Mesoscale anisotropic ice flow and stratigraphic disturbances

Mike Hay [1]
Committee:
Ed Waddington (advisor and chair), Twit Conway, Gerard Roe, and Randy Leveque (GSR)

Thanks to:
Don Voigt, Joan Fitzpatrick, and Dan Kluskiewicz

[1] Department of Earth and Space Sciences
University of Washington

Outline

1. Motivation
   - Strigraphic disturbances
   - Different kinds of disturbances
2. Current work
   - Ice fabric evolution
   - Coupled anisotropic ice flow/fabric evolution
3. Future work

Ice sheets and ice cores

- Ice cores in ice sheets are an important record of past climate.
- Accurate interpretation needs a depth-age relationship between ice depth and age.
- This is hard if the internal stratigraphy is disturbed.
- Stratigraphic disturbances happen a lot near the bed.
- But they also occur higher up.

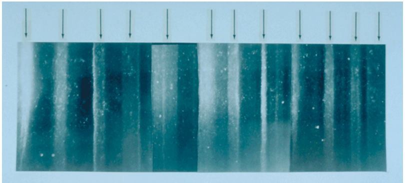
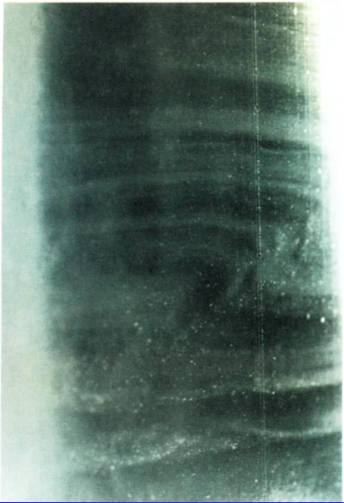


Figure: Annual layers from the GISP2 ice core

Observed statigraphic disturbances

- There are smaller-scale disturbances seen well off beds → Wavelengths too short to be due to bed.
- They must be due to factors internal to the ice.
- What can cause this?



Ice is very anisotropic

- Ice deforms mostly by shear parallel to the basal plane.
- Other slip systems 100x harder
- If grains orientations are random $\rightarrow$ bulk isotropy
- But during strain, grains rotate away from the axes of extension $\rightarrow$ anisotropic plasticity.

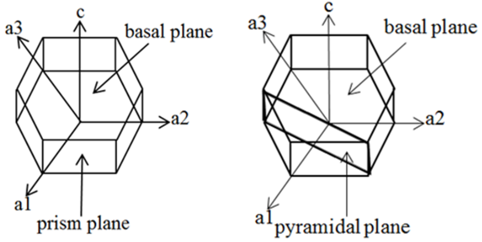


Figure: Ice crystal structure. The basal plane is the face of the hexagon, with the c-axis orthogonal. (Cappicioti et al.)

Ice is very anisotropic

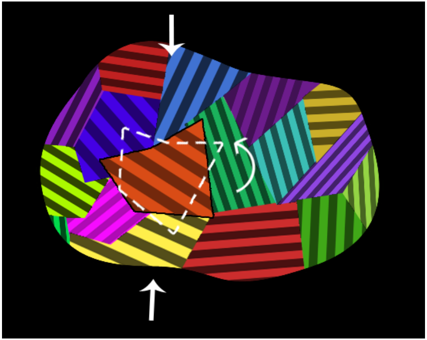


Figure: Rotation of a grain in uniaxial compression. To reduce interference between grains, c-axes line up. (Center for Ice and Climate, Univ. of Copenhagen)

Shear bands and boudinage

- Under both horizontal simple shear and uniaxial compression, c-axes go to vertical.
- Vertical c-axes are hard in compression, because there is little shear on the basal plane.
- Power-law fluids like ice are susceptible to boudinage, where strong layers get pinched out under compression.
- Big problem for stratigraphy.
- Anisotropy might help or hinder this.

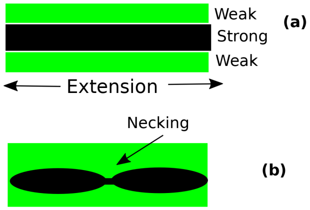


Figure: Boudinage under horizontal extension. (a) Undeformed strong layer within weak layers. (b) Necking as the strong layer is pulled apart.

