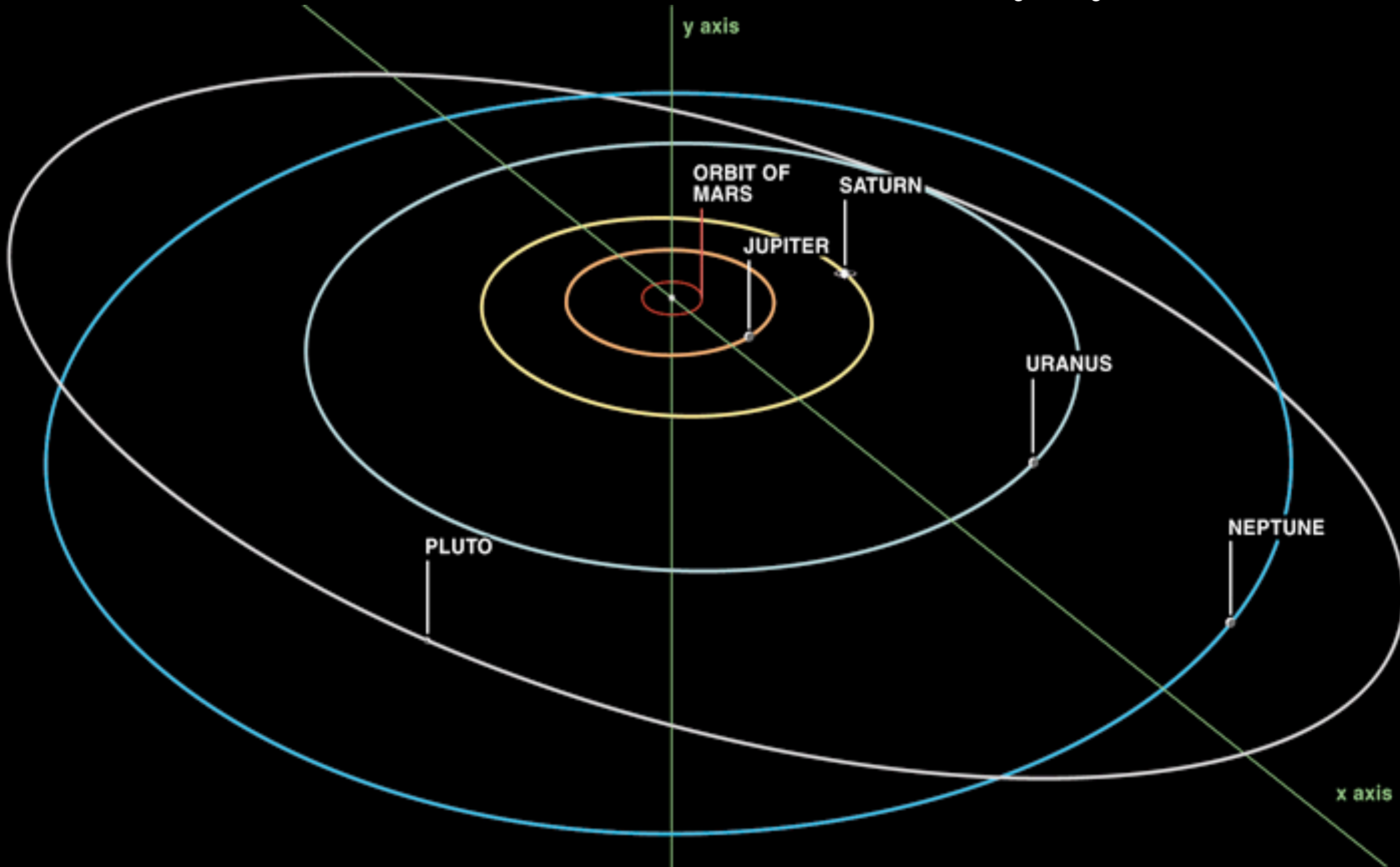
Dynamics:

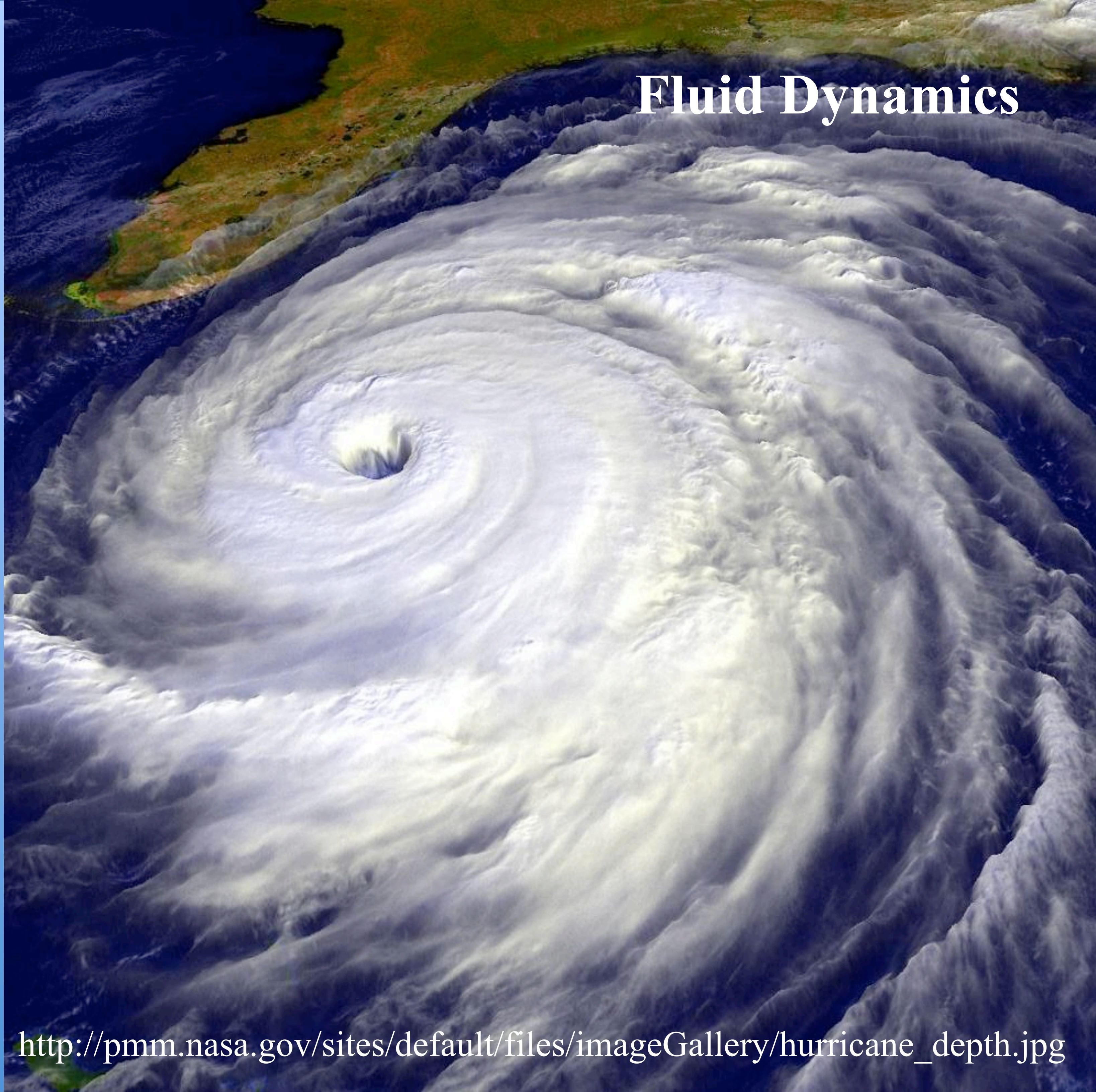
The general study of how systems change over time

