Examen d'optique quantique 2021

Exo 1

1 (a) $a|n\rangle = \sqrt{n}|n\rangle$, $a^+|n\rangle = \sqrt{n+1}\,|n+1\rangle$

(b) $a = \frac{U+iV}{\sqrt{2}}$, $a^+ = \frac{U-iV}{\sqrt{2}}$

$\Leftrightarrow U = \frac{a+a^+}{\sqrt{2}}$, $iV = \frac{a-a^+}{\sqrt{2}}$

(c) $|\psi\rangle = \frac{1}{\sqrt{2}}\left(|\alpha\rangle + i\,|1\rangle\right)$

$\Rightarrow U|\psi\rangle = \frac{1}{2}\left(\alpha|\alpha\rangle + a^+|\alpha\rangle + i\,|0\rangle + i\sqrt{2}\,|2\rangle\right)$

$\Rightarrow \langle\psi|U|\psi\rangle = \frac{1}{2\sqrt{2}}\left(\alpha+\alpha^*+0\right) = \frac{\alpha+\alpha^*}{2\sqrt{2}}$

$iV|\psi\rangle = \left(\alpha|\alpha\rangle - a^+|\alpha\rangle + \dots\right)$

FAUX

$\Rightarrow i\langle\psi|V|\psi\rangle = \frac{\alpha-\alpha^*}{2\sqrt{2}} \Rightarrow \langle V\rangle = \frac{i(\alpha^*-\alpha)}{2\sqrt{2}}$.

exclu du sujet

$U^2 = \frac{1}{2}\left(a^2+a^{+2}+aa^++a^+a\right) = \frac{1}{2}\left(a^2+a^{+2}+2a^+a+1\right)$

$\Rightarrow U^2|\psi\rangle = \frac{1}{2\sqrt{2}}\left(\alpha^2|\alpha\rangle + a^{+2}|\alpha\rangle + 2\alpha a^+|\alpha\rangle + |\alpha\rangle\right.$

$\left. + i\sqrt{6}\,|3\rangle + 3i\,|1\rangle\right)$

$\Rightarrow \langle\psi|U^2|\psi\rangle = \frac{1}{4}\left(\alpha^2+\alpha^{*2}+2|\alpha|^2+1+3\right)$

$= \frac{1}{4}|\alpha+\alpha^*|^2 + 1$

$-V^2 = \frac{1}{2}\left(a^2+a^{+2}-2a^+a-1\right)$

$\Rightarrow -V^2|\psi\rangle = \frac{1}{2\sqrt{2}}\left(\alpha^2|\alpha\rangle + a^{+2}|\alpha\rangle - 2\alpha a^+|\alpha\rangle - |\alpha\rangle\right.$

$\left. + i\sqrt{6}\,|3\rangle - 3i\,|1\rangle\right)$

$\Rightarrow \langle\psi|V^2|\psi\rangle = -\frac{1}{4}|\alpha-\alpha^*|^2 + 1$

(2) $S_p = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$ où $\left|\begin{array}{l}\cos^2\theta = 0{,}2 \\ \sin^2\theta = 0{,}8\end{array}\right.$

$= \begin{pmatrix} \sqrt{0{,}2} & \sqrt{0{,}8} \\ -\sqrt{0{,}8} & \sqrt{0{,}2} \end{pmatrix}$

(3) $\vec{B} = i\sum_{\vec{k}}\sqrt{\frac{\hbar}{2\varepsilon_0 \omega_k}}\left(\vec{k}\wedge\vec{e_s}\right)\left(\hat{a}\,e^{i\vec{k}\cdot\vec{r}} - a^+\,e^{-i\vec{k}\cdot\vec{r}}\right)$

(4) Bruit de grenaille : bruit d'un état cohérent : $\Delta N = \sqrt{\langle N\rangle}$.
c'est le bruit pour état cohérent.

