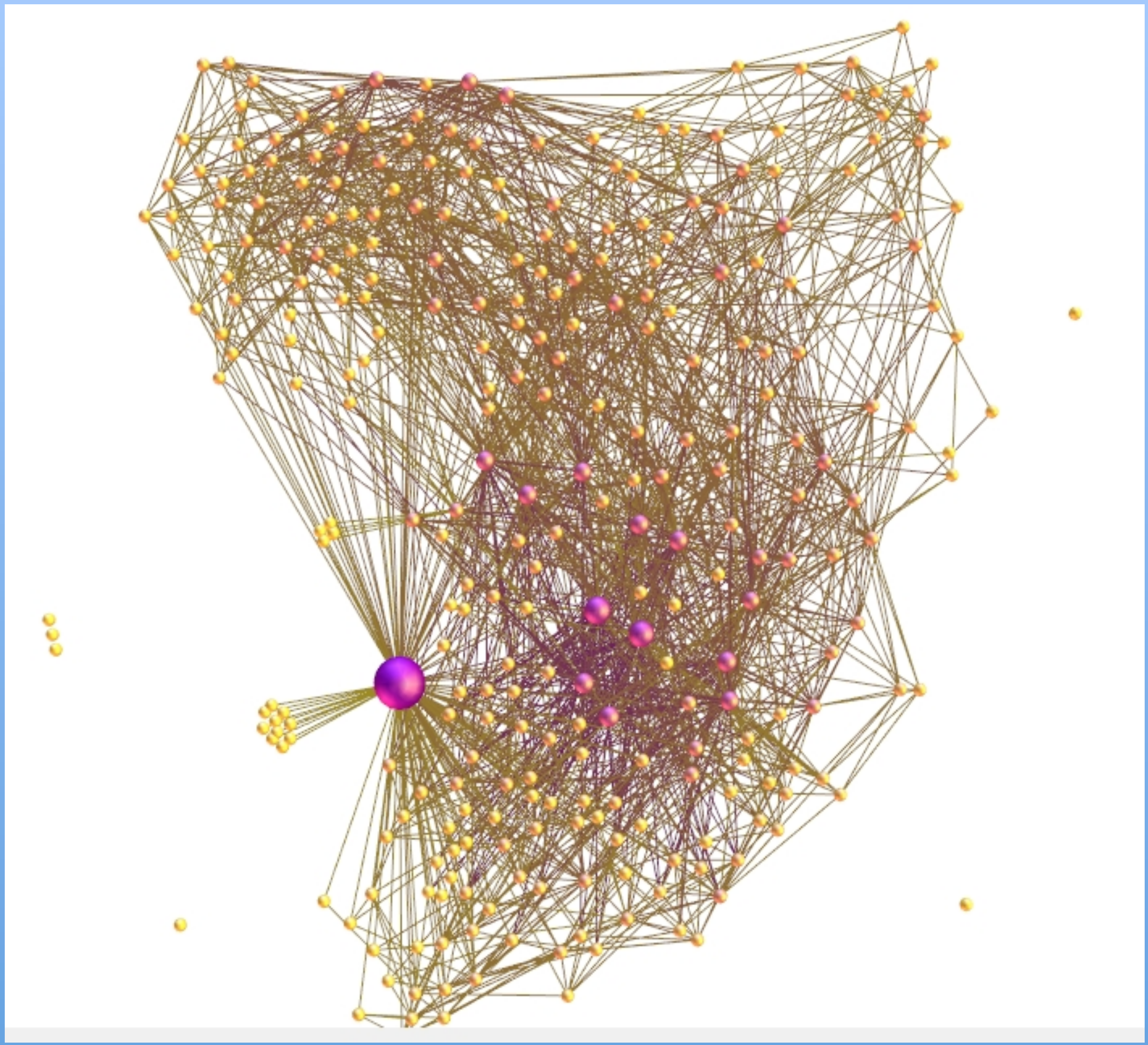
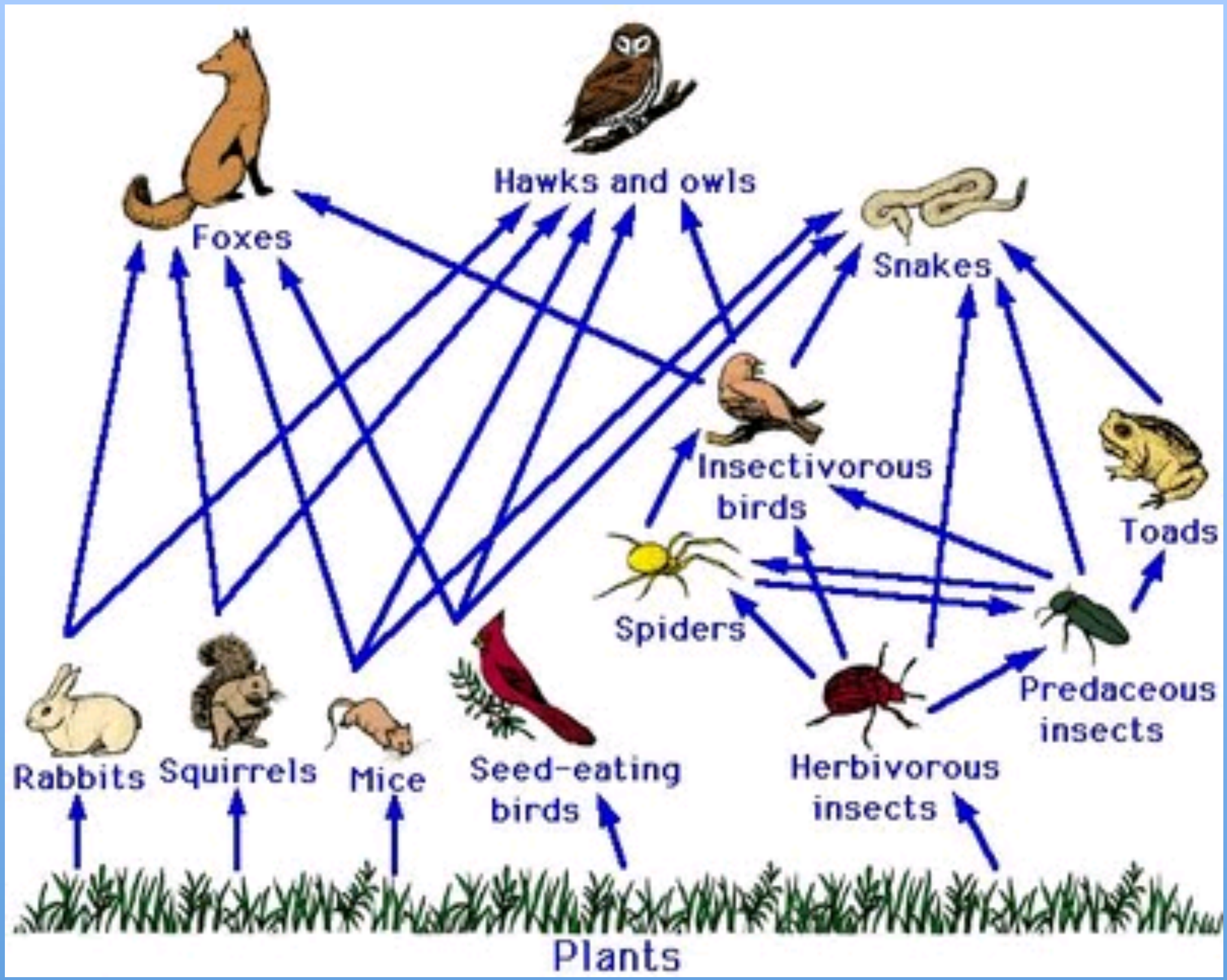
What are Networks?