Planetary dynamics

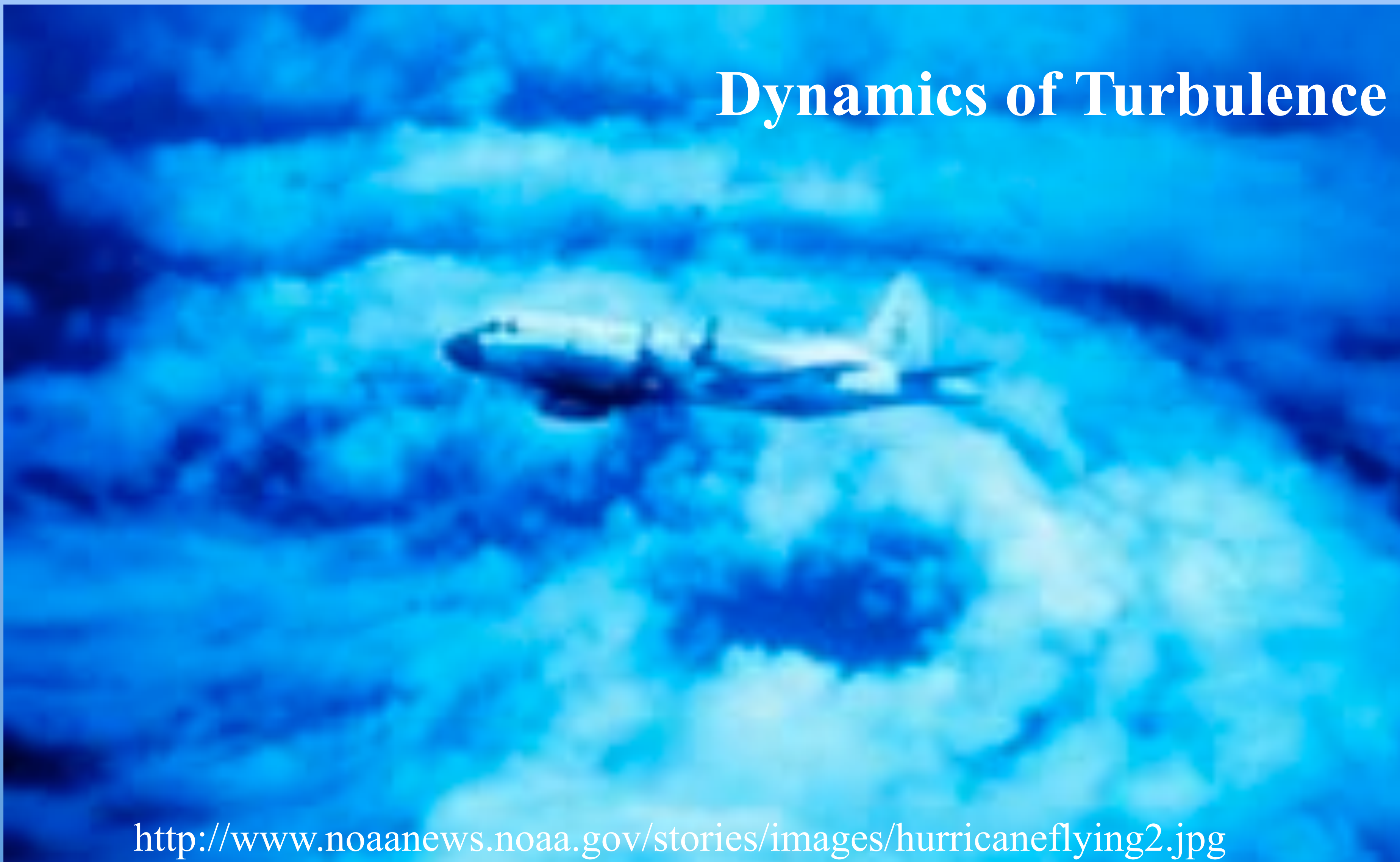


http://www.lpi.usra.edu/

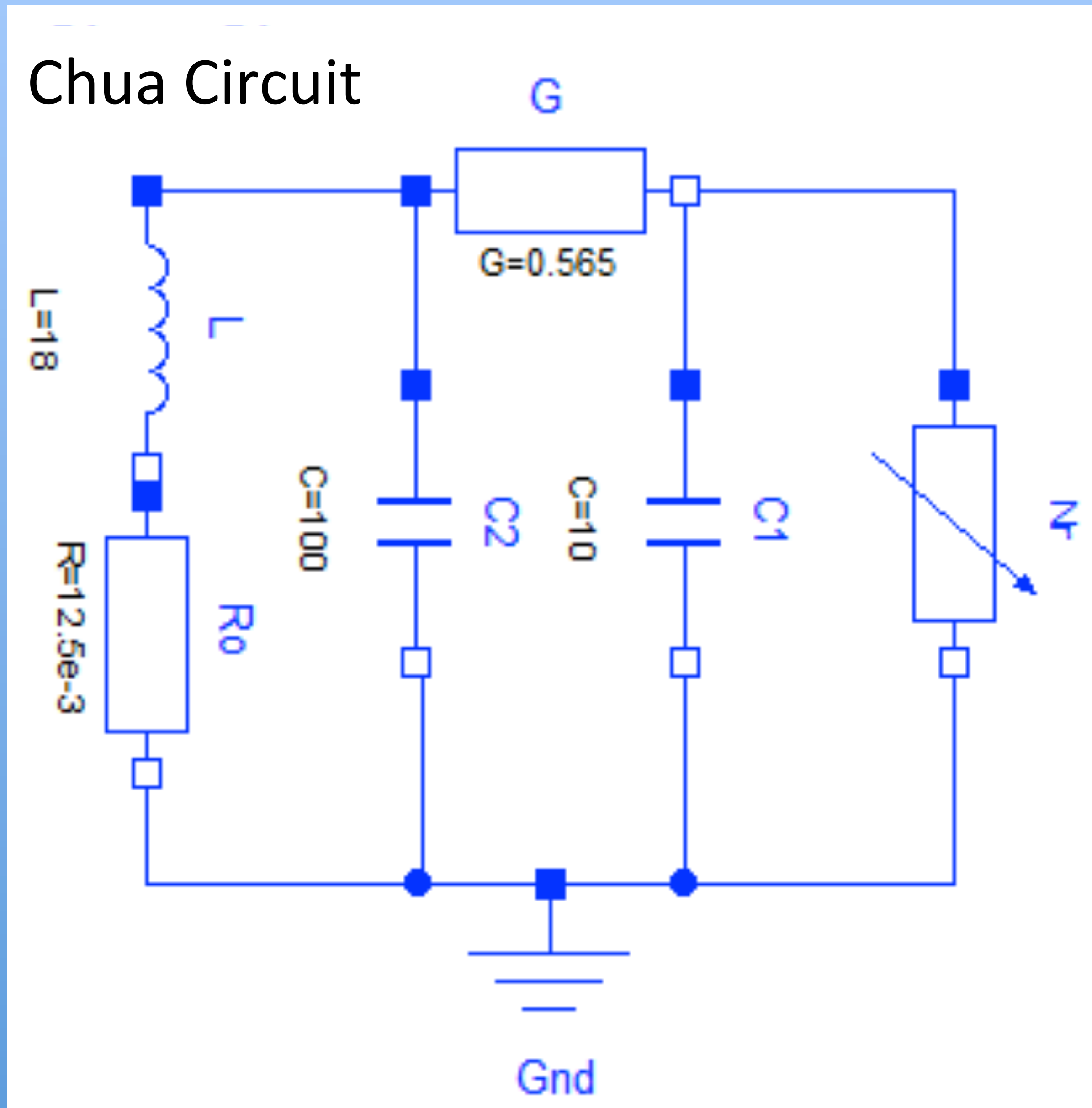
Fluid Dynamics



http://pmm.nasa.gov/sites/default/files/imageGallery/hurricane_depth.jpg

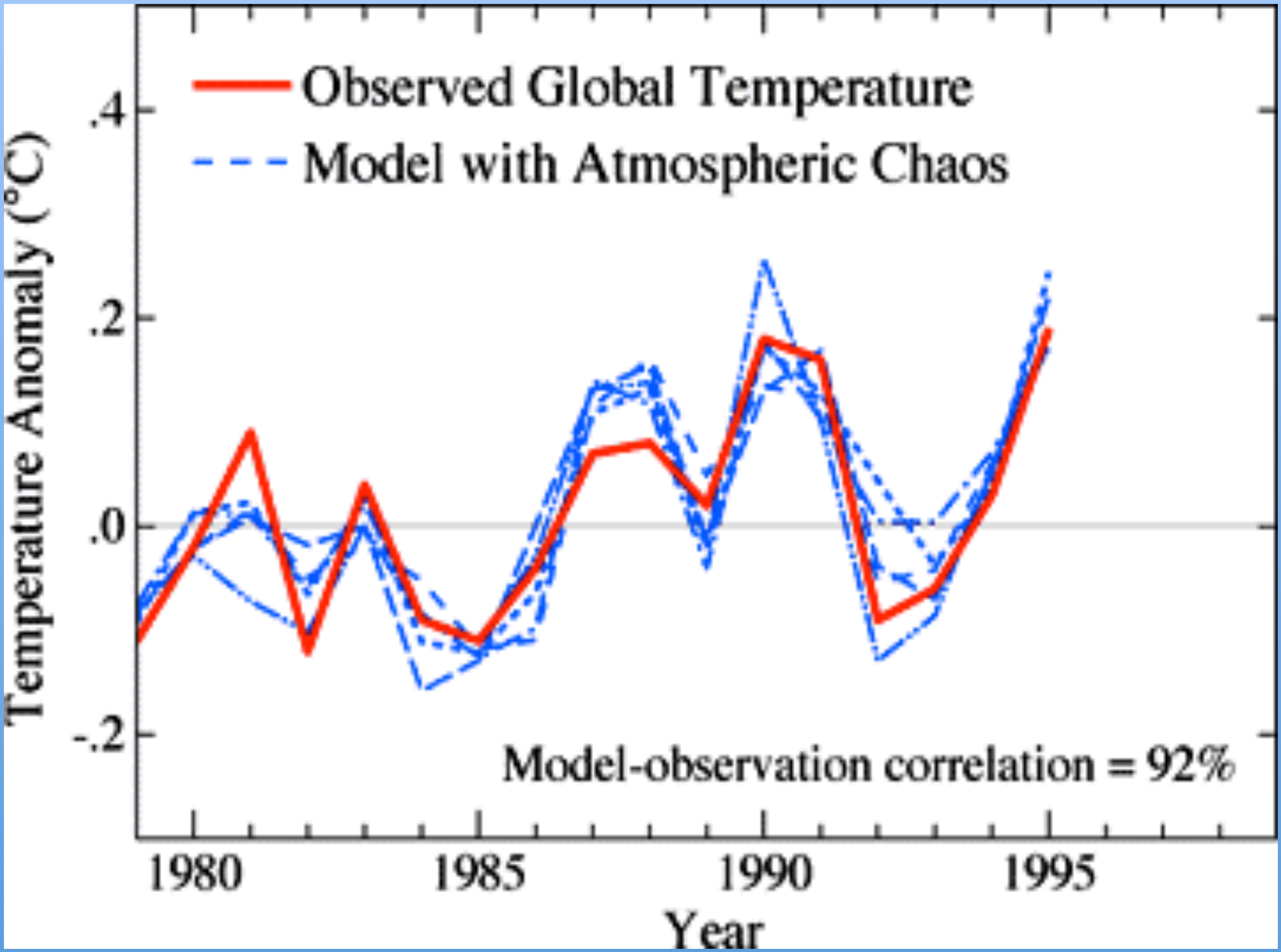

Dynamics of Turbulence
http://www.noaanews.noaa.gov/stories/images/hurricaneflying2.jpg

Electrical Dynamics



http://simulationresearch.lbl.gov/modelica/releases/msl/3.2/help/Modelica_Electrical_Analog_Examples.html

