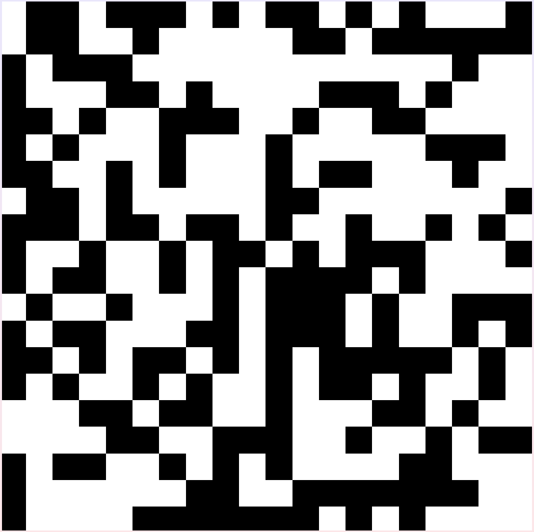
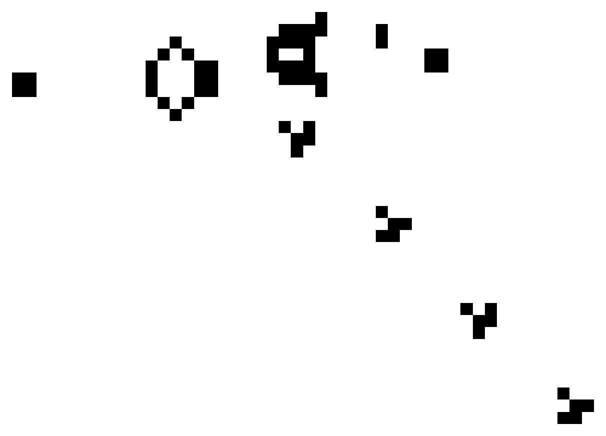
Game of life [John Horton Conway (1970)]



Game of life: special dynamic structures

Gosper's glider gun creating gliders

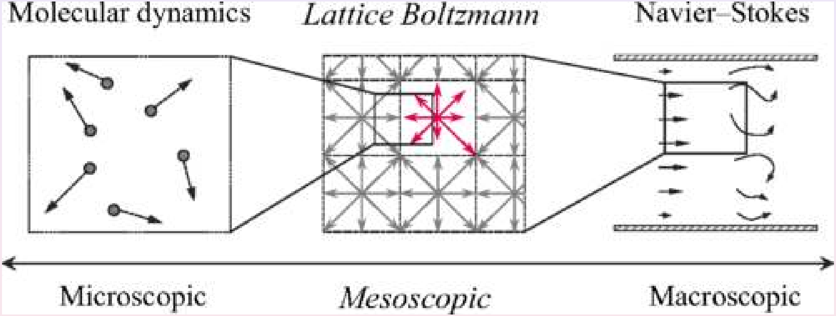


[From Wikimedia, by Lucas Vieira]

CAs are beautiful

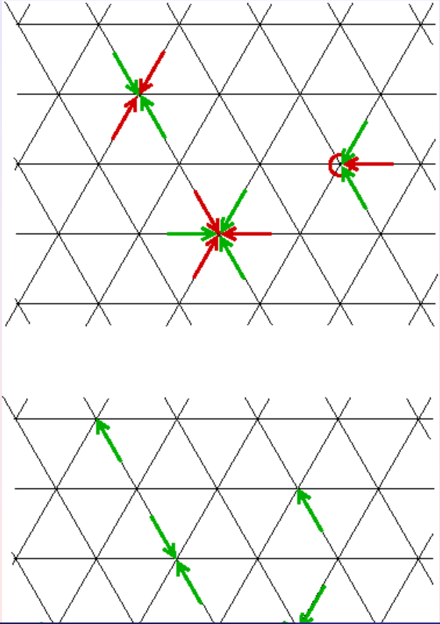
LGA for fluids

A mesoscopic model

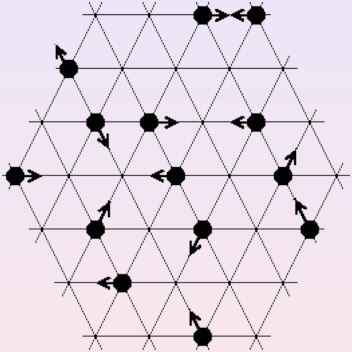


From [Saito et al, PRE (2017)]

