Mesoscale anisotropic ice flow and stratigraphic disturbances

Mike Hay [1]
Committee:
Ed Waddington (advisor and chair), Twit Conway, Gerard Roe, and Randy Leveque (GSR)
Thanks to:
Don Voigt, Joan Fitzpatrick, and Dan Kluskiewicz

[1]Department of Earth and Space Sciences
University of Washington

Outline

1. Motivation
   - Strigraphic disturbances
   - Different kinds of disturbances
2. Current work
   - Ice fabric evolution
   - Coupled anisotropic ice flow/fabric evolution
3. Future work

Ice is very anisotropic

- Ice deforms mostly be shear parallel to the basal plane.
- Other slip systems 100x harder
- If ice grains are random, no problem.
- But they usually aren't.
- Grains rotate away from the axes of extension $\rightarrow$ bulk anisotropic plasticity
- This can cause bulk flow to be highly anisotropic.

Ice is very anisotropic

- Ice deforms mostly be shear parallel to the basal plane.
- Other slip systems 100x harder
- If ice grains are random, no problem.
- But they usually aren't.
- Grains rotate away from the axes of extension $\rightarrow$ bulk anisotropic plasticity
- This can cause bulk flow to be highly anisotropic.

Ice is very anisotropic

- Ice deforms mostly be shear parallel to the basal plane.
- Other slip systems 100x harder
- If ice grains are random, no problem.
- But they usually aren't.
- Grains rotate away from the axes of extension $\rightarrow$ bulk anisotropic plasticity
- This can cause bulk flow to be highly anisotropic.

Shear bands and boudinage

- Under horizontal simple shear, c-axes go to vertical - soft ice
- A layer that is initially softer will shear faster, and get softer faster $\rightarrow$ a shear band.
- Boudinage is different. C-axes go to vertical with uniaxial compression, but that's the hard orientation.

Layer folding

- Shear overturns incipient wrinkles, horizontal extension flattens them.
- Alley and others (1997) showed that anisotropy exacerbates this by making ice soft in shear and hard in uniaxial compression.
- Incipient wrinkles could be caused by stripes, or transient flow.

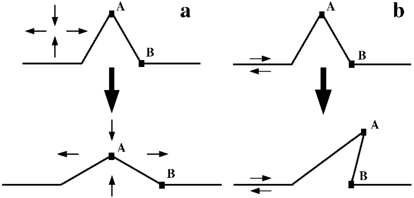
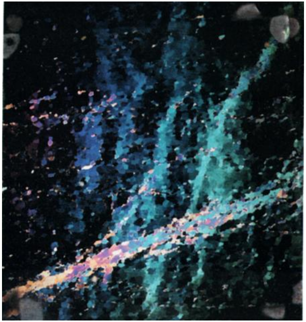


Figure: Thorsteinsson and Waddington (2002)

Observed statigraphic disturbances

- There are smaller-scale disturbances seen well off beds → Wavelengths too short to be due to bed.
- They must be due to factors internal to the ice.



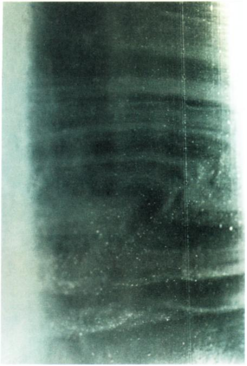


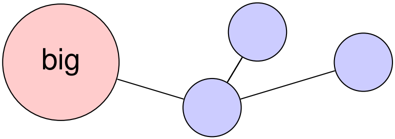
Figure: Alley and others (1997)

Fabric evolution model

- Thorsteinsson (2002) developed a fabric model including nearest-neighbor interaction.
- We have developed a new model for evolution of a finite number of grains.
- The model includes generalized nearest neighbor interaction, mass conservation, and recrystallization.

