Quantum information presentation

Quantum teleportation is a universal computational primitive

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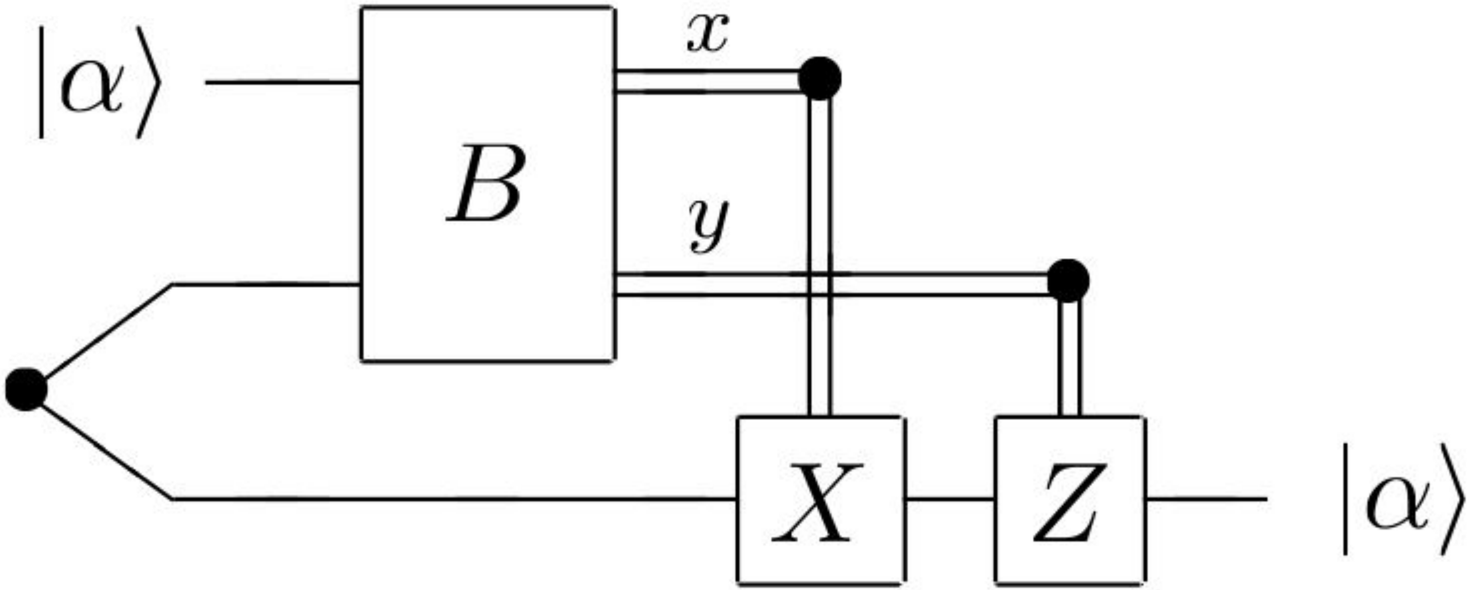
Quantum teleportation