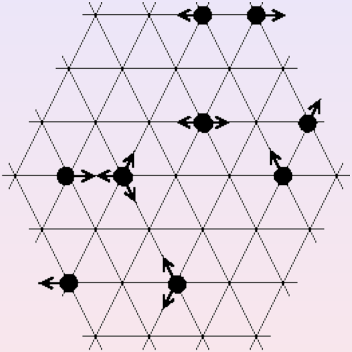
LGA for fluids



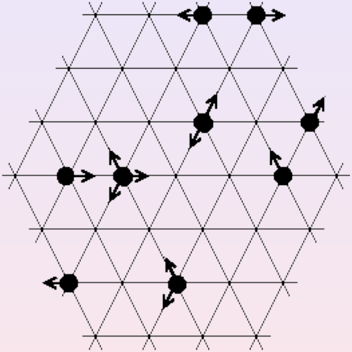
LGA for fluids



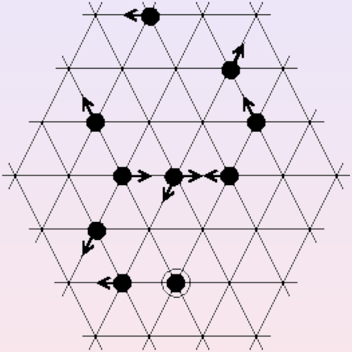
LGA for fluids



LGA for fluids

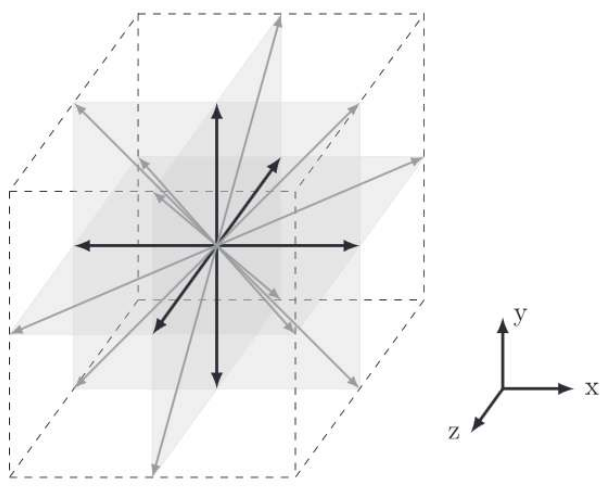


LGA for fluids



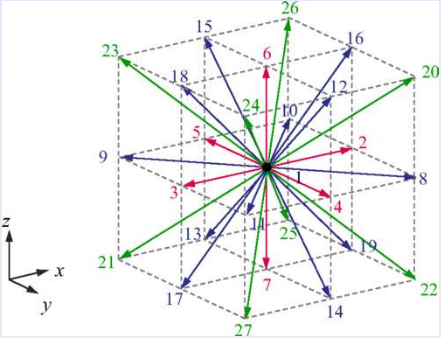
LGA for fluids

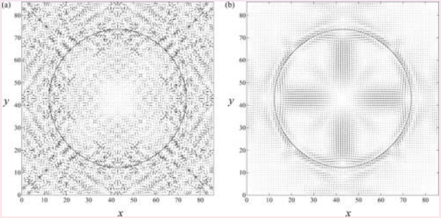
D3Q19



From [Owen et al, Advances in Physics: X (2023)]

Minimizing spurious flows





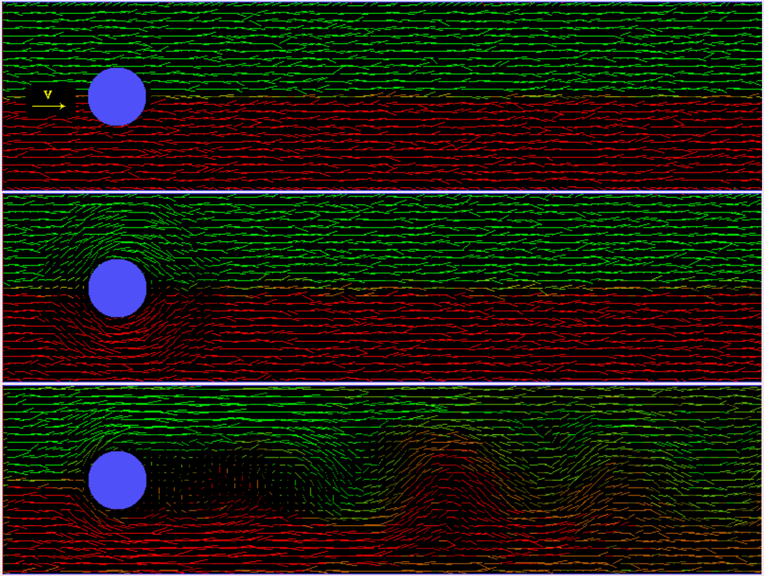
From [Saito et al, PRE (2017)]

Van Kármán vortex street

Satellite view of Jan Mayen and Rishiri islands with Von Kármán vortex street in clouds

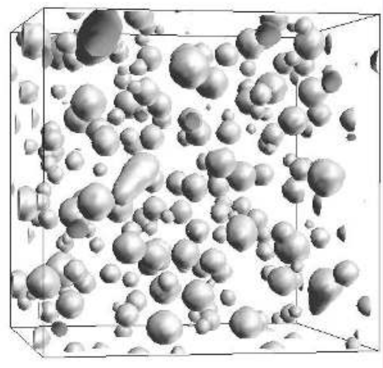
From NASA

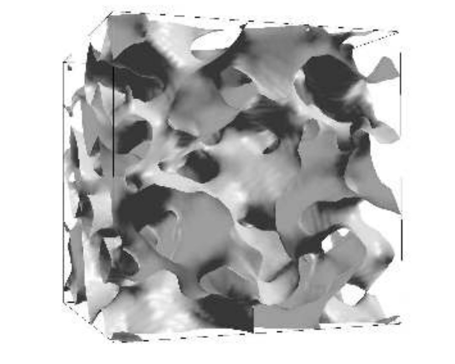
Simulation of Von Karman vortices with FHP-III model



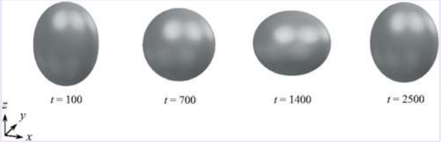
LGA for immiscible fluids

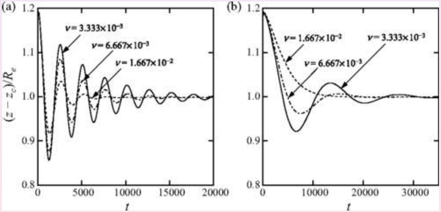
From the team of D. Rothman (MIT)





Oscillations of an initially elongated drop.

