

Stat 311 Homework 4

This assignment does not use any data sets but will include a few parts of problems where you will copy/paste R code with answers. Parts of a few problems are modified from McClave and Sincich, *Statistics*, Ninth Edition.

- You may handwrite or type up this assignment and submit as .docx or .pdf. Do **not** do this assignment in Rmarkdown.
 - If you handwrite your assignment, you can write down your R code with the answers in the few places where you are to show your R code.
 - Be sure to show work as needed. If you simply put down a numeric answer it will be marked wrong, even if the number is correct.
 - All requested interpretations should be in complete sentences.
1. A fair six-sided die is tossed, and the up face is noted. If the number is 1, the die is tossed again; if the number is > 1 , a fair coin is tossed. Define the events:
A: {A tail appears on the coin} and B: {The die is tossed only one time}
 - a) List the sample points in the sample space [Hint: there are 16 sample points].
 - b) Give the probability for each of the sample points.
 - c) Find $P(A)$ and $P(B)$.
 - d) Identify the sample points in A^c , B^c , $A \cap B$, and $A \cup B$.
 - e) Find $P(A^c)$, $P(B^c)$, $P(A \cap B)$, $P(A \cup B)$, $P(A | B)$, and $P(B | A)$.
 - f) Are A and B mutually exclusive events? Why? Justify
 - g) Are the events A and B independent? Justify your answer using an equation.
 2. A balanced six-sided die is thrown once. If a 4 appears, a ball is drawn from urn 1; otherwise, a ball is drawn from urn 2. Urn 1 contains four red and ten white balls. Urn 2 contains five red, four white, and five black balls.
 - a) Find the probability that a red ball is drawn.
 - b) Find the probability that urn 2 was used given that a red ball was drawn.
 3. Seventy-five percent of all women who submit to pregnancy tests are pregnant. A new pregnancy test gives a false positive result with probability 0.02 and a correct positive result with probability 0.97. Define the events: P : {a woman is pregnant} and $+$: {the pregnancy test is positive}.
 - a) Using the events P and $+$ as defined, write out the information given in this problem.
 - b) What is the sensitivity of this test? Explain in layperson terms what this means.
 - c) What is the specificity of this test? Explain in layperson terms what this means.
 - d) What is the false negative rate (FNR) for this test? Explain in layperson terms what this means.
 - e) What is the unconditional probability that this test produces a positive result? Explain in layperson terms what this means.
 - f) For a randomly chosen woman who receives a positive result using this test, what is the probability that she is pregnant? Interpret this result.

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4. City officials estimate that a dangerous intersection in the city has 0.75 accidents per month. Let X be the number of accidents that occur at this intersection over the next two months.
 - a) What probability distribution can you use to model X ?
 - b) Calculate the probability that at most three accidents occur at this intersection in the next two months. Write out the equation you need with numbers substituted but solve using R. Copy/paste your R code here, with the answer.
5. During a tight job market, recruiters have noticed that graduating seniors with an intermediate proficiency in a second language have a higher probability of getting a first interview following a screening call when seeking their first job out of college. For students with one language, the probability of getting a first interview following a screening call is 0.25, whereas for graduating seniors with a second language the probability is 0.35. Let X be the number of screening calls a graduating senior with an intermediate proficiency in a second language must go through before getting their first interview.
 - a) Name a probability distribution that you could use to find probabilities of X . Do not forget to include values of any parameters.
 - b) On average, how many screening calls must a graduating senior with an intermediate proficiency in a second language go through before getting their first interview?
 - c) What is the exact probability that a graduating senior with an intermediate proficiency in a second language must have at most three screening calls before getting their first interview? Write down the formula you would use with numbers substituted but use R to calculate the answer. Copy and paste your R code here with the answer.
6. Let X be a random variable that denotes the number of tails in a set of 25 tosses.
 - a) What is the sample space for X ?
 - b) Assume that the coin is unfair with $P(T) = 0.35$. Assuming independent toss outcomes, what probability distribution can you use to model X ? Make sure to name the distribution and the values of any parameters.
 - c) What are $E(X) = \mu_X$ and σ_X based on the distribution named in part (b) above? Interpret these numbers.
 - d) What is the exact probability that $X > 5$ in any given case? Write down the formula you would use with numbers substituted but use R to calculate the answer. Copy and paste your R code here with the answer.
 - e) Explain why the number of heads out in a set of 25 tosses can be modeled as a binomial random variable, Y , with parameters $n = 25$ and $p = 0.65$.
7. A hung jury is one that is unable to come to a unanimous decision regarding the guilt of the defendant. Suppose that there is a pool of 40 potential jurors, but 3 of the 40 potential jurors would never be willing to convict, regardless of the evidence presented. Twelve of the potential jurors will be randomly selected to be seated for the trial. Define X to be the number of jurors in the jury pool that would never be willing to convict.
 - a) Name a probability distribution that you could use to find probabilities of X . Do not forget to include values of any parameters.
 - b) What is the probability that the trial will result in a hung jury, regardless of the evidence, given a jury of 12 randomly selected jurors from the pool? Write out the equation you need with numbers substituted but solve using R. Copy/paste your R code here, with the answer.