Climate dynamics

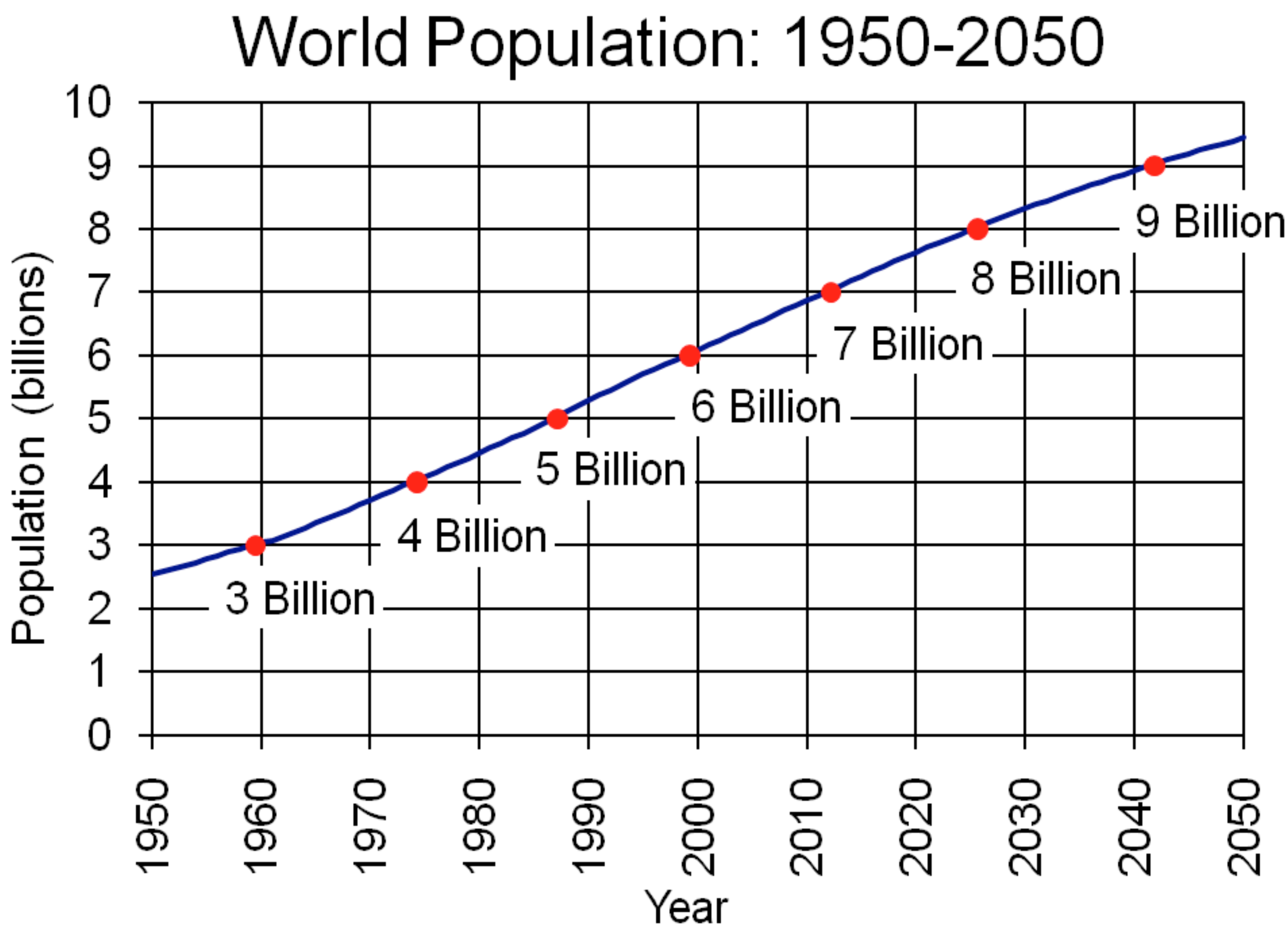


http://www.giss.nasa.gov/research/briefs/hansen_03/oceana_ts.gif

Crowd dynamics

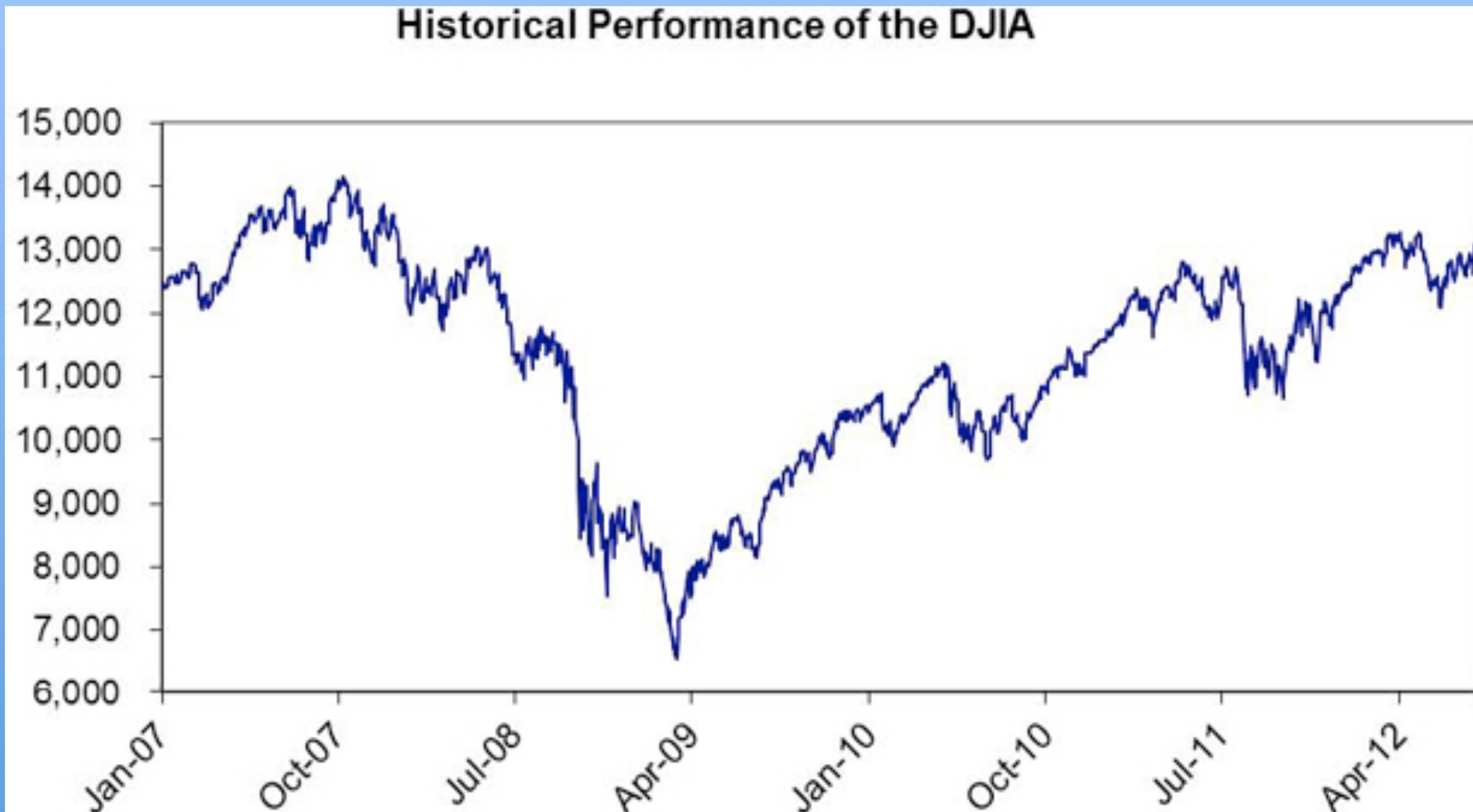
http://www.research.gov/common/images/PublicAffairs/img_22539_crowd%20control--rgov-800width.jpg

Population dynamics



Source: U.S. Census Bureau, International Data Base, June 2011 Update.

Dynamics of stock prices



http://www.sec.gov/Archives/edgar/data/70858/000119312512349971/g394492g73r41.jpg

Group dynamics

http://www.nsf.gov/news/mmg/media/images/lake_f1.jpg

http://www.nsf.gov/news/mmg/media/images/ruebeck2_h.jpg

