Ice fabric development with a new fabric evolution model

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Abstract

Ice crystal orientatation fabric has a strong influence on ice flow due to the plastic anisotropy of ice. Ice deforms by simple shear on the basal plane around 100 times more easily than other orientations. The evolution of crystal fabric is driven by strain-induced grain rotation, as well as recrystallization. Most fabric-evolution models ignore many of the physical processes involved, or are valid only for highly parameterized fabrics. In this paper, we outline a new fabric model that treats a variety of processes affecting fabric development. Results indicate that a large proportion of the observed variability in thin section samples is likely to be effectively stochastic in nature.

Introduction

Ice crystals are hexagonal close-packed crystals. An individual ice crystal has an anisotropic creep response, deforming most easily in shear parallel to the crystal basal-plane, orthogonal to the c-axis. Plastic deformation of an ice polycrystal depends on the orientations of its constituent grains. A polycrystal that is initially isotropic will develop a lattice-preferred orientation in response to applied strain, thus causing it to have a bulk anisotropic response. Due to interference between grains, there is a tendency for the c-axes to rotate away from the directions of principal extension to maintain spatial compatibility. In addition to rotation, recrystallization affects both grain size and orientation distribution. Near the melting point, dynamic recrystallization allows the nucleation of new, strain-free grains. These new grains grow rapidly at the expense of older grains with high strain energy. Polygonization is another recrystallization process in which dislocations in a highy strained grain arrange into a subgrain boundary, eventually producing two grains as the misalignment increases. Although the grains typically are misaligned by only a few degrees, this does have the effect of preventing the orientation distribution function from attaining a sharp maximum.

This model is essentially a Lagrangian model tracking the evolution of a single packet of grains. Previous work have used parameterized, non mass-conservative forms of grain growth. Unlike previous work, we incorporate explicit mass-conservation, with grain growth only occuring at the expense of neighboring grains. In addition, we allow for nearest-neighbor interactions to transfer stress between grains, in a similar, but more general way to Thorsteinsson et al. 2002. The inclusion of nearest-neighbor interactions tends to produce a more realistic, diffuse fabric.

Model formulation

The rate of c-axis rotation is calculated using Jeffery's equation:

$$\dot{c}_i = \zeta \left(W_{ij} c_j + D_{ij} c_j + c_i c_j c_k D_{jk} \right) \tag{1}$$

where c_i is the unit vector in the direction of the c-axis, D_{ij} is the strain rate tensor, and W_{ij} is the spin tensor. ζ is the softness parameter. The softness parameter allows strain to be transferred between a grain and its direct neighbors, such that grains in a hard orientation under the applied stress receive additional stress, and thus strain, from their neighbors. It is calculated as

$$\zeta = \xi \left((1-\gamma)\,\epsilon_0 + \gamma \sum_{i=1}^{n} \frac{A_i \epsilon_i}{\epsilon_0} \right), \tag{2}$$

where ϵ_0 and ϵ_i are the effective shear strain rates of the center grain and neighboring grain i, respectively. A_i is the normalized area fraction between the center grain and grain i. γ controls the share ζ determined by the grain's neighbors, and n is the number of nearest neighbors. ξ is a scaling factor taken over the polycrystal to maintain self-consistency with the bulk velocity gradient.

The evolution of each grain's radius due to continuous processes is found from the grain-boundary velocity between it and each of its neighbors. The aggregate grain-boundary velocity is the sum of grain-boundary velocities from all included processes. Grain-boundary velocity between two grains from normal grain growth is given by

$$\frac{dr_1}{dt} = K \left(\frac{1}{r_1} - \frac{1}{r_2} \right), \tag{3}$$

where r_1 and r_2 are the radii, and K is the grain-boundary mobility. Dislocation density also drives grain boundary velocity, with grains which have higher dislocation density losing mass to those with less dislocation density.

Dynamic recrytallization in ice is the nucleation of strain-free new grains, which grow by consuming older, more highly-strained grains. This weakens fabrics. This model treats initial nucleation stochastically, with potential nucleation sites nucleating new grains probabilistically depending on temperature and other factors. The new grains will then begin to consume neighboring grains if it is energetically favorable. Polygonization is a related process in which grains in a hard orientation, experiencing a bending moment, will split into two grains. This is handled by splitting the grains into two equal portions with a misorientation of several degrees if the ratio of resolved shear stress on the basal plane to effective applied stress exceeds a critical value.

