1 Regression models for Cylindrical data in Psychology

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13 Abstract

14 Cylindrical data are multivariate data which consist of a directional, in this paper circular,

15 and a linear component. Examples of cylindrical data in psychology include human

16 navigation (direction and distance of movement), eye-tracking research (direction and length

17 of saccades) and data from an interpersonal circumplex (type and strength of interpersonal

18 behavior). In this paper we adapt four models for cylindrical data to include a regression of

19 the circular and linear component onto a set of covariates. Subsequently, we illustrate how to

20 fit these models and interpret their results on a dataset on the interpersonal behavior of

21 teachers.

22 *Keywords:* cylindrical data, regression, interpersonal behavior

23 Word count: X

24 Regression models for Cylindrical data in Psychology

# 25 Introduction

26 In psychology there exist several types of data in which both a linear and a circular

27 outcome variable is measured. For example, in research on human navigation in the field of

28 cognitive psychology both the distance, a linear variable, and the direction, a circular

29 variable, of movement are of interest (Chrastil & Warren, 2017). In eye-tracking research,

30 saccade data are an example of cylindrical data, because both the direction (*i.e.*, the circular

31 variable) and the duration (*i.e.*, the linear variable) of the saccades are of interest (for a

32 review of eye-tracking research see Rayner (2009)). Apart from in psychology data with a

33 circular and linear outcome more commonly occur in meteorology (García-Portugués,

34 Crujeiras, & González-Manteiga, 2013), ecology (García-Portugués, Barros, Crujeiras,

35 González-Manteiga, & Pereira, 2014) or marine research (Lagona, Picone, Maruotti, &

36 Cosoli, 2015). The type of data that is used in the present study also contain a linear and a

37 circular outcome variable, namely data from circumplex measurement instruments. This

38 type of data is used in personality psychology (see the Teacher Data section for a more

39 detailed explanation).

40 Data that consist of a variable and a circular variable is called cylindrical data. A

41 circular variable is different from the linear variable in the sense that it is measured on a

42 different scale. Figure [1](#_bookmark14) shows the difference between a circular scale (right) and a linear

43 scale (left). The most important difference is that on a circular scale the datapoints 0*◦* and

44 360*◦* are connected and in fact represent the same number while on a linear scale the two

45 ends, −∞ and ∞, are not connected and consequently the values 0*◦* and 360*◦* are located on

46 different places on the scale. Both circular data and cylindrical data require special analysis

47 methods due to this periodicity in the scale of a circular variable (see e.g. Fisher (1995) for

48 an introduction to circular data and Mardia and Jupp (2000), Jammalamadaka and

49 Sengupta (2001) and Ley and Verdebout (2017) for a more elaborate overview).

50 In the literature, several methods have been put forward to model the relation between

51 the linear and circular component of a cylindrical variable. Some of these are based on

52 regressing the linear component onto the circular component using the following type of

53 relation:

*y* = *β*0 + *β*1 ∗ cos(*θ*) + *β*2 ∗ sin(*θ*) + *E,*

54 where *y* is the linear component and *θ* the circular component (Johnson & Wehrly, 1978;

55 Mardia & Sutton, 1978; Mastrantonio, Maruotti, & Jona-Lasinio, 2015). Others model the

56 relation in a different way, e.g. by specifying a multivariate model for several linear and

57 circular variables and modelling their covariance matrix (Mastrantonio, 2018) or by

58 proposing a joint cylindrical distribution. For example, Abe and Ley (2017) introduce a

59 cylindrical distribution based on a Weibull distribution for the linear component and a

60 sine-skewed von Mises distribution for the circular component and link these through their

61 respective shape and concentration parameters. However, none of the methods that have

62 been proposed thus far include additional covariates onto which both the circular and linear

63 component are regressed.

64 Our aim in this paper is twofold. Firstly, we intend to fill the gap in the literature on

65 cylindrical models by adapting four existing cylindrical models in such a way that they

66 include a regression of both the linear and circular component of a cylindrical variable onto a

67 set of covariates. From now on we will therefore refer to the components of the cylindrical

68 variable as outcome components. Secondly, we will show how using these models can benefit

69 the analysis of circumplex data and cylindrical data in psychology in general. More

70 specifically we will show these benefits for the teacher data, a dataset from the field of

71 educational psychology. In the teacher data, apart from modelling the dependence between

72 the linear and circular component of a cylindrical variable we would also like to predict the

73 two components from a set of covariates in a regression model. In our analysis we show how

74 this can be done in four different ways using the adapted cylindrical models.

75 The paper is organized as follows: First we describe the teacher data and outline why a

76 cylindrical analysis of these data is beneficial. Subsequently we present the four cylindrical

77 models and our associated adaptations to new regression models. We also discuss the model

78 fit criterion that we will use for the comparison of the four models. A detailed analysis of the

79 teacher data is also provided. We conclude the paper with a discussion, and the

80 Supplementary Material collects the technical details of the cylindrical models and MCMC

81 procedures to fit them.

# 82 Teacher Data

83 The motivating example for this article comes from the field of educational psychology

84 and was collected for the studies on classroom climate of Van der Want (2015), Claessens

85 (2016) and Pennings et al. (2018). An indicator of the quality of the classroom climate is the

86 students’ perception of their teachers’ interpersonal behavior. These interpersonal

87 perceptions, both in educational psychology as well as in other areas of psychology, can be

88 measured using circumplex measurement instruments (see Horowitz and Strack (2011) for an

89 overview of many such instruments).

90 The circumplex data used in this paper are measured using the Questionnaire on

91 Teacher Interaction (QTI) (Wubbels, Brekelmans, Brok, & Tartwijk, 2006) which is one such

92 circumplex measurement instrument. The QTI is designed to measure student perceptions of

93 their teachers’ interpersonal behavior and contains items that load on two interpersonal

94 dimensions: Agency and Communion. Agency refers to the degree of power or control a

95 teacher exerts in interaction with his/her students. Communion refers to the degree of

96 friendliness or affiliation a teacher conveys in interaction with his/her students. The loadings

97 on the two dimensions of the QTI can be placed in a two-dimensional space formed by

98 Agency (vertical) and Communion (horizontal), see Figure [2.](#_bookmark15) Different parts of this space

99 are characterized by different teacher behavior, e.g. “helpful” or “uncertain”. This

100 two-dimensional space is called the interpersonal circle/circumplex (IPC). The IPC is “a

101 continuous order with no beginning or end” (Gurtman, 2009, p. 2). We call such ordering a

102 circumplex ordering and the IPC is therefore often called the interpersonal circumplex. The

103 ordering also implies that scores on the IPC could be viewed as a circular variable. This

104 circular variable represents the type of interpersonal behavior that a teacher shows towards

105 his/her students.

106 Cremers et al. (2018a) explain the circular nature of the IPC data and analyze them as

107 such using a circular regression model. The two dimension scores, Agency and Communion,

108 can be converted to a circular score using the two-argument arctangent function in [(1)](#_bookmark0)1,

109 where *A* represents a score on the Agency dimension and *C* represents a score on the

110 Communion dimension. Note that when placing a unit circle on Figure [2](#_bookmark15) we see that the

111 Agency dimension is related to the sine of the circular score and the Communion dimension

112 is related to the cosine of the circular score.

 arctan ( *A* \ if *C >* 0

*C*



 arctan ( *A* \ + *π* if *C <* 0 & *A* ≥ 0

*C*



) =  arctan ( *A* \ − *π* if *C <* 0 & *A <* 0

*θ*

= atan2 (*A, C*

*C*

(1)

*C*

+*π*

2

 − *π*

 2



if *C* = 0 & *A >* 0 if *C* = 0 & *A <* 0

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 undefined if *C* = 0 & *A* = 0*.*

The resulting circular variable *θ* can then be modelled and takes values in the interval [0*,* 2*π*) or [0*◦,* 360*◦*). Note that the round brackets mean that 2*π* and 360*◦* are not included in the interval since these represent the same value as 0 as a result of periodicity.

