MCMC method 1:

Note that MCMC method 1 is similar to the one Nuñez-Antonio et.al. (2011) use. In a MH step use the following proposal distribution:

$$p(r_i) = N(m_{0i}, v_{0i})$$

where $m_{0i} = \log\left(\frac{1}{2}\left[b_i + (b_i^2 + 8)^{\frac{1}{2}}\right]\right)$ and $v_{0i} = (2 + \exp(2m_{0i}))^{-1}$. Define a value, $y_0$, as the (natural) logarithm of the current value of $r_i$ , $y_0 = \log(r_i)$. Next, complete the following steps 5 times:

- Sample a second value $y_1$ from the proposal distribution.
- Compute the kernel of $f(\ln(r_i)\,|\theta_i)$ for $y_1$:
    - $f(y_1) = 2y_1 - 0.5\exp(y_1)\,(\exp(y_1) - 2b_i)$.
- Compute the natural logarithm of the density of the proposal distribution for $y_1$.
- Compute the difference between $\log\big(f(y_1)\big)$ and $\log\big(p(y_1)\big)$:
    - $w_1 = \log\big(f(y_1)\big) - \log\big(p(y_1)\big)$.
- Repeat the previous three steps for $y_0$.
- Compute the acceptance ratio:
    - $\alpha = w_1 - w_0$,

  where $w_0 = \log\big(f(y_0)\big) - \log\big(p(y_0)\big)$.
- Sample a value $k$ from a uniform distribution:
    - $k \sim U(0,1)$.
- If $\alpha \geq \log(k)$ we accept $y_1$, so $y_0 = y_1$. Else, we do not accept $y_1$ and $y_0 = y_0$.

After five iterations $r_i = \exp(y_0)$.

MCMC method 2:

In a slice sampler the joint density for an auxiliary variable $v_i$ with $r_i$ for regression is:

- $p(r_i, v_i \mid \theta_i, \boldsymbol{\mu}_i = \boldsymbol{B}^t \boldsymbol{x}_i) \propto r_i \boldsymbol{I}(0 < v_i < \exp\{-.5(r_i - b_i)^2\}) \boldsymbol{I}(r_i > 0)$

The full conditionals for $v_i$ and $r_i$ are:

- $p(v_i \mid r_i = r_i, \boldsymbol{\mu}_i, \theta_i) \sim U(0, \exp\{-.5(r_i - b_i)^2\})$
- $p(r_i \mid v_i = v_i, \boldsymbol{\mu}_i, \theta_i) \sim r_i \boldsymbol{I}(b_i + \max\{-b_i, -\sqrt{-2 \ln v_i}\} < v_i < b_i + \sqrt{-2 \ln v_i})$

We thus sample $v_i$ from the uniform distribution specified above. Independently we sample a value $m$ from $U(0,1)$. We obtain a new value for $r_i$ by computing $r_i = \sqrt{(r_{i2}^2 - r_{i1}^2)m + r_{i1}^2}$ , where $r_{i1} = b_i + \max\{-b_i, -\sqrt{-2 \ln v_i}\}$ and $r_{i2} = b_i + \sqrt{-2 \ln v_i}$).