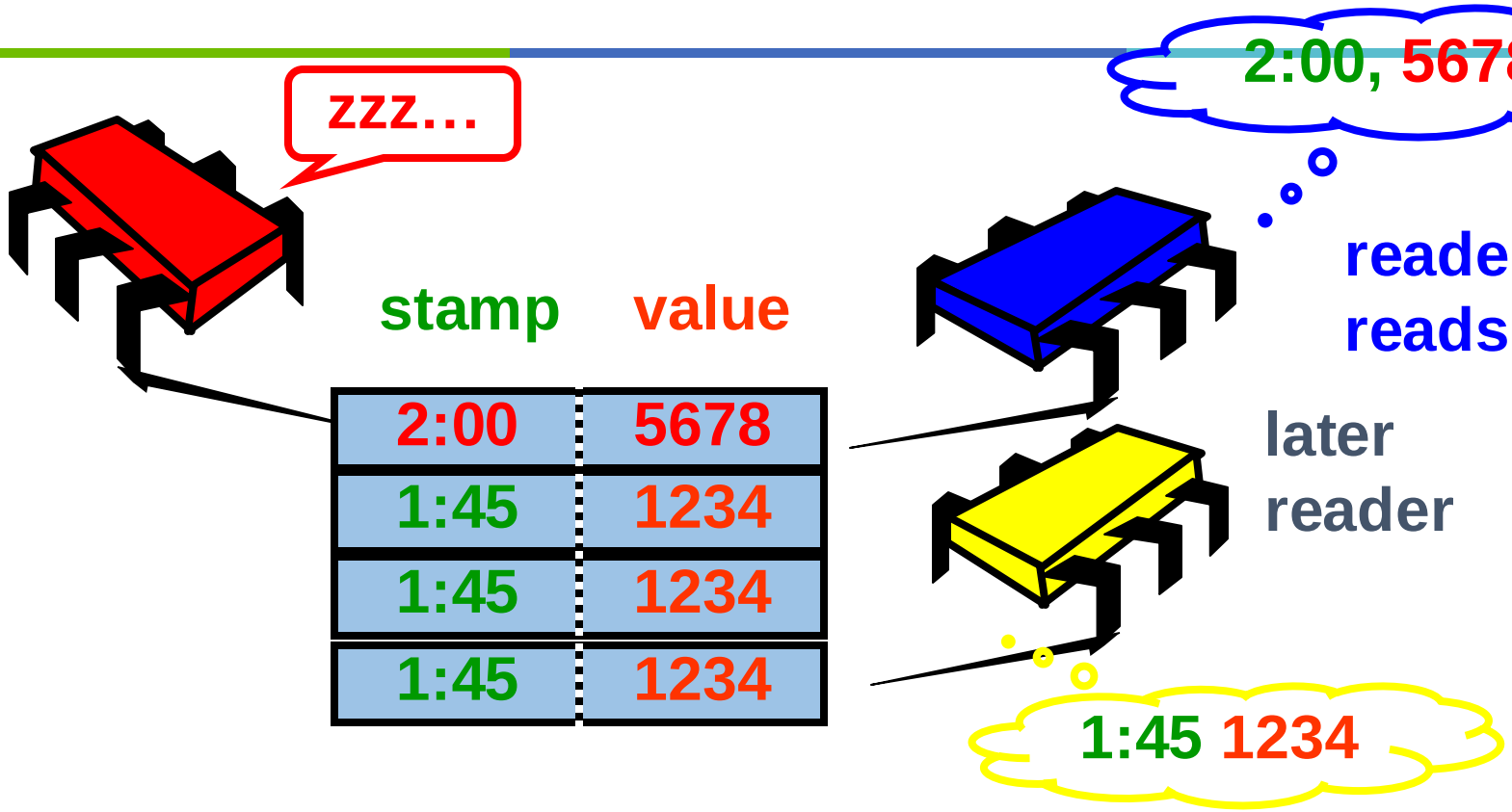
Atomic Single-Reader to Atomic Multi-Reader



Another Scenario

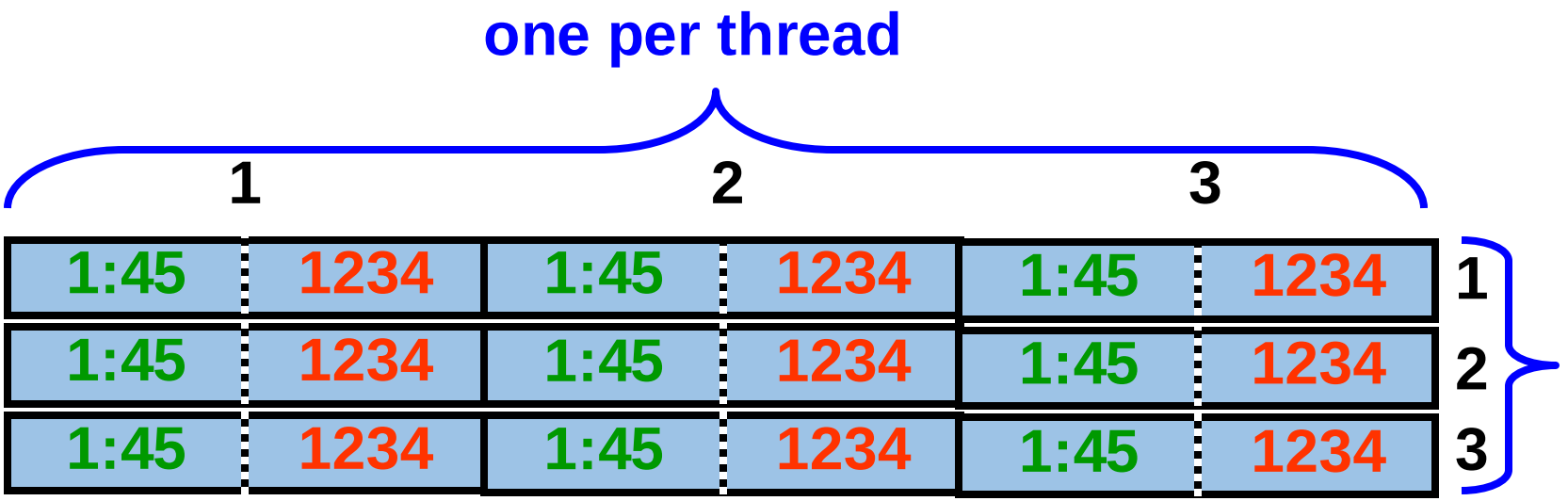


Another Scenario

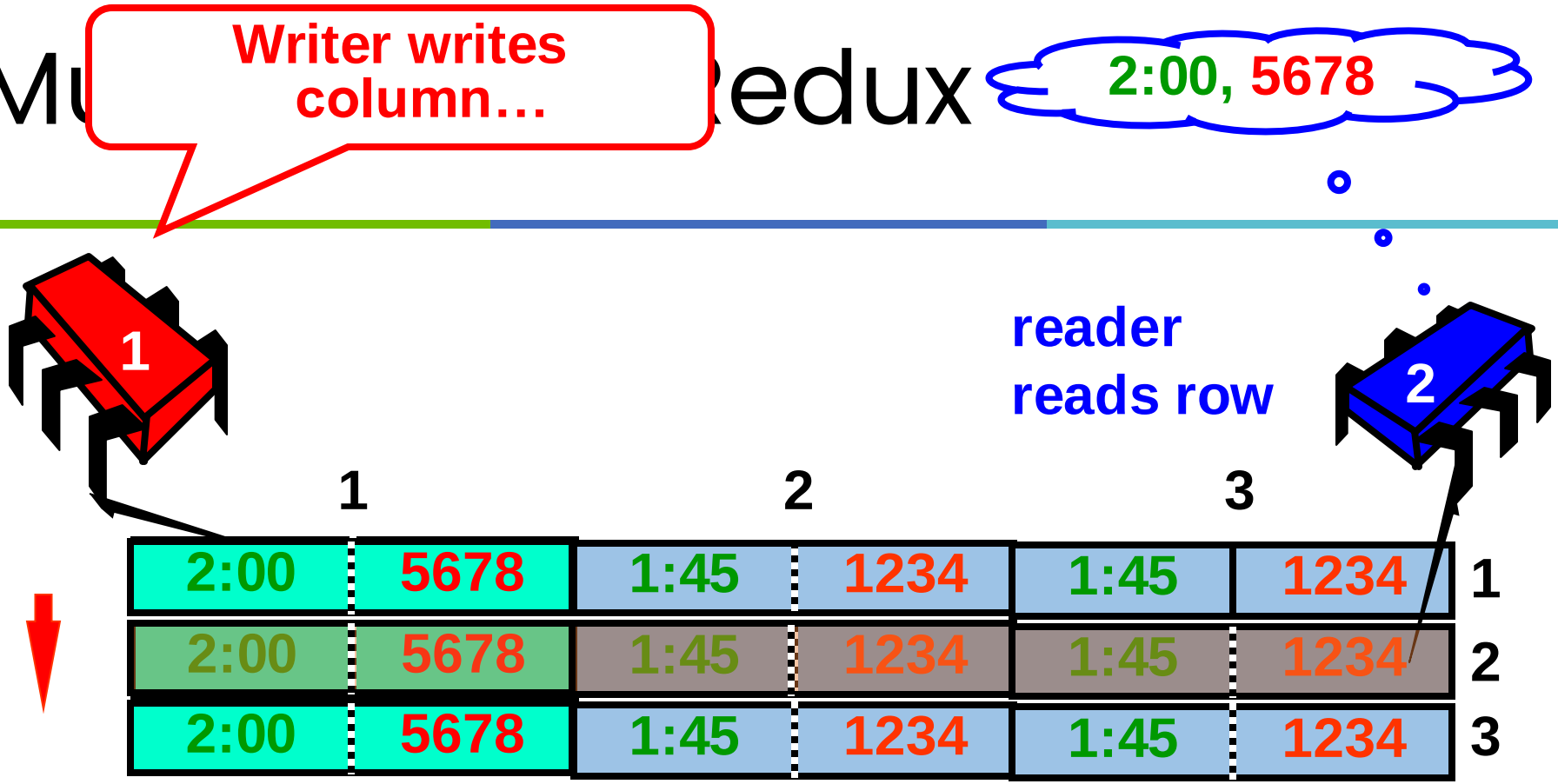


Yellow was completely after Blue but read earlier value…not linearizable!