A circular analysis of circumplex data has several benefits: it is more in line with its theoretical definition and it allows us to analyse the blend of the two dimensions Agency and

1 The selection of the origin in circumplex data depends on the scaling of the Agency and Communion scores; their respective 0 scores form the origin. The scaling of Agency and Communion is theoretically determined from AANVULLEN, waarvan. Scaling is however only considered an issue in those instances where cylindrical data is derived from measurements in bivariate space.

118 Communion instead of both dimensions separately providing us with new insights compared

119 to a separate analysis of the two dimensions that is standard in the literature (see e.g. HIER

120 REFERENTIES INVOEGEN). There is however one main drawback: when two-dimensional

121 data are converted to the circle we lose some information, namely the length of the

122 two-dimensional vector (*A, C*)*t*, *i.e.*, its Euclidean norm || (*A, C*)*t* ||. This length represents

123 the strength of the interpersonal behavior a teacher shows towards his/her students. In a

124 cylindrical model this strength (the linear outcome) can be modeled together with the type

125 of interpersonal behavior of a teacher (the circular outcome). This leads to an improved

126 analysis of interpersonal circumplex data, over either analyzing the two dimensions

127 separately or using a circular model, because we take all information, circular and linear,

128 into account. In the next section we introduce several cylindrical models that can be used to

129 analyze the teacher data. First however we will provide descriptives for the teacher data.

# 130 Data Description

131 The teacher data was collected between 2010 and 2015 and contains several repeated

132 measures on the IPC of 161 teachers. Measurements were obtained using the QTI and taken

133 in different years and classes. For this paper we only consider one measurement, the first

134 occasion (2010) and largest class if data for multiple classes were available. This results in a

135 sample of 151 teachers. In addition to the type of interpersonal behavior (IPC), the circular

136 outcome, and the strength of interpersonal behavior (IPC strength), the linear outcome, a

137 teachers’ self-efficacy (SE) concerning classroom management is used as covariate in the

138 analysis. HIER EEN STUK OVER THEORETISCHE RELEVANTIE SE, wellicht ook iets

139 over relatie tussen type and strength of interpersonal behavior? After listwise deletion of

140 missings (3 in total, only for the self-efficacy) we have a sample of 148 teachers. Table [1](#_bookmark7)

141 shows descriptives for the dataset. For the circular variable IPC we show sample estimates

142 for the circular mean *θ*¯ and concentration *ρ*ˆ. The circular concentration lies between 0,

143 meaning the data is not concentrated at all *i.e.* spread over the entire circle, and 1, meaning

144 all data is concentrated at a single point on the circle The population values of these

145 parameters are usually, and also in this paper, referred to as *µ* (circular location) and *κ*

146 (circular concentration). For the linear variables (strength IPC and SE) we show sample

147 estimates of the linear mean and standard deviation. Figure [3](#_bookmark16) is a scatterplot showing the

148 relation between the linear and circular outcome of the teacher data for teachers with low SE

149 (below 1 sd below the mean), average SE (between 1 sd below and 1 sd above the mean) and

150 high SE (above 1 sd above the mean).

# 151 Four Cylindrical Regression Models

152 One of the goals of this paper is to show the benefits of cylindrical methods for the

153 analysis of circumplex data and cylindrical data in psychology in general. To do so we decide

154 to focus on four cylindrical models. The models were selected for their relatively low

155 complexity and the ease with which a regression structure could be incorporated. But also

156 because they show different ways of modelling the the linear and circular outcome and

157 thereby illustrate a wider range of cylindrical models available in the literature. All four

158 cylindrical models contain a set of *q* predictors ***x*** = *x*1*, . . . , xq* and *p* predictors ***z*** = *z*1*, . . . , zq*

159 for the linear and circular outcomes, *Y* and Θ, respectively. The first two models are based

160 on a construction by Mastrantonio et al. (2015), while the other models are extensions of the

161 models from Abe and Ley (2017) and Mastrantonio (2018). The four cylindrical models are

162 introduced separately in the subsections below. However, to provide a more succinct

163 overview and comparison of the four models, [2](#_bookmark8) gives an overview of the similarities and

164 differences between the models.

# 165 The Modified Circular-Linear Projected Normal (CL-PN) and Modified

166 **Circular-Linear General Projected Normal (CL-GPN) Models**

167 Following Mastrantonio et al. (2015) we consider two models where the relation

168 between Θ ∈ [0*,* 2*π*) and *Y* ∈ (−∞*,* +∞) and *q* covariates is specified as

*Y* = *γ*0 + *γcos* ∗ cos(Θ) ∗ *R* + *γsin* ∗ sin(Θ) ∗ *R* + *γ*1 ∗ *x*1 + · · · + *γq* ∗ *xq* + *E,* (2)

169 where the random variable *R* ≥ 0 will be introduced below, the error term *E* ∼ *N* (0*, σ*2) with

170 variance *σ*2 *>* 0, *γ*0*, γcos, γsin, γ*1*, . . . , γq* are the intercept and regression coefficients and

171 *x*1*, . . . , xq* are the *q* covariates for the prediction of the linear outcome. We thus assume a

172 normal distribution for the linear outcome.

173 For the circular outcome we assume either a projected normal (PN) or a general

174 projected normal (GPN) distribution. These distributions arise from a projection of a

175 distribution defined in bivariate space onto the circle. Figure [4](#_bookmark17) represents this projection. In

176 the left plot of Figure [4](#_bookmark17) we see datapoints from the bivariate variable ***S*** that in the middle

177 plot are projected to form the circular outcome Θ in the right plot. Mathematically the

178 relation between ***S*** and Θ is defined as follows

*R****u***



*R* sin(Θ)

 *,*

 *SI*  =



***S*** =

*SII*



= *R* cos(Θ)

(3)

179 where *R* =|| ***S*** ||, the Euclidean norm of ***S***; the lines connecting the bivariate datapoints to

180 the origin in the middle plot. We call ***S*** the augmented representation of the circular

181 outcome, it is a variable that in contrast to Θ is not observed and thus considered latent or

182 auxiliary. This then means that we do not model Θ directly but indirectly through ***S***.

183 For both the PN and GPN distribution the circular location parameter *µ* ∈ [0*,* 2*π*) is

184 modeled as *µ*ˆ*i* = atan2(*µ*ˆ*II , µ*ˆ*I* ) = atan2(***β****II****z****i,* ***β****I****z****i*)[2](#_bookmark3) where ***β****I* and ***β****II* are vectors with

*i i*

2 Note that for the CL-GPN model the circular location parameter also depends on the variance-covariance matrix and the circular predicted values should be computed using numerical integretion or Monte Carlo methods because a closed form expression for the mean direction is not available.