Shear bands and boudinage

- Ice with vertical c-axes is soft in simple shear because the shear is along the basal plane
- A layer that is initially softer will shear faster, and get softer faster $\rightarrow$ a shear band.

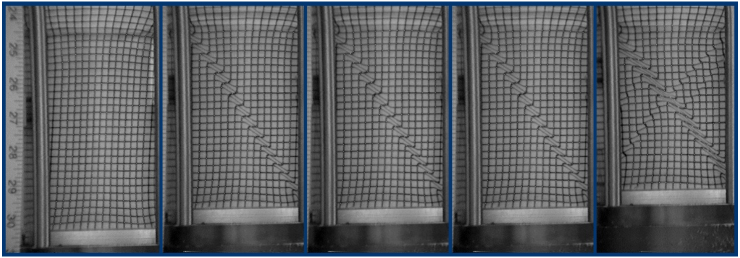


Figure: Shear banding in sand under uniaxial compression. Initial shearing in a layer causes runaway strain.

Stripes

- “Stripes” have been seen in GISP2.
- In hard vertical fabrics under uniaxial compression, stripes of grains are off-axis.
- This is soft under compression $\rightarrow$ faulting.

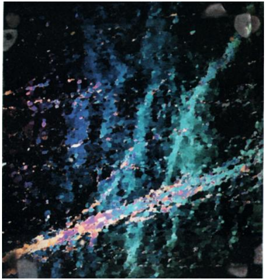


Figure: Thin section in polarized light. Dark areas have near-vertical c-axes. The stripe has c-axes off-vertical, along the dip.

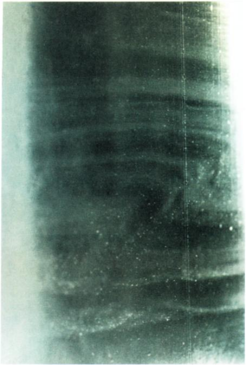


Figure: Alley and others (1997)

Layer folding

- Shear overturns incipient wrinkles, horizontal extension flattens them.
- Thorsteinsson and Waddington (2002) showed that anisotropy exacerbates folding by making ice soft in shear and hard in uniaxial compression.
- Incipient wrinkles could be caused by stripes, or transient flow.

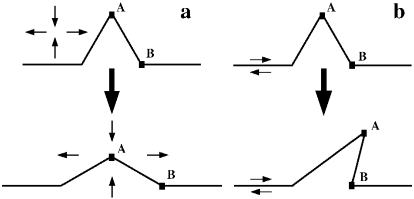


Figure: (a) Open fold is flattened by vertical compression. (b) It is overturned under shear. Thorsteinsson and Waddington (2002)

Fabric evolution

- Ice fabric is the properties of a polycrystal (orientations, radii, etc).
- This changes over time in response to stress, temperature, and other factors.
- During strain, c-axes rotate away from axes of extension. They also get rotated by any bulk vorticity.


Figure: Ice sheet thin sections (Center for Ice and Climate, Univ. of Copenhagen). Like colors indicate similar c-axis orientations (L) Shallow ice. (R) Deeper ice.

Quantifying fabric

- The orientation tensor for a single grain is $c \otimes c$.
- The orientation tensor for a collection of grains or a continuous orientation distribution function is $\mathbf{A}^{(2)} = < c \otimes c >$
- The strength of a fabric, and its principal orthogonal directions (strongest, middle, weakest) are the eigenvalues and eigenvectors of $\mathbf{A}^{(2)}$.

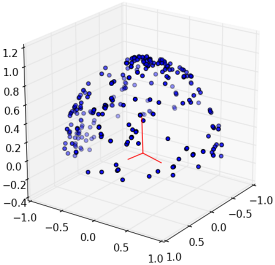


Figure: C-axes on the unit sphere. The red lines are the eigenvectors scaled by the corresponding eigenvalues

Quantifying fabric

- Fabric is plotted with Schmidt plots: The c-axes on the hemisphere are flatted down to an equal-area projection.

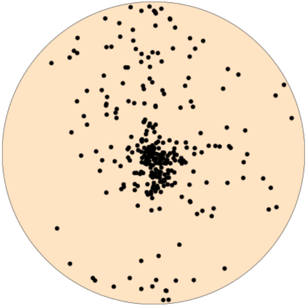


Figure: Schmidt plot of a fabric. The vertical axis comes out of the screen.