1 - The teleportation setup



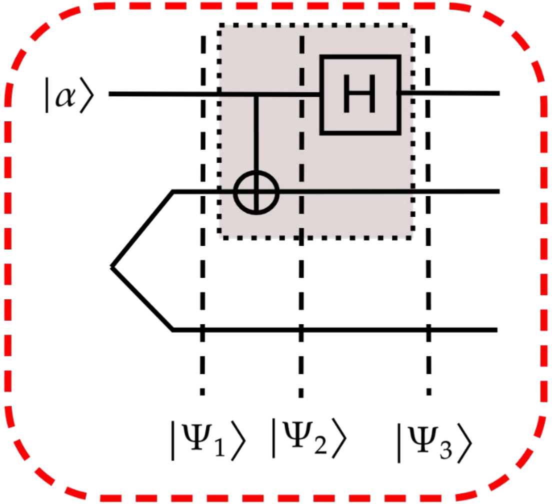
Quantum teleportation

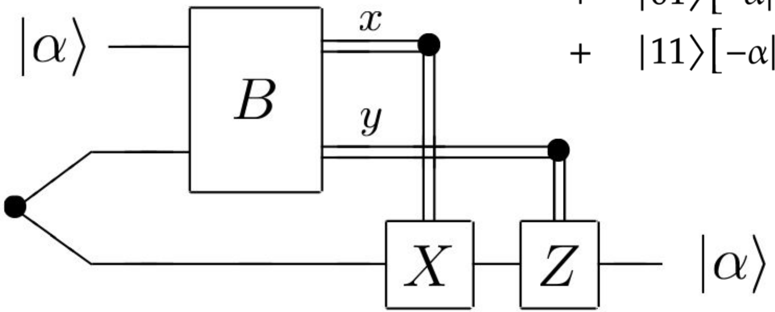
1 - The teleportation setup: first proof

$$|\Psi_1\rangle = |\alpha\rangle \otimes |\Psi\rangle = \left(\alpha|0\rangle + \beta|1\rangle\right)\frac{|00\rangle + |11\rangle}{\sqrt{2}}$$

$$|\Psi_2\rangle = \alpha\frac{|000\rangle + |011\rangle}{\sqrt{2}} + \beta\frac{|110\rangle + |101\rangle}{\sqrt{2}}$$

$$\begin{aligned} |\Psi_3\rangle = \frac{1}{2}(&|00\rangle[\ \alpha|0\rangle + \beta|1\rangle] \\ + &|10\rangle[\ \alpha|0\rangle - \beta|1\rangle] \\ + &|01\rangle[\ \alpha|1\rangle + \beta|0\rangle] \\ + &|11\rangle[-\alpha|1\rangle + \beta|0\rangle]) \end{aligned}$$





Quantum teleportation

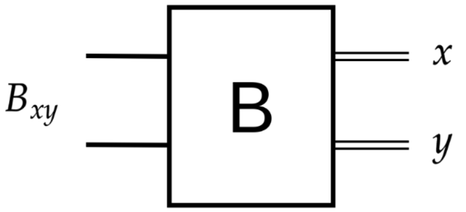
1 - The teleportation setup: Bell states

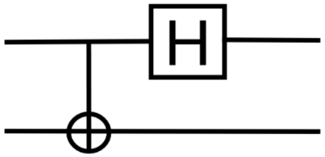
$$\underbrace{|\Psi\rangle}_{\text{epr}} = B_{00} = |\Phi^+\rangle = \frac{|00\rangle + |11\rangle}{\sqrt{2}}$$

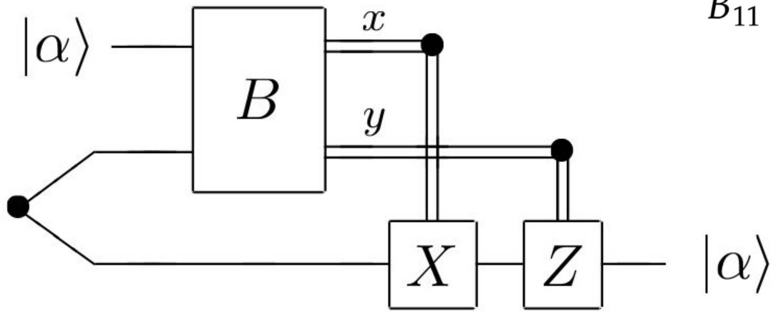
$$B_{10} = |\Phi^-\rangle = \frac{|00\rangle - |11\rangle}{\sqrt{2}}$$