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intercepts and regression coefficients for the prediction of ***S*** and ***z****i* is a vector with predictor values for each individual *i* ∈ 1*, . . . , n* where *n* is the sample size. Note that as a result of the augmented representation of the circular outcome we have two sets of regression coefficients and intercepts, one for each bivariate component of ***S***. This leads to problems when we want to interpret the effect of a covariate on the circle. A circular regression line is shown in Figure [5,](#_bookmark18) with covariate values on the x-axis and the predicted circular outcome on the

y-axis. As can be seen it is of a non-linear character meaning that the effect of a covariate is different at different values of the covariate. A circular regression line is usually described by the slope at the inflection point, the point at which the slope of the regression line starts flattening off (indicated with a square in Figure [5).](#_bookmark18) By default, the parameters from the PN and GPN model do not directly describe this inflection point. For the PN distribution however, Cremers et al. (2018b) solved this interpretation problem and introduce new circular regression coefficients. They introduce a new parameter *bc* that describes the slope at the inflection point of the regression line. For the GPN distrbution the interpretation problem however remains.

The main difference between the PN and GPN distribution lies in the definition of their covariance matrix. For the PN distribution this is an identity matrix, causing the

distribution to be unimodal and symmetric, whereas for the GPN distribution

*τ* 2 + *ξ*2 *ξ*



203

**Σ** = 

, allowing for multimodality and assymetry/skewness.

*ξ* 1

204 For the teacher dataset the predicted linear outcome, strengths of interpersonal

205 behavior, in the CL-PN and CL-GPN model is the following:

*y*ˆ*i* = *γ*0 + *γcos* cos(*θi*)*ri* + *γsin* sin(*θi*)*ri* + *γ*1SE*i.*

206 The predicted circular outcome, type of interpersonal behavior, equals:

*θ*ˆ*i* = *µi* = atan2(*βII* + *βII*SE*i, βI* + *βI*SE*i*)*.*

0 1 0 1

207 where SE*i* is the self-efficacy score of one individual. The CL-PN and CL-GPN models thus

208 allow us to assess the average type and strength of interpersonal behavior through the

209 parameters *βI*, *βII* and *γ*0 as well as the effect of self-efficacy on type and strength of teacher

0 0

210 behavior through the parameters, *βI*, *βII* and *γ*1. In addition, because the type and strength

1 1

211 of interpersonal behavior are modelled together via the regression in [(2)](#_bookmark2) we can assess the

212 effect of the type of interpersonal behavior on the strength through the parameters *γsin* and

213 *γcos*. In the teacher data these are the regression coefficients for the effect of the sine and

214 cosine of the type of behavior which are related to the scores on the Agency an Communion

215 dimensions respectively.

216 Both the CL-PN and CL-GPN models are estimated using Markov Chain Monte Carlo

217 (MCMC) methods based on Nuñez-Antonio, Gutiérrez-Peña, and Escarela (2011), Wang and

218 Gelfand (2013) and Hernandez-Stumpfhauser, Breidt, and Woerd (2016) for the regression of

219 the circular outcome. A detailed description of the Bayesian estimation and MCMC

220 samplers can be found in the Supplementary Material.

# 221 The Modified Abe-Ley Model

222 This model is an extension of the cylindrical model introduced in Abe and Ley (2017)

223 to the regression context. It concerns a combination of a Weibull distribution, with scale

224 parameter *ν >* 0 and shape parameter *α*, for the linear outcome and a sine-skewed von Mises

225 distribution, with location parameter *µ* ∈ [0*,* 2*π*), concentration parameter *κ >* 0 and

226 skewness *λ* ∈ [−1*,* 1], for the circular outcome. In contrast to the CL-PN and CL-GPN

227 models, the linear outcome *Y* is in this model defined only on the positive real half-line

228 [0*,* +∞) and thus can not be negative.

229 In this model we predict the linear scale parameter and circular location parameter,

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both of which we can express in terms of covariates:

*ν*ˆ*i* = exp(***x****t****γ***) *>* 0 and

231 *µ*ˆ*i* = *β*0 + 2 tan*−*1(***z****t****β***). The parameter ***γ*** is a vector of *q* regression coefficients

*i*

*i*

232 *γj* ∈ (−∞*,* +∞) for the prediction of *y* where *j* = 0*, . . . , q* and *γ*0 is the intercept. The

233 parameter *β*0 ∈ [0*,* 2*π*) is the intercept and ***β*** is a vector of *p* regression coefficients

234 *βj* ∈ (−∞*,* +∞) for the prediction of *θ* where *j* = 1*, . . . , p*. The vector ***x****i* is a vector of

235 predictor values for the prediction of *y* and ***z****i* is a vector of predictor values for the

236 prediction of *θ*.

237 For the teacher data, the predicted values for the circular outcome in the Abe-Ley

238 model are:

*θ*ˆ*i* = *µ*ˆ*i* = *β*0 + 2 ∗ tan*−*1(*β*1SE*i*)*.*

239 We do not directly predict the linear outcome. The conditional distribution for the linear

240 outcome is Weibull, meaning that we can use methods from survival analsis to interpret the

241 effect of a predictor. In survival analysis a “survival” function is used in which time is plotted

242 against the probability of survival of subjects suffering from a specific medical condition. In

243 the teacher data we can thus compute the probability of a teacher having a specific strength

244 on the IPC. This probability is computed using the “survival-function” defined as

1*/α*

exp(−*αyν*ˆ*i*(1*−*tanh(*κ*) cos(*θi−µ*ˆ*i*)) )*,*

*i*

245 with *ν*ˆ*i* = exp(*γ*0 + *γ*1SE*i*). From the survival function we also see that the circular

246 concentration parameter *κ* and the linear shape parameter *α* regulate the circular-linear

247 dependence in the Abe-Ley model. The Abe-Ley model thus allows us to assess the average

248 type and strength of interpersonal behavior through the parameters *β*0 and *γ*0 as well as the

249 effect of self-efficacy on type and strength of teacher behavior through the parameters, *β*1

250 and *γ*1.

251 We can use numerical optimization (Nelder-Mead) to find solutions for the maximum

252 likelihood (ML) estimates for the parameters of the model.

# 253 Modified Joint Projected and Skew Normal Model (GPN-SSN)

254 This model is an extension of the cylindrical model introduced by Mastrantonio (2018)

255 to the regression context. The model contains *m* independent circular outcomes and *w*

256 independent linear outcomes. The circular outcomes **Θ** = (**Θ**1*, . . . ,* **Θ***m*) are modelled

257 together by a multivariate GPN distribution. The linear outcomes ***Y*** = (***Y*** 1*, . . . ,* ***Y*** *w*) are

258 modelled together by a multivariate skew normal distribution (Sahu, Dey, & Branco, 2003).

259 Thus also in this model the linear outcome is defined constrained to be positive. Because the

260 GPN distribution is modelled using a so-called augmented representation (see also the

261 description of the CL-PN and CL-GPN models) it is convenient to use a similar tactic for

262 modelling the multivariate skew normal distribution. As in Mastrantonio (2018) dependence

263 between the linear and circular outcome is created by modelling the augmented

264 representations of **Θ** and ***Y*** together in a 2*m* + *w* dimensional normal distribution.

265 This means that we have a shared mean vector and variance-covariance matrix for the

266 linear and circular outcome(s), much like having multiple outcomes in a MANOVA

267 (multivariate analysis of variance) model. In our regression extension of the GPN-SSN model

268 we have *i* = 1*, . . . , n* observations of *m* circular outcomes, *w* linear outcomes and *g*

269 covariates. The mean vector then becomes ***M*** *i* = ***B****t****x****i* where ***B*** is a (*g* + 1) × (2*m* + *w*)

270 matrix with regression coefficients and intercepts and ***x****i* is a *g* + 1 dimensional vector

271 containing the value 1 to estimate an intercept and the *g* covariate values. This means that

272 in contrast to the other three models, we have to use the same set of predictors for the

273 circular and linear outcome.