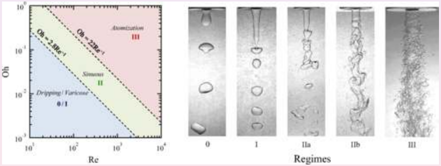


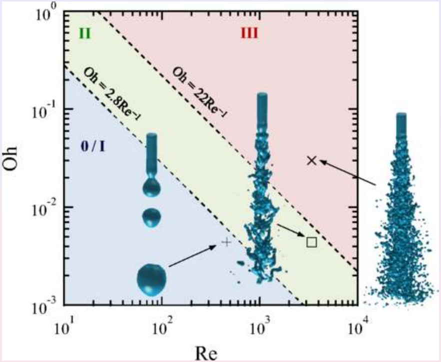


Time history of the interfacial location of an oscillating droplet, under (a) high or (b) low surface tension.

From [Saito et al, PRE (2017)]

LBE for interfacial flows



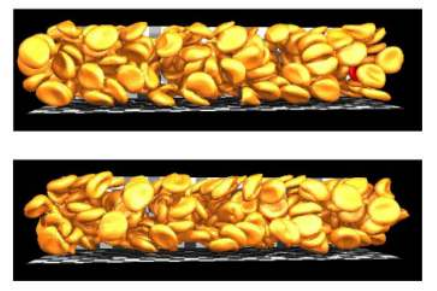


Real fluid

Simulated fluid

From [Saito et al, PRE (2017)]

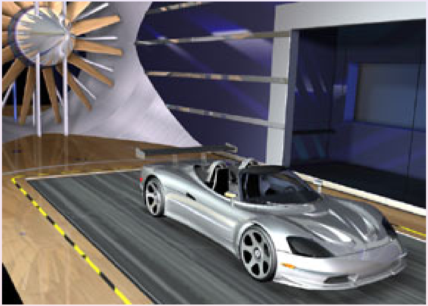
Microfluidic, Red blood cells

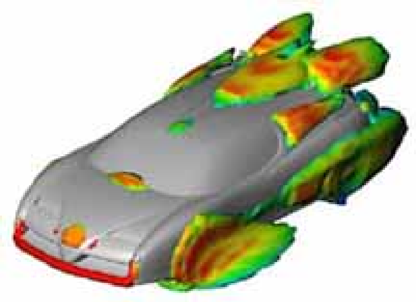


From [Dupin et al, PRE (2007)]

Commercial software for car industry

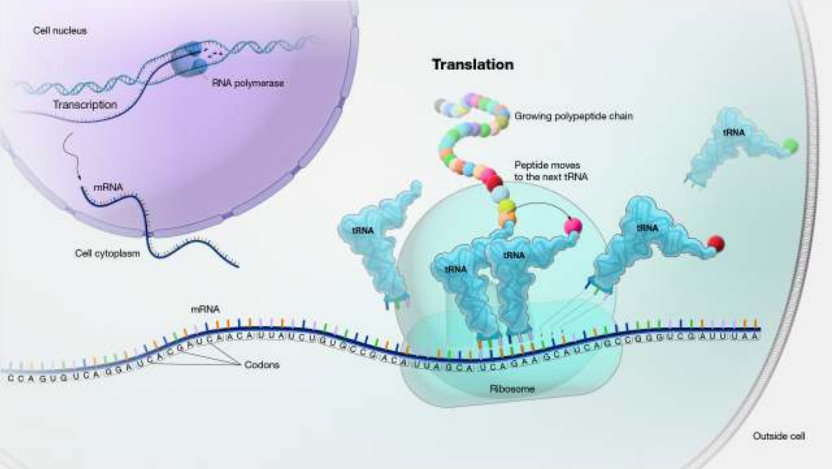
Commercialised by EXA Corporation.





Air flow around a vehicle.

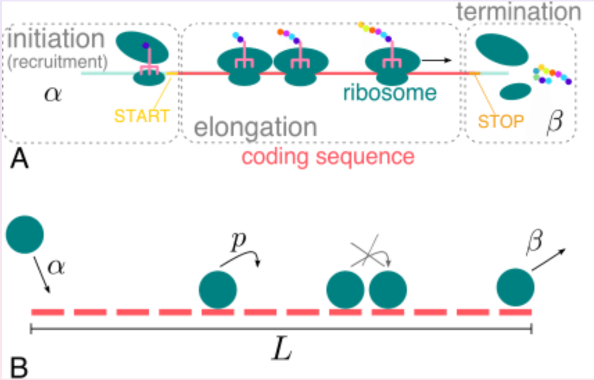
Translation = protein synthesis from mRNA



[From NHGRI (National Human Genome Research Institute), USA]
https://www.youtube.com/watch?v=oCp9IK6iBTo

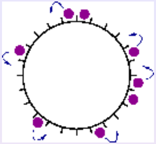
Exclusion process

First introduced to model protein synthesis by ribosomes on mRNA templates:

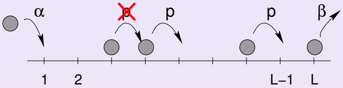


Model from [MacDonald et al (1968)], Sketch by [Fernandes et al, Sci. Rep. (2017)]

