

# Circular statistics in cognitive psychology: A short introduction

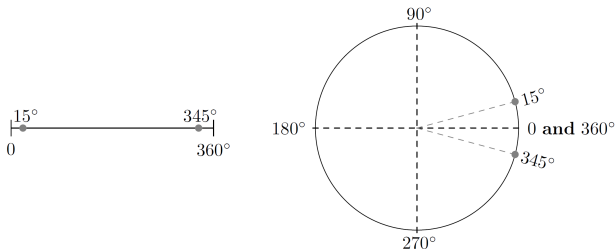
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# What is circular data?

A variable  $\theta$  defined on  $[0, 2\pi)$  radians or  $[0, 360)$  degrees:



# Three different types of circular data

- ▶ Directions in 2D space / Compass data
- ▶ Time of day / clock times
- ▶ Circular measurement scales

# Directions in 2D space / compass data

## Spatial cognition:

- ▶ Brunyé, T. T., Burte, H., Houck, L. A., & Taylor, H. A. (2015). The map in our head is not oriented north: Evidence from a real-world environment.
- ▶ Postma, A., Zuidhoek, S., Noordzij, M. L., & Kappers, A. M. (2008). Keep an eye on your hands: on the role of visual mechanisms in processing of haptic space.

## Eye movements:

- ▶ Mulder, K., Klugkist, I., van Renswoude, D., & Visser, I. (2020). Mixtures of peaked power Batschelet distributions for circular data with application to saccade directions.

# Time of day / clock times

## Time of day:

- ▶ Rocchi, M. B., & Perlini, C. (2002). Is the time of suicide a random choice? A new statistical perspective.
- ▶ Brunsdon, C., & Corcoran, J. (2006). Using circular statistics to analyse time patterns in crime incidence.

## Circadian rhythms:

- ▶ Peeters, F., Berkhof, J., Delespaul, P., Rottenberg, J., & Nicolson, N. A. (2006). Diurnal mood variation in major depressive disorder.

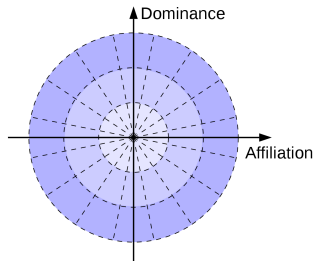
# Circular measurement scales

Rose of Leary:

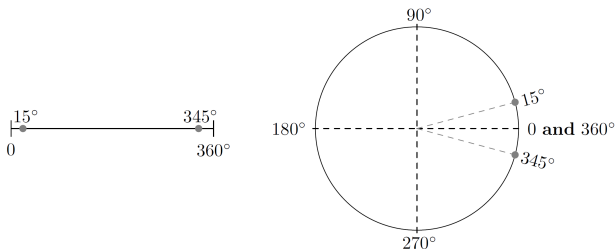
- ▶ Leary, T. (1958). Interpersonal diagnosis of personality.

Interpersonal circumplex:

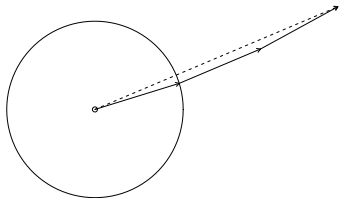
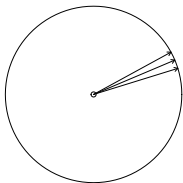
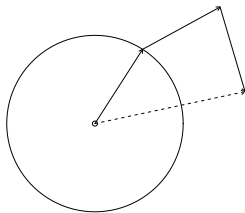
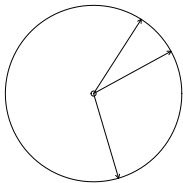
- ▶ Gurtman, M. B. (2009). Exploring personality with the interpersonal circumplex.



