Initiation à la matière quantique
Introduction

– Cours 6 –

Oscillateur harmonique revisité : 1D, 2D et électrons 2D dans un champ magnétique

Oscillateur harmonique quantique 1D

- Hamiltonien :

$$\hat{H} = \frac{\hat{p}^2}{2m} + \frac{1}{2}m\omega^2\hat{x}^2$$

- Opérateurs d'échelle : $\ell = \sqrt{\hbar/m\omega}$

$$\hat{a} = \frac{1}{\sqrt{2}}\left(\frac{\hat{x}}{\ell} + i\frac{\ell}{\hbar}\hat{p}\right) \qquad \hat{a}^\dagger = \frac{1}{\sqrt{2}}\left(\frac{\hat{x}}{\ell} - i\frac{\ell}{\hbar}\hat{p}\right), \qquad [\hat{a}, \hat{a}^\dagger] = 1$$

$$\hat{a}|n\rangle = \sqrt{n}|n-1\rangle \qquad \hat{a}^\dagger|n\rangle = \sqrt{n+1}|n+1\rangle$$

- Hamiltonien :

$$\hat{H} = \hbar\omega(\hat{N} + 1/2) \quad \text{où} \quad \hat{N} = \hat{a}^\dagger\hat{a}, \qquad \hat{N}|n\rangle = n|n\rangle$$

- Spectre d'énergie :

$$E_n = \hbar\omega(n + 1/2), \qquad n = 0, 1, 2, ...$$

Fonctions d'onde

Traduire condition d'état de plus basse énergie en équation différentielle :

$$\hat{a}|n=0\rangle = 0 \qquad \rightarrow \qquad \frac{1}{\sqrt{2}}\left(\frac{x}{\ell}+\ell\frac{\partial}{\partial x}\right)\phi_{n=0}(x)=0$$

Solution gaussienne :

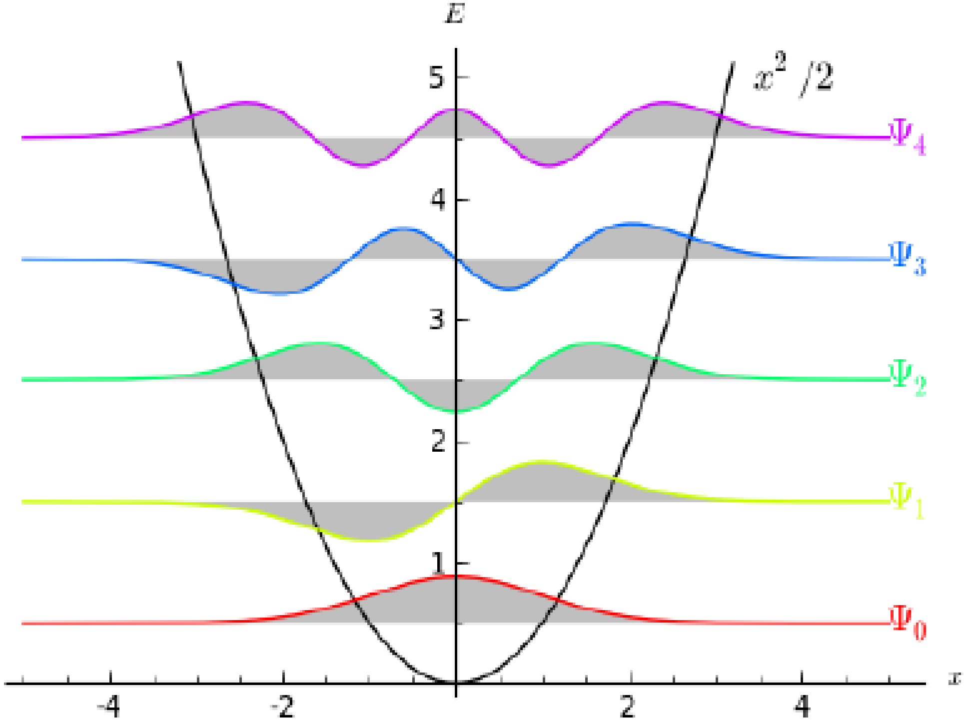
$$\phi_{n=0}(x)=\frac{1}{\sqrt{\sqrt{\pi}\ell}}e^{-x^2/2\ell^2}$$

