Mesoscale anisotropic ice flow and stratigraphic disturbances

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Outline

1. Motivation
   - Strigraphic disturbances
   - Different kinds of disturbances
   - Previous Work
2. Current work
   - Ice fabric evolution
   - Coupled anisotropic ice flow/fabric evolution
3. Future work

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- Use very short sentences or short phrases.

Ice is very anisotropic

- Ice deforms mostly be shear parallel to the basal plane.
- Other slip systems 100x harder
- If ice grains are random, no problem.
- But they usually aren't.
- Grains rotate away from the axes of extension $\rightarrow$ bulk anisotropic plasticity
- This can cause bulk flow to be highly anisotropic.

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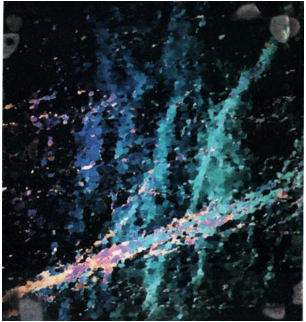
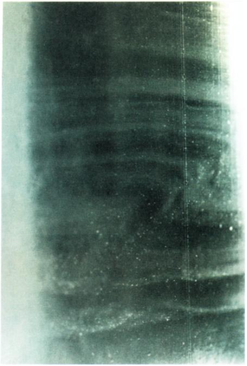
Observed statigraphic disturbances

- There are smaller-scale disturbances seen well off beds. $\rightarrow$ Too short wavelength to be due to bed.





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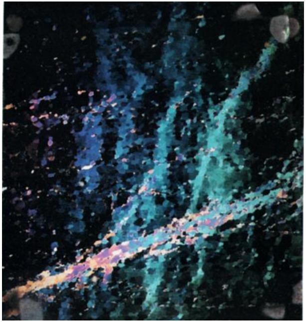
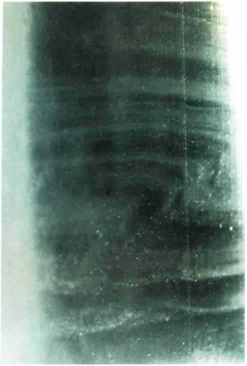


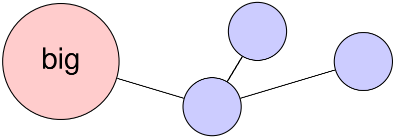


Introduction

- Azuma (1994)
- Thorsteinsson (2002) developed a fabric model including nearest-neighbor interaction.
- We have developed a new model for evolution of a finite number of grains.
- The model includes generalized nearest neighbor interaction, mass conservation, and recrystallization.

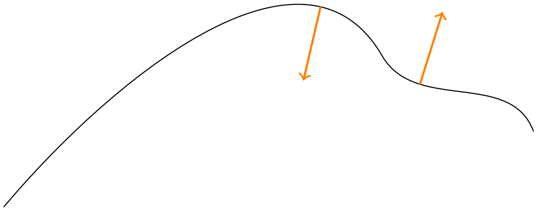
Neighbors and mass balance

- Each grain has a number of nearest neighbors in an undirected graph.
- Grains transfer mass between each other - one grain's loss is another's gain.
- Mass flux is determined by grain boundary velocity and shared boundary area.



Normal grain growth

- Average grain size increases with depth, until polygonization and recrystallization become important.
- This is driven by curvature energy: Highly curved interfaces have a lot of free energy $\rightarrow$ small grains shrink
- Grain-boundary velocity computed by estimating the curvature at the interface.



Flow and fabric

- Ice fabric development is driven by flow, so this is a coupled system.
- **?** found that Stokes flow of a fluid with slender fiber inclusions coupled to Jeffery's equation is unstable in response to perturbations of the orientation distribution function.
- **?** found a large-scale Kelvin-Helmholtz-looking instability in their coupled model Jefferys/linear anisotropic ice flow model.
- Can an initial random perturbation become a strong perturbation?

Shear bands and boudinage

- Under horizontal simple shear, c-axes go to vertical - soft ice
- A layer that is initially softer will shear faster, and get softer faster $\rightarrow$ a shear band.
- Boudinage is different. C-axes go to vertical with uniaxial compression, but that's the hard orientation.

Layer folding

- Shear overturns incipient wrinkles, horizontal extension flattens them.
- **?** showed that anisotropy exacerbates this by making ice soft in shear and hard in uniaxial compression.
- Incipient wrinkles could be caused by stripes, or transient flow.

GOLF flow

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- The General Orthotropic Linear Flow (GOLF) law is a constitutive relation depending on six viscosity parameters.
- It assumes the ODF is orthotropic.

$$S_{ij} = \eta_0 \left[\eta_r M_{rkl} D_{kl} \left(M^D_{rij} \right) + \eta_{r+3} \left(D_i \right. \right. \tag{1}$$

Fabric evolution

- **?** use a tensor-closure approximation for Jefferys equation. This very common in fiber injection molding modeling.
- $A^2 = < c \otimes c >$.

$$\frac{\partial A_{ij}}{\partial t} = -\frac{\partial}{\partial x_k} A_{ij} u_k + W_{ik} A_{kj} - A_{ik} W_{kj} - (D_{ik} W_{kj} + D_{kj} W_{ik}) + 2\mathbb{A}_{ijkl} C_{kl} \quad (2)$$

Perturbations

Other analytical work

- Anisotropic ice flow is still a relatively untapped area.
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Is GOLF sufficient to explain anisotropic flow?

- Fluids made up of linearly viscous transversely isotropic components do not depend on higher than 4^{th} order orientation tensors.
- GOLF assumes orthotropy, which does not hold in general for fourth order tensors.
- Also, ice is not linear, and nearest-neighber interactions seem to be important for ice.
- Does this mean that we need a more general flow model to address these shortcomings?
  I don't think so, but maybe.

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Is GOLF sufficient to explain anisotropic flow?

- Nearest neighbor interaction makes a big difference with fabric evolution. Some grains get $> 10\times$ softness.
- This only makes for differences of 1% in bulk viscosity. But is is possible to construct fabrics where it the difference is bigger.
- Strain softening might be a similar situation.
- I propose investigating this question numerically for different fabrics. This will determine whether it is necessary to expand on GOLF.
- Prediction of fabric orientation would be improved by a higher-order evolution equation, either up to the sixth-order orientation tensor or using spherical harmonics.

Numerical flow modeling

- To expand things beyond first order, I propose investigating mesoscale anisotropic flow with a 3d model in a box geometry.
- We can user Elmer/Ice FEM with the GOLF flow law.
- If we roll our own, we would use a simple finite volume scheme. Numerically, it is similar to to standard Stokes flow.

Summary

- This provides a plan for three or four papers.
- We will submit the paper on the fabric model shortly.
- The

References I

Azuma, N., 1994. A flow law for anisotropic ice and its application to ice sheets, *Earth and Planetary Science Letters*, **128**(3-4), 601–614.

Thorsteinsson, T., 2002. Fabric development with nearest-neighbor interaction and dynamic recrystallization, *J. Geophys. Res*, **107**(2014), 10–1019.