A joint model for the analysis of trajectories into retirement using the Danish labour market registry

Jolien Cremers

Section of Biostatistics, University of Copenhagen
&
Statistics Denmark

28 Feb 2020

Project

Postdoc project part of a larger project from a Challenge grant “Harnessing the Power of Big Data to Address the Societal Challenge of Ageing” funded by Novo Nordisk Fonden.

Focus: methods for analysing life-trajectories from register data.

Today: Occupational Trajectories and Time to Retirement in Later Life

Outline

- Early retirement pension
- Labour market registry & occupational trajectories
- Joint models
- Results from initial analysis
- Methodological challenges
- Further research

Early Retirement Pension Reform

Date of Birth	ERP age
$<$ 1954	60
$\geq$ 01-01-1954	60.5
$\geq$ 01-07-1954	61
$\geq$ 01-01-1955	61.5
$\geq$ 01-07-1955	62
$\geq$ 01-01-1956	62.5
$\geq$ 01-07-1956	63
$\geq$ 01-01-1959	63.5
$\geq$ 01-07-1959	64
$>$ 1963	computed in relation to life expectancy

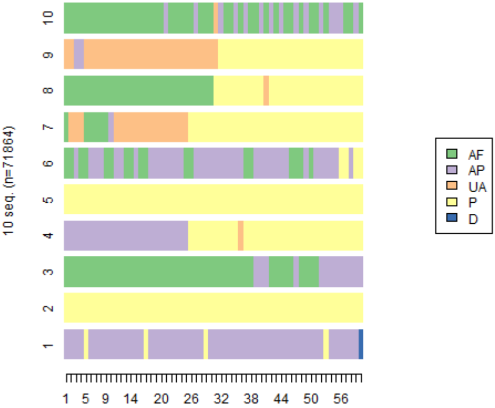
Design

- 3 cohorts, pre and post reform
  - 1950 (71,459 individuals)
  - 1953 (71,245 individuals)
  - 1954 (69,951 individuals)
- followed from age 58 until age 63 (5 years)
- monthly data (60 months)

Labour market registry (2008 - 2017)

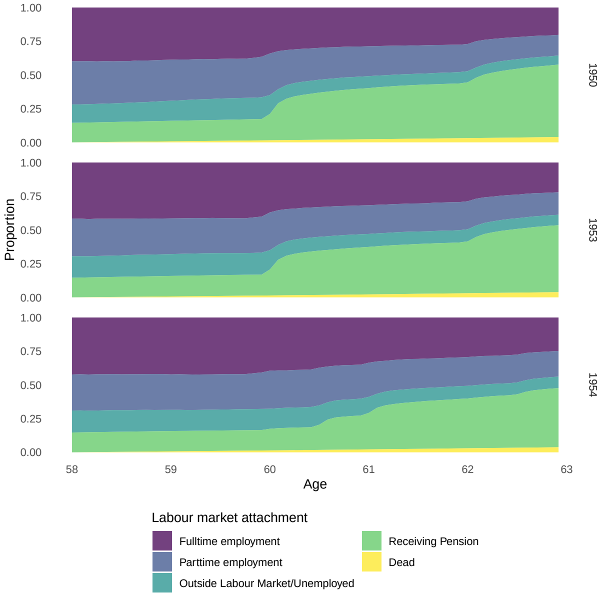
id	month	sex	birthdate	status	hours	start	end
1	January	M	08-03-1950	sick leave	148	01-01-2011	31-01-2011
2	January	F	21-09-1950	employed	10	20-01-2011	31-01-2011
2	Febuary	F	21-09-1950	employed	50	01-02-2011	28-02-2011
3	January	M	02-11-1950	employed	10	01-01-2011	15-01-2011
3	January	M	02-11-1950	employed	10	01-01-2011	15-01-2011
3	January	M	02-11-1950	unemployed	74	16-01-2011	31-01-2011
3	Febuary	M	02-11-1950	unemployed	37	01-02-2011	07-02-2011
3	Febuary	M	02-11-1950	pension	109	08-02-2011	28-02-2011
4	March	F	07-04-1950	pension	148	01-03-2011	31-03-2011
5	January	M	08-12-1950	employed	148	01-01-2011	31-01-2011

Occupational Trajectories for 10 individuals



AF = fulltime employment, AP = parttime employment, UA = outside the labour market/unemployed, P = pension (any), D = death.