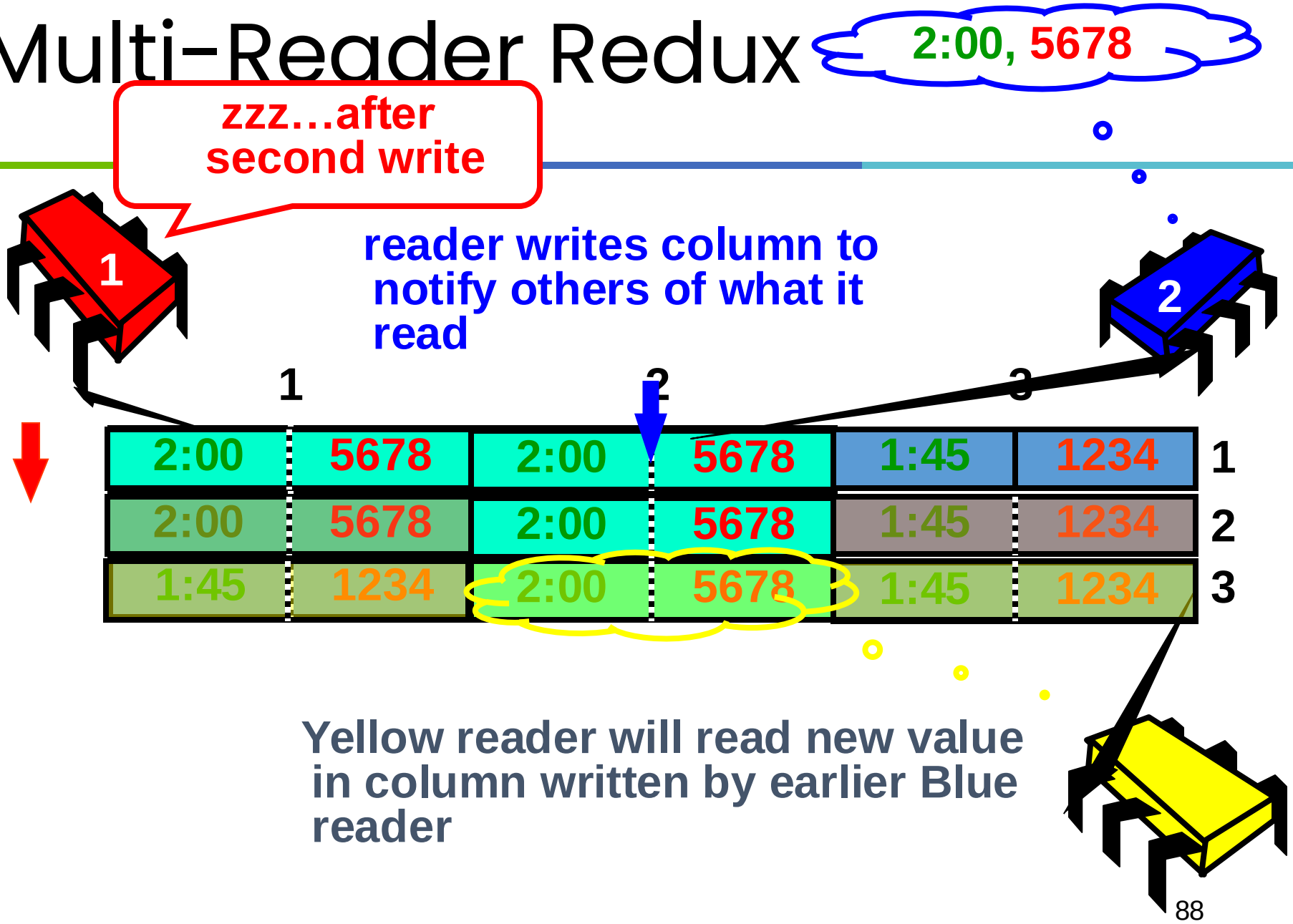
Multi-Reader Redux



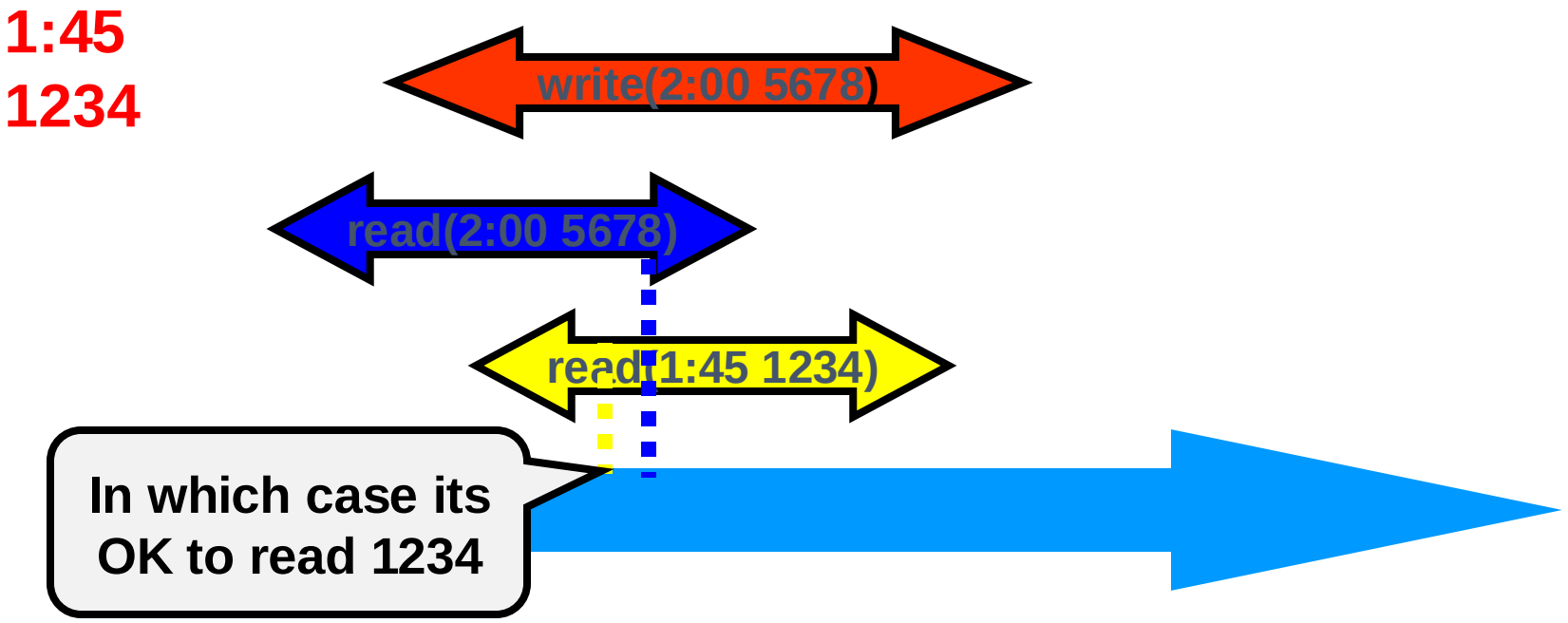
Mu…edux


Writer writes column…
2:00, 5678
1
reader reads row
2
1
2
3
2:00
5678
1:45
1234
1:45
1234
1
2:00
5678
1:45
1234
1:45
1234
2
2:00
5678
1:45
1234
1:45
1234
3