http://blogs.state.gov/images/Dipnote/behind_the_scenes/2011_0201_egypt_march_millions_m.jpg

Dynamics of conflicts

http://www.blogs.va.gov/VAntage/wp-
uploads/2011/10/afghantenblog.jpg

Dynamics of cooperation

http://gdb.voanews.com/9E85DB30-
EFD7-4BB5-845B-9E5722C1CA03_mw1024_n_s.jpg

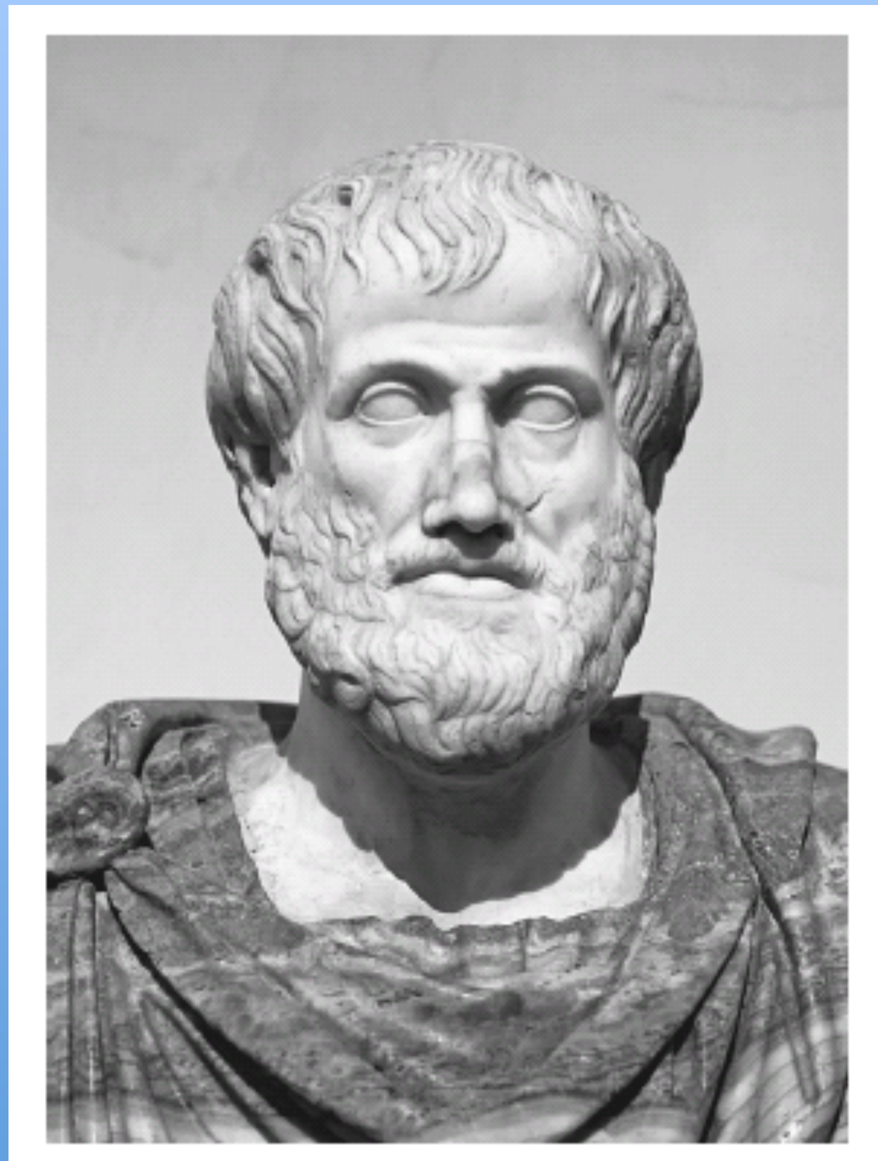
Dynamical Systems Theory:

- The branch of mathematics of how systems change over time
  - Calculus
  - Differential equations
  - Iterated maps
  - Algebraic topology
  - etc.

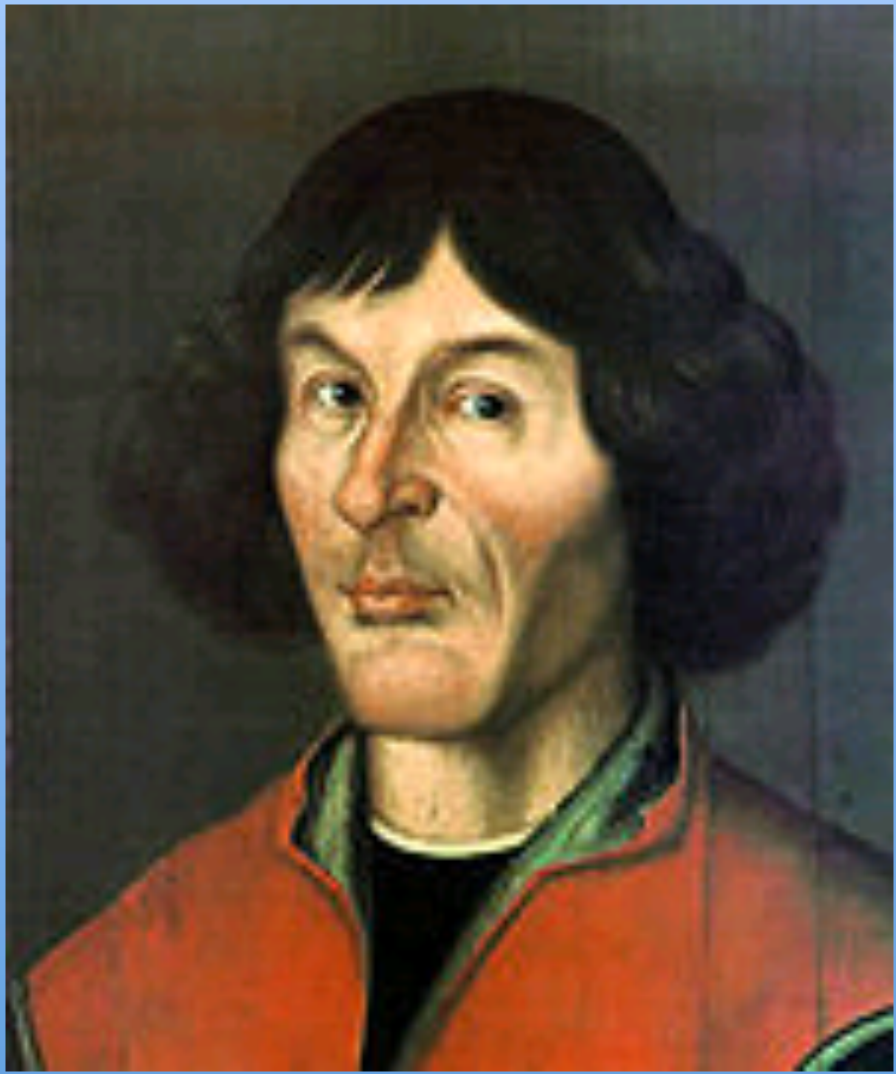
- The *dynamics of a system*: the manner in which the system changes