*β*0*sI β*0*sII β*0*y*  [3](#_bookmark4)

274 For the teacher data, ***B*** =  . The predicted circular

*s s*

and linear

*β*1 *I β*1 *II β*1*y*

275 outcomes in the GPN-SSN model are

*θ*ˆ*i* = atan2(*β*0 + *β*1 SE*i, β*0 + *β*1 SE*i*)*,*

*sII II I Is s s*

276 and

3 Note that for the GPN-SSN model the predicted circular outcome also depends on the variance-covariance matrix and the circular predicted values should be computed using numerical integretion or Monte Carlo methods because a closed form expression for the mean direction is not available.

*y*ˆ*i* = *β*0 + *β*1 SE*i.*

*yi yi*

277 The GPN-SSN model thus allows us to assess the average type and strength of interpersonal

278 behavior through the parameters *β*0 , *β*0 and *β*0 as well as the effect of self-efficacy on

*sI II ys*

279 type and strength of teacher behavior through the parameters, *β*1 , *β*1 and *β*1 . In

*sI II ys*

280 addition, because the type and strength of interpersonal behavior are modelled together

281 using a multivariate normal distribution we can through its variance-covariance matrix also

282 assess the dependence between the type and strength of interpersonal behavior.

283 We estimate the model using MCMC methods. A detailed description of these

284 methods is given in the Supplementary Material.

# 285 Model Fit Criterion

286 For the four cylindrical models we focus on their out-of-sample predictive performance

287 to determine the fit of the model. To do so we use k-fold cross-validation and split our data

288 into 10 folds. Each of these folds (10 % of the sample) is used once as a holdout set and 9

289 times as part of a training set. The analysis will thus be performed 10 times, each time on a

290 different training set.

291 A proper criterion to compare out-of-sample predictive performance is the Predictive

292 Log Scoring Loss (PLSL) (Gneiting & Raftery, 2007). The lower the value of this criterion,

293 the better the predictive performance of the model. Using ML estimates this criterion can be

294 computed as follows:

295

*M*

*P LSL* = 2 log *l*(*xi* ***ϑ***)*,*

 ˆ− |

*i*=1

where *l* is the model likelihood, *M* is the sample size of the holdout set, *xi* is the *ith*

296

datapoint from the holdout set and ***ϑ***ˆ

are the ML estimates of the model parameters. Using

297 posterior samples the criterion is similar to the log pointwise predictive density (lppd)

298 (Gelman et al., 2014, p. 169) and can be computed as:

(*j*)

*M*

*P LSL* = −2

1

*B*

*B j*=1 *i*=1

  log *l*(*x*

| ***ϑ*** )*,*

299 where *B* is the amount of posterior samples and ***ϑ***(*j*) are the posterior estimates of the model

*i*

300 parameters for the *jth* iteration. Because the joint density and thus also the likelihood for

301 the modified GPN-SSN model is not available in closed form (Mastrantonio, 2018) we

302 compute the PLSL for the circular and linear outcome separately for all models. Note that

303 although we fit the CL-PN, CL-GPN and GPN-SSN models using Bayesian statistics, we do

304 not take prior information into account when assessing model fit with the PLSL. According

305 to Gelman et al. (2014) this is not necessary since we are assessing the fit of a model to data,

306 the holdout set, only. They argue that the prior in such case is only of interest for estimating

307 the parameters of the model but not for determining the predictive accuracy.

308 For each of the four cylindrical models and for each of the 10 cross-validation analyses

309 we can then compute a PLSL for the circular and linear outcome by using the conditional

310 log-likelihoods of the respective outcome (see Supplementary Material for a definition of the

311 loglikelihoods). To evaluate the predictive performance we average across the PLSL criteria

312 of the cross-validation analyses. We also assess the cross-validation variability by means of

313 the standard deviations of the PLSL criteria.

314 **Data Analysis**

315 In this section we analyze the teacher data with the help of the four cylindrical models

316 from the previous section. We will present the results, posterior estimates and their

317 interpretation for all four models.

# 318 Results & Analysis

319 In the Supplementary Material we have described the starting values for the MCMC

320 procedures for the CL-PN, CL-GPN and GPN-SSN models, hence it remains to specify the

321 starting values for the maximum likelihood based Abe-Ley model:

322 *η*0 = 0*.*9*, η*1 = 0*.*9*, ν*0 = 0*.*9*, ν*1 = 0*.*9*, κ* = 0*.*9*, α* = 0*.*9*, λ* = 0. The initial number of iterations

323 for the three MCMC samplers was set to 2000. After convergence checks via traceplots we

324 concluded that some of the parameters of the GPN-SSN model did not converge. Therefore

325 we set the number of iterations of the MCMC models to 20,000 and subtracted a burn-in of

326 5000 to reach convergence. Note that we choose the same number of iterations for all three

327 models models estimated using MCMC prodedures to make their comparison via the PLSL

328 as fair as possible. Lastly, the predictor SE was centered before inclusion in the analysis as

329 this allows the intercepts to bear the classical meaning of average behavior.

330 Tables [3,](#_bookmark9) [4](#_bookmark10) and [5](#_bookmark11) show the results for the four cylindrical models that were fit to the

331 teacher data. For the models estimated using MCMC methods, CL-PN, CL-GPN and

332 GPN-SSN we show descriptives of the posterior of the estimated parameters (posterior mode

333 and lower and upper bound of the 95% highest posterior density (HPD) interval). For the

334 Abe-Ley model we show the maximum likelihood estimates of the parameters. To compare

335 the results of the four models we focus on the following aspects: the estimated average scores

336 (intercept) on the type and strength of interpersonal behavior (1), the effect of self-efficacy

337 on the type and strength of interpersonal behavior (2), the dependence between the type and

338 strength of interpersonal behavior (3) and the model fit (4).

339 **Average type and strength of interpersonal behavior.** The parameters *γ*0 in

340 the CL-PN, CL-GPN and Abe-Ley model and the parameter *β*0 in the GPN-SSN model

*y*

341 inform us about the strength of interpersonal behavior at the average self-efficacy. For the

342 CL-PN, CL-GPN and GPN-SSN model the parameters are estimated at 0.38, 0.37 and 0.30

343 respectively and are a direct prediction of the strength of interpersonal behavior at the

344 average self-efficacy. The estimate for the GPN-SSN model is notably lower and likely to be

345 caused by its skewed distribution for the strengths of interpersonal behavior. In the Abe-Ley

346 model, *γ*0 influences the shape parameter of the distribution of the strength of interpersonal

347 behavior and does not directly estimate the average strength. Instead we can use the

348 survival function to say something about the probability of having a certain strength of

349 interpersonal behavior. Figure [6](#_bookmark19) shows this function for several values of self-efficacy. We

350 look at the survival function at average values of self-efficacy. Note that this function is the

351 average of all survival functions for observations that fall within 1 standard deviation of the

352 mean. The survival function indicates that the probability of having a low strength of

353 interpersonal behavior is higher than having a high strength. We however can not make any

354 direct statement about the estimated strength using the Abe-Ley model.