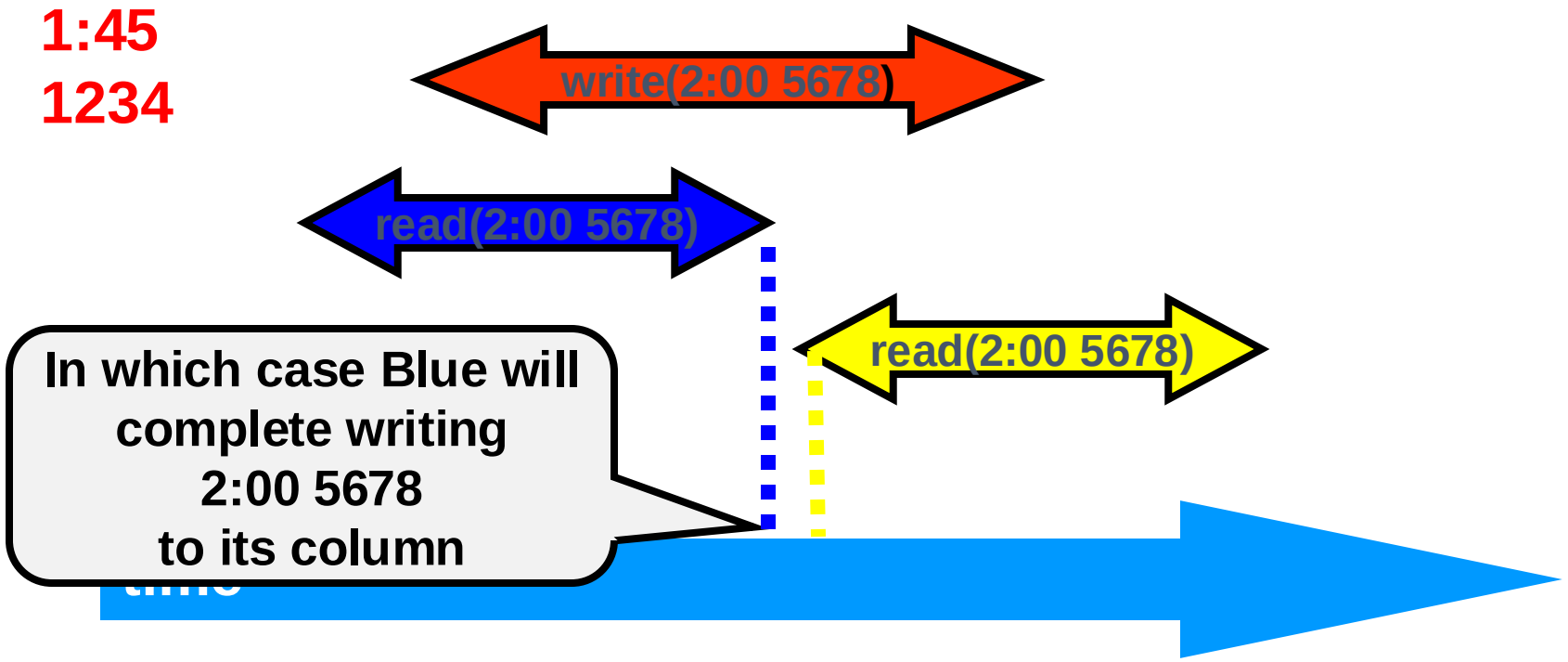
Multi-Reader Redux



Can't Yellow Miss Blue's Update? … Only if Readers Overlap…



Bad Case Only When Readers Don't Overlap



Road Map

- SRSW safe Boolean
- MRSW safe Boolean
- MRSW regular Boolean
- MRSW regular
- MRSW atomic
- MRMW atomic
- Atomic snapshot

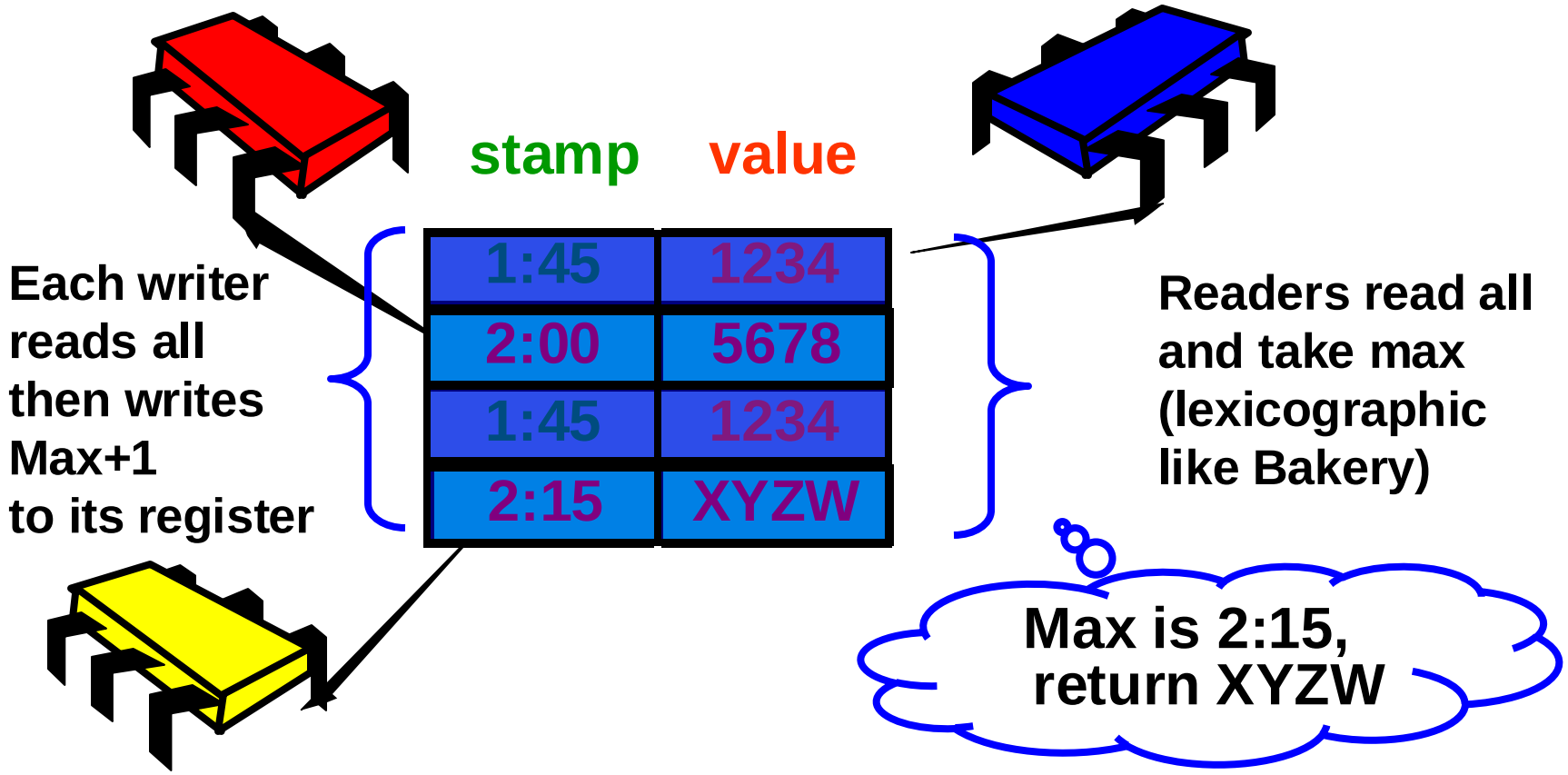
Questions?

Road Map

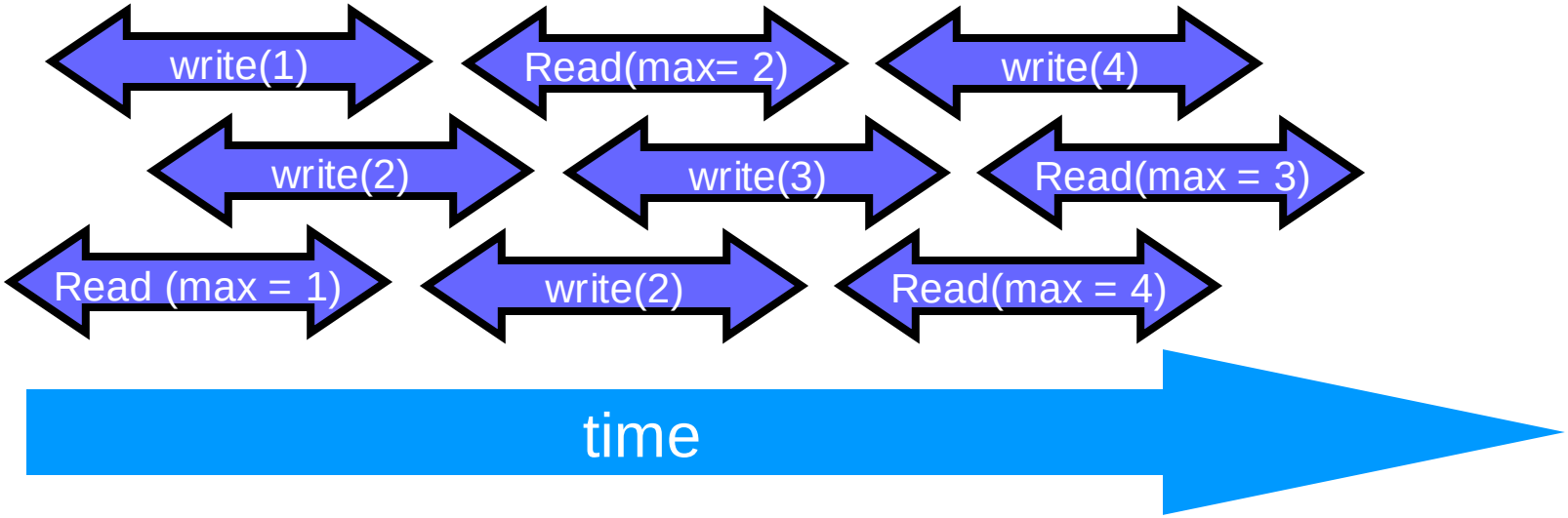
- SRSW safe Boolean
- MRSW safe Boolean
- MRSW regular Boolean
- MRSW regular
- MRSW atomic
- MRMW atomic
- Atomic snapshot

