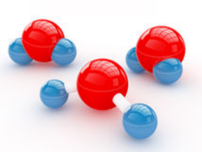
Macroscopic Models: Conservation Laws

Henri Navier
(1785 Dijon–1836 Paris)
- mechanical engineer
- mathematician
- physicist
- economist

George Gabriel Stokes
(1819 Ireland – 1903)
- mathematician
- physicist

Molecules



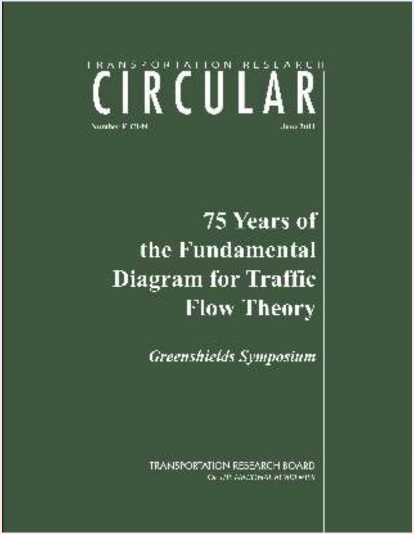
- Mass conservation
- Momentum conservation

Fluid

Navier-Stokes Equations (1823-1845)

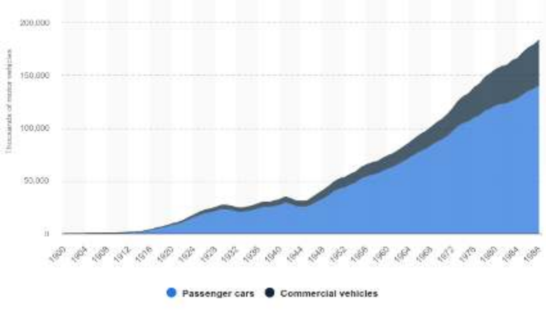
- 2 equations
  - Mass conservation
  - Momentum conservation
- 2 unknowns (density ρ, velocity u)

First Measurements for road traffic



Start of road traffic

1908



Statista 1993

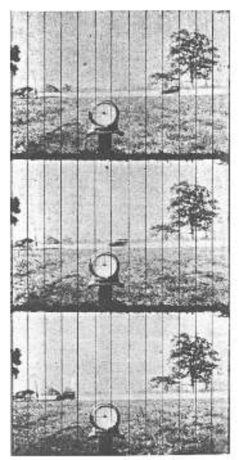
First Measurements for road traffic

[Charansonney et al, TFTC (2018)]

Johnson 1929

[Kühne, TRB (2011)]

Greenshield 1933



[Kühne, TRB (2011)]

Greenshield 1933

First Measurements for road traffic

[Charansonney et al, TFTC (2018)]

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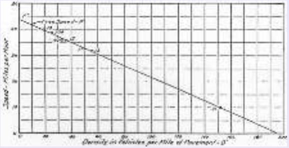
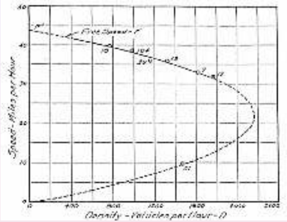
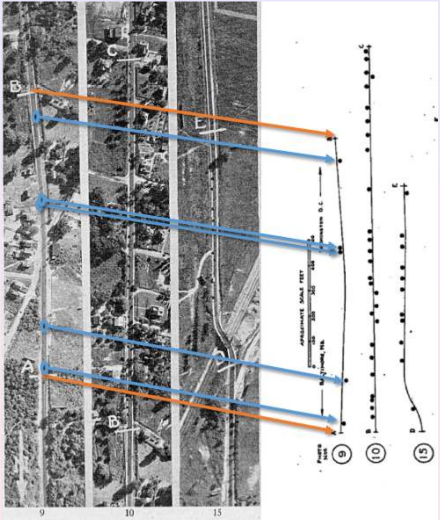




[Kühne, TRB (2011)]

Greenshield 1933

First Measurements for road traffic



[Charansonney et al, TFTC (2018)]

Johnson 1929
Aerial view

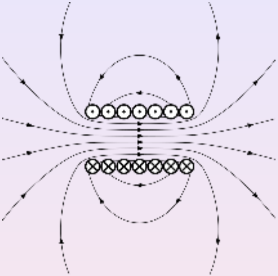
Vehicular Flux Measurements

Inductance loops

Vehicular Flux Measurements

Solenoid

Solenoid with iron core
➥ Electromagnet.



