Numerical Methods HW #04

1. 이산자료가 아래와 같이 주어졌을 때 각각의 Lagrange 3차 보간다항식을 구하고, 그 다항식을 이용하여 x=0.7에서 함수의 근사값을 구하라.

(a)

```c
#include<stdio.h>
int main()
{
	int i,j;
	double x=0.7;
	double v[4]={0,0.4,0.6,1.0};//variable
	double value[4]={1.0000,0.9604,0.9120,0.7652};
	double L[4]={1,1,1,1};
	for(i=0;i<=3;i++)
	{
		for(j=0;j<=i-1;j++)
		{
			L[i]=L[i]*(x-v[j])/(v[i]-v[j]);
		}
		for(j=i+1;j<=3;j++)
		{
			L[i]=L[i]*(x-v[j])/(v[i]-v[j]);
		}
	}
	double F=0;
	for(i=0;i<=3;i++)
	{
		F=F+L[i]*value[i];
	}
	printf("%lf",F);
	double a[4]={value[0],value[1],value[2],value[3]};//nominator
	for(i=0;i<=3;i++)
	{
		for(j=0;j<=i-1;j++)
		{
			a[i]=a[i]/(v[i]-v[j]);
		}
		for(j=i+1;j<=3;j++)
		{
			a[i]=a[i]/(v[i]-v[j]);
		}
	}
	double b[4][4]={0};//ab,ac,ad,bc,bd,cd
	for(i=0;i<=2;i++)
	{
		for(j=i+1;j<=3;j++)
		{
			b[i][j]=v[i]*v[j];
		}
	}
	double c[4]={1,1,1,1};//abc,abd,acd,bcd
	for(i=0;i<=3;i++)
	{
		for(j=0;j<=i-1;j++)
		{
			c[i]=c[i]*v[j];
		}
		for(j=i+1;j<=3;j++)
		{
			c[i]=c[i]*v[j];
		}
	}
	double A,B,C,D;//Coefficients of polynomial
	A=0;B=0; D=0;
		B=B-(v[0]+v[1]+v[2]+v[3]-v[i])*a[i];
		D=D-c[i]*a[i];
	}
```

```c
        for(i=0;i<=3;i++)
        {
                A=A+a[i];
                B=B-(v[0]+v[1]+v[2]+v[3]-v[i])*a[i];
                D=D-c[i]*a[i];
        }
        C=a[0]*(b[1][2]+b[1][3]+b[2][3])+a[1]*(b[0][2]+b[0][3]+b[2][3])+a[2]*(b[0][1]+b[0][3]+b[1][3])+a[3]*(b[0][1]+b[0][2]+b[1][2]);
        printf("\n%lf\n%lf\n%lf\n%lf",A,B,C,D);
}
```

