



FACULDADE DE  
CIÊNCIAS E TECNOLOGIA  
DEPARTAMENTO DE INFORMÁTICA

# Concurrent Objects

lectures 05 & 06 (2025-03-24)

**Master in Computer Science and Engineering**

— Concurrency and Parallelism / 2024-25 —

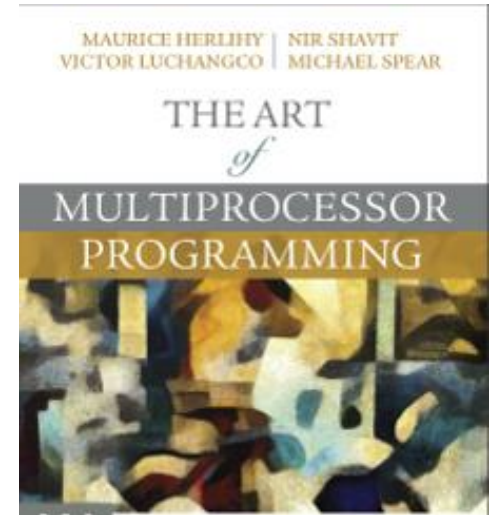
João Lourenço <[joao.lourenco@fct.unl.pt](mailto:joao.lourenco@fct.unl.pt)>

Based in slides the companion slides from “The Art of Multiprocessor Programming”

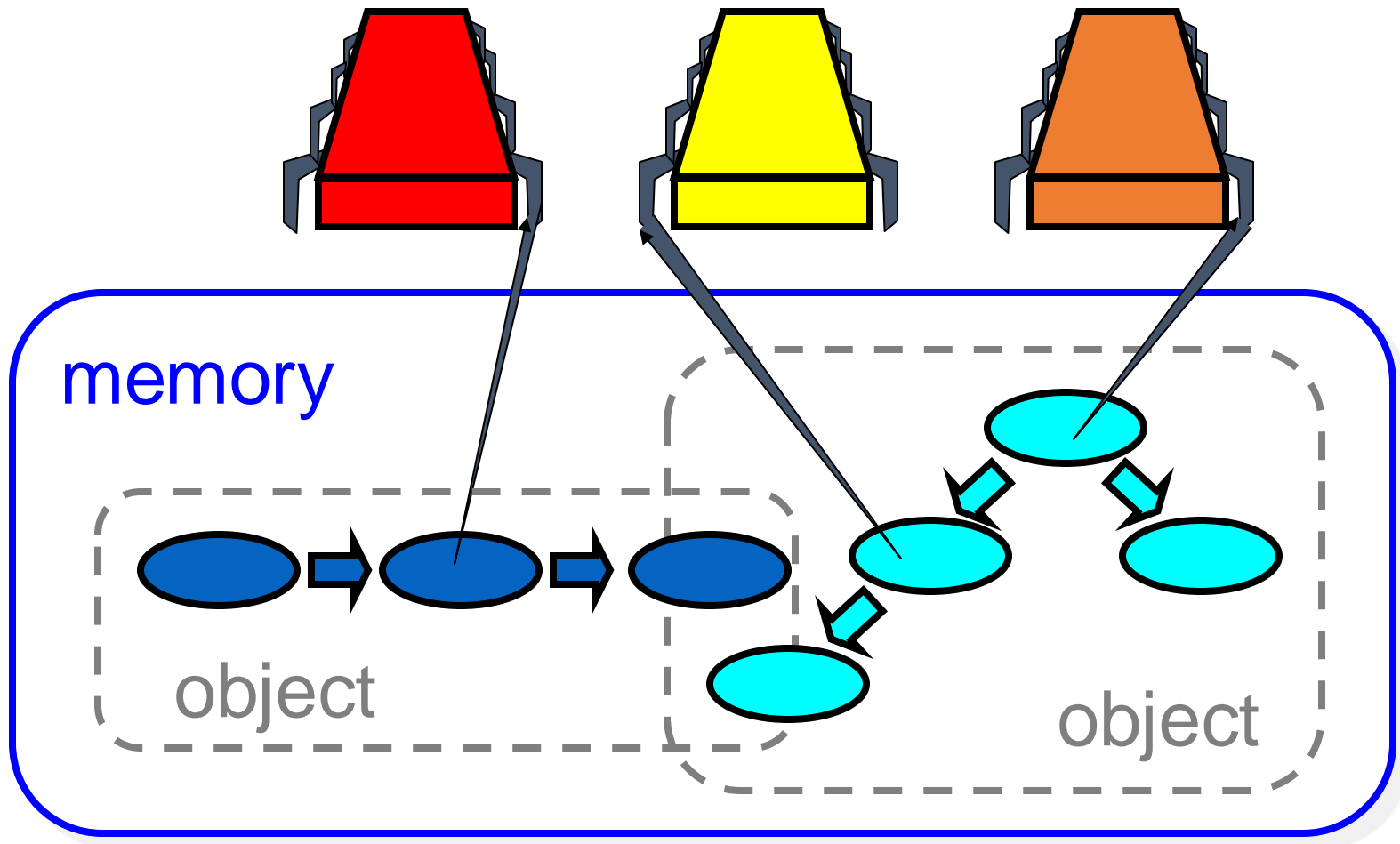
# Outline

- Concurrent Objects
  - Correctness
  - Sequential Objects
  - Quiescent and Sequential Consistency
  - Linearizability
  - Progress Conditions
- Bibliography:
  - **Chapters 3** of book

Herlihy M., Shavit N., Luchangco V., Spear M.;  
**The Art of Multiprocessor Programming**;  
Morgan Kaufmann (2020); ISBN: 978-0-12-415950-1



# Concurrent Computation



# Objectivism

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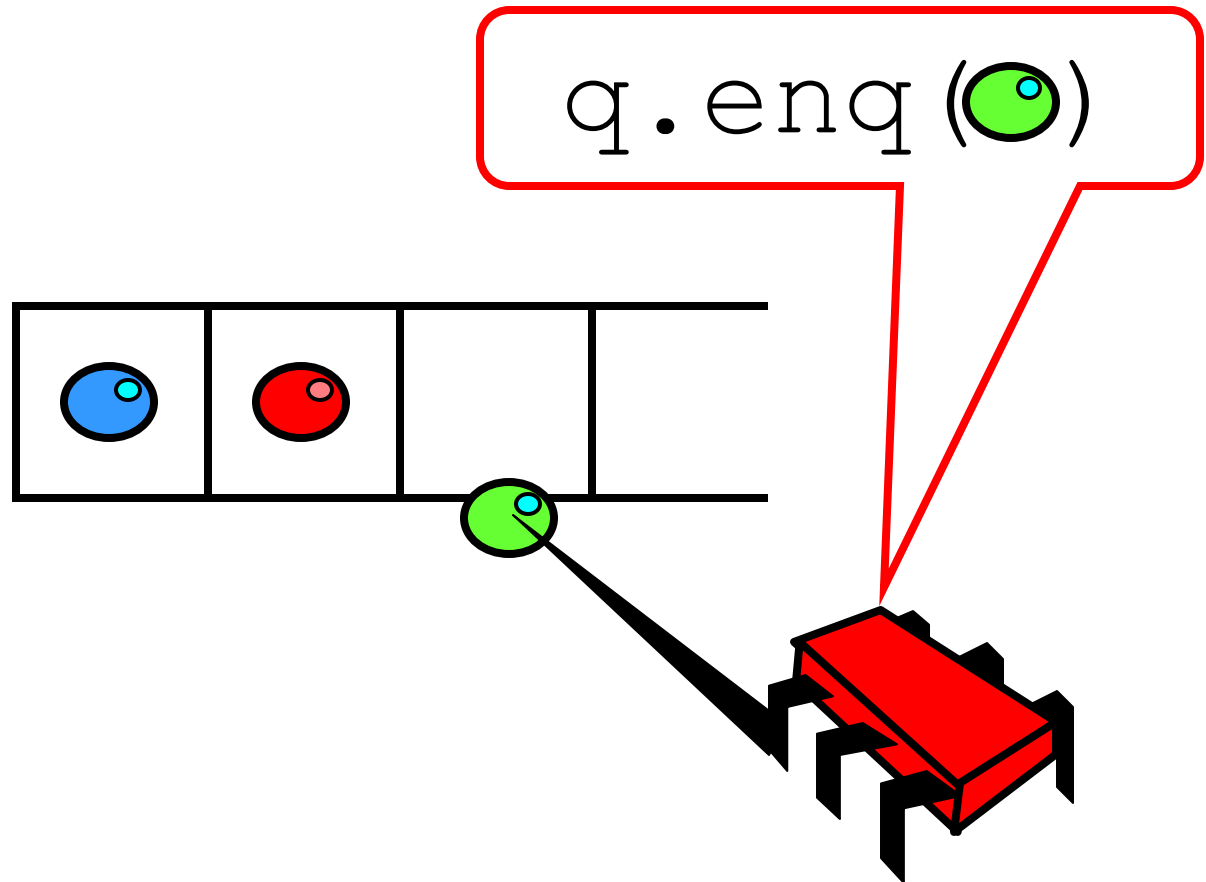
- What is a concurrent object?
  - How do we **describe** one?
  - How do we **implement** one?
  - How do we **tell if we're right**?

# Objectivism

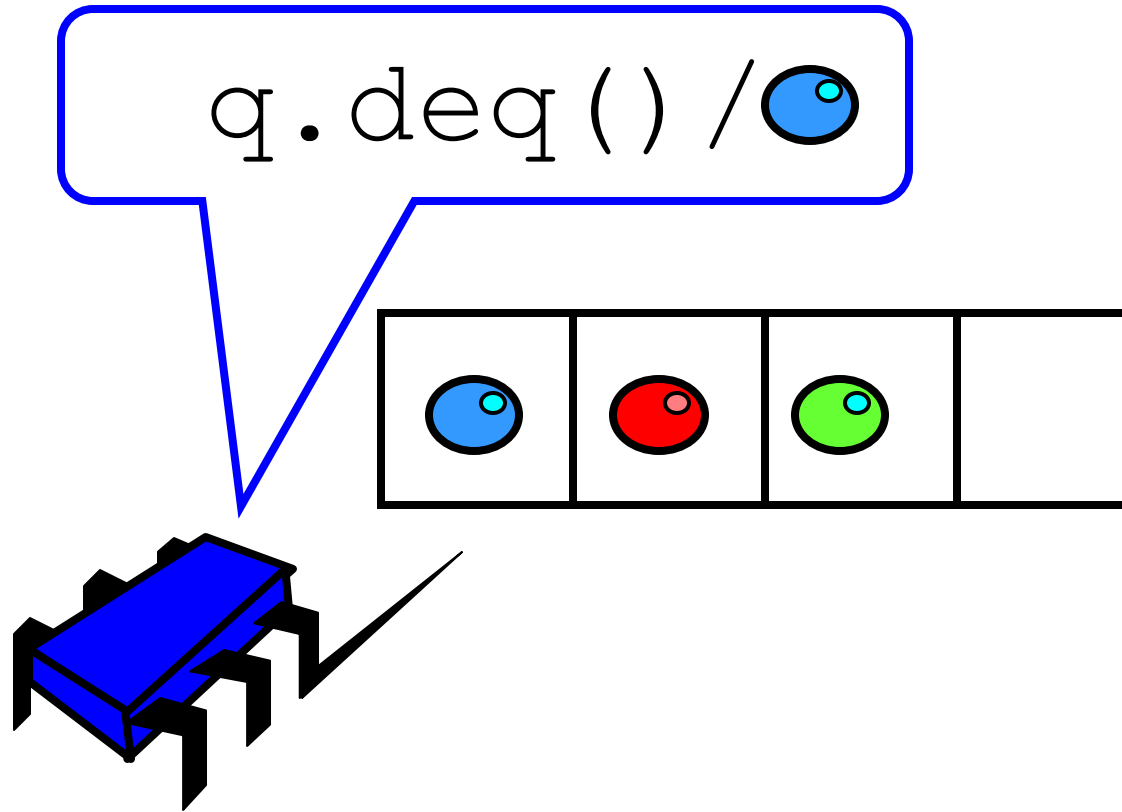
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- What is a concurrent object?
  - How do we **describe** one?
  - How do we **tell if we're right**?

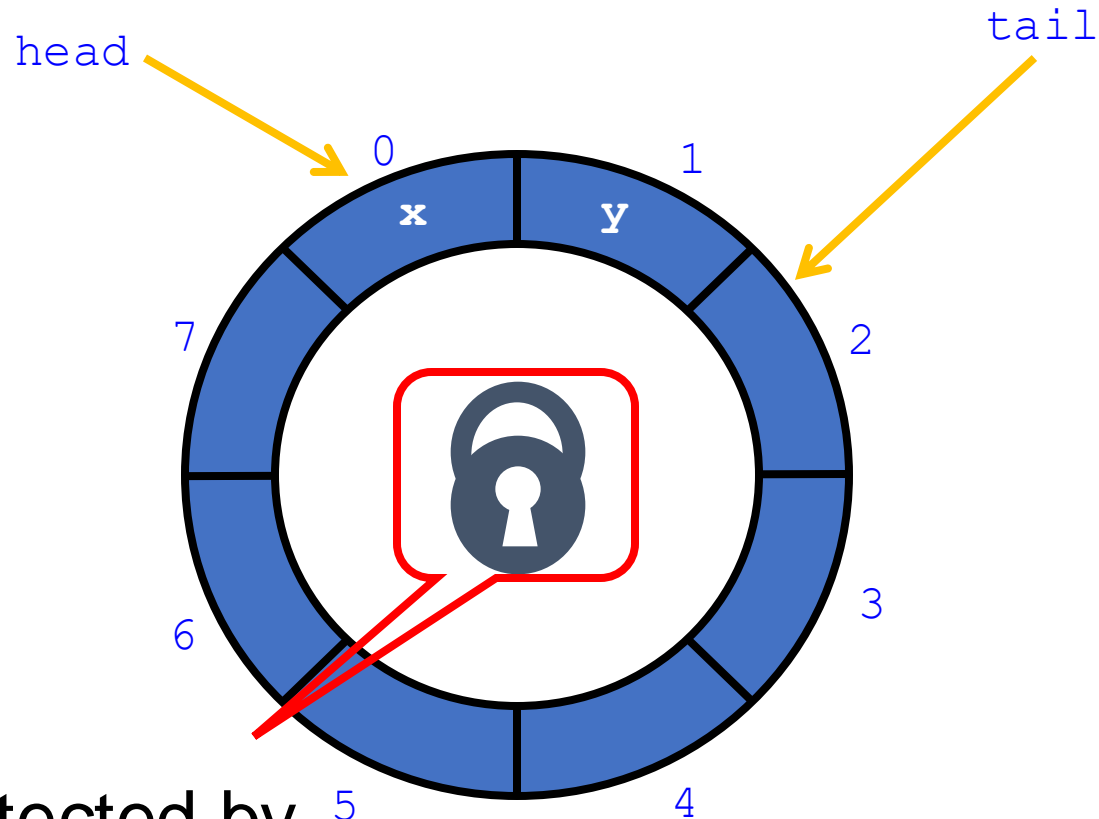
# FIFO Queue: Enqueue Method



# FIFO Queue: Dequeue Method



# Lock-Based Queue



Fields protected by  
single shared lock

capacity = 8

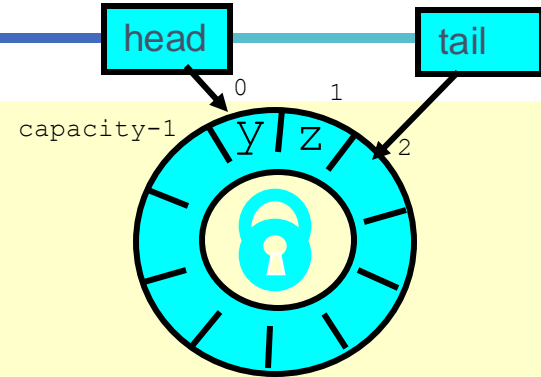


# A Lock-Based Queue

```
class LockBasedQueue<T> {  
    int head, tail;  
    T[] items;  
    Lock lock;  
    public LockBasedQueue(int capacity) {  
        head = 0; tail = 0;  
        lock = new ReentrantLock();  
        items = (T[]) new Object[capacity];  
    }  
}
```

# A Lock-Based Queue

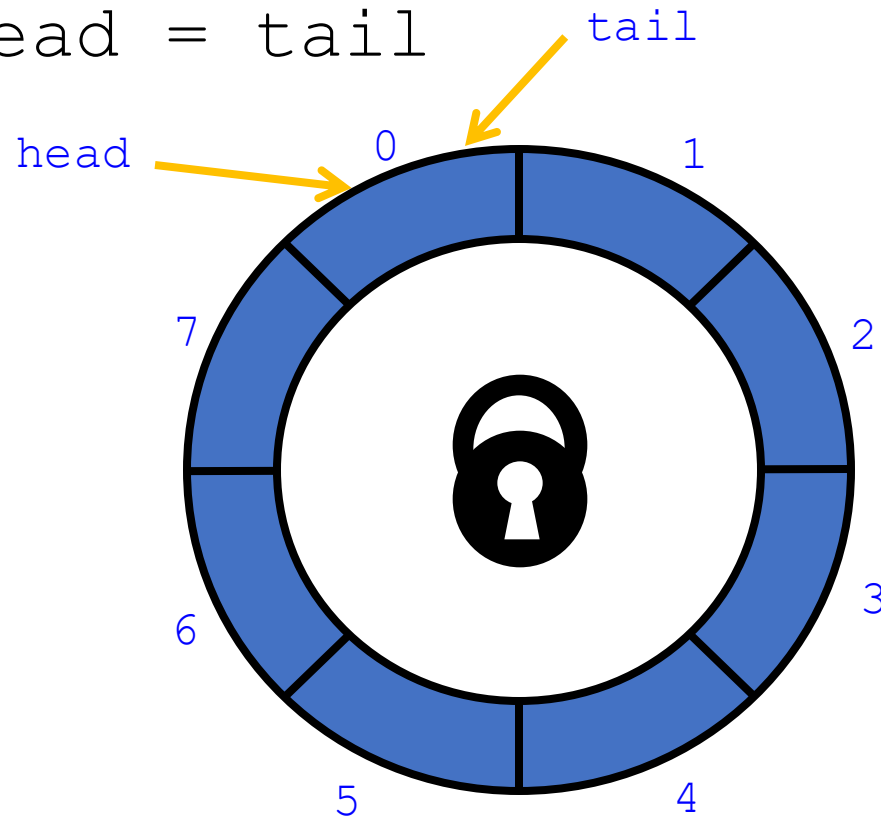
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Fields protected by  
single shared lock

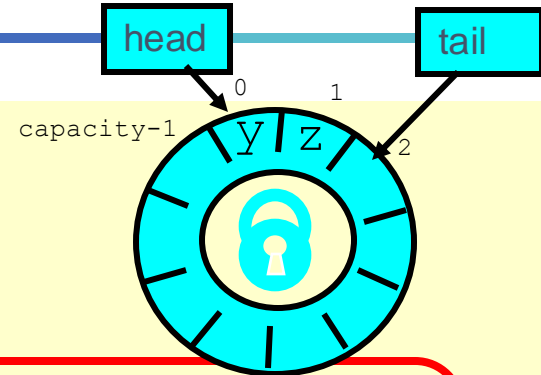
# Lock-Based Queue

Initially: `head = tail`



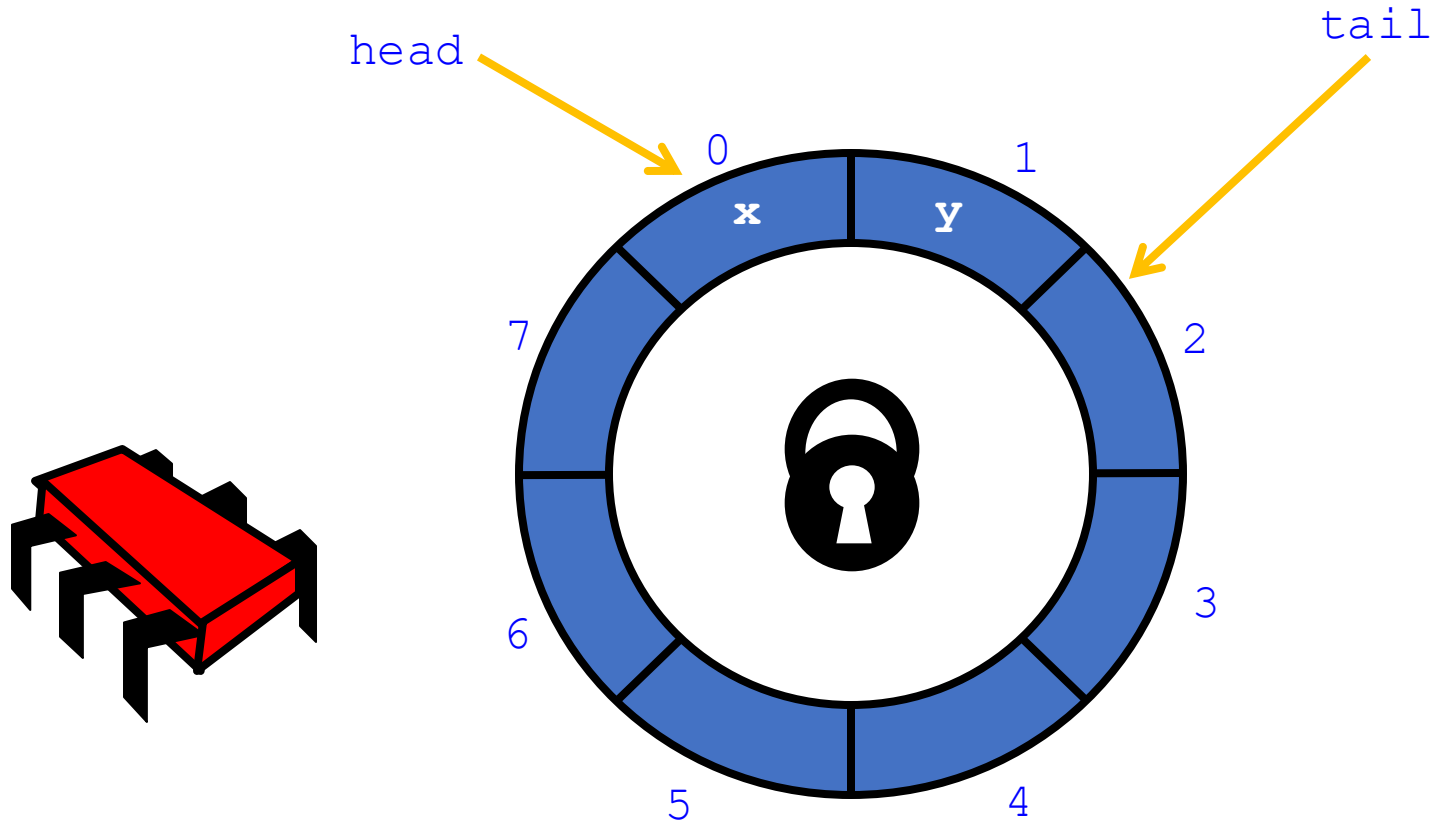
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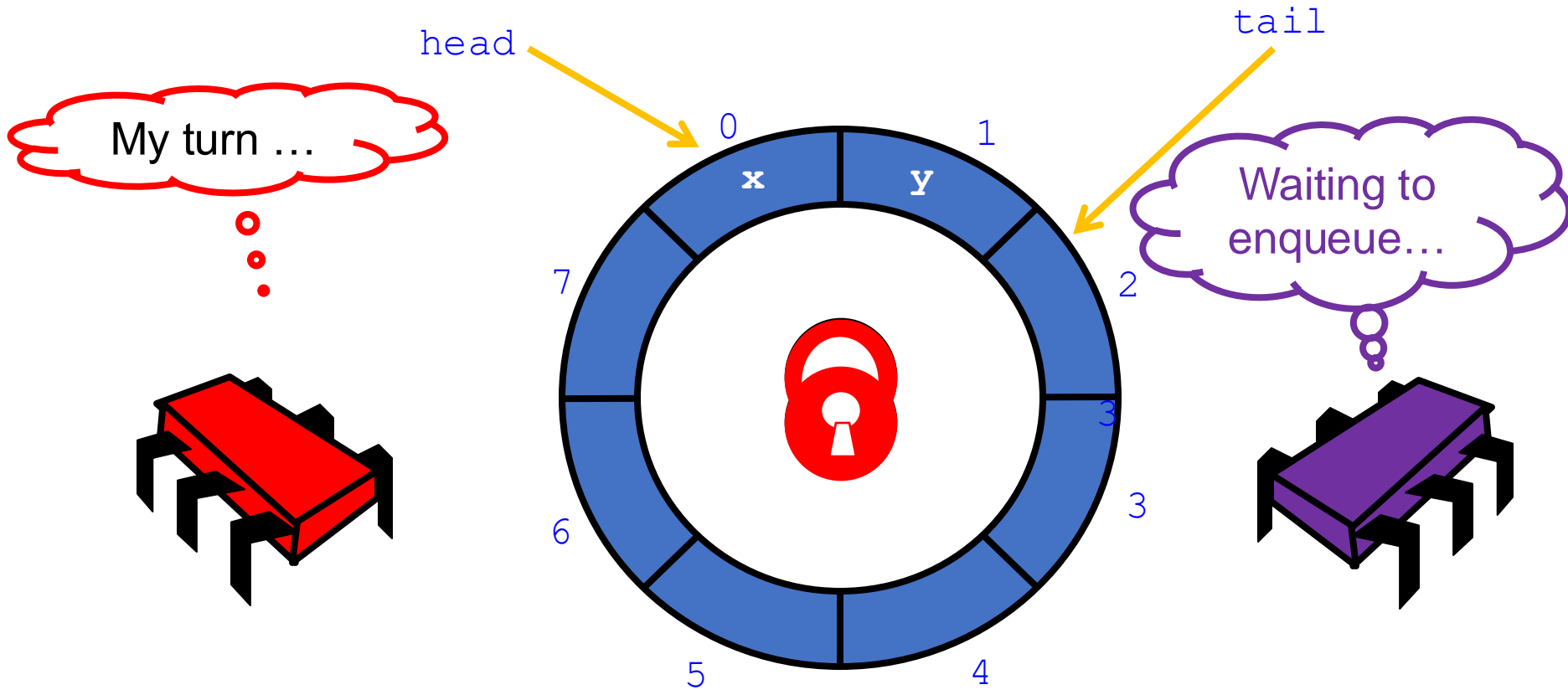


Initially head = tail

# Lock-Based `deq()`



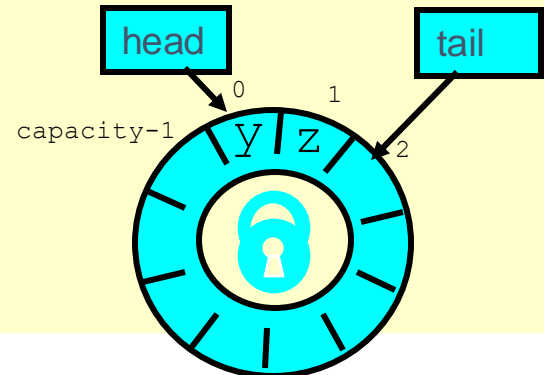
# Acquire Lock



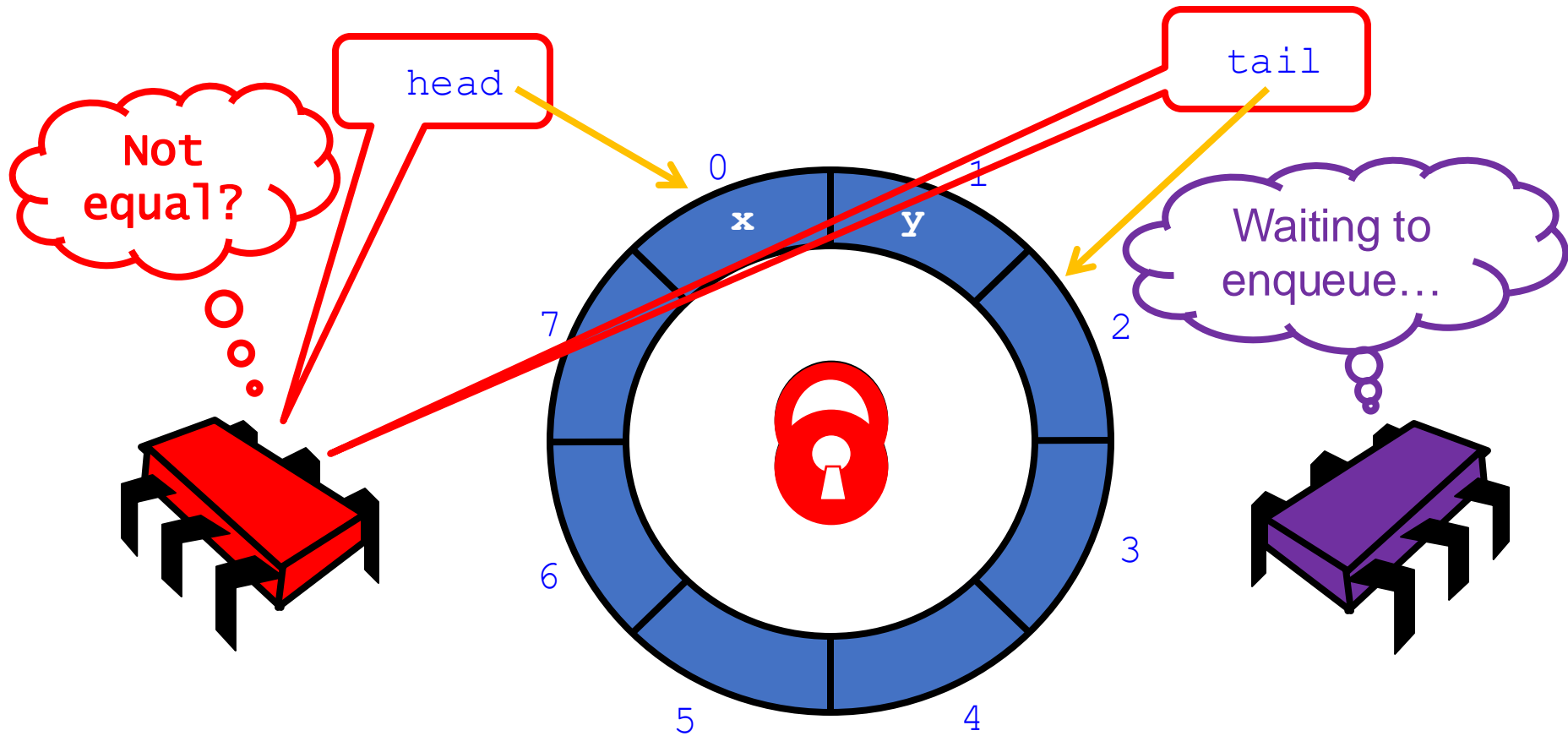
# Implementation: `deq()`

```
public T deq() throws EmptyException {  
    lock.lock();  
    try {  
        if (tail == head)  
            throw new EmptyException();  
        T x = items[head % items.length];  
        head++;  
        return x;  
    } finally {  
        lock.unlock();  
    }  
}
```

Acquire lock at  
method start



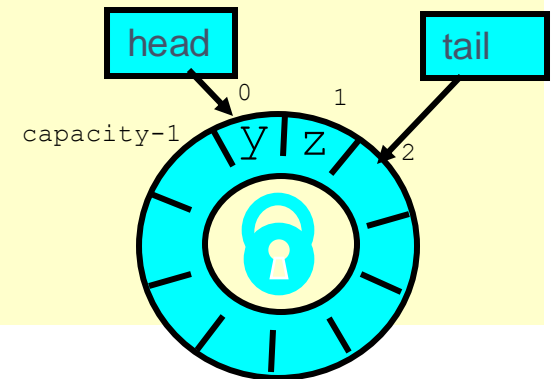
# Check if Non-Empty





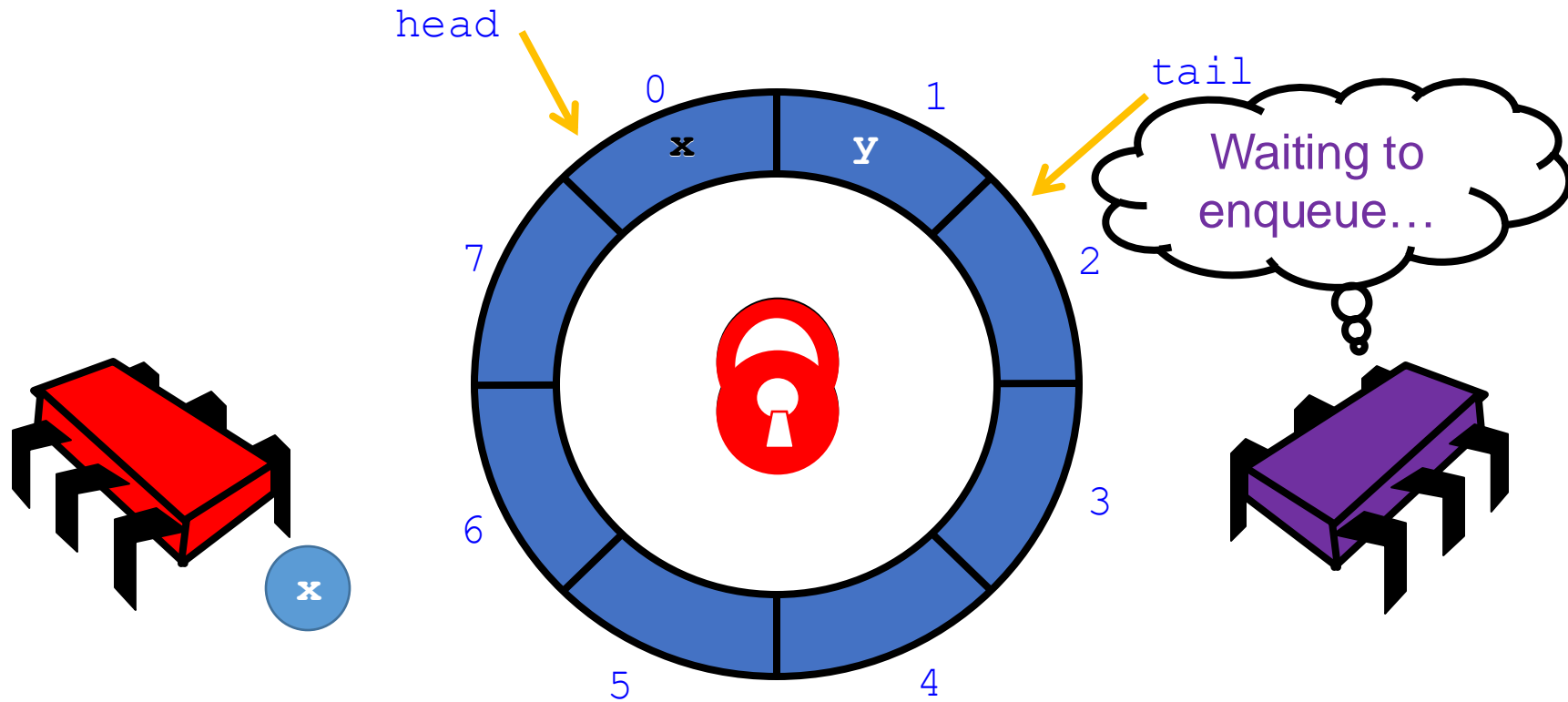
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        head++;  
        return x;  
    } finally {  
        lock.unlock();  
    }  
}
```



If queue empty  
throw exception

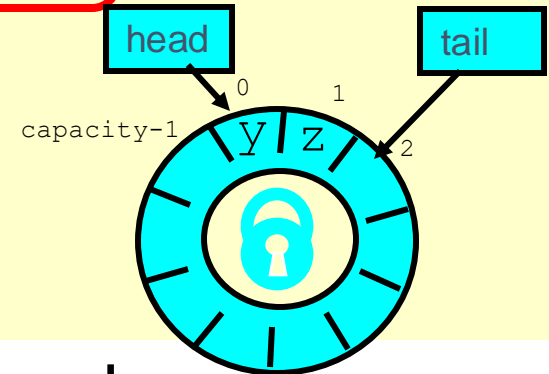
# Modify the Queue



# Implementation: `deq()`