# Why do we need different analysis methods?



# Circular summary statistics





## Circular mean direction

$$\bar{\theta} = \begin{cases} \tan^{-1} \left( \frac{\sum_{i=1}^n \sin \theta_i}{\sum_{i=1}^n \cos \theta_i} \right) & \sum_{i=1}^n \sin \theta_i > 0, \sum_{i=1}^n \cos \theta_i > 0 \\ \tan^{-1} \left( \frac{\sum_{i=1}^n \sin \theta_i}{\sum_{i=1}^n \cos \theta_i} \right) + \pi & \sum_{i=1}^n \cos \theta_i < 0 \\ \tan^{-1} \left( \frac{\sum_{i=1}^n \sin \theta_i}{\sum_{i=1}^n \cos \theta_i} \right) + 2\pi & \sum_{i=1}^n \sin \theta_i < 0, \sum_{i=1}^n \cos \theta_i > 0 \end{cases}$$

## Mean resultant length

$$R = \sqrt{\left(\sum_{i=1}^n \cos \theta_i\right)^2 + \left(\sum_{i=1}^n \sin \theta_i\right)^2}$$
$$\bar{R} = R/n$$

1 = All datapoints have the same value

0  $\neq$  uniform, e.g. bimodality at exactly opposite parts of the circle (0°, 180°) also leads to a zero value.

## Other summary statistics

- ▶ circular variance,  $V = 1 - \bar{R}$
- ▶ circular standard deviation  $\nu = -2 \log(1 - V)^{0.5}$
- ▶ circular median direction
- ▶ circular modal direction

see Fisher (1995) for an overview.

# Circular distributions

Three classes:

- ▶ Intrinsic distributions (e.g. von Mises)
- ▶ Wrapped distributions ( $\theta \sim N(\mu, \sigma) \bmod 2\pi$ )
- ▶ Projected distributions  
( $(\sin \theta, \cos \theta)^t = \mathbf{y} \sim N_2(\boldsymbol{\mu}, \boldsymbol{\Sigma}) / \|\mathbf{y}\|$ )

# Intrinsic distributions

Directly defined on the circle, e.g.:

- ▶ von Mises distribution

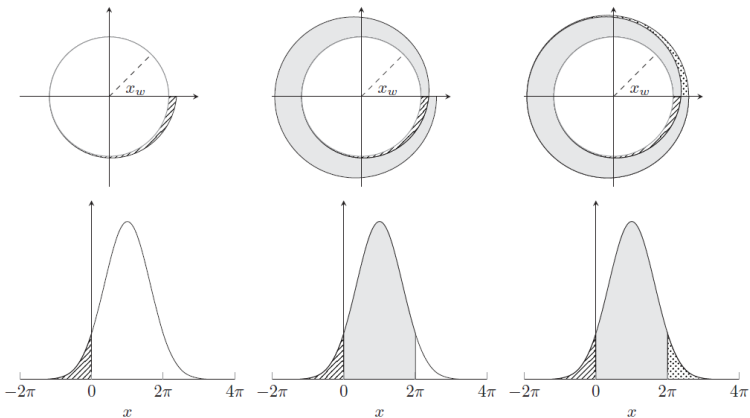
$$f(\theta) = [2\pi I_0(\kappa)]^{-1} \exp[\kappa \cos(\theta - \mu)]$$

mean direction =  $\mu$

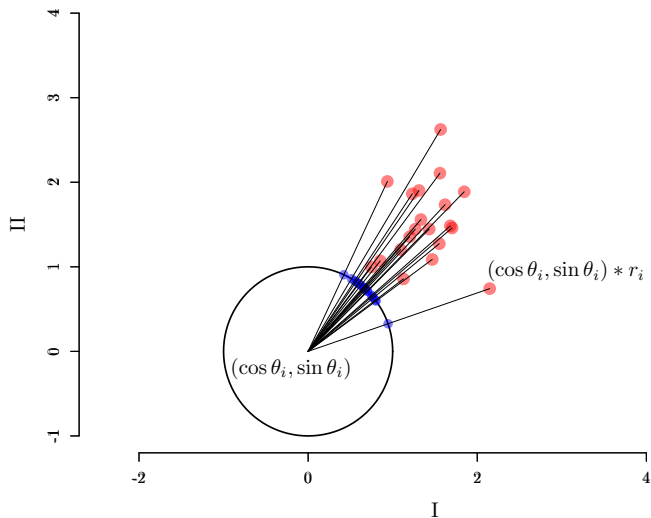
concentration parameter =  $\kappa$

mean resultant length =  $A_1(\kappa) = I_1(\kappa)/I_0(\kappa)$ , see Fisher (1995)