$$B_{01} = |\Psi^+\rangle = \frac{|01\rangle + |10\rangle}{\sqrt{2}}$$

$$B_{11} = |\Psi^-\rangle = \frac{|01\rangle - |10\rangle}{\sqrt{2}}$$



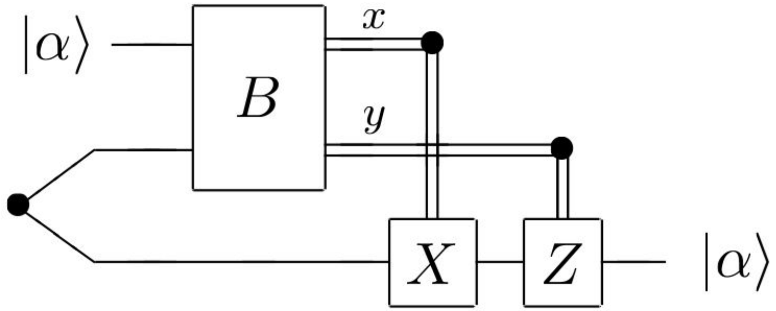




Quantum teleportation

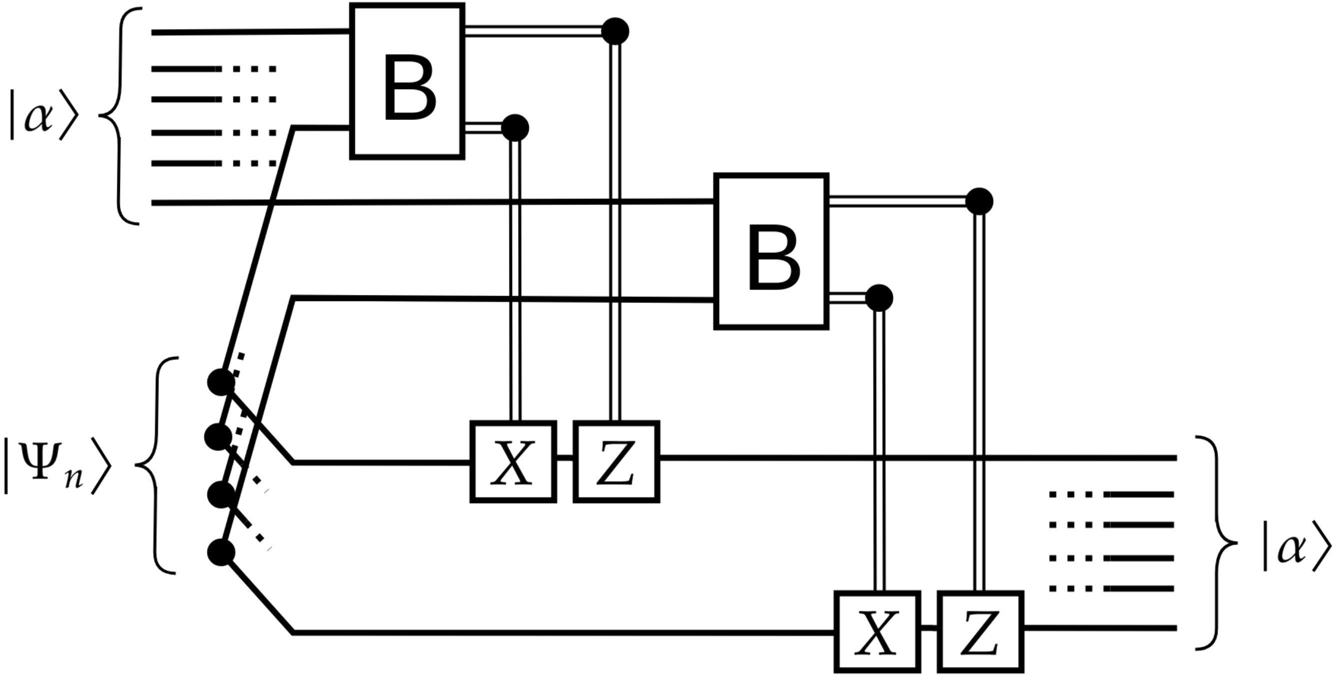
1 - The teleportation setup: better proof

$$
\begin{aligned}
|\alpha\rangle \otimes |\Psi\rangle = \frac{1}{2}(B_{00} &\otimes \quad\; |\alpha\rangle \\
+ \quad B_{10} &\otimes X \;\; |\alpha\rangle \\
+ \quad B_{01} &\otimes Z \;\; |\alpha\rangle \\
+ \quad B_{11} &\otimes XZ|\alpha\rangle)
\end{aligned}
$$