[Wikimedia, by Geek3]

Vehicular velocity Measurements

Double loops

From [Treiber, M2 Course]

Vehicular velocity Measurements

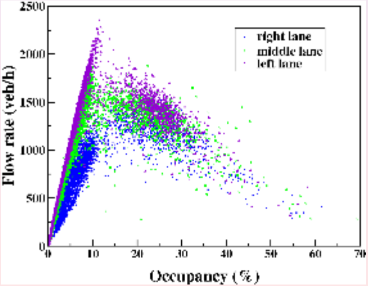
Double loops

From [Coifman (2018), Traffic Flow Theory and Characteristics Committee Mid-Year Meeting]

Macroscopic Models for road traffic

Model LWR (1955-1956)

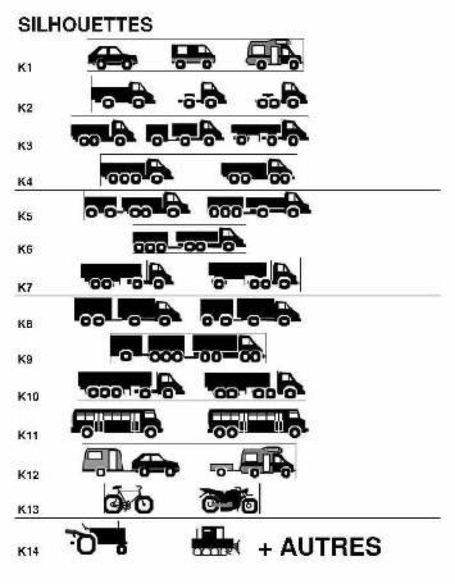
$$\partial_t \rho + \partial_x(\rho u) = 0 \quad \text{(Mass conservation)}$$
$$u = V(\rho) = \frac{F(\rho)}{\rho} \quad \text{(Fundamental Diagram)}$$



Single vehicle data

- Passage time
- velocity
- length
- SETRA category

SETRA categories

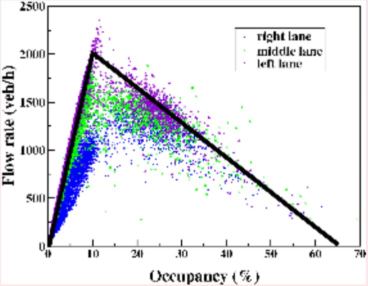


Macroscopic Models for road traffic

Model LWR (1955-1956)

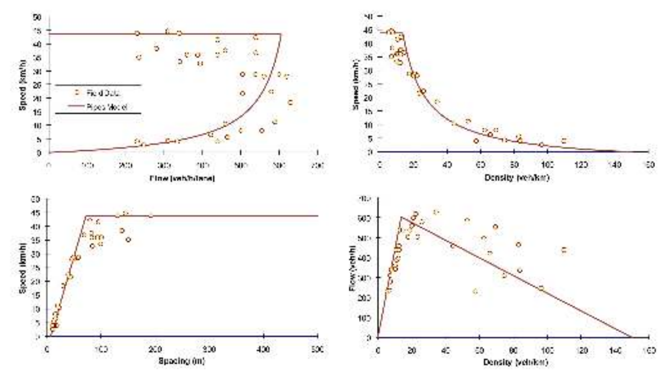
$$\partial_t \rho + \partial_x (F(\rho)) = 0 \quad \text{(Mass conservation)}$$

$$F(\rho) \quad or \quad V(\rho) \equiv \frac{F(\rho)}{\rho} \quad \text{(Fundamental diagram)}$$



