Mesoscale anisotropic ice flow and stratigraphic disturbances

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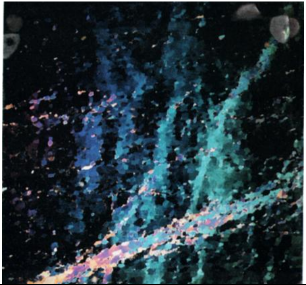
Outline

1. Motivation
   - Strigraphic disturbances
   - Different kinds of disturbances
2. Current work
   - Ice fabric evolution
   - Coupled anisotropic ice flow/fabric evolution
3. Future work

Observed statigraphic disturbances

- There are smaller-scale disturbances seen well off beds $\rightarrow$ Wavelengths too short to be due to bed.
- They must be due to factors internal to the ice.
- What can cause this?



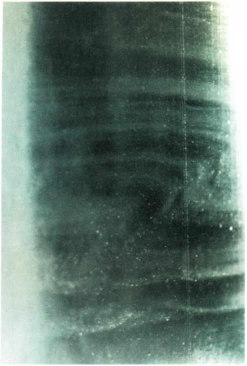


Figure: Alley and others (1997)

Ice is very anisotropic

- Ice deforms mostly be shear parallel to the basal plane.
- Other slip systems 100x harder
- If the distribution of grain orientations (orientation distribution function) is random, no problem.
- But they usually aren't.
- Grains rotate away from the axes of extension → bulk anisotropic plasticity.

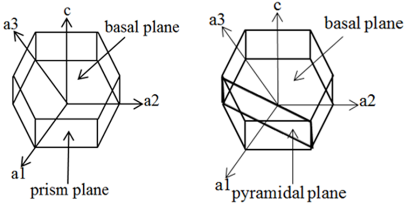


Figure: Ice crystal structure. (Cappicioti et al.)

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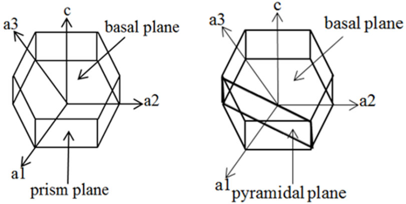

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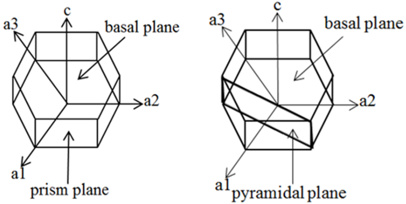
- This can cause bulk flow to



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Shear bands and boudinage

- Under both horizontal simple shear and uniaxial compression, c-axes go to vertical.
- The ice is soft in simple shear because the shear is along the basal plane
- A layer that is initially softer will shear faster, and get softer faster $\rightarrow$ a shear band.
- It's opposite with vertical compression: No shear on horizontal basal planes.

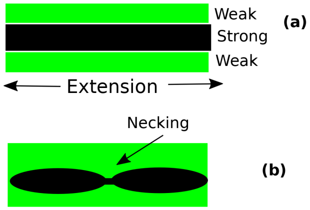


Figure: Boudinage under horizontal extension. (a) Undeformed strong layer within weak layers. (b) Necking as the strong layer is pulled apart.

Shear bands and boudinage

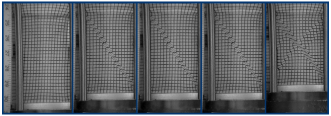


Figure: Shear banding in sand under uniaxial compression. Initial shearing in a layer causes runaway strain.

Layer folding

- Shear overturns incipient wrinkles, horizontal extension flattens them.
- Alley and others (1997) showed that anisotropy exacerbates this by making ice soft in shear and hard in uniaxial compression.
- Incipient wrinkles could be caused by stripes, or transient flow.

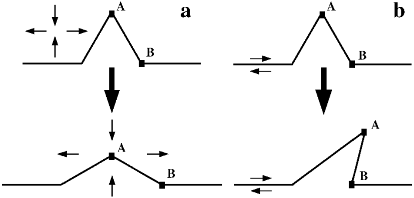


Figure: Thorsteinsson and Waddington (2002)

Quantifying fabric

- The orientation tensor for a single grain is $c \otimes c$.
- The orientation tensor for a collection of grains or a continuous orientation distribution function is $\mathbf{A}^{(2)} < c \otimes c >$
- The strength of a fabric, and its principal orthogonal directions (strongest, middle, weakest) are the eigenvalues and eigenvectors of $\mathbf{A}^{(2)}$.