# Wrapped distributions



# Projected distributions



## Projected & wrapped distributions

Instead of inference on the circular outcome,  $\theta$ , directly we perform inference on underlying bivariate data or univariate linear data.

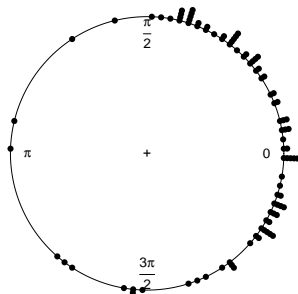
- ▶ Advantage: **flexible** approach, make use of already existing models for linear data.
- ▶ Disadvantage:  $Y$  and  $r$  are latent.
- ▶ Disadvantage: **interpretation** results need to be transformed back to the circle.



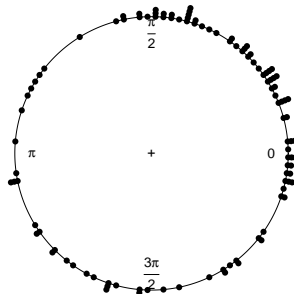
# A short example analysis using the R-package bpnreg

Brunyé, T. T., Burte, H., Houck, L. A., & Taylor, H. A. (2015). The map in our head is not oriented north: Evidence from a real-world environment. PLoS One, 10(9), e0135803.

males



females



## Fit a circular regression model

```
fit <- bpnr(theta ~ sex + age + experience + sbsod,  
            data = data,  
            its = 10000, burn = 100,  
            seed = 101)
```

# Results

Estimated circular means for males and females with average age, experience and Santa Barbara Sense of Direction (SBSOD) score:

```
coef_circ(fit, type = "categorical", units = "degrees")
```

##	mean	mode	sd	LB	UB
## (Intercept)	10.92	12.52	7.29	-3.88	24.71
## sexfemale	32.04	35.23	9.90	12.42	51.39

## Results

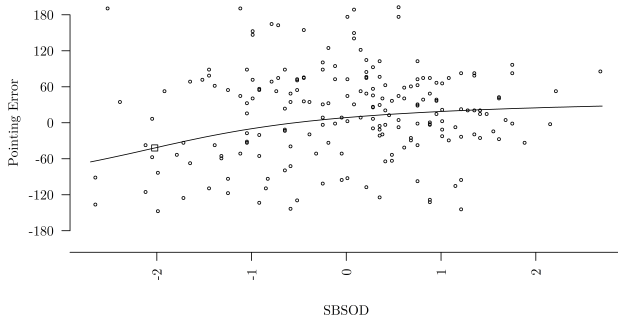
Estimated mean resultant length for males and females with average age, experience and Santa Barbara Sense of Direction (SBSOD) score:

##	mean	mode	sd	LB	UB
## (Intercept)	0.55	0.56	0.06	0.43	0.65
## sexfemale	0.38	0.38	0.06	0.26	0.50

# Results

##	mean	mode	sd	LB HPD	UB HPD
## age ax	166.00	187.14	815.97	-788.58	1026.88
## experience ax	-198.54	-148.36	225.27	-590.93	45.05
## sbsod ax	-140.80	-114.51	61.76	-250.45	-51.63
## age ac	13.76	85.00	60.89	-76.79	100.48
## experience ac	39.38	84.91	71.39	-82.12	102.05
## sbsod ac	-44.74	-47.95	22.42	-74.67	-15.71
## age bc	1.36	8.65	13.75	-23.30	24.56
## experience bc	-5.90	-16.54	21.30	-37.20	34.70
## sbsod bc	23.70	22.74	11.11	9.02	43.29
## age AS	10.58	-5.24	5148.08	-179.94	143.21
## experience AS	-11.55	8.68	1453.90	-341.68	372.51
## sbsod AS	18.99	12.49	605.77	4.19	27.17
## age SAM	-35.50	-5.21	1599.56	-206.92	162.68
## experience SAM	0.33	8.87	2131.26	-398.87	477.85
## sbsod SAM	13.85	11.81	32.38	5.03	21.35
## age SSDO	-6.58	-53.40	51.71	-66.49	65.19
## experience SSDO	21.29	67.12	66.73	-84.29	87.65
## sbsod SSDO	-69.12	-72.24	24.94	-92.14	-54.15

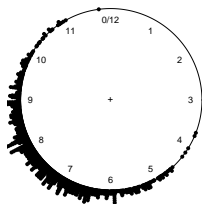
# Results



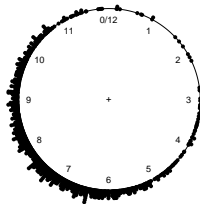
Interpretation of coefficients, see Cremers, Mulder, & Klugkist (2018).

# Some thoughts on circular statistics and the Libet clock

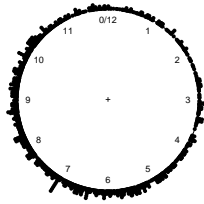
Group 1



Group 2



Group 3



# Some thoughts on circular statistics and the Libet clock

## Location

### Linear analysis:

- ▶ Group 1 mean: 7.623
- ▶ Group 2 mean: 7.601
- ▶ Group 3 mean: 8.026

### Circular analysis:

- ▶ Group 1 mean: 7.625
- ▶ Group 2 mean: 7.613
- ▶ Group 3 mean: 7.632



# Some thoughts on circular statistics and the Libet clock

## Accuracy

### Linear analysis:

- ▶ Scores converted to 0-180 degree scale, deviance from correct time
- ▶ linear regression

### Circular analysis:

- ▶ Models that allow for simultaneous modeling of mean and variance (heterogenous)
- ▶ Projected normal regression (available in R-package 'bpnreg')
- ▶ Heterogenous von Mises models

# Some thoughts on circular statistics and the Libet clock

## Linear analysis:

- ▶ Careful with scale and results
- ▶ Analysis methods easily available

## Circular analysis:

- ▶ Often quite similar to linear analysis
- ▶ Scale is correct
- ▶ Analysis methods not always readily available

# Software

- ▶ R-packages:
  - ▶ `circular`: general purpose package
  - ▶ `bpnreg`: Bayesian circular regression and mixed-effects models based on projected normal distribution
  - ▶ `circglmbayes`: Bayesian circular regression based on von Mises distribution
- ▶ Other:
  - ▶ `CircStat`: general purpose MATLAB toolbox

# Suggested literature

## ► Introductory

- Batschelet (1981). Circular statistics in biology.
- Fisher (1995). Statistical analysis of circular data.
- Pewsey, Neuhäuser & Ruxton (2013). Circular statistics in R.
- Ley & Verdebout (Eds.). (2018). Applied directional statistics: Modern methods and case studies.
- Cremers & Klugkist (2018). One direction? A tutorial for circular data analysis using R with examples in cognitive psychology.

## ► Advanced (requires mathematical background)

- Pewsey & García-Portugués (2020). Recent advances in directional statistics.
- Mardia & Jupp (2000). Directional statistics.
- Ley & Verdebout (2017). Modern directional statistics.

## References

Cremers, J., Mulder, K., & Klugkist, I. (2018). Circular interpretation of regression coefficients. *British Journal of Mathematical and Statistical Psychology*, 71(1), 75–95.  
doi:10.1111/bmsp.12108

Fisher, N. I. (1995). *Statistical analysis of circular data*. Cambridge: Cambridge University Press.