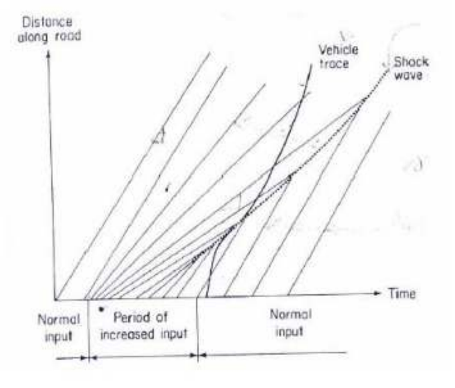
Fundamental diagrams

Various equivalent representations of the fundamental diagram



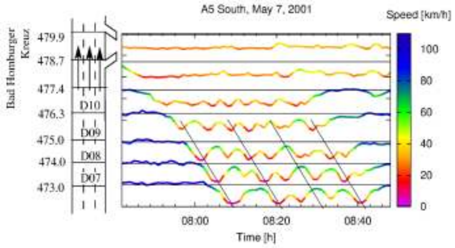
From [Rakha & Gao (2011), 75 Years of FD]

Characteristics and shocks



From [Dingra et al (2011)]

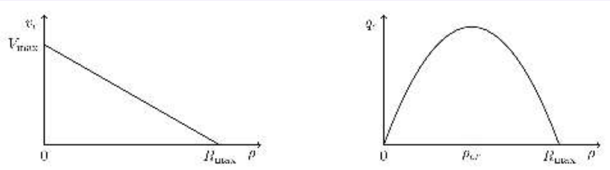
Measuring the wave speed w

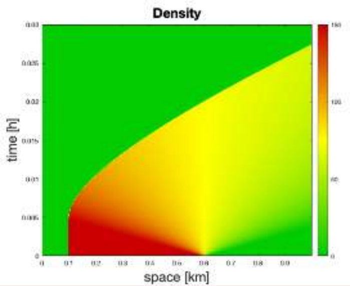


From [Treiber, M2 course]

Example of simulation of the LWR model

Model LWR with fundamental diagram $V(\rho) = V_{max}\left(1 - \frac{\rho}{\rho_{max}}\right)$ (Greenshield's fundamental diagram)





From [Goatin (2023), review paper]

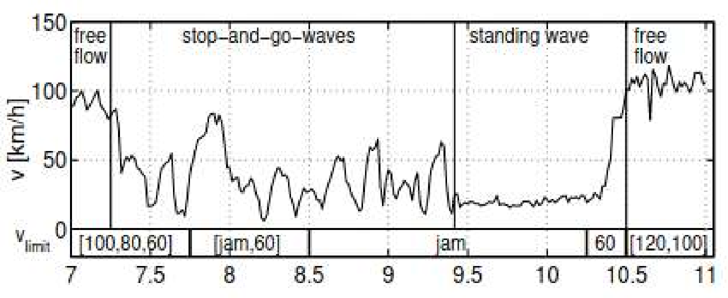
Ex: traffic light becomes green in $x = 0.6$.

Stop and Go waves

From [Sugiyama et al, New J. Phys. (2008)]

Drivers were asked to cruise at about 30 km/h.