Neural Network
(*C. Elegans*)

http://gephi.org/wp-content/uploads/2008/12/screenshot-celegans.png



Food Web

http://1.bp.blogspot.com/_vIFBm3t8boU/SBhzqbchIeI/AAAAAAAAAXk/RsC-Pj45Avc/s400/food%2Bweb.bmp

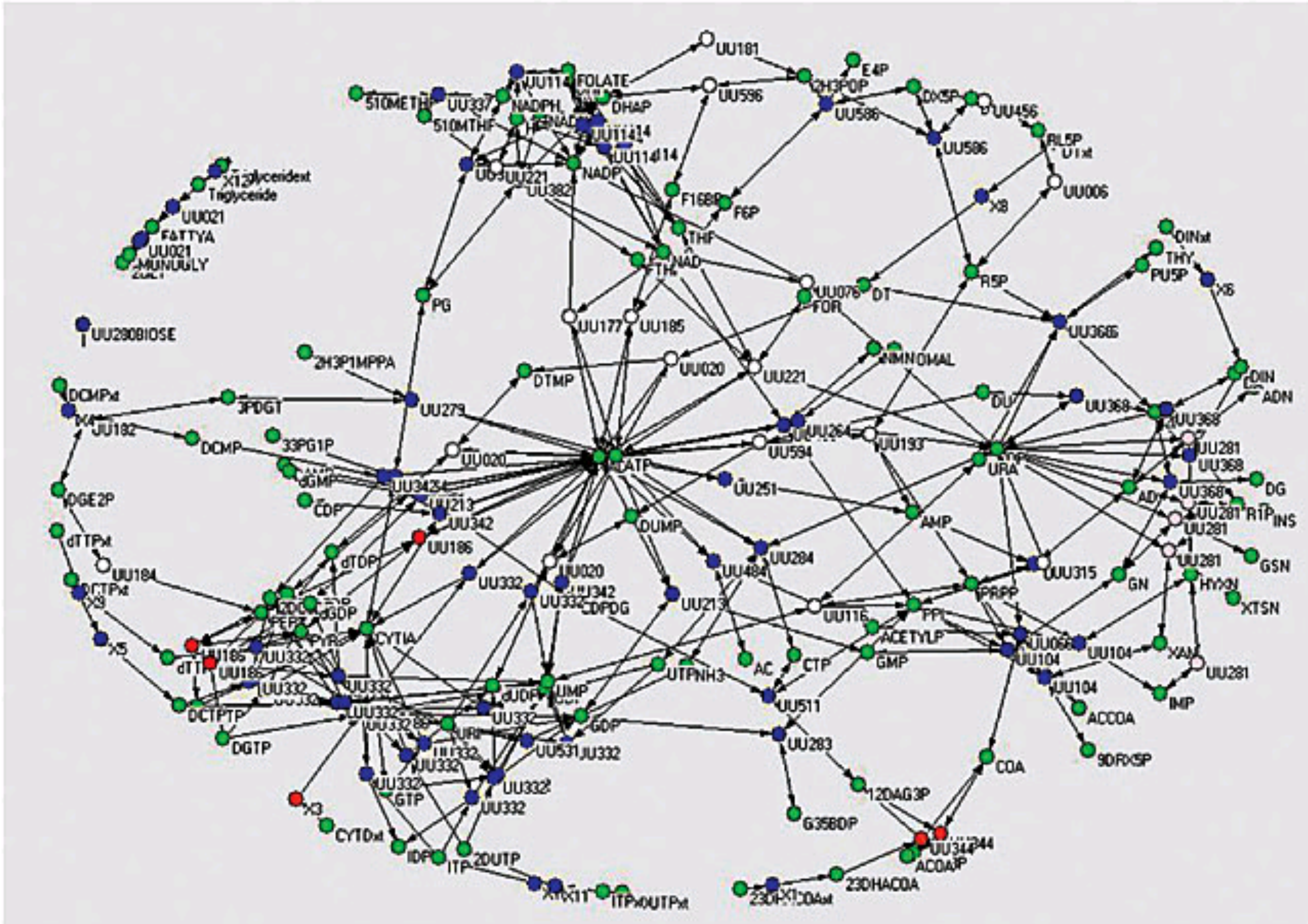