- Dynamical systems theory gives us a ***vocabulary*** and ***set of tools*** for describing dynamics

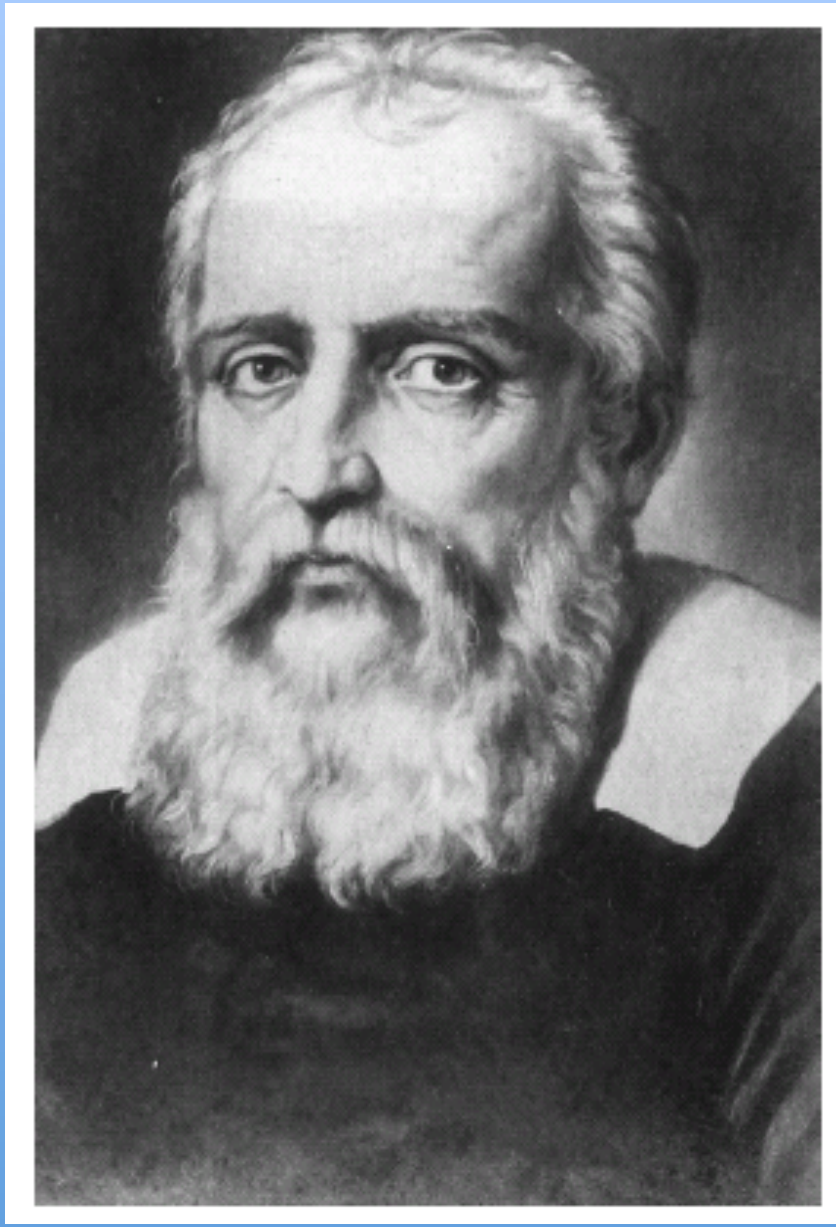
A brief history of the science of dynamics



Aristotle, 384 – 322 BC



Nicolaus Copernicus, 1473 – 1543



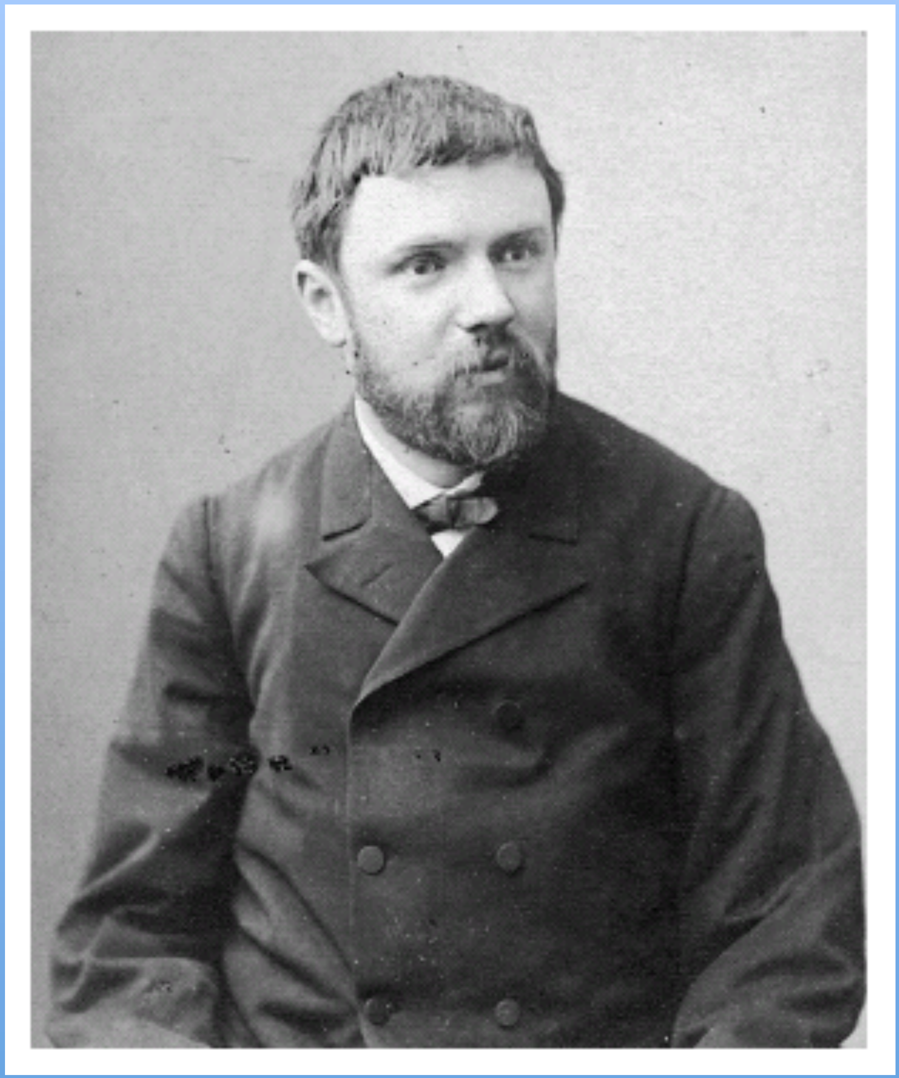
Galileo Galilei, 1564 – 1642

Isaac Newton, 1643 – 1727

Pierre- Simon Laplace, 1749 – 1827

“We may regard the present state of the universe as the effect of its past and the cause of its future. An intellect which at a certain moment would know all forces that set nature in motion, and all positions of all items of which nature is composed, if this intellect were also vast enough to submit these data to analysis, it would embrace in a single formula the movements of the greatest bodies of the universe and those of the tiniest atom; for such an intellect nothing would be uncertain and the future just like the past would be present before its eyes.”