Occupational Trajectories for 3 Cohorts



Joint Models

Individuals $i = 1, \ldots, N$, Timepoints $t = 1, \ldots, n_i$

Longitudinal submodel:

$$y_{it} = \boldsymbol{x}_{it}^{(1)t} \boldsymbol{\beta}^{(1)} + \boldsymbol{z}_{it}^{t} \boldsymbol{b}_i + \epsilon_{ij}$$

Survival submodel:

$$\lambda_i(t) = \lambda_0(t) \exp(\boldsymbol{x}_i^{(2)^t} \boldsymbol{\beta}^{(2)} + w_i(t))$$

Joint Models

Parameterization $w_i(t)$:

- current value: $w_i(t) = \alpha \boldsymbol{z}_{it}^t \boldsymbol{b}_i$, implemented in stan_jm/JM/JMbayes
- correlated random-effects: $w_i(t) = \alpha\theta_i$, where $\begin{bmatrix} \boldsymbol{b}_i \\ \theta_i \end{bmatrix} \sim N\left(\begin{bmatrix} \mathbf{0} \\ 0 \end{bmatrix}, \begin{bmatrix} \Sigma_{bb}\Sigma_{b\theta} \\ \Sigma_{b\theta}\sigma_\theta^2 \end{bmatrix} \right)$

See Hickey et.al. (2018)[1] for more parameterizations.

[1] Hickey, G.L., Philipson, P., Jorgensen, A. & Kolamunnage-Dona, R. (2018). A comparison of joint models for longitudinal and competing risks data, with application to a epilepsy drug randomized controlled trial. Journal of the Royal Statistical Society A, 181(4), p. 1105-1123.

Joint Model: Longitudinal Submodel

Hierarchical Multinomial:

Probability state
$k \in \{1, \ldots, K\} = \{\text{Fulltime}, \text{Parttime}, \text{Outside the labour market}\}$:

$$\pi_{itk} = P(Y_{it} = k \mid \boldsymbol{x}_{it}, b_{ih}) = \begin{array}{l} \dfrac{1}{1 + \sum_{h=1}^{K-1} \exp(\boldsymbol{x}_{it}\boldsymbol{\beta}_h + b_{ih})} \text{ if } k = K \\ \dfrac{\exp(\boldsymbol{x}_{it}\boldsymbol{\beta}_h + b_{ih})}{1 + \sum_{h=1}^{K-1} \exp(\boldsymbol{x}_{it}\boldsymbol{\beta}_h + b_{ih})} \text{ if } k = 1, \ldots, K-1, \end{array}$$

$\boldsymbol{x}_{it}\boldsymbol{\beta}_h = \beta_{0h} + \beta_{1h} * \text{Sex}_i + \beta_{2h} * \text{Education}_i$
$b_{ih} = \text{random intercept}$

Joint Model: Survival Submodel

Hierarchical Poisson log-linear model (proportional hazards model with piece-wise constant baseline hazard):

$$\log \mu_{it} = \log t_{it} + \boldsymbol{x}_{it}\boldsymbol{\eta} + \alpha_t + u_i,$$

μ_{it} = hazard
$\alpha_t = \log \lambda_t = \mu_\lambda + N(\log \lambda_{t-1}, \sigma_\lambda)$ (random walk baseline hazard)
$\boldsymbol{x}_{it}\boldsymbol{\eta} = \eta_1 * \text{Sex}_i + \eta_2 * \text{Education}_i$
u_i = frailty
t_{it} = offset

Joint Model: Link

Random intercept b_{ih} and frailty u_i assumed to follow a multivariate normal distribution with the following variance-covariance matrix:

$$\mathbf{\Sigma} = \begin{bmatrix} \Sigma_{\boldsymbol{b}} & \Sigma'_{\boldsymbol{b}u} \\ \Sigma_{\boldsymbol{b}u} & \sigma_u^2 \end{bmatrix},$$

$$\Sigma_{\boldsymbol{b}} = \begin{bmatrix} \sigma^2_{\boldsymbol{b}_1} & \sigma_{\boldsymbol{b}_1,\boldsymbol{b}_2} \\ \sigma_{\boldsymbol{b}_2,\boldsymbol{b}_1} & \sigma^2_{\boldsymbol{b}_2} \end{bmatrix}, \;\; \Sigma_{\boldsymbol{bu}} = (\sigma_{\boldsymbol{b}_1,\boldsymbol{u}}, \sigma_{\boldsymbol{b}_2,\boldsymbol{u}})^t$$

