Homework 8

AMATH 352, Fall 2022

Due on Nov 28, 2022 at midnight.

Directions, Reminders and Policies

Read these instructions carefully:

- **You are required to upload a PDF report to Canvas along with a zip of your code.**
- The report should be a maximum of 3 pages long with references included. Minimum font size 10pts and margins of at least 1inch on A4 or standard letter size paper.
- Do not include your code in the report. Simply create a zip file of your main scripts and functions, without figures or data sets included, and upload the zip file to Canvas.
- Your report should be formatted as follows:
  - Title/author: Title of report, your name and email address. This is not meant to be a separate title page.
  - Sec. 1. Introduction and overview of the problem.
  - Sec. 2. Theoretical background and description of algorithms.
  - Sec. 3. Computational Results
  - Sec. 4. Summary and Conclusions
  - References
- I suggest you use LaTeX(Overleaf is a great option) to prepare your reports. A template is provided on Canvas under the Syllabus tab. You are also welcome to use Microsoft Word or any other software that properly typesets mathematical equations.
- I encourage collaborations, however, everything that is handed in (both your report and your code) should be your work.
- Your homework will be graded based on how completely you solved it as well as neatness and little things like: did you label your graphs and include figure captions. **The homework is worth 10 points. 5 points will be given for the overall layout, correctness and neatness of the report, and 5 additional points will be for specific technical things and computational results that the TAs will look for in the report itself.**

Problem Description

Your goal in this HW is to explore the use of SVD in image processing and compression.

- Python users: Download the file `faces.npz` from Canvas and load it using `numpy.load`.
- MATLAB users: Download the file `faces.mat` from Canvas and load it using the `load` command.

The provided files contain a matrix A of size 766×713. Each row of A is an image of size 31×23 of former President George W Bush and former Secretary of State Colin Powell, stored as a one dimensional array. You can use the following code snippet to plot the j-th image and see what they look like:

```
import matplotlib.pyplot as plt
plt.imshow(A[j,:].reshape((31, 23), cmap='gray'))
```

Note the reshape command is used to convert the rows of A, which are one dimensional vectors to two dimensional matrices that can be plotted as images. Henceforth, "plotting as an image" refers to using the above snippet to reshape and plot a one dimensional vector as an image.

1. Use `numpy.mean` to compute the mean of the rows of A and plot it as an image.
2. Define the 766×713 matrix B by subtracting the mean image from the rows of A.
3. Use `numpy.linalg.svd` to compute the SVD of B. Let σ_j for $j = 1, \ldots, 713$ denote the singular values of B. Present a plot of $\log(\sigma_j)$ vs j. Discuss your observations.
4. Plot the first 5 right singular vectors of B as images. Comment on your observations.
5. Let B_r be the rank-r approximation to B obtained by truncating its SVD and consider the relative error
$$\frac{\|B - B_r\|_F}{\|B\|_F}.$$
Find the smallest value of r needed for achieving the relative error of $30\%, 20\%, 10\%$ and 1%. Present your findings in a table.
6. Plot the first row of B along with the first row of the matrix B_r as images, for the values of r that you found in Step 4. Comment on the quality of the approximate images and how they compare to the original.