—Pierre Simon Laplace, *A Philosophical Essay on Probabilities*

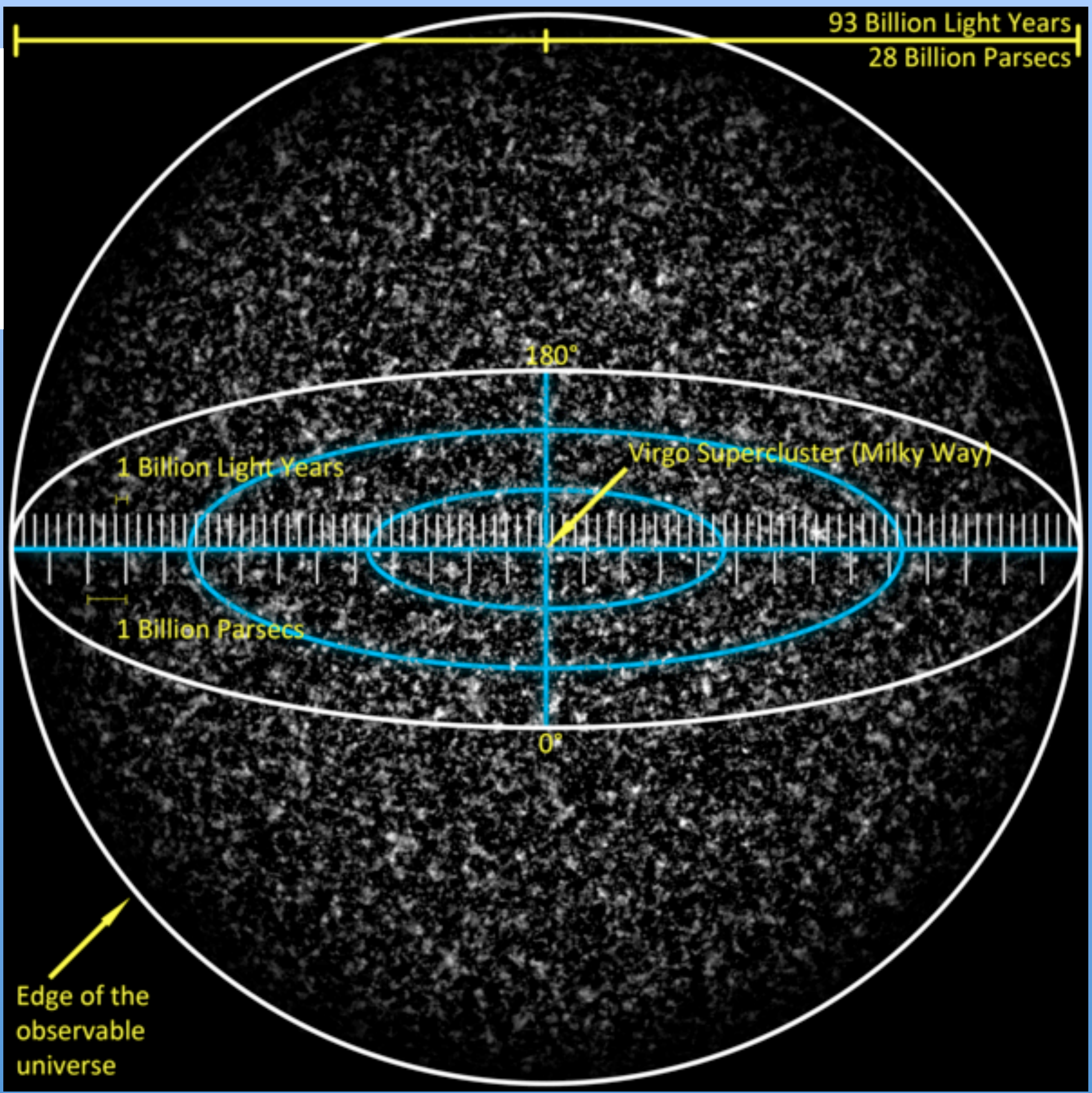


Henri Poincaré, 1854 – 1912

“If we knew exactly the laws of nature and the situation of the universe at the initial moment, we could predict exactly the situation of that same universe at a succeeding moment. but even if it were the case that the natural laws had no longer any secret for us, we could still only know the initial situation *approximately*. If that enabled us to predict the succeeding situation with *the same approximation*, that is all we require, and we should say that the phenomenon had been predicted, that it is governed by laws. But it is not always so; it may happen that **small differences in the initial conditions produce very great ones in the final phenomena**. A small error in the former will produce an enormous error in the latter. Prediction becomes impossible...”

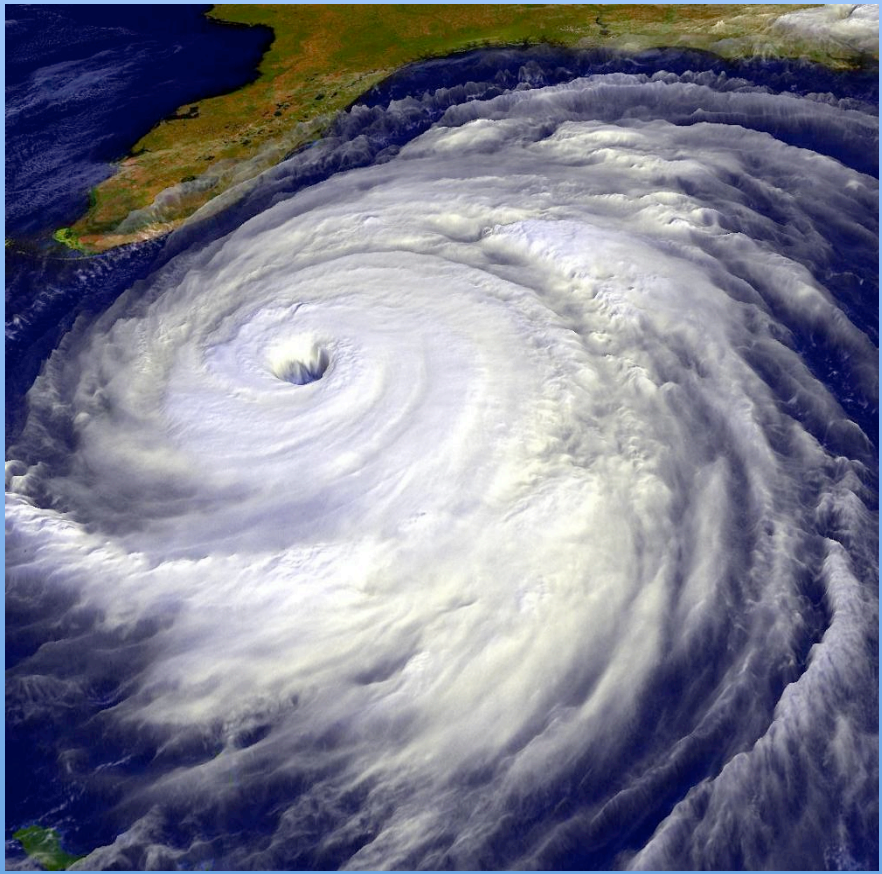
$$F = m\,a$$

$$F = G\,m_1\,m_2\,/\,d^2$$



Azcolvin429, Wikimedia Commons

“Sensitive dependence on initial conditions”



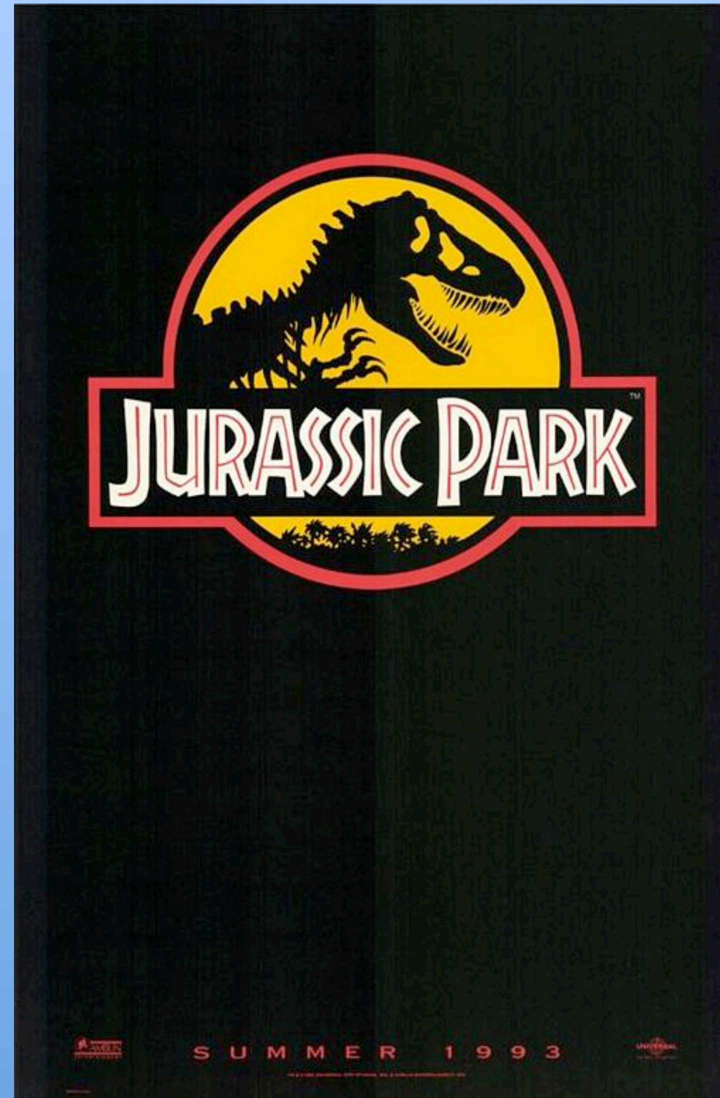
http://www.fws.gov/sacramento/ES_Kids/Mission-Blue-Butterfly/Images/mission-blue-butterfly_header.jpg