Normal grain growth

- Average grain size increases with depth, until polygonization and recrystallization become important.
- This is driven by curvature energy: Highly curved interfaces have a lot of free energy $\rightarrow$ small grains shrink.
- Large grains eat smaller grains. Average grain size increases.

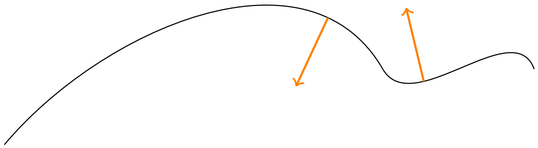
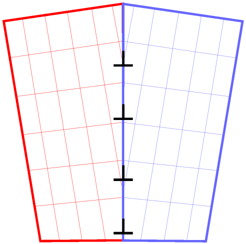


Figure: Boundary between two grains. The boundary migrates towards the concave side.

Polygonization

- Poorly oriented grains with a large bending moment can split by polygonization.
- Dislocations line up, forming a new grain boundary where the two split grains differ in orientation by a small amount.
- This limits grain growth, and makes the fabric more diffuse.



Dynamic recrystallization

- Above about $-10°$C, dynamic recrytallization is important.
- New grains can nucleate at grain boundaries.
- Old, highly strained grains have high strain energy.
- The new, unstrained grains eat the old ones.
- This can weaken fabrics deep in cores, and make average grain size smaller.

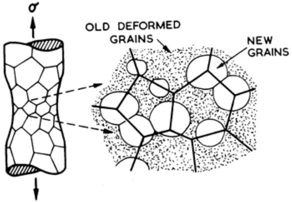


Figure: Hardwick *et al. 1961*

Fabric evolution model

- It is important to know how ice fabric evolves, as it affects stratigraphy and flow.
- Thorsteinsson (2002) developed a fabric model including nearest-neighbor interactions between grains.
- We have developed a new model for evolution of a finite number of grains.
- The model includes generalized nearest neighbor interaction, mass conservation, and recrystallization.

Neighbors and mass balance

- Each grain has a number of nearest neighbors in an undirected graph.
- This does not have a notion of space (like 2D, 3D); just connectivity between grains.
- Grains transfer mass between each other - one grain's loss is another's gain.
- Mass flux is determined by the total grain boundary velocity between two grains, and the shared area between those two grains.

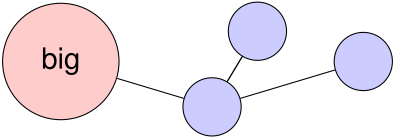


Figure: Connectivity between grains.

Stress homogenization

- The bulk stress of the polycrystal has to be partitioned between grains.
- I use homogeneous stress, the usual assumption for ice: The stress in every grain is the bulk stress.
- Each grain gets its viscosity divided by a softness parameter ζ.
- Hard grains surrounded by soft grains get a big ζ, so they deform faster.
- The stress on each grain is rotated randomly by a small amount - essentially diffusion.
- Thus, each grain gets different strain.

Mass flux

- Grain-boundary velocity from curvature energy is proportional to the curvature of the interface.
- Differences in accumulated strain energy also cause grains to migrate.
- Polygonization is handled by splitting a grain into two halves, with half the volume, and changing the orientation of each.
- Dynamic recrystallization is done by probabilistically nucleating new grains, taking mass from their neighbors.
- If the neighbors are highly strained, the new grain will probably grow.

Grain rotation and nearest-neighbor interaction

Grain rotation is done with a modified Jeffery's equation (Jeffery, 1922).

$$\dot{c}_i = W_{ij}c_j + \zeta\left(D^g_{ij}c_j + c_ic_jc_kD^g_{jk}\right), \tag{1}$$

W_{ij} is the bulk vorticity tensor, D^g_{ij} is the local strain rate tensor of the grain. ζ is the softness parameter.

Fabrics

The model does a good job at reproducing various fabric types, like girdles and single maximum.

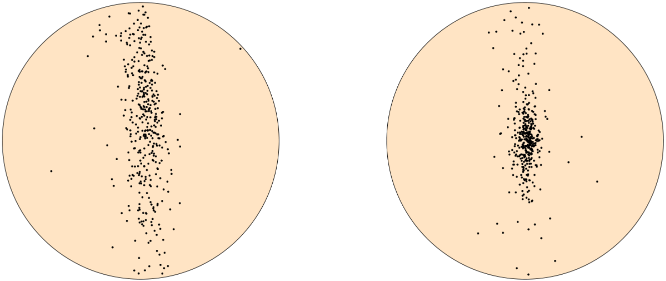


Figure: Girdle (left) and single maximum fabrics produced from uniaxial compression and simple shear, respectively.