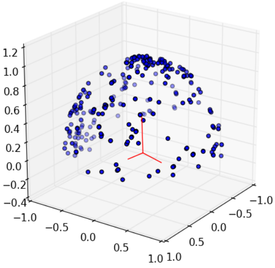


Figure: C-axes on the unit sphere. The red lines are the eigenvectors scaled by the corresponding eigenvalues

Quantifying fabric

- Fabric is plotted with Schmidt plots: The c-axes on the hemisphere are flatted down to an equal-area projection.

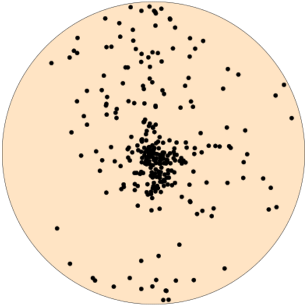


Figure: Schmidt plot of a fabric. The vertical axis comes out of the screen.

Fabric evolution

- Ice fabric is the properties of a polycrystal (orientations, radii, etc).
- This changes over time in response to stress, temperature, and other factors.
- During strain, c-axes rotate away from axes of extension. They also get rotated by any bulk vorticity.

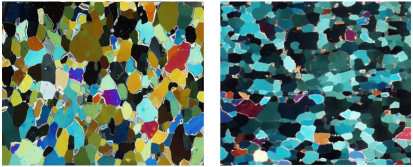
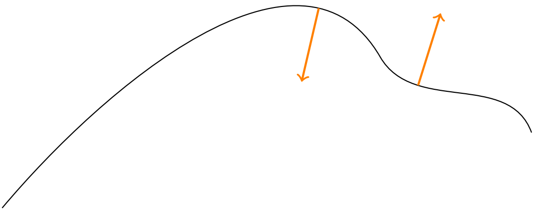


Figure: Ice sheet thin sections (Center for Ice and Climate, Univ. of Copenhagen. Like colors indicate similar c-axis orientations (L) Shallow ice. (R) Deeper ice.

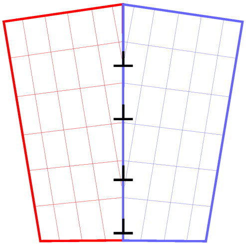
Normal grain growth

- Average grain size increases with depth, until polygonization and recrystallization become important.
- This is driven by curvature energy: Highly curved interfaces have a lot of free energy $\rightarrow$ small grains shrink
- Large grains eat smaller grains. Average grain size increases.



Polygonization

- Poorly oriented grains with a large bending moment can split by polygonization.
- This limits grain growth, and makes the fabric more diffuse.
- The model does this by splitting grains if the bending moment gets too large. The new orientations are Fisher-distributed about the original c-axis.



Dynamic recrystallization

- Above about 10°C, dynamic recrytallization is important.
- Old, highly strained grains have high strain energy, and get eaten by new grains.
- The model does this by probabilistically nucleating new grains. They take initial mass from their neighbors.

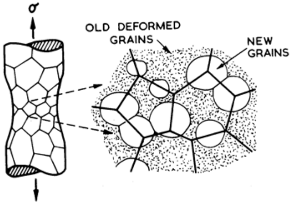


Figure: Hardwick *et al.* *1961*

Fabric evolution model

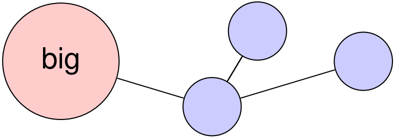
- Thorsteinsson (2002) developed a fabric model including nearest-neighbor interactions between grains.
- We have developed a new model for evolution of a finite number of grains.
- The model includes generalized nearest neighbor interaction, mass conservation, and recrystallization.

Stress homogenization

- The bulk stress of the polycrystal has to be partitioned between grains.
- I use homogeneous stress, the usual assumption for ice: The stress in every grain is the bulk stress.
- Each grain gets its viscosity divided by a softness parameter ζ. Hard grains surrounded by soft grains get a big ζ, because they'd get concentrated stress.
- The stress on each grain is rotated randomly by a small amount - essentially diffusion.
- Thus, each grain gets different strain.

Neighbors and mass balance

- Each grain has a number of nearest neighbors in an undirected graph.
- This does not have a notion of space (like 2D, 3D); just connectivity between grains.
- Grains transfer mass between each other - one grain's loss is another's gain.
- Mass flux is determined by the total grain boundary velocity between two grains, and the shared area between those two grains..



