

## Introduction to Complexity

### Final exam

You may use any course materials, websites, Netlogo simulations, calculators, etc. for this exam. Just don't ask another person for the answers.

We strongly suggest writing down all the answers first, then submitting them on the web page.

1. For the logistic map, let  $R = 3.2$ . What kind of attractor does this yield?

- A. Fixed point attractor
- B. Period 2 attractor
- C. Period 4 attractor
- D. Period 8 attractor
- E. Chaotic attractor

2. What is the fixed point for the logistic map with  $R = 2.8$ ?

- A. About 0.512
- B. About 0.6
- C. About 0.643
- D. About 0.699
- E. It doesn't have a fixed point

3. Which of the following can you conclude from the logistic map bifurcation diagram?

- A. When  $R$  is between 2.4 and 3.0, the logistic map has a fixed point that increases as  $R$  increases.
- B. When  $R$  is between 3.0 and 3.4, the logistic map has a period-2 attractor in which, as  $R$  increases, the two values in the attractor get closer together.
- C. The onset of chaos for the logistic map is between 3.4 and 3.5.

4. Recall the creation of the “Koch curve” fractal from Unit 3 (see Unit 3.2, video 1). The process starts at level 0 with a single line segment of length  $L$ . Recall that at level 1, the length of the curve is  $(4/3) \times L$ . What is the total length of the curve at level  $N$ ?

A.  $(4/3) \times N \times L$

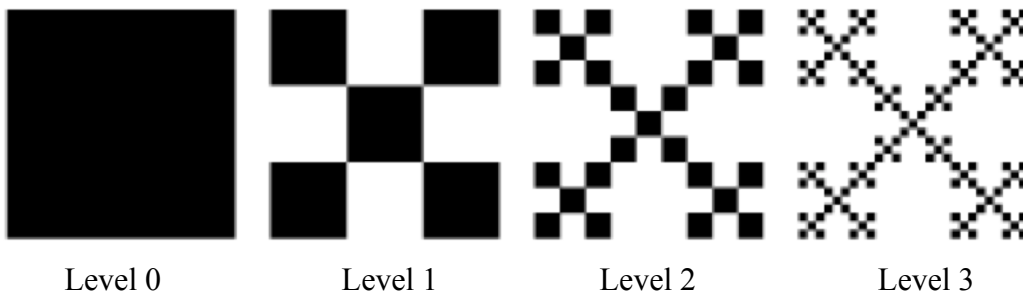
B.  $(4/3)^N \times L$

C.  $(4/3) \times L^N$

D.  $(4/3)^N$

5. Consider the Box Fractal, which is defined as follows.

Start with a single square with sides of length  $L$  (level 0). To get to level 1, divide the single square into 9 smaller squares, each with side  $L/3$ , and delete the center square from each side. To get to subsequent levels, apply this same rule to each square in the figure.



What is the fractal (Hausdorff) dimension of the Box Fractal?

(Hint: See Unit3Slides.pdf, slide 44. Note that here the reduction factor is the reduction factor in the size of a line segment from one level to the next.)

A.  $\log 3 / \log 5$

B.  $\log 9 / \log 5$

C.  $\log 5 / \log 9$

D.  $\log 5 / \log 3$

6. Recall Unit 4, which discussed the notions of “macrostate” and “microstate”, and illustrated this notion using a slot machine with three windows (see SlotMachine.nlogo from the Course Materials page). In each window, there were five possible fruit pictures that could appear. Which of the following macrostates of the slot machine has the most microstates associated with it? (You can use SlotMachine.nlogo to answer this problem.)

- A. **Macrostate:** Three of the same kind (e.g., “pear pear pear” or “apple apple apple”)
- B. **Macrostate:** At least one lemon
- C. **Macrostate:** Exactly two pears

7. Recall Unit 4, which defined Shannon information in terms of a “message source” and the messages it produces. Assume that the following sequences were generated by different message sources (with individual letters being the messages).

**Message Source 1:** A C G T B W M R

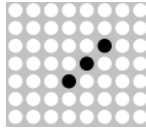
**Message Source 2:** C C C T C C C T

**Message Source 3:** T C T C T C T C

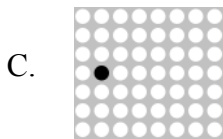
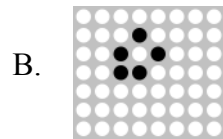
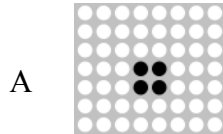
Based on the texts above, which source seems to have the **highest** Shannon information content?

- A. Message Source 1
- B. Message Source 2
- C. Message Source 3

8. Consider this grid in the Game of Life:



What will the grid look like after two time steps?



9. Using Wolfram's numbering for elementary cellular automata, what is the Wolfram number for the following rule table?

□ □ □	→	■
□ □ ■	→	□
□ ■ □	→	□
□ ■ ■	→	□
■ □ □	→	□
■ □ ■	→	□
■ ■ □	→	□
■ ■ ■	→	■

(Hint: see Unit 6 Slides, slide 12 for an example.)

A. 193

B. 101

C. 128

D. 129

**10.** According to the definitions given in Unit 6 (see slide 14), which Wolfram class does the behavior of elementary cellular automaton rule 128 fall into? (You can investigate this by running ElementaryCAs.nlogo with this rule on several random initial configurations.)

- A. Class 1
- B. Class 2
- C. Class 3
- D. Class 4

**11.** According to the definitions given in Unit 6 (see slide 14), which Wolfram class does the behavior of elementary cellular automaton rule 200 fall into? (You can investigate this by running ElementaryCAs.nlogo with this rule on several random initial configurations.)