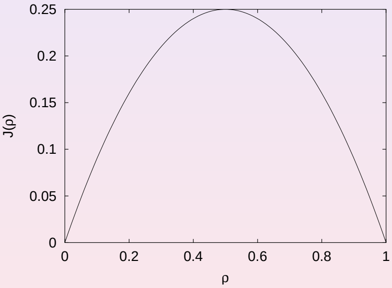
Exclusion Processes



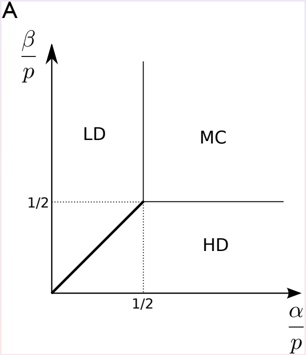
Periodic Boundary Conditions



Open Boundary Conditions



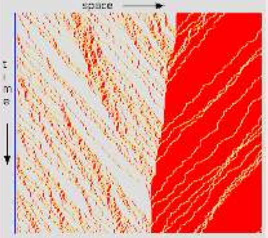
Fundamental diagram



Phase Space

Domain Wall Theory

Ising model

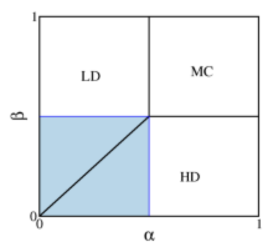


$\alpha > \beta$

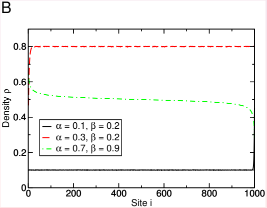
- microscopic state
  $\longrightarrow$ macroscopic uniform domains
- Domains separated by *domain walls*, supposed of negligible thickness
- Detailed dynamics of the system $\longleftrightarrow$ Motion of the walls

Domain wall picture

Valid only in a region of the phase diagram:



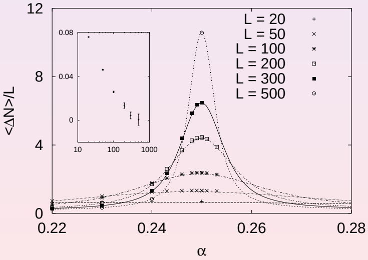
Predicts correctly the density profile with the localization length.

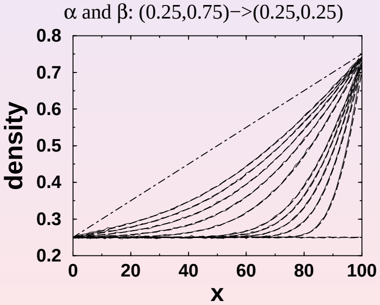


[C. Appert-Rolland, M. Ebbinghaus, and L. Santen, Phys. Rep. **593**, 1-59 (2015)]

There are many exact results for the stationary state of TASEP.
So why is it interesting to have a phenomenological picture?

- Calculations are more easy and give a physical interpretation;
- Calculations can be extended to non-stationary states, to variants of the ASEP





[L. Santen et al, J. Stat. Phys. **106** (2002) p 187]

Updates

Cellular automaton = geometry + rules + update

- Continuous time (Gillespie algorithm).
- random sequential update
  - At each microtimestep, one particle is randomly chosen and updated with a rate dependent probability
  - ➥ close to a continuous time
  - ➥ large fluctuations

Rescaling of time $\Longrightarrow p = 1$ without loss of generality

- parallel update
  - All particles are updated in parallel
  - State of a particle at time $t + \Delta t$ determined by the state of the system at time t
  - ➥ road traffic (Δt = reaction time)
  - ➥ conflicts are possible (crossing, lane changing...).

$p = 1 \Longrightarrow$ Deterministic TASEP : all possible moves are accepted

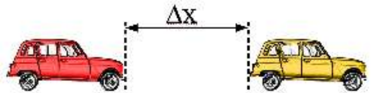
Exclusion Processes

- ordered sequential update (forward, backward...)
- shuffle updates
  - Each particle is updated once per time step
  - random shuffle update [Wölki et al (2006); Smith & Wilson (2007) J. Phys. A]
    - The order of the updates is chosen randomly at each time step
  - frozen shuffle update [C. A-R, Cividini & Hilhorst, J. Stat. Mech. (2011)]
    - Particles are updated in a predefined order according to their phase in the stepping cycle.

➥ no conflicts, no large fluctuations
➥ proposed to model pedestrian traffic

Models for road traffic

Car-following models

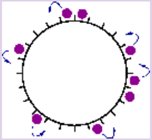


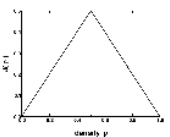
acceleration = fonction (distance, velocity difference, $\cdots$)

Fluid-like models

Equations for the density ρ and mean velocity u of the vehicles

Cellular Automata





Cellular automata simulations

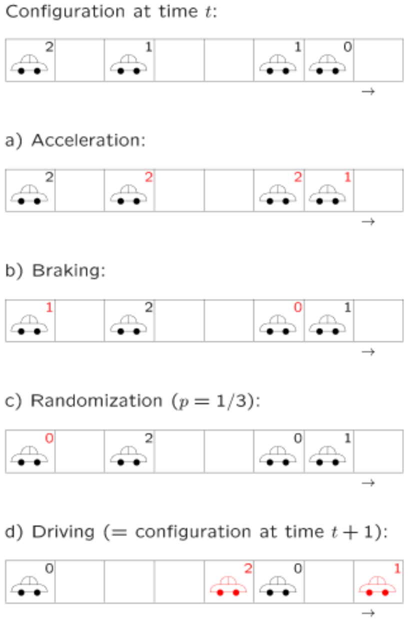
Road = divided into cells

Particle = vehicle

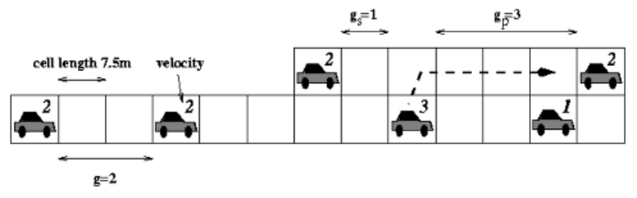
State = speed (between 0 and v_{MAX})