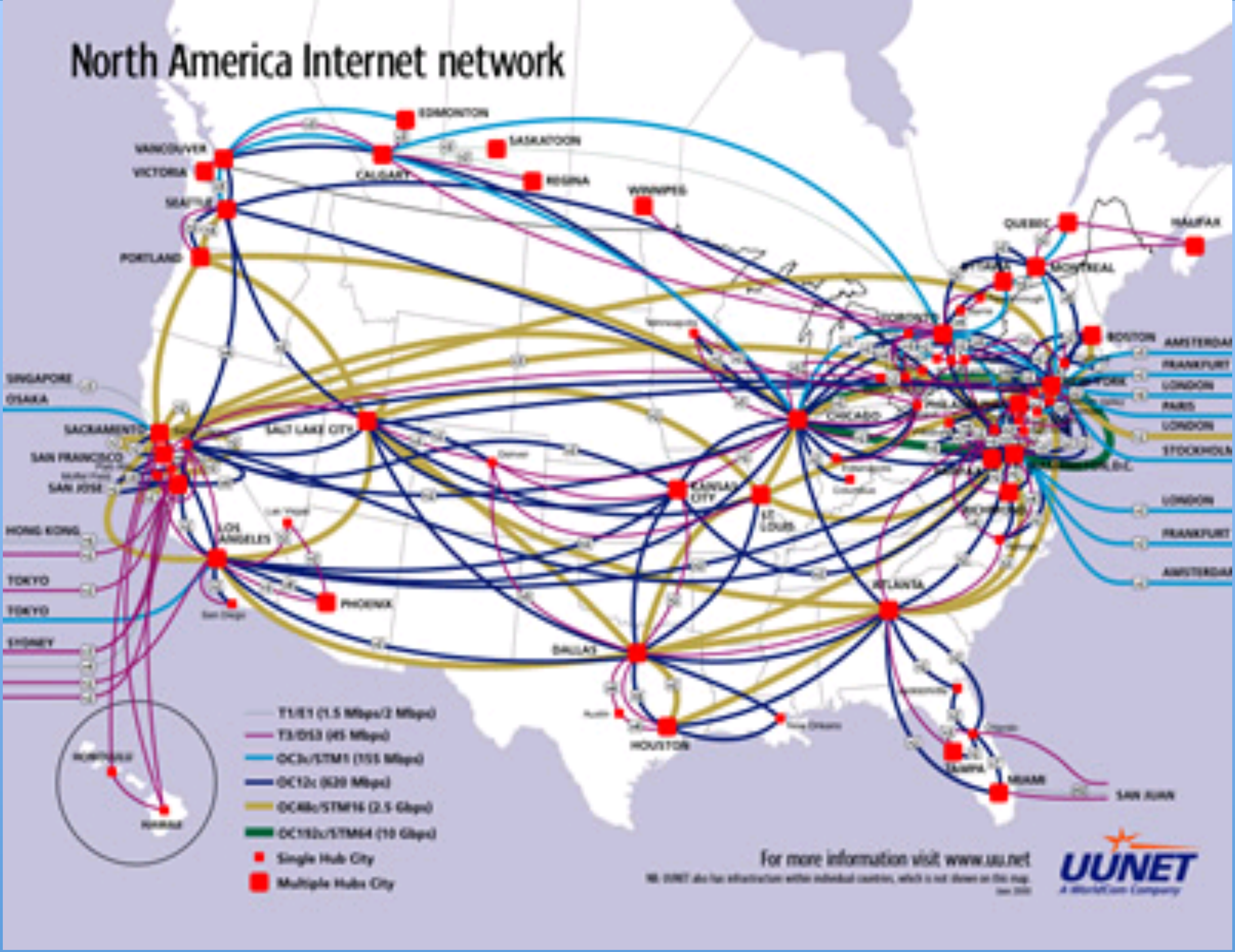
Figure 2. Bipartite graph of the metabolic network of *Ureaplasma urealyticum*. Dark gray and white nodes represent enzymes and light gray nodes represent metabolites (Lemke et al., 2004).

