

Title

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Johan Larsson



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Thesis for the degree of Doctor of Philosophy
Thesis advisors: Jonas Wallin and Małgorzata Bogdan
Faculty opponent: TBA

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A doctoral thesis at a university in Sweden takes either the form of a single, cohesive research study (monograph) or a summary of research papers (compilation thesis), which the doctoral student has written alone or together with one or several other author(s).

In the latter case the thesis consists of two parts. An introductory text puts the research work into context and summarizes the main points of the papers. Then, the research publications themselves are reproduced, together with a description of the individual contributions of the authors. The research papers may either have been already published or are manuscripts at various stages (in press, submitted, or in draft).

Cover illustration front: A really pretty thing that looks great on the front but has only a remote connection to do with my research (Credits: The awesome person who made this picture).

Cover illustration back: Picture showing my research (Paper v).

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MADE IN SWEDEN 

Dedicated to my siblings
Name – Name – Name

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Abstract

Need more languages? Go to `preamble.tex` and add them to the `usepackage[...]`babel line. Install the corresponding packages on your system.

List of Publications

This thesis is based on the following publications.

- I Johan Larsson, Małgorzata Bogdan, and Jonas Wallin. “The Strong Screening Rule for SLOPE”. in: *Advances in Neural Information Processing Systems 33*. 34th Conference on Neural Information Processing Systems (NeurIPS 2020). Ed. by Hugo Larochelle, Marc’Aurelio Ranzato, Raia Hadsell, Maria-Florina Balcan, and Hsuan-Tien Lin. Vol. 33. Virtual: Curran Associates, Inc., Dec. 6, 2020–12, pp. 14592–14603. ISBN: 978-1-71382-954-6
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- III Johan Larsson and Jonas Wallin. “The Hessian Screening Rule”. In: *Advances in Neural Information Processing Systems 35*. 36th Conference on Neural Information Processing Systems (NeurIPS 2022). Ed. by S. Koyejo, S. Mohamed, A. Agarwal, D. Belgrave, K. Cho, and A. Oh. Vol. 35. New Orleans, USA: Curran Associates, Inc., Nov. 28–Dec. 9, 2022, pp. 15823–15835. ISBN: 978-1-71387-108-8
- IV Thomas Moreau, Mathurin Massias, Alexandre Gramfort, Pierre Ablin, Pierre-Antoine Bannier, Benjamin Charlier, Mathieu Dagr  ou, Tom Dupr   la Tour, Ghislain Durif, Cassio F. Dantas, Quentin Klopfenstein, Johan Larsson, En Lai, Tanguy Lefort, Benoit Mal  zieux, Badr Moufad, Binh T. Nguyen, Alain Rakotomamonjy, Zaccharie Ramzi, Joseph Salmon, and Samuel V  ter. “Benchopt: Reproducible, Efficient and Collaborative Optimization Benchmarks”. In: *Advances in Neural Information Processing Systems 35*. 36th Conference on Neural Information Processing Systems (NeurIPS 2022). Ed. by S. Koyejo, S.

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I Introduction

I.I Foo bar

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Figure 1: The Faculty of Science and the Astronomy tower. (Figure: Lund University)

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1.2 Baz Baf

Explaining something cool (Figure 1), which can be seen in a fantastic reference [Bog+15]. Figures and tables are placed automatically, such that they are close by but not necessarily on the same page¹.

The concepts are summarized in Table 1.

Table 1: Table caption with a CAPS word, no small caps can be used in table and figure captions because sans serif fonts don’t support it.

	Superman	Spiderman
Gender	male	male
Species	Homo Sapien	Human/spider
Homeworld	Gotham City	Earth
Publisher	DC Comics	Marvel Comics