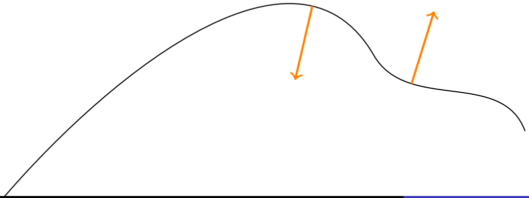
Neighbors and mass balance

- Each grain has a number of nearest neighbors in an undirected graph.
- Grains transfer mass between each other - one grain's loss is another's gain.
- Mass flux is determined by grain boundary velocity and shared boundary area.



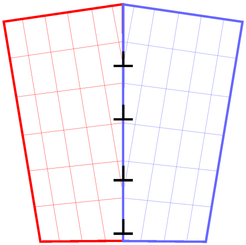
Normal grain growth

- Average grain size increases with depth, until polygonization and recrystallization become important.
- This is driven by curvature energy: Highly curved interfaces have a lot of free energy $\rightarrow$ small grains shrink
- Grain-boundary velocity computed by by estimating the area of the interface, and asssuming that grain curvatures are normally distributed with a mean inversely proportional to the radius.
- Large grains eat smaller grains. Average grain size increases.



Polygonization

- Poorly oriented grains with a large bending moment can split by polygonization.
- This limits grain growth, and makes the fabric more diffuse.
- The model does this by splitting grains if the bending moment gets too large. The new orientations are Fisher-distributed about the original c-axis.



Dynamic recrystallization

- Above about 10°C, dynamic recrytallization is important.
- Old, highly strained grains have high strain energy, and get eaten by new grains.
- The model does this by probabilistically nucleating new grains. They take initial mass from their neighbors.

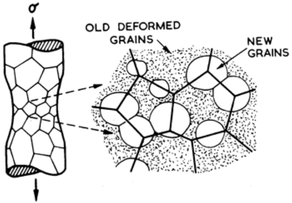


Figure: Hardwick *et al.* *1961*

Grain rotation and nearest-neighbor interaction

- Grain rotation is done with by Jeffery's equation (Jeffery, 1922).
- Each grain gets its viscosity multiplied by a softness parameter ζ. Hard grains surrounded by soft grains get a big ζ.
- I assume *homogeneous stress* between grains.
- The stress on each grain is rotated randomly by the Fisher distribution - rotary diffusion.

$$\dot{c}_i = W_{ij} c_j + \zeta \left(D^g_{ij} c_j + c_i c_j c_k D^g_{jk} \right), \\ \zeta = \xi \left((1 - \gamma)\, \sigma_0 + \gamma \sum_{i=1}^{n} \frac{A_i \sigma_i}{\sigma_0} \right) \quad (1)$$

Fabrics

The model does a good job at reproducing various fabric types, like girdles and single maximum.

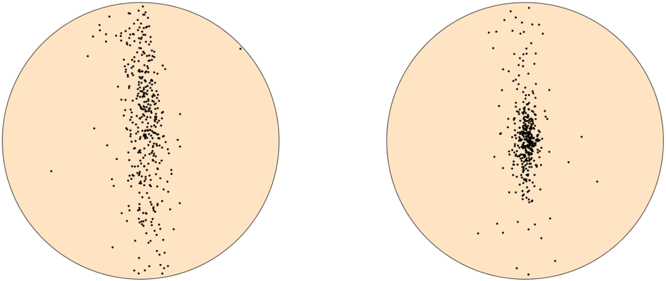


Figure: Girdle (left) and single maximum fabrics produced from uniaxial compression and simple shear, respectively

Comparison to WAIS ice core

- I forced the model with temperature and velocity from the WAIS divide core.
- As an initial condition, I used randomly selected grains (with replacement) from thin sections of WDC from Don Voigt.
- The model does a pretty good job of fitting the core, capturing the transition from girdle to single maximum, as well as the onset of recrystallization.