```
public T deq() throws EmptyException {  
    lock.lock();  
    try {  
        if (tail == head)  
            throw new EmptyException();  
        T x = items[head % items.length];  
        head++;  
        return x;  
    } finally {  
        lock.unlock();  
    }  
}
```

**T x = items[head % items.length];  
head++;**

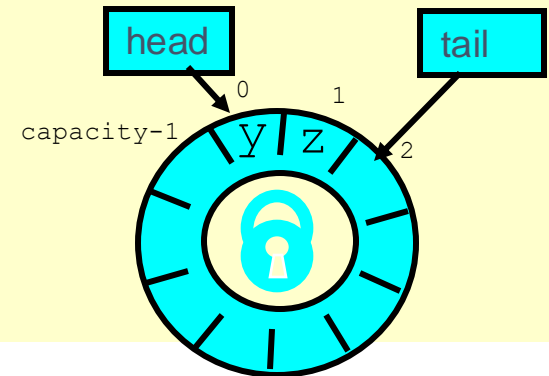


Queue not empty?  
Remove item and update head

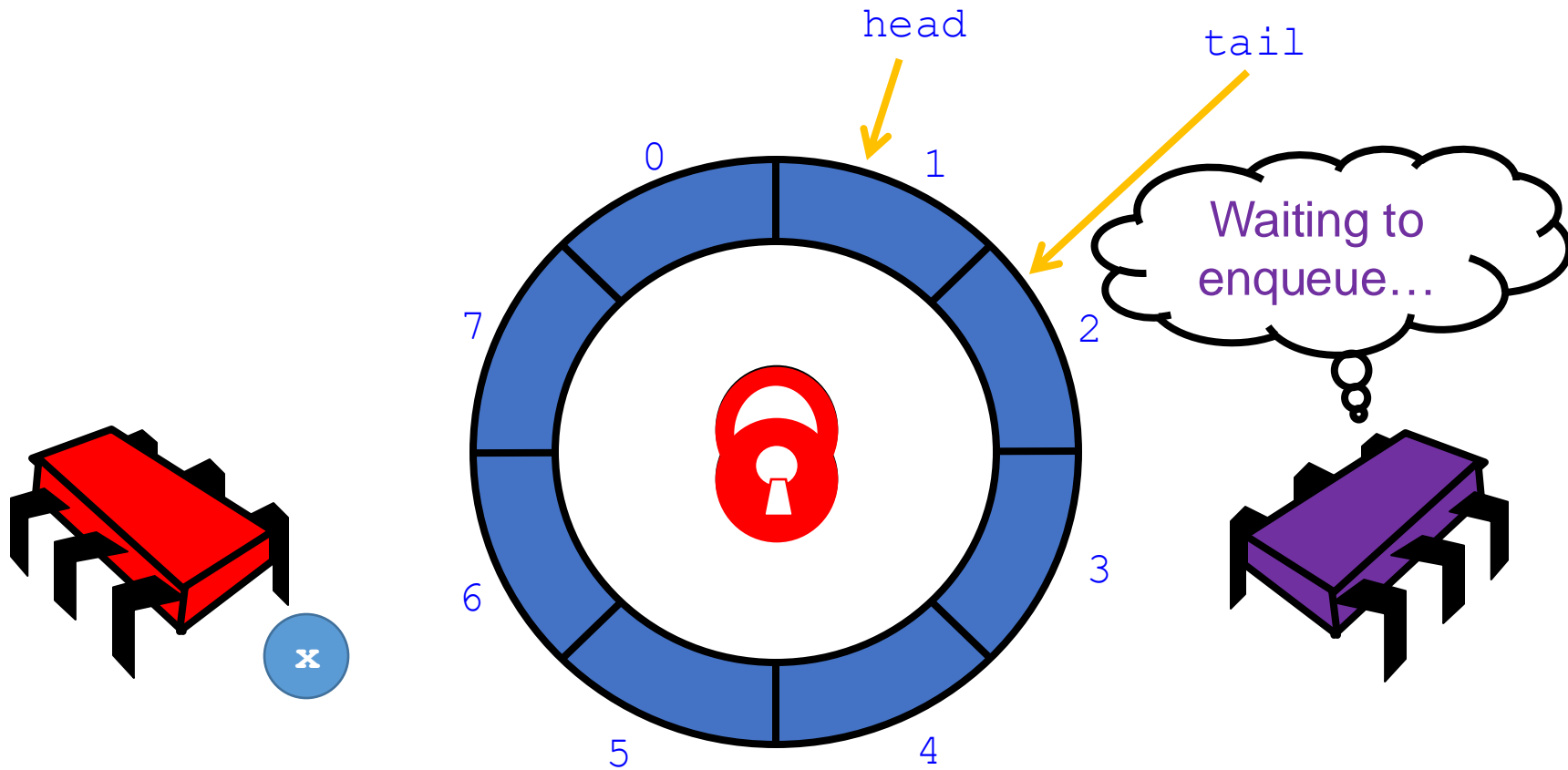
# Implementation: `deq()`

```
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        head++;  
        return x;  
    } finally {  
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    }  
}
```

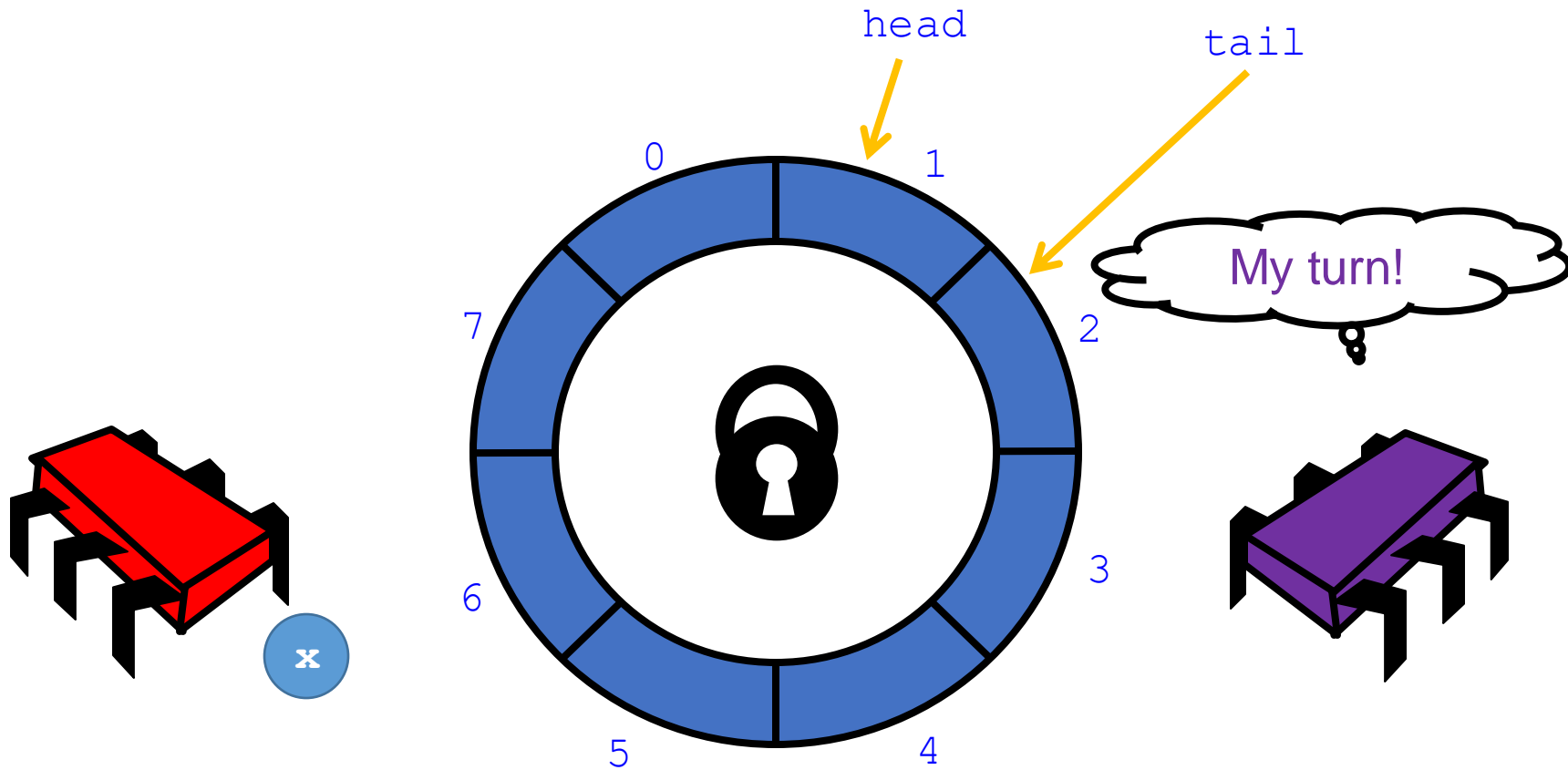
Return result



# Release the Lock

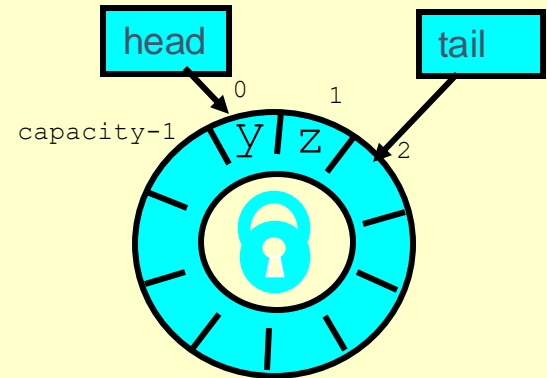


# Release the Lock



# Implementation: `deq()`

```
public T deq() throws EmptyException {  
    lock.lock();  
    try {  
        if (tail == head)  
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        T x = items[head % items.length];  
        head++;  
        return x;  
    } finally {  
        lock.unlock();  
    }  
}
```



Release lock no  
matter what!

# Implementation: `deq()`

```
public T deq() throws EmptyException {  
    lock.lock();  
    try {  
        if (tail == head)  
            throw new EmptyException();  
        T x = items[head % items.length];  
        head++;  
        return x;  
    } finally {  
        lock.unlock();  
    }  
}
```

Should be correct because  
modifications are mutually exclusive...

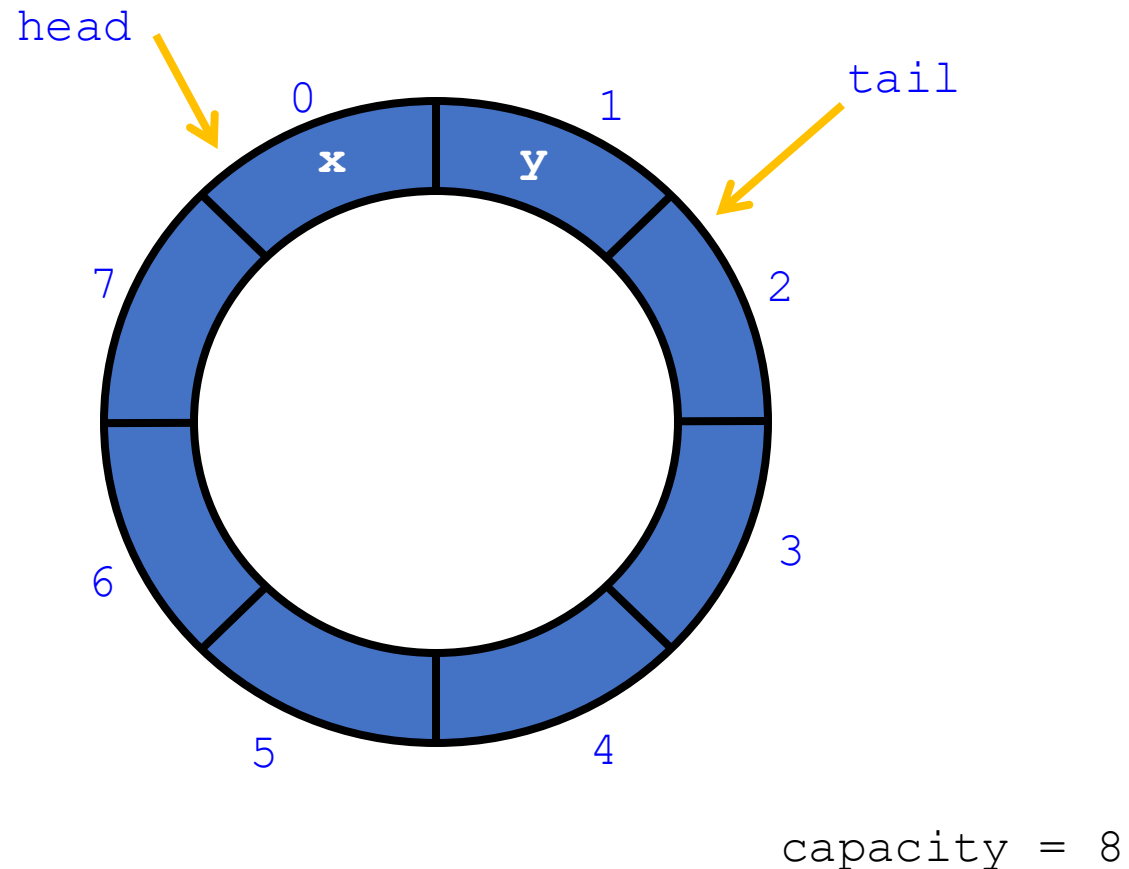


# Now consider the following implementation

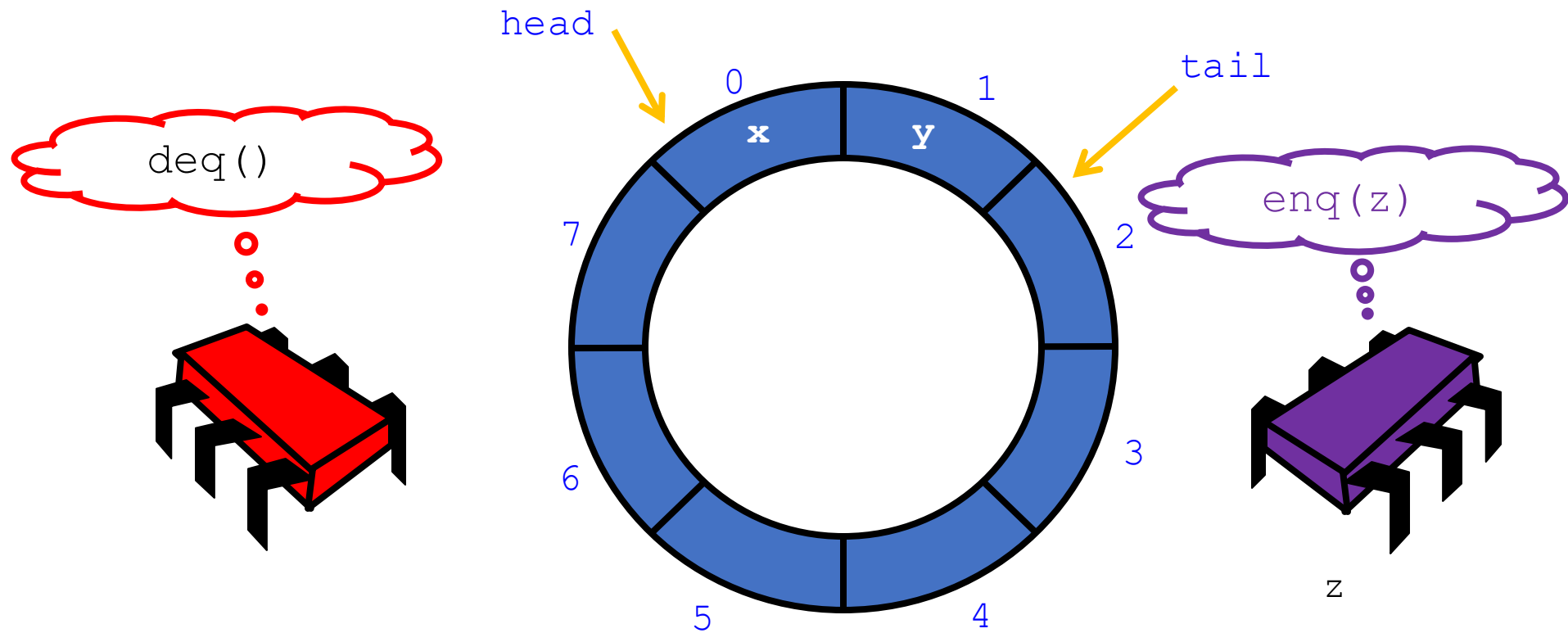
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- The same thing without mutual exclusion
- For simplicity, only two threads
  - One thread **enq only**
  - The other **deq only**

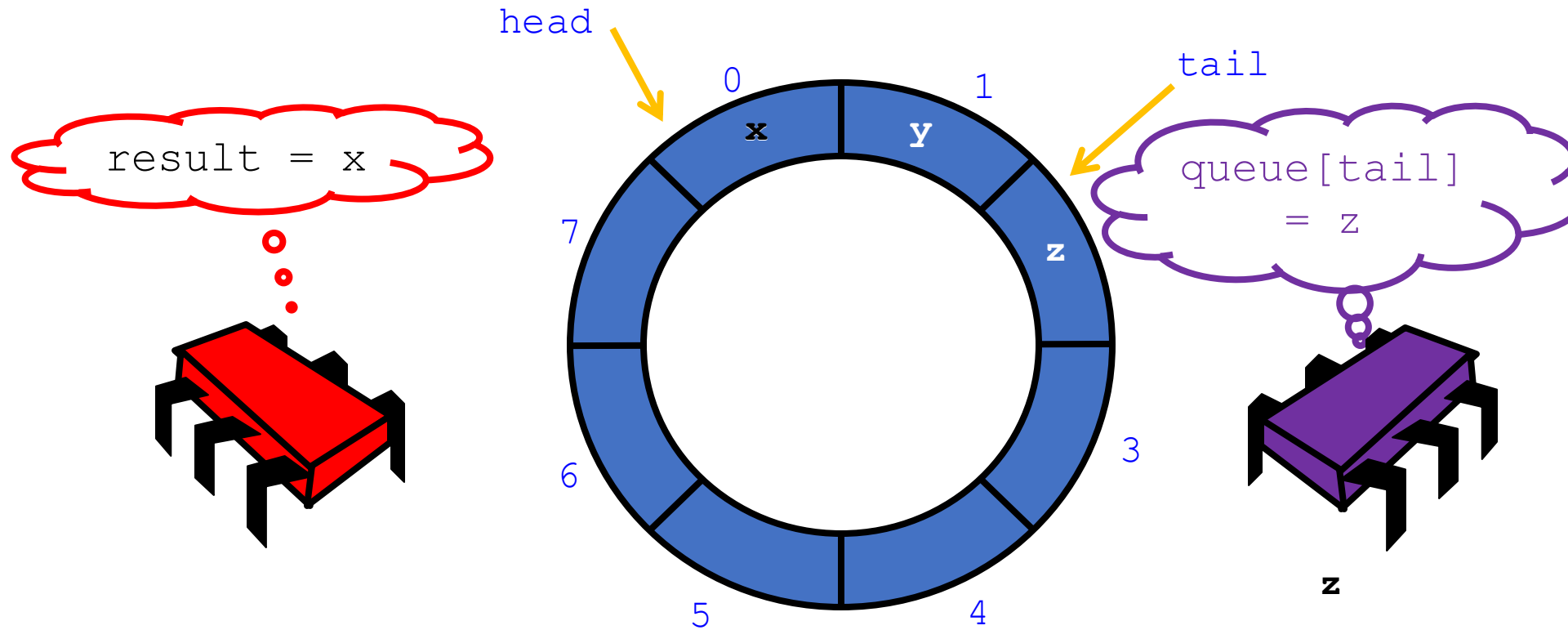
# Wait-free 2-Thread Queue



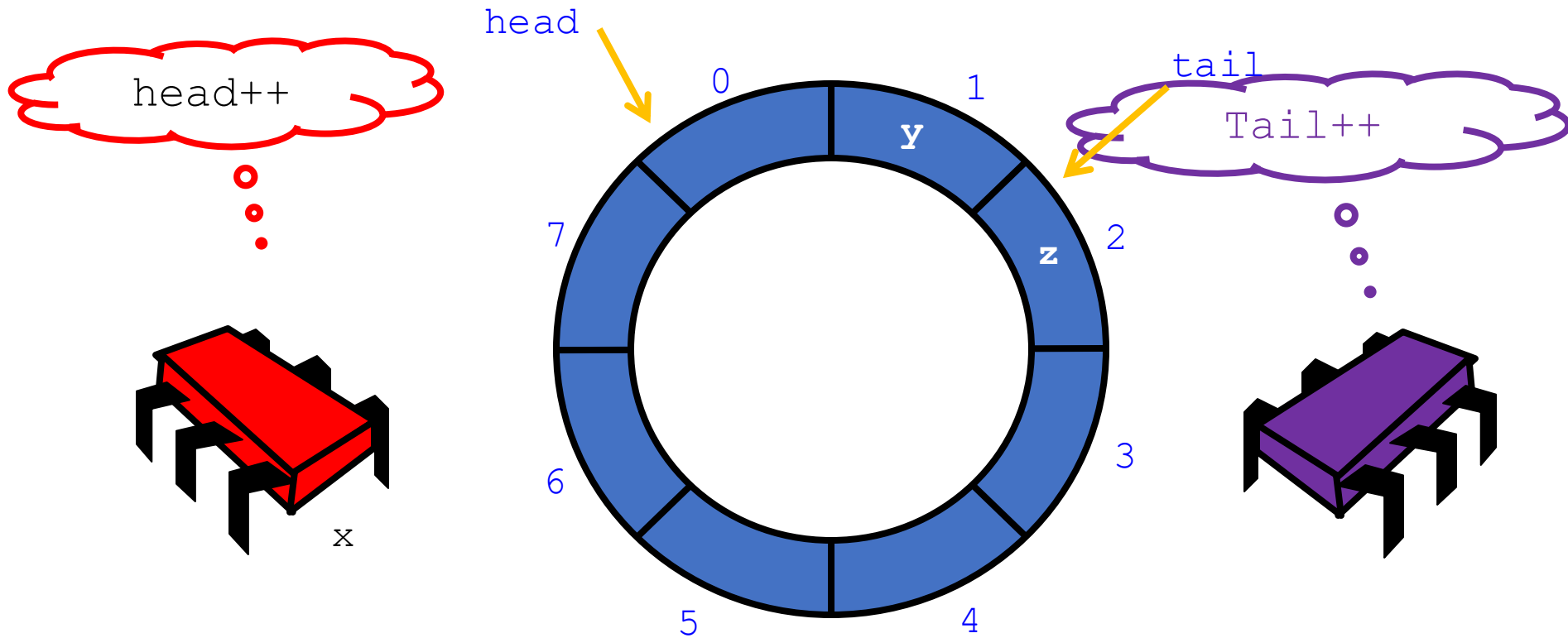
# Wait-free 2-Thread Queue



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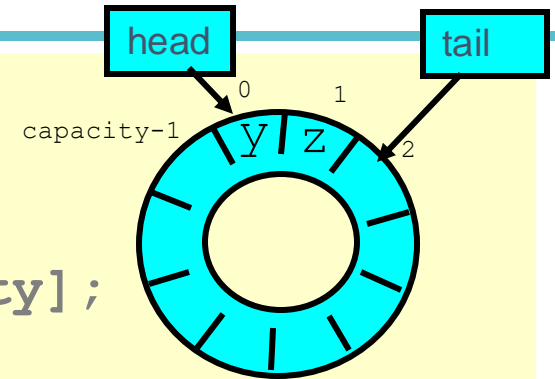


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# Wait-free 2-Thread Queue

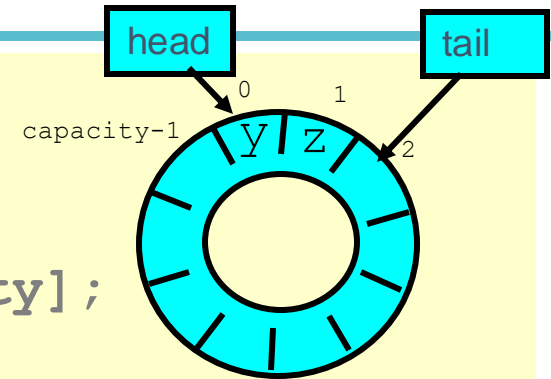
```
public class WaitFreeQueue {  
  
    int head = 0, tail = 0;  
    items = (T[]) new Object[capacity];  
  
    public void enq(Item x) {  
        if (tail-head == capacity) throw  
            new FullException();  
        items[tail % capacity] = x; tail++;  
    }  
  
    public Item deq() {  
        if (tail == head) throw  
            new EmptyException();  
        Item item = items[head % capacity]; head++;  
        return item;  
    }  
}
```



**No lock needed?!**

# Wait-free 2-Thread Queue

```
public class WaitFreeQueue {  
  
    int head = 0, tail = 0;  
    items = (T[]) new Object[capacity];  
  
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        if (tail-head == capacity) throw  
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        items[tail % capacity] = x; tail++;  
    }  
  