Quantum teleportation

1 - Multiple qubit teleportation



Quantum teleportation

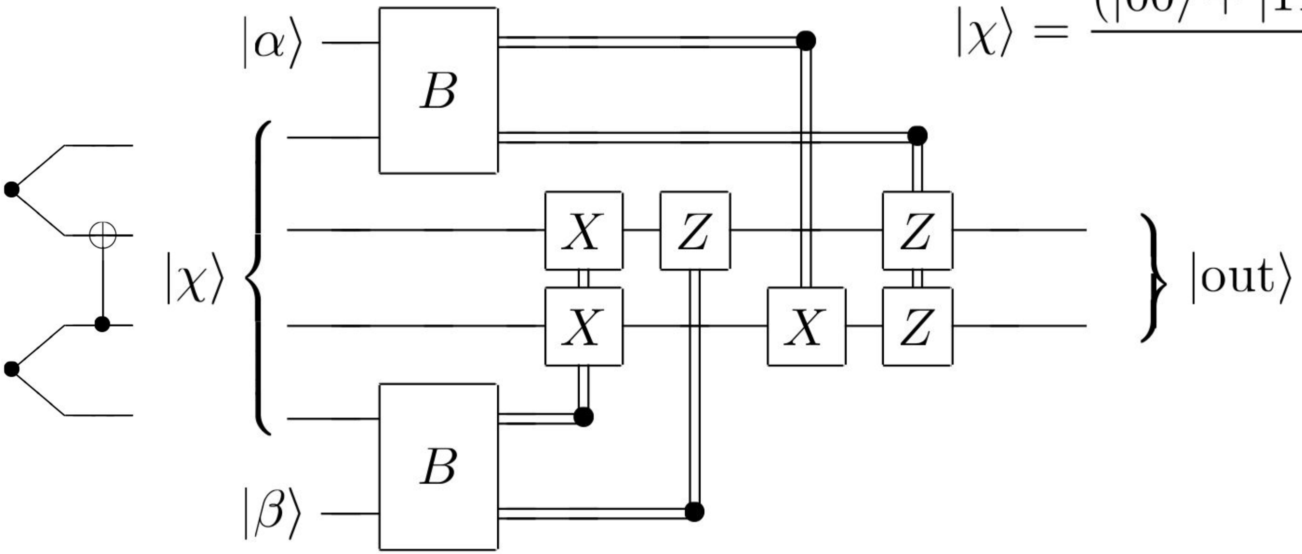
2 - Why use quantum teleportation ?

- Use different types of qubit for computation and storage
- Long range qubit transportation

Using quantum teleportation to build quantum gates

3 - From a quantum teleporter to the CNOT gate:

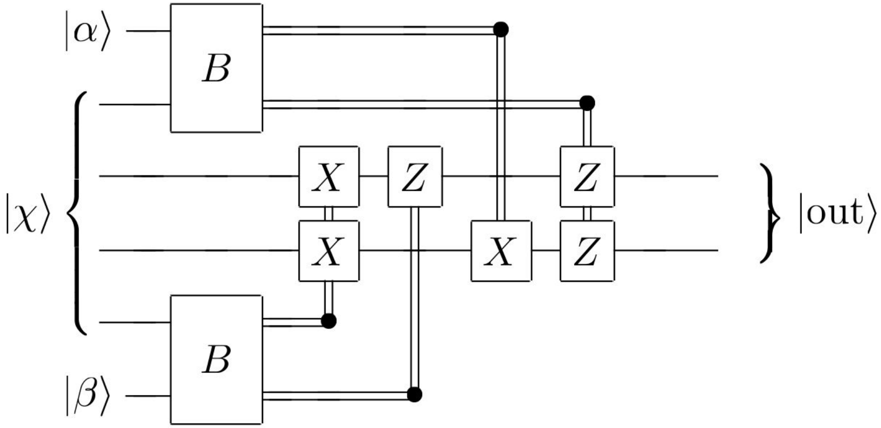
$$|\chi\rangle = \frac{(|00\rangle + |11\rangle)|00\rangle + (|01\rangle + |10\rangle)|11\rangle}{\sqrt{2}}$$