<Result>

$$P_3(x) = 0.03x^3 - 0.2683x^2 + 0.0035x + 1 \quad , \quad P_3(0.7) = 0.8813$$

(b)

```c
#include<stdio.h>
int main()
{
        int i,j;
        double x=0.7;
        double v[4]={0,0.2,0.6,1.0};//variable
        double value[4]={0.0000,0.2227,0.6039,0.8427};
        double L[4]={1,1,1,1};
        for(i=0;i<=3;i++)
        {
                for(j=0;j<=i-1;j++)
                {
                        L[i]=L[i]*(x-v[j])/(v[i]-v[j]);
                }
                for(j=i+1;j<=3;j++)
                {
                        L[i]=L[i]*(x-v[j])/(v[i]-v[j]);
                }
        }
        double F=0;
        for(i=0;i<=3;i++)
        {
                F=F+L[i]*value[i];
        }
        printf("%lf",F);
        double a[4]={value[0],value[1],value[2],value[3]};//nominator
        for(i=0;i<=3;i++)
        {
                for(j=0;j<=i-1;j++)
                {
                        a[i]=a[i]/(v[i]-v[j]);
                }
                for(j=i+1;j<=3;j++)
                {
                        a[i]=a[i]/(v[i]-v[j]);
                }
        }
        double b[4][4]={0};//ab,ac,ad,bc,bd,cd
        for(i=0;i<=2;i++)
        {
                for(j=i+1;j<=3;j++)
                {
                        b[i][j]=v[i]*v[j];
                }
        }
        double c[4]={1,1,1,1};//abc,abd,acd,bcd
        for(i=0;i<=3;i++)
        {
                for(j=0;j<=i-1;j++)
                {
                        c[i]=c[i]*v[j];
                }
```

```
                  for(j=i+1;j<=3;j++)
                  {
                           c[i]=c[i]*v[j];
                  }
         }
         double A,B,C,D;//Coefficients of polynomial
         A=0;B=0; D=0;
         for(i=0;i<=3;i++)
         {
                  A=A+a[i];
                  B=B-(v[0]+v[1]+v[2]+v[3]-v[i])*a[i];
                  D=D-c[i]*a[i];
         }
         C=a[0]*(b[1][2]+b[1][3]+b[2][3])+a[1]*(b[0][2]+b[0][3]+b[2][3])+a[2]*(b[0][1]+b[0][3]+b[1]
[3])+a[3]*(b[0][1]+b[0][2]+b[1][2]);
         printf("\n%lf\n%lf\n%lf\n%lf",A,B,C,D);
}
```

<Result>

$$P_3(x) = -0.1775x^3 - 0.1255x^2 + 1.1457x \quad , \quad P_3(0.7) = 0.6796$$

(c)

```
#include<stdio.h>
int main()
{
         int i,j;
         double x=0.7;
         double v[4]={0.0,0.2,0.8,1.0};//variable
         double value[4]={0.0000,0.1996,0.7721,0.9461};
         double L[4]={1,1,1,1};
         for(i=0;i<=3;i++)
         {
                  for(j=0;j<=i-1;j++)
                  {
                           L[i]=L[i]*(x-v[j])/(v[i]-v[j]);
                  }
                  for(j=i+1;j<=3;j++)
                  {
                           L[i]=L[i]*(x-v[j])/(v[i]-v[j]);
                  }
         }
         double F=0;
         for(i=0;i<=3;i++)
         {
                  F=F+L[i]*value[i];
         }
         printf("%lf",F);
         double a[4]={value[0],value[1],value[2],value[3]};//nominator
         for(i=0;i<=3;i++)
         {
                  for(j=0;j<=i-1;j++)
                  {
                           a[i]=a[i]/(v[i]-v[j]);
                  }
                  for(j=i+1;j<=3;j++)
                  {
                           a[i]=a[i]/(v[i]-v[j]);
                  }
         }
         double b[4][4]={0};//ab,ac,ad,bc,bd,cd
         for(i=0;i<=2;i++)
         {
                  for(j=i+1;j<=3;j++)
                  {
                           b[i][j]=v[i]*v[j];
                  }
         }
         double c[4]={1,1,1,1};//abc,abd,acd,bcd
```

```
        for(i=0;i<=3;i++)
        {
                for(j=0;j<=i-1;j++)
                {
                        c[i]=c[i]*v[j];
                }
                for(j=i+1;j<=3;j++)
                {
                        c[i]=c[i]*v[j];
                }
        }
        double A,B,C,D;//Coefficients of polynomial
        A=0;B=0; D=0;
        for(i=0;i<=3;i++)
        {
                A=A+a[i];
                B=B-(v[0]+v[1]+v[2]+v[3]-v[i])*a[i];
                D=D-c[i]*a[i];
        }
        C=a[0]*(b[1][2]+b[1][3]+b[2][3])+a[1]*(b[0][2]+b[0][3]+b[2][3])+a[2]*(b[0][1]+b[0][3]+b[1]
[3])+a[3]*(b[0][1]+b[0][2]+b[1][2]);
        printf("\n%lf\n%lf\n%lf\n%lf",A,B,C,D);
}
```