    public Item deq() {  
        if (tail-head == 0) throw  
            new EmptyException();  
        Item item = items[head % capacity]; head++;  
        return item;  
    }  
}
```



How do we define "correct" when  
modifications are not mutually exclusive?

**No lock needed?!**

# *What is* a Concurrent Queue?

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- Need a way to specify a concurrent queue object
- Need a way to prove that an algorithm implements the object's specification
- Let's talk about object specifications...



# Correctness and Progress

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- In a concurrent setting, we need to specify both the **safety** and the **liveness** properties of an object
- Need a way to define
  - when an **implementation is correct**
  - the conditions under which it **guarantees progress**

Let's begin with correctness

# Sequential Objects

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- Each object has a **state**
  - Usually given by a set of **fields**
  - Queue example: *sequence of items*
- Each object has a set of **methods**
  - Only way to manipulate state
  - Queue example: **enq()** and **deq()** methods

# Sequential Specifications

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- If (precondition)
  - the object is in such-and-such a state
  - before you call the method,
- Then (postcondition)
  - the method will return a particular value
  - or throw a particular exception.
- and (postcondition, con't)
  - the object will be in some other state
  - when the method returns,

# Pre and PostConditions for Dequeue

---

- Precondition:
    - Queue is non-empty
  - Postcondition:
    - Returns first item in queue
  - Postcondition:
    - Removes first item in queue
- Precondition:
    - Queue is empty
  - Postcondition:
    - Throws Empty exception
  - Postcondition:
    - Queue state unchanged

# Why Sequential Specifications Totally Rock

---

- Interactions among methods captured by side-effects of methods on object state
  - State meaningful between method calls
- Documentation size linear in number of methods
  - Each method described in isolation
- Can add new methods
  - Without changing descriptions of old methods

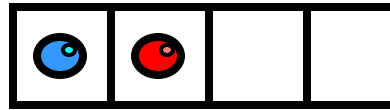
# What About Concurrent Specifications ?

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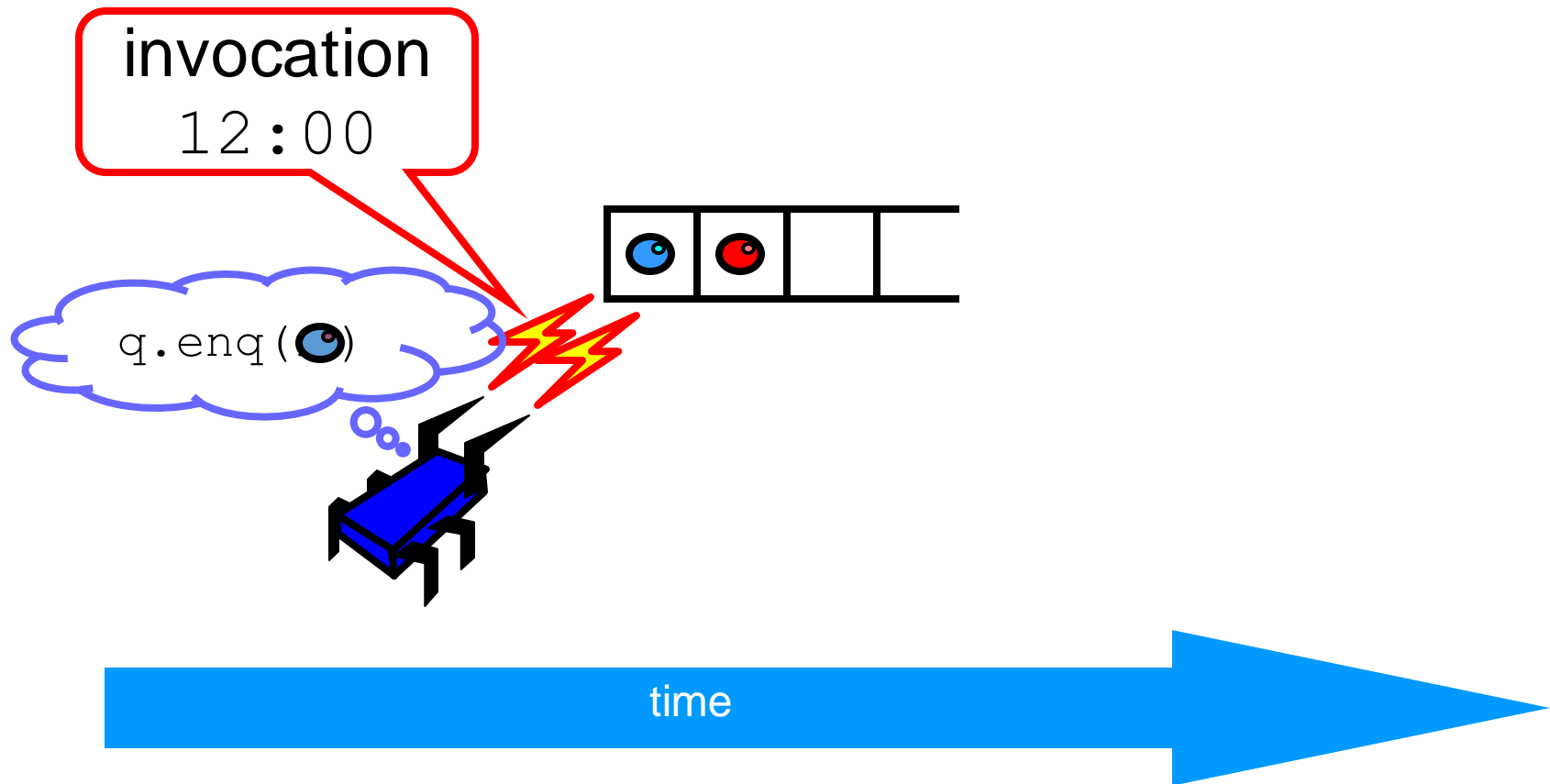
- Methods?
- Documentation?
- Adding new methods?

# Methods Take Time

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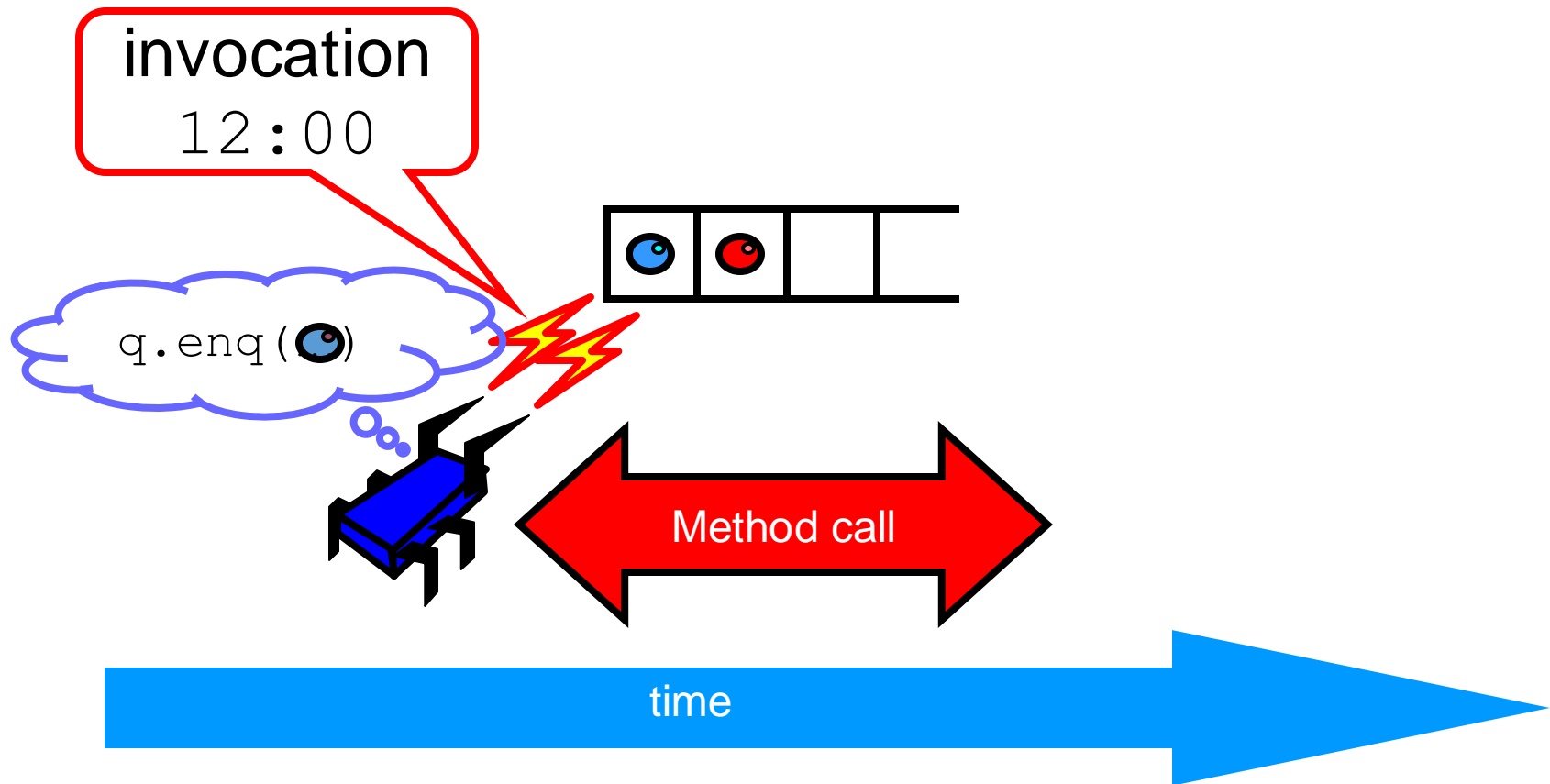


# Methods Take Time

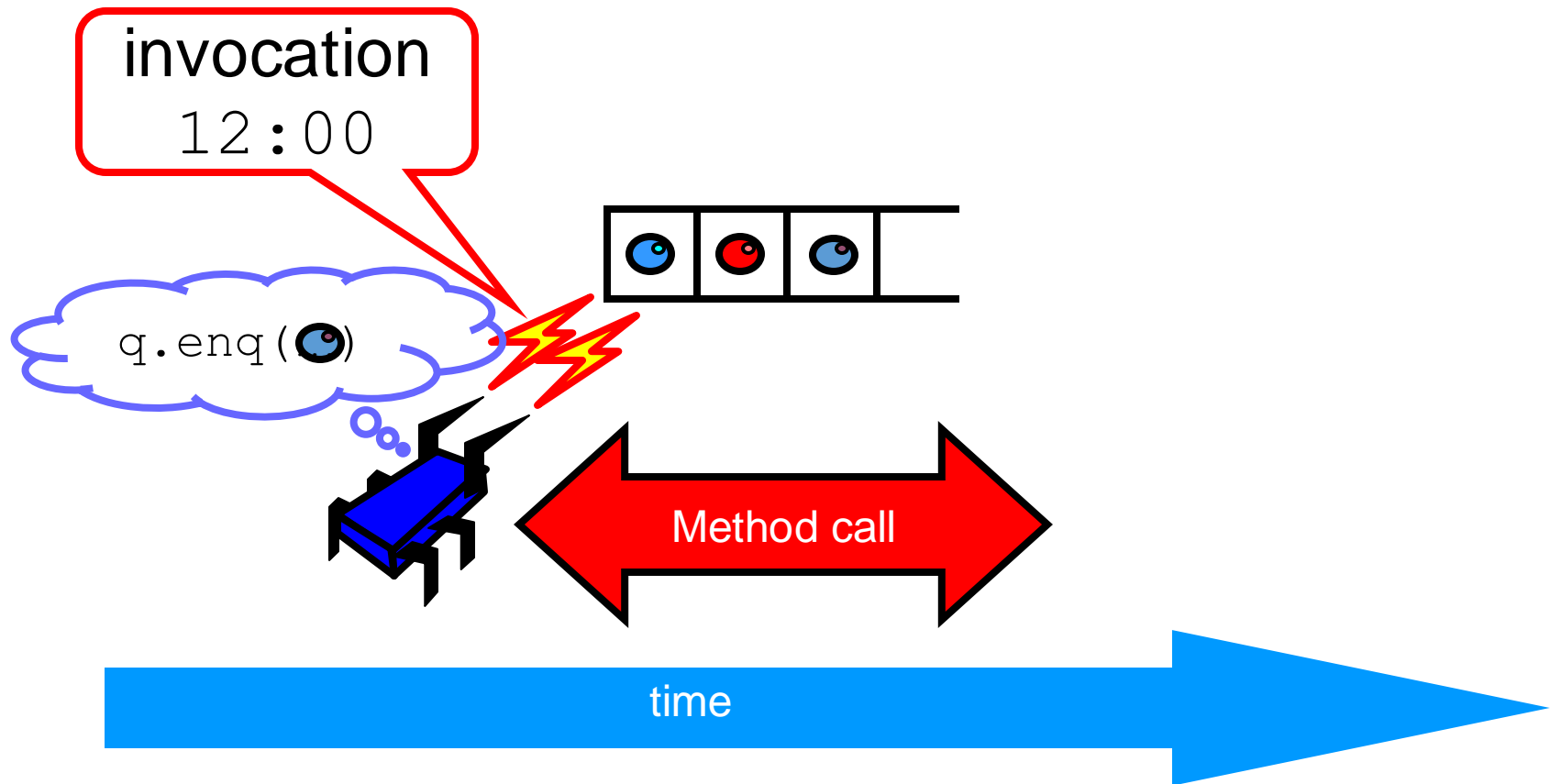




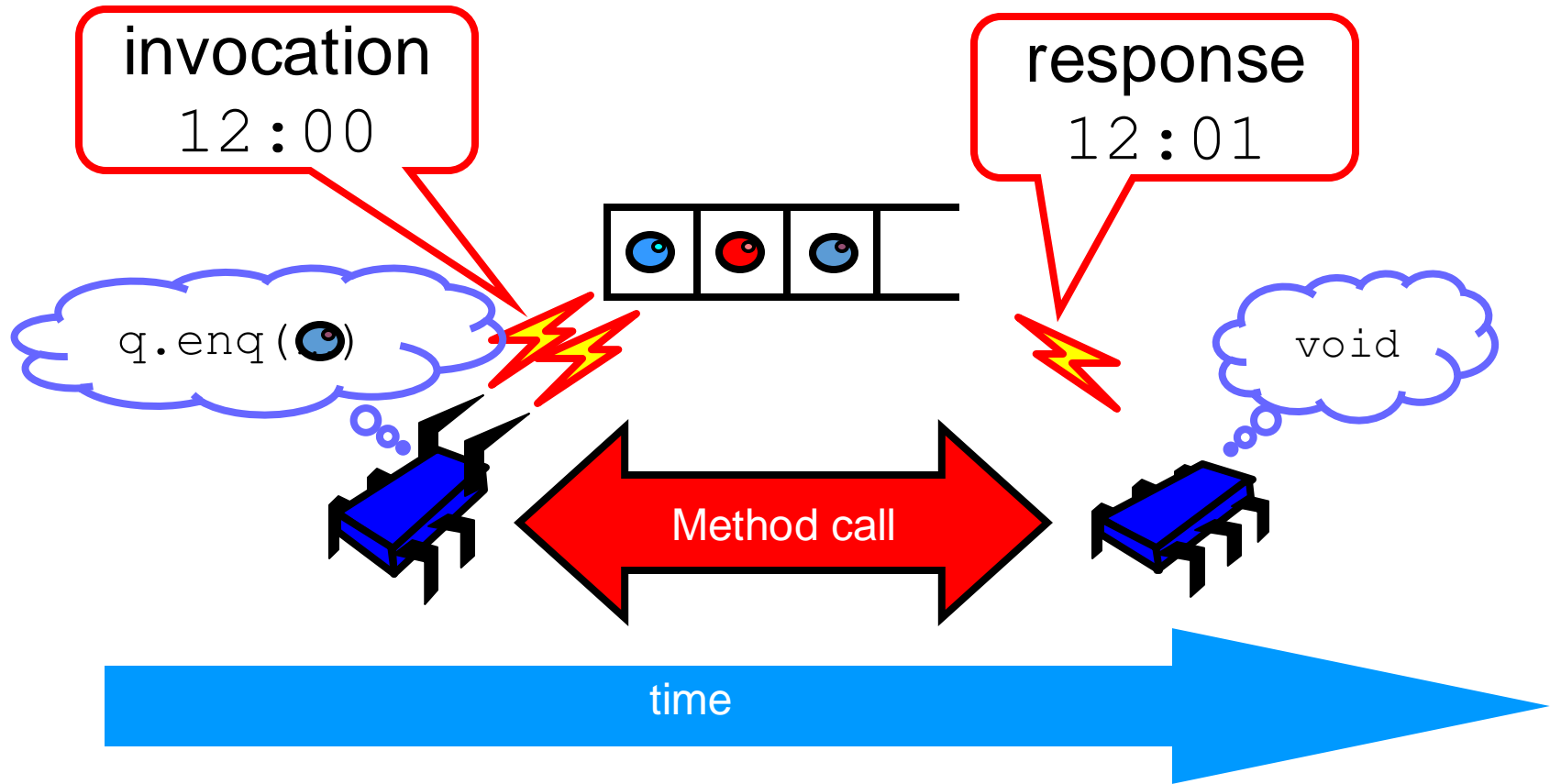
# Methods Take Time



# Methods Take Time



# Methods Take Time



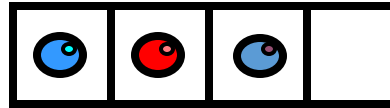
# Sequential vs Concurrent

---

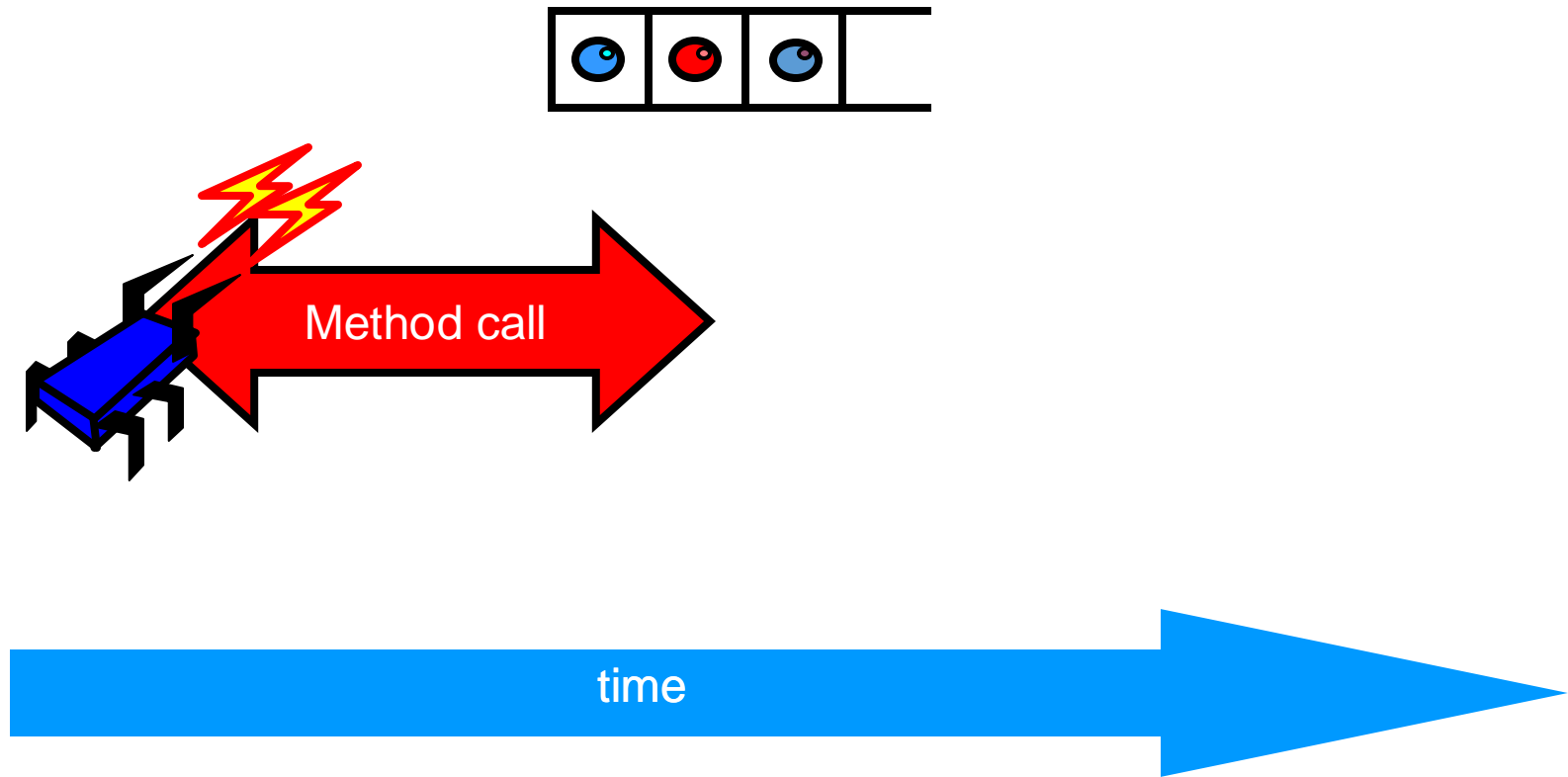
- Sequential
  - Methods take time? Who knew?
- Concurrent
  - Method call is not an event
  - Method call is an interval

# Concurrent Methods Take Overlapping Time

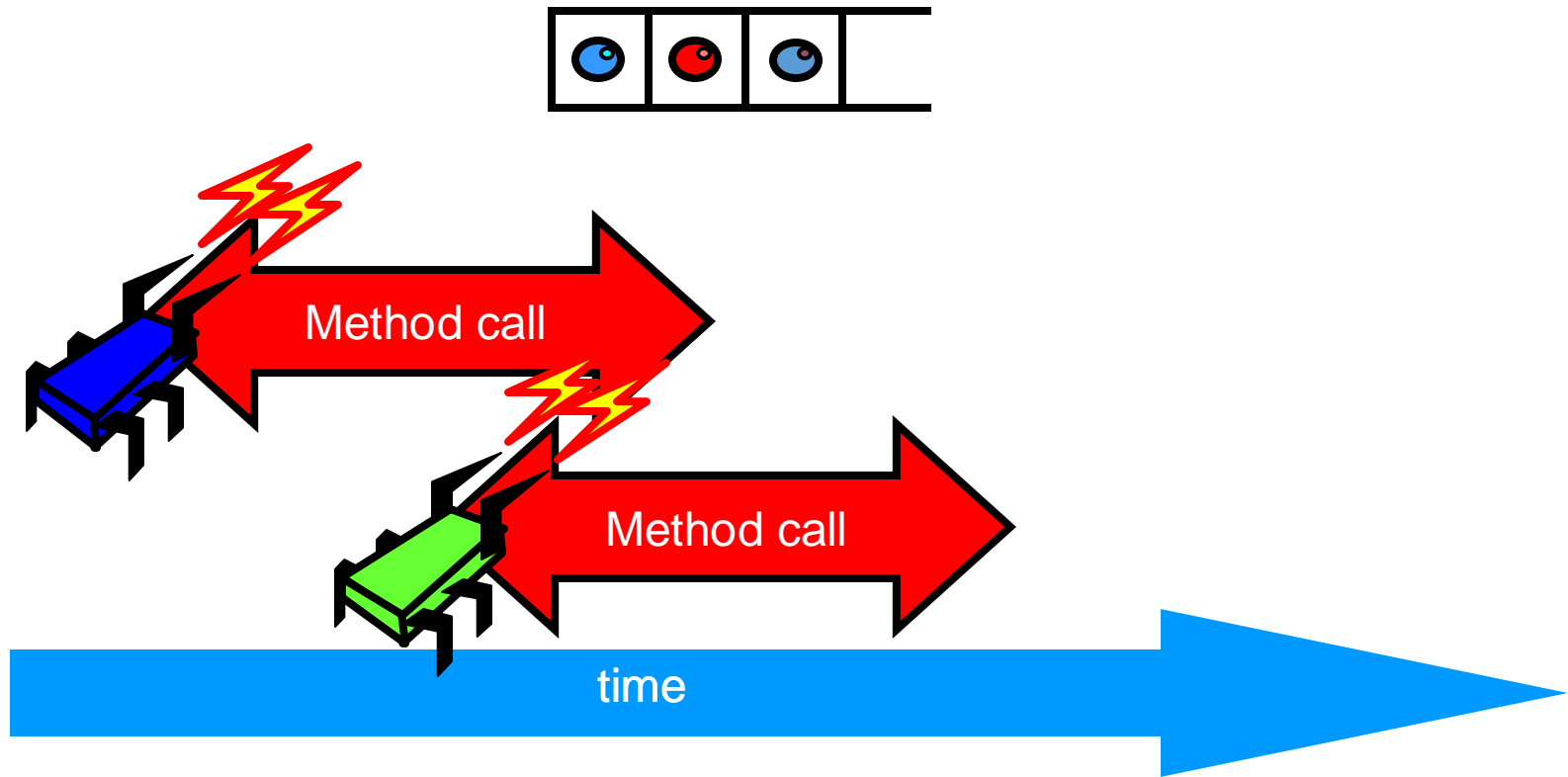
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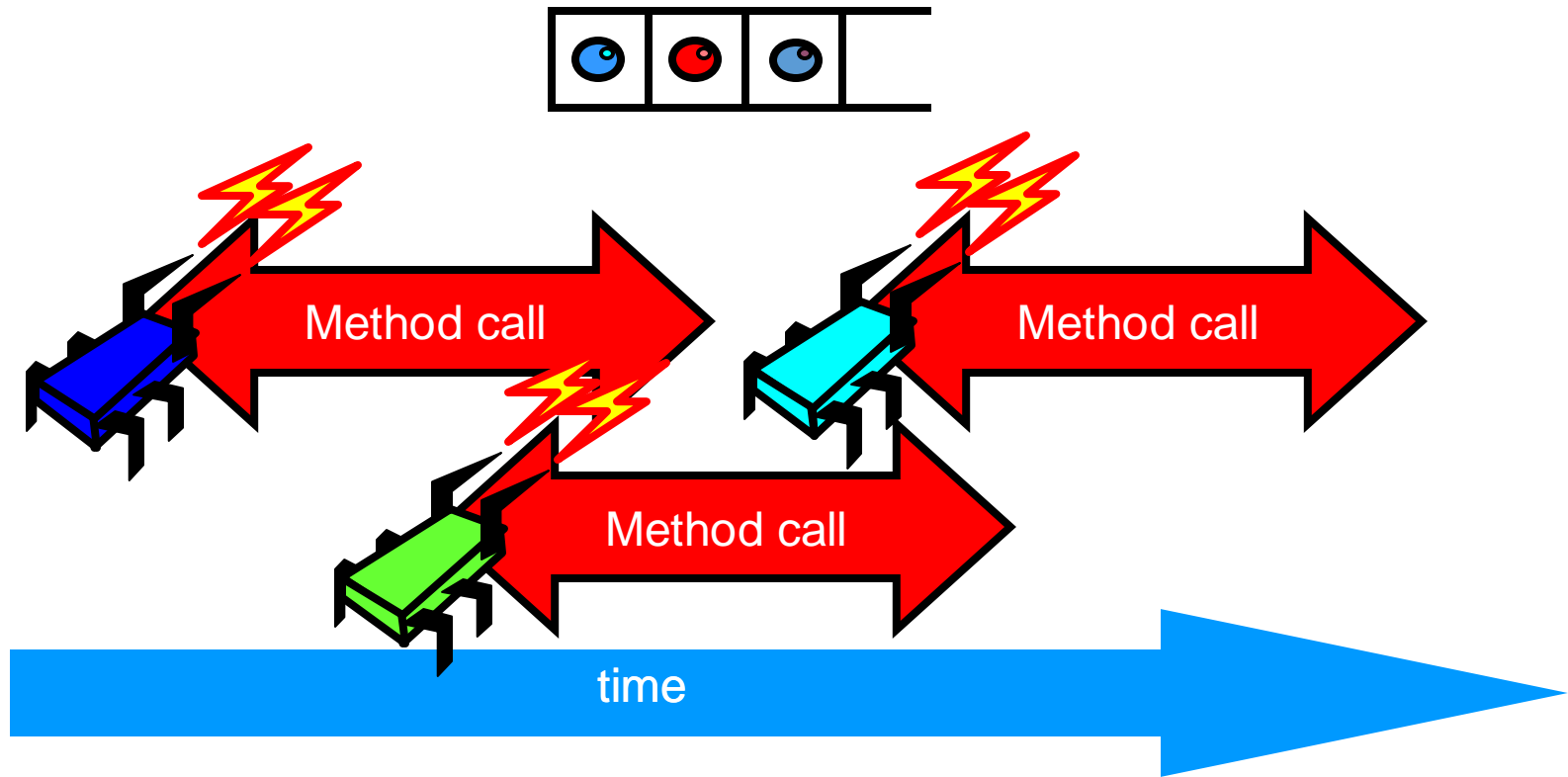
# Concurrent Methods Take Overlapping Time



# Concurrent Methods Take Overlapping Time



# Concurrent Methods Take Overlapping Time





# Sequential vs Concurrent: **time**

---

- Sequential:
  - Object needs meaningful state only **between** method calls
- Concurrent
  - Because method calls overlap, object might **never** be between method calls

# Sequential vs Concurrent: doc

- Sequential:
  - Each method described in isolation
- Concurrent
  - Must characterize **all** possible interactions with concurrent calls
    - What if two **enq()** calls overlap?
    - Two **deq()** calls? **enq()** and **deq()**? ...

# Sequential vs Concurrent: add

- Sequential:
  - Can add new methods without affecting older methods
- Concurrent:
  - Everything can potentially interact with everything else



# The Big Question

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- What does it **mean** for a *concurrent* object to be correct?
  - *What is* a concurrent FIFO queue?
  - FIFO means strict temporal order
  - Concurrent means ambiguous temporal order

# Intuitively...

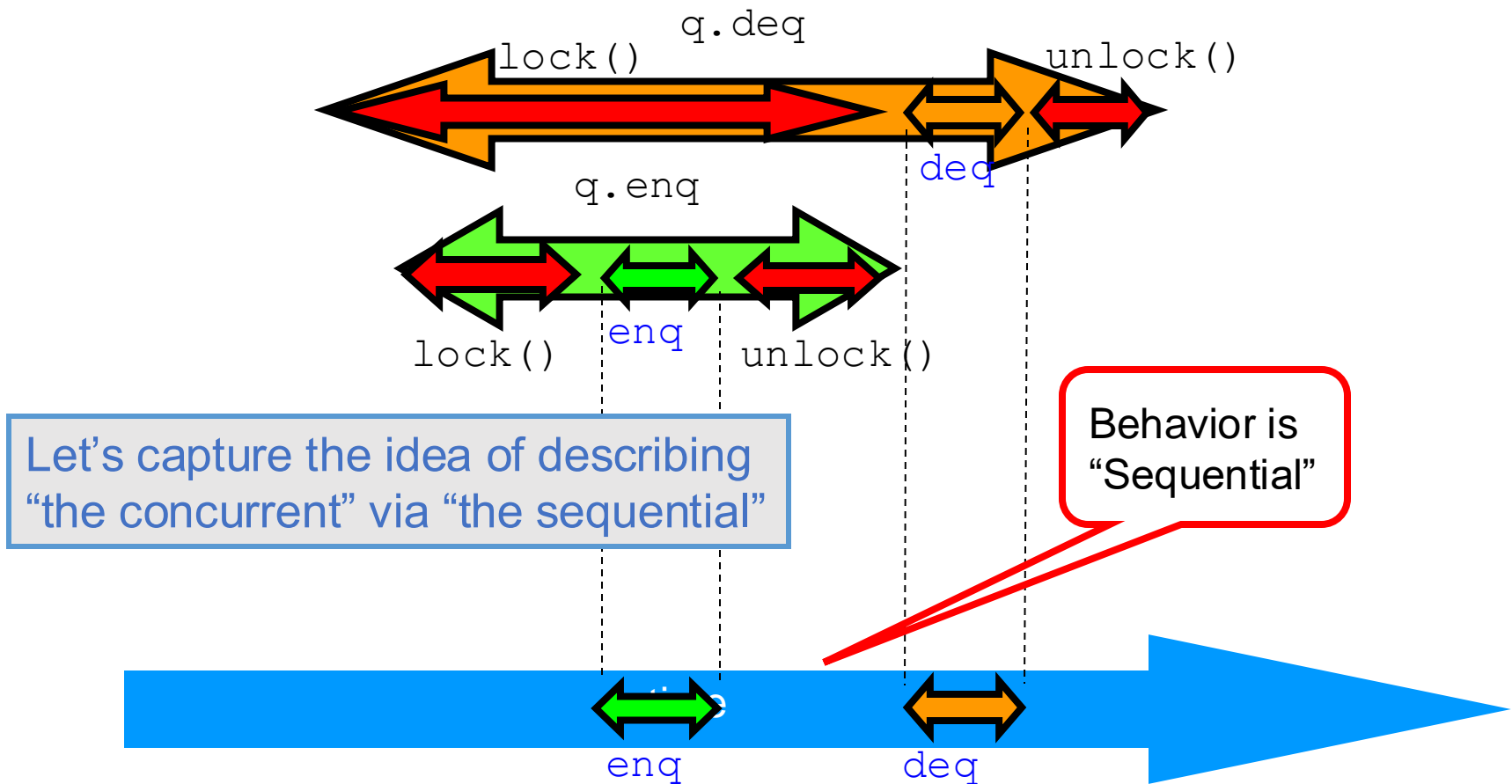
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            throw new EmptyException();  
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        head++;  
        return x;  
    } finally {  
        lock.unlock();  
    }  
}
```

# Intuitively...

```
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    lock.lock();  
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            throw new EmptyException();  
        T x = items[head % items.length];  
        head++;  
        return x;  
    } finally {  
        lock.unlock();  
    }  
}
```

All queue modifications  
are mutually exclusive

# Intuitively



# Linearizability

---

- Each method should
  - “Take effect”
  - Instantaneously
  - Between invocation and response events
- Object is correct if this “sequential” behavior is correct
- Any such concurrent object is
  - **Linearizable**

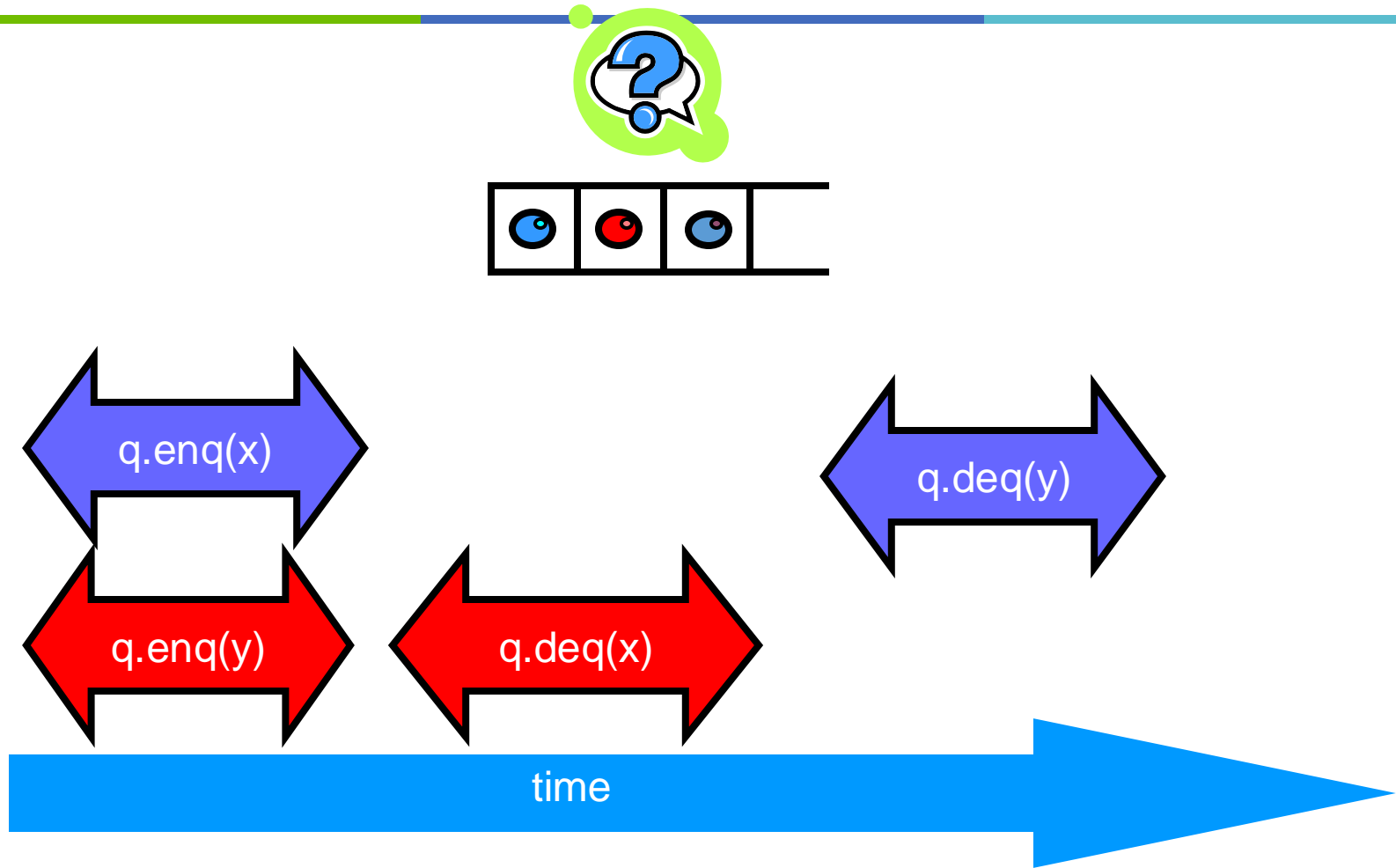


# Is it really about the object?

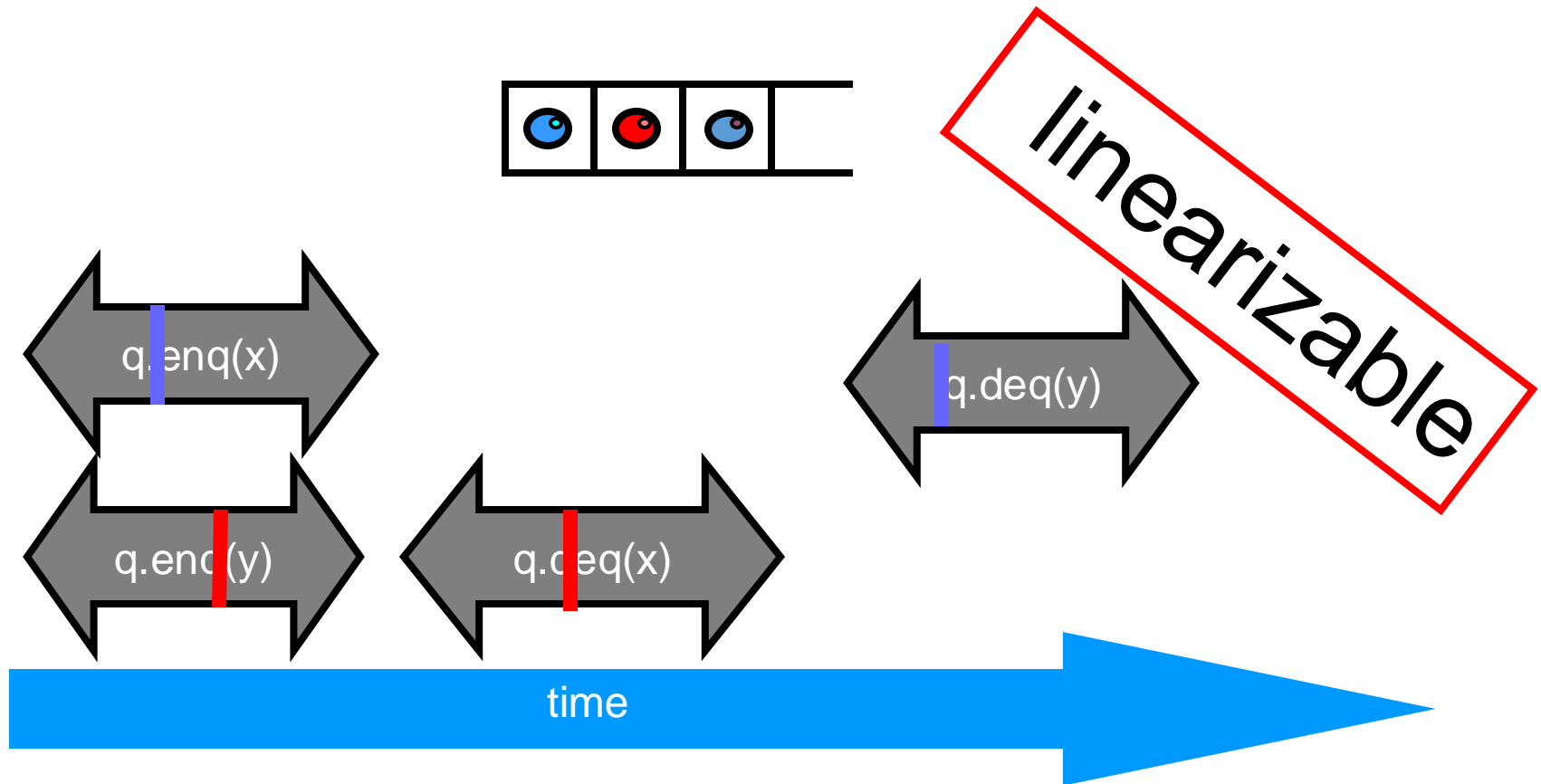
---

- Each method should
  - “Take effect”
  - Instantaneously
  - Between invocation and response events
- Sounds like a property of an execution...
  - Some methods may be linearizable, others don't
- A linearizable object is...
  - *One whose methods are all linearizable*
  - *One whose all possible executions are linearizable*