Results

The model does well at reproducing characteristic fabrics of different flow regimes, such as girdles and single-maximum fabric. To validate the model, we compared an ensemble of model runs against thin sections from the West Antarctic Ice Sheet (WAIS) divide ice-core. The model was forced with the contemporary temperature profile, and a current estimate of velocity. The initial conditions of each run in the ensemble was taken to be a resample, with replacement, of the orientation data from the thin section at the top of the core. Figure 2 shows the 95% confidence interval of the earth mover's distance (a distance measure between distributions) from a perfect vertical single maximum. The EMDs of two samples as well as the thin-section profile are also included. The variability of the resampled fabrics indicates that smaller excursions in the thin-section data are likely to be due to sampling error. However, there is a large short-wavelength component to the variations in the fabric thin-section EMD. This likely indicates that varying initial conditions over relatively short timespans can affect fabric greatly. The model qualitatively reproduces the main fabric types seen in the WAIS divide core, which is a girdle fabric higher in the core, transitioning to a diffuse single maximum (with recrystallization) near the bed. However, due to using an initial condition of fabric resampled from the top thin section, it does not capture the short wavelength variability seen in the WAIS core.

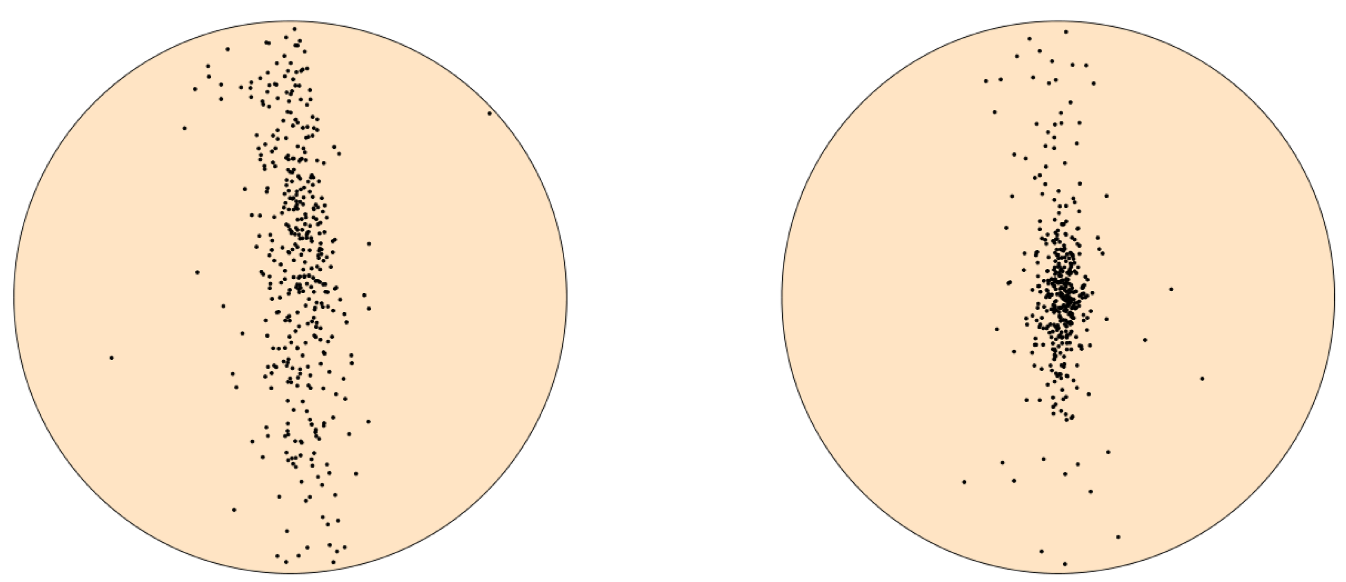

Figure 1: Left: Schmidt plot (equal-area projection of a hemisphere of c-axis directions) of a fabric under a uniaxial extension regime. Right: Schmidt plot under simple shear.