(5) $\rho = \frac{1}{2}\left(|+\rangle\langle+| + |-\rangle\langle-|\right)$ $\quad Tr(\rho^2) \leq 1$ et état pur $Tr(\rho^2) = 1$

Exo 2 | Partie 1

$|\psi_B\rangle = \frac{1}{\sqrt{2}}\left(a_k^+ a_{-k}^+ + e^{i\gamma} a_q^+ a_{-q}^+\right)|0\rangle$

(1) $a_k' = e^{i\varphi} a_k$, $a_{-q}' = e^{i\varphi'} a_{-q}$

(2) $a_1 = (a_k' + a_q)/\sqrt{2} \Rightarrow a_2 = (\alpha a_k' + \beta a_q)$

où $[a_1^+, a_2] = 0$, $[a_2, a_2^+] = 1$

$\Rightarrow \frac{1}{\sqrt{2}}(\alpha+\beta) = 0$ $\quad |\alpha|^2 + |\beta|^2 = 1$

$\Rightarrow \alpha = -\beta = 1/\sqrt{2}$.

$\Rightarrow a_2 = \frac{1}{\sqrt{2}}\left(a_k' - a_q\right)$

(3) $a_3 = \frac{1}{\sqrt{2}}\left(a_{-q}' + a_{-k}\right) \Rightarrow a_4 = \frac{1}{\sqrt{2}}\left(a_{-k} - a_{-q}'\right)$

④ $a_1^+ a_1 = \frac{1}{\sqrt{2}}\left(a_k^+ a_k + a_q^+ a_q + e^{i\varphi} a_q^+ a_k + e^{-i\varphi} a_k^+ a_q\right)$

$a_2^+ a_2 =$ ~ – , – –)

$\Rightarrow \hat{N}_1 + \hat{N}_2 = \hat{N}_q + \hat{N}_k$ indep de φ.

⑤ VP $|n_k, n_q\rangle$ de N_p $n_k + n_q$.

$$\langle \psi_D | n_k n_q \rangle = \frac{1}{\sqrt{2}} \langle 0 | a_k a_{-k} + e^{-i\chi} a_q a_{-q} | n_k n_q \rangle$$

$n_k \; n_{-q}$

$$= \frac{1}{\sqrt{2}}\left(\delta_{n_k 1}\, \delta_{n_{-k} 1} + e^{-i\chi} \delta_{n_q 1}\, \delta_{n_{-q} 1}\right)$$

$$\Rightarrow \sum_{n_k\, n_q} |\langle \psi_D | n_k n_q \rangle|^2 = \frac{1}{2}\left(\delta_{n_k 1} + \delta_{n_q 1}\right)$$

$\Rightarrow P(1\,0) = P(0,1) = 1/2$. Le reste est nul.

⑥ $a_1 |\psi_D\rangle = \frac{1}{2}\left(e^{i\varphi} a_{-k}^+ |0\rangle + e^{i\chi} a_{-q}^+ |0\rangle\right)$

$\Rightarrow \langle N_1 \rangle = \frac{1}{4}(1+1) = 1/2 = \langle N_i \rangle$.

⑦ $a_3 a_1 = \frac{1}{2\sqrt{2}}\left(e^{i\varphi'} a_{-q} + a_{-k}\right) \cancel{a(\psi_D)} \left(e^{i\varphi} a_{-k}^+ |0\rangle + e^{i\chi} a_{-q}^+ |0\rangle\right)$

$$= \frac{1}{2\sqrt{2}}\left[e^{i\varphi} + e^{i(\chi + \varphi')}\right]$$

$$\Rightarrow \langle N_1 N_3 \rangle = \frac{1}{2} \cos^2\left(\frac{\varphi' + \chi - \varphi}{2}\right)$$

$a_4 a_1 = \frac{1}{2\sqrt{2}}\left(a_{-k} - e^{i\varphi'} a_{-q}\right)\left(e^{i\varphi} a_{-k}^+ |0\rangle + e^{i\chi} a_{-q}^+ |0\rangle\right)$

$$= \frac{1}{2\sqrt{2}}\left(e^{i\varphi} - e^{i(\varphi' + \chi)}\right)|0\rangle$$

$$\Rightarrow \langle N_1 N_4 \rangle = \frac{1}{2} \sin^2\left(\frac{\varphi' + \chi - \varphi}{2}\right)$$

et $\langle N_2 N_4 \rangle = \langle N_1 N_3 \rangle$

$\langle N_2 N_3 \rangle = \langle N_1 N_4 \rangle$

⑧ Double détection.