Next

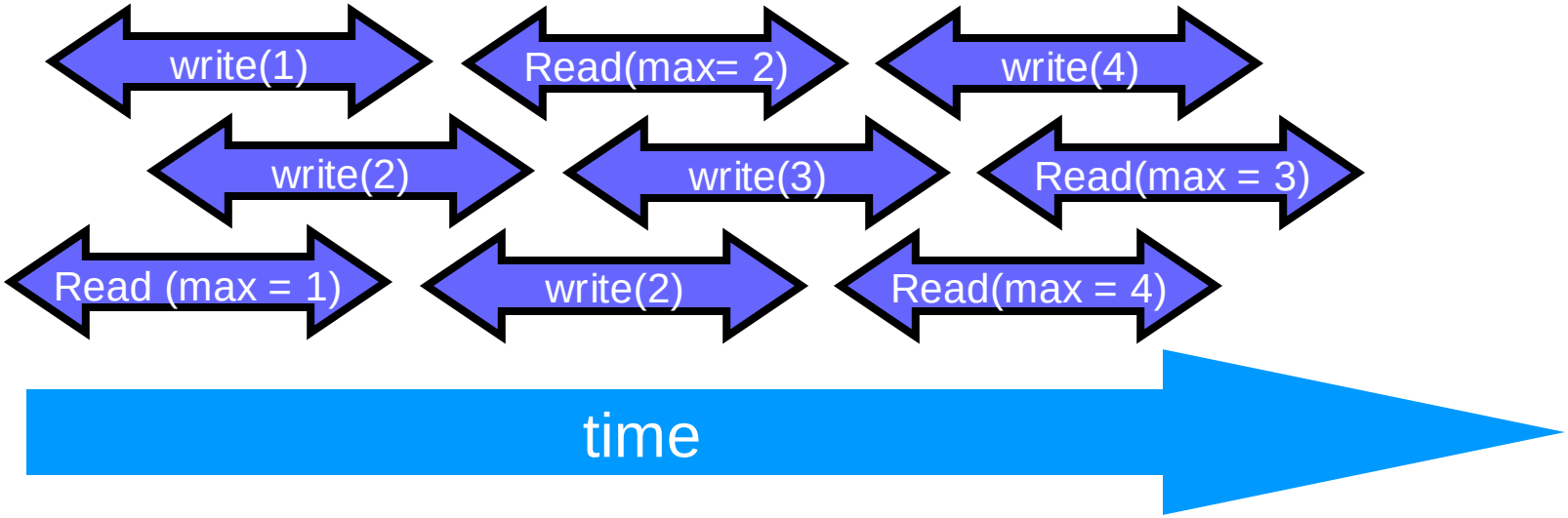
Multi-Writer Atomic From Multi-Reader Atomic



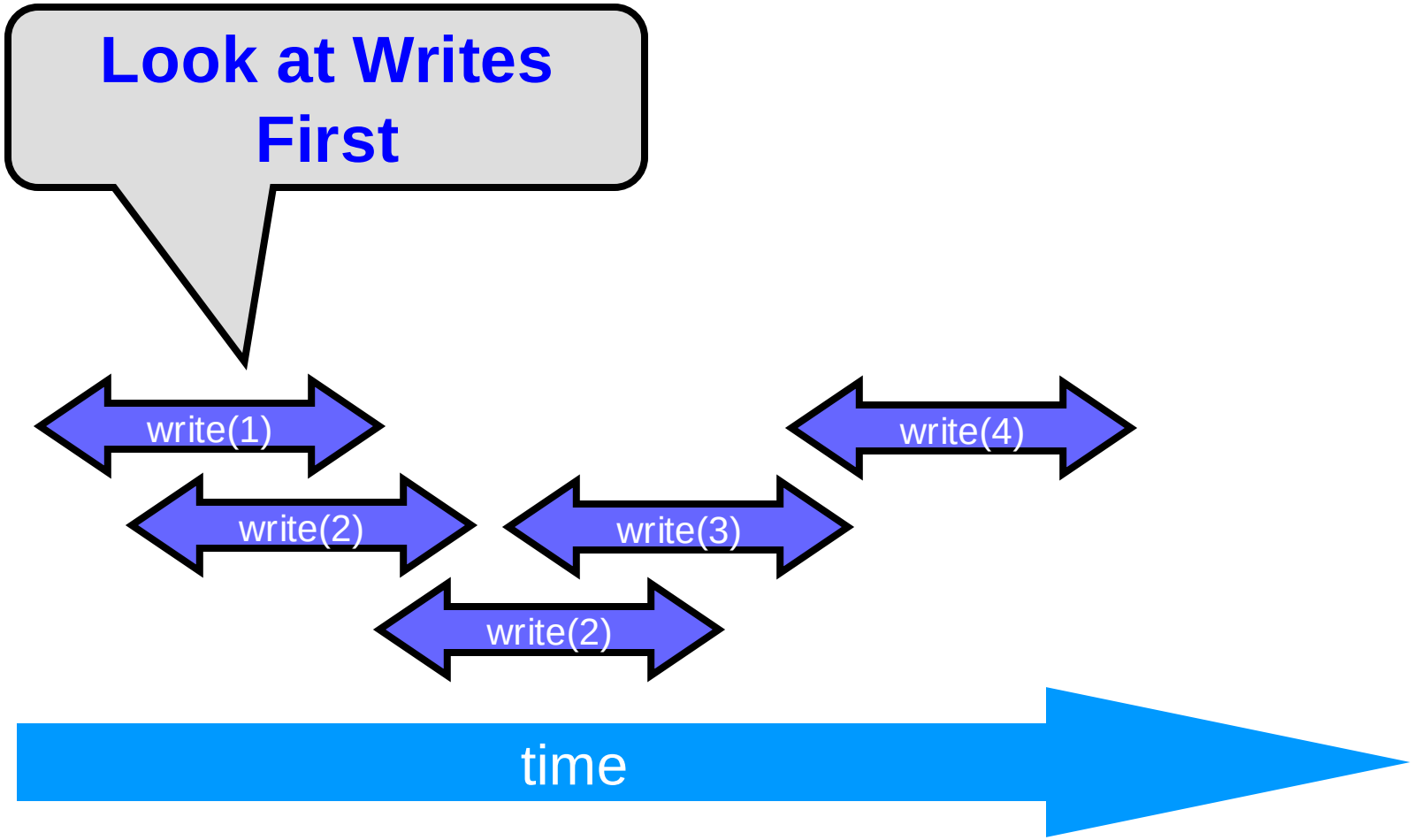
Atomic Execution
Means it is Linearizable