Metabolic Network

http://www.funpecrp.com.br/gmr/year2005/vol3-4/wob01_full_text.htm

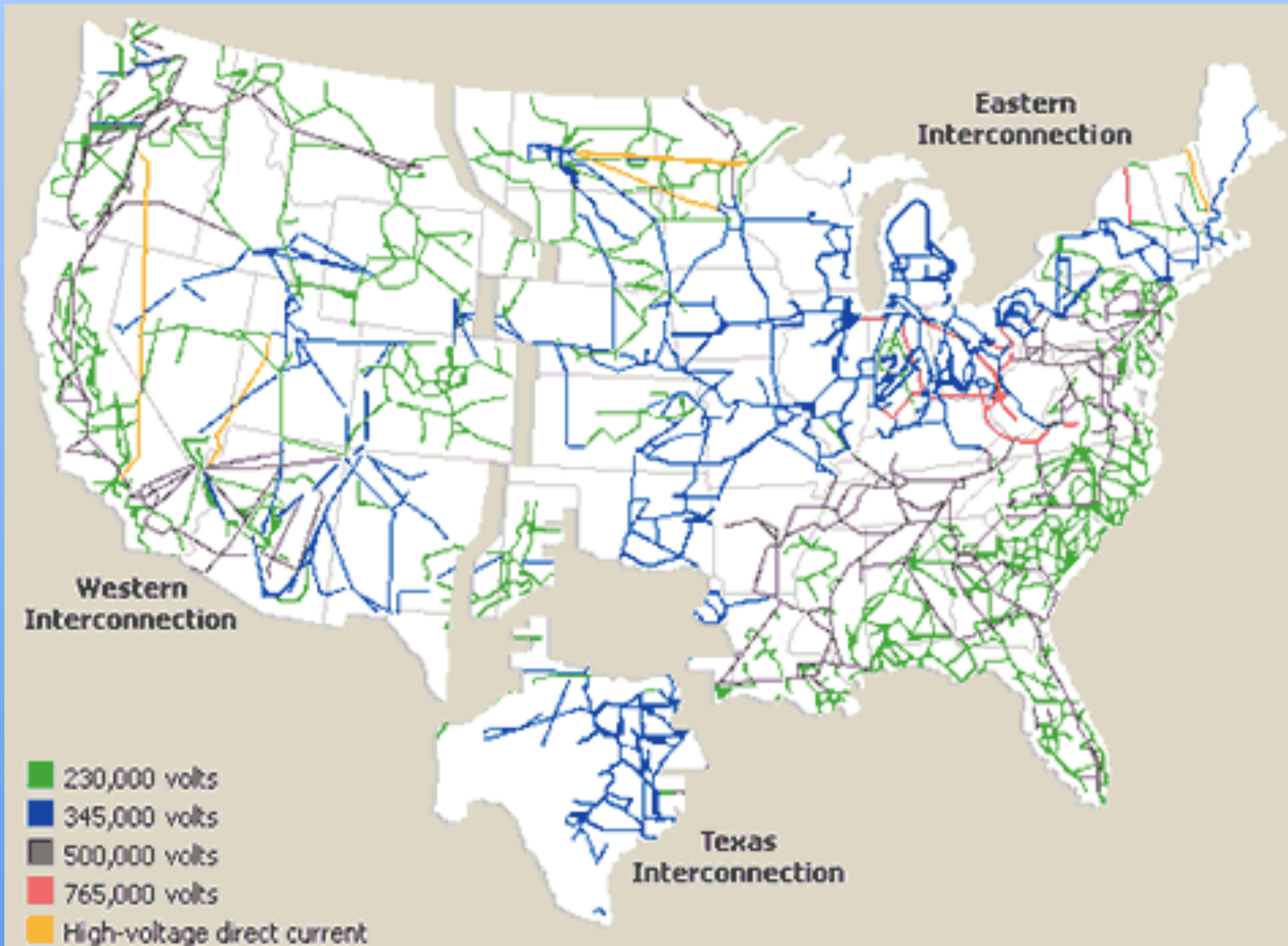
Airline Routes

http://virtualskies.arc.nasa.gov/research/tutorial/images/12routemap.gif



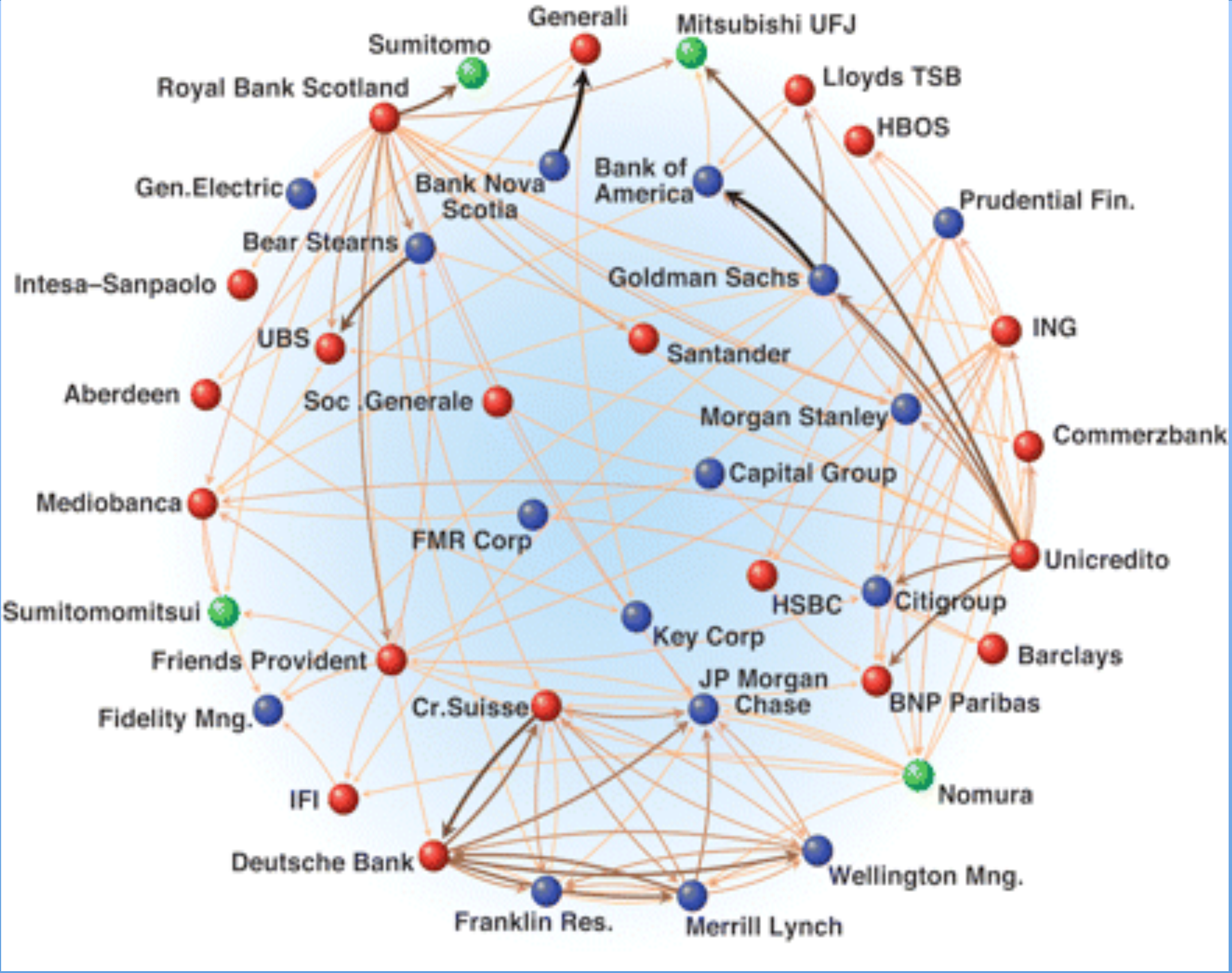
Internet

http://www.visualcomplexity.com/vc/images/270_big01.jpg



US Power Grid

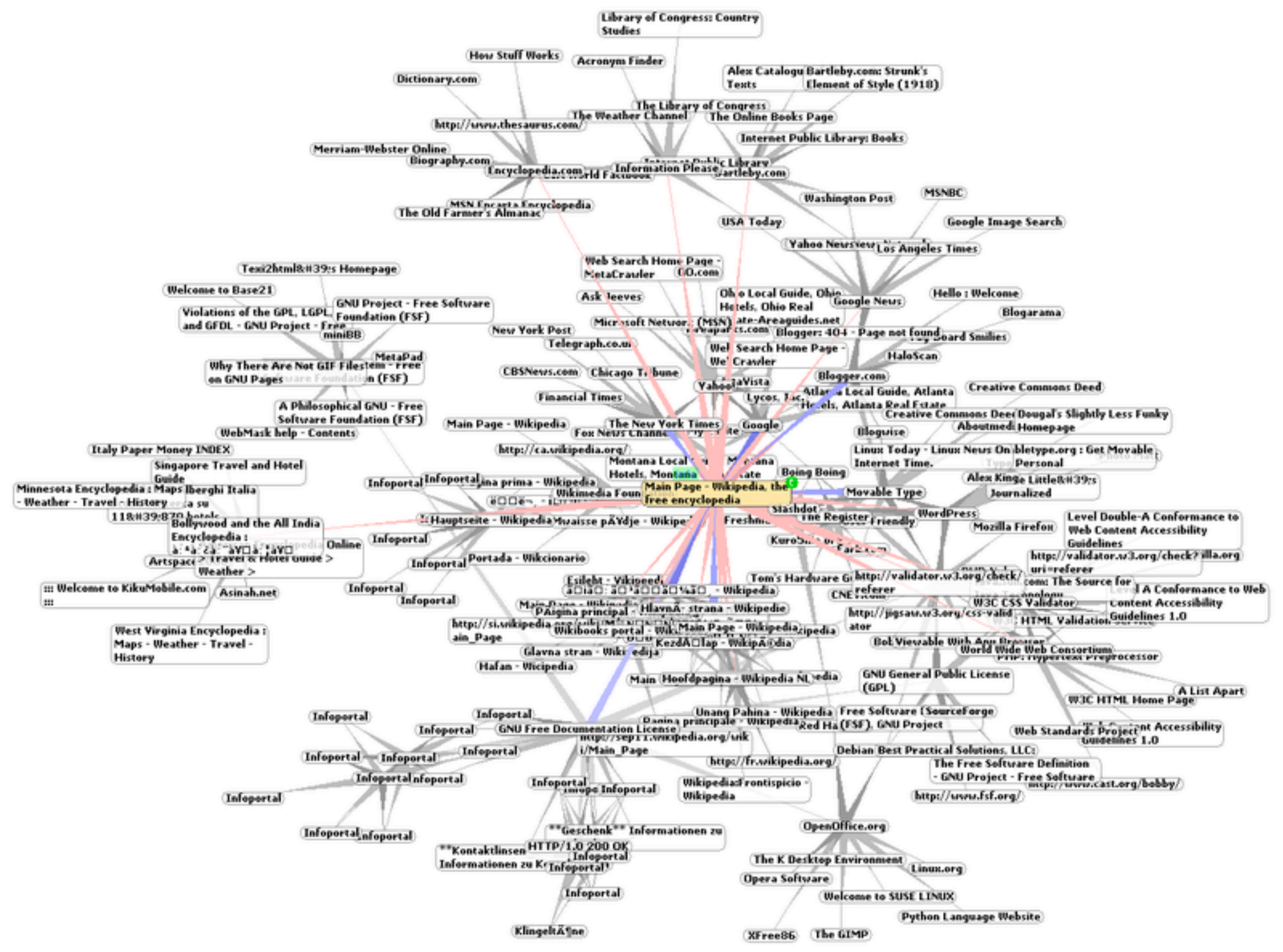
http://images.encarta.msn.com/xrefmedia/aencmed/targets/maps/map/000a5302.gif



Bank Network

From Schweitzer et al., Science, 325, 422-425, 2009
http://www.sciencemag.org/cgi/content/full/325/5939/422

The World Wide Web
(some hyperlinks from Wikipedia.org)



http://en.wikipedia.org/wiki/File:WorldWideWebAroundWikipedia.png

Social Networks

The Science of Networks

Are there properties common to all complex networks?

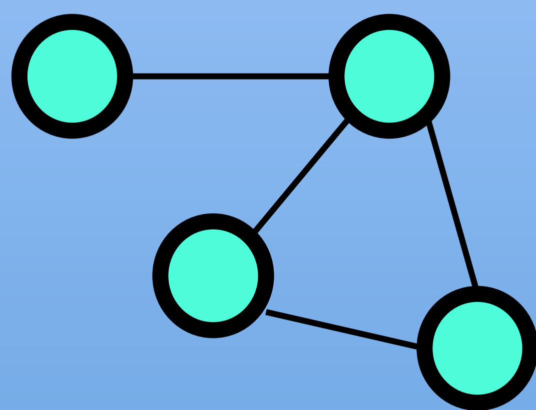
If so, why?