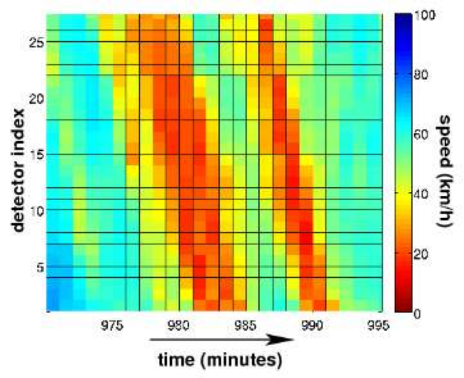
Stop and Go waves



From [Lenz et al, ECC, 2001 APCA]

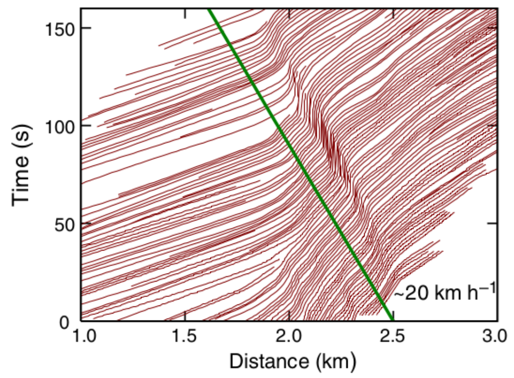
Stop and Go waves

Spatiotemporal plot of speed (averaged across three running lanes) for the high coverage section of M42 ATM showing two stop-and-go waves.



From [E. Wilson (2011), 75 Years of FD]

Stop and Go waves



From [Sugiyama et al, New J. Phys. (2008)]

- Trajectories of vehicles on the highway (aerial photograph taken in 1967). From [Treiterer and Myers, Transp. Traffic Theory (1974)]
- Green line : corresponds to a backward cluster velocity of 20 km/h, as measured in [Sugiyama (2008)].

Macroscopic Models for road traffic

1st order macroscopic model

- Modèle LWR (1955-1956)

2nd order macroscopic model

- Payne-Whitham model (1971)
  - ➥ “Requiem for second-order fluid approximations of traffic flow”
    [Daganzo 1995]

Macroscopic Models for road traffic

1st order macroscopic model

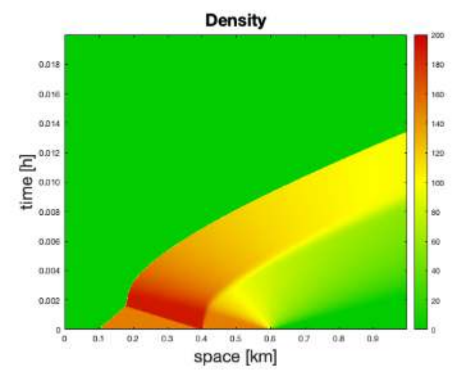
- Modèle LWR (1955-1956)

2nd order macroscopic model

- Payne-Whitham model (1971)
  - ➥ "Requiem for second-order fluid approximations of traffic flow"
    [Daganzo 1995]
- Aw-Rascle model (2000)
  - ➥ "Resurrection of "Second Order" Models of Traffic Flow and numerical simulation"

ARZ Model with the choice $p(\rho) = \rho$.
• ignoring the relaxation term ($\tau \to \infty$)
• with a group of more agressive drivers (higher w) behind slow ones.
• ρ^0 = 150 veh/km for $0.1 < x < 0.6$;

v^0 = 50 km/h for $0.1 < x < 0.4$;
v^0 = 10 km/h for $0.4 < x < 0.6$;

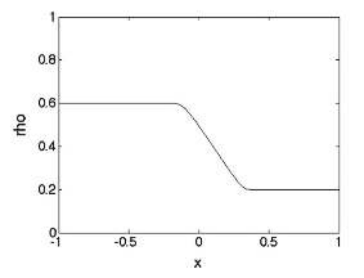


From [Goatin (2023), review paper]

Rieman pbl黎曼问题

$\rho_L = 0.6$ for $x < 0$

$\rho_R = 0.2$ for $x \geq 0$



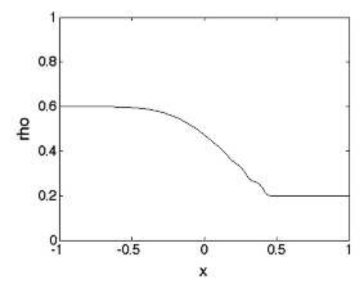
[Blandin & Goatin, Numer. Math.(2016)]