Evolution rules = acceleration and deceleration + propagation

- Pionnering work [Nagel & Schreckenberg (1992)]
- Model by [Knospe et al (2000)]
  - finite braking capacity
  - anticipation
  - slow-to-start rule -> metastability



Lane changes



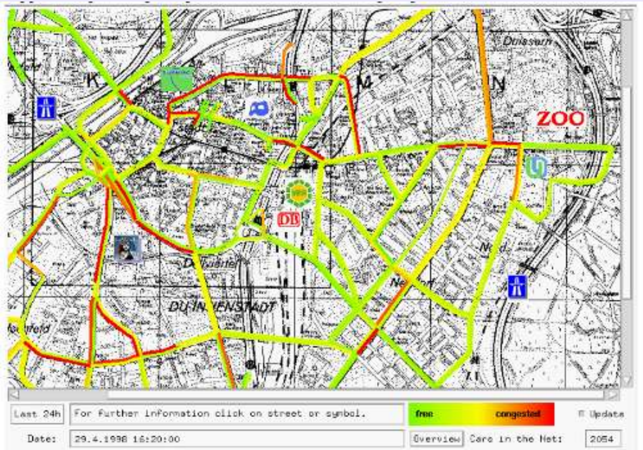
From [Barlovic et al (1999)]

Many other improvements ➥ real life applications

Duisburg, Germany

Road traffic by cellular automata

Duisburg, Germany



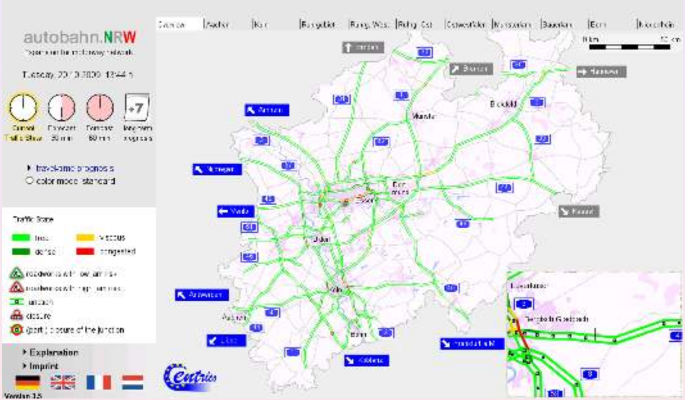
From [Barlovic et al (1999)]

Online simulation - Interactive site

`http://www.traffic.uni-duisburg.de/`

Road traffic by cellular automata

Many improvements, real life applications

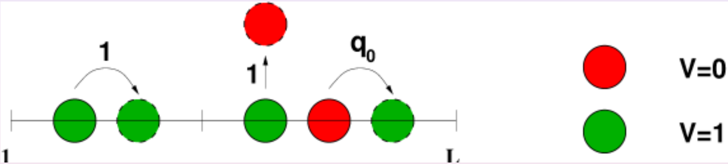


https://www.verkehr.nrw

Cellular automata

- can be used for realistic simulations
- can be used to understand a mechanism
    - Ex: Understanding the consequences of the VDR rule (Velocity Dependent Randomization)
      [*Barlović et al, Eur. Phys. J.* **B5** *793 (1998)*].