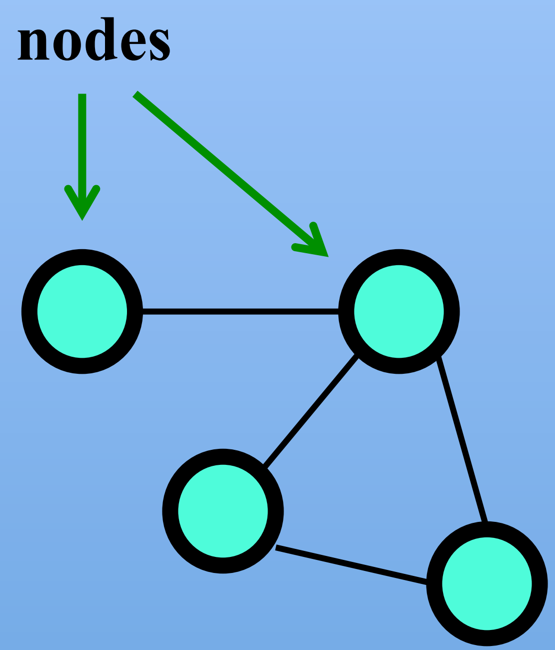
Can we formulate a general theory of the structure, evolution, and dynamics of networks?

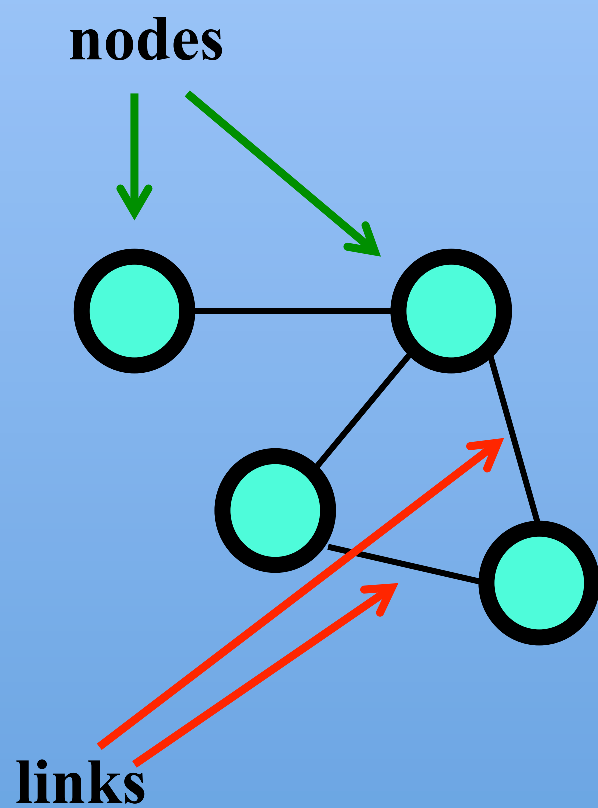
Proposed common properties:

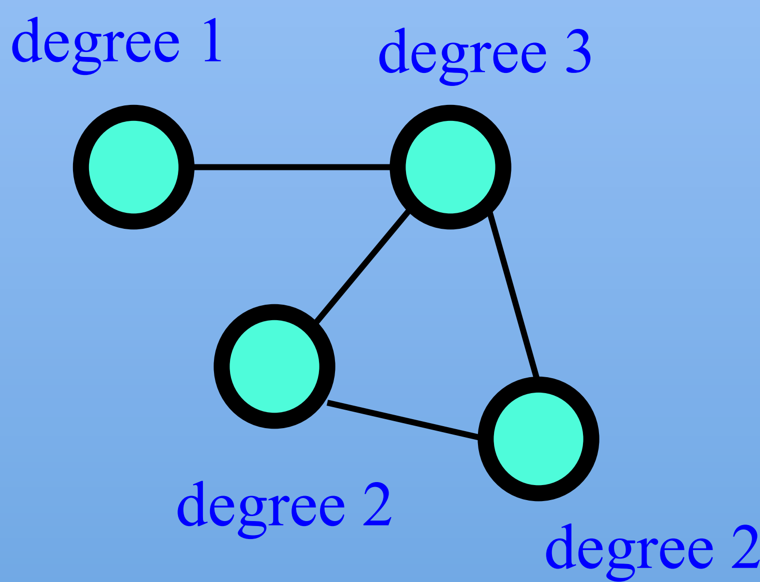
- Small world property
- Long-tailed degree distribution
  - Special case: *Scale-free* structure
- Clustering and community structure
- Robustness to random node failure
- Vulnerability to targeted hub attacks
- Vulnerability to cascading failures

