# Analysis of trajectories into retirement using the Danish labour market registry

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## The Danish labour market registry

id	month	sex	birthdate	status	hours	start	end
1	January	М	08-03-1950	sick leave	148	01-01-2011	31-01-2011
2	January	F	21-09-1950	employed	10	20-01-2011	31-01-2011
2	Febuary	F	21-09-1950	employed	50	01-02-2011	28-02-2011
3	January	M	02-11-1950	employed	10	01-01-2011	15-01-2011
3	January	M	02-11-1950	employed	10	01-01-2011	15-01-2011
3	January	M	02-11-1950	unemployment benefits	74	16-01-2011	31-01-2011
3	Febuary	M	02-11-1950	unemployment benefits	37	01-02-2011	07-02-2011
3	Febuary	M	02-11-1950	early retirement pension	109	08-02-2011	28-02-2011
4	March	F	07-04-1950	early retirement pension	148	01-03-2011	31-03-2011
5	January	M	08-12-1950	employed	148	01-01-2011	31-01-2011

Table 1: Simulated example of the data contained in the labour market register.

## Early Retirement Pension (ERP)

Table 2: Ages at which Early Retirement Pension (ERP) is available according to date of birth.

Date of Birth	ERP age
< 1954	60
$\geq 0$ 1-January-1954	60.5
$\geq$ 01-July-1954	61
$\geq 0$ 1-January-1955	61.5
$\geq$ 01-July-1955	62
$\geq 0$ 1-January-1956	62.5
$\geq$ 01-July-1956	63
$\geq 0$ 1-January-1959	63.5
$\geq$ 01-July-1959	64
> 1963	computed in relation
	to life expectancy

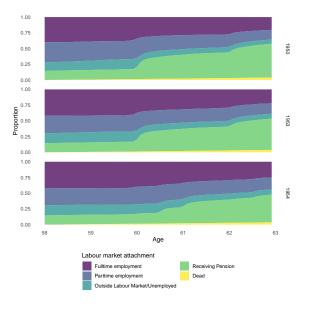


Figure 1: Attachment to the labour market over time for three cohorts.

# Sequence analysis

## Sequence analysis

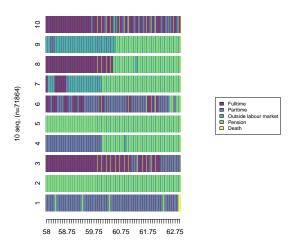


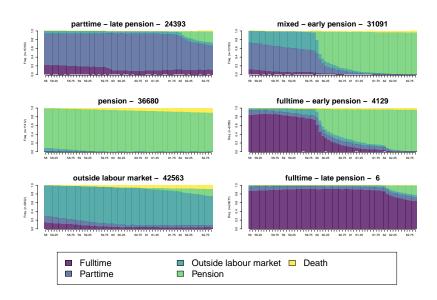
Figure 2: Ten example sequences for the 1950 cohort.

## Implementation in R

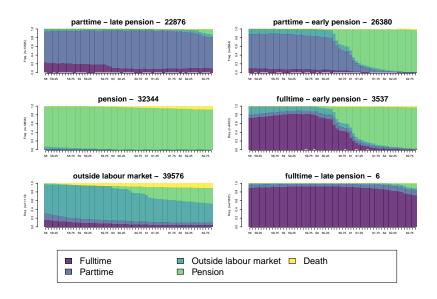
### R-packages:

- TraMineR
- ▶ TraMineRextras
- WeightedCluster

#### Clusters 1950 cohort



#### Clusters 1954 cohort



## Crosstabulations

Cluster	1950	1954
fulltime - late pension	0.28	0.31
fulltime - early pension	0.09	0.09
parttime - late pension	0.21	0.19
parttime - early pension	_	0.09
outside labour market	0.07	0.16
mixed - early pension	0.18	-
pension	0.16	0.15

## Crosstabulations II

Category	Male	Female
fulltime - late pension	14236	8143
outside labour market - late pension	2999	2929
fulltime - early pension	6566	8114
parttime - late pension	4274	7461
mixed - early pension	4765	6277
pension	3076	2908

But

What if we want to model the time to pension?

Joint Models for Longitudinal and Survival data

## Joint Models for Longitudinal and Survival data

Individuals  $i=1,\ldots,N$ , Timepoints  $t=1,\ldots,n_i$ 

Longitudinal submodel:

$$y_i(t) = \boldsymbol{x}_i(t)'\boldsymbol{\beta} + \boldsymbol{z}_i(t)'\boldsymbol{b}_i + \epsilon_i(t)$$

Survival submodel:

$$\lambda_i(t) = \lambda_0(t) \exp(\mathbf{s}_i(t)' \gamma + w_i(t))$$

Why?