Mass flux

- Grain-boundary velocity from curvature energy is proportional to the curvature of the interface.
- Differences in accumulated strain energy also cause grains to migrate.
- Polygonization is handled by splitting a grain into two halves, with half the volume, and changing the orientation of each.
- Dynamic recrystallization is done by probabilistically nucleating new grains, taking mass from their neighbors.
- If the neighbors are highly strained, the new grain will probably grow.

Grain rotation and nearest-neighbor interaction

Grain rotation is done with by Jeffery's equation (Jeffery, 1922).

$$\dot{c}_i = W_{ij} c_j + \zeta \left(D^g_{ij} c_j + c_i c_j c_k D^g_{jk} \right),$$

(1)

W_{ij} is the bulk vorticity tensor, D^g_{ij} is the local strain rate tensor of the grain. ζ is the softness parameter.

Fabrics

The model does a good job at reproducing various fabric types, like girdles and single maximum.

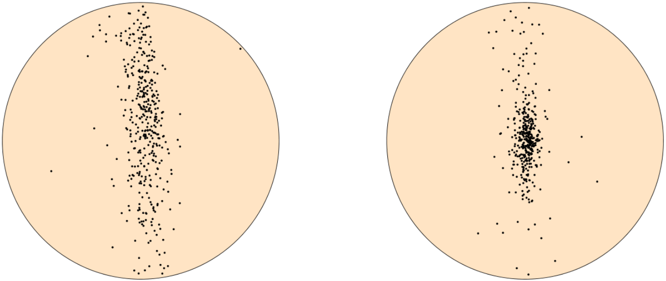
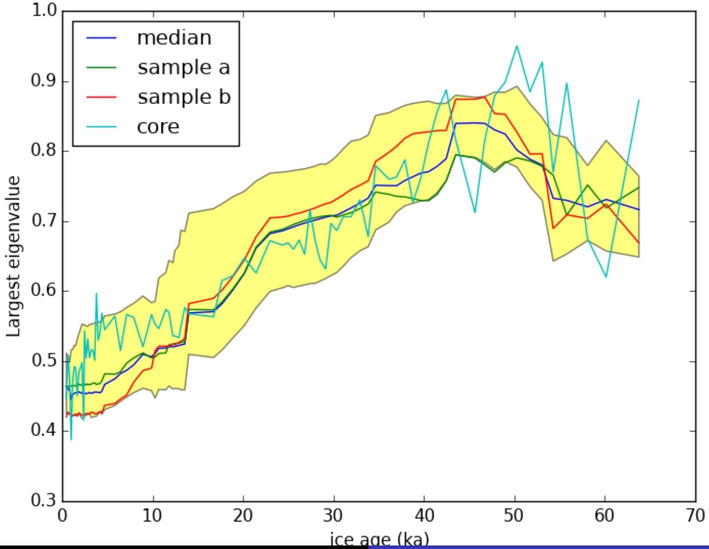


Figure: Girdle (left) and single maximum fabrics produced from uniaxial compression and simple shear, respectively

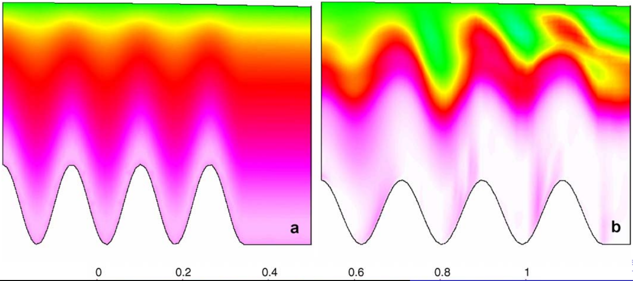
Comparison to WAIS ice core

- I forced the model with temperature and velocity from the WAIS divide core.
- As an initial condition, I used randomly selected grains from the top thin section at WAIS from Don Voigt.
- The model does a pretty good job of fitting the core, capturing the transition from girdle to single maximum, as well as the onset of recrystallization.


median
sample a
sample b
core
Largest eigenvalue
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0
10
20
30
40
50
60
70
ice age (ka)

Flow and fabric

- Ice fabric development is driven by flow, so this is a coupled system.
- Stokes flow with fiber inclusions (similar to ice) is unstable to pertubrations in the ODF (Montgomery-Smith, 2011).
- Gillet-Chaulet and others (2006) found large-scale stratigraphic disruptions in a numerical model.
- Can an initial random perturbation become a strong perturbation?