Some Terminology




nodes

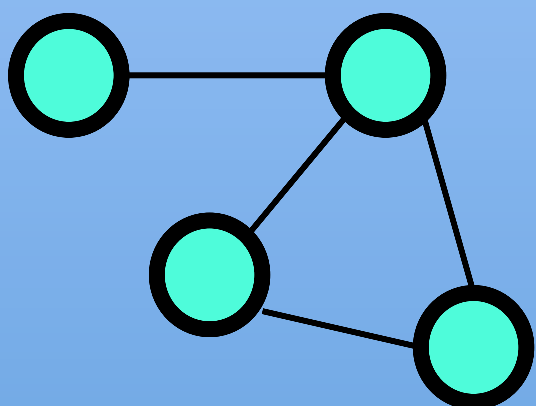

nodes
links



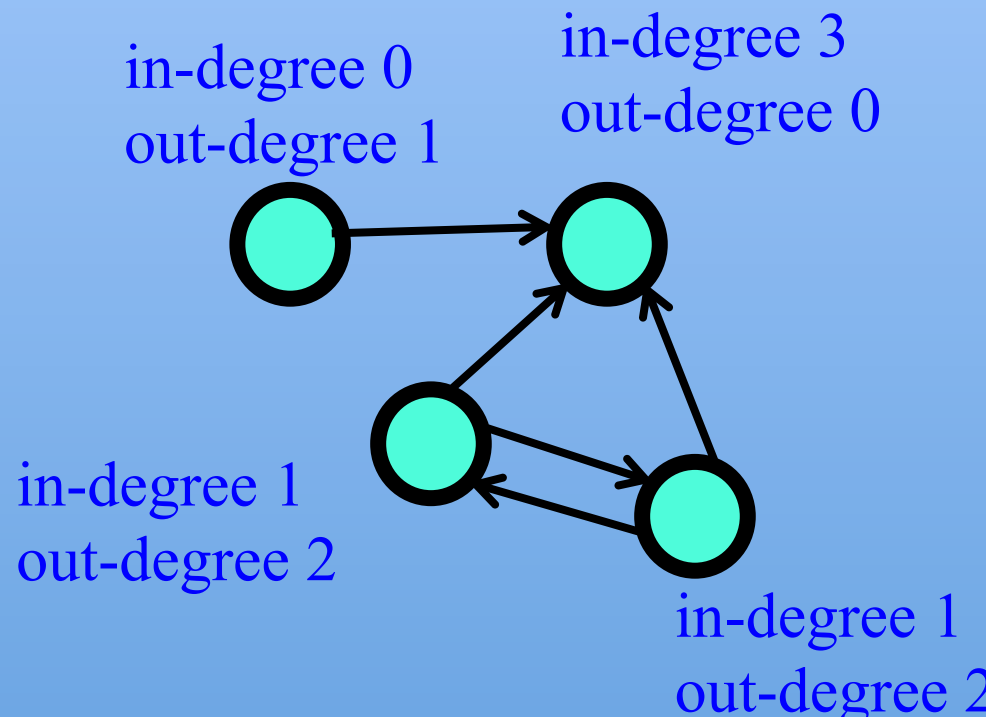
***Degree* of a node:**

Number links coming into (or out of) node

Links can be undirected or directed

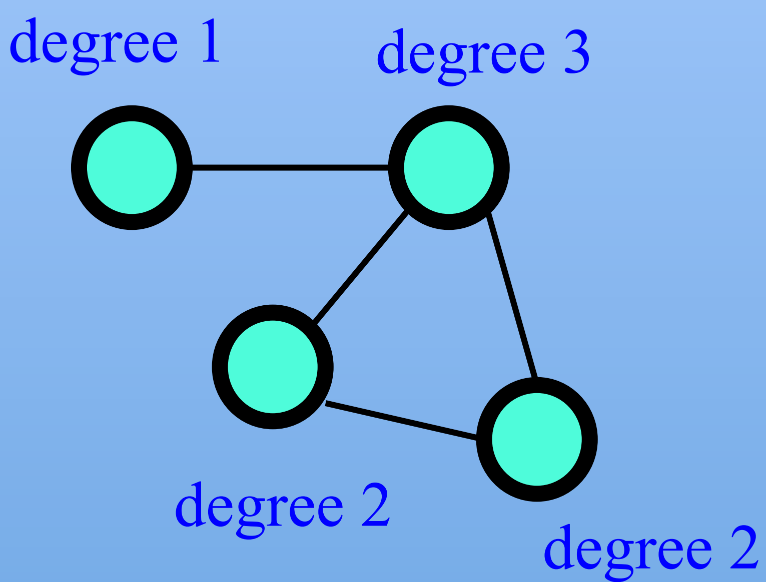


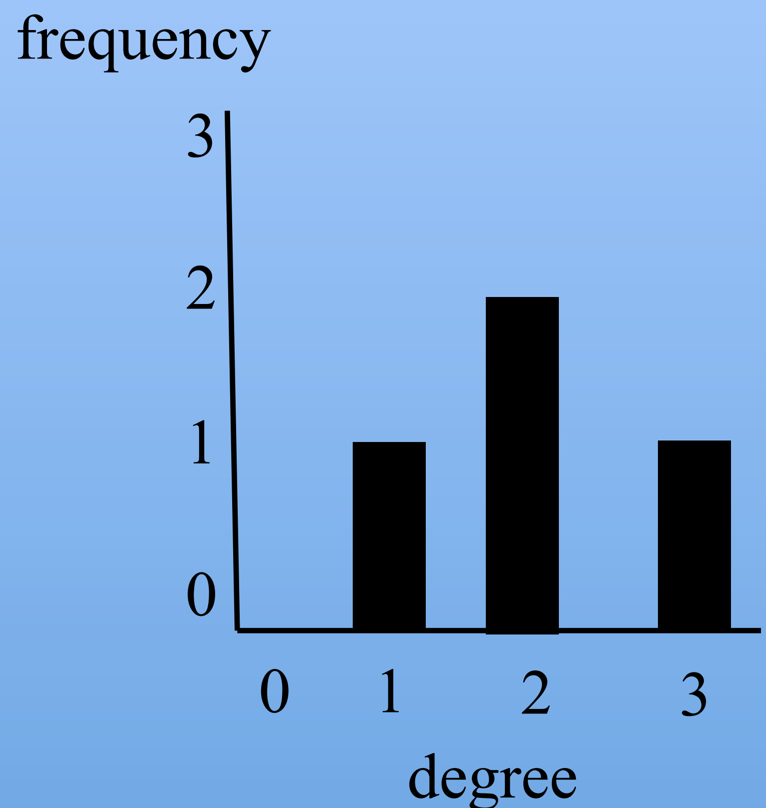
Undirected links
(e.g., Facebook friendship links)