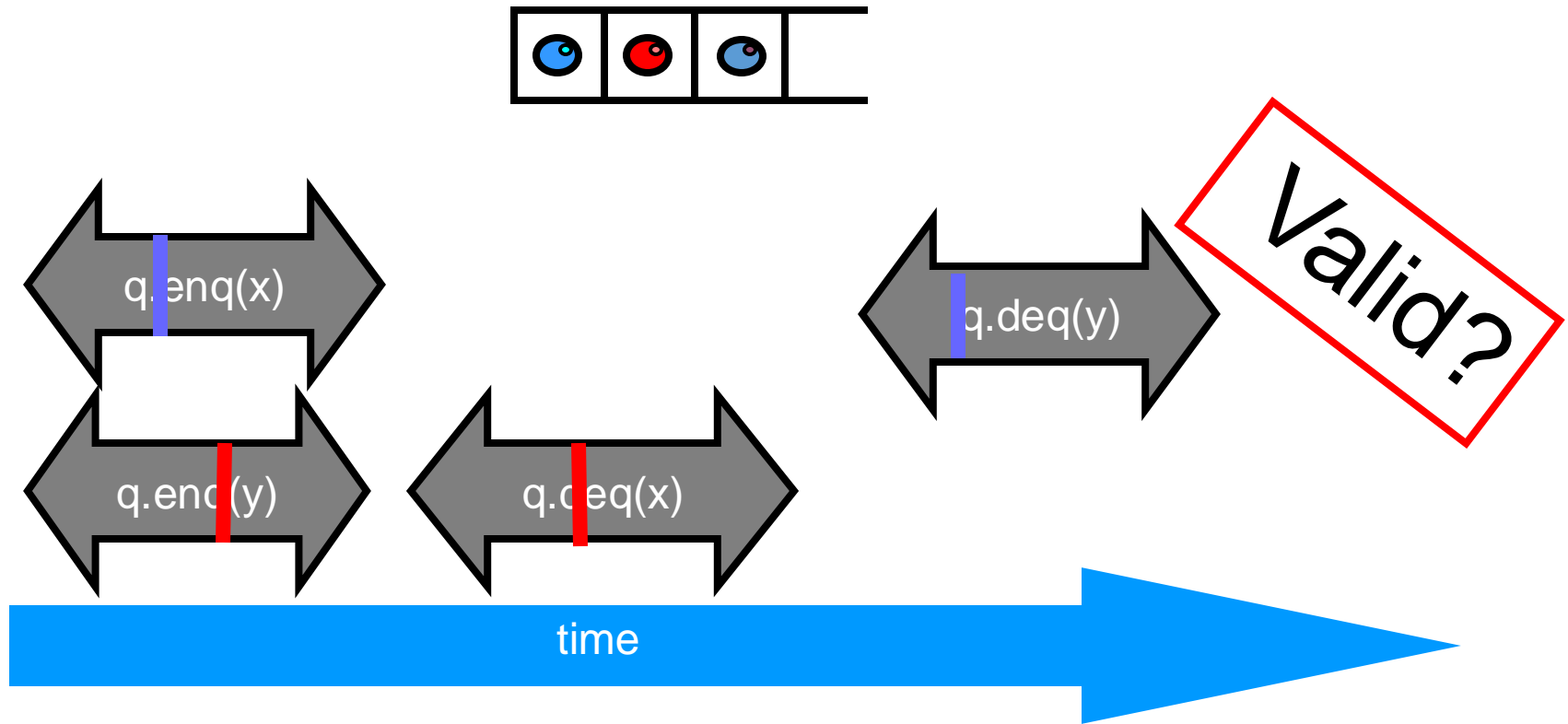
# Example



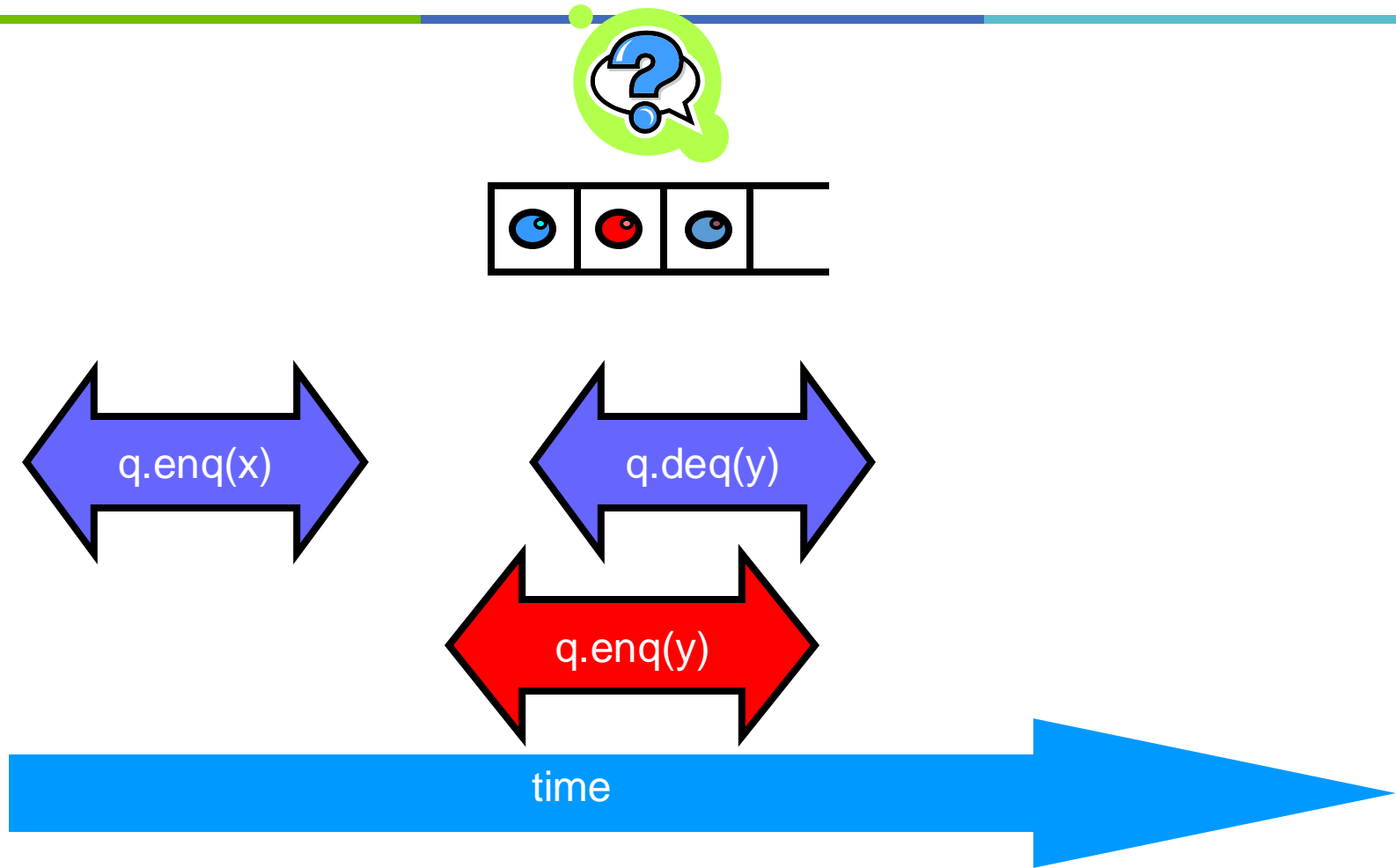
# Example



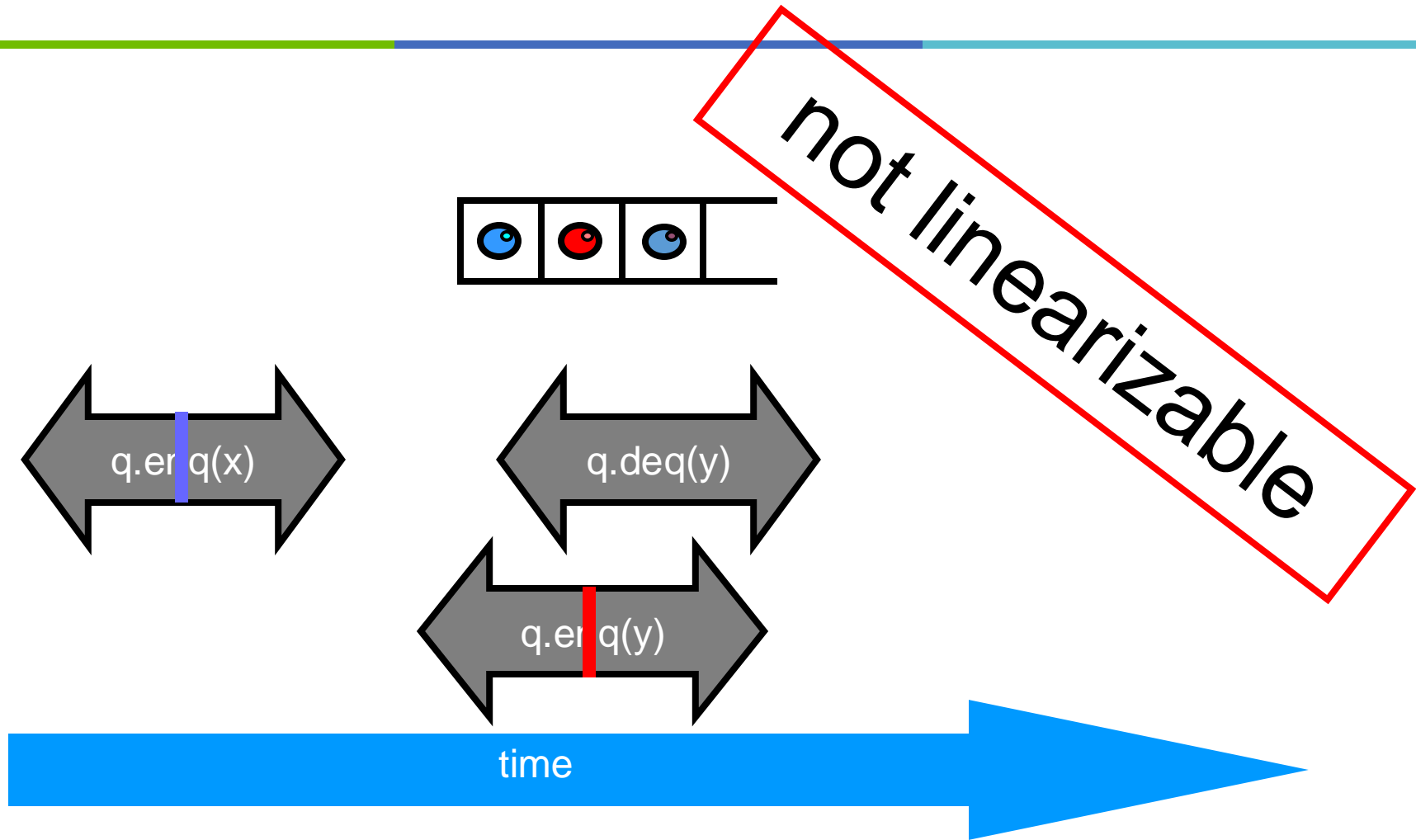
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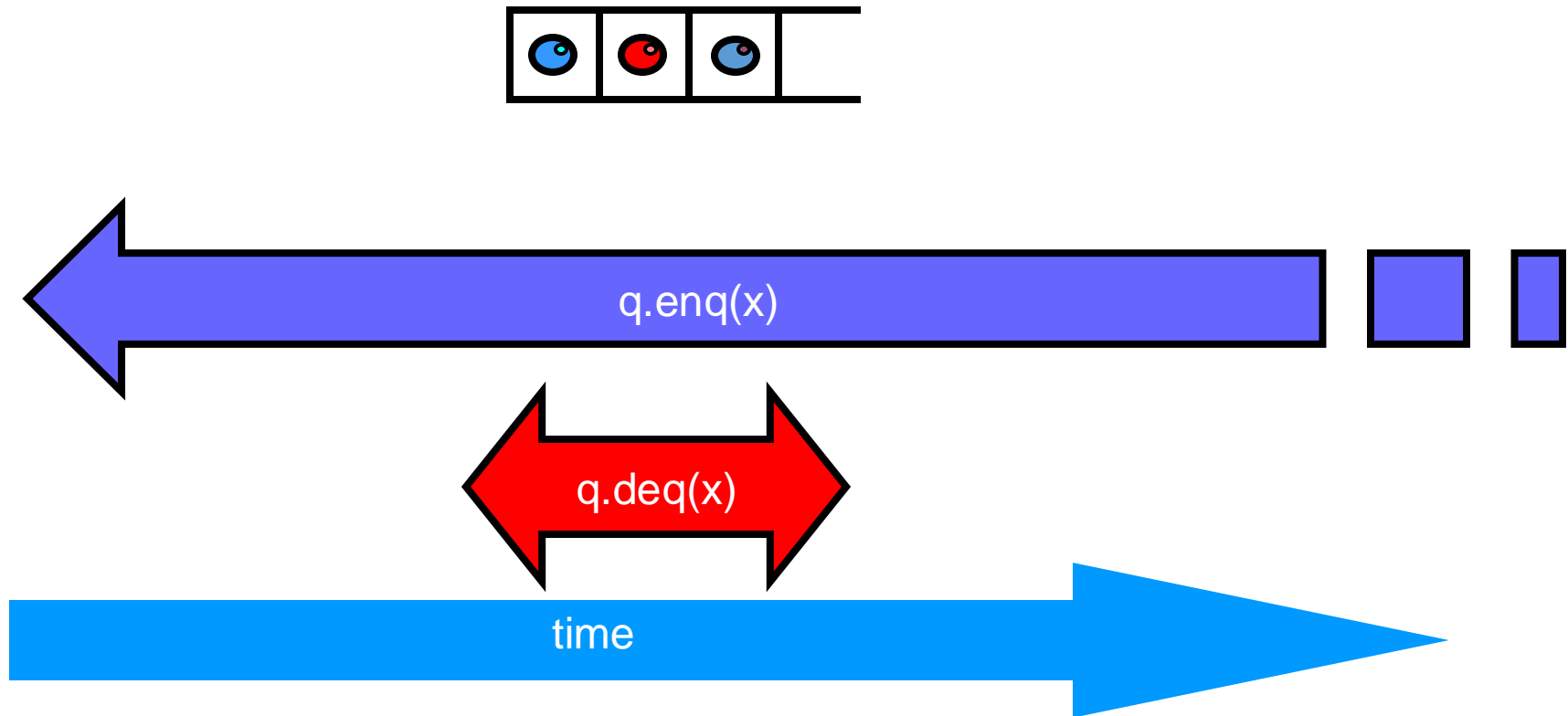
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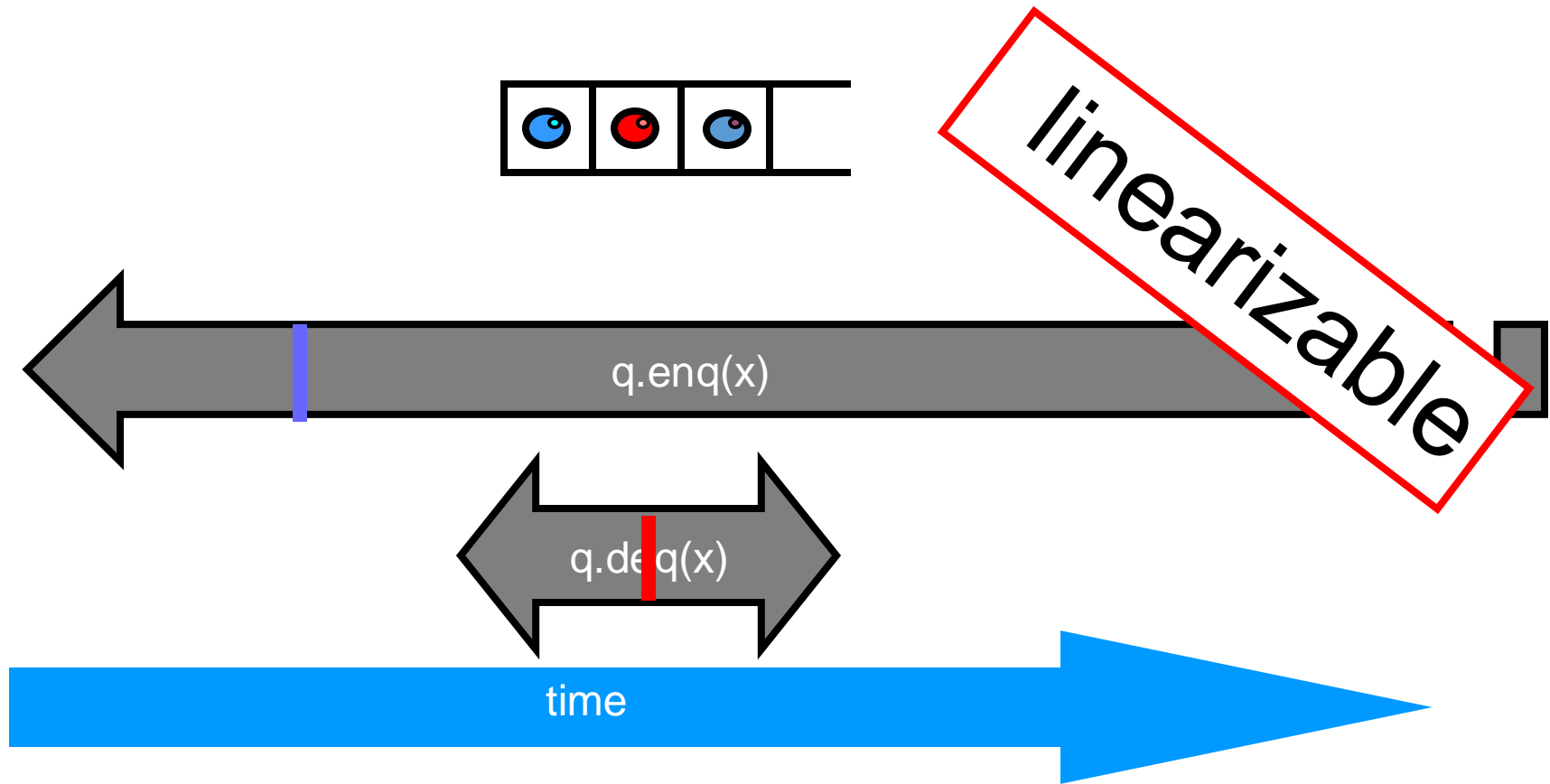
# Example