Remarques :

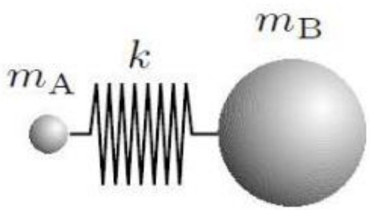
- solution unique (car équation différentielle de premier ordre)
- tous les états sont non dégénérés par conséquent :

$$|n\rangle=\frac{(\hat{a}^\dagger)^n}{\sqrt{n!}}|n=0\rangle \qquad \rightarrow \qquad \phi_n(x)=\frac{1}{\sqrt{\sqrt{\pi}2^n n!\ell}}\left(\frac{x}{\ell}-\ell\frac{\partial}{\partial x}\right)^n e^{-x^2/2\ell^2}$$

Spectre de l'oscillateur harmonique



La spectroscopie Infrarouge



Molécule AB

$$\bar{\nu} = \frac{1}{2\pi c}\sqrt{\frac{k}{\mu}}$$

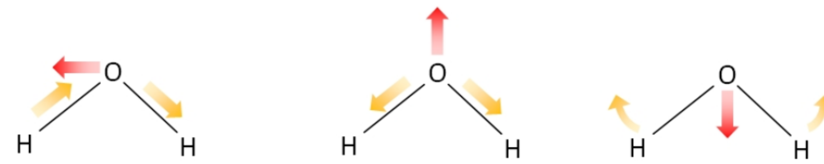
$$\mu = \frac{m_1 \cdot m_2}{m_1 + m_2}$$

$\nu_{C=C}$ en cm^{-1}

k en dynes.cm^{-1} (1N = 10^5 dynes)

μ en g (μ/ 6.023 10^{23})