Cellular automata



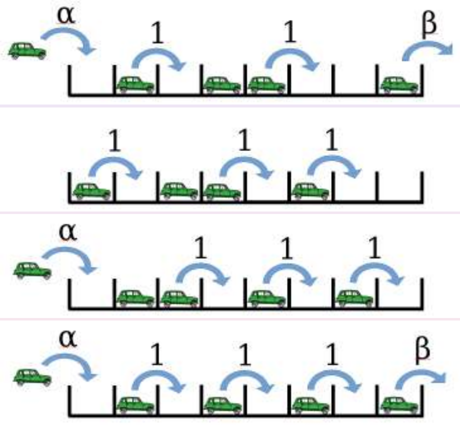
Cellular automata

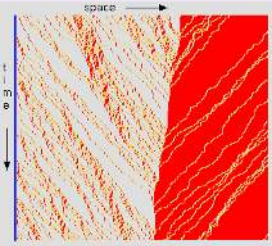
$t = 0$

$t = 1$

$t = 2$

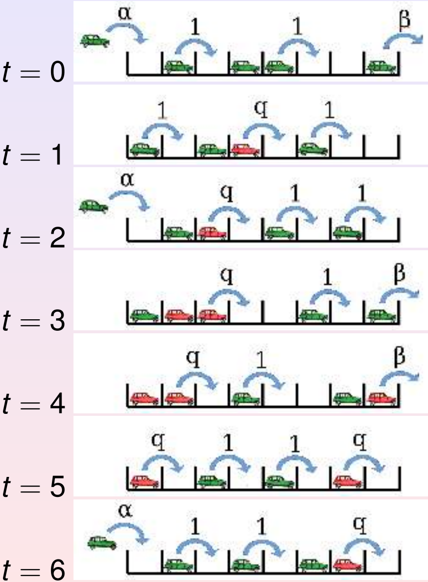
$t = 3$



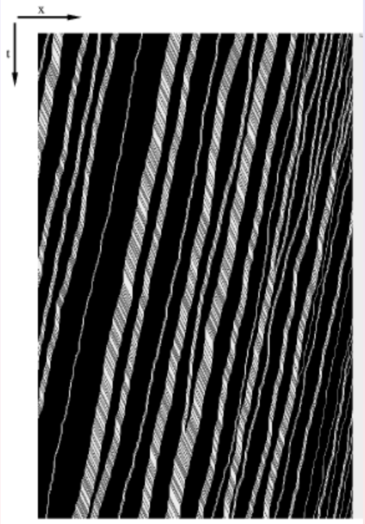


$\alpha > \beta$

Cellular automata with reaction time

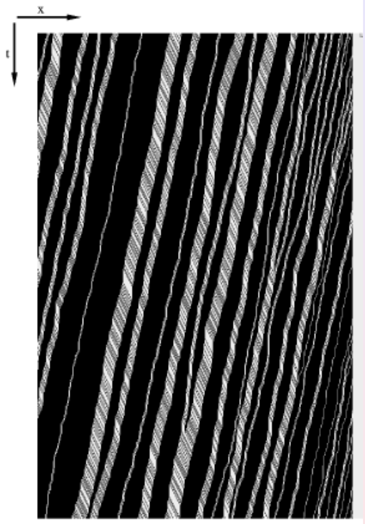


$q < 1$



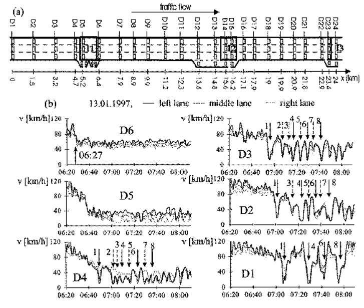
[Appert & Santen, PRL (2001)]

Cellular automata with reaction time



Simulation

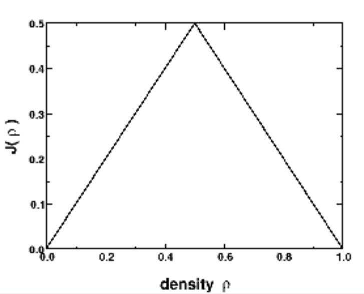
[Appert & Santen, PRL (2001)]



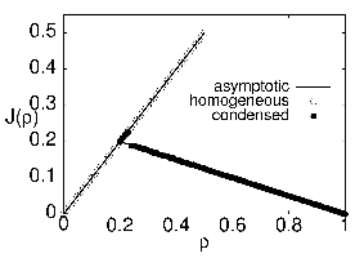
Real data

[Kerner, PRL (1998)]

Cellular automata with reaction time



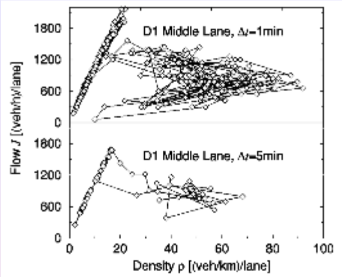
Without reaction time



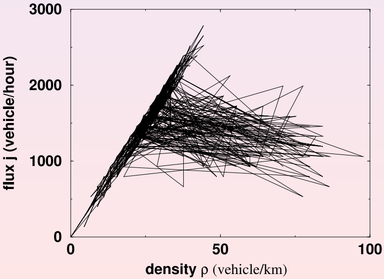
With reaction time

- Metastability
- Hysteresis

Local measurements by an inductive loop

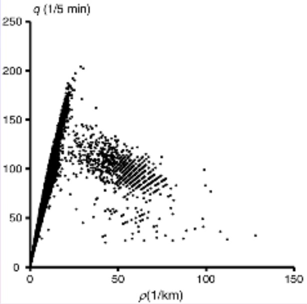


From [*Neubert et al, PRE (1999)*]

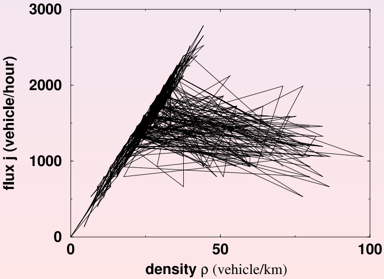


- ➥ stochasticity
- ➥ metastability

Local measurements by an inductive loop

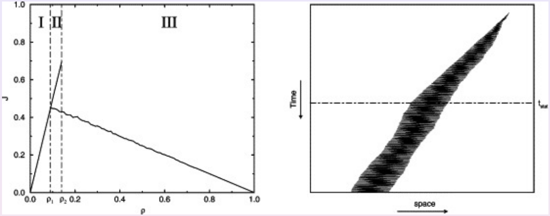


From [Sugiyama et al, New J. Phys. (2008)]

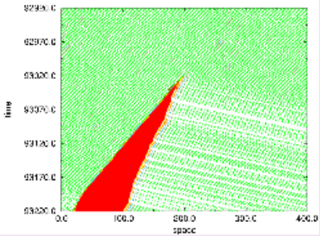


- ➥ stochasticity
- ➥ metastability

Consequences of the VDR rule



From [*Barlovic et al, (2001)*]



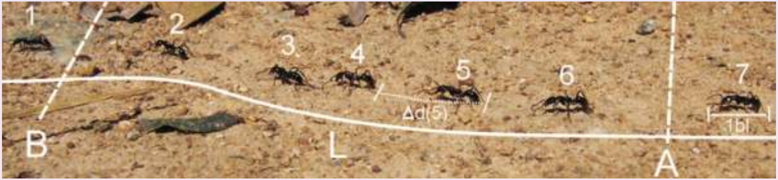
From [*Barlovic et al, (1998)*]

VDR model for $\rho = 0.15$, $p = 0.01$, $p_0 = 0.5$, v_{max}
"The homogeneous initial state is not destroyed immediately".
"In the outflow regime of the jam the density is reduced compared to the average density".