# Example

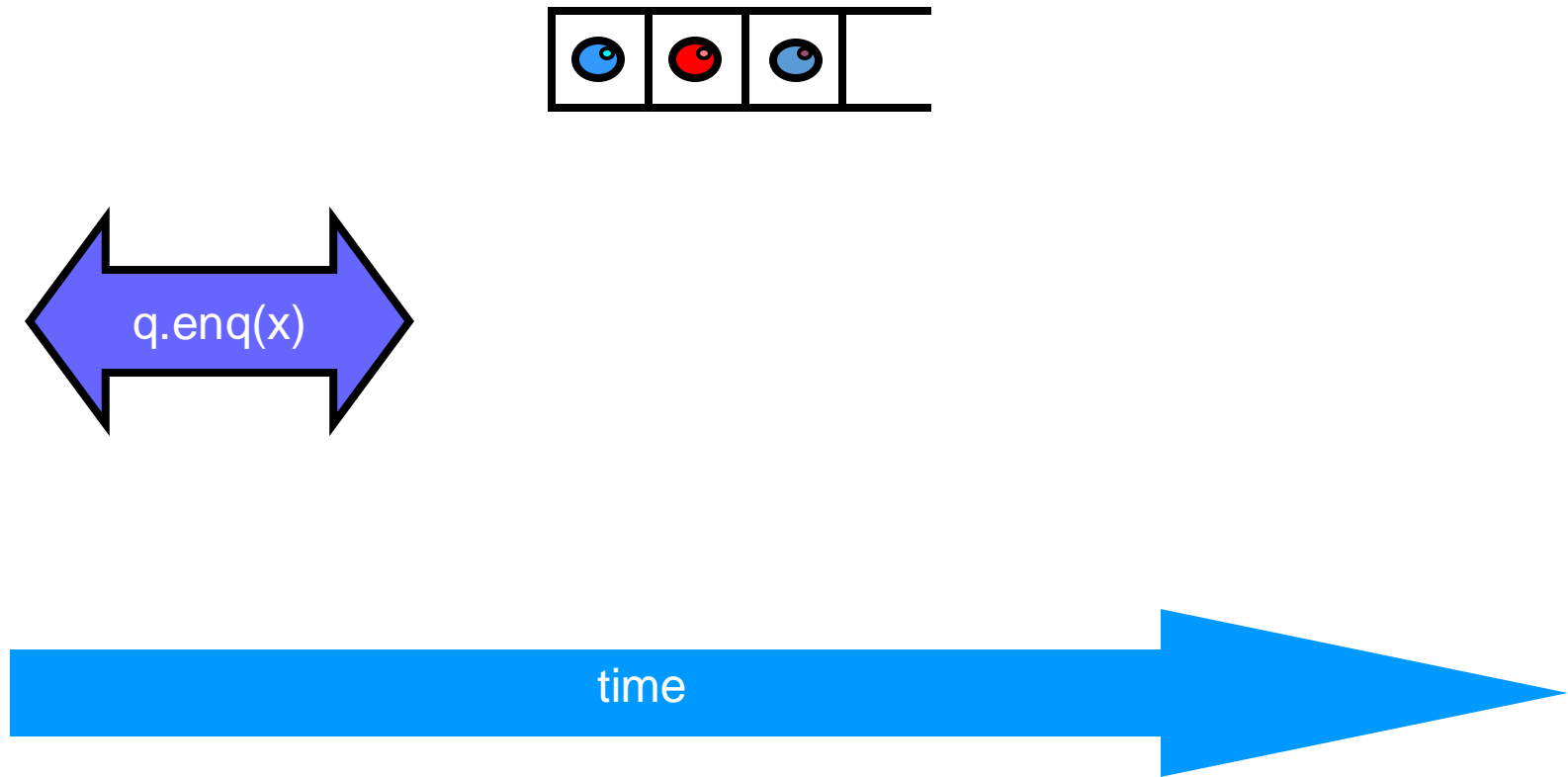


# Example

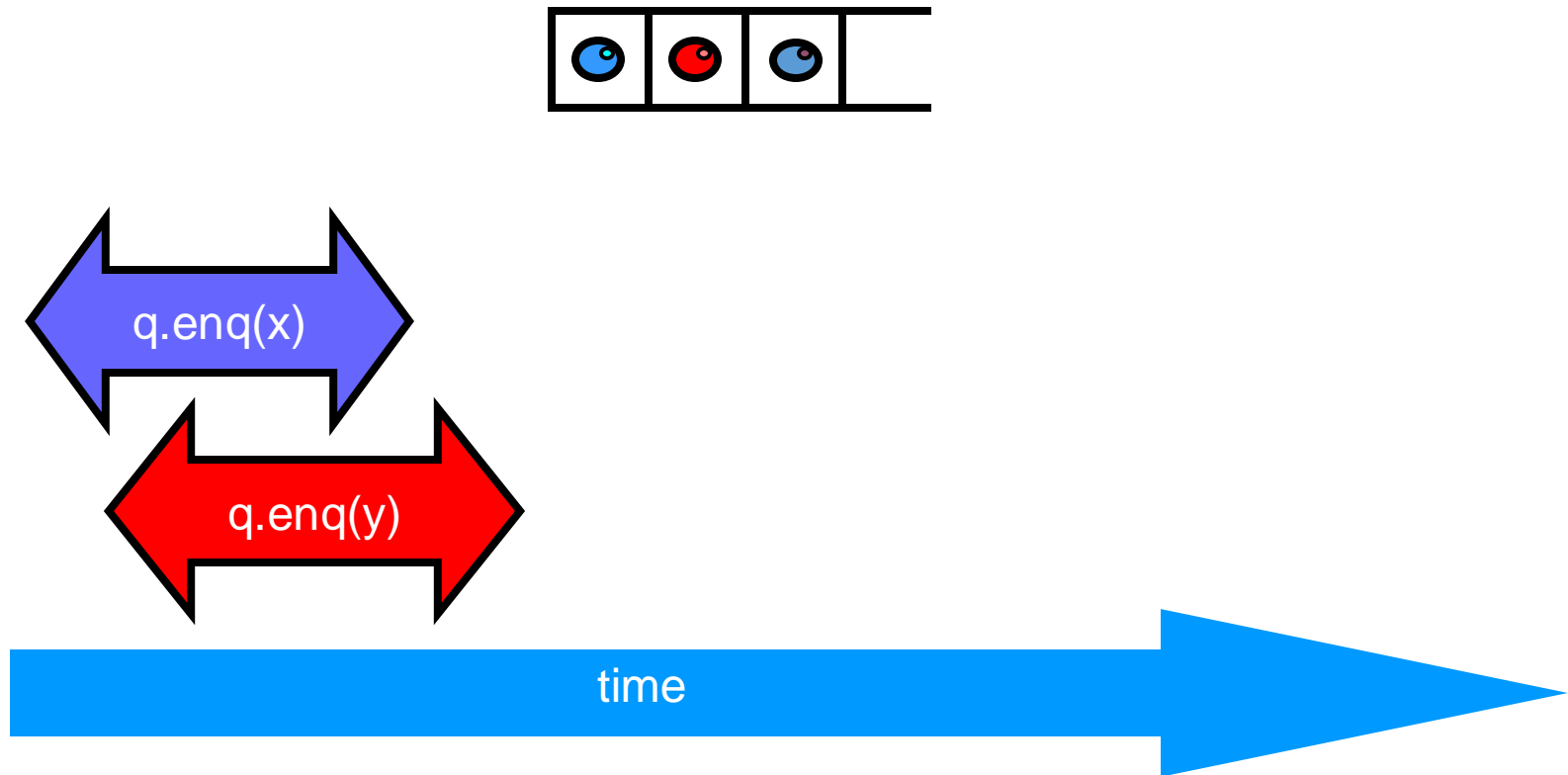




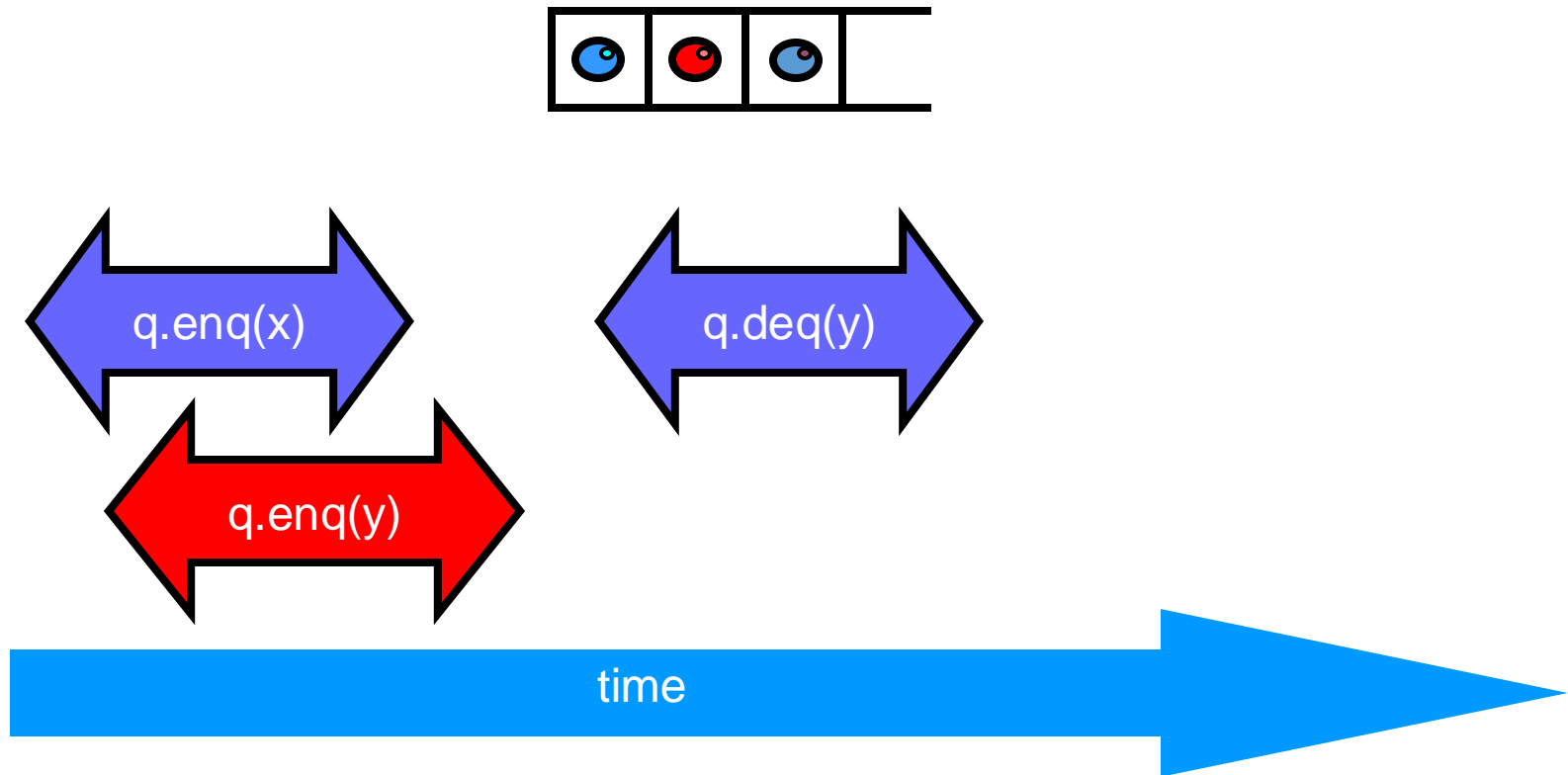
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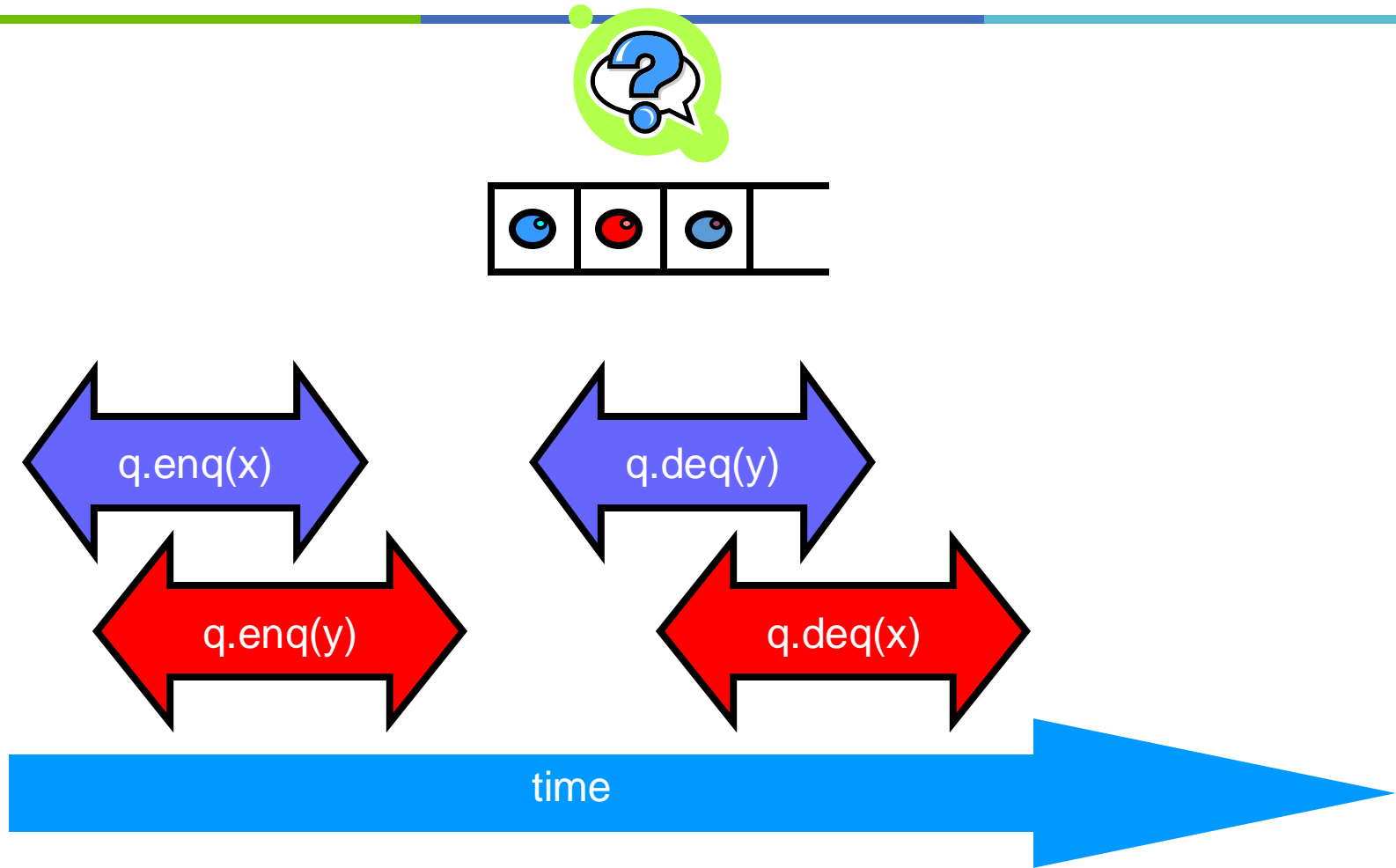
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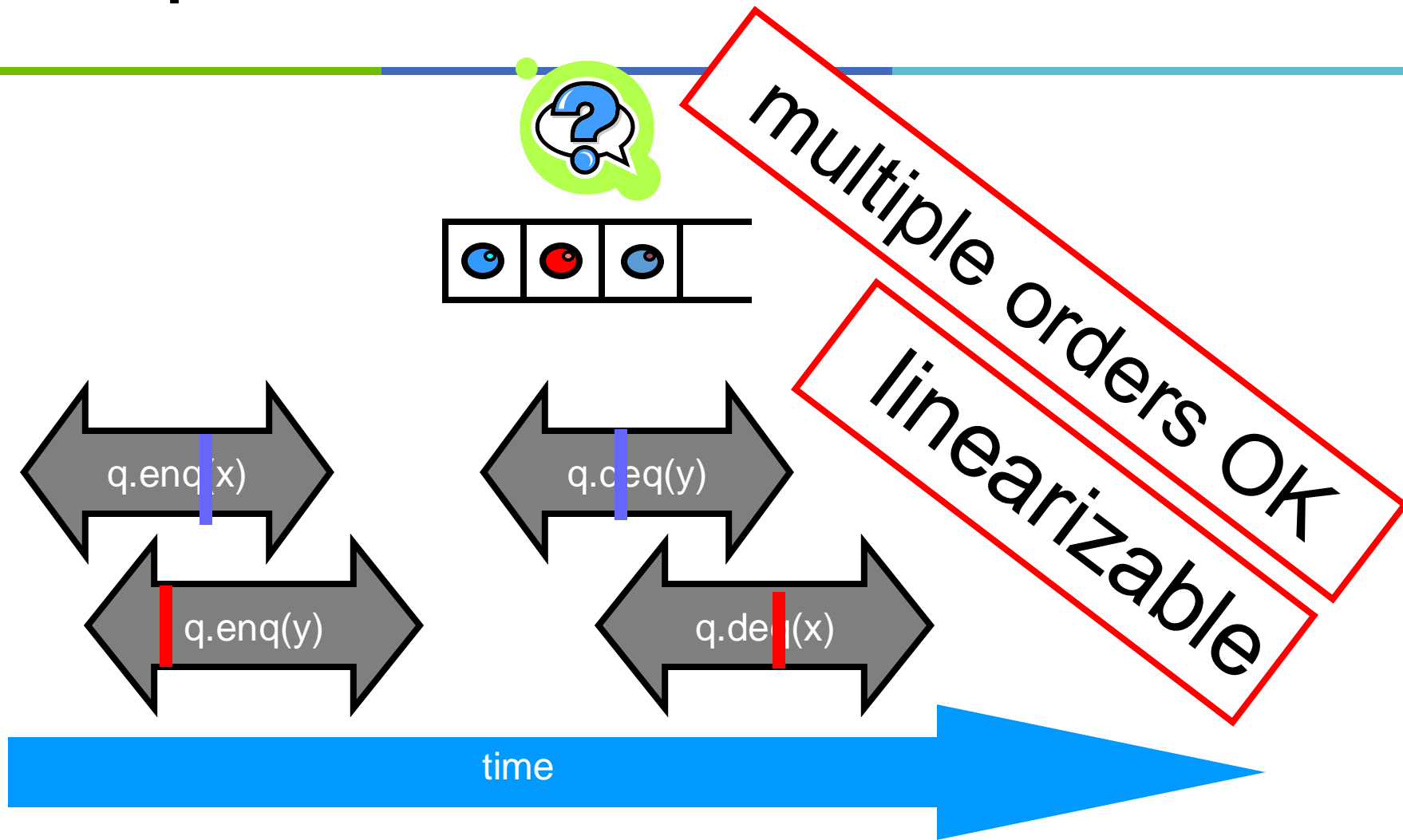
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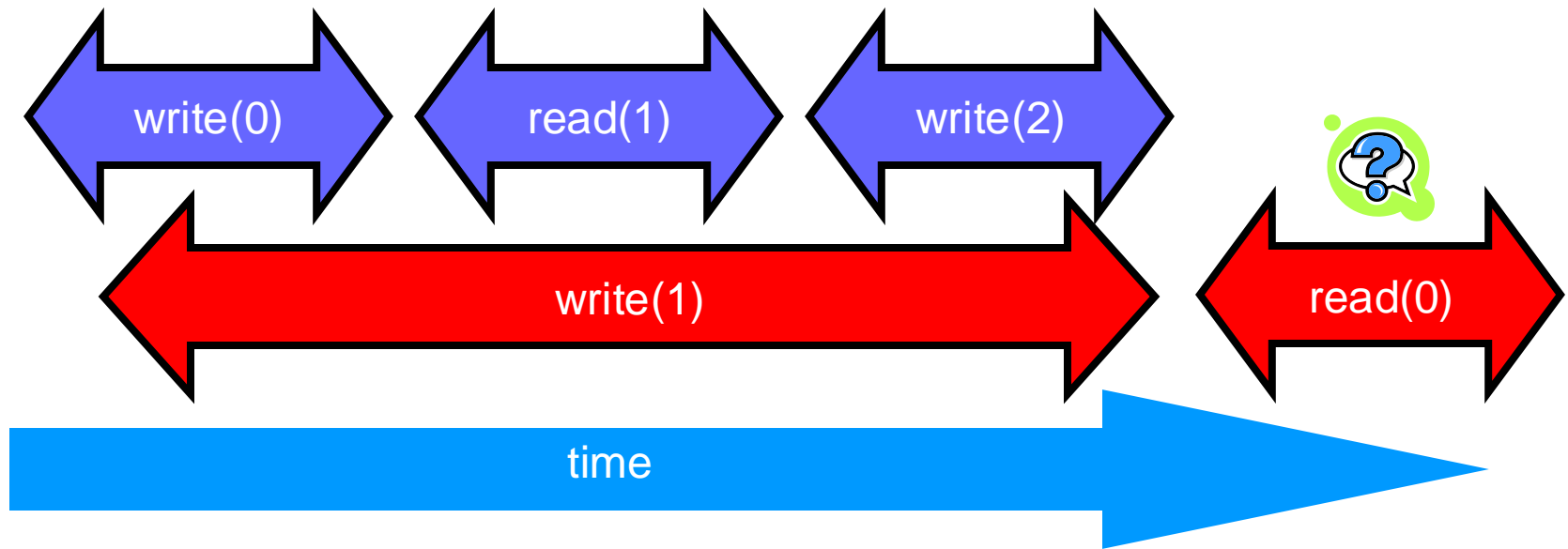
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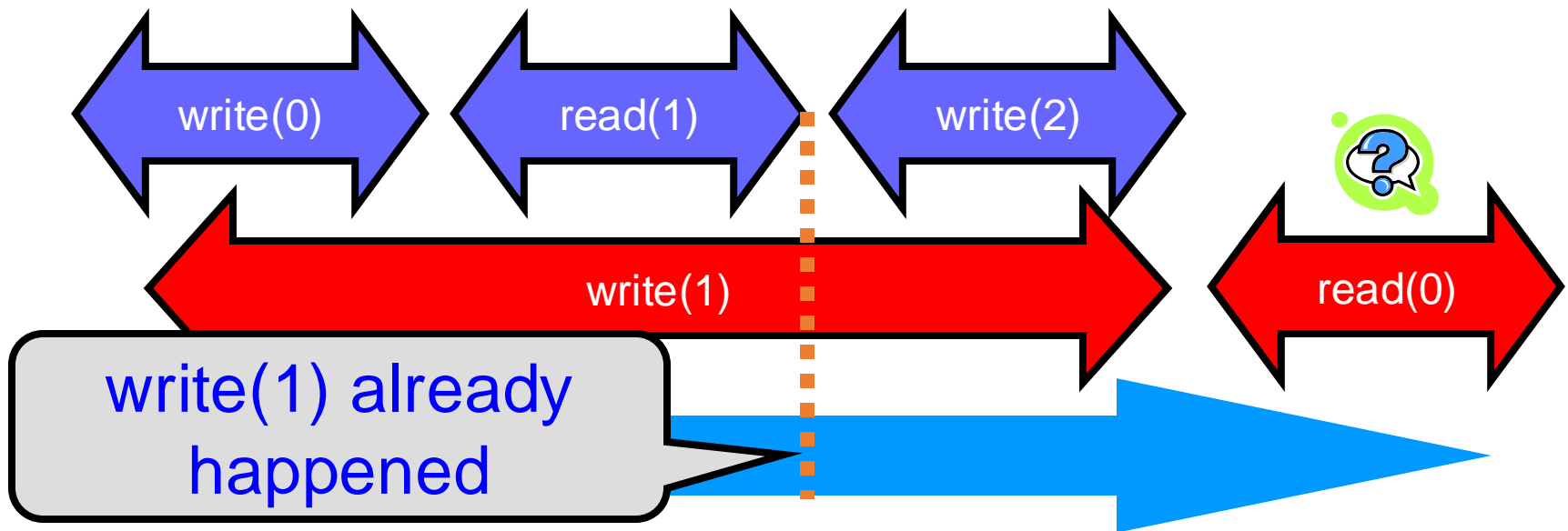
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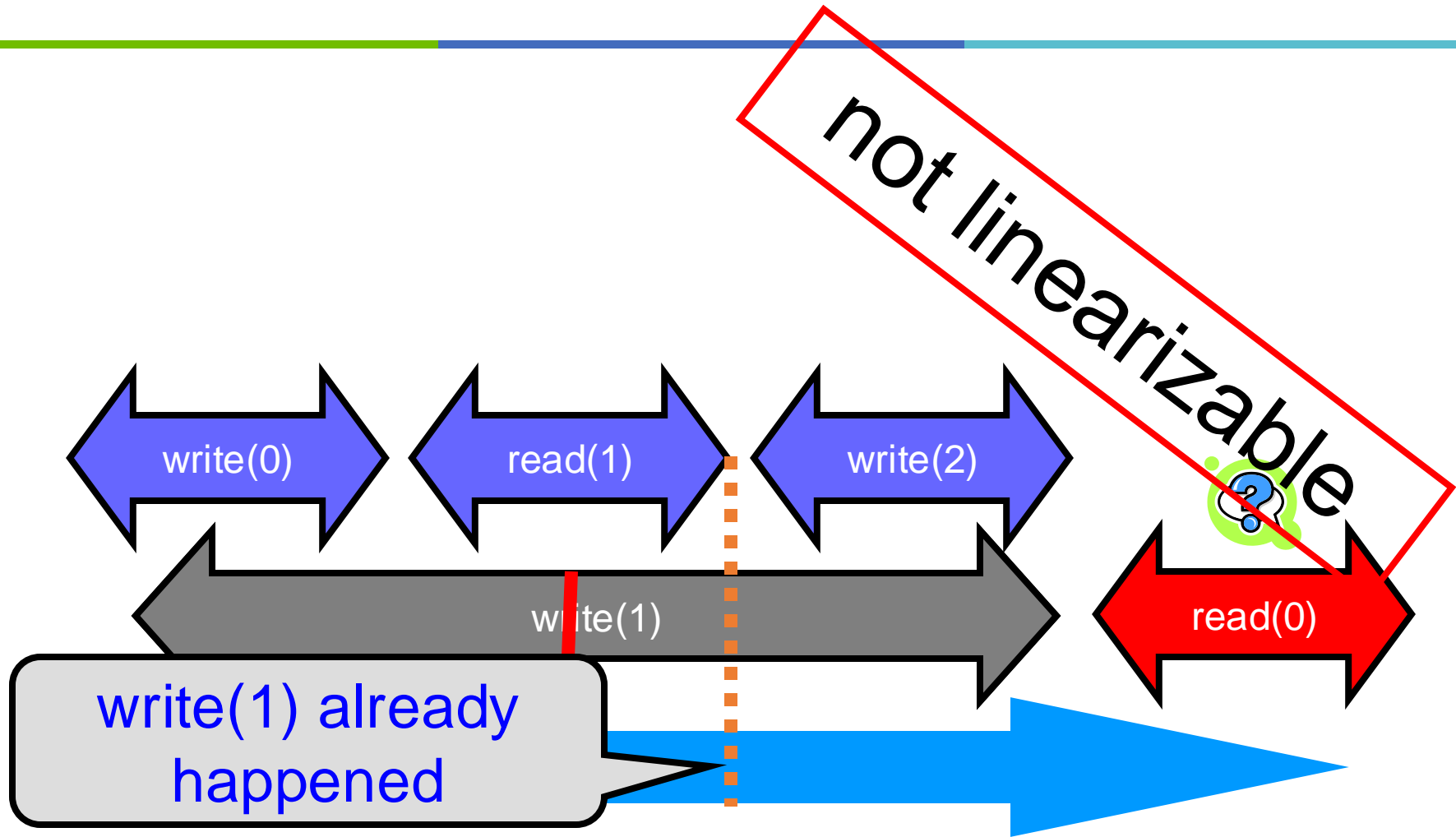
# Read/Write Register Example



# Read/Write Register Example

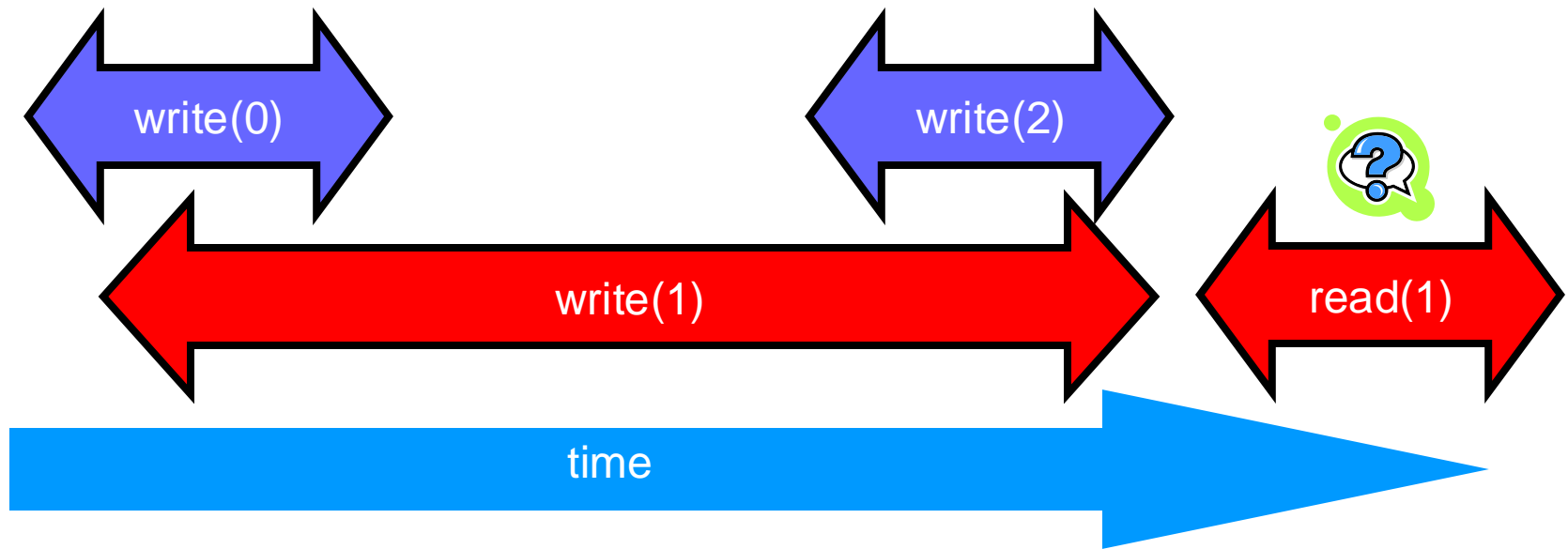


# Read/Write Register Example

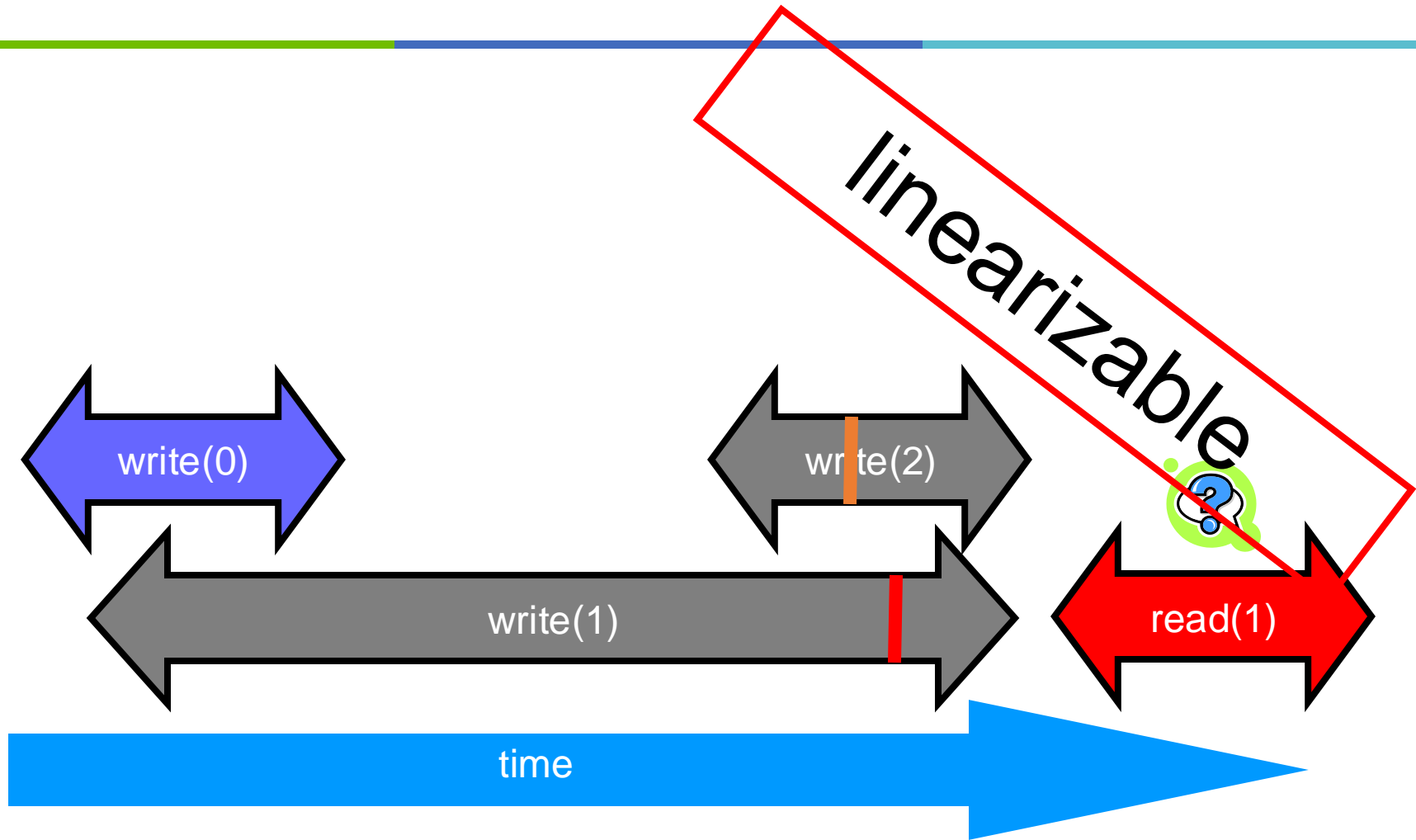




# Read/Write Register Example



# Read/Write Register Example



# Talking About Executions

---

- Why?
  - Can't we specify the linearization point of each operation without describing an execution?
- Not Always
  - In some cases, linearization point ***depends on the execution***

# Formal Model of Executions

---

- Define precisely what we mean
  - Ambiguity is bad when intuition is weak
- Allow reasoning
  - Formal
  - But mostly informal
    - In the long run, actually is equally (or even more) important
    - Why?!

# Split Method Calls into Two Events

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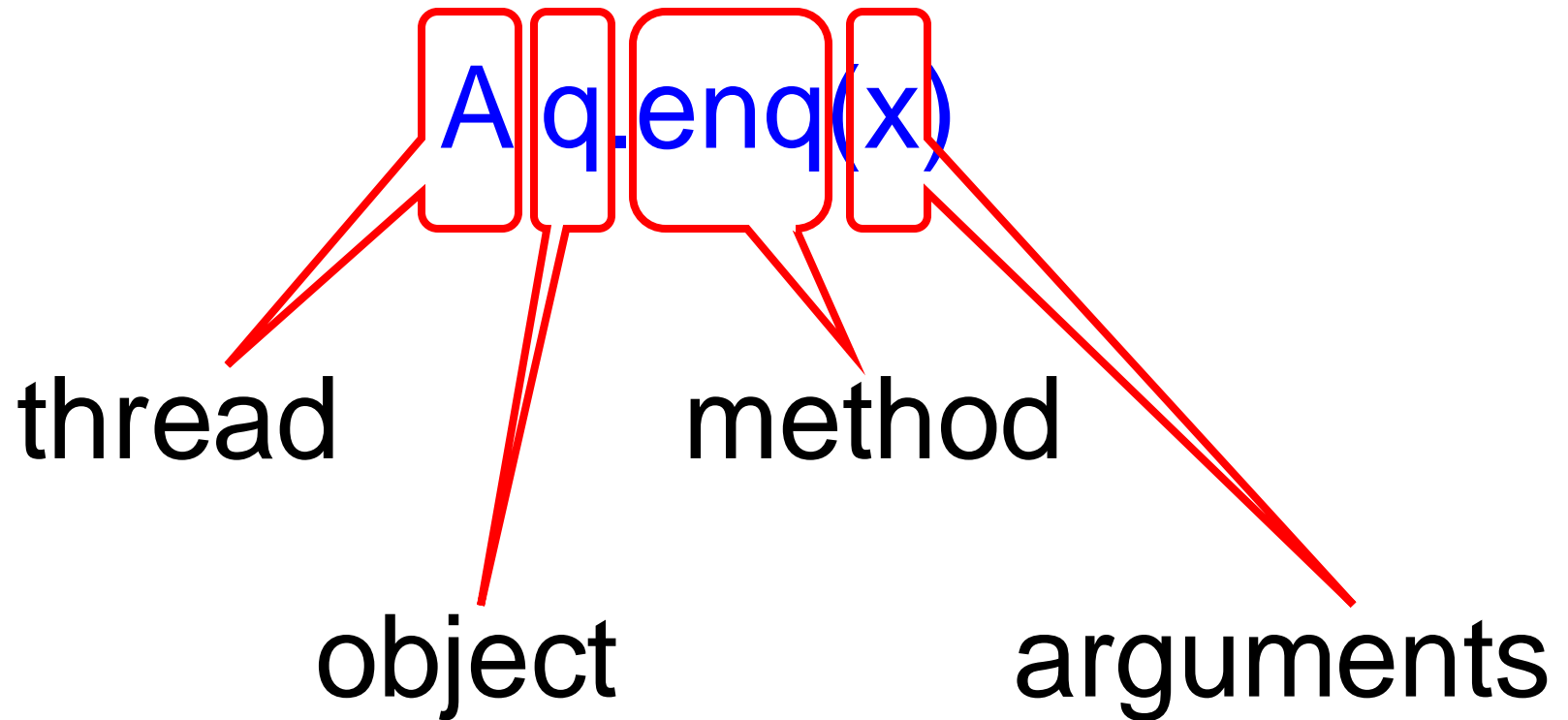
- Invocation
  - method name & args
  - `q.enq(x)`
- Response
  - result or exception
  - `q.enq(x)` returns `void`
  - `q.deq()` returns `x`
  - `q.deq()` throws `empty`

# Invocation Notation

---

$A \ q.\text{enq}(x)$

# Invocation Notation



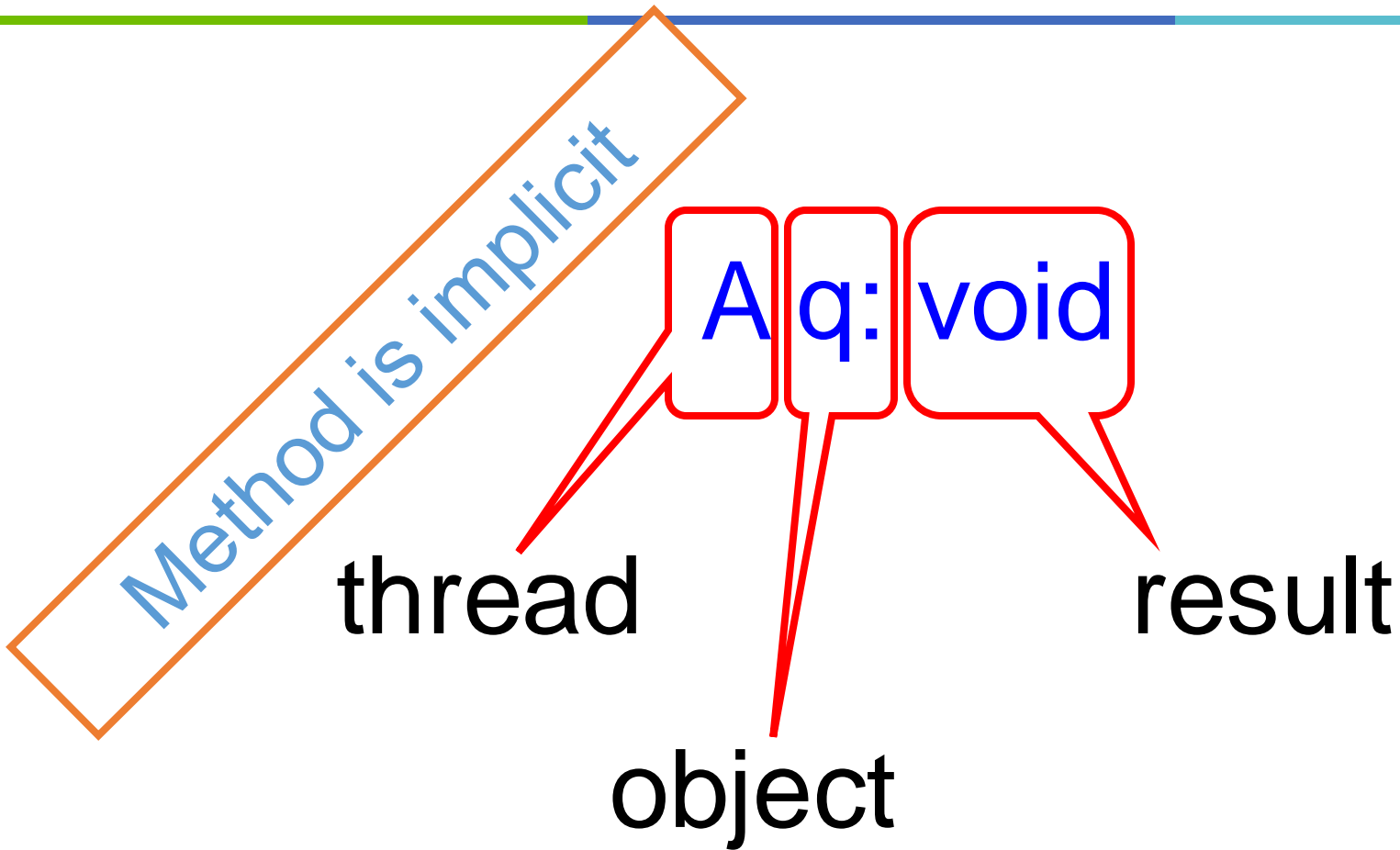
# Response Notation

---

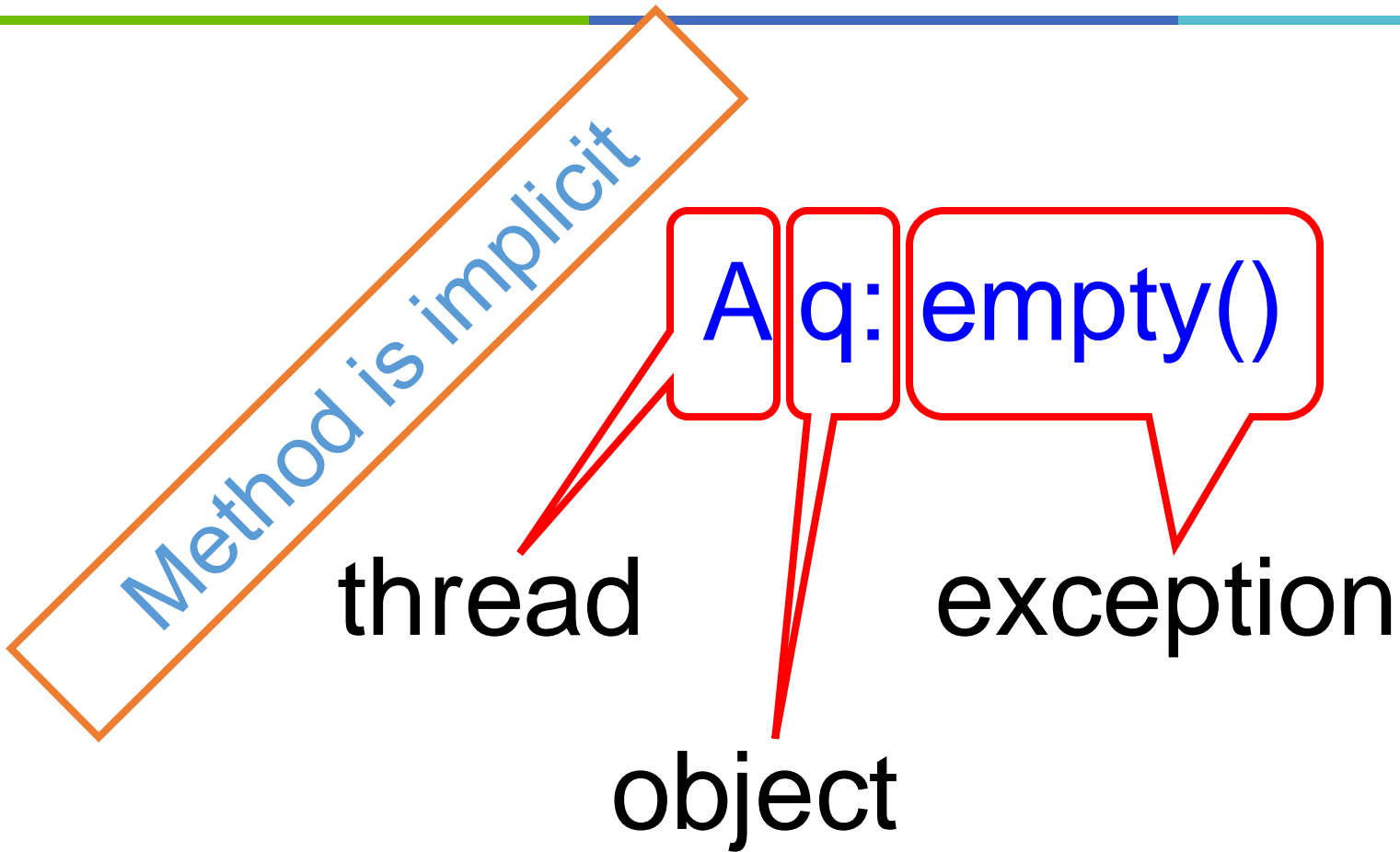
A q: void



# Response Notation



# Response Notation



# History – Describing an Execution

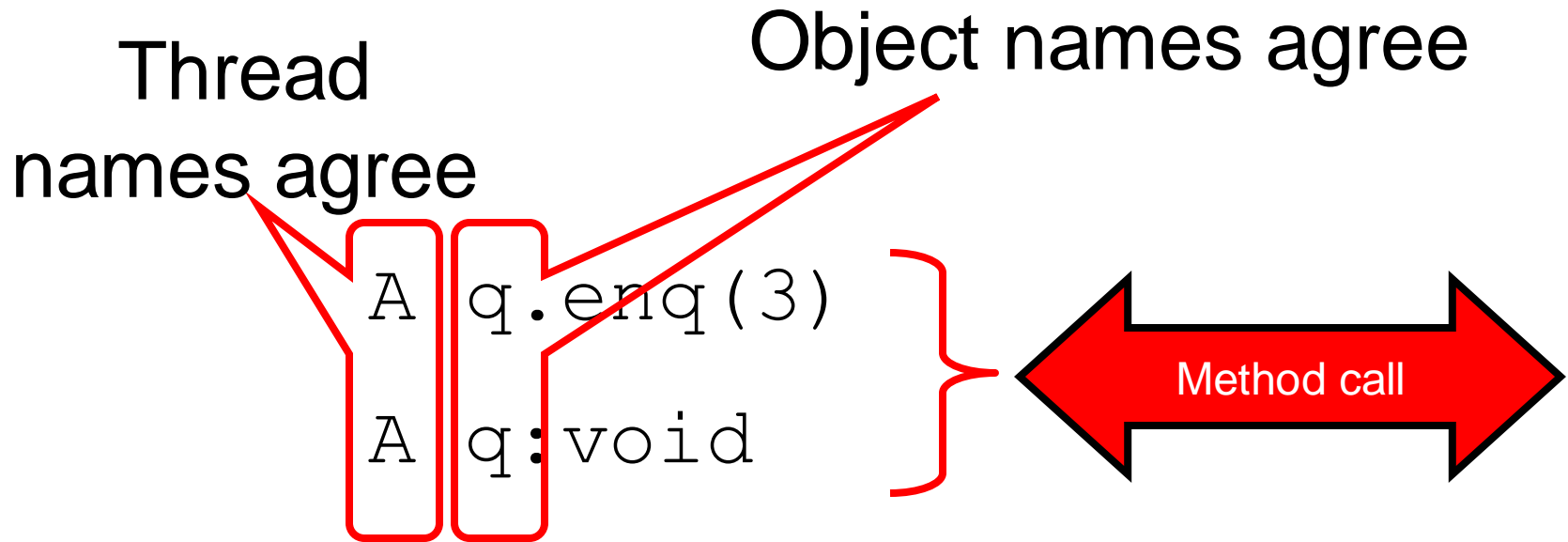
Sequence of invocations  
and responses

$H =$

- A `q.enq(3)`
- A `q:void`
- A `q.enq(5)`
- B `p.enq(4)`
- B `p:void`
- B `q.deq()`
- B `q:3`