<Result>

$$P_3(x) = -0.0504x^3 - 0.0044x^2 + 1.0009x \quad , \quad P_3(0.7) = 0.6812$$

(d)

```
#include<stdio.h>
int main()
{
        int i,j;
        double x=0.7;
        double v[4]={0.0,0.3,0.6,1.0};//variable
        double value[4]={0.0000,0.5477,0.7746,1.0000};
        double L[4]={1,1,1,1};
        for(i=0;i<=3;i++)
        {
                for(j=0;j<=i-1;j++)
                {
                        L[i]=L[i]*(x-v[j])/(v[i]-v[j]);
                }
                for(j=i+1;j<=3;j++)
                {
                        L[i]=L[i]*(x-v[j])/(v[i]-v[j]);
                }
        }
        double F=0;
        for(i=0;i<=3;i++)
        {
                F=F+L[i]*value[i];
        }
        printf("%lf",F);
        double a[4]={value[0],value[1],value[2],value[3]};//nominator
        for(i=0;i<=3;i++)
        {
                for(j=0;j<=i-1;j++)
                {
                        a[i]=a[i]/(v[i]-v[j]);
                }
                for(j=i+1;j<=3;j++)
                {
                        a[i]=a[i]/(v[i]-v[j]);
                }
        }
        double b[4][4]={0};//ab,ac,ad,bc,bd,cd
```

```
	for(i=0;i<=2;i++)
	{
		for(j=i+1;j<=3;j++)
		{
			b[i][j]=v[i]*v[j];
		}
	}
	double c[4]={1,1,1,1};//abc,abd,acd,bcd
	for(i=0;i<=3;i++)
	{
		for(j=0;j<=i-1;j++)
		{
			c[i]=c[i]*v[j];
		}
		for(j=i+1;j<=3;j++)
		{
			c[i]=c[i]*v[j];
		}
	}
	double A,B,C,D;//Coefficients of polynomial
	A=0;B=0; D=0;
	for(i=0;i<=3;i++)
	{
		A=A+a[i];
		B=B-(v[0]+v[1]+v[2]+v[3]-v[i])*a[i];
		D=D-c[i]*a[i];
	}
	C=a[0]*(b[1][2]+b[1][3]+b[2][3])+a[1]*(b[0][2]+b[0][3]+b[2][3])+a[2]*(b[0][1]+b[0][3]+b[1][3])+a[3]*(b[0][1]+b[0][2]+b[1][2]);
	printf("\n%lf\n%lf\n%lf\n%lf",A,B,C,D);
}
```

<Result>

$$P_3(x) = 1.5067x^3 - 3.1383x^2 + 2.6315x \quad , \quad P_3(0.7) = 0.8211$$

2. $0 < x \le 1$에 대한 함수 $f(x) = \frac{1}{x}$의 차분표를 구간간격 h=0.1로 하여 작성하고 x=0.48에서 함수의 근사값 $P_n(0.48)$과 절대 오차를 다음 방법으로 계산하라.

(a) Newton의 전진차분 보간다항식을 사용하되 기준점을 0.4로 하라.

x=0.48	참 값	2.0833			
x	f(x)	df	d2f	d3f	d4f
0.1	10.0000	-5.0000	3.3333	-2.5000	2.0000
0.2	5.0000	-1.6667	0.8333	-0.5000	0.3333
0.3	3.3333	-0.8333	0.3333	-0.1667	0.0952
0.4	2.5000	-0.5000	0.1667	-0.0714	0.0357
0.5	2.0000	-0.3333	0.0952	-0.0357	0.0159
0.6	1.6667	-0.2381	0.0595	-0.0198	0.0079
0.7	1.4286	-0.1786	0.0397	-0.0119	
0.8	1.2500	-0.1389	0.0278		
0.9	1.1111	-0.1111		전진차분	
1	1.0000				

		항	P(0.48)	절단오차	절대오차
		2.5000	2.5000	-0.4000	0.4167
s	0.8	-0.4000	2.1000	-0.0133	0.0167
s-1	-0.2	-0.0133	2.0867	-0.0023	0.0033
s-2	-1.2	-0.0023	2.0844	-0.0006	0.0010
s-3	-2.2	-0.0006	2.0838		0.0004