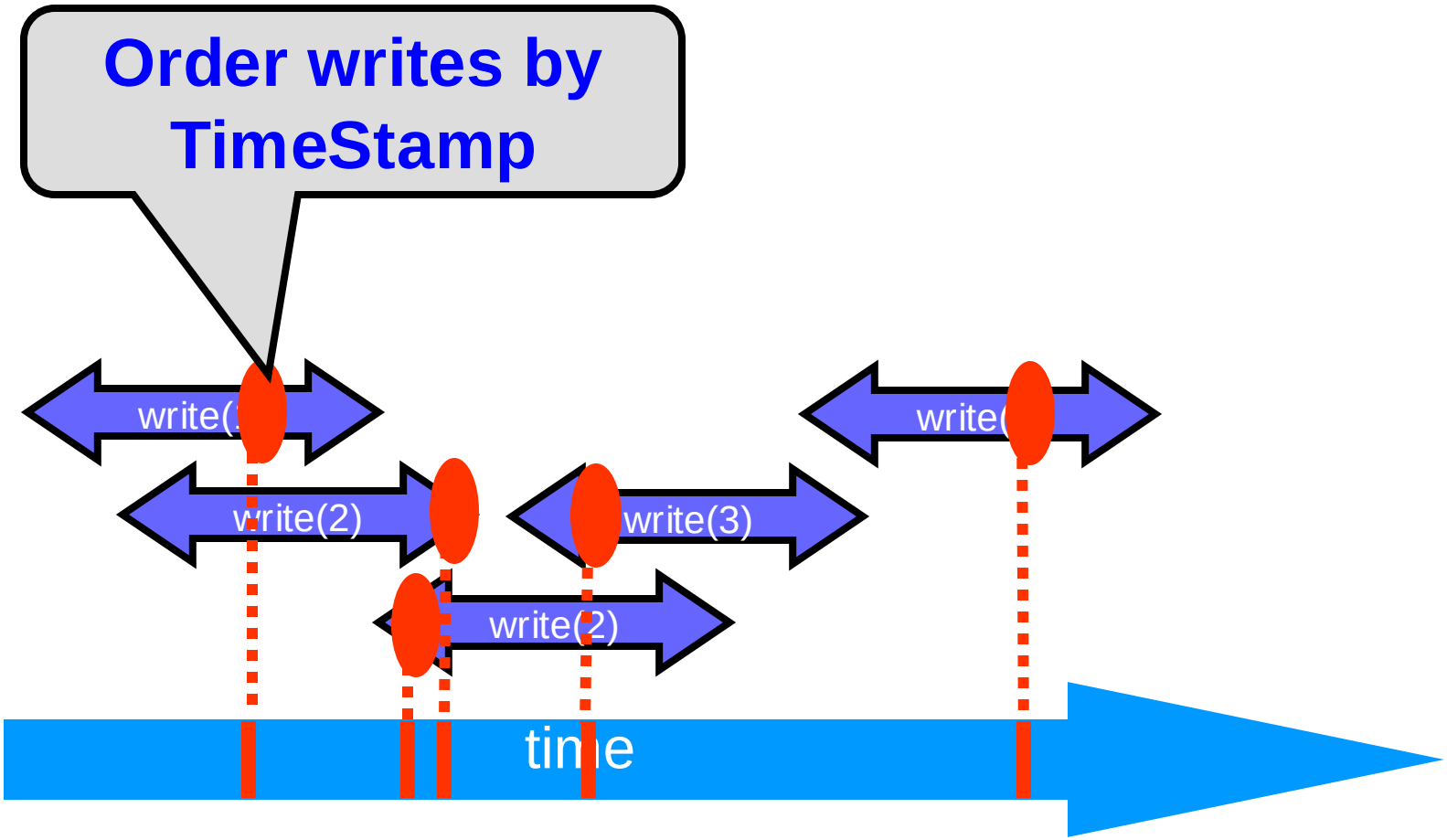
Linearization Points



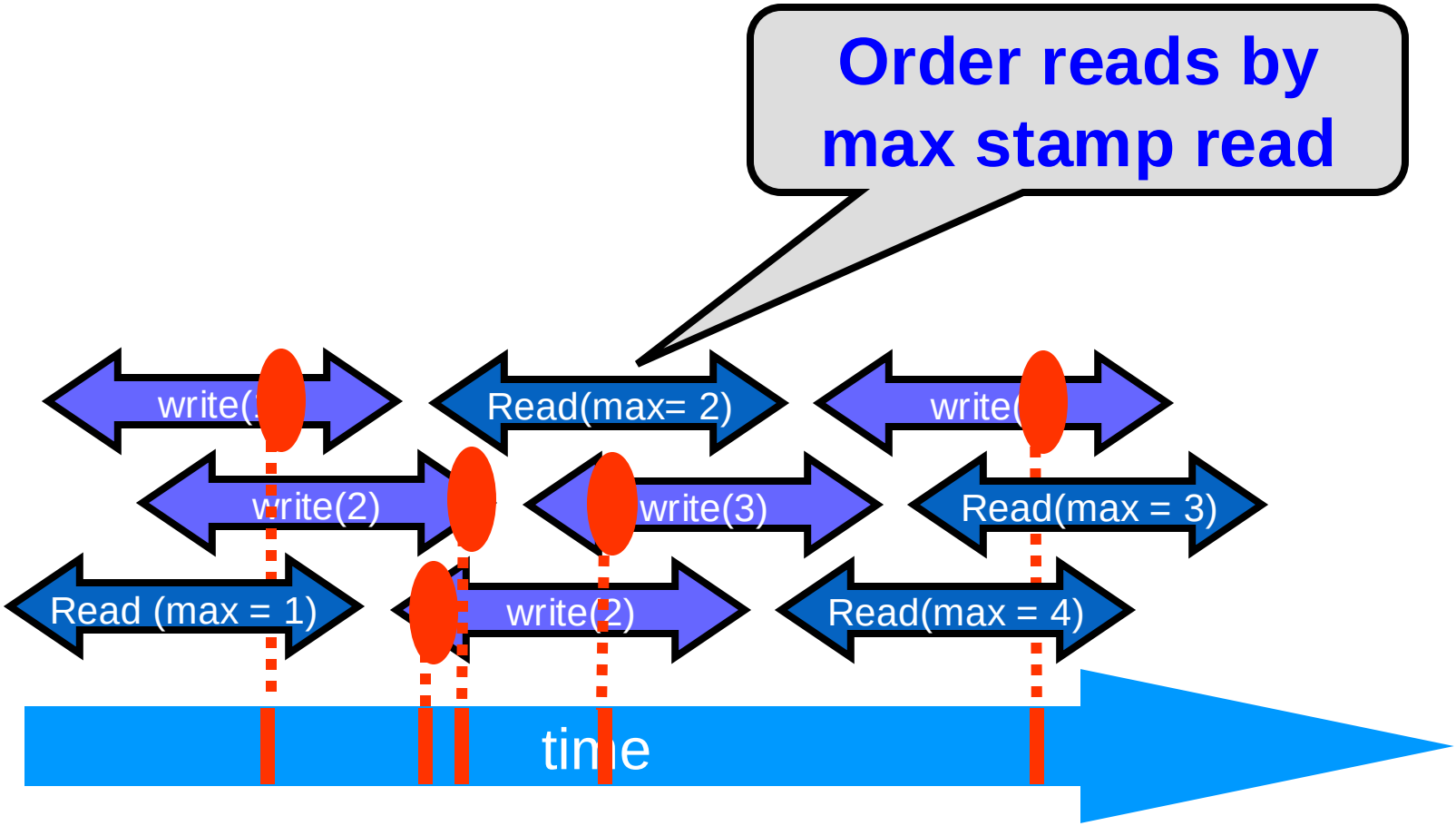
Linearization Points



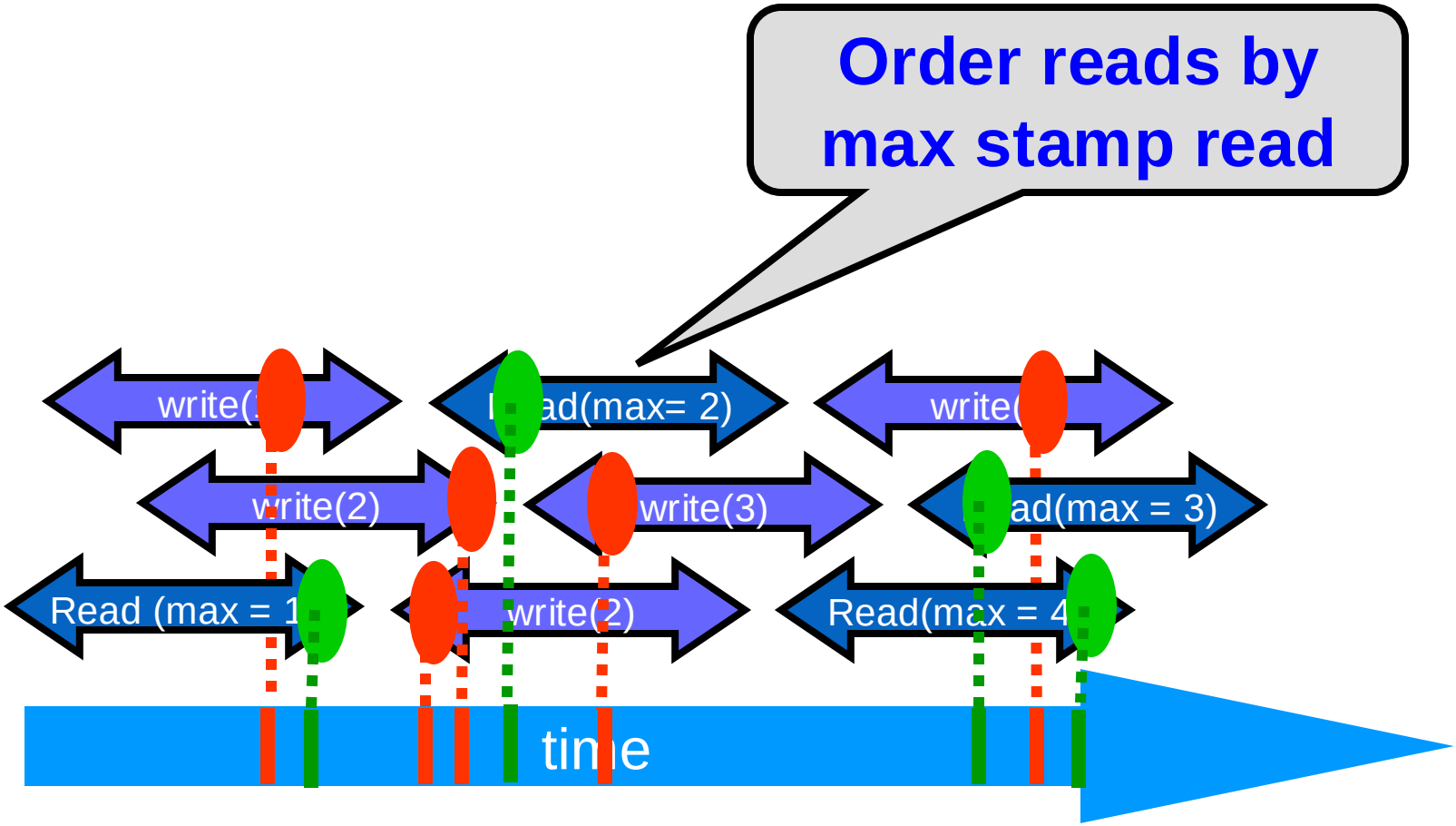
Linearization Points



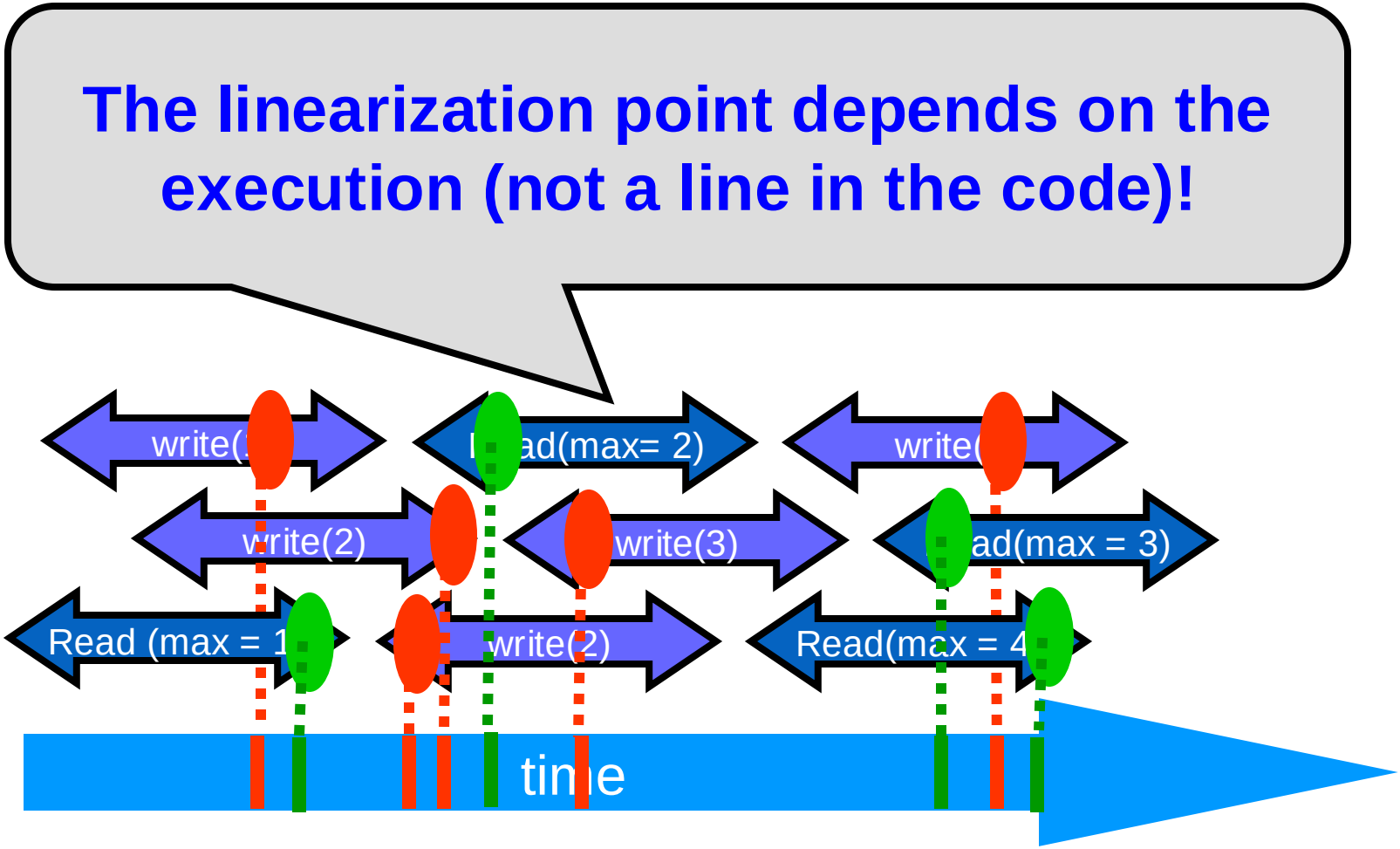
Linearization Points



Linearization Points



Linearization Points



Road Map

- SRSW safe Boolean
- MRSW safe Boolean
- MRSW regular Boolean
- MRSW regular
- MRSW atomic
- MRMW atomic
- Atomic snapshot

Questions?

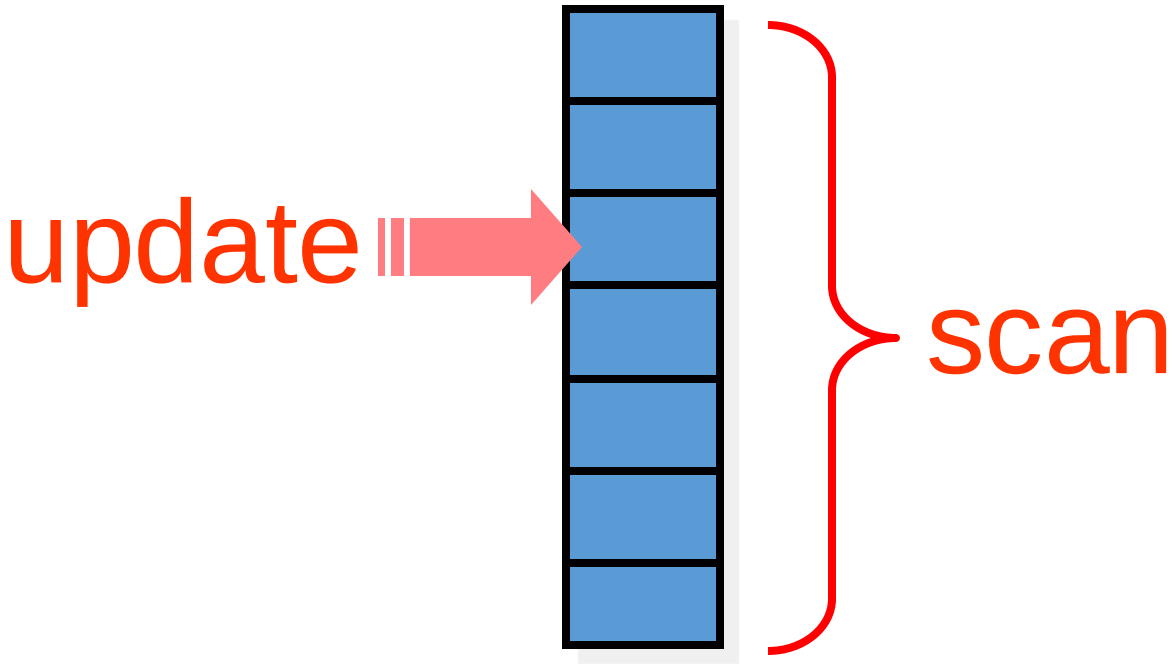
Road Map

- SRSW safe Boolean
- MRSW safe Boolean
- MRSW regular Boolean
- MRSW regular
- MRSW atomic
- MRMW atomic
- Atomic snapshot

Next

Atomic Snapshot

- Atomic multiple reads



Atomic Snapshot

- Array of SWMR atomic registers
- Take instantaneous snapshot of all