# Definition

- Invocation & response *match* if



# Object Projections

---

**H** =

- A q.enq(3)
- A q:void
- B p.enq(4)
- B p:void
- B q.deq()
- B q:3

# Object Projections

---

A `q.enqueue(3)`

A `q: void`

$H|q =$

B `q.dequeue()`

B `q: 3`

# Thread Projections

---

**H =**

A	q.enq(3)
A	q:void
B	p.enq(4)
B	p:void
B	q.deq()
B	q:3

# Thread Projections

---

$H|B =$

- B p.enq(4)
- B p:void
- B q.deq()
- B q:3

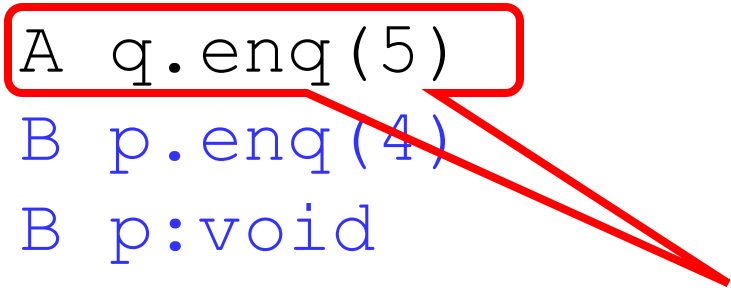


# Complete Subhistory

H =

A	q.enq(3)
A	q:void
A	q.enq(5)
B	p.enq(4)
B	p:void
B	q.deq()
B	q:3

An invocation is *pending* if it has no matching response



# Complete Subhistory

H =

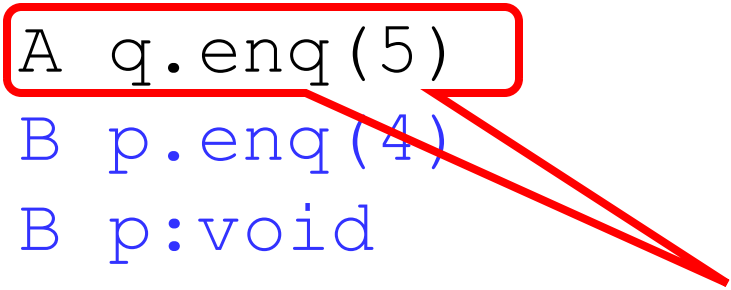
A	q.enq(3)	
A	q:void	
A	q.enq(5)	
B	p.enq(4)	
B	p:void	
B	q.deq()	
B	q:3	May or may not have taken effect

# Complete Subhistory

H =

A	q.enq(3)	
A	q:void	
A	q.enq(5)	
B	p.enq(4)	
B	p:void	
B	q.deq()	
B	q:3	

Discard pending  
invocations



# Complete Subhistory

---

```
A  q.enqueue(3)
A  q: void
```

```
Complete(H) = B  p.enqueue(4)
               B  p: void
               B  q.dequeue()
               B  q: 3
```

# Sequential Histories

---

A q.enqueue(3)

A q:void

B p.enqueue(4)

B p:void

B q.dequeue()

B q:3

A q.enqueue(5)

# Sequential Histories

A q.enqueue(3)  
A q:void

B p.enqueue(4)  
B p:void

B q.dequeue()  
B q:3

A q.enqueue(5)

match

match

match

Final pending  
invocation OK

Method calls of different  
threads do not interleave

# Well-Formed Histories

---

H=  
A q.enq(3)  
B p.enq(4)  
B p:void  
B q.deq()  
A q:void  
B q:3

# Well-Formed Histories

H=

```
A q.enqueue(3)
B p.enqueue(4)
B p:void
B q.dequeue()
A q:void
B q:3
```

H | B=

```
B p.enqueue(4)
B p:void
B q.dequeue()
B q:3
```

Per-thread projections  
are sequential

H | A=

```
A q.enqueue(3)
A q:void
```



# Equivalent Histories

Threads see the same  
thing in both projections

$$\left\{ \begin{array}{l} H \mid A = G \mid A \\ H \mid B = G \mid B \end{array} \right.$$

H=

A q.enq(3)	A q.enq(3)
A q:void	A q:void
B p.enq(4)	
B p:void	
B q.deq()	B p.enq(4)
	B p:void
B q:3	B q.deq()
	B q:3

G=

A q.enq(3)	A q.enq(3)
B p.enq(4)	A q:void
B p:void	
B q.deq()	
A q:void	B p.enq(4)
B q:3	B p:void
	B q.deq()
	B q:3

# Sequential Specifications

---

- A sequential specification is some way of telling whether a
  - Single-thread, single-object history **is legal**
- For example:
  - Pre and post-conditions
  - But plenty of other techniques exist ...

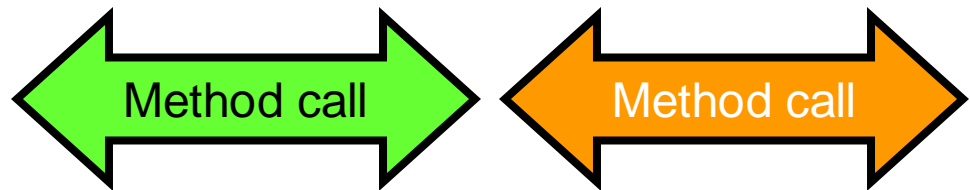
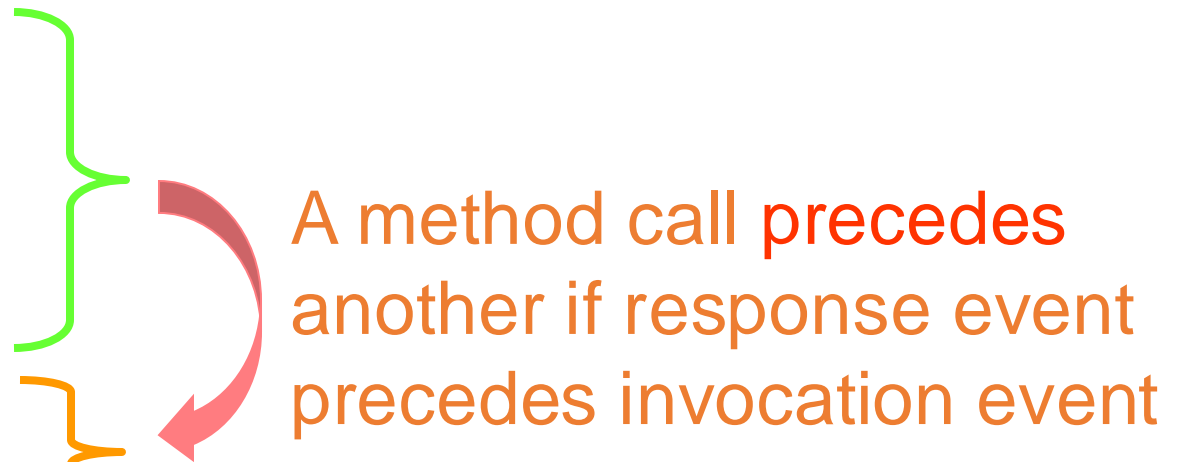
# Legal Histories

---

- A sequential (multi-object) history  $H$  is legal if
  - For every object  $x$
  - $H \mid x$  is in the sequential spec for  $x$

# Precedence

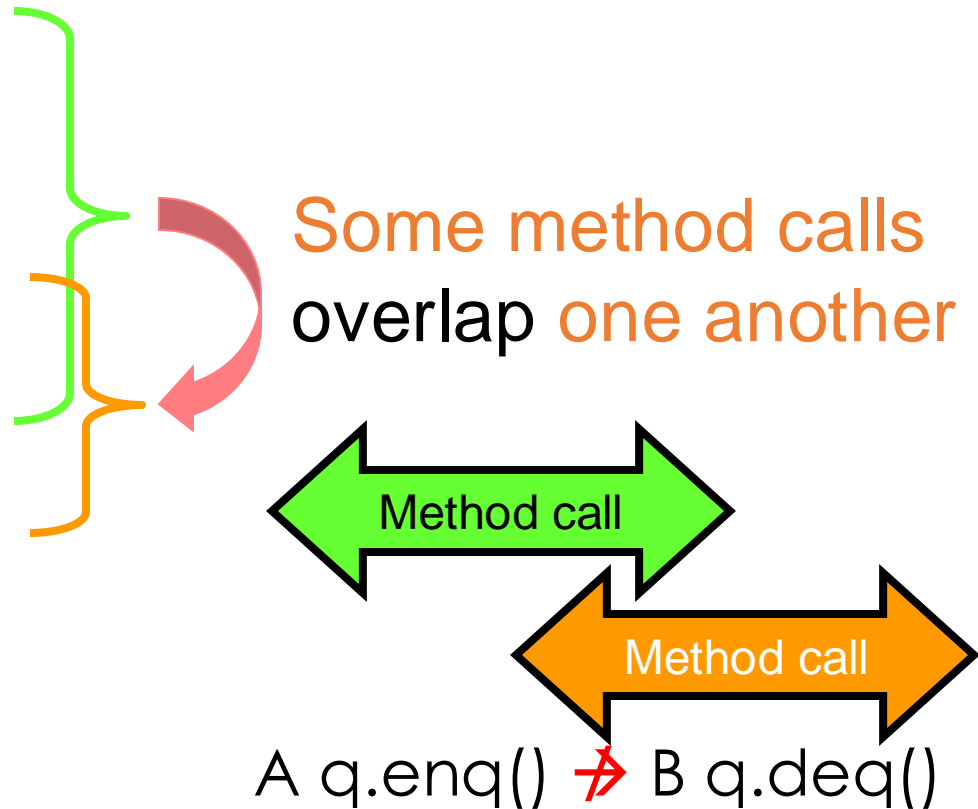
```
A q.enq(3)
B p.enq(4)
B p.void
A q:void
B q.deq()
B q:3
```



A q.enq() → B q.deq()

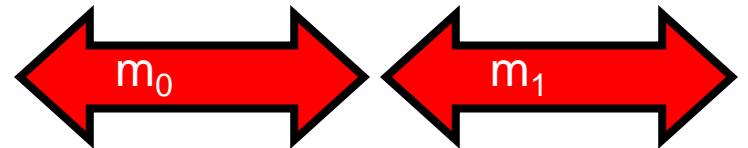
# Non-Precedence

```
A q.enq(3)
B p.enq(4)
B p.void
B q.deq()
A q: void
B q: 3
```



# Notation

- Given
  - History  $H$
  - method executions  $m_0$  and  $m_1$  in  $H$
- We say  $m_0 \rightarrow_H m_1$ , if
  - $m_0$  precedes  $m_1$
- Relation  $m_0 \rightarrow_H m_1$  is a
  - Partial order
  - Total order if  $H$  is sequential



# Linearizability

- History **H** is **linearizable** if it can be extended to **G** by
  - Appending zero or more responses to pending invocations
  - Discarding other pending invocations
- So that **G** is equivalent to
  - Legal sequential history **S**
  - where  $\rightarrow_G \subset \rightarrow_S$

# Remarks

---

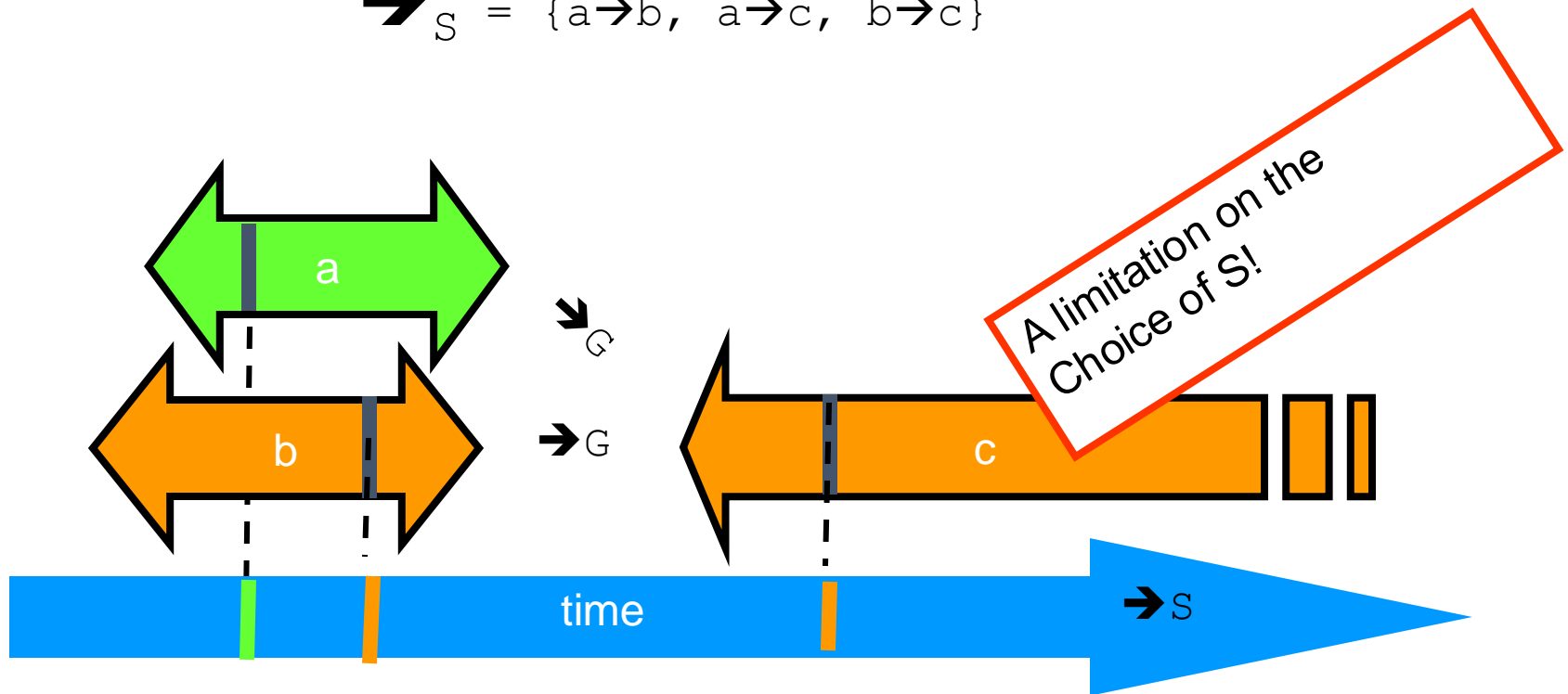
- Some pending invocations
  - Took effect, so keep them
  - Discard the rest
- Condition  $\rightarrow_G \subset \rightarrow_S$ 
  - Means that **S** respects “real-time order” of **G**



# Ensuring $\rightarrow_G \subset \rightarrow_S$

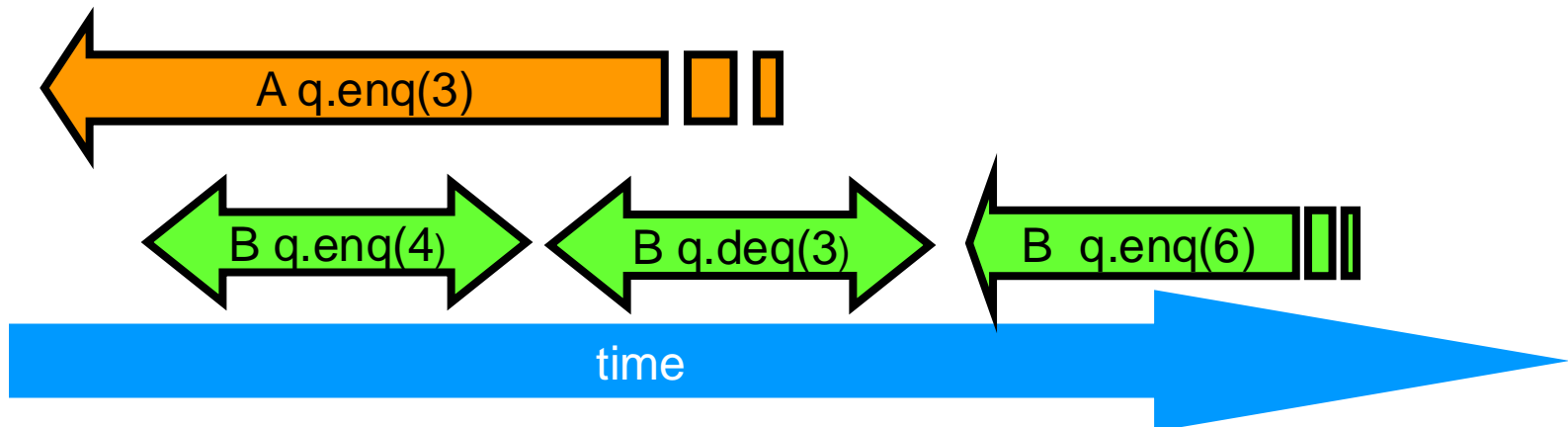
$$\rightarrow_G = \{a \rightarrow c, b \rightarrow c\}$$

$$\rightarrow_S = \{a \rightarrow b, a \rightarrow c, b \rightarrow c\}$$



# Example

```
A q.enq(3)
B q.enq(4)
B q:void
B q.deq()
B q:4
B q:enq(6)
```



# Example

A q.enq(3)

B q.enq(4)

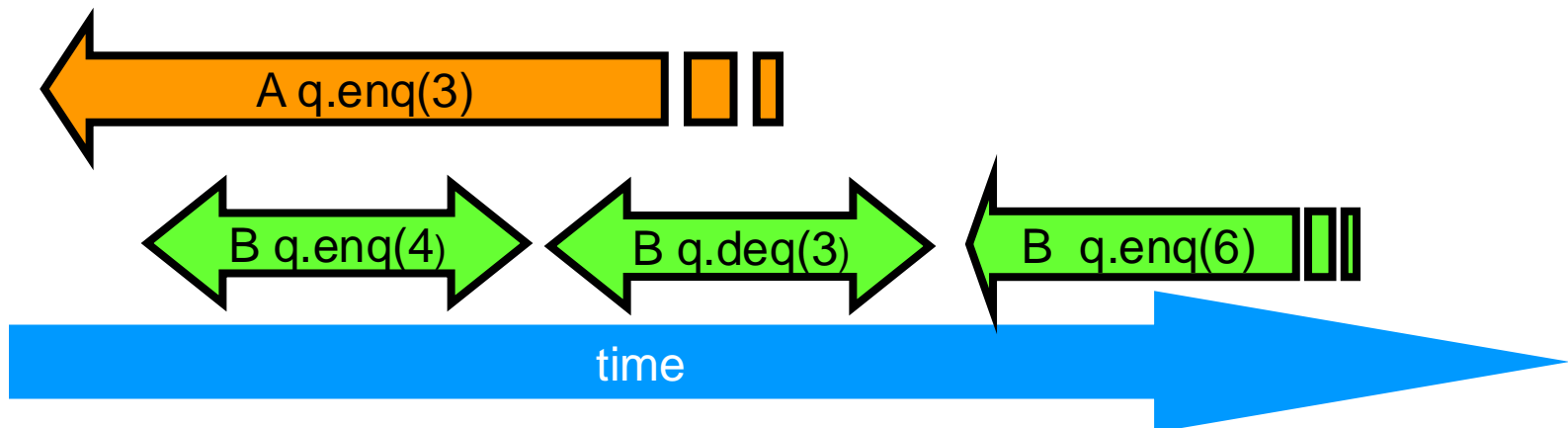
B q:void

B q.deq()

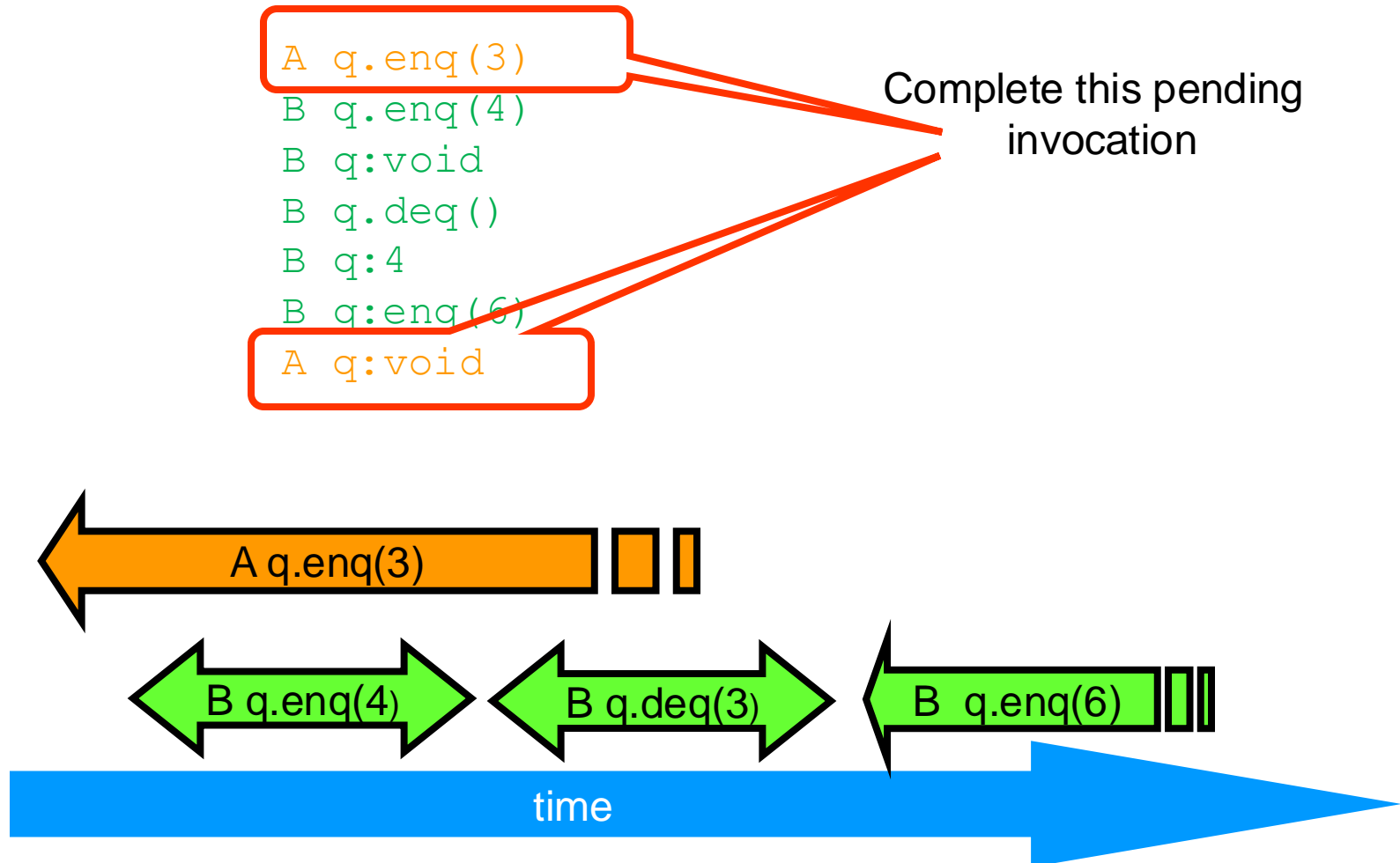
B q:4

B q:enq(6)

Complete this pending invocation



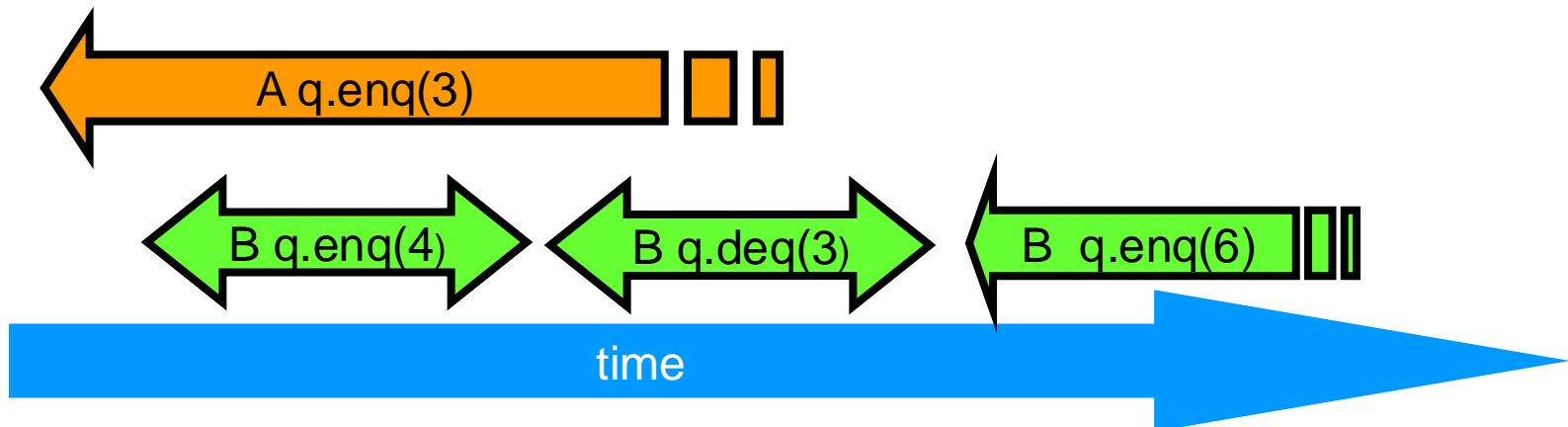
# Example



# Example

discard this one

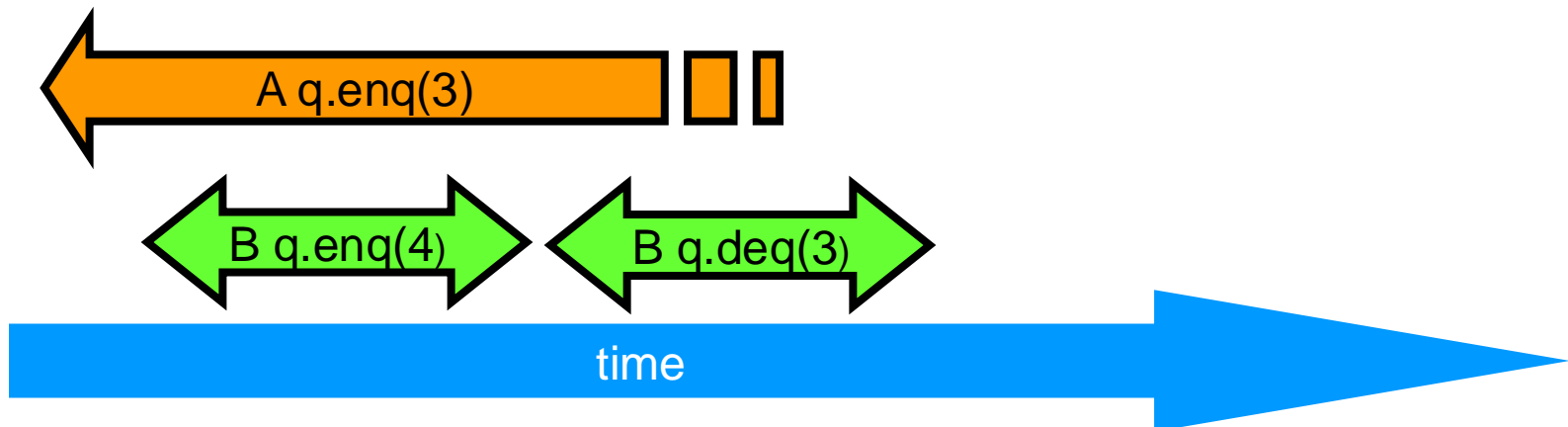

```
A q.enq(3)
B q.enq(4)
B q:void
B q.deq()
B q:4
B q:enq(6)
A q:void
```



# Example

discard this one

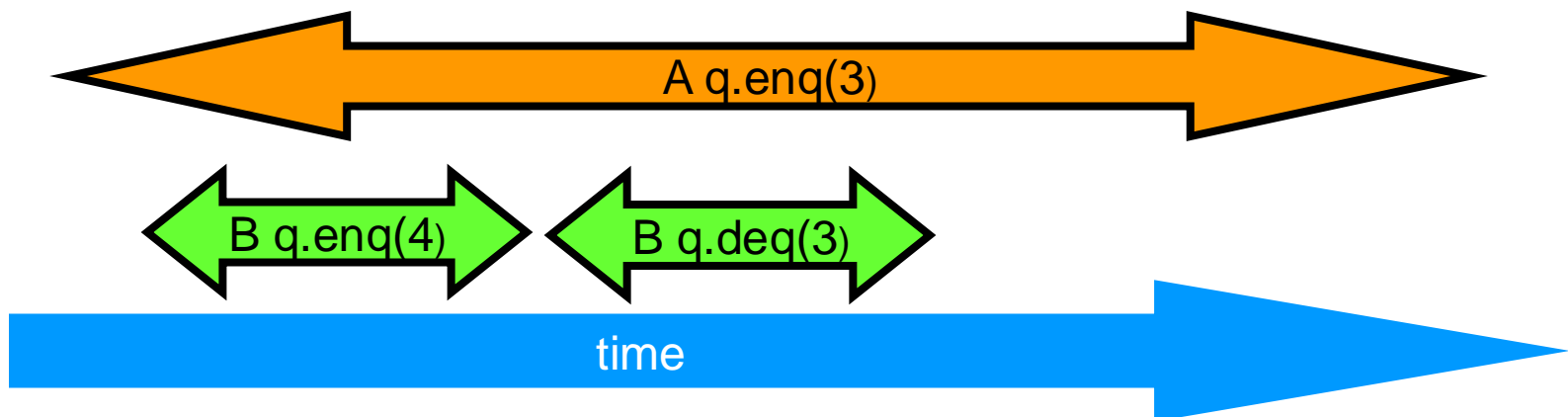
```
A q.enq(3)
B q.enq(4)
B q:void
B q.deq()
B q:4
A q:void
```



# Example

A q.enq(3)  
B q.enq(4)  
B q:void  
B q.deq()  
B q:4  
A q:void

B q.enq(4)  
B q:void  
A q.enq(3)  
A q:void  
B q.deq()  
B q:4

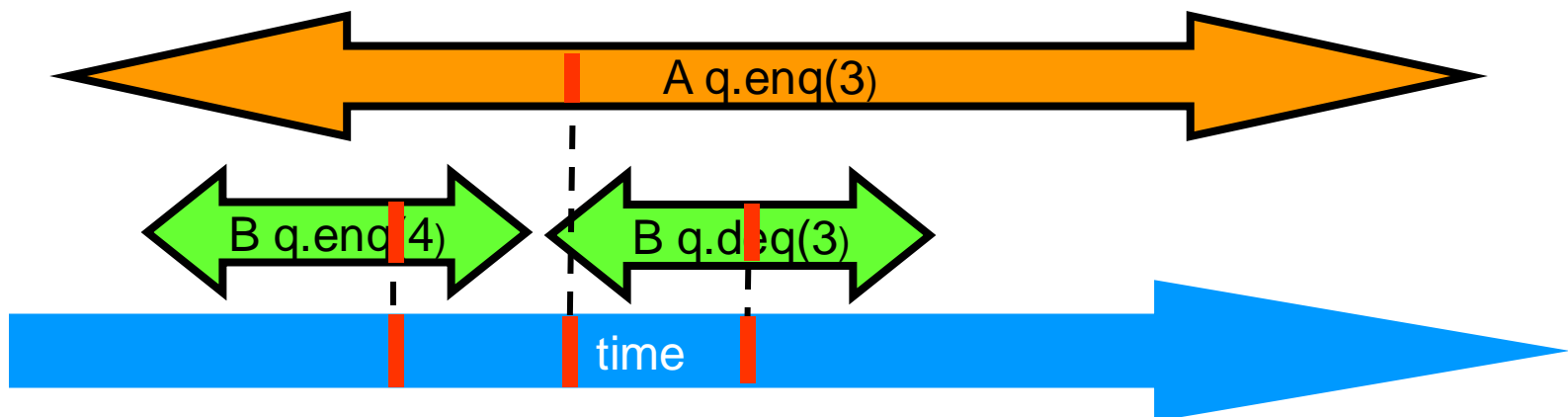


# Example

Equivalent sequential history

A q.enq(3)  
B q.enq(4)  
B q:void  
B q.deq()  
B q:4  
A q:void

B q.enq(4)  
B q:void  
A q.enq(3)  
A q:void  
B q.deq()  
B q:4





# Concurrency

---

- How much concurrency does linearizability allow?
- When must a method invocation block?