(b) Newton의 후진차분 보간다항식을 사용하되 기준점을 0.5로 하라.

x=0.48	참 값	2.0833			
x	f(x)	df	d2f	d3f	d4f
0.1	10.0000			후진차분	
0.2	5.0000	-5.0000			
0.3	3.3333	-1.6667	3.3333		
0.4	2.5000	-0.8333	0.8333	-2.5000	
0.5	2.0000	-0.5000	0.3333	-0.5000	2.0000
0.6	1.6667	-0.3333	0.1667	-0.1667	0.3333
0.7	1.4286	-0.2381	0.0952	-0.0714	0.0952
0.8	1.2500	-0.1786	0.0595	-0.0357	0.0357
0.9	1.1111	-0.1389	0.0397	-0.0198	0.0159
1	1.0000	-0.1111	0.0278	-0.0119	0.0079

		항	P(0.48)	절단오차	절대오차
		2.0000	2.0000	0.1000	0.0833
s	-0.2	0.1000	2.1000	-0.0267	0.0167
s+1	0.8	-0.0267	2.0733	0.0240	0.0100
s+2	1.8	0.0240	2.0973	-0.0672	0.0140
s+3	2.8	-0.0672	2.0301		0.0532

3. 다음 자료에 대한 3차 spline 곡선을 구하라. 단, 양 끝점에서는 natural cubic spline을 이용하라.

x	0	0.5	1	1.5
y	1	1.65	2.72	4.48

y_0	1.3
y_1	2.14
y_2	3.52

여기에서 y_0=$\bar{\Delta} y_1 = \left(\frac{y_1 - y_0}{h}\right)$

식(4.36)을 얻을 수 있다. $\sigma_i = s_i$라 하면

$$4s_1 + s_2 = \frac{6}{h}(\bar{\Delta}y_1 - \bar{\Delta}y_{0)}$$
$$s_1 + 4s_2 = \frac{6}{h}(\bar{\Delta}y_2 - \bar{\Delta}y_{1)}$$

두 식을 가우스 소거법으로 풀면 $s_0 = s_3 = 0$

$$s_1 = 1.5840 \quad , \quad s_2 = 3.7440$$

값을 대입하여 식(4.33)을 얻을 수 있다.

$$f_0(x) = \frac{1.5840}{6}\left\{\frac{x^2}{0.5} - 0.5x\right\} + \left\{\frac{0.5-x}{0.5}\right\} + \frac{1.65x}{0.5}$$

$$f_1(x) = \frac{1.5840}{6}\left\{\frac{(1-x)^3}{0.5} - 0.5(1-x)\right\} + \frac{3.7440}{6}\left\{\frac{(x-0.5)^2}{0.5} - 0.5(x-0.5)\right\} + \frac{1.65(1-x)}{0.5} + \frac{2.72(x-0.5)}{0.5}$$

$$f_2(x) = \frac{3.7440}{6}\left\{\frac{(1.5-x)^3}{0.5} - 0.5(1.5-x)\right\} + \frac{2.72(1.5-x)}{0.5} + \frac{4.48(x-1)}{0.5}$$

4. 다음은 열교환기를 지나는 공기유량과 압력강하 값을 측정한 것이다. 공기 유량과 압력강하의 관계는 2차식으로 나타낼 수 있다. 최소자승법을 이용하여 관계식을 구하라.

$$Q = a_0 + a_1(\Delta P) + a_2(\Delta P)^2$$

ΔP	6.272	7.056	7.84	8.232	9.212	11.368	11.956	13.132
Q	0.012825	0.012494	0.012327	0.012272	0.011941	0.011664	0.011499	0.011387