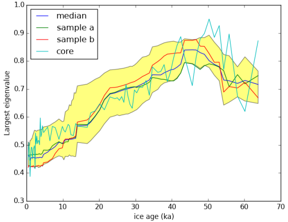
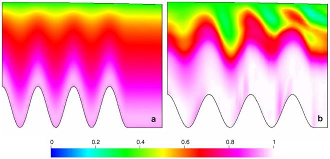


Figure: Largest normalized eigenvalue

Flow and fabric

- Ice fabric development is driven by flow, so this is a coupled system.
- Montgomery-Smith (2011) found that Stokes flow of a fluid with slender fiber inclusions coupled to Jeffery's equation is unstable in response to perturbations of the orientation distribution function.
- Gillet-Chaulet and others (2006) found a large-scale stratigraphic disruptions in their coupled model Jefferys/linear anisotropic ice flow model.
- Can an initial random perturbation become a strong perturbation?
- I'm doing linear stability analysis to find out.



A33 component of second-order orientation tensor. (a) initial; (b) steady state

Gillet-Chaulet and others (2006)

The flow model

- The General Orthotropic Linear Flow (GOLF) law (Gillet-Chaulet and others, 2005) is a constitutive relation depending on six viscosity parameters.
- It assumes the ODF is orthotropic (three axes of symmetry).
- Under some assumptions, I found an expression for the fabric parameters in terms of the fabric eigenvalues.

$$S_{ij} = \eta_0 \left[\eta_r M_{rkl} D_{kl} \left(M_{rij}^D \right) + \eta_{r+3} \left(D_{ik} M_{rkj} + D_{kj} M_{rik} \right)^D \right], \qquad (2)$$

Fabric evolution

- I use a tensor-closure approximation for Jefferys equation. This very common in fiber injection molding modeling.
- $\mathbf{A}^{(2)} = < c \otimes c >$ is the second order orientation tensor.

$$\frac{\partial A_{ij}}{\partial t} = -\frac{\partial}{\partial x_k} A_{ij} u_k + W_{ik} A_{kj} - A_{ik} W_{kj} - (D_{ik} W_{kj} + D_{kj} W_{ik}) + 2\mathbb{A}_{ijkl} C_{kl} \quad (3)$$

Setup and perturbations

- I assume an infinite 3D space with an unperturbed velocity gradient U_{ij}.
- Fabric is spatially homogeneous.
- Then, we put in in a small perturbation of $A^{(2)}_{ij}$ with a wavenumber κ_i.

$$
\begin{aligned}
A_{ij} &\to \bar{A}_{ij} + \epsilon \hat{A}_{ij} e^{i\kappa_k x_k} \\
u_j &\to \bar{U}_{ij} x_j + \epsilon \hat{u}_j e^{i\kappa_k x_k} \\
p &\to \bar{p} + \epsilon \hat{p} e^{i\kappa_k x_k} \\
S_{ij} &\to \bar{S}_{ij} + \epsilon \hat{S}_{ij} e^{i\kappa_k x_k}
\end{aligned}
$$

Stability

- Using the previous replacements, the exponential terms go away. It leaves an ODE for $A_{ij}^{(2)}$.
- It is a long ODE, but it is nice and linear nonetheless.
- This can be used to look at stability of fabric perturbations under many different scenarios.

$$\begin{aligned}\frac{\partial \hat{A}_{ij}}{\partial t} &= -\kappa_k \hat{A}_{ij}\bar{u}_k + \kappa_k \bar{A}_{ij}\hat{u}_k + \bar{W}_{ik}\hat{A}_{kj} + \hat{W}_{ik}\bar{A}_{kj} - \bar{A}_{ik}\hat{W}_{kj} \\ &- \hat{A}_{ik}\bar{W}_{kj} - \hat{D}_{ik}\bar{W}_{kj} - \bar{D}_{ik}\hat{W}_{kj} \\ &- \hat{D}_{kj}\bar{W}_{ik} - \bar{D}_{kj}\hat{W}_{ik} + 2(\bar{\mathbb{A}}_{ijkl}\hat{D}_{kl} + \hat{\mathbb{A}}_{ijkl}\bar{D}_{kl})\end{aligned} \tag{4}$$

.