# Concurrency

---

- Focus on **total** methods
  - Defined in every state
- Example:
  - `deq()` that throws `Empty` exception
  - Versus `deq()` that waits ...
- Why?
  - Otherwise, blocking unrelated to synchronization

# Concurrency

---

- **Question:** When does linearizability require a method invocation to block?
- **Answer:** never.
- Linearizability is *non-blocking*

# Non-Blocking Theorem

---

If method invocation

$A \text{ } q.\text{inv}(\dots)$

is pending in history  $H$ , then there exists a response

$A \text{ } q:\text{res}(\dots)$

such that

$H + A \text{ } q:\text{res}(\dots)$

is linearizable

# Proof

- Pick linearization  $S$  of  $H$
- If  $S$  already contains
  - Invocation  $A \text{ } q.\text{inv}(\dots)$  and response,
  - Then we are done.
- Otherwise, pick a response such that
  - $S + A \text{ } q.\text{inv}(\dots) + A \text{ } q:\text{res}(\dots)$
  - Possible because object is **total**



All methods are total



Methods are defined for  
every object state

# Composability Theorem

---

- History  $H$  is linearizable if and only if
  - For every object  $x$
  - $H \mid x$  is linearizable
- This means that linear histories are composable

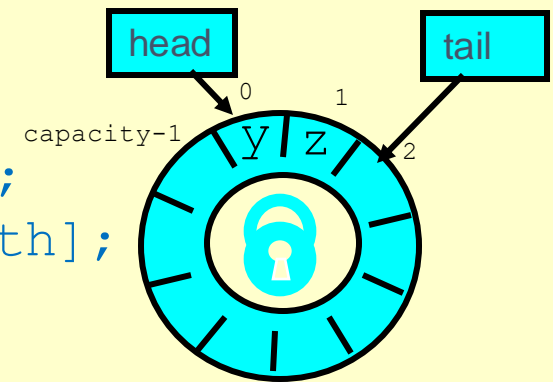
# Why Does Composability Matter?

---

- Modularity
- Can prove linearizability of objects in isolation
- Can compose independently-implemented objects

# Reasoning About Linearizability: Locking

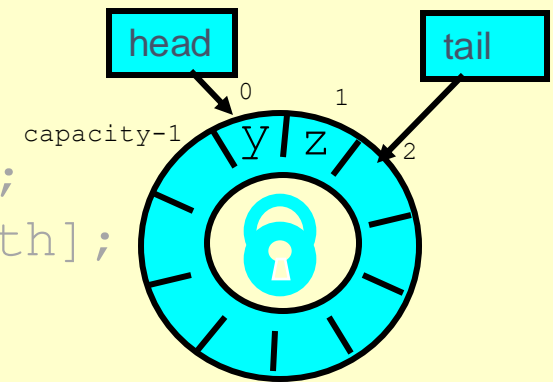
```
public T deq() throws EmptyException {  
    lock.lock();  
    try {  
        if (tail == head)  
            throw new EmptyException();  
        T x = items[head % items.length];  
        head++;  
        return x;  
    } finally {  
        lock.unlock();  
    }  
}
```





# Reasoning About Linearizability: Locking

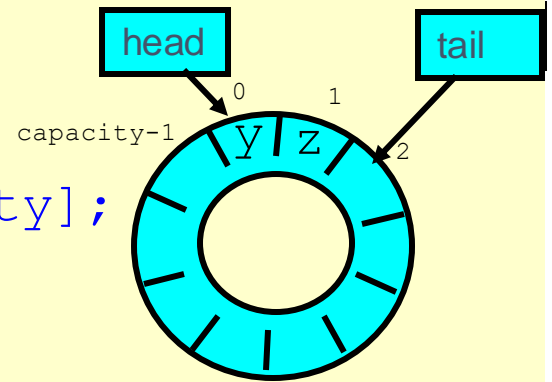
```
public T deq() throws EmptyException {  
    lock.lock();  
    try {  
        if (tail == head)  
            throw new EmptyException();  
        T x = items[head % items.length];  
        head++;  
        return x;  
    } finally {  
        lock.unlock();  
    }  
}
```



Linearization points  
are when locks are  
released

# More Reasoning: Wait-free

```
public class WaitFreeQueue {  
  
    int head = 0, tail = 0;  
    items = (T[]) new Object[capacity];  
  
    public void enq(Item x) {  
        if (tail-head == capacity) throw  
            new FullException();  
        items[tail % capacity] = x; tail++;  
    }  
  
    public Item deq() {  
        if (tail == head) throw  
            new EmptyException();  
        Item item = items[head % capacity]; head++;  
        return item;  
    }  
}
```



# More Reasoning: Wait-free

Remember that there  
is only one enqueuer  
and only one dequeuer

Linearization order is  
when head and tail  
fields modified

```
public class FreeQueue {  
    int head = 0;  
    int tail = 0;  
    Item[] items = new Item[capacity];  
  
    public void enq(Item x) {  
        while (tail == head) throw  
            new FullException();  
        items[tail % capacity] = x;  
        tail++;  
    }  
  
    public Item deq() {  
        if (tail == head) throw  
            new EmptyException();  
        Item item = items[head % capacity];  
        return item;  
    }  
}
```

tail++;

head++;

# Strategy

---

- Identify one atomic step where method “happens”
  - Critical section
  - Machine instruction
- Doesn't always work
  - Might need to define several different steps for a given method

# Linearizability: Summary


---

- Powerful specification tool for shared objects
- Allows us to capture the notion of objects being “atomic”
- Don’t leave home without it 😊

# Alternative: Sequential Consistency

- History  $H$  is *Sequentially Consistent* if it can be extended to  $G$  by
  - Appending zero or more responses to pending invocations
  - Discarding other pending invocations
- So that  $G$  is equivalent to a
  - Legal sequential history  $S$
  - ~~– Where  $\rightarrow_G \subseteq \rightarrow_S$~~

Differs from  
linearizability

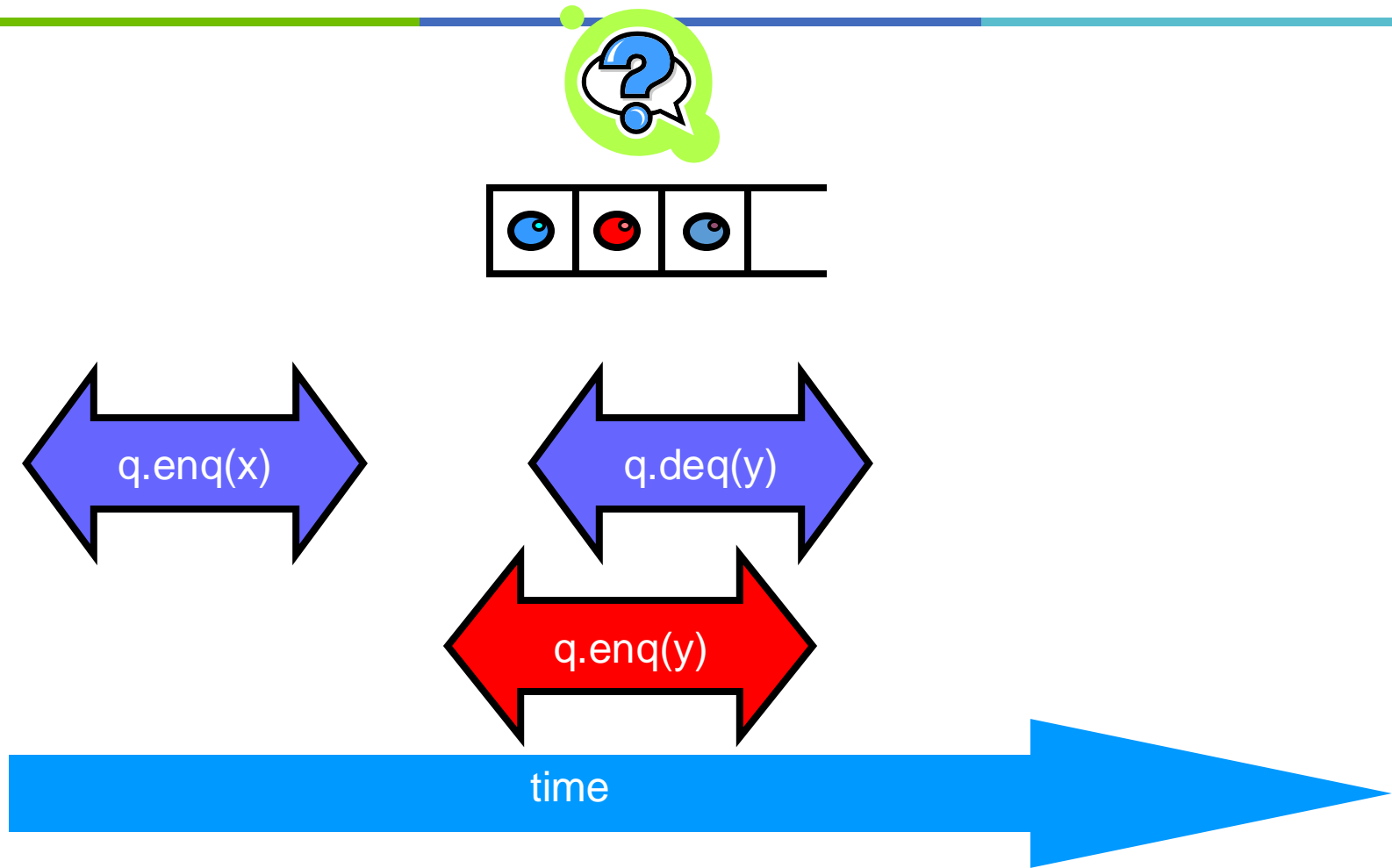


# Sequential Consistency

---

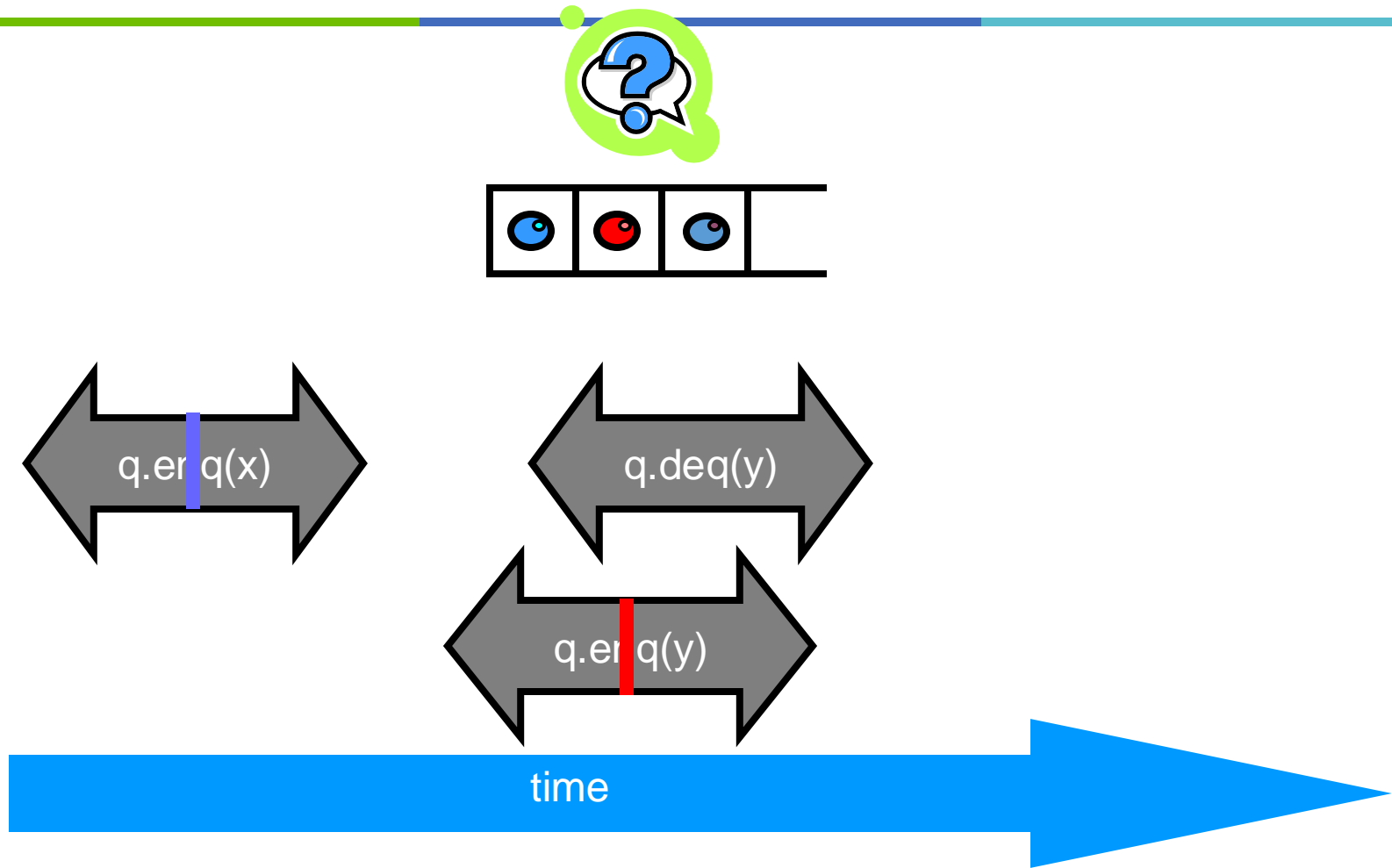
- No need to preserve real-time order
  - **Cannot** re-order operations done by the same thread
  - **Can** re-order non-overlapping operations done by different threads
- Often used to describe multiprocessor memory architectures