355

The parameters *βI*, *βII*, *β*0, *β*0

and *β*0

*sII*

inform us about the type of interpersonal

356 behavior at the average self-efficacy for the CL-PN, CL-GPN, Abe-Ley and GPN-SSN model

0 0 *sI*

357 respectively. For the CL-PN, CL-GPN and GPN-SSN model we need to combine the

358 estimates for the underlying bivariate components {*I, II*} into one circular estimate using

359 the double arctangent function4. Table [6](#_bookmark12) shows that these circular estimates are similar for

360 the three models at 32.29*◦*, 33.70*◦* and 35.53*◦*. In the Abe-Ley model the type of

361 interpersonal behavior at the average self-efficacy is estimated at 0.36 radians or 20.63*◦*.

362 **The effect of self-efficacy.** The parameters *γ*1 in the CL-PN, CL-GPN, Abe-Ley

363 model and *β*1 in the GPN-SSN model inform us about the effect of self-efficacy on the

*y*

364 strength of interpersonal behavior. For the CL-PN, CL-GPN and GPN-SSN model the

365 parameters are estimated at 0.03, 0.03 and 0.09 respectively and are a direct estimate of the

366 effect of self-efficacy on the strength of interpersonal behavior, *i.e.* an increase of 1 unit in

367 self-efficacy leads to an increase of 0.09 units in the strength of interpersonal behavior

368 according to the GPN-SSN model. These estimates are however quite small and only

4 atan2(*β*0*II , β*0*I* ) or atan2(*β*0*sII , β*0*sI* )

369 different from 0 (the HPD interval does not contain 0) in the GPN-SSN model. It is hard to

370 say which of the three models, CL-PN, CL-GPN or GPN-SSN, to use to base our conclusions

371 on. The model CL-GPN and CL-PN fit the linear outcome best according to the model fit in

372 Table [7.](#_bookmark13) In these models the linear outcome has a symmetric distribution whereas in the

373 GPN-SSN the distribution of the linear outcome is skewed. However, the effect of

374 self-efficacy is different from 0 only in the GPN-SSN model which does not seem to match

375 with its lower model fit.

376 In the Abe-Ley model, *γ*1 influences the shape parameter of the distribution of the

377 strength of interpersonal behavior and does not directly estimate the effect of self-efficacy.

378 Instead we can use the survival function to say something about the probability of having a

379 certain strength of interpersonal behavior for different values of self-efficacy. Figure [6](#_bookmark19) shows

380 this function for low, average and high values of self-efficacy (as defined in Figure [3).](#_bookmark16) This

381 function indicates that the effect of self-efficacy on the strength of interpersonal behavior is

382 not linear. The probability of having a higher strength of interpersonal behavior is highest

383 for low self-efficacy and lowest for average self-efficacy.

384

The parameters *βI*, *βII*, *β*1, *β*1

and *β*1

*sII*

inform us about the effect of self-efficacy on

385 the type of interpersonal behavior in the CL-PN, CL-GPN, Abe-Ley and GPN-SSN model

1 1 *sI*

386 respectively. For the CL-PN and Abe-Ley model we have drawn the circular regression lines

387 for this effect in Figure [7](#_bookmark20) (see the description of the CL-PN and CL-GPN models). For the

388 CL-PN model the inflection point is indicated with a square in Figure [7.](#_bookmark20) The inflection point

389 for the Abe-Ley model falls outside the bounds of the plot and is therefore not displayed.

390 The slope at the inflection point, *bc*, for the CL-PN model is computed by using methods

391 from Cremers et al. (2018b) and is equal to 1.67 (-24.66, 29.33)5 The parameter *β*1 is the

392 slope at the inflection point for the Abe-Ley model and is equal to -0.03. Even though these

5 Note that this is a linear approximation to the circular regression line representing the slope at a specific point. Therefore it is possible for the HPD interval to be wider than 2*π*. In this case the interval is much wider and covers 0, indicating there is no evidence for an effect.

393 slopes are quite different, the regression lines in Figure [7](#_bookmark20) are quite similar in the data range.

394 Both the regression line of the Abe-Ley model and the CL-PN model show slopes that are

395 not very steep in the range of the data indicating that the effect of self-efficacy on the type

396 of interpersonal behavior is not large.

397 In the CL-GPN and GPN-SSN model we cannot compute circular regression

398 coefficients due to the fact that not only the mean vector of the GPN distribution but also

399 the covariance matrix influences the predicted value on the circle. Instead, we will compute

400 posterior predictive distributions for the predicted circular outcome of individuals scoring the

401 minimum, maximum and median self-efficacy. The modes and 95% HPD intervals of these

402 posterior predictive distributions are *θ*ˆ*SE* = 215*.*74*◦*(147*.*36*◦,* 44*.*49*◦*),

*min*

403 *θ*ˆ*SE* = 25*.*93*◦*(337*.*02*◦,* 138*.*59*◦*), *θ*ˆ*SE* = 30*.*86*◦*(8*.*63*◦,* 72*.*19*◦*) for the CL-GPN model.

*median max*

404 Note that we display the modes and HPD intervals for the posterior predictive distributions

405 on the interval [0*◦,* 360*◦*) and that 44*.*49*◦* = 404*.*49*◦* due to the periodicity of a circular

406 variable. The posterior mode estimate of 215*.*74*◦* thus lies within its HPD interval

407 (147*.*36*◦,* 44*.*49*◦*). For the GPN-SSN model the modes and 95% HPD intervals of the

408 posterior predictive distributiona are *θ*ˆ*SE* = 206*.*87*◦*(117*.*12*◦,* 72*.*02*◦*),

*min*

409 *θ*ˆ*SE* = 24*.*68*◦*(334*.*73*◦,* 128*.*27*◦*), *θ*ˆ*SE* = 29*.*81*◦*(0*.*74*◦,* 80*.*61*◦*). For both the CL-GPN

*median max*

410 and GPN-SSN model the HPD intervals of the mode of the posterior predictive intervals of

411 individuals scoring the minimum, median and maximum self-efficacy overlap. This indicates

412 that the effect of self-efficacy, if there is any, on the type of interpersonal behavior a teacher

413 shows is not expected to be strong. Had the HPD intervals not overlapped we could have

414 concluded that as the self-efficacy increases, the score of the teacher on the IPC moves

415 counterclockwise.

416

# 417 Dependence between type and strength of interpersonal behavior. The

418 relation between the type and strength of interpersonal behavior in the CL-PN and CL-GPN

419 model, is described by the parameters *γ*cos and *γ*sin. The HPD interval of *γ*cos does not

420 include 0 for both the CL-PN and CL-GPN models, meaning that the cosine component of

421 the type of interpersonal behavior has an effect on the strength of interpersonal behavior.

422 In the teacher data the sine and cosine components have a substantive meaning. This

423 is illustrated in Figure [2.](#_bookmark15) In a unit circle the horizontal axis (Communion) represents the

424 cosine of and the vertical axis (Agency) represents the sine of an angle. For the teacher data

425 this means that the Communion (cosine) dimension of the IPC positively effects the strength

426 of a teachers’ type of interpersonal behavior, in plain words: teachers exhibiting

427 interpersonal behavior types with higher communion scores (e.g., “helpful” and

428 “understanding” in Figure 2) are stronger in their interpersonal behavior.