Ants

[Ryohei Yamaoka et al]

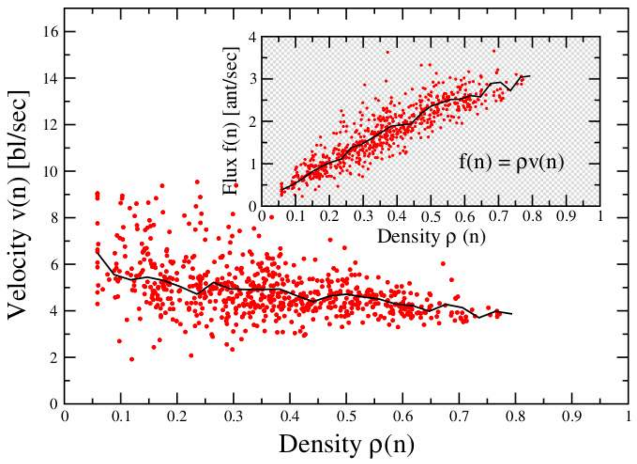
Monomorphic ant species *Leptogenys processionalis* ➥ Same body length



[John et al, PRL (2009)]

For the observed species, 1 bl (body length) = 18 mm.
Observed trail section: length L = 17 bl.

Ants

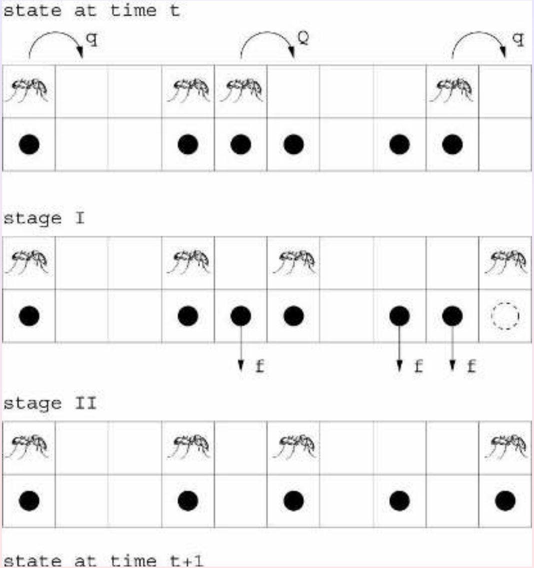


[John et al, PRL (2009)]

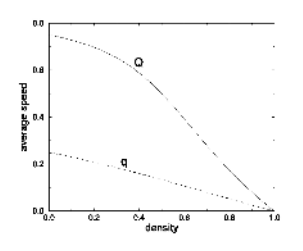
Average velocity almost independent of density.
No jammed phase.

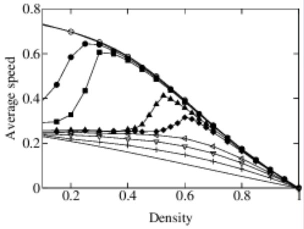
Ants

[Schadschneider et al, 2003]



Variation of speed and flux with density

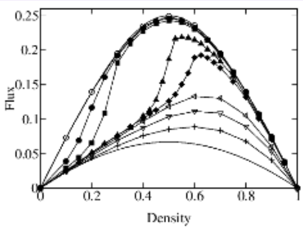


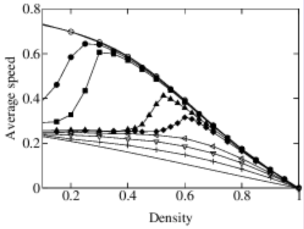


[Schadschneider et al (2003)]

cars ants

Variation of speed and flux with density



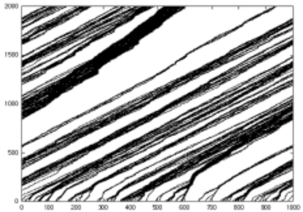


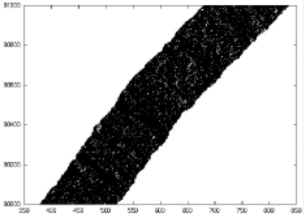
[Schadschneider et al (2003)]

ants

ants

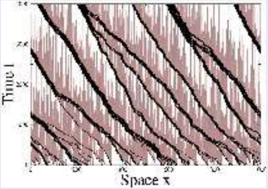
Clustering in ants model

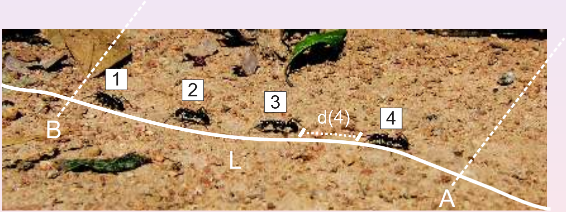




[Schadschneider et al (2003)]

Clustering in ants model

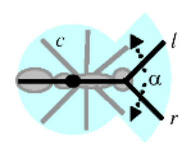


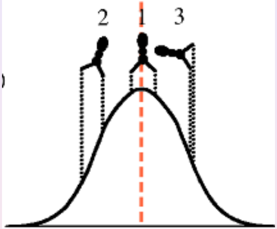


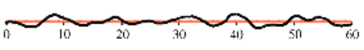
[John et al, TGF2007]

Platoons of ants (black) & traces of pheromone (purple grey)

Lane formation and optimized traffic flow in army ants

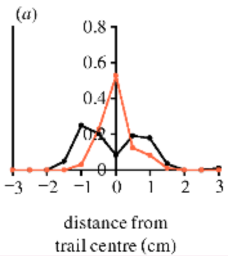


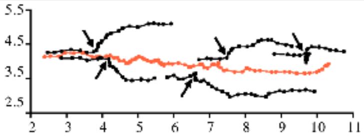


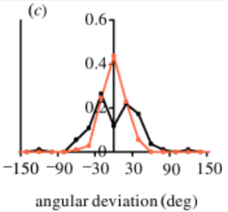


[Couzin et al (2003)]