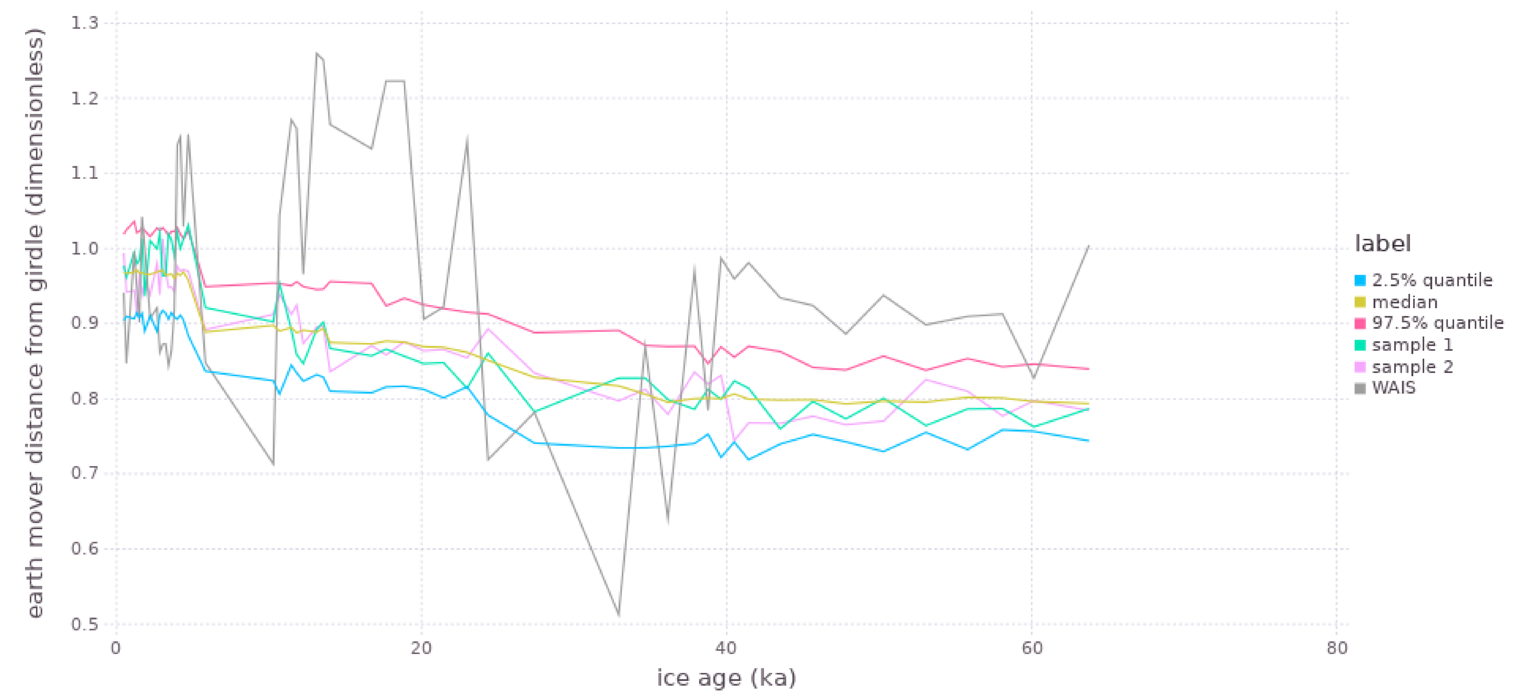

Figure 2: 90% confidence interval and median of evolution of the earth mover's distance of distribution of the fabric sample from a perfect single maxima at the top of the core, inferred from repeated bootstrap samples of the thin section data. Two sample realizations are also included.

Conclusions

- This model does a good job of reproducing observed fabric types.
- Magnitude of initial variability in samples of ice-core fabrics does not diminish over time.
- Effectively random variability of recrystallization and connectivity between grains are important sources of fabric variability.
- Only very large excursion of fabric in thin sections are likely to be statistically significant, rather than sampling error. However, it is likely that seasonal cycles or other effects may have a long-term influence on fabric development. Feedbacks between flow and fabric development may also be important.
- Random fabric variability is probably important for small-scale flow. The rheology of the ice does not care if the variability is random.

Forthcoming Research

Plastic anisotropy is likely the cause behind much observed smaller-scale flow disturbances in ice cores, such as boudinage and shear bands. Random fabric variability could be an initial source for these disturbances. In addition, recent results show that coupled Stokes-Jefferys equations are unstable, in that initial variations in the orientation distribution function can grow significantly in short amounts of time. This may also be the case for ice fabric. Future work will couple a fabric-evolution model to a a fully anisotropic ice-flow model to study these effects.

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