Using quantum teleportation to build quantum gates

3 - From a quantum teleporter to the CNOT gate:

Using the CNOT gate we have built, and single-qubit gates we have a universal set of quantum gates !



Circuit functionality is proven later on

Fault tolerant quantum gates

4 - introducing fault tolerance

Redondance block to correct faults

-> Avoid operations within blocks to avoid error propagation

-> Prefere <u>transversal operations</u>

Fault tolerant quantum gates

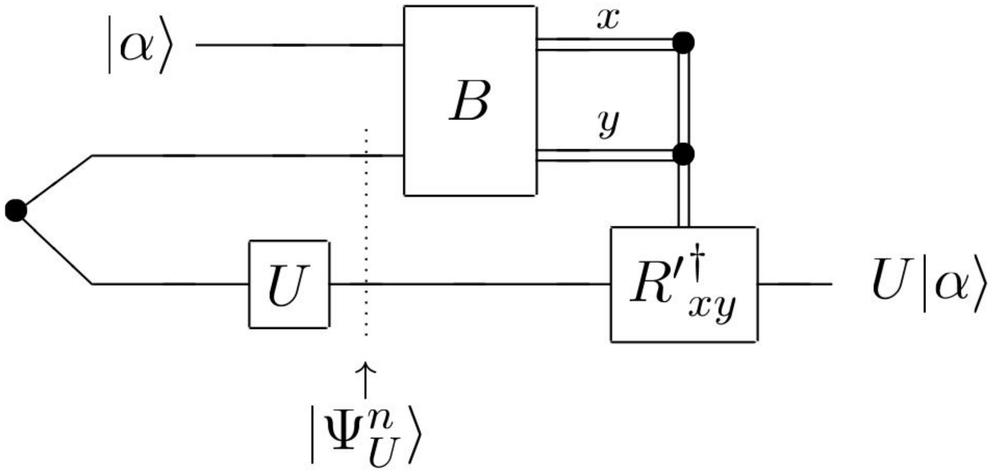
4 - introducing fault tolerance: building gates from the C3 group

$$C_1 = \{X,\ Y,\ Z\} \qquad \text{(Pauli gates)}$$
$$C_2 = \left\{U | U C_1 U^\dagger \subseteq C_1\right\} = \{S,\ H,\ CNOT...\} \qquad \text{(Clifford gates)}$$
$$C_3 = \left\{U | U C_1 U^\dagger \subseteq C_2\right\}$$

C1 and C2 gates can be applied transversely, while C3 gates cannot in general.