Comparison to WAIS ice core

- I forced the model with temperature and velocity from the WAIS divide core.
- As an initial condition, I used randomly selected grains from the top thin section at WAIS from Don Voigt.
- The model does a pretty good job of fitting the core, capturing the transition from girdle to single maximum, as well as the onset of recrystallization.

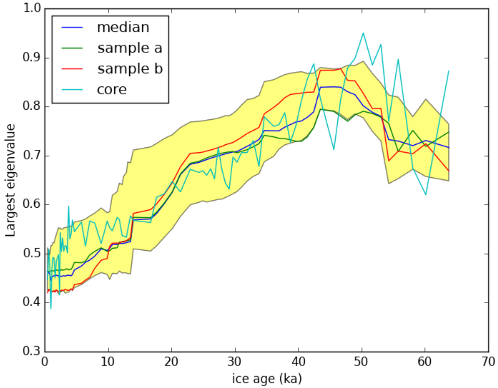
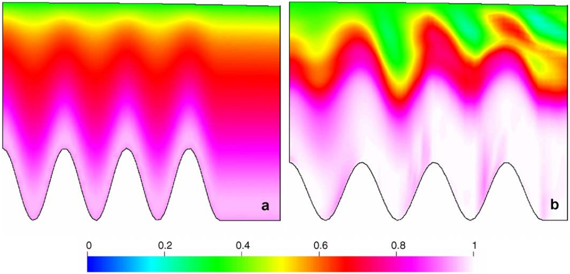


Figure: Largest eigenvalues of modeled and observed fabric thin sections through the WAIS core. The shaded area is the 95% interval of all model runs. Two individual runs are included.

Flow and fabric

- Fabric evolution and flow form a coupled system.
- Stokes flow with fiber inclusions (similar to ice) is unstable to pertubrations in the ODF (Montgomery-Smith, 2011).
- Gillet-Chaulet and others (2006) found large-scale stratigraphic disruptions in a numerical model.
- Can an initial small perturbation become a strong perturbation?



A_{33} component of second-order orientation tensor with initial fabric from GRIP. (a) initial; (b) steady state Gillet-Chaulet and others (2006)

The flow model

- The General Orthotropic Linear Flow (GOLF) law (Gillet-Chaulet and others, 2005) is a constitutive relation depending on six viscosity parameters.
- It assumes the ODF is orthotropic (three axes of symmetry).
- Under some assumptions, I found an expression for the fabric parameters in terms of the fabric eigenvalues.

GOLF constitutive relation

$$S_{ij} = \eta_0 \left[\eta_r M_{rkl} D_{kl} \left(M_{rij}^D \right) + \eta_{r+3} \left(D_{ik} M_{rkj} + D_{kj} M_{rik} \right)^D \right], \qquad (2)$$

S_{ij} is the stress tensor. D_{ij} is the strain rate tensor. $M_{rij} = v_{ri} v_{rj}$ (no sum in r), where v_{ri} is the r^{th} unit fabric eigenvector. η_i are six viscosity parameters depending on fabric.

Fabric evolution

- I use an approximation for Jeffery's equation in terms of A_{ij}. This very common in fiber injection molding modeling.

Jeffery's equation

$$\frac{\partial A_{ij}}{\partial t} = -\frac{\partial}{\partial x_k} A_{ij} u_k + W_{ik} A_{kj} - A_{ik} W_{kj} - (D_{ik} W_{kj} + D_{kj} W_{ik}) + 2 A_{ij} A_{ij} D_{kl} \quad (3)$$

D_{ij} is the strain rate tensor, and W_{ij} is the vorticity tensor.

Setup and perturbations

- I assume an infinite 3D space with an unperturbed velocity gradient $\bar{U}_{ij}$.
- Fabric is spatially homogeneous.
- Then, we put in in a small perturbation of $A^{(2)}_{ij}$ with a wavenumber κ_i.
- In response, other quantities get perturbed by the same wavenumber as well.