Other analytical ideas

- Stripe growth.
- Is boudinage inhibited or helped by anisotropy?

Is GOLF sufficient to explain anisotropic flow?

- Fluids made up of linearly viscous transversely isotropic components do not depend on higher than 4^{th} order orientation tensors.
- GOLF assumes orthotropy, which does not hold in general for fourth order tensors.
- Also, ice is not linear, and nearest-neighbor interactions seem to be important for ice.
- Does this mean that we need a more general flow model to address these shortcomings?

  I don't think so, but maybe.

Is GOLF sufficient to explain anisotropic flow?

- Fluids made up of linearly viscous transversely isotropic components do not depend on higher than 4^{th} order orientation tensors.
- GOLF assumes orthotropy, which does not hold in general for fourth order tensors.
- Also, ice is not linear, and nearest-neighber interactions seem to be important for ice.
- Does this mean that we need a more general flow model to address these shortcomings?
  I don't think so, but maybe.

Is GOLF sufficient to explain anisotropic flow?

- Nearest neighbor interaction makes a big difference with fabric evolution. Some grains get more than 10x softness.
- This only makes for differences of 1% in bulk viscosity. But is is possible to construct fabrics where it the difference is bigger.
- Strain softening might be a similar situation.
- I propose investigating this question numerically for different fabrics. This will determine whether it is necessary to expand on GOLF.
- Prediction of fabric orientation would be improved by a higher-order evolution equation, either up to the sixth-order orientation tensor or using spherical harmonics.

Numerical flow modeling

- To expand things beyond first order, I propose investigating mesoscale anisotropic flow with a 3d model in a box geometry.
- We can user Elmer/Ice FEM with the GOLF flow law.
- If we roll our own, we would use a simple finite volume scheme. Numerically, it is similar to to standard Stokes flow.

Outlook

- We will submit the paper on the fabric model shortly.
- The perturbatative coupled model will form another paper.
- The work on constitutive relations and numerical flow modeling will make for a total of three or four.
- I am funded on NSF Grant 0636996 through June 2016.
- I anticipate finishing in 2016.

References I

Alley, RB, AJ Gow, DA Meese, JJ Fitzpatrick, ED Waddington and JF Bolzan, 1997. Grain-scale processes, folding, and stratigraphic disturbance in the GISP2 ice core, *Journal of Geophysical Research*, **102**(C12), 26819–26.

Gillet-Chaulet, Fabien, Olivier Gagliardini, Jacques Meyssonnier, Maurine Montagnat and Olivier Castelnau, 2005. A user-friendly anisotropic flow law for ice-sheet modelling, *Journal of glaciology*, **51**(172), 3–14.

Gillet-Chaulet, F., O. Gagliardini, J. Meyssonnier, T. Zwinger and J. Ruokolainen, 2006. Flow-induced anisotropy in polar ice and related ice-sheet flow modelling, *Journal of Non-Newtonian Fluid Mechanics*, **134**(1), 33–43.

References II

Jeffery, George B, 1922. The motion of ellipsoidal particles immersed in a viscous fluid, *Proceedings of the Royal Society of London. Series A, Containing papers of a mathematical and physical character*, 161–179.

Montgomery-Smith, Stephen, 2011. Perturbations of the coupled Jeffery-Stokes equations, *Journal of Fluid Mechanics*, **681**, 622–638.

Thorsteinsson, T., 2002. Fabric development with nearest-neighbor interaction and dynamic recrystallization, *J. Geophys. Res*, **107**(2014), 10–1019.

Thorsteinsson, T. and E.D. Waddington, 2002. Folding in strongly anisotropic layers near ice-sheet centers, *Annals of Glaciology*, **35**(1), 480–486.