- Generalizes to MRMW registers …

Snapshot Interface

```
public interface Snapshot {
  public int update(int v);
  public int[] scan();
}
```

Snapshot Interface

Thread i writes v to its register

```
public interface Snapshot {
  public int update(int v);
  public int[] scan();
}
```

Snapshot Interface

Instantaneous snapshot of all theads' registers

```
public interface Snapshot {
  public int update(int v);
  public int[] scan();
}
```

Atomic Snapshot

- Collect
  - Read values one at a time
- Problem
  - Incompatible concurrent collects
  - Result not linearizable

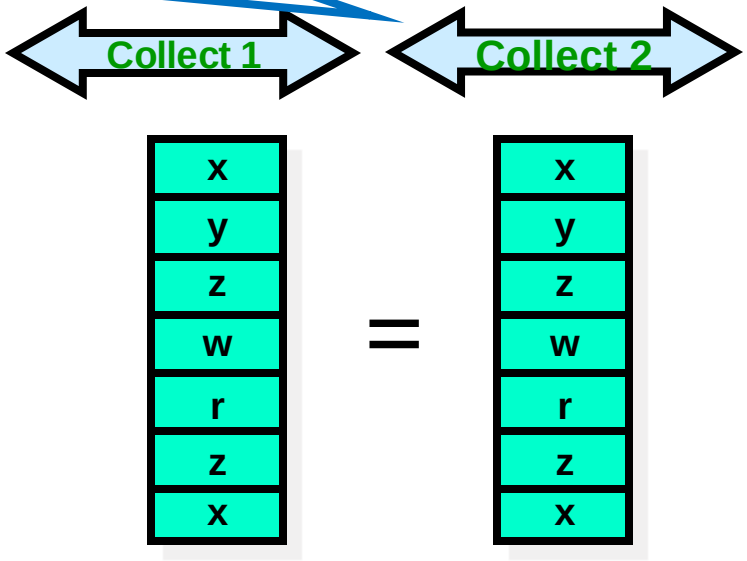
Clean Collects

- Clean Collect
  - Collect during which nothing changed
  - Can we make it happen?
  - Can we detect it?