Stability

- Using the previous replacements, the exponential terms go away. It leaves an ODE for $A_{ij}^{(2)}$.
- It is a long ODE, but it is nice and linear nonetheless.
- This can be used to look at stability of fabric perturbations under many different scenarios.
- If the system matrix **M** has positive eigenvalues, it is linearly unstable $\rightarrow$ perturbations can grow.

ODE for the fabric perturbation

$$\frac{d\hat{\mathbf{a}}}{dt} = \mathbf{M}\left(\bar{\mathbf{A}}, \bar{\mathbf{U}}, t\right)\hat{\mathbf{a}} \tag{4}$$

$\hat{a}$ is a vector of of the six independent components of $\hat{A}_{ij}$. M is the matrix of the system. **U** and **A** are the unperturbed velocity gradient and orientation tensor, respectively.

Other analytical ideas

- Stripe growth.
- Is boudinage inhibited or helped by anisotropy?

Is GOLF sufficient to explain anisotropic flow?

- Fluids made up of linearly viscous transversely isotropic components do not depend on higher than 4^{th} order orientation tensors.
- GOLF assumes orthotropy, which does not hold in general for fourth order tensors.
- Also, ice is not linear, and nearest-neighbor interactions seem to be important for ice.
- Does this mean that we need a more general flow model to address these shortcomings?

  I don't think so, but maybe.

Is GOLF sufficient to explain anisotropic flow?

- Fluids made up of linearly viscous transversely isotropic components do not depend on higher than 4^{th} order orientation tensors.
- GOLF assumes orthotropy, which does not hold in general for fourth order tensors.
- Also, ice is not linear, and nearest-neighbor interactions seem to be important for ice.
- Does this mean that we need a more general flow model to address these shortcomings?
  I don't think so, but maybe.

Is GOLF sufficient to explain anisotropic flow?

- Nearest neighbor interaction makes a big difference with fabric evolution. Some grains get more than 10x softness.
- This only makes for differences of 1% in bulk viscosity. But is is possible to construct fabrics where it the difference is bigger.
- Strain softening might be a similar situation.
- I propose investigating this question numerically for different fabrics. This will determine whether it is necessary to expand on GOLF.
- Prediction of fabric orientation would be improved by a higher-order evolution equation, either up to the sixth-order orientation tensor or using spherical harmonics.

Numerical flow modeling

- To expand things beyond first order, I propose investigating mesoscale anisotropic flow with a 3d model in a box geometry.
- We can user Elmer/Ice FEM with the GOLF flow law.
- If we roll our own, we would use a simple finite volume scheme. Numerically, it is similar to to standard Stokes flow.

Outlook

- We will submit the paper on the fabric model shortly.
- The perturbatative coupled model will form another paper.
- The work on constitutive relations and numerical flow modeling will make for a total of three or four.
- I am funded on NSF Grant 0636996 through June 2016.
- I anticipate finishing in 2016.

References I

Alley, RB, AJ Gow, DA Meese, JJ Fitzpatrick, ED Waddington and JF Bolzan, 1997. Grain-scale processes, folding, and stratigraphic disturbance in the GISP2 ice core, *Journal of Geophysical Research*, **102**(C12), 26819–26.

Gillet-Chaulet, Fabien, Olivier Gagliardini, Jacques Meyssonnier, Maurine Montagnat and Olivier Castelnau, 2005. A user-friendly anisotropic flow law for ice-sheet modelling, *Journal of glaciology*, **51**(172), 3–14.

Gillet-Chaulet, F., O. Gagliardini, J. Meyssonnier, T. Zwinger and J. Ruokolainen, 2006. Flow-induced anisotropy in polar ice and related ice-sheet flow modelling, *Journal of Non-Newtonian Fluid Mechanics*, **134**(1), 33–43.

References II

Jeffery, George B, 1922. The motion of ellipsoidal particles immersed in a viscous fluid, *Proceedings of the Royal Society of London. Series A, Containing papers of a mathematical and physical character*, 161–179.

Montgomery-Smith, Stephen, 2011. Perturbations of the coupled Jeffery-Stokes equations, *Journal of Fluid Mechanics*, **681**, 622–638.

Thorsteinsson, T., 2002. Fabric development with nearest-neighbor interaction and dynamic recrystallization, *J. Geophys. Res*, **107**(2014), 10–1019.

Thorsteinsson, T. and E.D. Waddington, 2002. Folding in strongly anisotropic layers near ice-sheet centers, *Annals of Glaciology*, **35**(1), 480–486.