Local model局域模型

$$\partial_t \rho(t,x) + \partial_x \left[\rho(t,x)\left(1 - \rho(t,x)\right)\right] = 0$$

Non-local models

Rieman pbl
$\rho_L = 0.6$ for $x < 0$
$\rho_R = 0.2$ for $x \geq 0$



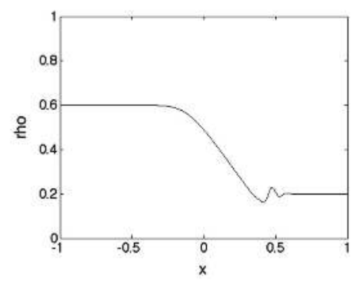
[Blandin & Goatin, Numer. Math.(2016)]

Downstream non-local model

$$\partial_t \rho(t,x) + \partial_x \left[\rho(t,x) V \left(\int_x^{x+\eta} \rho(t,y) w_\eta(y-x) dy \right) \right] = 0$$

Non-local models

Rieman pbl
$\rho_L = 0.6$ for $x < 0$
$\rho_R = 0.2$ for $x \geq 0$



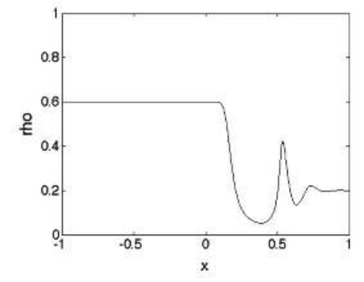
[Blandin & Goatin, Numer. Math.(2016)]

Centered non-local model

$$\partial_t \rho(t,x) + \partial_x \left[\rho(t,x) V \left(\int_{x-\eta/2}^{x+\eta/2} \rho(t,y) w_\eta(y-x) dy \right) \right] = 0$$

Non-local models

Rieman pbl
$\rho_L = 0.6$ for $x < 0$
$\rho_R = 0.2$ for $x \geq 0$



[Blandin & Goatin, Numer. Math.(2016)]

Upstream non-local model

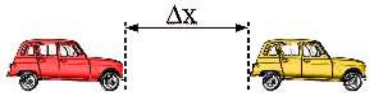
$$\partial_t \rho(t,x) + \partial_x \left[\rho(t,x) V \left(\int_{x-\eta}^{x} \rho(t,y) w_\eta (y-x) dy \right) \right] = 0$$

Models for road traffic

Fluid-like models

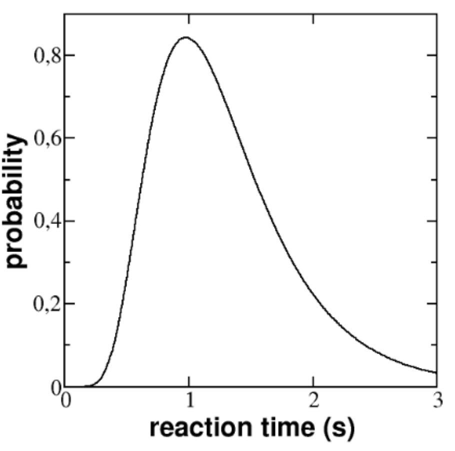
Equations for the density ρ and mean velocity u of the vehicles

Car-following models



acceleration = fonction (distance, velocity difference, $\cdots$)

