1 Champs

$$\vec{E} = i\sum_{k,\lambda}\omega_k\vec{\epsilon}_{k,\lambda}\left(A_{k,\lambda}e^{i(\vec{k}.\vec{r}-\omega_k t)} - A^*_{k,\lambda}e^{-i(\vec{k}.\vec{r}-\omega_k t)}\right)$$

$$\vec{B} = i\sum_{k,\lambda}\vec{k}\times\vec{\epsilon}_{k,\lambda}\left(A_{k,\lambda}e^{i(\vec{k}.\vec{r}-\omega_k t)} - A^*_{k,\lambda}e^{-i(\vec{k}.\vec{r}-\omega_k t)}\right)$$

Quantification : $A_{k,\lambda}e^{-i\omega t} \equiv \alpha_{k,\lambda} + i\beta_{k,\lambda}$

2 Pour un oscillateur harmonique on a :

2.1 Opérateurs de quadratures

$$U_{k,\lambda} \equiv 2\sqrt{\frac{\epsilon_0 V\omega_k}{\hbar}}\alpha_{k,\lambda} \qquad V_{k,\lambda} \equiv 2\sqrt{\frac{\epsilon_0 V\omega_k}{\hbar}}\beta_{k,\lambda}$$

$$\hat{a}_{k,\lambda} = \frac{\hat{U}_{k,\lambda} + i\hat{V}_{k,\lambda}}{\sqrt{2}} \qquad \hat{a}^\dagger_{k,\lambda} = \frac{\hat{U}_{k,\lambda} - i\hat{V}_{k,\lambda}}{\sqrt{2}} \qquad [\hat{a}_{k,\lambda}, \hat{a}^\dagger_{k,\lambda}] = 1$$

$$\hat{H} = \sum_{k,\lambda}\hbar\omega_k(\hat{a}^\dagger_{k,\lambda}\hat{a}_{k,\lambda} + 1/2) \qquad \mathcal{E}_{k,\lambda} = \hbar\omega_k(n + 1/2)$$

2.2 Pour les Opérateurs champs :

$$A_{k,\lambda}e^{-i\omega t} \rightarrow \hat{A}_{k,\lambda} = \sqrt{\frac{\hbar}{2\epsilon_0 V\omega_k}}\hat{a}_{k,\lambda}(t)$$

$$\hat{\vec{A}}(\vec{r},t) = \sqrt{\frac{\hbar}{2\epsilon_0 V}}\sum_{k,\lambda}\frac{1}{\sqrt{\omega_k}}\vec{\epsilon}_{k,\lambda}\left(e^{i\vec{k}.\vec{r}}\hat{a}_{k,\lambda} + e^{-i\vec{k}.\vec{r}}\hat{a}^\dagger_{k,\lambda}\right)$$

$$\hat{\vec{E}}(\vec{r},t) = i\sqrt{\frac{\hbar}{2\epsilon_0 V}}\sum_{k,\lambda}\sqrt{\omega_k}\vec{\epsilon}_{k,\lambda}\left(e^{i\vec{k}.\vec{r}}\hat{a}_{k,\lambda} - e^{-i\vec{k}.\vec{r}}\hat{a}^\dagger_{k,\lambda}\right)$$

$$\hat{\vec{B}}(\vec{r},t) = i\sqrt{\frac{\hbar}{2\epsilon_0 V}}\sum_{k,\lambda}\frac{\vec{k}\times\vec{\epsilon}_{k,\lambda}}{\sqrt{\omega_k}}\left(e^{i\vec{k}.\vec{r}}\hat{a}_{k,\lambda} - e^{-i\vec{k}.\vec{r}}\hat{a}^\dagger_{k,\lambda}\right)$$

3 Etats Quantiques

Vide : $\langle 0|\hat{\vec{E}}|0\rangle = 0$ $\quad \langle 0|\hat{U}|0\rangle = \langle 0|\hat{V}|0\rangle = 0$ $\quad \langle 0|\hat{E}^2|0\rangle = \frac{\hbar\omega_k}{2\epsilon_0 V}$ $\quad \Delta U\Delta V = 1/2$