The flow model

- The General Orthotropic Linear Flow (GOLF) law (Gillet-Chaulet and others, 2005) is a constitutive relation depending on six viscosity parameters.
- It assumes the ODF is orthotropic (three axes of symmetry).
- Under some assumptions, I found an expression for the fabric parameters in terms of the fabric eigenvalues.

$$S_{ij} = \eta_0 \left[\eta_r M_{rkl} D_{kl} \left(M_{rij}^D \right) + \eta_{r+3} \left(D_{ik} M_{rkj} + D_{kj} M_{rik} \right)^D \right], \qquad (2)$$

Fabric evolution

- I use a tensor-closure approximation for Jefferys equation. This very common in fiber injection molding modeling.
- $\mathbf{A}^{(2)} = < c \otimes c >$ is the second order orientation tensor.

$$\frac{\partial A_{ij}}{\partial t} = -\frac{\partial}{\partial x_k} A_{ij} u_k + W_{ik} A_{kj} - A_{ik} W_{kj} - (D_{ik} W_{kj} + D_{kj} W_{ik}) + 2\mathbb{A}_{ijkl} C_{kl} \tag{3}$$

Setup and perturbations

- I assume an infinite 3D space with an unperturbed velocity gradient U_{ij}.
- Fabric is spatially homogeneous.
- Then, we put in in a small perturbation of $A_{ij}^{(2)}$ with a wavenumber κ_i.

$$A_{ij} \rightarrow \bar{A}_{ij} + \epsilon \hat{A}_{ij} e^{i\kappa_k x_k}$$

$$u_j \rightarrow \bar{U}_{ij} x_j + \epsilon \hat{u}_j e^{i\kappa_k x_k}$$

$$p \rightarrow \bar{p} + \epsilon \hat{p} e^{i\kappa_k x_k}$$

$$S_{ij} \rightarrow \bar{S}_{ij} + \epsilon \hat{S}_{ij} e^{i\kappa_k x_k}$$

Stability

- Using the previous replacements, the exponential terms go away. It leaves an ODE for $A_{ij}^{(2)}$.
- It is a long ODE, but it is nice and linear nonetheless.
- This can be used to look at stability of fabric perturbations under many different scenarios.

$$\begin{aligned}\frac{\partial \hat{A}_{ij}}{\partial t} = &-\kappa_k \hat{A}_{ij}\bar{u}_k + \kappa_k \bar{A}_{ij}\hat{u}_k + \bar{W}_{ik}\hat{A}_{kj} + \hat{W}_{ik}\bar{A}_{kj} - \bar{A}_{ik}\hat{W}_{kj} \\ &- \hat{A}_{ik}\bar{W}_{kj} - \hat{D}_{ik}\bar{W}_{kj} - \bar{D}_{ik}\hat{W}_{kj} \\ &- \hat{D}_{kj}\bar{W}_{ik} - \bar{D}_{kj}\hat{W}_{ik} + 2(\bar{\mathbb{A}}_{ijkl}\hat{D}_{kl} + \hat{\mathbb{A}}_{ijkl}\bar{D}_{kl})\end{aligned} \tag{4}$$

.

Other analytical ideas

- Stripe growth.
- Is boudinage inhibited or helped by anisotropy?

Is GOLF sufficient to explain anisotropic flow?

- Fluids made up of linearly viscous transversely isotropic components do not depend on higher than 4^{th} order orientation tensors.
- GOLF assumes orthotropy, which does not hold in general for fourth order tensors.
- Also, ice is not linear, and nearest-neighbor interactions seem to be important for ice.
- Does this mean that we need a more general flow model to address these shortcomings?
  I don't think so, but maybe.

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Is GOLF sufficient to explain anisotropic flow?

- Nearest neighbor interaction makes a big difference with fabric evolution. Some grains get more than 10x softness.
- This only makes for differences of 1% in bulk viscosity. But is is possible to construct fabrics where it the difference is bigger.
- Strain softening might be a similar situation.
- I propose investigating this question numerically for different fabrics. This will determine whether it is necessary to expand on GOLF.
- Prediction of fabric orientation would be improved by a higher-order evolution equation, either up to the sixth-order orientation tensor or using spherical harmonics.

Numerical flow modeling

- To expand things beyond first order, I propose investigating mesoscale anisotropic flow with a 3d model in a box geometry.
- We can user Elmer/Ice FEM with the GOLF flow law.
- If we roll our own, we would use a simple finite volume scheme. Numerically, it is similar to to standard Stokes flow.

Outlook

- We will submit the paper on the fabric model shortly.
- The perturbatative coupled model will form another paper.
- The work on constitutive relations and numerical flow modeling will make for a total of three or four.
- I am funded on NSF Grant 0636996 through June 2016.
- I anticipate finishing in 2016.

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