http://pmm.nasa.gov/sites/default/files/imageGallery/hurricane_depth.jpg

Chaos:

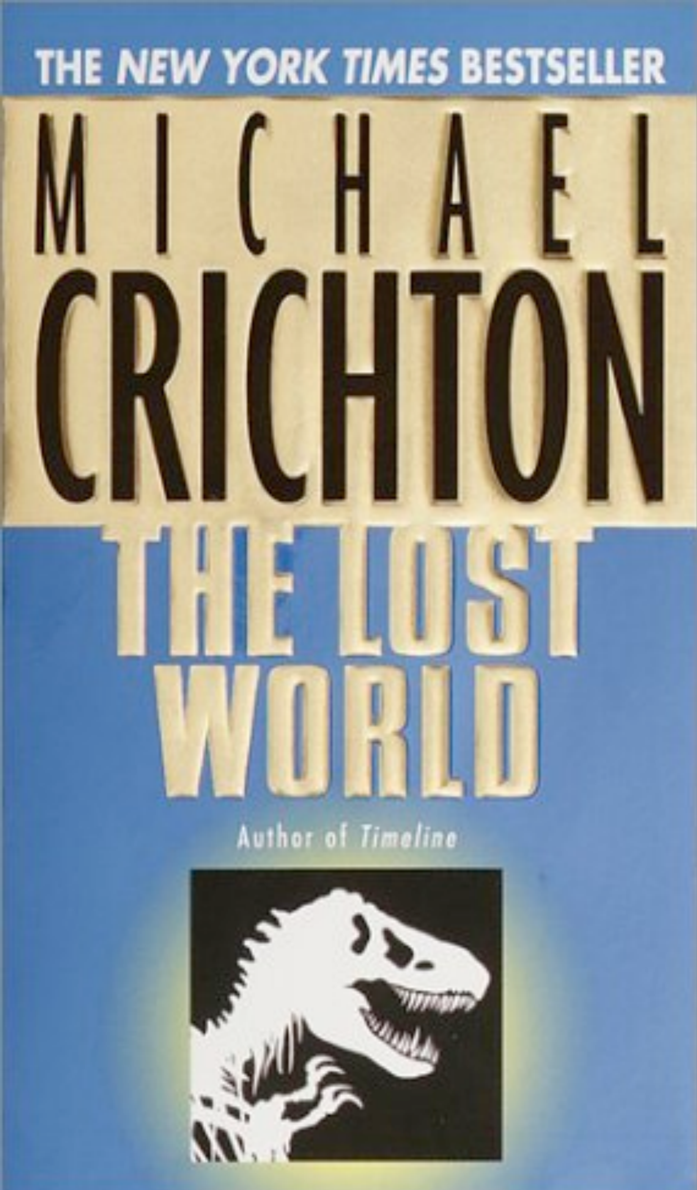
- One particular type of dynamics of a system

- Defined as “sensitive dependence on initial conditions”



Dr. Ian Malcolm

"You've never heard of Chaos theory? Non-linear equations? Strange attractors?"



Prologue:

"Life at the Edge of Chaos"

The Santa Fe Institute was housed in a series of buildings on Canyon Road which had formerly been a convent, and the Institute's seminars were held in a room which had served as a chapel. Now, standing at the podium, with a shaft of sunlight shining down on him, Ian Malcolm paused dramatically before continuing his lecture.

Dressed entirely in black, leaning on a cane, Malcolm gave the impression of severity. He was known within the Institute for his unconventional analysis, and his tendency to pessimism. His talk that August, entitled "Life at the Edge of Chaos," was typical of his thinking. In it, Malcolm presented his analysis of chaos theory as it applied to evolution.

Ian Malcolm

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Ian Malcolm, malcolm@santafe.edu

Position: Research professor

Affiliations:

- Santa Fe Institute
- Department of Mathematics, University of Western Costa Rica

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Research interests:

- Non-linear dynamics and chaotic systems
- Evolution theory
- Experimental entomological genetics (especially flies)
- Vertebrate paleontology

My recent publications are listed here.

Chaos in Nature

- Dripping faucets
- Electrical circuits
- Solar system orbits
- Weather and climate (the “butterfly effect”)
- Brain activity (EEG)
- Heart activity (EKG)
- Computer networks
- Population growth and dynamics
- Financial data

What is the difference between *chaos* and *randomness*?

Notion of "deterministic chaos"

Lord Robert May
b. 1936

Mitchell Feigenbaum
b. 1944

“The fact that the simple and deterministic equation [i.e., the Logistic Map] can possess dynamical trajectories which look like some sort of random noise has disturbing practical implications. It means, for example, that apparently erratic fluctuations in the census data for an animal population need not necessarily betoken either the vagaries of an unpredictable environment or sampling errors; they may simply derive from a rigidly deterministic population growth relationship...Alternatively, it may be observed that in the chaotic regime, arbitrarily close initial conditions can lead to trajectories which, after a sufficiently long time, diverge widely. This means that, even if we have a simple model in which all the parameters are determined exactly, long-term prediction is nevertheless impossible”

—— Robert May, 1976

Chaos: Seemingly random behavior with *sensitive dependence on initial conditions*

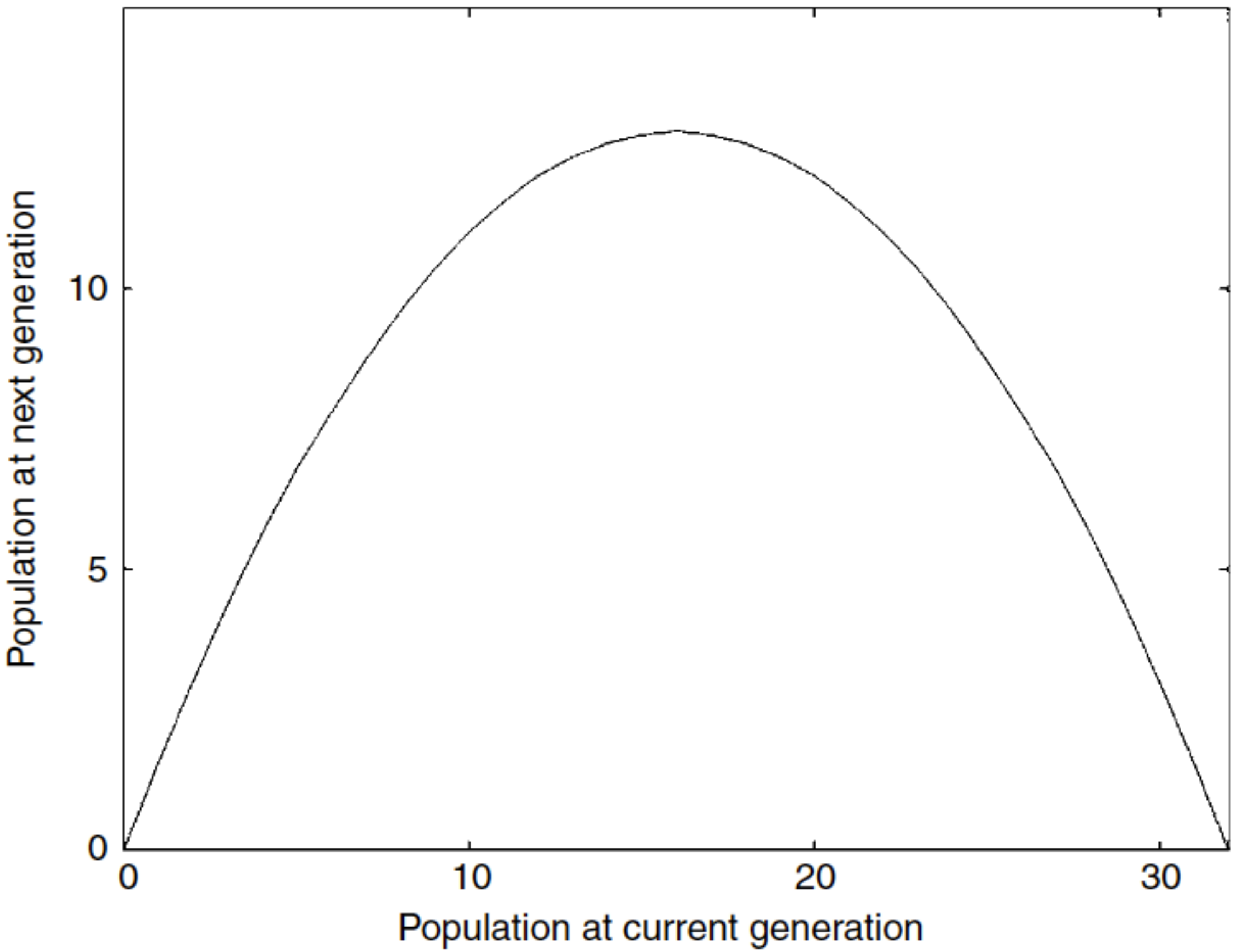
Logistic map: A simple, completely deterministic equation that, when iterated, can display chaos (depending on the value of R).