EA02 - Partie 2

① $E = \frac{\cos^2 - \sin^2}{\cos^2 + \sin^2} = \cos\left(\varphi' + \chi - \varphi\right)$

② $\varphi_a = \pi/4$, $\varphi_b = -\pi/4$.

$\Rightarrow$ on peut prendre $\varphi'_a = 0 - \chi$, $\varphi'_b = \pi/2 - \chi$

car alors : $\cos \pi/4 + \cos(\pi/4) + \cos(-\pi/4) - \cos(-3\pi/4) = 2\sqrt{2}$.

③ ⓐ une séparatrice de transmission en intensité η.

ⓑ : $a_i \rightarrow a_i' = \sqrt{\eta}\, a_i + \sqrt{1-\eta}\, v$

$\Rightarrow \langle N_i \rangle_\eta = \eta \langle N_i \rangle$ ⓒ et $\langle N_i N_j \rangle_\eta = \eta^2 \langle N_i N_j \rangle$ car $i \neq j$.

ⓓ : E reste inchangé : cool pour les manips.

EA02 - Partie 3

$$-1 \leq x_a y_a + x_a y_b + x_b y_a - x_b y_b - x_a - y_a \leq 0.$$

① $x_a = \frac{N_{1a}}{N_{12}}$ $x_b = \frac{N_{1b}}{N_{12}}$ $y_a = \frac{N_{3a}}{N_{34}}$ $y_b = \frac{N_{3b}}{N_{34}}$

$$\Rightarrow_{(i)} -1 \leq \frac{N_{1a} N_{3a} + N_{1a} N_{3b} + N_{1b} N_{3a} - N_{1b} N_{3b}}{N_{12} N_{34}} - \frac{N_{1a}}{N_{12}} - \frac{N_{3a}}{N_{34}} \leq 0.$$

2) $-1 \le \dfrac{N_{2a}N_{4a}+N_{2a}N_{4b}+N_{2b}N_{4a}-N_{2b}N_{4b}}{N_{12}\,N_{34}} - \dfrac{N_{2a}}{N_{12}} - \dfrac{N_{4a}}{N_{12}} \le 0$ ii

$-1 \le \dfrac{N_{1a}N_{4a}+N_{1a}N_{4b}+N_{1b}N_{4a}-N_{1b}N_{4b}}{N_{12}\,N_{34}} - \dfrac{N_{1a}}{N_{12}} - \dfrac{N_{4a}}{N_{34}} \le 0$ iii

$-1 \le \dfrac{N_{2a}N_{3a}+N_{2a}N_{3b}+N_{2b}N_{3a}-N_{2b}N_{3b}}{N_{12}\,N_{34}} - \dfrac{N_{2a}}{N_{12}} - \dfrac{N_{3a}}{N_{34}} \le 0$ iv

3) i + ii − iii − iv donne $-2 \le \dots \le +2$

4) $e = \dfrac{(N_1-N_2)(N_3-N_4)}{(N_1+N_2)(N_3+N_4)} = \dfrac{N_1N_3+N_2N_4-N_2N_3-N_1N_4}{N_{12}N_{34}}$

$e(a\,a) + e(a\,b) + e(b\,a) - e(b\,b) =$

$N_{1a}N_{3a}+N_{1a}N_{3b}+N_{1b}N_{3a}-N_{1b}N_{3b}$

$+\,N_{2a}N_{4a}+N_{2a}N_{4b}+N_{2b}N_{4a}-N_{2b}-N_{4b}$

$-\,(N_{2a}N_{3a}+N_{2a}N_{3b}+N_{2b}N_{3a}-N_{2b}N_{3b})$

$-\,(N_{1a}N_{4a}+N_{1a}N_{4b}+N_{1b}N_{4a}-N_{1b}N_{4b})$

5) $\Rightarrow -2 \le \dots \le 2$ oui.

6) $-2\,N_{12}N_{34} \le \dots \le 2\,N_{12}N_{34}$

$\Rightarrow$ on peut multiplier par $p(\lambda)$ et faire l'intégrale $(p>0)$ puis rediviser $\Rightarrow -2 \le E.. \le 2$.