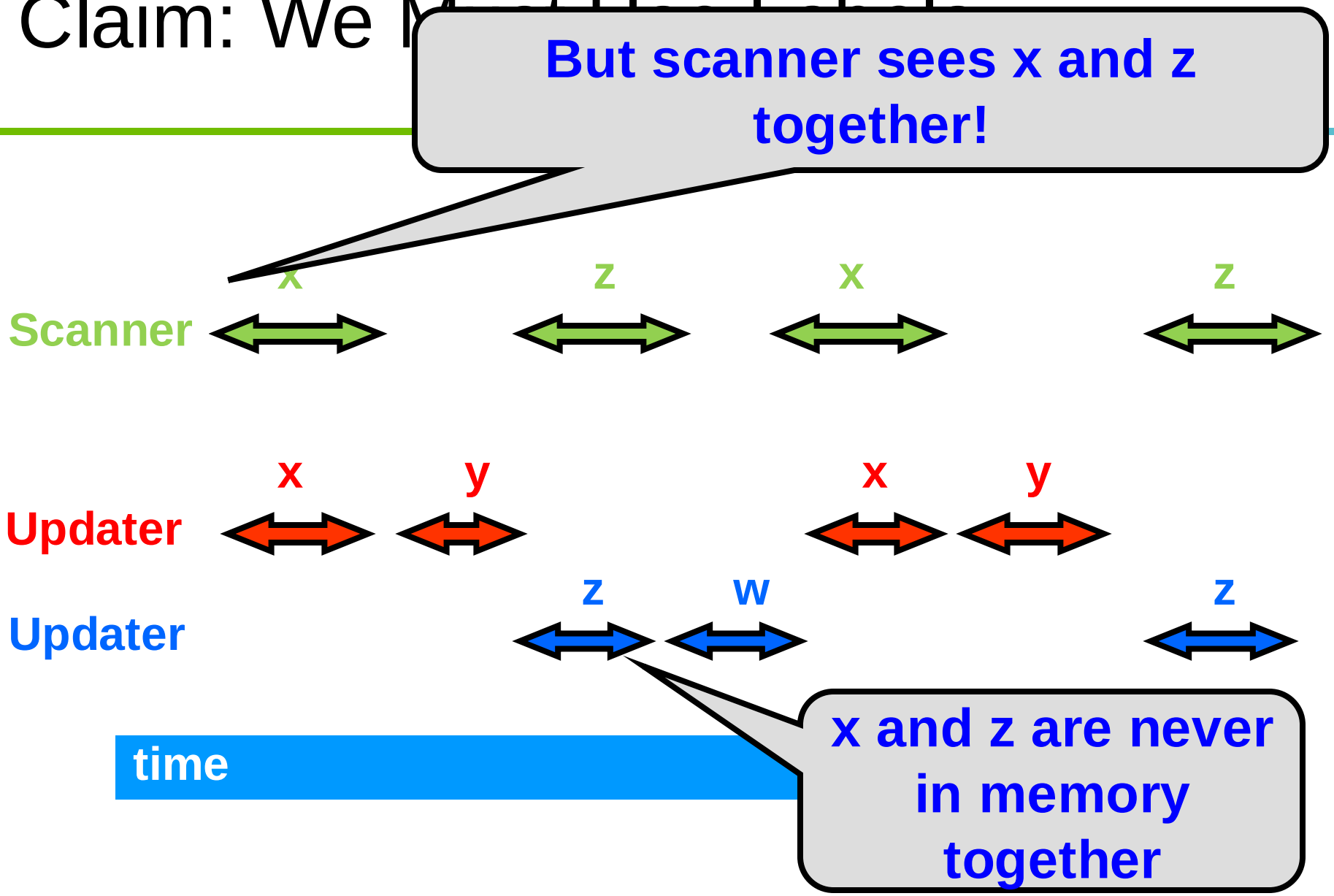
Simple Snapshot

- Put increasing labels on each entry
- C
- I
  - We're done
- Otherwise,
  - Try again

Problem: Scanner might not be collecting a snapshot!

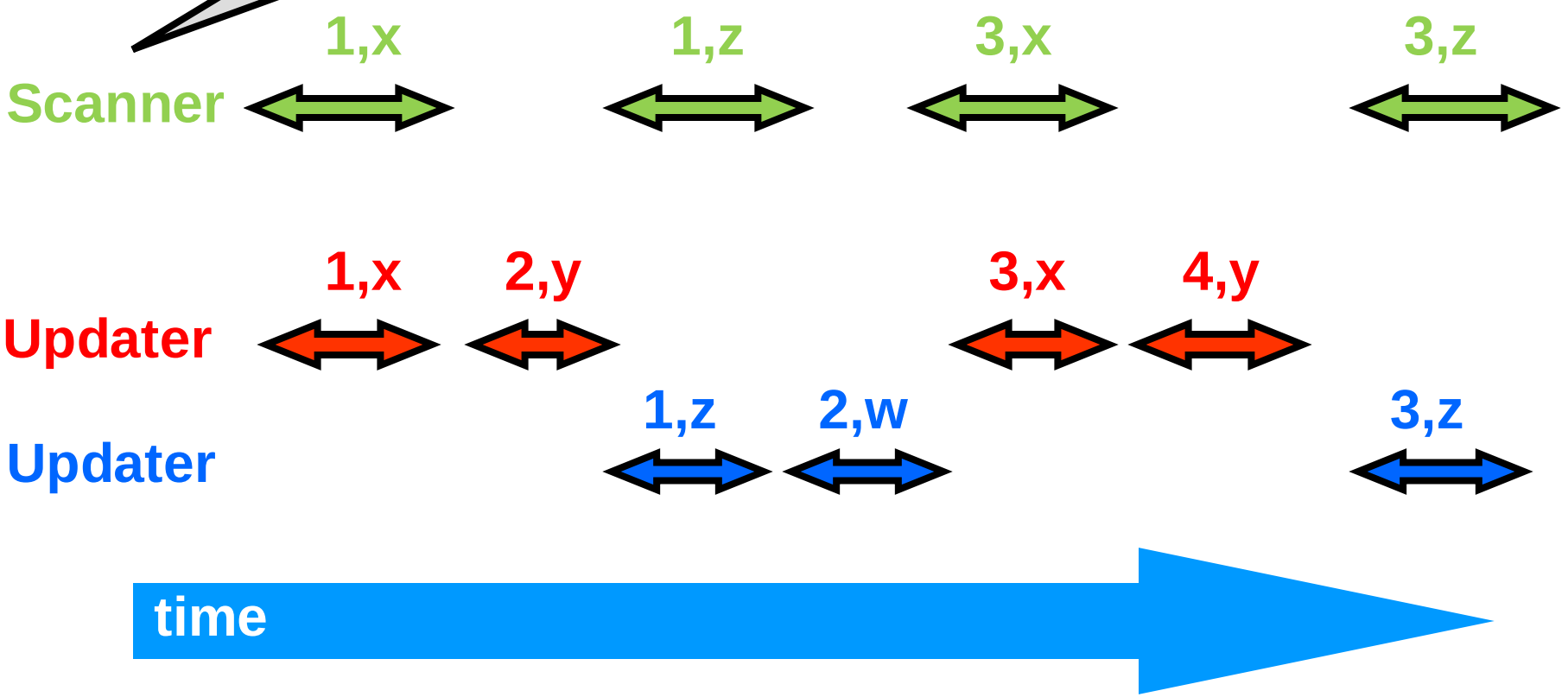


Claim: We M


But scanner sees x and z together!
x
z
x
z
Scanner
x
y
x
y
Updater
z
w
z
Updater
time
x and z are never in memory together

