OV model :

$$\ddot{x}_i(t) = \frac{1}{\tau}\left[V(\Delta x_i(t)) - \dot{x}_i(t)\right]$$

$\Delta x_i = x_{i-1} - x_i$

Local stability :

- Uniform solu$^\circ$

where $v = V(d)$

$$\Delta x_k^u = d \quad \forall k$$

- Small perturba$^\circ$ applied only on the follower i

$$x_i(t) = x_i^u(t) + y(t)$$
$$= \underbrace{x_{i-1}^u(t) - d}_{\text{uniform transla}^\circ} + y(t)$$

and also $\longrightarrow$ $\Delta x_i(t) = x_{i-1}(t) - x_i(t)$
$= x_{i-1}^U(t) - x_i(t)$
$= d - y(t)$

$$\dot{x}_i(t) = \underbrace{\dot{x}_{i-1}^u(t)}_{= v} + \dot{y}(t)$$

$$\ddot{x}_i(t) = \ddot{y}(t)$$

We put this in the model

$$\ddot{y}(t) = \frac{1}{\tau}\left[V(d - y(t)) - v - \dot{y}(t)\right]$$

$$= \frac{1}{\tau}\left[\underbrace{V(d)}_{= v} - y(t)V'(d) - v - \dot{y}(t)\right]$$

Taylor exp

$$= \frac{1}{\tau}\left[-y(t)V'(d) - \dot{y}(t)\right]$$

<=>

$$\tau\ddot{y}(t) + \dot{y}(t) + V'(d)\,y(t) = 0$$

characteristic eq$^\circ$:

$$\tau r^2 + r + V'(d) = 0$$

$$\Delta = 1 - 4\tau V'(d)$$

The solu$^{\circ}$ is non-oscillating if $\Delta \geq 0$

$\Leftrightarrow 1 \geq 4\tau V'(d)$

$\Leftrightarrow \boxed{\tau V'(d) \leq \frac{1}{4}}$

- If $\Delta \geq 0$

$r^{\pm} = \dfrac{-1 \pm \sqrt{1-4\tau V'(d)}}{2\tau}$

Local linear stability only if $\begin{cases} r^+ < 0 \\ r^- < 0 \leftarrow \text{always true for } \tau > 0 \end{cases}$

We have $\tau > 0$

$r^+ < 0 \Leftrightarrow -1 + \sqrt{1-4\tau V'(d)} < 0$

$\Leftrightarrow \sqrt{1-4\tau V'(d)} < 1$

$\Leftrightarrow -4\tau V'(d) < 0$

$\underset{\tau > 0}{\Leftrightarrow} V'(d) > 0$

distance

local linear stability if $\boxed{V'(d) > 0}$ (τ is taken >0).

→ reasonable condi$^{\circ}$.

- If $\Delta < 0$: $\left(\sqrt{\Delta} = i\sqrt{4\tau V'(d) - 1}\right)$

the roots can be written as $\dfrac{-1 \pm i\sqrt{4\tau V'(d)-1}}{2\tau}$

the real part $-\frac{1}{2\tau}$ is always <0 → local linear stability.

but oscillating

→ risk of collision

If $t_h = \dfrac{d}{V(d)} = 2s$ $\quad V(d) = \dfrac{d}{2s}$ $\quad V'(d) = \frac{1}{2}\ s^{-1}$

donc $\tau V'(d) \leq \frac{1}{4} \Leftrightarrow \tau \leq \frac{1}{2}\ s$ very small