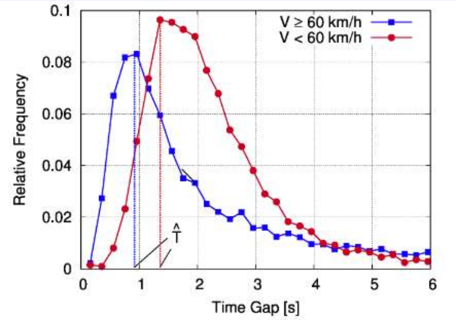
Reaction times



Rapport LCPC / Appert

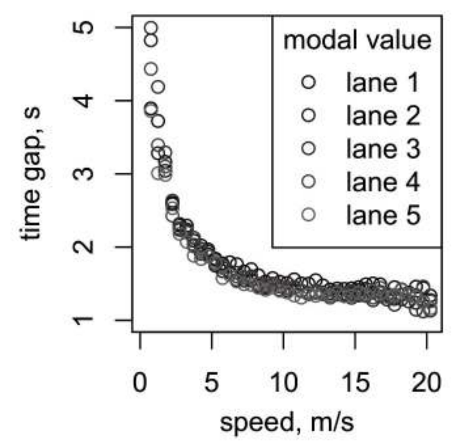
Log-normal law (Probability density): $\sigma = 0.44$, $\mu = 0.17$

Time headways



From [Treiber, M2 Course]

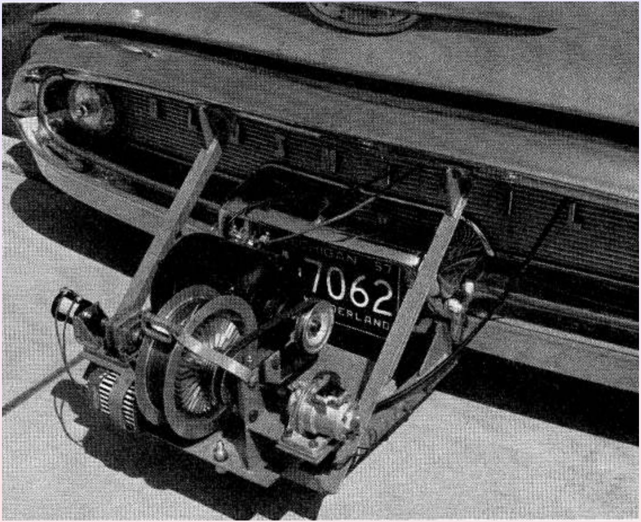
Time headways



Data from Hollywood Freeway

From [A. Tordeux et al, Transp. Res. B (2010)]

Car-following experiment



From [Chandler et al (1958)]

Other car-following models

GHR model (1959-1961)

$$a_f(t) = c v_f^m(t) \frac{\Delta v(t-\tau)}{\Delta x^l(t-\tau)}$$

From Gazis, Herman, Rothery

Other car-following models

Gibbs model (1981)

$$\dot{x}_n(t+\tau^n_{Gipps}) = \min\begin{cases} \underbrace{b^n \cdot \tau^n_{Gipps} + \sqrt{(b^n \cdot \tau^n_{Gipps})^2 - b^n \cdot [2 \cdot (\Delta x^{n-1,n}(t) - \delta^n_0) - \dot{x}_n(t) \cdot (\tau^n_{Gipps} - \dot{x}_n(t)^2/\hat{b}]}}_{\text{Régime congestionné}} \\ \underbrace{\dot{x}_n(t) + 2.5 \cdot a^n \cdot \tau^n_{Gipps} \cdot (1 - \dot{x}_n(t)/u^n_f) \times \sqrt{0.025 + \dot{x}_n(t)/u^n_f}}_{\text{Régime fluide}} \end{cases}$$

[Gibbs, Transp. Res. B (1981)],
Formula rewritten in [Gomez-Patiño, PhD thesis (2022)]

Other car-following models

Tampere model (2004)

$$\frac{dv}{dt} = \min\left(\frac{v_n^f - v_n}{\tau_f}; \quad \frac{s_n - s^d(v_n)}{\tau_s} + \frac{v_{n-1} - v_n}{\tau_v}\right)$$

with

$$\begin{aligned} a_{min} &< \frac{dv}{dt} < a_{max} \\ s^d(v) &= s_1^d v + s_2^d v^2 \end{aligned}$$

From [Tampere, PhD thesis TU Delft (2004)]