Fault tolerant quantum gates

4 - introducing fault tolerance: building gates from the C3 group

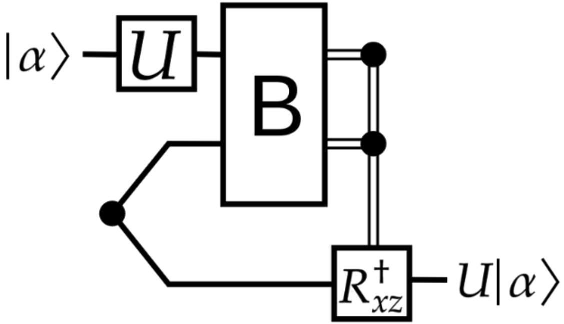


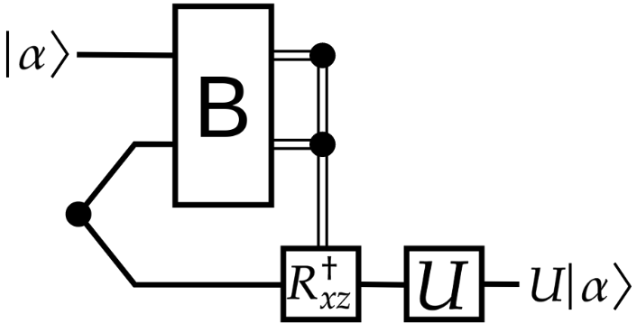
$$|\Psi^n_U\rangle = (I \otimes U)|\Psi^n\rangle$$

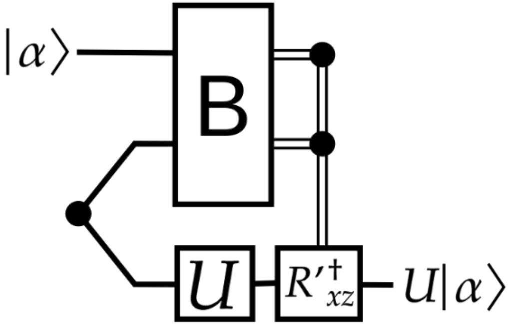
$$|\psi_{out}\rangle = U R_{xz}|\psi\rangle = R'_{xz} U|\psi\rangle$$

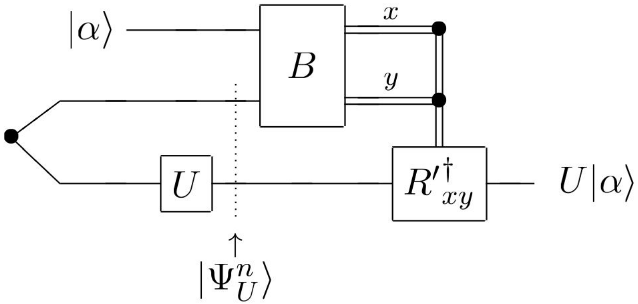
Fault tolerant quantum gates

4 - introducing fault tolerance: building gates from the C3 group









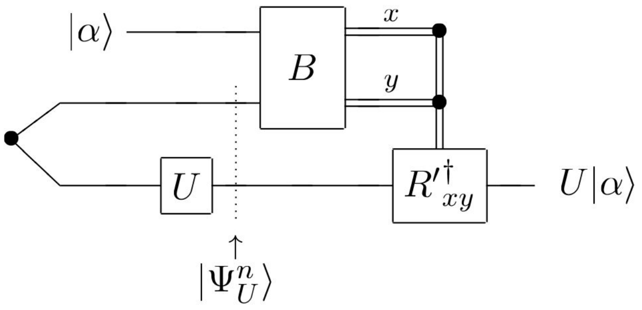
$$|\psi_{out}\rangle = U R_{xz}|\psi\rangle = R'_{xz}U|\psi\rangle$$

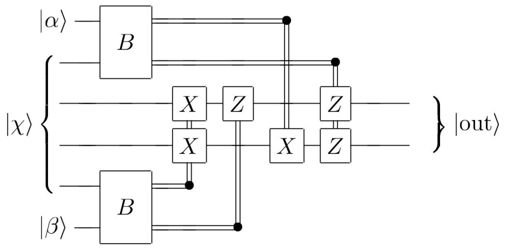
Fault tolerant quantum gates

4 - introducing fault tolerance: building gates from the C3 group

We can build gates from the C3 group (Cn) only by applying gates from the C2 group (Cn-1) to the actual Qubits !

Can also be used to prove the CNOT gate circuit





Conclusion

Advantages of this method:

- Typical advantages of quantum teleportation:
    - Apply gates on “volatile qubit”, while storing “stable qubit”
- Use already existing quantum teleportation device
- Apply “complex” gates (for fault tolerant comp.) using “simpler” gates