429 In the GPN-SSN model the dependence between the type and strengths of

430 interpersonal behavior is modelled through the covariances between the linear outcome and

431 the sine and cosine of the circular outcome *sy*2*,*3 and *sy*1*,*3 . Both covariances, *sy*2*,*3 = 0*.*09

432 and *sy*1*,*3 = 0*.*23, are different from zero, but the one of the cosine component, and thus the

433 correlation with the Communion dimension, is larger. This mean that teachers scoring both

434 high on Communion and Agency show stronger behavior. Together with the results from the

435 CL-PN and CL-GPN models in the previous paragraph this translates to the conclusion that

436 teachers with the strongest interpersonal behavior have a type of interpersonal behavior

437 between 0*◦* and 90*◦*. To get these scores on the circle both the Agency and the Communion

438 score of a Teacher have to be positive (see [(1)](#_bookmark0)). This corresponds to the pattern observed in

439 the teacher data in Figure [2.](#_bookmark15) At a strength of 0.4 and up we see that the scores on the circle

440 range on average between 0*◦* and 100*◦*.

441 **Model fit.** Table [7](#_bookmark13) shows the values fof the PLSL criterion for the linear and circular

442 outcomes of the four cylindrical models fit to the teacher data.

443 The CL-PN and CL-GPN models have the best out-of-sample predictive performance

444 for the linear outcome. They show roughly the same performance because they model the

445 linear outcome in the same way. We should note that even though the predictive

446 performance of the Abe-Ley model for the linear outcome is worst on average, the standard

447 deviation of the cross-validation estimates is rather large. This means that in some samples,

448 the Abe-Ley model shows a lower PLSL value than the average of 25.49

449 The Abe-Ley model has the best out-of-sample predictive performance for the circular

450 outcome. This would suggest that for the circular variable a slightly skewed distribution fits

451 best. However, both the GPN-SSN and the CL-GPN models fit much worse even though the

452 distribution for the circular outcome in these models can also take a skewed shape. It should

453 be noted that the standard deviation of the cross-validation estimates is rather large for the

454 Abe-Ley and the CL-GPN model. It is possible that these large standard deviations for the

455 PLSL are caused by the fact that they are computed for a relatively small sample size, but

456 this does not explain why the PLSL has a large standard deviation for only a few cylindrical

457 models and not for all.

458 In this situation, where one model fits the linear outcome best and another one fits the

459 circular outcome best, it is hard to determine which model we should choose. In this case

460 the results for the CL-PN /CL-GPN and Abe-Ley model are quite different regarding the

461 effect of self-efficacy on the linear outcome (strength of interpersonal behavior). Because the

462 Abe-Ley fit for the linear part is worst we would choose to trust the results for the CL-PN

463 and CL-GPN model here. For the circular part however the results of the CL-PN/CL-GPN

464 model do not differ as much from the Abe-Ley model and we reach the same conclusion for

465 both models, namely that the effect of self-efficacy on type of interpersonal behavior is not

466 very strong. Therefore we would prefer the CL-PN/CL-GPN models in this case because

467 where it matters in terms of interpretation (the linear part) they show better fit.

468 **Discussion**

469 In this paper we modified four models for cylindrical data in such a way that they

470 include a regression of both the linear and circular outcome onto a set of covariates.

471 Subsequently we have shown how these four methods can be used to analyze a dataset on

472 the interpersonal behavior of teachers. In this final section we will first comment on what

473 researchers can gain by using cylindrical models for the teacher data. Subsequently we will

474 comment on the differences between the cylindrical models that were introduced in this

475 paper.

476 Concerning the teacher data, the advantage of cylindrical data analysis is that we were

477 able to analyze the information about the type and strength of interpersonal behavior

478 simultaneously. In previous research, the two components of the interpersonal circumplex

479 (*i.e.*, Agency and Communion) were analyzed separately. Such an approach also provides

480 information about the strength of teachers’ score on Agency and Communion, yet a large

481 portion of information about the combination of Agency and Communion, which describes

482 the type of behavior that is observed, gets lost. A first solution to include both dimensions

483 as a circular variable in data analysis was described by Cremers et al. (2018a). A downside

484 of that analysis was that information about the strength of the specific type of interpersonal

485 behavior could not be retained. In the present study, we have shown how using cylindrical

486 models can simultaneously model the information about the type of and strength of

487 interpersonal behavior and how these are influenced by teachers’ self-efficacy in classroom

488 management. Although we do not find any strong effects of self-efficacy on either the type or

489 strength of behavior, the four cylindrical models do provide a way of analyzing and

490 interpreting this effect. This is beneficial for future research in which we may want to

491 investigate the effect of further covariates on data from the circumplex. Furthermore, in

492 addition to being able to assess the influence of covariates, the cylindrical models also

493 provide information about the dependence between the type and strength of interpersonal

494 behavior. We found that stronger behavior is associated with those types of behavior that

495 show higher scores on the Communion dimension (and also on the Agency dimension). This

496 translates to stronger behavior being shown by teachers whose type of interpersonal behavior

497 ranges between 0*◦* and 90*◦*, the “helpful” and “directing” subtypes according to Figure [2.](#_bookmark15) Is

498 dit nieuw/zijn er papers die hier iets over zeggen?

499 As mentioned in the introducion data from the interpersonal circumplex is not the only

500 type of cylindrical data that occurs in psychology. The methods presented in this paper are

501 also of use for research on human navigation and eye-tracking research. Furthermore, even

502 though cylindrical models are already used in fields outside of psychology, the addition of a

503 regression structure to the models is of use in these fields as well.

504 In terms of interpretability, the CL-PN and Abe-Ley models perform best out of the

505 four cylindrical models. In the CL-GPN and GPN-SSN models the interpretation of the

506 parameters of the circular outcome component is not straightforward, if at all possible. This

507 is caused by the fact that in addition to the mean vector the covariance matrix of the GPN

508 distribution affects the location of the circular data, making it difficult to compute regression

509 coefficients on the circle. Wang and Gelfand (2013) state that Monte Carlo integration can

510 be used to compute a circular mean and variance for the GPN distribution. In future

511 research, this solution might be applied to the methods of Cremers et al. (2018b) in order to

512 compute circular coefficients for GPN models.

513 In terms of flexibility the GPN-SSN model scores best. Multiple linear and circular

514 outcomes can be included and we can thus apply the model to multivariate cylindrical data.

515 In addition the GPN-SSN, the CL-GPN and CL-PN models are extendable to a

516 mixed-effects structure and can thus also be fit to longitudinal data (see Nuñez-Antonio and

517 Gutiérrez-Peña (2014) and Hernandez-Stumpfhauser et al. (2016) for

518 hierarchical/mixed-effects models for the PN and GPN distributions respectively). For the

519 Abe-Ley model this may also be possible but has not been done in previous research for the

520 conditional distribution of its circular outcome (sine-skewed von Mises). Concerning

521 asymmetry, both the GPN-SSN as well as the Abe-Ley model allow for non-symmetrical

522 shapes of the distributions of both the linear and circular outcome, while the CL-GPN model

523 permits an asymmetric circular outcome.