$C = 3.10^{10}$ cm.s−1



Elongation antisymétrique

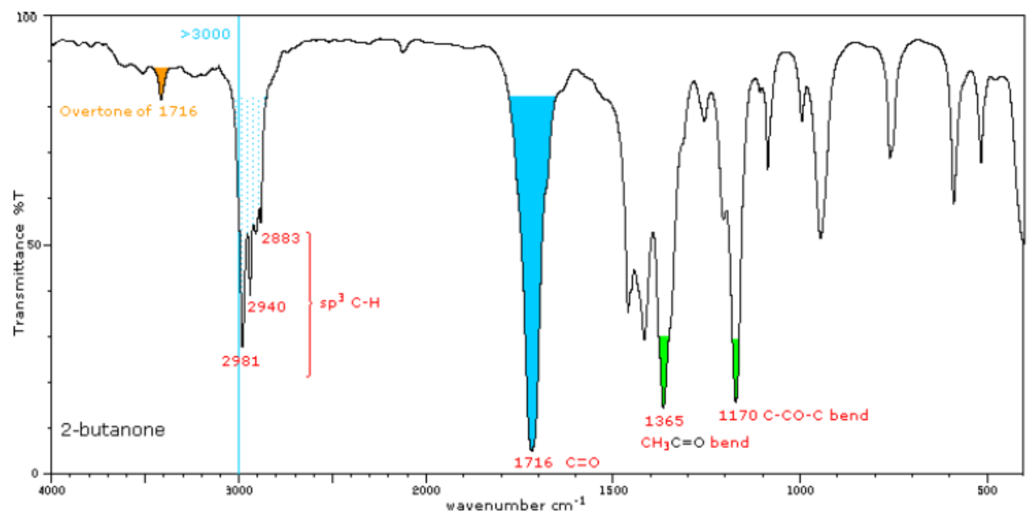
σ = 3756 cm^{-1} ; λ = 2,66 µm

Elongation symétrique

σ = 3652 cm^{-1} ; λ = 2,74 µm

Déformation angulaire

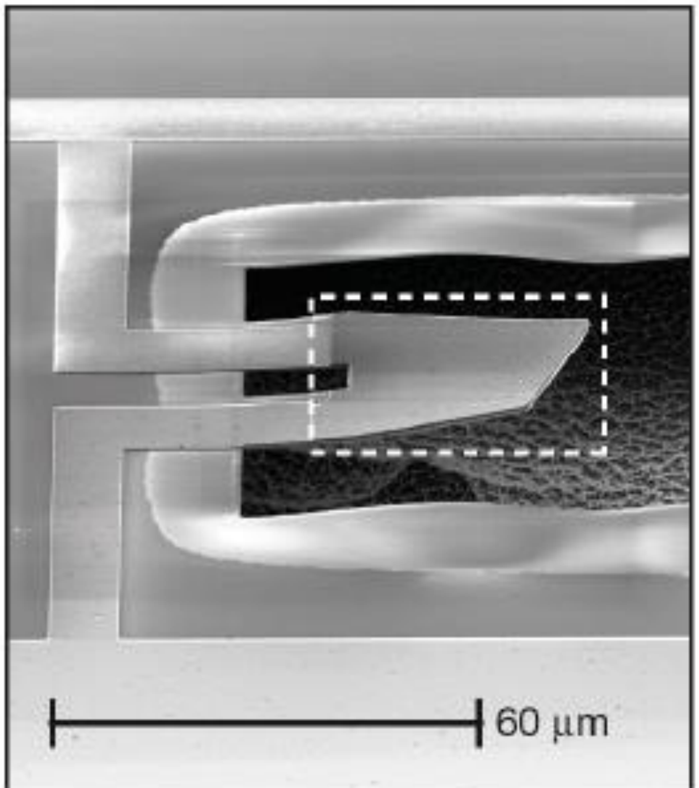
σ = 1595cm^{-1} ; λ = 6,27 µm



Refroidissement d'un résonateur mécanique

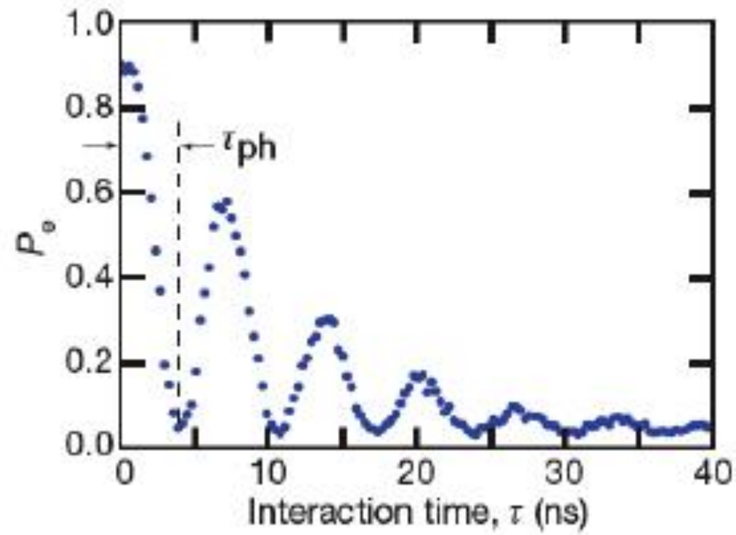
Quantum ground state and single-phonon control of a mechanical resonator

A. D. O'Connell[1], M. Hofheinz[1], M. Ansmann[1], Radoslaw C. Bialczak[1], M. Lenander[1], Erik Lucero[1], M. Neeley[1], D. Sank[1], H. Wang[1], M. Weides[1], J. Wenner[1], John M. Martinis[1] & A. N. Cleland[1]