Joint Model: Estimation

Bayesian Model in Stan on a subset of the data ($\pm 10\%$ stratified sample on sex and education)

Priors:

- regression coefficients ($\boldsymbol{\beta}$, $\boldsymbol{\eta}$):
  - $N(\mu_0 = 0, \sigma_0 = 10000)$
- baseline hazards:
  - $\mu_\lambda \sim N(0, 1)$
  - $\sigma_\lambda \sim N(0, 1)$
  - $\log(\lambda_0) \sim N(0, 1)$
- random effects:
  - $(\boldsymbol{b}_i, u_i) \sim MVN(\mathbf{0}, \mathbf{\Sigma})$
- variance-covariance random effects ($\mathbf{\Sigma}$):
  - $LKJ(2)$ prior for correlations ($LKJ(1) = U(-1, 1)$)
  - $\exp(\lambda = 0.5)$ prior for standard deviation

Results

3 different sets of results:

1. relation between employment status and retirement timing for the three cohorts
2. differences between the three cohorts in the effect of sex and education level on retirement timing
3. differences between the three cohorts with regard to the probability of having a certain employment status and the effects of sex and education level on this status

Random Effects/Frailties

Instead of looking at the variance-covariance matrix we look at a matrix with correlations on the off-diagnonals and standard deviations on the diagonals:

$$\boldsymbol{R} = \begin{bmatrix} \sigma_{\boldsymbol{b}_1} & r_{(\boldsymbol{b}_1,\boldsymbol{b}_2)} & r_{(\boldsymbol{b}_1,\boldsymbol{u})} \\ r_{(\boldsymbol{b}_1,\boldsymbol{b}_2)} & \sigma_{\boldsymbol{b}_2} & r_{(\boldsymbol{b}_2,\boldsymbol{u})} \\ r_{(\boldsymbol{b}_1,\boldsymbol{u})} & r_{(\boldsymbol{b}_2,\boldsymbol{u})} & \sigma_{\boldsymbol{u}} \end{bmatrix} \tag{1}$$

Random Effects/Frailties

	1950 cohort		1953 cohort		1954 cohort	
	mean	CI	mean	CI	mean	CI
$r_{(\boldsymbol{b}_1,\boldsymbol{b}_2)}$	0.61	(0.53, 0.69)	0.64	(0.56, 0.71)	0.64	(0.56, 0.72)
$r_{(\boldsymbol{b}_1,\boldsymbol{u})}$	-0.79	(-0.95, -0.58)	-0.82	(-0.96, -0.63)	-0.77	(-0.95, -0.53)
$r_{(\boldsymbol{b}_2,\boldsymbol{u})}$	-0.35	(-0.59, -0.11)	-0.53	(-0.73, -0.31)	-0.49	(-0.76, -0.20)
$\sigma_{\boldsymbol{b}_1}$	5.67	(5.16, 6.25)	5.97	(5.41, 6.59)	6.31	(5.73, 6.98)
$\sigma_{\boldsymbol{b}_2}$	3.90	(3.55, 4.28)	4.90	(4.45, 5.42)	5.21	(4.70, 5.79)
$\sigma_{\boldsymbol{u}}$	0.50	(0.32, 0.70)	0.57	(0.38, 0.79)	0.44	(0.25, 0.65)

Fixed Effects: Survival Submodel

Hazard Ratios:

	1950 cohort		1953 cohort		1954 cohort	
	mean	CI	mean	CI	mean	CI
sex	1.47	(1.14, 1.87)	1.51	(1.17, 1.91)	1.92	(1.45, 2.49)
education	0.78	(0.59, 1.01)	0.78	(0.59, 1.01)	0.64	(0.46, 0.85)

Fixed Effects: Longitudinal Submodel

Coefficients and odds ratios:

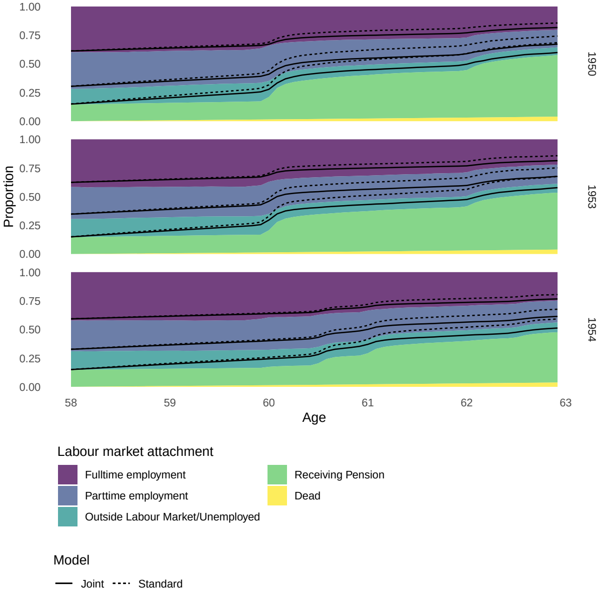
		1950 cohort			1954 cohort		
		mean		CI	mean		CI
		β	$\exp(\beta)$		β	$\exp(\beta)$	
fulltime employment	intercept	2.65	14.15	(1.91, 3.37)	3.00	20.09	(2.12, 3.90)
	sex	-2.23	0.11	(-3.28, -1.23)	-2.06	0.13	(-3.25, -0.88)
	education	1.81	6.11	(0.69, 2.97)	2.29	9.87	(1.05, 3.52)
parttime employment	intercept	1.51	4.53	(1.00, 2.05)	1.19	3.29	(0.43, 1.97)
	sex	0.65	1.92	(-0.07, 1.34)	0.31	1.36	(-0.68, 1.30)
	education	0.53	1.70	(-0.28, 1.39)	1.71	5.53	(0.60, 2.77)

Fixed Effects: Longitudinal Submodel

Probabilities:

		1950 cohort		1954 cohort	
		mean	CI	mean	CI
mean	CI				
fulltime	male + low	0.71	(0.59, 0.82)	0.82	(0.71, 0.90)
	male + high	0.90	(0.81, 0.96)	0.90	(0.81, 0.96)
	female + low	0.14	(0.21, 0.57)	0.32	(0.18, 0.48)
	female + high	0.37	(0.10, 0.39)	0.49	(0.29, 0.70)
parttime	male + low	0.23	(0.14, 0.34)	0.14	(0.08, 0.22)
	male + high	0.09	(0.04, 0.17)	0.09	(0.03, 0.18)
	female + low	0.77	(0.67, 0.85)	0.55	(0.40, 0.70)
	female + high	0.59	(0.40, 0.74)	0.49	(0.29, 0.69)
outside	male + low	0.05	(0.03, 0.09)	0.04	(0.02, 0.09)
	male + high	0.01	(0.00, 0.03)	0.01	(0.00, 0.01)
	female + low	0.09	(0.05, 0.15)	0.13	(0.06, 0.23)
	female + high	0.04	(0.02, 0.08)	0.02	(0.01, 0.05)

Comparison to standard survival and longitudinal models



Methodological challenges

Computational speed:

- Replace multinomial setup with multiple logistic models
- Parallell MCMC computation through Wasserstein barycenters of sub-posteriors (WASP)[1]
- Deep learning models (e.g. dynamic-DeepHit[2]), but:
  - Interpretation (nonlinear functions of variables)
  - Variable importance?

[1] Srivastava, S., Cevher, V., Dinh, Q., & Dunson, D. (2015). WASP: Scalable Bayes via barycenters of subset posteriors. *Artificial Intelligence and Statistics, 38*, 912–920.

[2] Lee, C., Yoon, J & van der Schaar, M. (2020). Dynamic-DeepHit: A deep learning approach for dynamic survival analysis with competing risks based on longitudinal data. *IEEE Transactions on Biomedical Engineering, 67* (1), 122-133.

Further research

- Include death in a competing risks setup
- Inclusion of health related variables (e.g. disease diagnosis, medicine usage) as additional longitudinal or survival outcomes
    - Advantage of registry data (patient registry) instead of subjective health measurements[1]
    - But, how to operationalize/quantify 'health'?
    - Use reform as an instrumental variable.
- Sequence analysis to identify typical employment trajectories (exploratory).

[1] See e.g. Christensen, B.J. & Kallestrup-Lamb, M. (2012). The impact of health changes on labor suppy: Evdence from merge data on individual objective medical diagnosis codes and early retirement behavior. *Health Economics, 21*, 56-100.