524 The four cylindrical models that were modified to the regression context in this paper

525 are not the only cylindrical distributions available from the literature. Other interesting

|  |  |
| --- | --- |
| 526 | cylindrical distributions have been introduced by Fernández-Durán (2007), Kato and |
| 527 | Shimizu (2008) and Sugasawa (2015) (for more references we refer to Chapter 2 of Ley and |
| 528 | Verdebout (2017)). In the present study we have decided not to include these distributions |
| 529 | for reasons of space, complexity of the models and ease of implementing a regression |
| 530 | structure. In future research however it would be interesting to investigate other types of |
| 531 | cylindrical distributions as well in order to compare the interpretability, flexibility and model |
| 532 | fit to the models developed in the present study. |
| 533 |  |
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Table 1

*Descriptives for the teacher dataset.*

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Variable | mean/*θ*¯ | sd/*ρ*ˆ | Range | Type |
| IPC | 33.22*◦* | 0.76 | - | Circular |
| strength IPC | 0.43 | 0.15 | 0.08 - 0.80 | Linear |
| SE | 5.04 | 1.00 | 1.5 - 7.0 | Linear |

Note: For the circular variable IPC we show sample estimates for the circular mean *θ*¯ and concentration *ρ*ˆ. For the linear variable we show the sample mean, standard deviation and range.

Table 2

*Comparison of the four cylindrical regression models*

Aspect CL-PN CL-GPN Abe-Ley GPN-SSN

Θ

Distribution PN GPN Sine-skewed vM GPN Domain [0*,* 2*π*) [0*,* 2*π*) [0*,* 2*π*) [0*,* 2*π*) Shape symmetric, assymetric, assymetric, assymetric,

unimodal multimodal unimodal multimodal

*Y*

Distribution Normal Normal Weibull skewed-Normal Domain (−∞*,* +∞) (−∞*,* +∞) (0*,* +∞) (0*,* +∞)

Shape symmetric, symmetric, assymetric, assymetric,

unimodal unimodal unimodal unimodal

Θ-*Y* dependence

*y* regressed on *y* regressed on *α* and *κ* multivariate sin(*θ*) and cos(*θ*) sin(*θ*) and cos(*θ*) distribution

Note: PN and GPN refer to the projected normal and general projected normal distribution. vM refers to the von-Mises distribution

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30

Table 3

*Results, cross-validation mean and standard deviation, for the modified CL-PN and CL-GPN models*

Parameter CL-PN CL-GPN

0

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | | Mode | HPD LB | HPD UB |  | Mode | HPD LB | HPD UB |
| *βI* |  | 1.76 (0.09) | 1.50 (0.07) | 2.03 (0.09) |  | 2.43 (0.12) | 1.91 (0.10) | 3.05 (0.17) |
| *βI* |  | 0.65 (0.07) | 0.42 (0.06) | 0.90 (0.08) |  | 0.84 (0.11) | 0.45 (0.09) | 1.29 (0.15) |
| *βII* |  | 1.15 (0.05) | 0.92 (0.04) | 1.37 (0.04) |  | 1.47 (0.05) | 1.16 (0.04) | 1.78 (0.05) |
| *βII* |  | 0.58 (0.03) | 0.38 (0.04) | 0.79 (0.04) |  | 0.70 (0.06) | 0.47 (0.05) | 0.96 (0.08) |
| *γ*0 |  | 0.38 (0.01) | 0.31 (0.01) | 0.44 (0.01) |  | 0.37 (0.01) | 0.31 (0.01) | 0.42 (0.01) |
| *γcos* |  | 0.04 (0.00) | 0.01 (0.00) | 0.06 (0.00) |  | 0.03 (0.00) | 0.01 (0.00) | 0.04 (0.00) |
| *γsin* |  | -0.01 (0.00) | -0.04 (0.00) | 0.02 (0.00) |  | -0.00 (0.00) | -0.03 (0.00) | 0.03 (0.00) |
| *γ*1 |  | 0.03 (0.01) | -0.00 (0.00) | 0.07 (0.01) |  | 0.03 (0.00) | -0.00 (0.00) | 0.06 (0.00) |
| *σ* |  | 0.14 (0.00) | 0.12 (0.00) | 0.16 (0.00) |  | 0.14 (0.00) | 0.12 (0.00) | 0.16 (0.00) |
| 1*,*1 |  | NA (NA) | NA (NA) | NA (NA) |  | 3.04 (0.29) | 1.85 (0.13) | 5.00 (0.41) |
| 1*,*2 |  | NA (NA) | NA (NA) | NA (NA) |  | 0.47 (0.12) | 0.12 (0.12) | 0.80 (0.10) |
| 2*,*2 |  | NA (NA) | NA (NA) | NA (NA) |  | 1.00 (0.00) | 1.00 (0.00) | 1.00 (0.00) |

1

0

1

Note: *βI*, *βII* and *γ*0 inform us about the type and strength of interpersonal behavior

0 0

at the average self-efficacy. *βI*, *βII* and *γ*1 inform us about the effect of self-efficacy on the

1 1

type and strength of interpersonal behavior. *γcos* and *γsin* inform us about the dependence between the type and strength of interpersonal behavior. 1*,*1, 1*,*2 and 2*,*2 are are elements of the variance-covariance matrix of the type of interpersonal behavior in the

CL-GPN model and *σ* is the error standard deviation of the strength of interpersonal behavior.

Table 4

*Results, cross-validation mean and standard deviation (SD), for the modified Abe-Ley model*

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | *β*0 | *β*1 | *γ*0 | *γ*1 | *α* | *κ* | *λ* |
| Mean | 0.36 | -0.03 | 1.17 | 0.04 | 3.66 | 1.51 | 0.70 |
| SD | 0.02) | 0.01 | 0.02 | 0.02 | 0.12 | 0.08 | 0.05 |

Note: *β*0 and *γ*0 inform us about the type and strength of interpersonal behavior at the average self-efficacy. *β*1 and *γ*1 inform us about the effect of self-efficacy on the type and strength of interpersonal behavior. *α* is the shape parameter of the distribution of the strength

of interpersonal bahavior. *κ* and *λ* are the concentration and skewness parameters for the distribution of the type of interpersonal behavior.

Table 5

*Results, cross-validation mean and standard deviation, for the GPN-SSN model*

Parameter Unconstrained Constrained

*s*

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | | Mode | HPD LB | HPD UB |  | Mode | HPD LB | HPD UB |
| *β*0*I* |  | 0.30 (0.01) | 0.26 (0.01) | 0.34 (0.01) |  | 2.11 (0.11) | 1.75 (0.09) | 2.50 (0.11) |
| *β*0*II* |  | 0.19 (0.00) | 0.17 (0.01) | 0.21 (0.00) |  | 1.34 (0.06) | 1.10 (0.05) | 1.57 (0.06) |
| *β*0*y* |  | 0.33 (0.01) | 0.30 (0.30) | 0.36 (0.01) |  | 0.33 (0.01) | 0.30 (0.01) | 0.36 (0.01) |
| *β*1*I* |  | 0.09 (0.01) | 0.05 (0.01) | 0.13 (0.01) |  | 0.60 (0.06) | 0.33 (0.05) | 0.90 (0.06) |
| *β*1*II* |  | 0.07 (0.00) | 0.04 (0.00) | 0.09 (0.01) |  | 0.48 (0.03) | 0.30 (0.04) | 0.66 (0.04) |
| *β*1*y* |  | 0.09 (0.01) | 0.06 (0.06) | 0.12 (0.01) |  | 0.09 (0.01) | 0.06 (0.01) | 0.12 (0.01) |
| *s*1*,*1 |  | 0.05 (0.00) | 0.04 (0.00) | 0.06 (0.00) |  | 2.44 (0.15) | 1.72 (0.07) | 3.46 (0.14) |
| *s*2*,*2 |  | 0.02 (0.00) | 0.02 (0.00) | 0.03 (0.00) |  | 1.00 (0.00) | 1.00 (0.00) | 1.00 (0.00) |
| *y*3*,*3 |  | 0.03 (0.00) | 0.02 (0.02) | 0.04 (0.00) |  | 0.03 (0.00) | 0.02 (0.00) | 0.04 (0.00) |
| *s*1*,*2 |  | 0.00 (0.00) | -0.00 (0.00) | 0.01 (0.00) |  | 0.08 (0.06) | -0.20 (0.06) | 0.34 (0.06) |
| *sy*1*,*3 |  | 0.03 (0.00) | 0.02 (0.00) | 0.04 (0.00) |  | 0.23 (0.01) | 0.17 (0.00) | 0.32 (0.01) |
| *sy*2*,*3 |  | 0.01 (0.00) | 0.01 (0.01) | 0.02 (0.00) |  | 0.09 (0.01) | 0.06 (0.01) | 0.12 (0.01) |
| *λ* |  | 0.16 (0.01) | 0.14 (0.01) | 0.18 (0.01) |  | 0.16 (0.01) | 0.14 (0.01) | 0.18 (0.01) |