Fréquence de vibration = 6 GHz

Oscillation n = 0 - n =1



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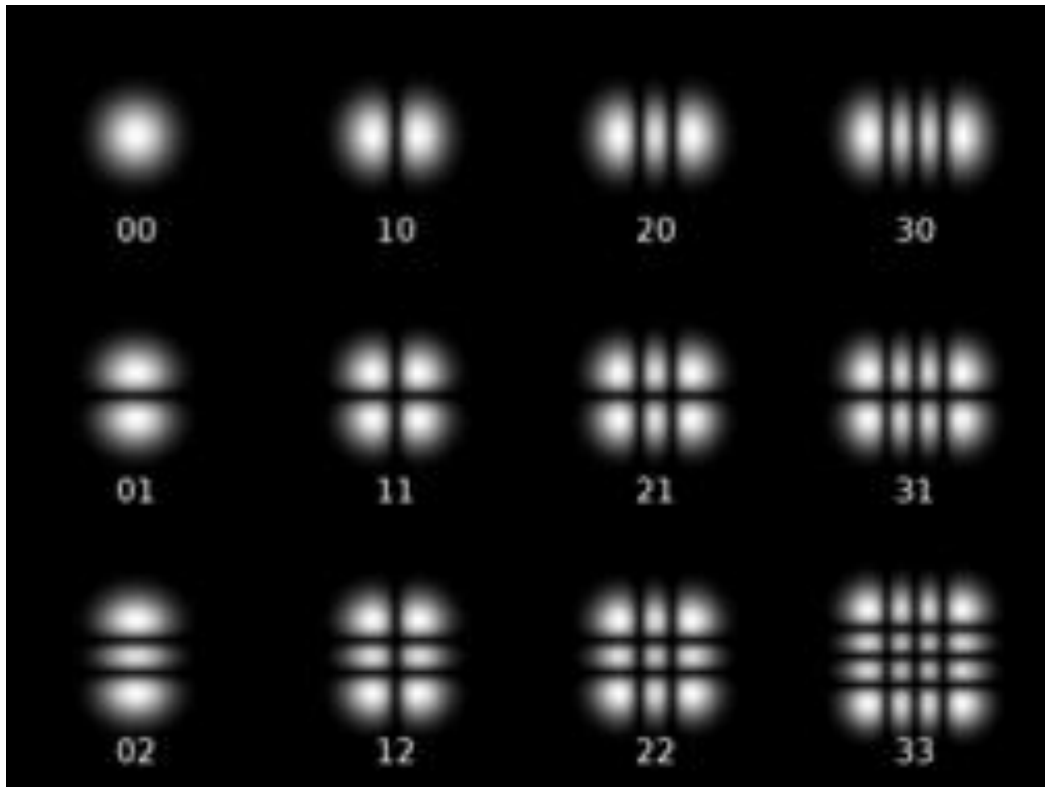
Oscillateur bidimensionnel



Oscillateur bidimensionnel

$$\hat{a}_g = \frac{1}{\sqrt{2}}\left(\hat{a}_x + i\hat{a}_y\right) \qquad \hat{a}_g^\dagger = \frac{1}{\sqrt{2}}\left(\hat{a}_x^\dagger - i\hat{a}_y^\dagger\right)$$

$$\hat{a}_d = \frac{1}{\sqrt{2}}\left(\hat{a}_x - i\hat{a}_y\right) \qquad \hat{a}_d^\dagger = \frac{1}{\sqrt{2}}\left(\hat{a}_x^\dagger + i\hat{a}_y^\dagger\right).$$

- États

$$|n_d, 0\rangle \propto (\hat{a}_x^\dagger + i\hat{a}_y^\dagger)^{n_d}|0,0\rangle \qquad |0, n_g\rangle \propto (\hat{a}_x^\dagger - i\hat{a}_y^\dagger)^{n_d}|0,0\rangle$$