Etats de Fock : $\langle n|\hat{\vec{E}}|n\rangle = 0$ $\quad \langle n|\hat{E}^2|n\rangle = \frac{\hbar\omega_k}{2\epsilon_0 V}(2n+1)$ $\quad \Delta U\Delta V = n + 1/2$

Etat cohérent : $|\alpha\rangle = e^{-\frac{|\alpha|^2}{2}}\sum_n \frac{\alpha^n}{\sqrt{n!}}|n\rangle$ $\quad \alpha = |\alpha|e^{i\phi}$ $\quad \hat{a}|\alpha\rangle = \alpha|\alpha\rangle$

$$\langle U\rangle = \sqrt{2}|\alpha|\cos\phi \qquad \langle V\rangle = \sqrt{2}|\alpha|\sin\phi$$

4 Operateurs matriciels

Séparatrice : $\begin{pmatrix}\hat{a}_3\\ \hat{a}_4\end{pmatrix} = \begin{pmatrix}\sqrt{T}\,e^{i\epsilon} & i\sqrt{R}\,e^{-i\gamma}\\ i\sqrt{R}\,e^{i\gamma} & \sqrt{T}\,e^{i\epsilon}\end{pmatrix}\begin{pmatrix}\hat{a}_1\\ \hat{a}_2\end{pmatrix}$ si 50/50: $M = \frac{1}{\sqrt{2}}\begin{pmatrix}1 & i\\ i & 1\end{pmatrix}$ *ou* $\frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\ -1 & 1\end{pmatrix}$

Miroir : $M = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ ou $\begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}$

$\rightarrow$ **Avec perte** $\hat{a}' = \sqrt{\eta}\,\hat{a} + \sqrt{1-\eta} \;\Rightarrow\; \langle \hat{a}^{\dagger\prime}\hat{a}'\rangle = \eta\langle \hat{a}^\dagger \hat{a}\rangle \;\Rightarrow\; (\Delta N')^2 = (\Delta N)^2 + \eta(1-\eta)N$

4.1 Interaction lumiere-matiere

moment dipolaire : $\hat{\vec{d}} = \sum_{n,m}\langle\phi_n|\hat{\vec{d}}|\phi_m\rangle|\phi_n\rangle\langle\phi_m| \qquad \vec{d}_{n,n} = 0$

Semi-classique : $\hat{W} = -\hat{\vec{d}}.\vec{E} \qquad P_{0\to N}(t) = \left(\frac{d_{N,0}E_0}{2\hbar}\right)^2 sinc((\Omega_N - \Omega_0 - \omega)t/2)^2$

Si continuum (règle d'or) : $P \propto \rho(\mathcal{E} = \hbar(\Omega_0+\omega))d_{0,\hbar(\Omega_0+\omega)}t = \Gamma t$

Représentation Heinsenberg : $\hat{M}_H = e^{i\hat{H}t/\hbar}\,\hat{M}_S\,e^{-i\hat{H}t/\hbar}$

Représentation interaction : $|\Psi_I(t)\rangle = e^{i\hat{H}_0 t/\hbar}|\Psi_S(t)\rangle \qquad \hat{M}_I(t) = e^{i\hat{H}_0 t/\hbar}\,\hat{M}_S\,e^{-i\hat{H}_0 t/\hbar}$

$$\hat{H} = \hat{H}_0 + \hat{W} \;\Rightarrow\; i\hbar\frac{d|\Phi_I\rangle}{dt} = \hat{W}_I(t)|\Phi\rangle$$

$$\hat{E}_I = i\sum_{k,\lambda}\sqrt{\frac{\hbar\omega_k}{2\epsilon_0 V}}\vec{\epsilon}_{k,\lambda}(\hat{a}_{k,\lambda}e^{i(\vec{k}.\vec{r}-\omega_k t)} - \hat{a}^\dagger_{k,\lambda}e^{-i(\vec{k}.\vec{r}-\omega_k t)})$$