Experimental observations



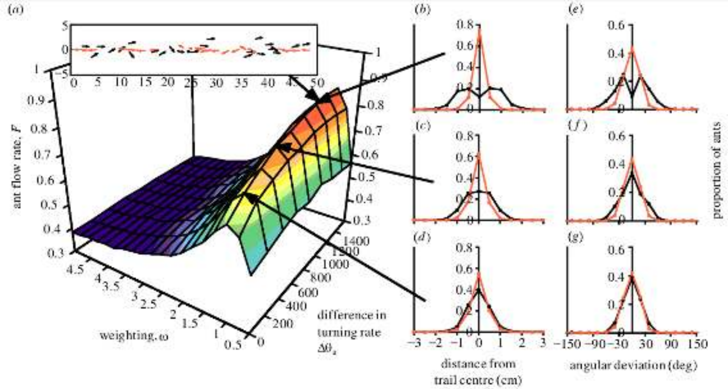




[Couzin et al (2003)]

- red: ants returning to the nest
- black: outbound foraging ants

Experimental observations



[Couzin et al (2003)]

- red: ants returning to the nest
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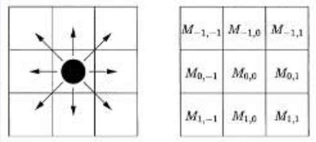
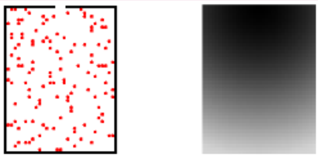
Floor field model



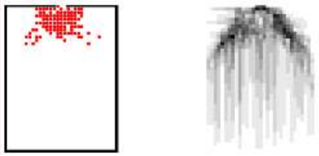
[C. Burstedde et al, Physica A **295** (2001) 507-525]

- Discrete model
- Inspired by the pheromones left by ants
- Dynamical field: When a pedestrian leaves a cell (i,j)

  the dynamic field is increased
  d(i,j) = d(i,j) + 1
  Diffusion of the field d
  Decay of d with a certain rate
- Static field s(i,j): path planning



Static floor field

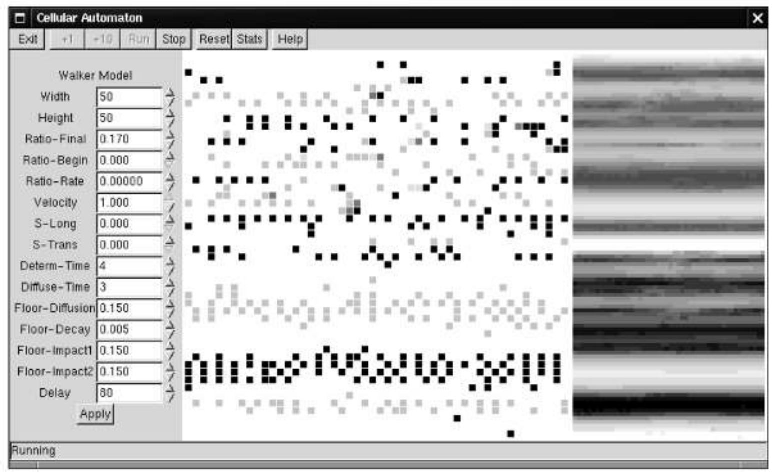


Dynamic floor field

[C. Burstedde et al, PED (2001)]

Floor field model

Bidirectional flow



[C. Burstedde et al, PED (2001)]

Floor field model

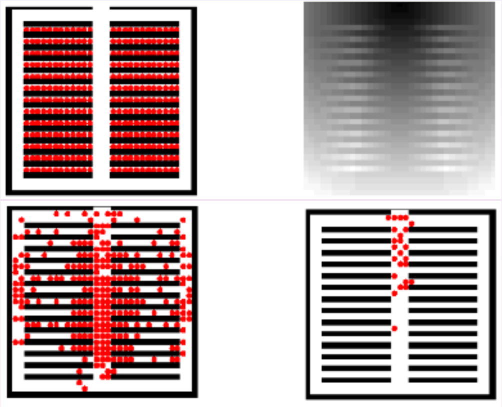
Bidirectional flow

[C. Burstedde et al, PED (2001)]

PBC; Random initial positions

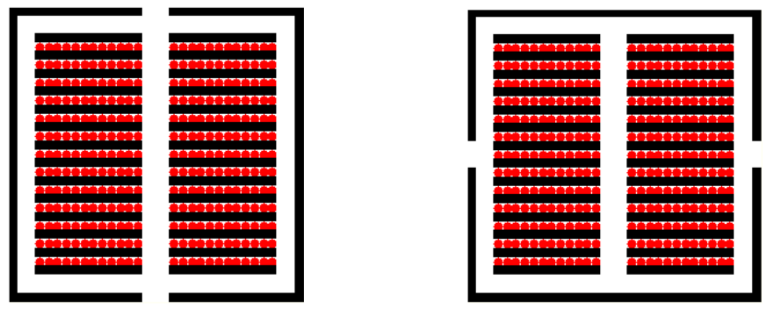
Floor field model

Static floor field



[C. Burstedde et al, PED (2001)]

Floor field model



[C. Burstedde et al, PED (2001)]

Comparing mean evacuation times T and corresponding variances σ

	hall A	hall B
T	560	363
σ	85	24

Cellular automata models for pedestrians

- Commercial software
  - Ex: PEDGO Software