- A. Class 1
- B. Class 2
- C. Class 3
- D. Class 4

**12.** Which elementary cellular automaton has been proved to be a universal computer?

- A. Rule 96
- B. Rule 110
- C. Rule 111

**13.** Using Fireflies.nlogo from Unit 8 with the default parameters, look at the effects of increasing *cycle-length*. Which of the following statements is true? (Remember: You can use the speed bar to increase the simulation speed.)

- A. The shortest cycle length tends to result in the fastest time for the whole population to synchronize.
- B. The longest cycle length tends to result in the fastest time for the whole population to synchronize
- C. All cycle lengths result in about equal time for the whole population to synchronize

**14.** Using Fireflies.nlogo from Unit 8 with the default parameters (set cycle-length back to 10), look at the effects of population size (the *number* slider) on time to synchronize. Testing the model with populations 100, 1000, and 2000, which of the following statements is true?

- A. The largest population tends to result in the fastest time for the whole population to synchronize.
- B. The smallest population tends to result in the fastest time for the whole population to synchronize.
- C. Population size does not affect time to synchronize.

**15.** Zipf's law says that the frequency of a given word is a power-law function of its rank: Frequency is proportional to  $1 / \text{rank}$ . According to Wikipedia, the three most frequent English words are "the", "be", and "to", in that order. Which is the correct statement about what Zipf's law predicts?

- A. "the" occurs three times more often than "be" and two times more often than "to".
- B. "the" occurs two times more often than "be" and three times more often than "to".
- C. "the" occurs ten times more often than "be" and one hundred times more often than "to".
- D. "the" occurs half as often as "be" and one third as often as "to".

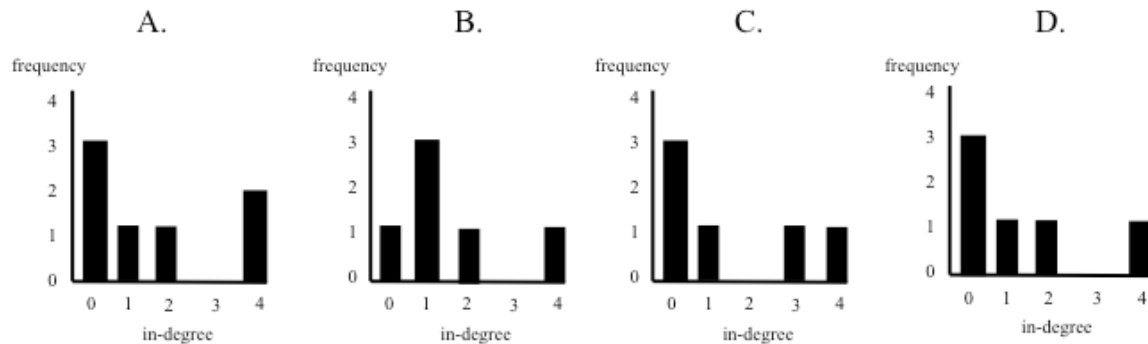
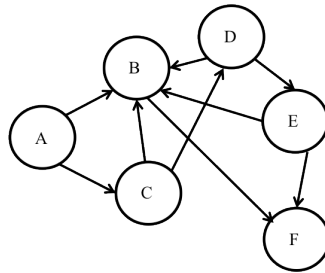
**16.** Unit 9 discussed a comparison of the "Surface Hypothesis" with "Kleiber's law". What is the difference?

- A. Kleiber's law is based on mathematical reasoning and the Surface Hypothesis is based on data.
- B. The Surface Hypothesis posits that metabolic rate is proportional to body mass raised to the  $2/3$  power, and Kleiber's law posits that metabolic rate is proportional to body mass raised to the  $3/4$  power.
- C. Kleiber's law applies to large mammals but not to small mammals.

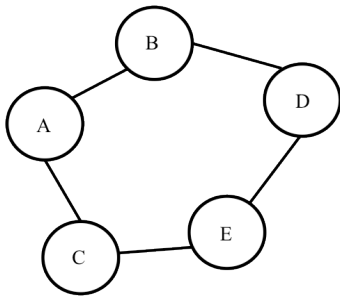
17. Suppose, as discussed in Unit 9, that empirical data about metabolic rate supports Kleiber's hypothesis rather than the Surface Hypothesis. Which of the following does this imply?

- A. The metabolic rate of large organisms is greater than would be predicted by the Surface Hypothesis.
- B. The metabolic rate of large organisms is less than would be predicted by the Surface Hypothesis.
- C. The metabolic rate of large organisms is about equal to what would be predicted by the ratio of surface area to volume.

18. Which is the correct in-degree distribution of the following network?



**19.** What is the clustering coefficient of the 5-node network below?



- A. 0
- B. 0.5
- C. 0.75
- D. 1

**20.** Which statement is true?

- A. Small-world networks are distinguished by having high average path-length and low clustering.
- B. Small-world networks are distinguished by having low average path-length and high clustering.
- C. Small-world networks are distinguished by having low average path-length and low clustering.

**21.** True or False: “A scale-free network has a power-law degree distribution.”

- A. True
- B. False



**22.** How many years has Patrisia Brunello been the receptionist at SFI?

A. 11 years

B. 21 years

C. 30 years

**23.** Simon DeDeo never wears glasses.

A. True

B. False

**24.** Cheese is sometimes served at teatime at the Santa Fe Institute.

A. True

B. False