$$\hat{\vec{d}}_I = \sum_{n,m}\vec{d}_{n,m}e^{i(\Omega_n-\Omega_m)t}|\phi_n\rangle\langle\phi_m| = \hat{\vec{d}}_I = \sum_{n,m}\vec{d}_{n,m}e^{i(\Omega_n-\Omega_m)t}\hat{P}_{n,m}$$

$$\hat{W}_I = -\sum_{n,m}\sum_{k,\lambda}i\sqrt{\frac{\hbar\omega_k}{2\epsilon_0 V}}\vec{\epsilon}_{k,\lambda}(\hat{a}_{k,\lambda}e^{i(\vec{k}.\vec{r}-\omega_k t)} - \hat{a}^\dagger_{k,\lambda}e^{-i(\vec{k}.\vec{r}-\omega_k t)}).\vec{d}_{n,m}e^{i(\Omega_n-\Omega_m)t}|\phi_n\rangle\langle\phi_m|$$

4.2 approximation ondes tournantes

$n > m \Rightarrow \Omega_n > \Omega_m$ alors on ne garde que $(\Omega_n - \Omega_m - \omega_k)$ résonnants

$$\hat{W}_I = -\sum_{n>m}\sum_{k,\lambda}i\sqrt{\frac{\hbar\omega_k}{2\epsilon_0 V}}\vec{\epsilon}_{k,\lambda}.\left(\vec{d}_{n,m}\hat{P}_{n,m}\hat{a}_{k,\lambda}e^{i(\vec{k}.\vec{r}+(\Omega_n-\Omega_m-\omega_k)t)} - \vec{d}_{m,n}\hat{P}_{m,n}\hat{a}^\dagger_{k,\lambda}e^{-i(\vec{k}.\vec{r}+(\Omega_n-\Omega_m-\omega_k)t)}\right)$$

Probabilité de transition/détection : $|\Psi_0\rangle = |\phi_0\rangle\otimes|\chi_0\rangle \rightarrow |\Psi_F\rangle = |\phi_N\rangle\otimes|\chi_F\rangle$

$$P_{0\to F} = \frac{1}{\hbar^2}t^2 sinc^2((\Omega_N-\Omega_0-\omega_{k0})t/2)\frac{\hbar\omega_{k0}}{2\epsilon_0 V}|\vec{d}_{N,0}.\vec{k}_{k0}|^2|\langle\chi_F|\hat{a}_{k0,\lambda0}|\chi_0\rangle|^2$$

$$P_{\Psi_0\to F=\phi_N} = \frac{1}{\hbar^2}t^2 sinc^2((\Omega_N-\Omega_0-\omega_{k0})t/2)\frac{\hbar\omega_{k0}}{2\epsilon_0 V}|\vec{d}_{N,0}.\vec{k}_{k0}|^2\langle\chi_0|\hat{a}^\dagger_{k0,\lambda0}\hat{a}_{k0,\lambda0}|\chi_0\rangle$$

Probabilité de double detection : $|\Psi_0\rangle = |\phi_0^1\rangle\otimes|\phi_0^2\rangle\otimes|\chi_0\rangle \rightarrow |\Psi_F\rangle = |\phi_N^1\rangle\otimes|\phi_M^2\rangle\otimes|\chi_F\rangle$

$$P_{\Psi_0\to\phi_{N,M}} = \frac{1}{\hbar^4}t^4 sinc^2((\Omega_N-\Omega_0-\omega_{k0})\frac{t}{2})sinc^2((\Omega_M-\Omega_0-\omega_{k0})\frac{t}{2})\left(\frac{\hbar\omega_{k0}}{2\epsilon_0 V}\right)^2|\vec{d}_{N,0}.\vec{k}_{k0}|^2|\vec{d}_{M,0}.\vec{k}_{k0}|^2\langle\chi_0|\hat{a}^{\dagger 2}_{k0,\lambda0}\hat{a}^2_{k0,\lambda0}|\chi_0\rangle$$