The Bell-CHSH inequality

Bell's theorem

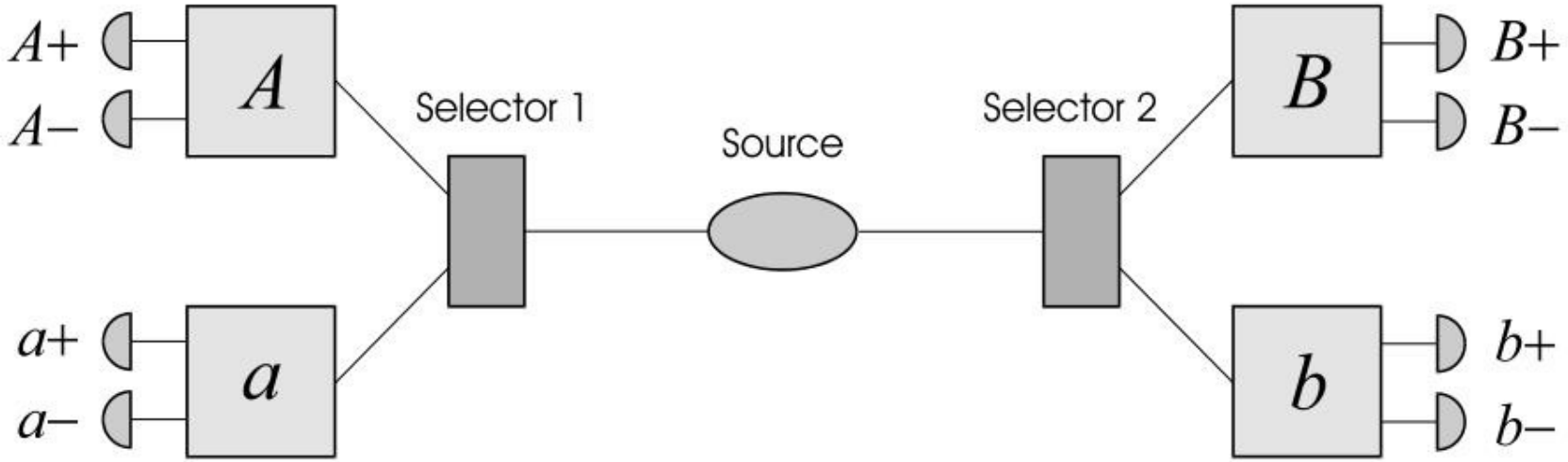
Some predictions of QM are incompatible with EPR's elements of reality (no local hidden-variable theory can reproduce QM).

"The most profound discovery of science."

H. P. Stapp,
Nuovo Cimento **29B**, 270 (1975).

Scenario for the Bell-CHSH inequality



$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|01\rangle - |10\rangle\right)$$

The Bell-CHSH inequality

$$A, a, B, b \in \{-1, 1\}$$

$$(A - a, A + a) \in \{(0, \pm 2), (\pm 2, 0)\}$$

$$(A - a)B - (A + a)b \in \{-2, 2\}$$

$$-2 \leq \langle AB - Ab - aB - ab \rangle \leq 2$$

$$\boxed{\left| \langle AB \rangle - \langle Ab \rangle - \langle aB \rangle - \langle ab \rangle \right| \leq 2}$$

Predictions of QM for the *singlet* state

$$\left|\psi^{-}\right\rangle = \frac{1}{\sqrt{2}}\left(\left|01\right\rangle - \left|10\right\rangle\right)$$

$$\left\langle AB\right\rangle = \left\langle \psi^{-}\left|\hat{A}\otimes\hat{B}\right|\psi^{-}\right\rangle$$
$$= -\cos\theta_{AB}$$

QM violates the Bell-CHSH inequality

$$F_{\mathrm{QM}} = \left|\langle AB\rangle - \langle Ab\rangle - \langle aB\rangle - \langle ab\rangle\right|$$

$$= \left|-\cos\theta_{AB} + \cos\theta_{Ab} + \cos\theta_{aB} + \cos\theta_{ab}\right|$$

$$\hat{A} = \sigma_x$$

$$\hat{a} = \sigma_y$$

$$\hat{B} = (\sigma_y - \sigma_x)/\sqrt{2}$$

$$\hat{b} = (\sigma_y + \sigma_x)/\sqrt{2}$$

$$F_{\mathrm{QM}} = 2\sqrt{2} > 2\,!!!$$

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Experimental Tests of Realistic Local Theories via Bell's Theorem

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(Received 30 March 1981)

We have measured the linear polarization correlation of the photons emitted in a radiative atomic cascade of calcium. A high-efficiency source provided an improved statistical accuracy and an ability to perform new tests. Our results, in excellent agreement with the quantum mechanical predictions, strongly violate the generalized Bell's inequalities, and rule out the whole class of realistic local theories. No significant change in results was observed with source-polarizer separations of up to 6.5 m.

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Experimental Realization of Einstein-Podolsky-Rosen-Bohm *Gedankenexperiment*: A New Violation of Bell's Inequalities

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(Received 30 December 1981)

The linear-polarization correlation of pairs of photons emitted in a radiative cascade of calcium has been measured. The new experimental scheme, using two-channel polarizers (i.e., optical analogs of Stern-Gerlach filters), is a straightforward transposition of Einstein-Podolsky-Rosen-Bohm *gedankenexperiment*. The present results, in excellent agreement with the quantum mechanical predictions, lead to the greatest violation of generalized Bell's inequalities ever achieved.

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Experimental Test of Bell's Inequalities Using Time-Varying Analyzers

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(Received 27 September 1982)

Correlations of linear polarizations of pairs of photons have been measured with time-varying analyzers. The analyzer in each leg of the apparatus is an acousto-optical switch followed by two linear polarizers. The switches operate at incommensurate frequencies near 50 MHz. Each analyzer amounts to a polarizer which jumps between two orientations in a time short compared with the photon transit time. The results are in good agreement with quantum mechanical predictions but violate Bell's inequalities by 5 standard deviations.