Car-following models in commercial softwares

TABLE 1 Software Car-Following Model Formulations

Software	Model	Formulation
CORSIM	Pitt Model	$u_n(t+\Delta t) = \min\left(3.6\cdot\left[\frac{s_n(t)-s_j}{c_3} - b\left(u_n(t)-u_{n-1}(t)\right)^2\right], u_f\right)$
VISSIM	Wiedemann74	$u_n(t+\Delta t) = \min\left\{\begin{matrix}3.6\cdot\left(\frac{s_n(t)-s_j}{BX}\right)^2 \\ 3.6\cdot\left(\frac{s_n(t)-s_j}{BX\cdot EX}\right)^2\end{matrix}, u_f\right\}$
	Wiedemann99	$u_n(t+\Delta t) = \min\left\{\begin{matrix}u_n(t)+3.6\cdot\left[CC8+\frac{CC8-CC9}{80}u_n(t)\right]\Delta t \\ 3.6\cdot\frac{s_n(t)-CC0-L_{n-1}}{u_n(t)}\end{matrix}, u_f\right\}$
Paramics	Fritzsche	$u_n(t+\Delta t) = \min\left\{\begin{matrix}3.6\cdot\left(\frac{AD-A_0}{T_D}\right) \\ 3.6\cdot\left(\frac{AR-A_0}{T_r}\right)\end{matrix}, u_f\right\}$
AIMSUN2	Gipps	$u_n(t+T) =$ $\min\left\{\begin{matrix}u_n(t)+3.6\left[2.5a_{max}T\left(1-\frac{u_n(t)}{u_f}\right)\sqrt{0.025+\frac{u_n(t)}{u_f}}\right], \\ 3.6\left[-bT+\sqrt{b^2T^2+b\left(2\left[s_n(t)-L_{n-1}\right]-\frac{u_n(t)}{3.6}T+\frac{u_{n-1}(t)^2}{3.6^2\times b'}\right)}\right]\end{matrix}\right\}$
INTEGRATION	Van Aerde	$u_n(t+\Delta t) =$ $\min\left\{\begin{matrix}u_n(t)+3.6\cdot\frac{F_n(t)-R_n(t)}{m}\Delta t, \\ \frac{-c_1'+c_3u_f+\tilde{s}_n(t)-\sqrt{\left[c_1'-c_3u_f-\tilde{s}_n(t)\right]^2-4c_3\left[\tilde{s}_n(t)u_f-c_1'u_f-c_2\right]}}{2c_3}\end{matrix}\right\}$ Where: $\tilde{s}_n(t) = s_n(t)+\left[u_{n-1}(t+\Delta t)-u_n(t)\right]\Delta t+0.5a_{n-1}(t+\Delta t)\Delta t^2$

From [Rakha & Gao (2011), 75 Years of FD]

Data collection

Magnetic loops

Limitations:

- Hypothesis: homogeneous vehicles
  - ➥ results depend on the length of vehicles, on their free velocity [cf Coifman (2018)]
  - ➥ possible to put corrections if single-vehicle data

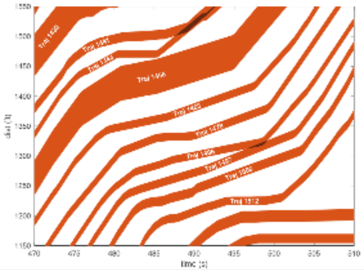
Videos

- NGSIM project (Next Generation Simulation):
  - by U.S. Department of Transportation Federal Highway Administration
  - Extracted vehicle trajectories
  - Made available for the community

Data collection: NGSIM Project

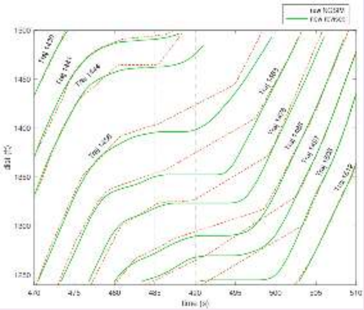
- Measurements on several highways/freeways (2005...)
- Many lanes
- Onramps and intersections
- Typically 500 meters long
- Typically hour duration