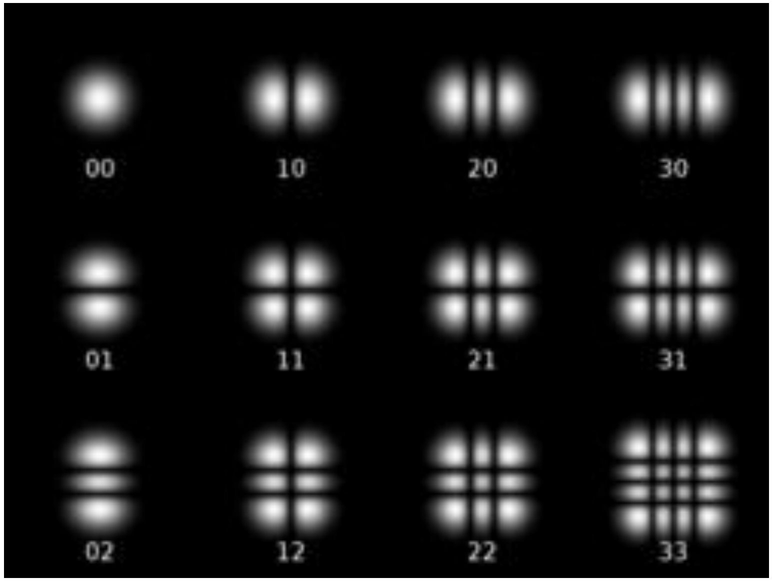
$$\phi_{n_d,0}(z,\bar{z}) \propto \left(\frac{z}{\ell} - 2\ell\partial_{\bar{z}}\right)\phi_{0,0}(z,\bar{z}) \qquad \phi_{0,n_g}(z,\bar{z}) \propto \left(\frac{\bar{z}}{\ell} - 2\ell\partial_z\right)\phi_{0,0}(z,\bar{z})$$

- Fonctions d'onde :

$$\phi_{n_d,0}(z,\bar{z}) \propto z^{n_d} e^{-\frac{z\bar{z}}{2\ell^2}} = r^{n_d} e^{in_d\phi} e^{-r^2/2\ell^2}$$

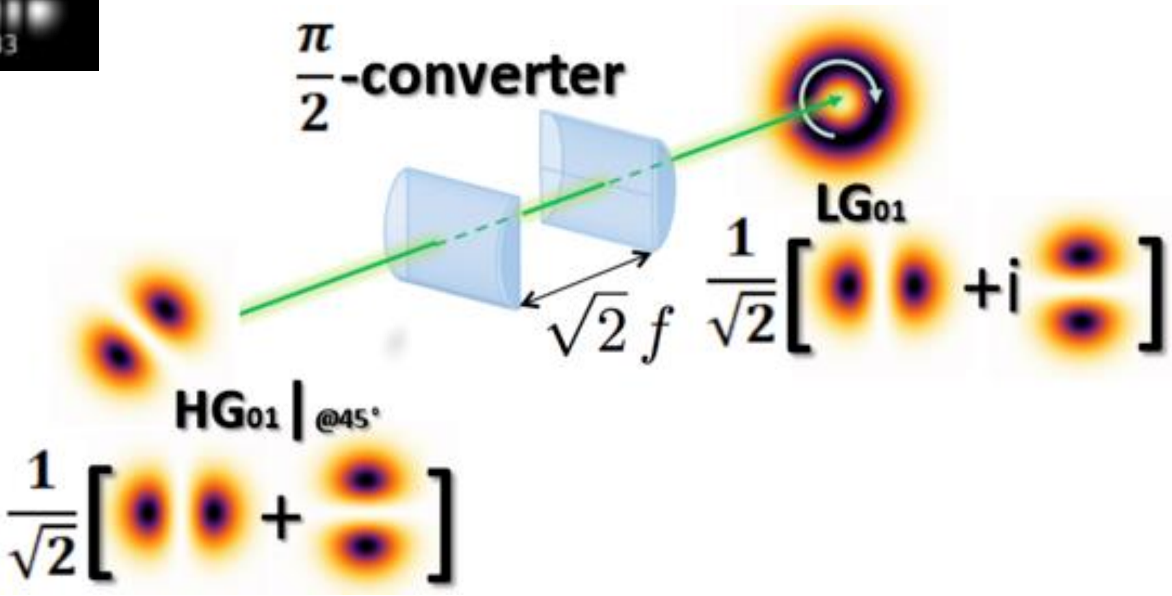
$$\phi_{0,n_g}(z,\bar{z}) \propto \bar{z}^{n_g} e^{-\frac{z\bar{z}}{2\ell^2}} = r^{n_g} e^{-in_g\phi} e^{-r^2/2\ell^2}$$