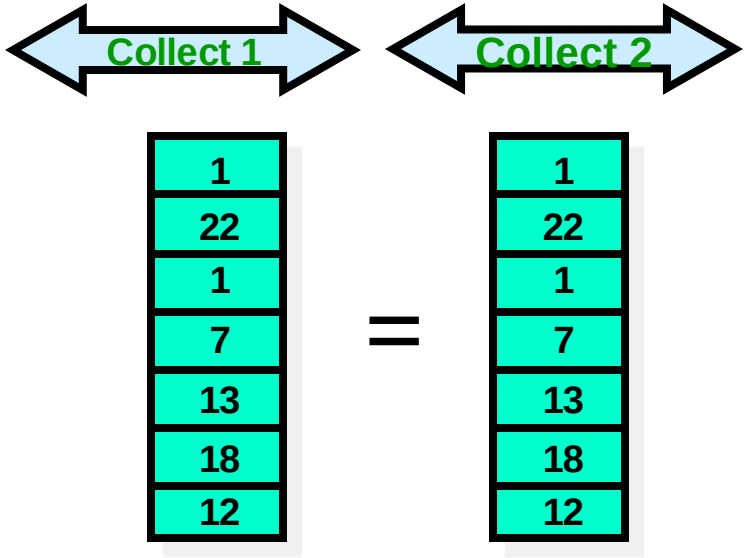
Must Use L

Scanner reads x and z with different labels and recognizes collect not clean



Simple Snapshot

- Collect twice
- If both agree,
  - We're done
- Otherwise,
  - Try again



Simple Snapshot: Update

```
public class SimpleSnapshot implements Snapshot {
  private AtomicMRSWRegister[] register;

  public void update(int value) {
    int i = Thread.myIndex();
       LabeledValue oldValue = register[i].read();
    LabeledValue newValue =
     new LabeledValue(oldValue.label+1, value);
    register[i].write(newValue);
  }
```

Simple Snapshot: Update

```
public class SimpleSnapshot implements Snapshot {
  private AtomicMRSWRegister[] register;

  public void update(int value) {
    int i = Thread.myIndex();
      LabeledValue oldValue = register[i].read();
    LabeledValue newValue =
     new LabeledValue(oldValue.label+1, value);
    register[i].write(newValue);
  }
```

One single-writer register per thread

Simple Snapshot: Update

```
public class SimpleSnapshot implements Snapshot {
  private AtomicMRSWRegister[] register;

  public void update(int value) {
    int i = Thread.myIndex();
    LabeledValue oldValue = register[i].read();
    LabeledValue newValue =
     new LabeledValue(oldValue.label+1, value);
    register[i].write(newValue);
  }
```

Write each time with higher label

Simple Snapshot: Collect

```
private LabeledValue[] collect() {
 LabeledValue[] copy =
  new LabeledValue[n];
 for (int j = 0; j < n; j++)
  copy[j] = this.register[j].read();
 return copy;
}
```

Simple Snapshot

```
private LabeledValue[] collect() {
 LabeledValue[] copy =
  new LabeledValue[n];
 for (int j = 0; j < n; j++)
  copy[j] = this.register[j].read();
 return copy;
}
```

Just read each register into array

Simple Snapshot: Scan

```
public int[] scan() {
 LabeledValue[] oldCopy, newCopy;
 oldCopy = collect();
 collect: while (true) {
  newCopy = collect();
  if (!equals(oldCopy, newCopy)) {
     oldCopy = newCopy;
     continue collect;
  }
  return getValues(newCopy);
}}
```

Simple Snapshot: Scan

```
public int[] scan() {
 LabeledValue[] oldCopy, newCopy;
 oldCopy = collect();
 collect: while (true) {
  newCopy = collect();
  if (!equals(oldCopy, newCopy)) {
     oldCopy = newCopy;
     continue collect;
  }
  return getValues(newCopy);
}}
```

Collect once

Simple Snapshot: Scan

```
public int[] scan() {
 LabeledValue[] oldCopy, newCopy;
 oldCopy = collect();
 collect: while (true) {
  newCopy = collect();
   if (!equals(oldCopy, newCopy)) {
      oldCopy = newCopy;
      continue collect;
   }
   return getValues(newCopy);
}}
```

Collect once

Collect twice

Simple Snapshot: Scan

```
public int[] scan() {
 LabeledValue[] oldCopy, newCopy;
 oldCopy = collect();
 collect: while (true) {
  newCopy = collect();
  if (!equals(oldCopy, newCopy)) {
     oldCopy = newCopy;
     continue collect;
  }
  return getValues(newCopy);
}}
```

Collect once

Collect twice

On mismatch, try again

Simple Snapshot: Scan

```
public int[] scan() {
 LabeledValue[] oldCopy, newCopy;
 oldCopy = collect();
 collect: while (true) {
  newCopy = collect();
   if (!equals(oldCopy, newCopy)) {
      oldCopy = newCopy;
      continue collect;
   }
  return getValues(newCopy);
}}
```

Collect once

Collect twice

On match, return values

Simple Snapshot

- Linearizable
- Update is wait-free
  - No unbounded loops
- But Scan can starve
  - If interrupted by concurrent update

Wait-Free Snapshot

- Add a scan before every update
- Write resulting snapshot together with update value
- If scan is continuously interrupted by updates, scan can take the update's snapshot

Implementation of a wait-free snapshot is quite complex!

Study book's Chapter 4!

Observations

- Uses unbounded counters
  - can be replaced with 2 bits
- Assumes SWMR registers
  - for labels
  - can be extended to MRMW

Implementation of a wait-free snapshot is quite complex!

Study book's Chapter 4!

Summary

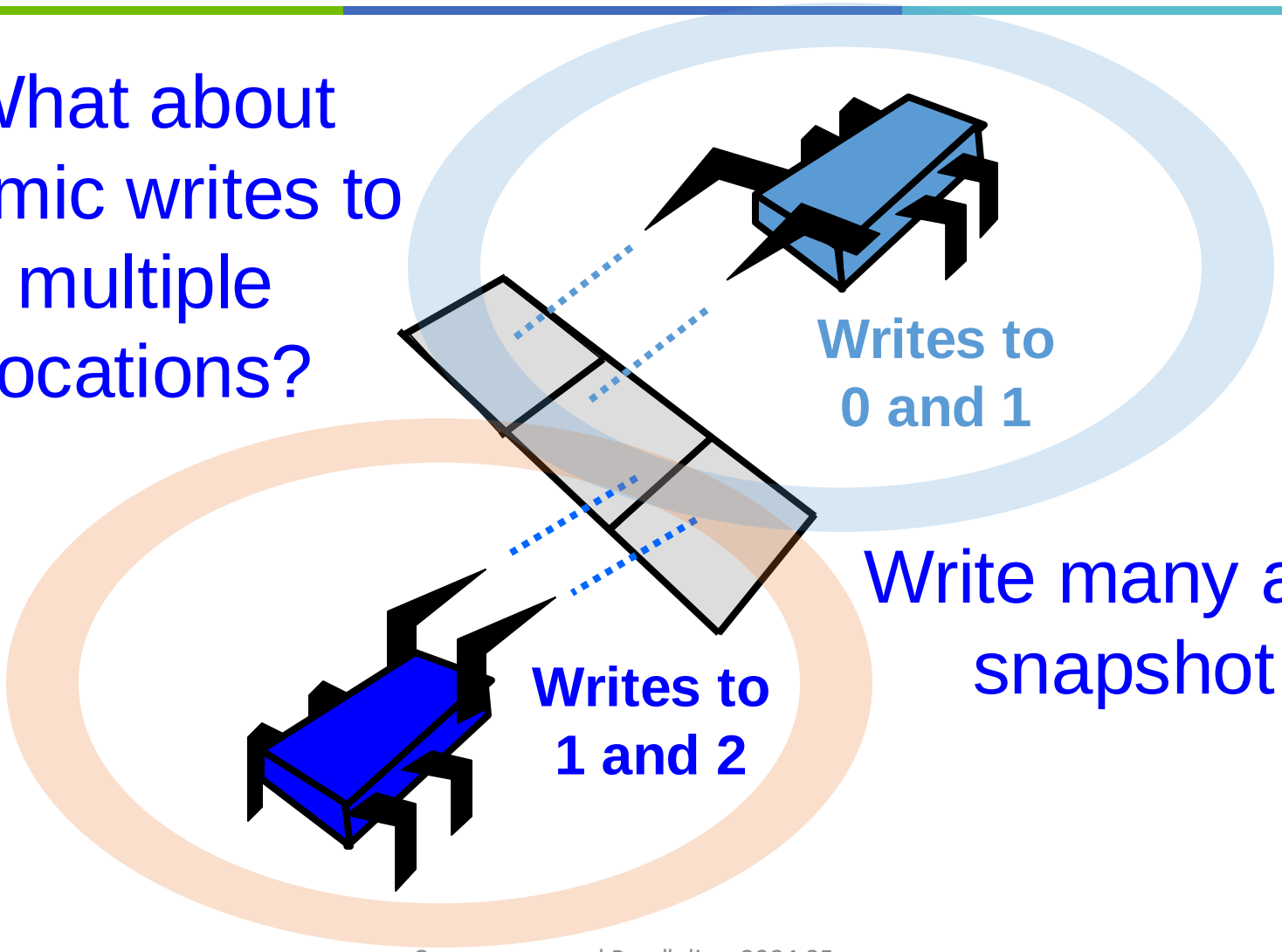
- We saw we could implement MRMW multi-valued snapshot objects
- From SRSW binary safe registers (simple flipflops)
- But what is the next step to attempt with read-write registers?

Grand Challenge

- Snapshot means
  - Write any one array element
  - Read multiple array elements

Grand Challenge

What about atomic writes to multiple locations?



Write many and snapshot

The END