Data collection: NGSIM Project



[Coifman and Li, Transp. Res. B (2017)]

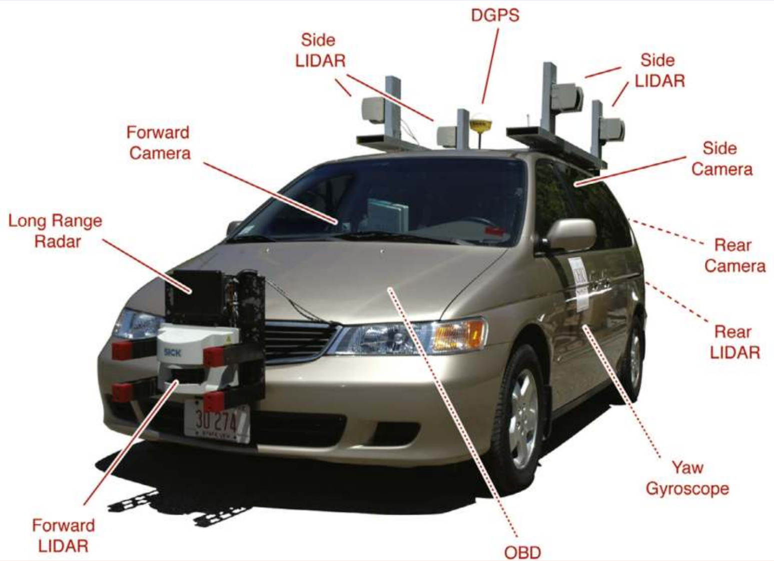
Data collection: NGSIM Project



- Dashed lines: raw NGSIM trajectories
- Solid lines: new trajectories

[Coifman and Li, Transp. Res. B (2017)]

Data collection



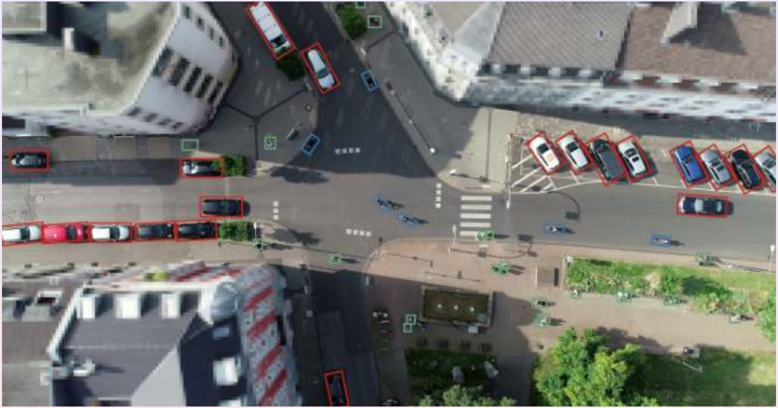
[Coifman et al, Transp. Res. C (2016)]

Data collection

Data

- Benjamin Coifman homepage
  - NGSIM revisited and others (LIDAR, etc)
- TU Delft (Hoogendoorn, Daamen, etc) : trajectories at on ramp
- ZEN Traffic Data (2018...) : trajectories on several kilometers on highway (Japan)
- Open ACC Database from data.europa.eu
  - Car-following experiments involving 28 vehicles, among which 22 with commercial Adaptive Cruise Control systems
- πneuma project (2020) at open-traffic.epfl.ch
  - 10 drones, congested area of a 1.3 km^2 area, more than 100 km-lanes, $\sim$ half a million trajectories.
- inD Dataset (2020) at www.ind-dataset.com
  - Drones, 13 500 road users (vehicles, bicyclists and pedestrians), 4 intersections, 10 hours of measurement.

Data collection: inD Dataset



[Bock et al, 2020 IEEE Intelligent Vehicles Symposium]