Deterministic chaos: Perfect prediction, *a la* Laplace's deterministic "clockwork universe", is impossible, even in principle, if we're looking at a chaotic system.

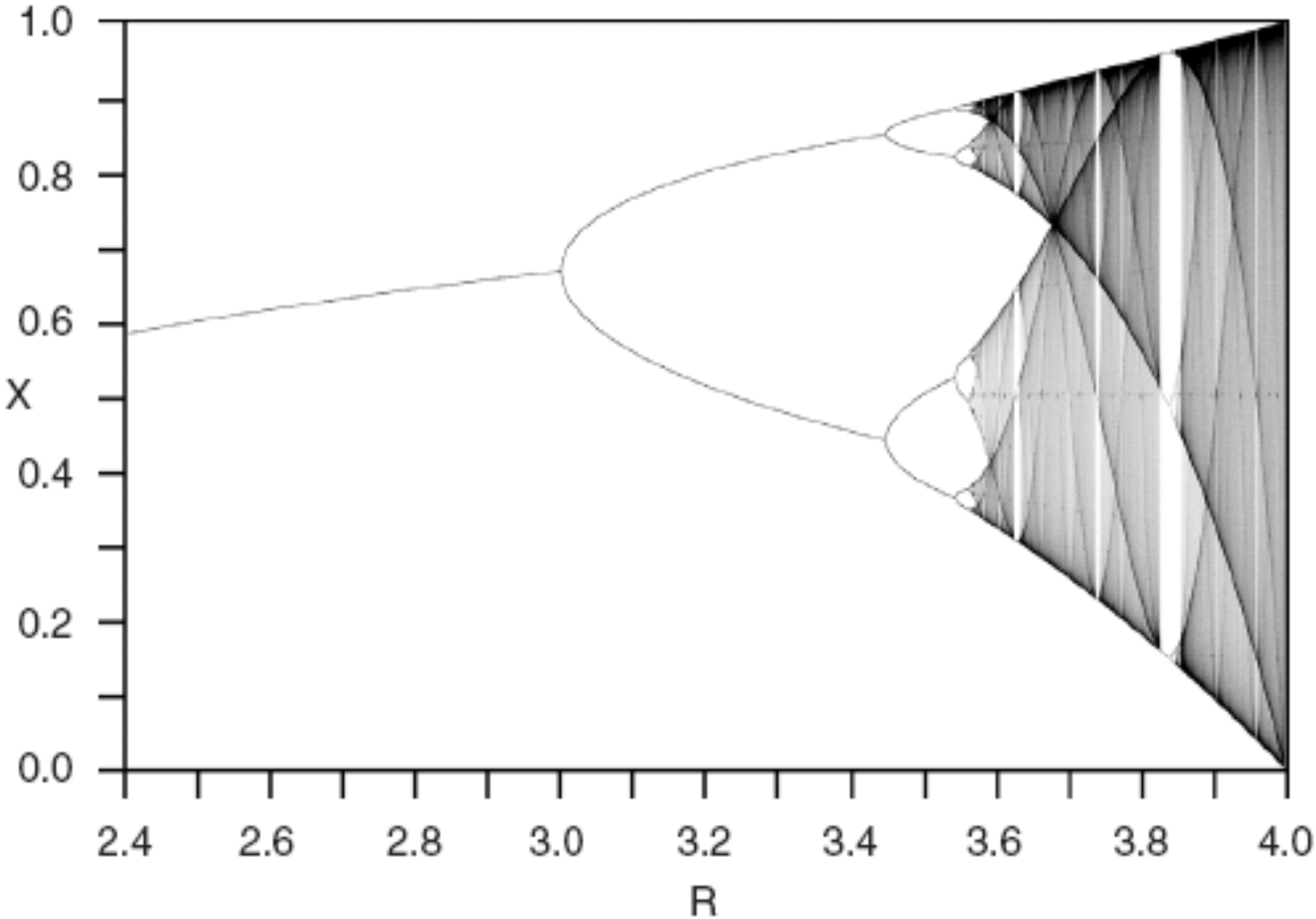
Universality in Chaos

While chaotic systems are not predictable in detail, a wide class of chaotic systems has highly predictable, “universal” properties.

A *Unimodal* (“one humped”) Map



Logistic Map Bifurcation Diagram



Bifurcations in the Logistic Map

$R_1 \approx 3.0$: period 2

$R_2 \approx 3.44949$ period 4

$R_3 \approx 3.54409$ period 8

$R_4 \approx 3.564407$ period 16

$R_5 \approx 3.568759$ period 32

$R_\infty \approx 3.569946$ period ∞ (onset of chaos)

Bifurcations in the Logistic Map

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$R_5 \approx 3.568759$ period 32

$R_\infty \approx 3.569946$ period ∞ (chaos)

Rate at which distance between bifurcations is shrinking:

$$\frac{R_2 - R_1}{R_3 - R_2} = \frac{3.44949 - 3.0}{3.54409 - 3.44949} = 4.75147992$$

$$\frac{R_3 - R_2}{R_4 - R_3} = \frac{3.54409 - 3.44949}{3.564407 - 3.54409} = 4.65619924$$

$$\frac{R_4 - R_3}{R_5 - R_4} = \frac{3.564407 - 3.54409}{3.568759 - 3.564407} = 4.66842831$$

$$\vdots$$

$$\lim_{n\to\infty}\left(\frac{R_{n+1} - R_n}{R_{n+2} - R_{n+1}}\right) \approx 4.6692016....$$

Bifurcations in the Logistic Map

$R_1 \approx 3.0$: period 2

$R_2 \approx 3.44949$ period 4

$R_3 \approx 3.54409$ period 8

$R_4 \approx 3.564407$ period 16

$R_5 \approx 3.568759$ period 32

$R_\infty \approx 3.569946$ period ∞ (chaos)

Rate at which distance between bifurcations is shrinking:

R_2 ... 992

R_3

...4

...31

In other words, each new bifurcation appears about 4.6692016 times faster than the previous one.

Feigenbaum derived this constant mathematically!

He also showed that any unimodal (one-humped) map will have the same value for this rate.

A "universal" constant!!!

$$\lim_{n\to\infty}\left(\frac{R_{n+1}-R_n}{R_{n+2}-R_{n+1}}\right) \approx 4.6692016....$$

Feigenbaum's constant

Amazingly, at almost exactly the same time, the same constant was independently discovered (and mathematically derived by) another research team, the French mathematicians Pierre Collet and Charles Tresser.

Summary
Significance of dynamics and chaos for complex systems

- Complex, unpredictable behavior from simple, deterministic rules

- Dynamics gives us a vocabulary for describing complex behavior

- There are fundamental limits to detailed prediction

- At the same time there is universality: “Order in Chaos”