Introduction to Complexity

Homework 4

Graded Part: This part you will submit on the course webpage, by doing the exercises, and then clicking on the "Homework 4" link in Unit 4.6, filling out the answers, and then clicking on the "Submit" button at the bottom of the page.

1. Maxwell's demon was invented by James Clerk Maxwell in order to show that

A. The second law of thermodynamics is wrong

B. The second law of thermodynamics is true only in the statistical sense

C. Entropy can be decreased if work is applied

D. Entropy can be increased if work is applied

E. Molecules do not obey the laws of thermodynamics

2. Recall the three-window slot machine example described in Unit 4. 3. Recall that a microstate is a specific set of three fruits (one in each window), and that we calculated that there are 125 possible microstates, since there are five possible fruits that could show up in each window, and $5 \times 5 \times 5 = 125$. How many possible microstates would there be for a slot machine with four windows, and five possible fruits that could show up in each window?

A. 125

B. 250

C. 625

D. 1024

E. 256

3. For the three-window slot machine with five possible fruits per window, consider the macrostate **"Exactly two pears"**. Select the true statement A, B, or C below.

[Note: you can either calculate this mathematically, or by downloading SlotMachine.nlogo from the Course Materials page and running it on this macrostate. To do so, select "Exactly two pears" under "Macrostates"), and run for 1000 pulls. OPTIONAL: If you use the Netlogo model, try to predict the correct answer below before you run it and see if your prediction is correct.]

If you downloaded SlotMachine.nlogo before March 11, please redownload it.

A. The number of microstates corresponding to this macrostate is *much higher* than the number of microstates **not** corresponding to this macrostate.

B. The number of microstates corresponding to this macrostate is *much lower* than the number of microstates **not** corresponding to this macrostate.

C. The number of microstates corresponding to this macrostate is *about equal* to the number of microstates **not** corresponding to this macrostate (i.e, for 1000 pulls, they are within 200 of each other).

4. Same as question 3, but this time for the macrostate "**At least two of the same kind**". Remember to do "Reset" before clicking on "Go". Select the true statement below.

A. The number of microstates corresponding to this macrostate is *much higher* than the number of microstates **not** corresponding to this macrostate.

B. The number of microstates corresponding to this macrostate is *much lower* than the number of microstates **not** corresponding to this macrostate.

C. The number of microstates corresponding to this macrostate is *about equal* to the number of microstates **not** corresponding to this macrostate (i.e, for 1000 pulls, they are within 200 of each other).

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5. Same as question 3, but this time for the macrostate "**One orange and one cherry**". Remember to do "Reset" before clicking on "Go". Select the true statement below.

A. The number of microstates corresponding to this macrostate is *much higher* than the number of microstates **not** corresponding to this macrostate.

B. The number of microstates corresponding to this macrostate is *much lower* than the number of microstates **not** corresponding to this macrostate.

C. The number of microstates corresponding to this macrostate is *about equal* to the number of microstates **not** corresponding to this macrostate (i.e, for 1000 pulls, they are within 200 of each other).

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6. Same as question 3, but this time for the macrostate "**No apples or cherries**". Remember to do "Reset" before clicking on "Go". Select the true statement below.

   A. The number of microstates corresponding to this macrostate is *much higher* than the number of microstates **not** corresponding to this macrostate.

   B. The number of microstates corresponding to this macrostate is *much lower* than the number of microstates **not** corresponding to this macrostate.

   C. The number of microstates corresponding to this macrostate is *about equal* to the number of microstates **not** corresponding to this macrostate (i.e, for 1000 pulls, they are within 200 of each other).

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7. Same as question 3, but this time for the macrostate "**At least one lemon**". Remember to do "Reset" before clicking on "Go". Select the true statement below.

   A. The number of microstates corresponding to this macrostate is *much higher* than the number of microstates **not** corresponding to this macrostate.

   B. The number of microstates corresponding to this macrostate is *much lower* than the number of microstates **not** corresponding to this macrostate.

   C. The number of microstates corresponding to this macrostate is *about equal* to the number of microstates **not** corresponding to this macrostate (i.e, for 1000 pulls, they are within 200 of each other).

8. Suppose you have two biased coins, Coin A, which comes up heads 90% of the time, and Coin B, which comes up heads 55% of the time. If you consider each independently as a message source, which one will have the **higher** Shannon information content? (Hint: For questions 8–10, you don't have to calculate the information content – just use what you know about the intuitive definition of information content.)

   A. Coin A

   B. Coin B

9. Which will have higher Shannon information content: A fair (six-sided) die, or a biased die for which "1" comes up more often than 1/6 of the time?

A. Fair die

B. Biased die

10. Which will have higher Shannon information content: A fair six-sided die or a fair twelve-sided die?

A. Six-sided die

B. Twelve-sided die

Ungraded Part:

Beginner option:

Experiment with TextInformationContent.nlogo, which can be downloaded from the Course Materials page. Use the model to see if information content is a good method for distinguishing, say, Spanish and English text, or any other dichotomy you can think of.

Intermediate option:

In addition to the Beginner options:

A. Calculate mathematically the number of microstates corresponding to the different macrostates given in questions 3–7 above. How does your mathematical result compare with the result given by the SlotMachine.nlogo model?

B. Make up a macrostate different from those given in questions 3–7 above. Modify SlotMachine.nlogo to include your macrostate.

C. Modify TextInformationContent.nlogo to include a fair 12-sided die, and use this to test your answer to question 10 above.

Advanced option:

In addition to the Intermediate option:

Modify the model LogisticMap.nlogo (on the Course Materials page under Unit 2) to compute information content of the logistic map's symbolic dynamics, as described in the Unit 4.4 video. That is, at each time step, have the modified logistic map output "1" if $x_{t+1} > 0.5$, and output "0" otherwise. Have the model calculate the Shannon information content of the stream of 0s and 1s that results, for given values of R and x_0.