N=8

#	1	2	3	4	5	6	7	8	평균
Q	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.10
ΔP	6.27	7.06	7.84	8.23	9.21	11.37	11.96	13.13	75.07
2	39.34	49.79	61.47	67.77	84.86	129.23	142.95	172.45	747.84
3	246.73	351.30	481.89	557.85	781.74	1469.10	1709.06	2264.61	7862.27
4	1547.48	2478.76	3778.02	4592.21	7201.38	16700.76	20433.54	29738.80	86470.95
QP1	0.080	0.088	0.097	0.101	0.110	0.133	0.137	0.150	0.896
QP2	0.505	0.622	0.758	0.832	1.013	1.507	1.644	1.964	8.844
QP3	3.164	4.389	5.940	6.846	9.335	17.136	19.652	25.787	92.250
St	5.99.E-07	1.96.E-07	7.61.E-08	4.88.E-08	1.21.E-08	1.50.E-07	3.05.E-07	4.41.E-07	1.83.E-06
S	1.10.E-07	1.26.E-08	3.20.E-09	3.28.E-09	1.81.E-08	1.11.E-08	3.48.E-08	1.73.E-08	2.10.E-07

8.0	75.1	747.8	0.0964
75.1	747.8	7862.3	0.8959
747.8	7862.3	86470.9	8.8440

Q 평균	0.0121

8.0	75.1	747.8	0.0964
0.0	-6269.6	-727107.3	0.8851
0.0	-65913.4	-7996879.4	8.7374

8	75.1	747.8	0.0964	>	a0	1.3385E-02
0	-6269.6	-727107.3	0.8851		a1	-1.4227E-04
0	0.0	919436018.9	8.7386		a2	9.5043E-09

결정계수	0.8849

5. 아래의 자료는 온도의 변화에 따른 물의 열전도계수를 나타낸 것이다. 이 자료에 대해 최소자승법을 사용하여 1~4차 다항식의 형으로 적합곡선을 구하고, 그 중 어느 것이 최적의 적합곡선인가를 판별하라.

$T(°C)$	0	10	27	32	49	60	71	82
$k(W/m\ \cdot °C)$	0.556	0.585	0.614	0.623	0.644	0.654	0.665	0.673

k	0.556	0.585	0.614	0.623	0.644	0.654	0.665	0.673	5.0140
T	0	10	27	32	49	60	71	82	331.00
T^2	0	100	729	1024	2401	3600	5041	6724	19619
T^3	0	1000	19683	32768	117649	216000	357911	551368	1296379
T^4	0	10000	531441	1048576	5764801	12960000	25411681	45212176	90938675
T^5	0	100000	14348907	33554432	282475249	777600000	1804229351	3707398432	6619706371
T^6	0	1000000	387420489	1073741824	13841287201	46656000000	1.281E+11	3.04007E+11	494066404859
T^7	0	10000000	10460353203	34359738368	6.78223E+11	2.79936E+12	9.09512E+12	2.49285E+13	37546080379579
T^8	0	100000000	2.8243E+11	1.09951E+12	3.32329E+13	1.67962E+14	6.45754E+14	2.04414E+15	2892470961634590
kT	0	5.85	16.578	19.936	31.556	39.24	47.215	55.186	215.56
kT2	0	58.5	447.606	637.952	1546.244	2354.4	3352.265	4525.252	12922
kT3	0	585	12085.362	20414.464	75765.956	141264	238010.815	371070.664	859196
kT4	0	5850	326304.774	653262.848	3712531.844	8475840	16898767.87	30427794.45	60500352
kT5	0	58500	8810228.898	20904411.14	181914060.4	508550400	1199812518	2495079145	4415129264
kT6	0	585000	237876180.2	668941156.4	8913788957	30513024000	85186688807	2.04596E+11	330117393970