*s*

*s*

*s*

Note: *β*0*I* , *β*0*II* and *β*0 inform us about the type and strength of interpersonal behavior

*y*

*s s*

at the average self-efficacy. *β*1*I* , *β*1*II* and *β*1 inform us about the effect of self-efficacy

*y*

*s s*

on the type and strength of interpersonal behavior. *s*1*,*1 , *s*1*,*2 , *s*2*,*2 , *y*3*,*3 , *sy*1*,*3 , and *sy*2*,*3

are elements of the variance-covariance matrix of which *sy*1*,*3 , and *sy*2*,*3 inform us about the dependence between the type and strength of interpersonal behavior.

*λ* is the skewness parameter of the distribution of the strengths of interpersonal behavior.

Table 6

*Posterior estimates (in degrees) for the circular mean (at SE = 0) in the CL-PN, CL-GPN and GPN-SSN models*

|  |  |  |  |
| --- | --- | --- | --- |
|  | Mode | HPD LB | HPD UB |
| CL-PN | 32.29 | 24.81 | 39.71 |
| CL-GPN | 33.70 | 26.72 | 41.15 |
| GPN-SSN | 35.53 | 28.40 | 43.30 |

Note that these means are based on their posterior predictive distribution following (Wang and Gelfand, 2013)

Table 7

*PLSL criteria, cross-validation mean and standard deviation, for the circular and linear outcome in the four cylindrical models*

Model Circular Linear

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | | mean | sd |  | mean | sd |
| CL-PN |  | 82.96 | (9.47) |  | -17.65 | (3.70) |
| CL-GPN |  | 86.05 | (16.63) |  | -18.30 | (3.00) |
| Abe-Ley |  | 31.97 | (22.07) |  | 25.49 | (17.46) |
| GPN-SSN |  | 107.10 | (10.52) |  | -2.37 | (7.01) |

90*◦*

0*◦* 360*◦*

-*∞* +*∞*

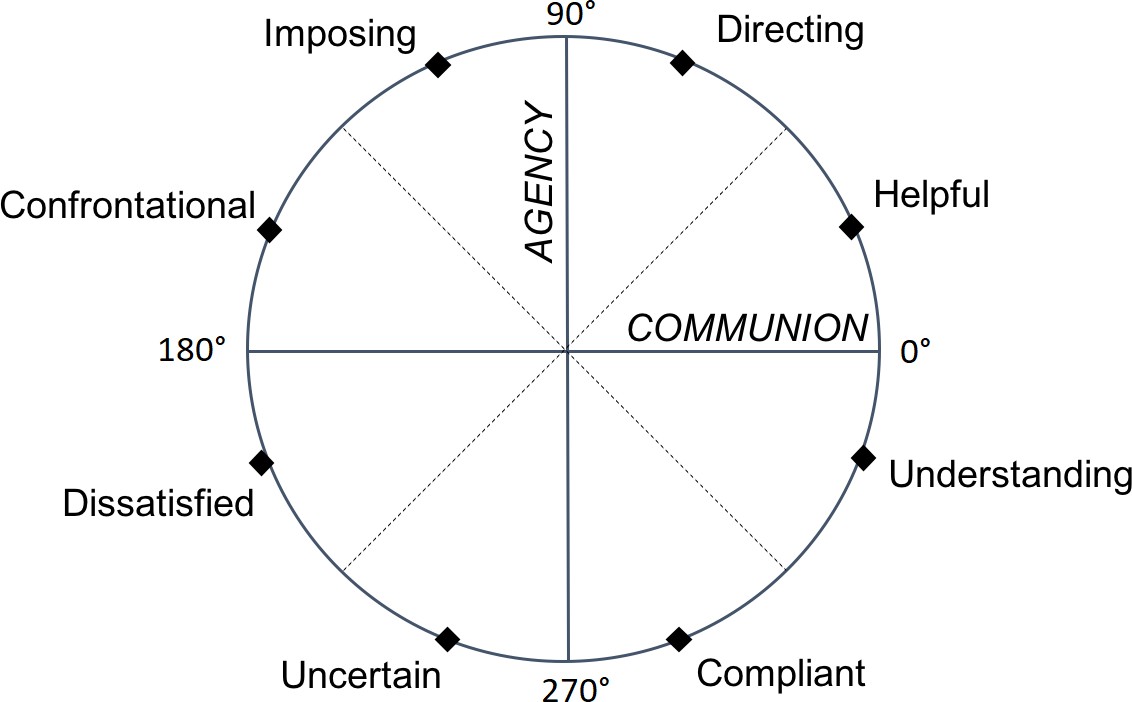
180*◦* 0 **and** 360*◦*

270*◦*

*Figure 1* . The difference between a linear scale (left) and a circular scale (right).

IPC

-50 0 50



*Figure 2* . The interpersonal circle for teachers (IPC-T). The words presented in the circum- ference of the circle are anchor words to describe the type of behavior located in each part of the IPC.

150

low SE average SE high SE

0.1 0.2 0.3 0.4 0.5 0.6

-150

IPC strength

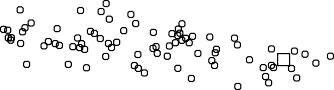
*Figure 3* . Plot showing the relation between the linear and circular outcome component (in degrees) of the teacher data.

*Figure 4* . Plot showing the projection of datapoints in bivariate space, ***S***, (left) to the circle (right). The lines connecting the bivariate datapoints to the circular datapoints represent the euclidean norm of the bivariate datapoints, the random variable *R*.



180

120



Circular Outcome

60

0

-60

-120

-180

0

1

2

3

4

5

Predictor

-5

-4

-3

-2

-1

*Figure 5* . Circular regression line for the relation between a covariate and a circular outcome with the data the regression line was fit to. The square indicates the inflection point of the regression live.

0.8

1.0

0.0 0.2 0.4 0.6 0.8 1.0

low SE

average SE

high SE

P(strength IPC)

0.0

0.2

0.4

0.6

strength IPC

*Figure 6* . Plot showing the probability of having a particular strength of interpersonal behavior (survival plot) for the minimum, mean and maximum self-efficacy in the data.

80

160

-5 -4 -3 -2 -1 0 1 2

*θ*

-160

-80

0

self-efficacy

*Figure 7* . Plot showing circular regression lines for the effect of self-efficacy as predicted by the Abe-Ley model (solid line) and CL-PN model (dashed line). The black square indicates the inflection point of the circular regression line for the CL-PN model.