De la lumière avec un moment angulaire



Modes de Hermite Gauss

Modes de Laguerre Gauss



Pinces optiques : https://www.youtube.com/watch?v=-HOWmGQhTu8

Vortex Formation in a Stirred Bose-Einstein Condensate

K. W. Madison, F. Chevy, W. Wohlleben,* and J. Dalibard

Laboratoire Kastler Brossel,† Département de Physique de l'Ecole Normale Supérieure, 24 rue Lhomond, 75005 Paris, France

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Using a focused laser beam we stir a Bose-Einstein condensate of ^{87}Rb confined in a magnetic trap and observe the formation of a vortex for a stirring frequency exceeding a critical value. At larger rotation frequencies we produce states of the condensate for which up to four vortices are simultaneously present. We have also measured the lifetime of the single vortex state after turning off the stirring laser beam.

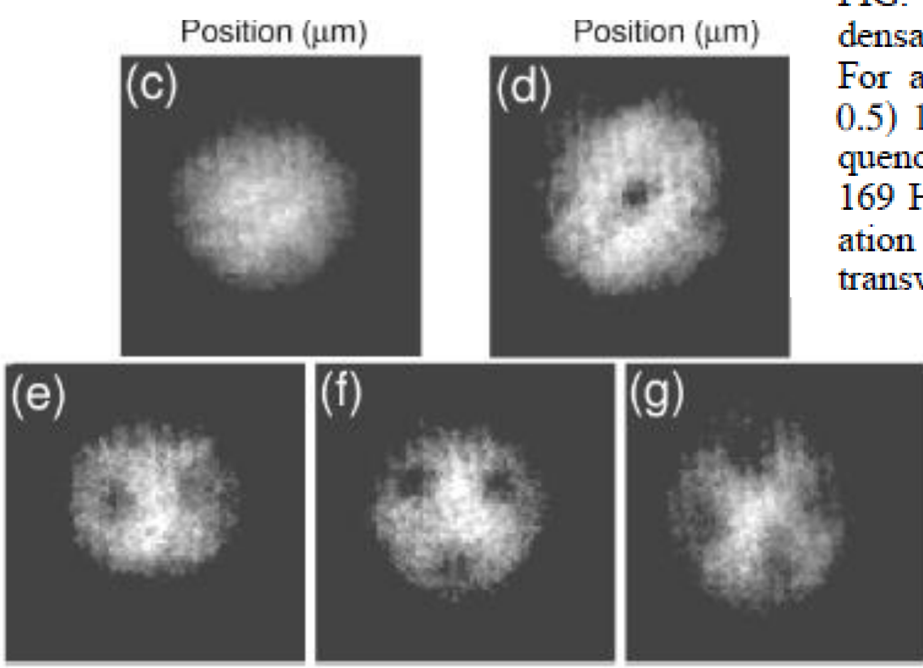

FIG. 1. Transverse absorption images of a Bose-Einstein condensate stirred with a laser beam (after a 27 ms time of flight). For all five images, the condensate number is $N_0 = (1.4 \pm 0.5)\ 10^5$ and the temperature is below 80 nK. The rotation frequency $\Omega/(2\pi)$ is, respectively, (c) 145 Hz, (d) 152 Hz, (e) 169 Hz, (f) 163 Hz, (g) 168 Hz. In (a) and (b) we plot the variation of the optical thickness of the cloud along the horizontal transverse axis for the images (c) (0 vortex) and (d) (1 vortex).