N=8

1 차 다항식

8	331.00	5.0140
331.00	19619	215.56

8	331.00	5.0140	>	a0	0.630953
0.00	-792117.13	80.46		a1	-0.000102

$$P_1(x) = 0.631 - 0.0001x$$

2 차 다항식

8	331.00	19619	5.0140
331.00	19619	1296379	215.56
19619	1296379	90938675	12922

8	331	19619	5.014
0	-792117.125	-3177911071	80.45814325
0	-52341302.13	-2.22925E+11	4823.218242

8	331	19619	5.014	>	a0	0.630953488
0	-792117.125	-3177911071	80.45814325		a1	-0.000101595
0	0	8.94134E+14	4823.708153		a2	5.39484E-12

$$P_2(x) = 0.631 - 0.0001x + (5.39484E-12)x^2$$

3 차 다항식

8	331	19619	1296379	5.0140
331	19619	1296379	90938675	215.56
19619	1296379	90938675	6619706371	12922
1296379	90938675	6619706371	494066404859	859196

8	331	19619	1296379	5.014
0	-792117.125	-3177911071	-1.47363E+13	80.45814325
0	-52341302.13	-2.22925E+11	-1.0727E+15	4823.218242
0	-3671649003	-1.62274E+13	-8.00617E+16	320695.0044

8	331	19619	1296379	5.014
0	-792117.125	-3177911071	-1.47363E+13	80.45814325
0	0	8.94134E+14	1.99561E+22	4823.708153
0	0	6.50867E+16	1.48944E+24	320727.5785

8	331	19619	1296379	5.014	>	a0	0.630953488
0	-792117.125	-3177911071	-1.47363E+13	80.45814325		a1	-0.000101595
0	0	8.94134E+14	1.99561E+22	4823.708153		a2	5.39484E-12
0	0	0	-3.32428E+31	320727.5785		a3	-9.64804E-27

$$P_3(x) = 0.631 - 0.0001x + (5.39484E-12)x^2 - (9.64804E-27)x^3$$

4 차다항식

8	331	19619	1296379	90938675	5.014
331	19619	1296379	90938675	6619706371	215.561
19619	1296379	90938675	6619706371	4.94066E+11	12922.219
1296379	90938675	6619706371	4.94066E+11	3.75461E+13	859196.261
90938675	6619706371	4.94066E+11	3.75461E+13	2.89247E+15	60500351.78

8	331	19619	1296379	90938675	5.014
0	-792117.125	-3177911071	-1.47363E+13	-7.52484E+16	80.45814325
0	-52341302.13	-2.22925E+11	-1.0727E+15	-5.61622E+18	4823.218242
0	-3671649003	-1.62274E+13	-8.00617E+16	-4.26799E+20	320695.0044
0	-2.67271E+11	-1.21114E+15	-6.08421E+18	-3.28797E+22	22581756.3

8	331	19619	1296379	90938675	5.014
0	-792117.125	-3177911071	-1.47363E+13	-7.52484E+16	80.45814325
0	0	8.94134E+14	1.99561E+22	5.33521E+29	4823.708153
0	0	6.50867E+16	1.48944E+24	4.05444E+31	320727.5785
0	0	4.85779E+18	1.13189E+26	3.12346E+33	22584050.01

8	331	19619	1296379	90938675	5.014
0	-792117.125	-3177911071	-1.47363E+13	-7.52484E+16	80.45814325
0	0	8.94134E+14	1.99561E+22	5.33521E+29	4823.708153
0	0	0	-3.32428E+31	-2.41925E+46	320727.5785
0	0	0	-2.52625E+33	-1.86374E+48	22584050.01

8	331	19619	1296379	90938675	5.014		a0	0.630953488
0	-792117.125	-3177911071	-1.47363E+13	-7.52484E+16	80.45814325		a1	-0.000101595
0	0	8.94134E+14	1.99561E+22	5.33521E+29	4823.708153	>	a2	5.39484E-12
0	0	0	-3.32428E+31	-2.41925E+46	320727.5785		a3	-9.64804E-27
0	0	0	0	1.35634E+63	22584050.01		a4	1.66507E-56

$$P_4(x) = 0.631 - 0.0001\mathrm{x} + (5.39484\mathrm{E} - 12)\mathrm{x}^2 - (9.64804\mathrm{E} - 27)\mathrm{x}^3 + (1.66507\mathrm{E} - 56)\mathrm{x}^4$$