¹to optimize page space

2 Some more background

Lorem ipsum dolor sit amet, consectetur adipiscing elit. Etiam lobortis facilisis sem. Nullam nec mi et neque pharetra sollicitudin. Praesent imperdiet mi nec ante. Donec ullamcorper, felis non sodales commodo, lectus velit ultrices augue, a dignissim nibh lectus placerat pede. Vivamus nunc nunc, molestie ut, ultricies vel, semper in, velit. Ut porttitor. Praesent in sapien. Lorem ipsum dolor sit amet, consectetur adipiscing elit. Duis fringilla tristique neque. Sed interdum libero ut metus. Pellentesque placerat. Nam rutrum augue a leo. Morbi sed elit sit amet ante lobortis sollicitudin. Praesent blandit blandit mauris. Praesent lectus tellus, aliquet aliquam, luctus a, egestas a, turpis. Mauris lacinia lorem sit amet ipsum. Nunc quis urna dictum turpis accumsan semper. Lorem ipsum dolor sit amet, consectetur adipiscing elit. Etiam lobortis facilisis sem. Nullam nec mi et neque pharetra sollicitudin. Praesent imperdiet mi nec ante. Donec ullamcorper, felis non sodales commodo, lectus velit ultrices augue, a dignissim nibh lectus placerat pede. Vivamus nunc nunc, molestie ut, ultricies vel, semper in, velit. Ut porttitor. Praesent in sapien. Lorem ipsum dolor sit amet, consectetur adipiscing elit. Duis fringilla tristique neque. Sed interdum libero ut metus. Pellentesque placerat. Nam rutrum augue a leo. Morbi sed elit sit amet ante lobortis sollicitudin. Praesent blandit blandit mauris. Praesent lectus tellus, aliquet aliquam, luctus a, egestas a, turpis. Mauris lacinia lorem sit amet ipsum. Nunc quis urna dictum turpis accumsan semper. Lorem ipsum dolor sit amet, consectetur adipiscing elit. Etiam lobortis facilisis sem. Nullam nec mi et neque pharetra sollicitudin. Praesent imperdiet mi nec ante. Donec ullamcorper, felis non sodales commodo, lectus velit ultrices augue, a dignissim nibh lectus placerat pede. Vivamus nunc nunc, molestie ut, ultricies vel, semper in, velit. Ut porttitor. Praesent in sapien. Lorem ipsum dolor sit amet, consectetur adipiscing elit. Duis fringilla tristique neque. Sed interdum libero ut metus. Pellentesque placerat. Nam rutrum augue a leo. Morbi sed elit sit amet ante lobortis sollicitudin. Praesent blandit blandit mauris. Praesent lectus tellus, aliquet aliquam, luctus a, egestas a, turpis. Mauris lacinia lorem sit amet ipsum. Nunc quis urna dictum turpis accumsan semper. Lorem ipsum dolor sit amet, consectetur adipiscing elit. Etiam lobortis facilisis sem.

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3 Main results of the research papers

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Papers

ON THE INTERPRETATION OF THE SPECTROSCOPICALLY OBSERVED ROTATIONS OF GALAXIES

Erik Holmberg

(Communicated by the Astronomer Royal)

I.—Observations of Spectroscopic Rotation

1. A great deal of observational work has been done on the spectroscopic rotation of extragalactic objects. During the last ten years the rotational effects to be found within our own galactic system have been very much discussed. However, the first definite evidence of rotation was found, not in our own system, but in external galaxies. The first spectroscopic observations of the rotation of a spiral object were performed by V. M. Slipher * in 1914. Later on evidence has been obtained by Slipher, F. G. Pease †, M. Wolf ‡ and H. W. Babcock §, for the rotations of several galaxies.

According to Slipher, rotations have been definitely established for six objects. These are N.G.C. 221, 224, 1068, 2683, 3623 and 4594. Measurements of the rotations represent extremely difficult work. In general Slipher gives no numerical values of the rotational change of radial velocity. The present paper is intended as a contribution to the interpretation of the observed spectroscopic rotations. When numerical values become accessible for the above six objects it will be possible to enlarge considerably the material of the present investigation.

Very detailed informations about spectroscopic rotation are given by Pease for the two objects N.G.C. 224 and N.G.C. 4594. The rotational change of radial velocity is measured along the major axis at different distances from the centre of the object. For the Andromeda spiral the rotation is determined along the minor axis, too. For both objects the measurements extend over an interval of about 2'·5 on each side of the centre.

The measurements of Babcock refer to the Andromeda spiral. They extend over the very large interval of 30' on each side of the centre of the system. Within the central core the observed rotation agrees with that found by Pease.

2. In this connection the largest interest lies in the relation found between radial velocity and distance from the centre of the object. The above measurements by Pease result in a *linear* relation. This means that the observed angular velocity of rotation is about the same at different distances from the centre. For N.G.C. 224 the rotational change of radial velocity amounts to about 29 km./sec. per 1', whereas a value of 167 km./sec. per 1' is found for N.G.C. 4594.

It is true that the measurements by Babcock show that the above linear

* *Lowell Bull.* II, 65, 1914.

† Cf. *Handb. d. Astroph.*, V, 2, 851, 1933.

‡ See *Upsala Medd.*, 40, 1928.

§ *P.A.S.P.*, 50, 174, 1938.

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