# Example

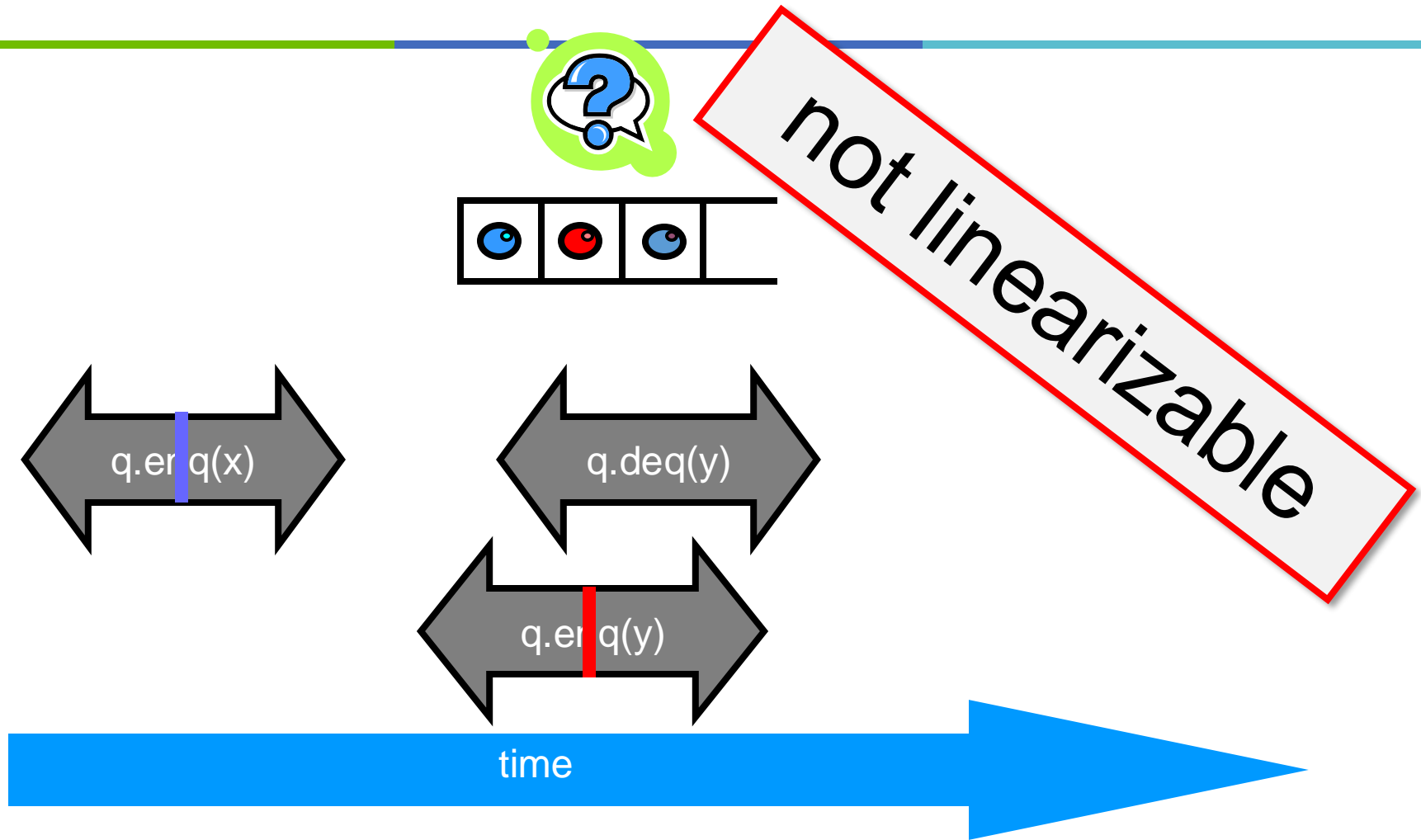




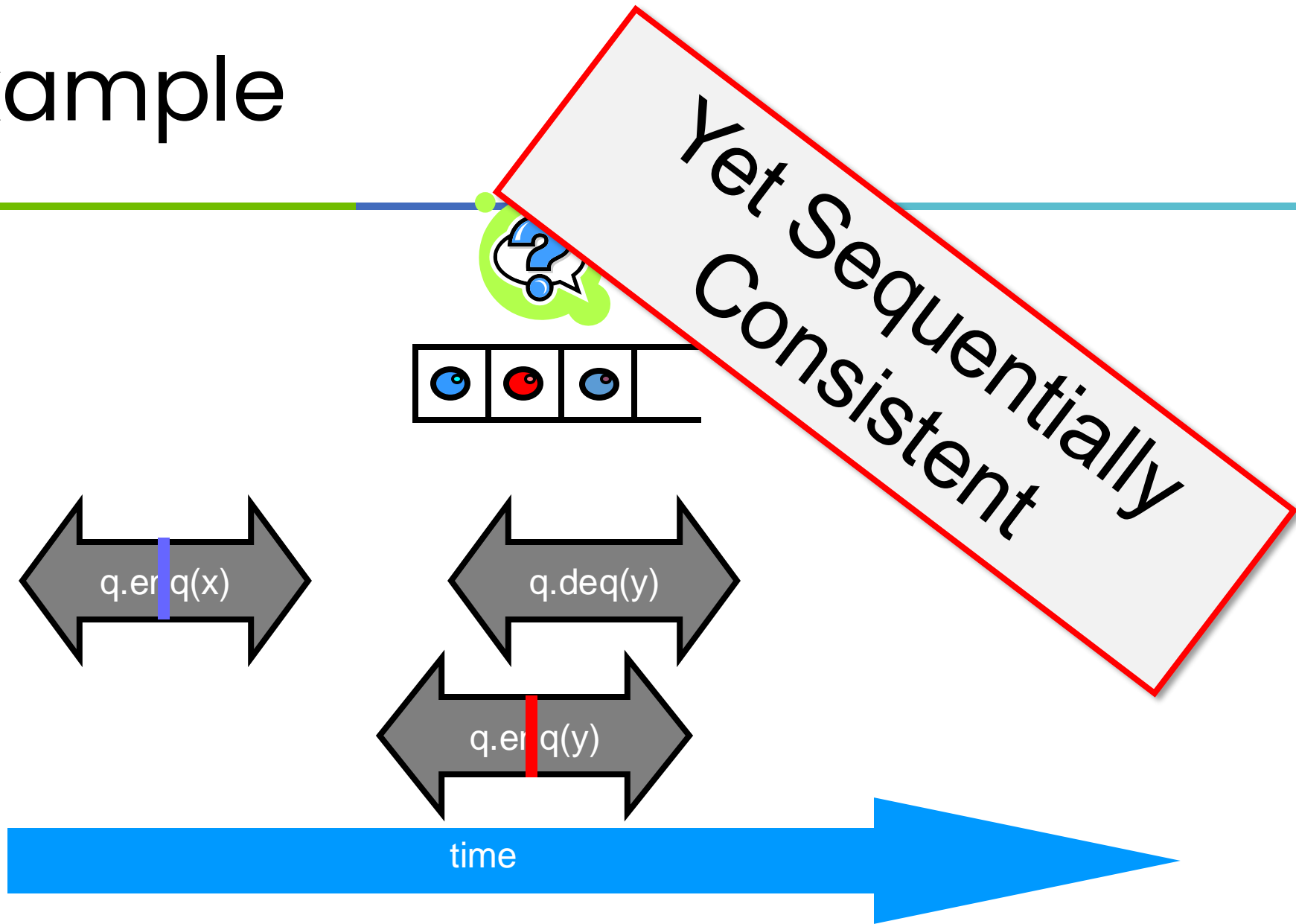
# Example



# Example



# Example

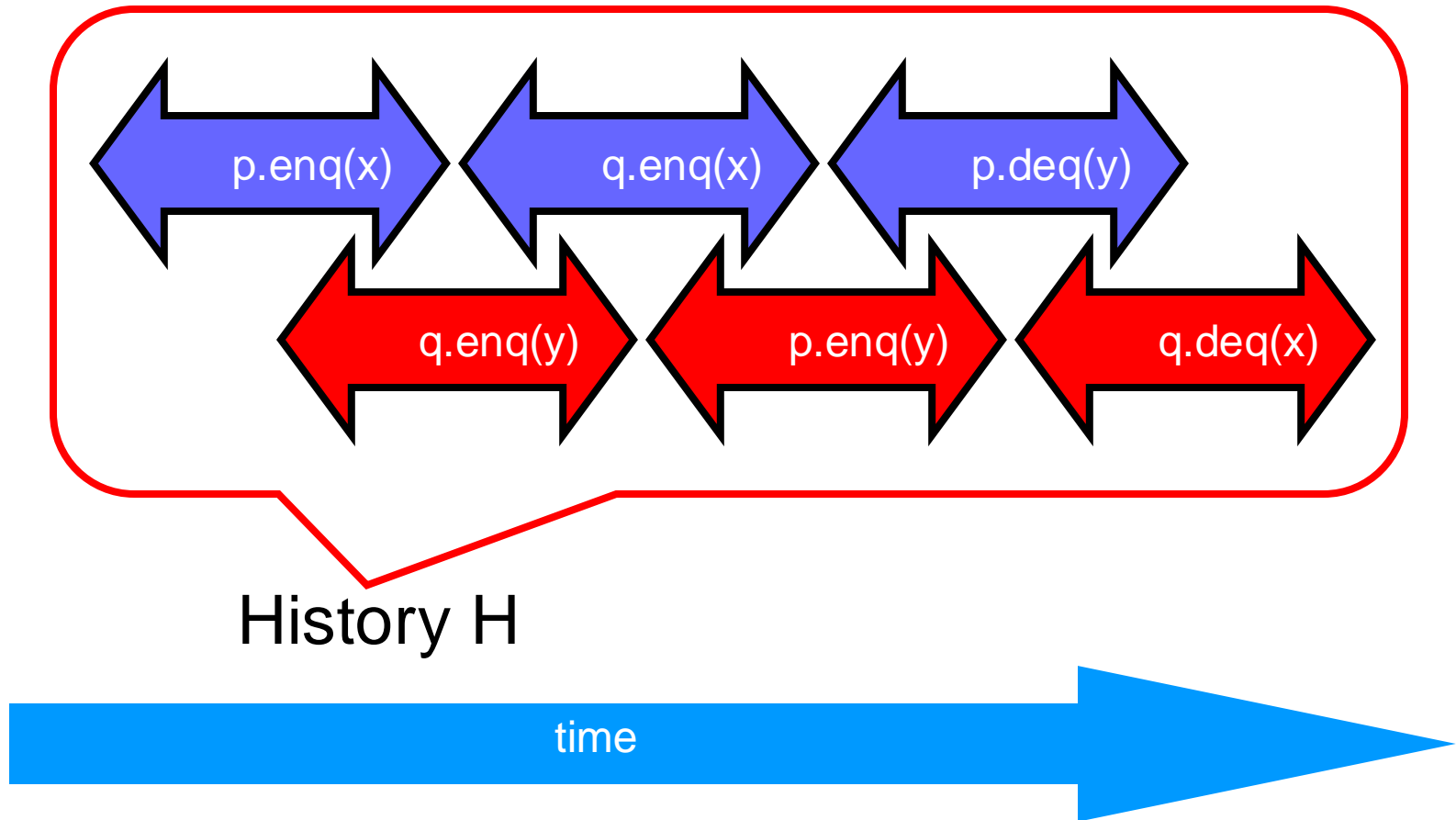


# Theorem

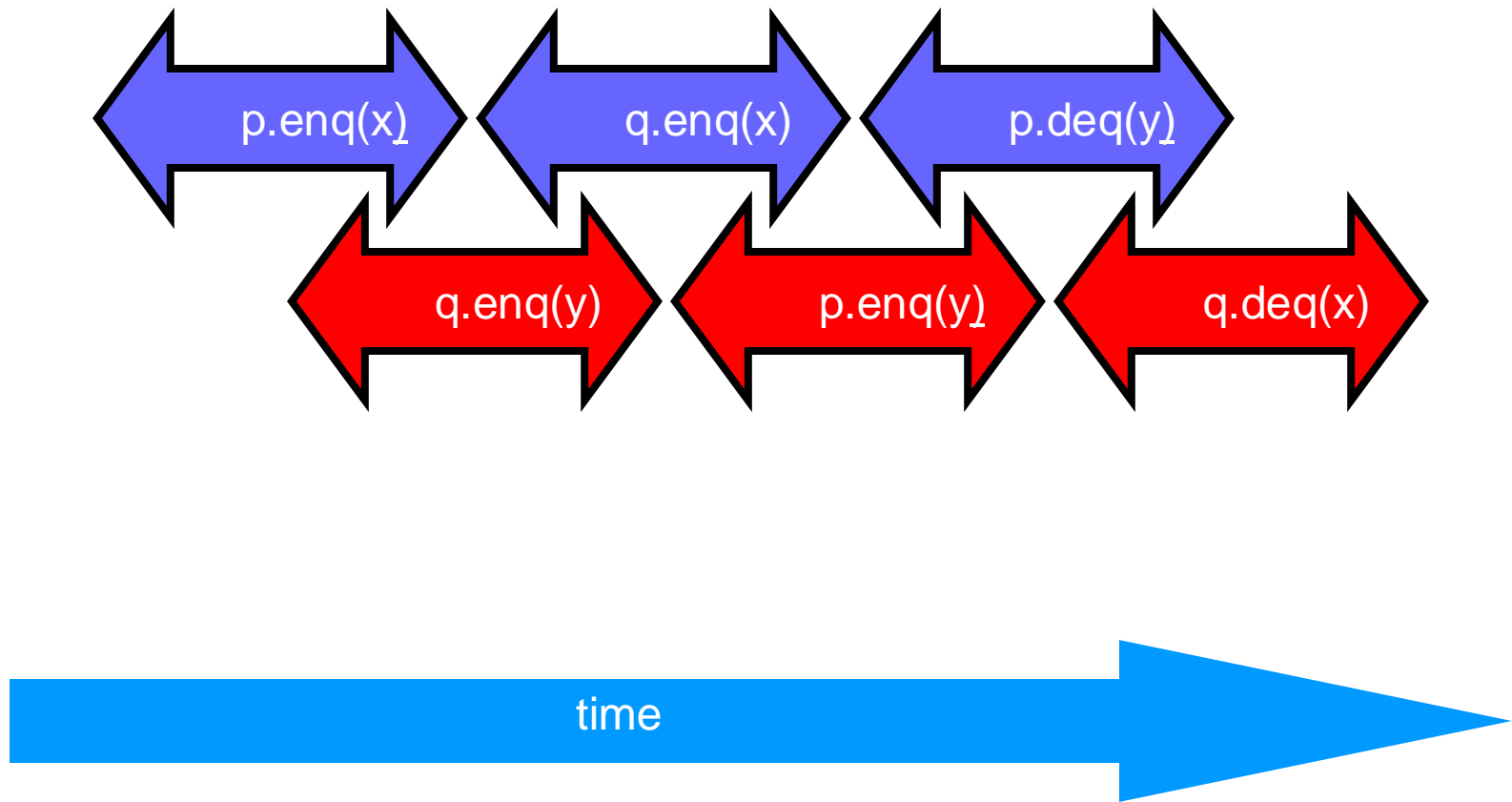
---

Sequential Consistency is not composable

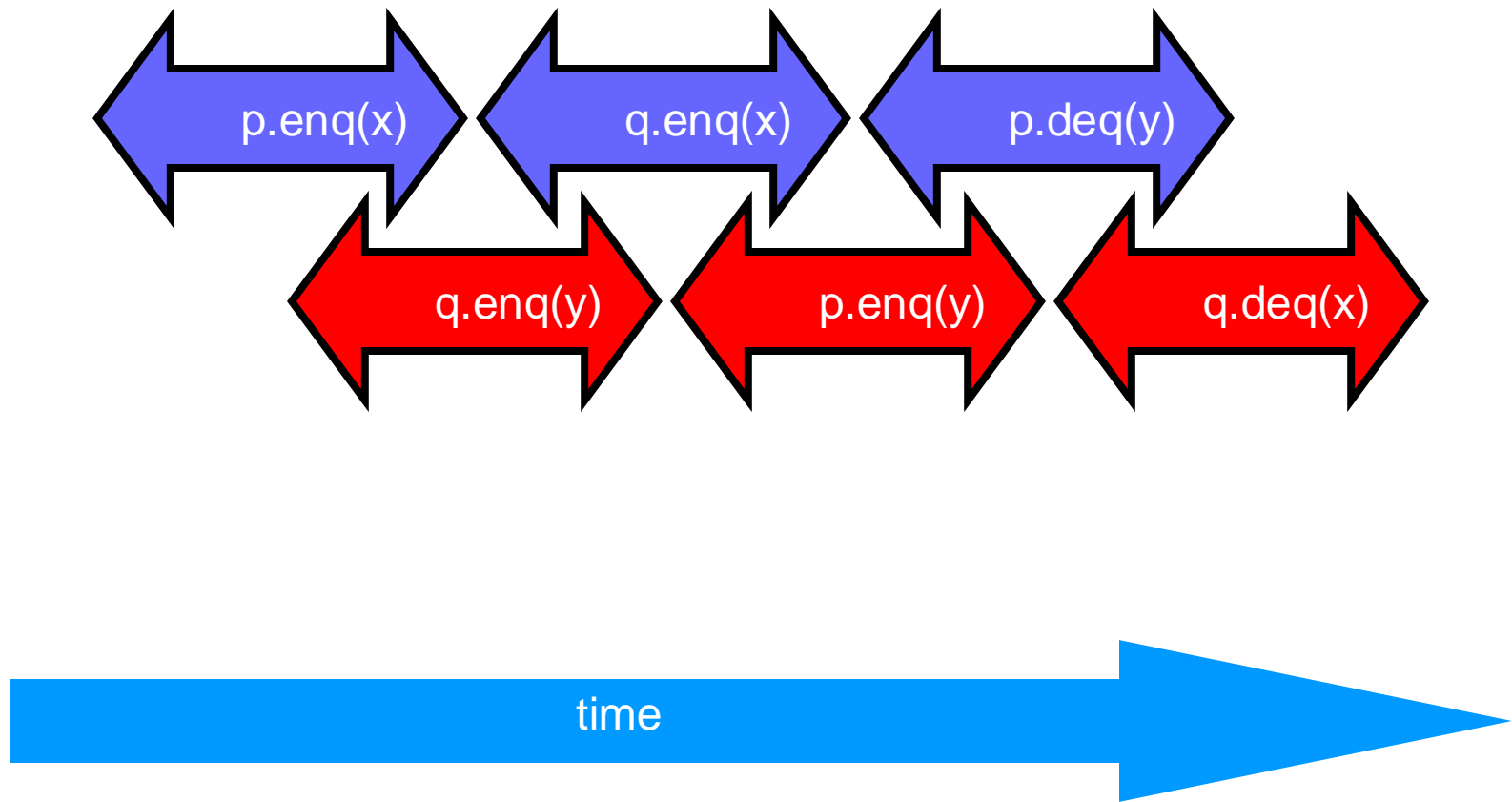
# FIFO Queue Example



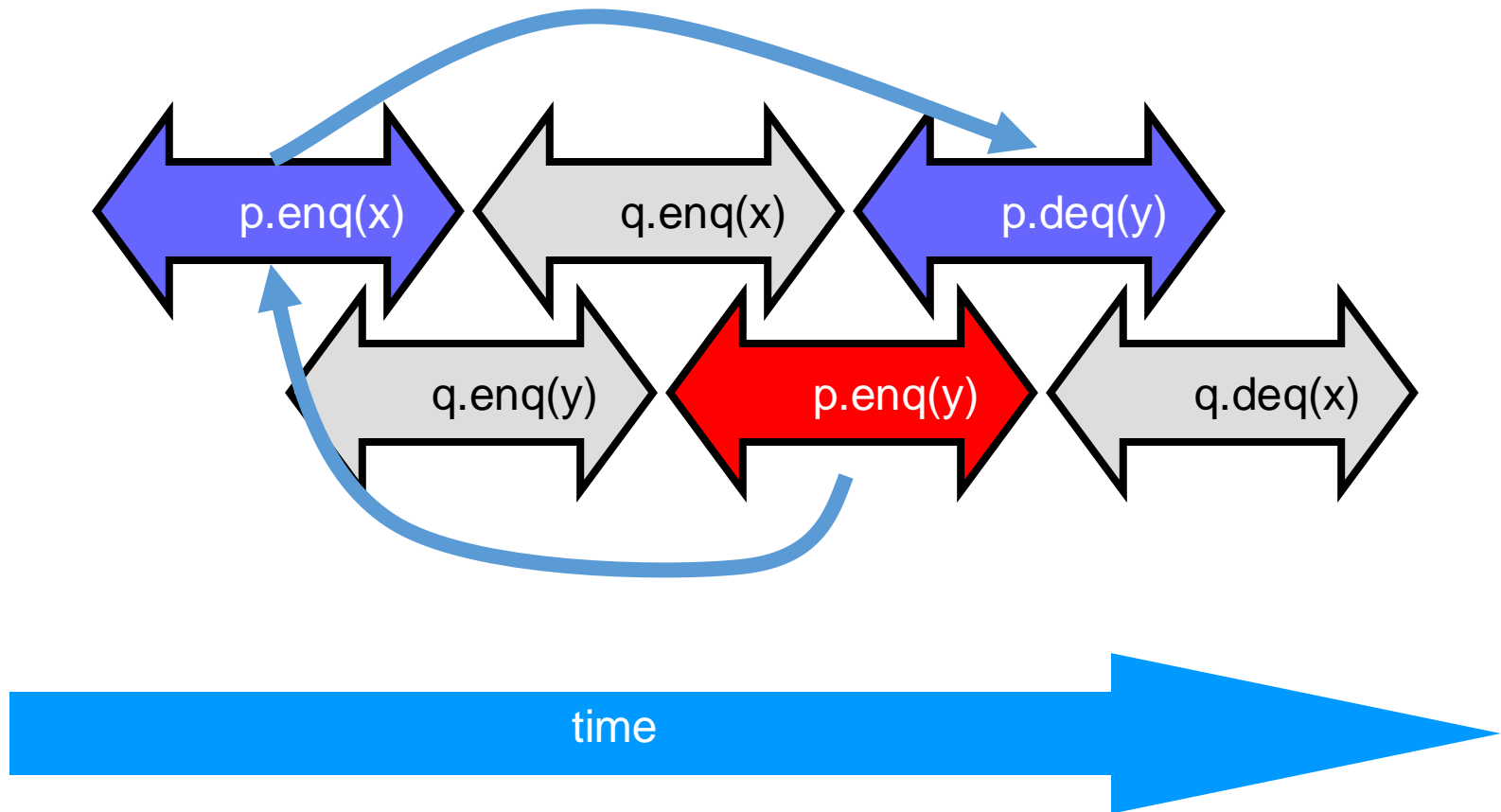
# Hlp Sequentially Consistent



# H/q Sequentially Consistent

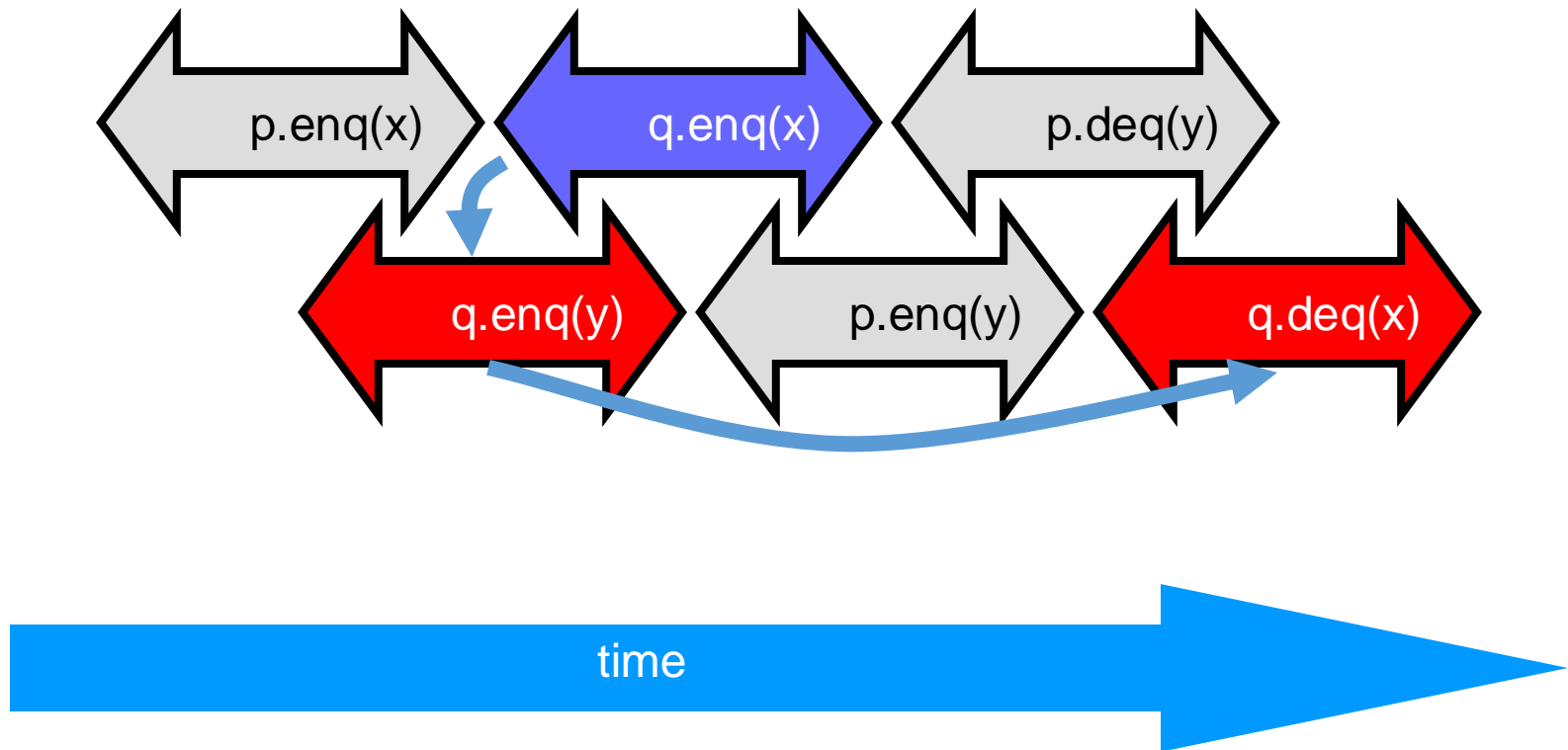


# Ordering imposed by p

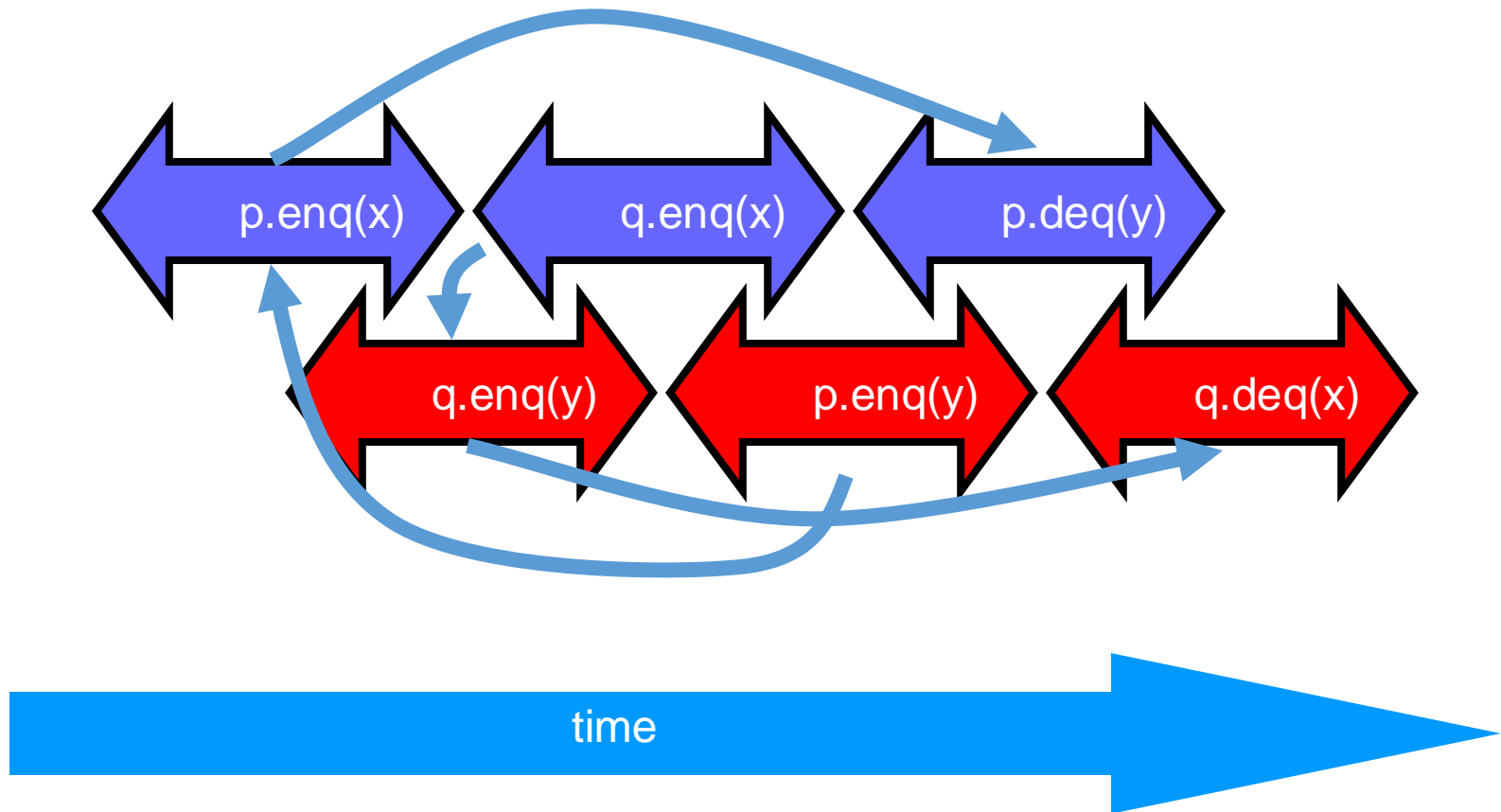




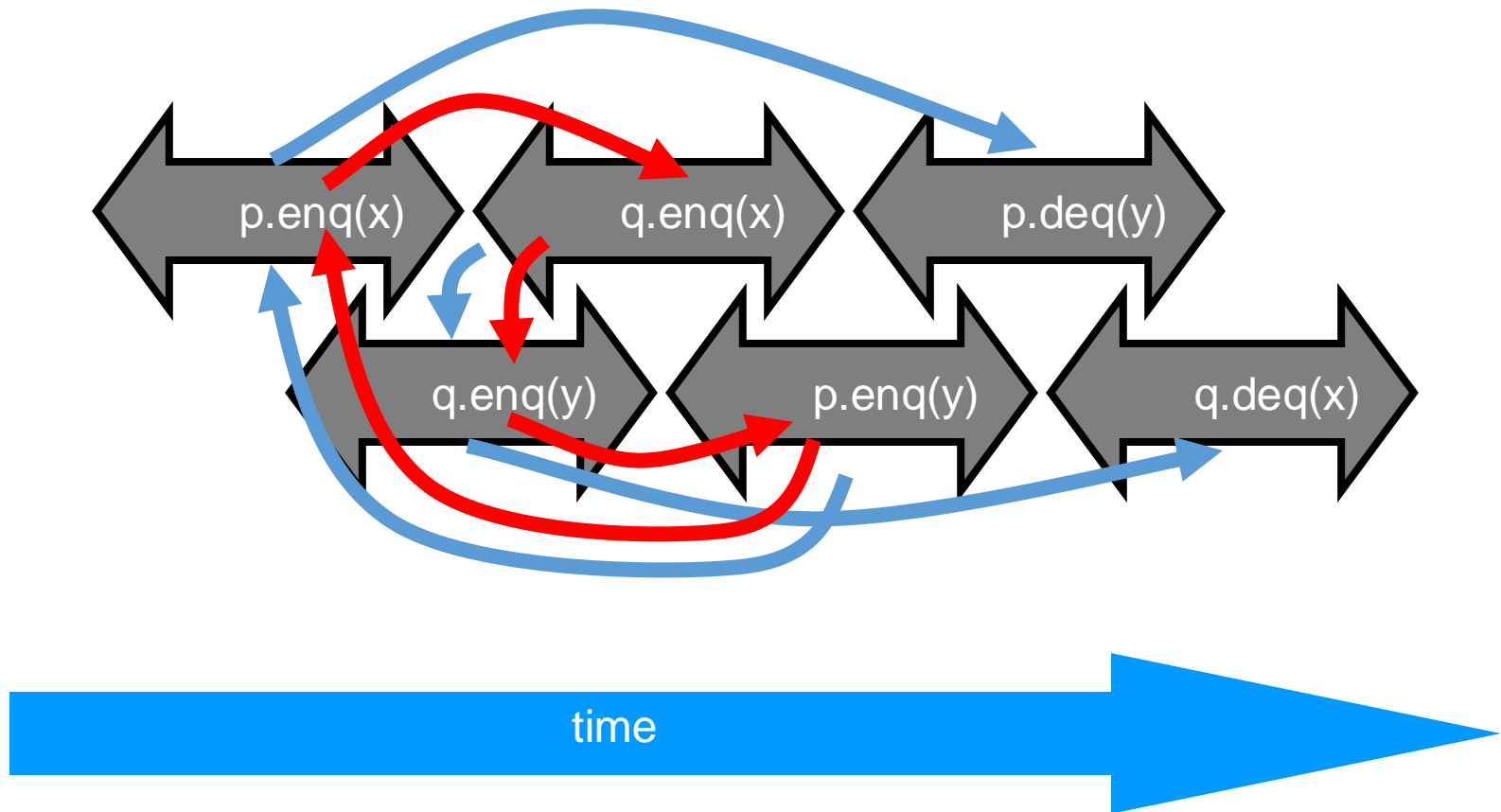
# Ordering imposed by q



# Ordering imposed by both



# Combining orders

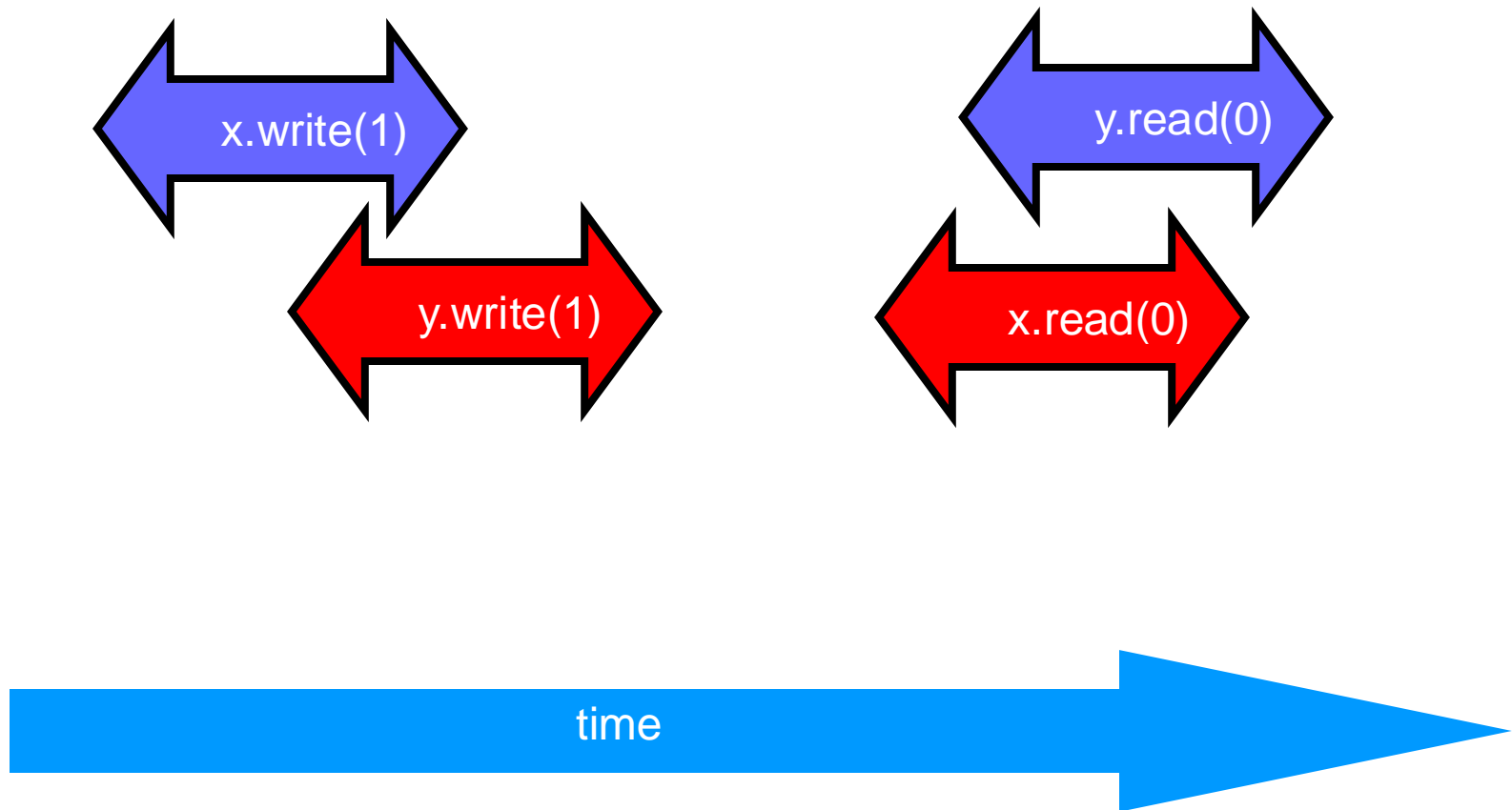


# Fact

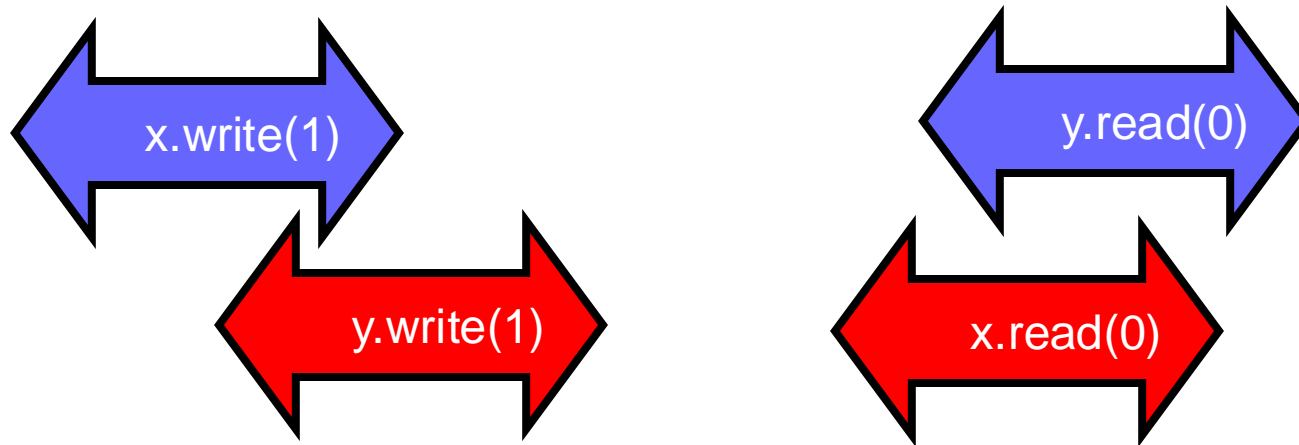
---

- Most hardware architectures don't support sequential consistency
- Because they think it's too strong
- Here's another story...

# The Flag Example

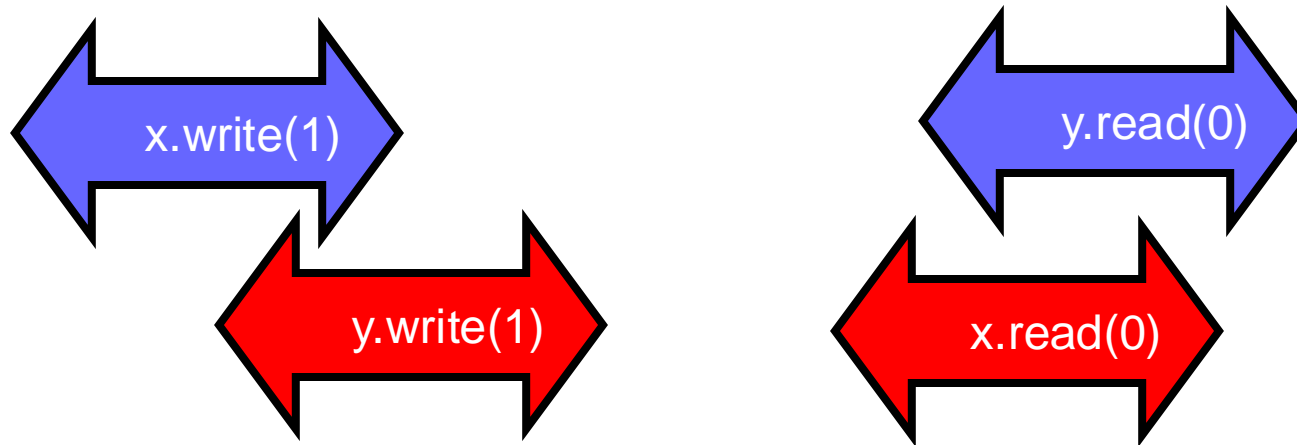


# The Flag Example



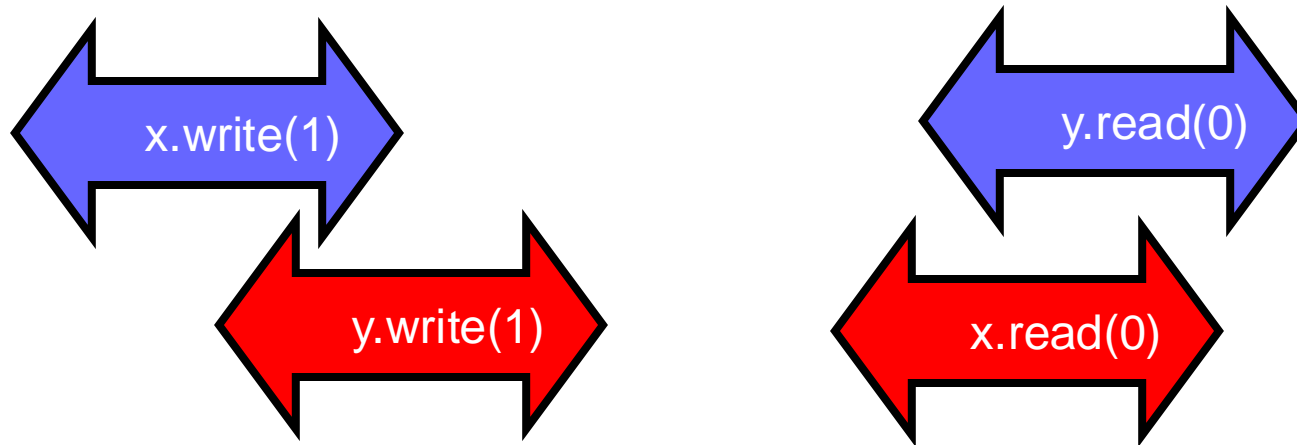
- Each thread's view is sequentially consistent
  - It went first

# The Flag Example



- Entire history isn't sequentially consistent
  - Can't both go first

# The Flag Example



- Is this behavior really so wrong?
  - We can argue either way ...



# Opinion: It's Wrong

---

- This pattern
  - Write mine, read yours
- Is exactly the flag principle
  - Beloved of Alice and Bob
  - Heart of mutual exclusion
    - Peterson
    - Bakery, etc.
- It's non-negotiable!

# Peterson's Algorithm

```
public void lock() {  
    flag[i] = true;  
    victim = i;  
    while (flag[j] && victim == i) {};  
}  
public void unlock() {  
    flag[i] = false;  
}
```

# Crux of Peterson Proof

---

(1)  $\text{write}_B(\text{flag}[B]=\text{true}) \rightarrow$

(3)  $\text{write}_B(\text{victim}=B) \rightarrow$

(2)  $\text{write}_A(\text{victim}=A) \rightarrow \text{read}_A(\text{flag}[B])$   
 $\rightarrow \text{read}_A(\text{victim})$

# Crux of Peterson Proof

(1)  $\text{write}_B(\text{flag}[B]=\text{true}) \rightarrow$

(3)  $\text{write}_B(\text{victim}=B) \rightarrow$

(2)  $\text{write}_A(\text{victim}=A) \rightarrow \text{read}_A(\text{flag}[B])$   
 $\rightarrow \text{read}_A(\text{victim})$

**Observation:** proof relied on fact that if a location is stored, a later load by some thread will return this or a later stored value.

# Opinion: But It Feels So Right ...

---

- Many hardware architects think that sequential consistency is too strong
- Too expensive to implement in modern hardware
- OK if flag principle
  - violated by default
  - Honored by explicit request

# Hardware Consistency

Initially,  $a = b = 0$ .

## Processor 0

```
mov 0, %ebx ;Init  
mov 1, a    ;Store  
mov b, %ebx ;Load
```

## Processor 1

```
mov 0, %eax ;Init  
mov 1, b    ;Store  
mov a, %eax ;Load
```

- What are the final possible values of `%eax` and `%ebx` after both processors have executed?
- Sequential consistency implies that no execution ends with  $\%eax = \%ebx = 0$

# Hardware Consistency

---

- No modern-day processor implements sequential consistency
- Hardware actively reorders instructions
- Compilers may reorder instructions, too
- Why?
- Because most of performance is derived from a single thread's unsynchronized execution of code

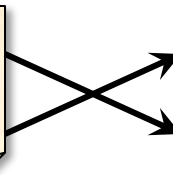
# Instruction Reordering

## Program Order

```
mov 1, a      ;Store  
mov b, %ebx   ;Load
```

## Execution Order

```
mov b, %ebx   ;Load  
mov 1, a      ;Store
```



Q. Why might the hardware or compiler decide to reorder these instructions?

A. To obtain higher performance by covering load latency — *instruction-level parallelism*.



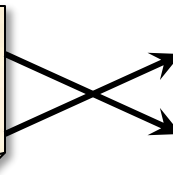
# Instruction Reordering

## Program Order

```
mov 1, a      ;Store  
mov b, %ebx   ;Load
```

## Execution Order

```
mov b, %ebx   ;Load  
mov 1, a      ;Store
```

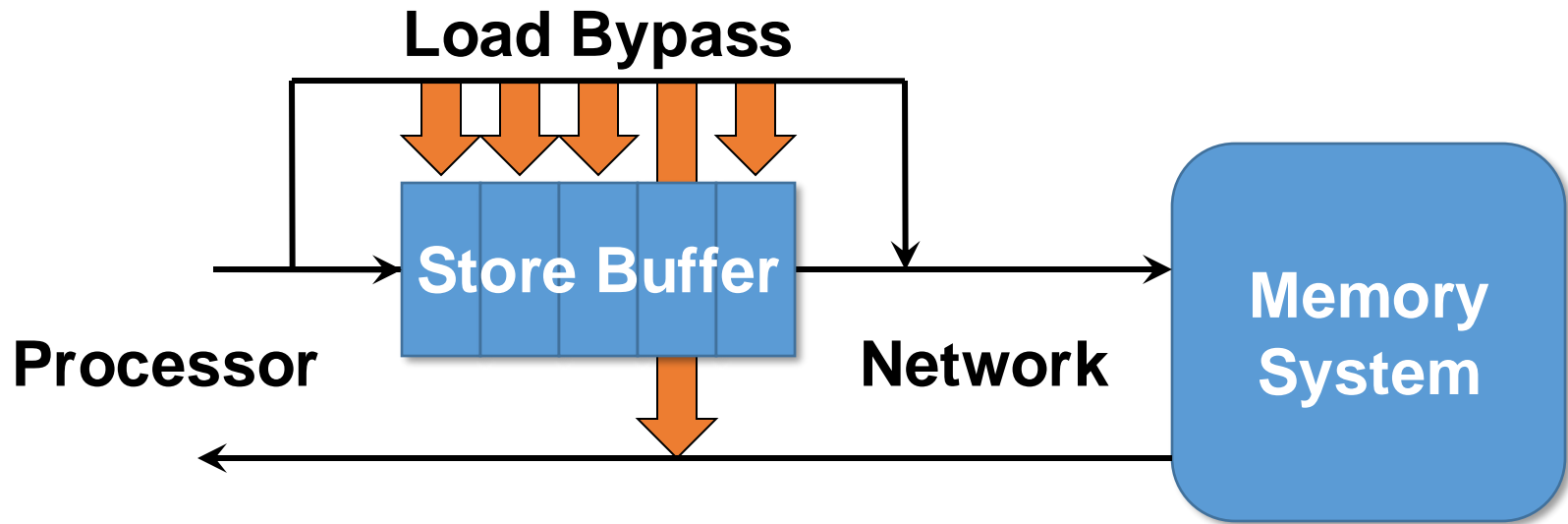


Q. When is it safe for the hardware or compiler to perform this reordering?

A. When  $a \neq b$

A'. And there's no concurrency

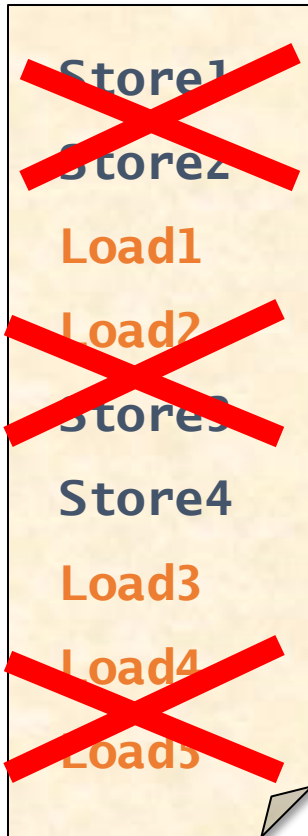
# Hardware Reordering



- Processor can issue stores faster than the network can handle them  $\Rightarrow$  **store buffer**
- **Loads take priority**, bypassing the store buffer
- Except if a load address matches an address in the store buffer, **the store buffer returns the result**

# X86: Memory Consistency

Thread's  
Code



1. Loads *are not* reordered with loads.
2. Stores *are not* reordered with stores.
3. Stores *are not* reordered with prior loads.
4. A load *may* be reordered with a prior store to a different location *but not* with a prior store to the same location.
5. Stores to the same location *respect a global total order*.

# X86: Memory Consistency

Thread's  
Code

Store1

Store2

Load1

Load2

Store3

Store4

Load3

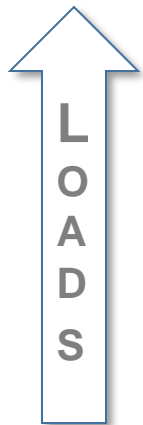
Load4

Load5

1. Loads *are not* reordered with loads.
2. Stores *are not* reordered with stores.

3. **Total Store Ordering (TSO)...weaker than sequential consistency**
4. *or*
5. Stores to the same location *respect a* global total order.

OK!



# Memory Barriers (Fences)

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- A *memory barrier* (or *memory fence*) is a hardware action that enforces an ordering constraint between the instructions before and after the fence.
- A memory barrier can be issued explicitly as an instruction (x86: mfence)
- The typical cost of a memory fence is comparable to that of an L2-cache access.

# X86: Memory Consistency

## Thread's Code

Store1

Store2

Load1

Load2

Store3

Store4

Barrier

Load3

Load4

1. Loads *are not* reordered with loads.
2. Stores *are not* reordered with stores.
3. Stores *are not* reordered with loads.
4. A load *is not* reordered with a store to the same location *but not* with a store to a different location.
5. Stores to the same location *respect a global total order*.

**Total Store Ordering + properly placed memory barriers = sequential consistency**

# Memory Barriers

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- Explicit Synchronization
- Memory barrier will
  - Flush write buffer
  - Bring caches up to date
- Compilers often do this for you
  - Entering and leaving critical sections

# Volatile Variables

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- In Java (and C), can ask compiler to keep a variable up-to-date by declaring it volatile
- Adds a memory barrier after each store
- Inhibits reordering, removing from loops, & other “compiler optimizations”
- Will talk about it in detail in later lectures



# Summary: Real-World

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- Hardware weaker than sequential consistency
- Can get sequential consistency at a price
- Linearizability better fit for high-level software

# Linearizability

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- Linearizability
  - Operation takes effect instantaneously between invocation and response
  - Uses sequential specification, locality implies composability

# Summary: Correctness

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- Sequential Consistency
  - Not composable
  - Harder to work with
  - Good way to think about hardware models
- We will use *linearizability* as our consistency condition in the remainder of this course unless stated otherwise

# Progress

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- We saw an implementation whose methods were lock-based (deadlock-free)
- We saw an implementation whose methods did not use locks (lock-free)
- How do they relate?

# Progress Conditions

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- *Deadlock-free*: **some** thread trying to acquire the lock eventually succeeds.
- *Starvation-free*: **every** thread trying to acquire the lock eventually succeeds.
- *Lock-free*: **some** thread calling a method eventually returns.
- *Wait-free*: **every** thread calling a method eventually returns.

# Progress Conditions

	Non-Blocking	Blocking
Everyone makes progress	Wait-free	Starvation-free
Someone makes progress	Lock-free	Deadlock-free

# Summary

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- Later in this course, will look at *linearizable blocking* and *non-blocking* implementations of objects

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# The END

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