#### Bias:

- Endogeneity
- non-random dropout
- measurement error

#### Joint Model

#### Longitudinal:

$$\pi_{itk} = P(Y_{it} = k) = \frac{1}{1 + \sum_{h=1}^{K-1} \exp(\mathbf{X}_{it}\beta + b_{ih})} \text{ if } k = K$$

$$\frac{\exp(\mathbf{X}_{it}\beta + b_{ik})}{1 + \sum_{h=1}^{K-1} \exp(\mathbf{X}_{it}\beta + b_{ih})} \text{ if } k = 1, \dots, K-1,$$

 $k \in \{1, \dots, K\} = \{$ Fulltime, Parttime, Outside the labour market $\}$ ,  $b_{ih} =$  random intercepts.

#### Survival:

$$\log \mu_{it} = \log t_{it} + \mathbf{X}_{it} \boldsymbol{\eta} + \alpha_t + u_i,$$

 $\mu_{it}=$  hazard,  $\alpha_t=\log \lambda_t=\log$  baseline hazard,  $t_{it}=$  time at risk (offset),  $u_i=$  frailty,  $\log \lambda_t=\mu_\lambda+N(\log \lambda_{t-1},\sigma_\lambda)$ .

### Joint Model

Random intercept  $b_{ih}$  and frailty  $u_i$  assumed to follow a multivariate normal distribution with the following variance-covariance matrix:

$$\mathbf{\Sigma} = \begin{bmatrix} \Sigma_{\boldsymbol{b}} & \Sigma'_{\boldsymbol{b}\boldsymbol{u}} \\ \Sigma_{\boldsymbol{b}\boldsymbol{u}} & \sigma_{\boldsymbol{u}}^2 \end{bmatrix},$$

$$\Sigma_{\boldsymbol{b}} = \begin{vmatrix} \sigma_{\boldsymbol{b}_1}^2 & \sigma_{\boldsymbol{b}_1, \boldsymbol{b}_2} \\ \sigma_{\boldsymbol{b}_2, \boldsymbol{b}_1} & \sigma_{\boldsymbol{b}_2}^2 \end{vmatrix}, \ \Sigma_{\boldsymbol{b}\boldsymbol{u}} = (\sigma_{\boldsymbol{b}_1, \boldsymbol{u}}, \sigma_{\boldsymbol{b}_2, \boldsymbol{u}})^t$$

## Implementation in R

- Bayesian model
- Rstan

## Results

## 1. Fixed Effects: Survival Submodel

	19	50 cohort	1954 cohort		
	HR	CI	HR	CI	
sex	1.47	(1.14, 1.87)	1.92	(1.45, 2.49)	
education	0.78	(0.59, 1.01)	0.64	(0.46, 0.85)	

## 2. Fixed Effects: Longitudinal Submodel

		1950 cohort		1954 cohort			
		β	OR	CI	β	OR	CI
	intercept	2.65	14.15	(1.91, 3.37)	3.00	20.09	(2.12, 3.90)
fulltime employment	sex	-2.23	0.11	(-3.28, -1.23)	-2.06	0.13	(-3.25, -0.88)
	education	1.81	6.11	(0.69, 2.97)	2.29	9.87	(1.05, 3.52)
	intercept	1.51	4.53	(1.00, 2.05)	1.19	3.29	(0.43, 1.97)
parttime employment	sex	0.65	1.92	(-0.07, 1.34)	0.31	1.36	(-0.68, 1.30)
	education	0.53	1.70	(-0.28, 1.39)	1.71	5.53	(0.60, 2.77)

## 3. Random Effects/Frailties

	19	950 cohort	1954 cohort		
	mean	CI	mean	CI	
$r_{(\boldsymbol{b}_1,\boldsymbol{b}_2)}$	0.61	(0.53, 0.69)	0.64	(0.56, 0.72)	
$r_{(\boldsymbol{b}_1,\boldsymbol{u})}$	-0.79	(-0.95, -0.58)	-0.77	(-0.95, -0.53)	
$r_{(\boldsymbol{b}_2,\boldsymbol{u})}$	-0.35	(-0.59, -0.11)	-0.49	(-0.76, -0.20)	
$\sigma_{\boldsymbol{b}_1}$	5.67	(5.16, 6.25)	6.31	(5.73, 6.98)	
$\sigma_{\boldsymbol{b}_2}$	3.90	(3.55, 4.28)	5.21	(4.70, 5.79)	
$\sigma_{u}$	0.50	(0.32, 0.70)	0.44	(0.25, 0.65)	

## Sequence analysis vs. Joint Models

- typification of trajectories
- duration and pattern
- inclusion of covariates