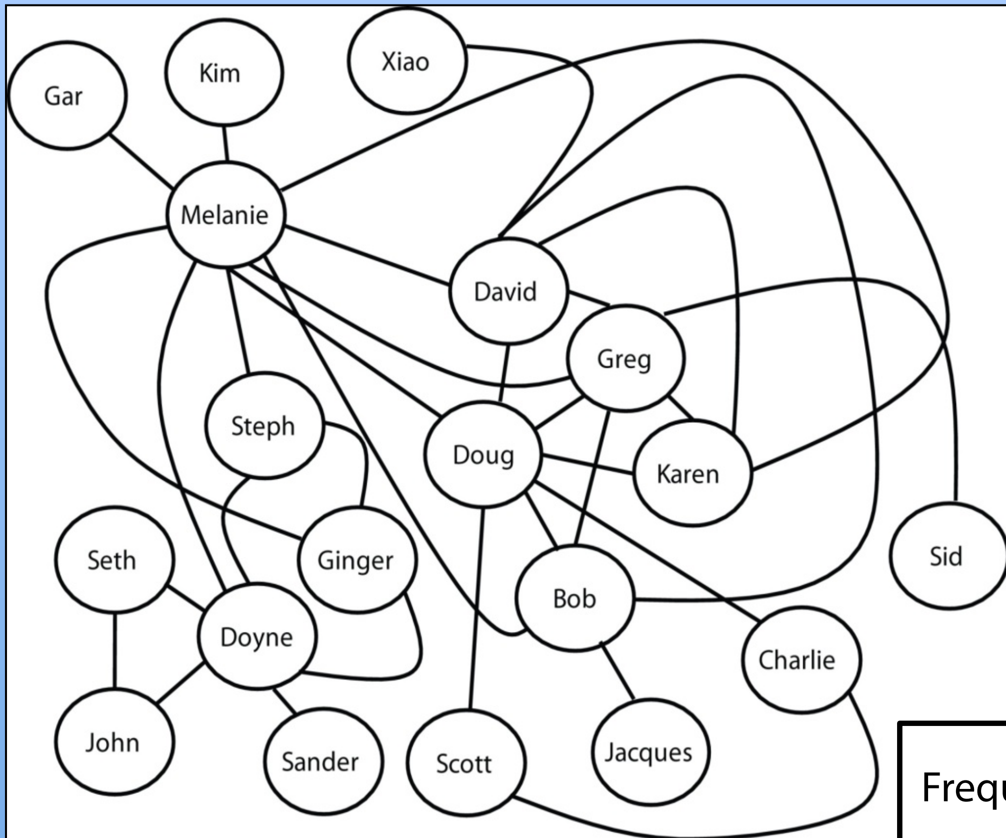
Directed links
(e.g., Twitter “following” links)

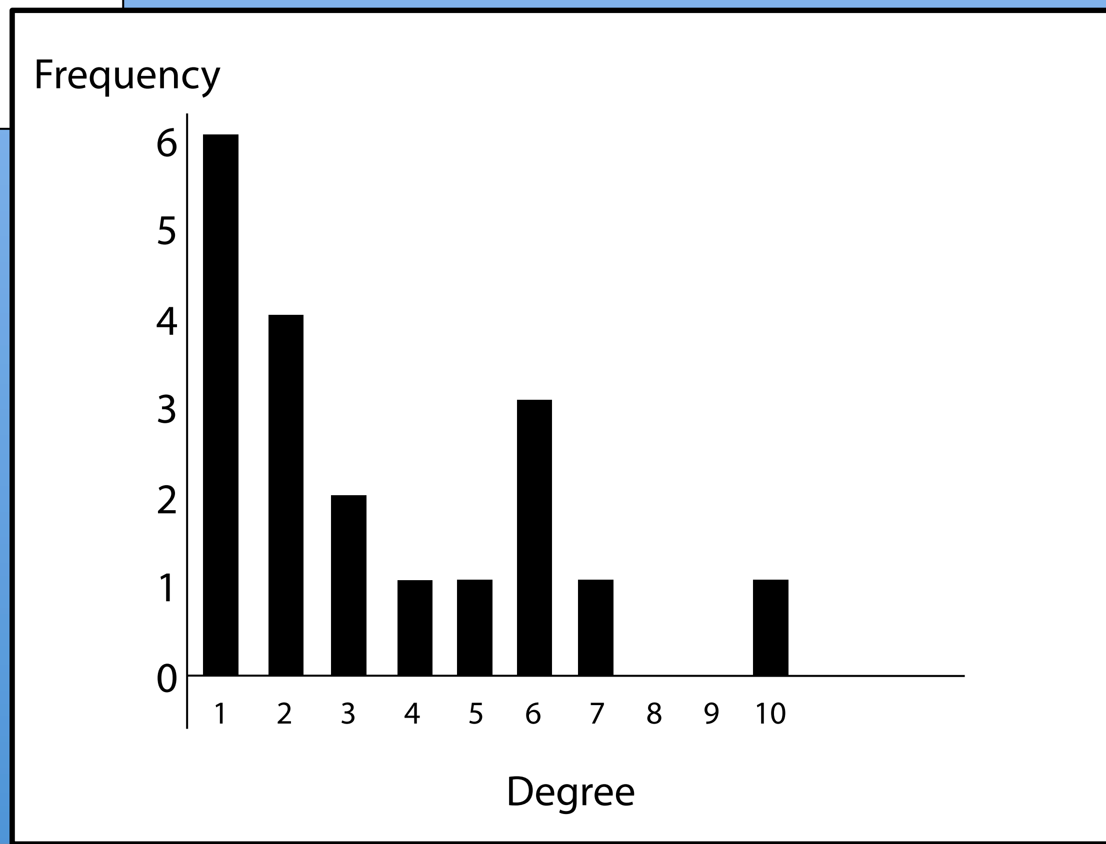
Degree distribution of a network



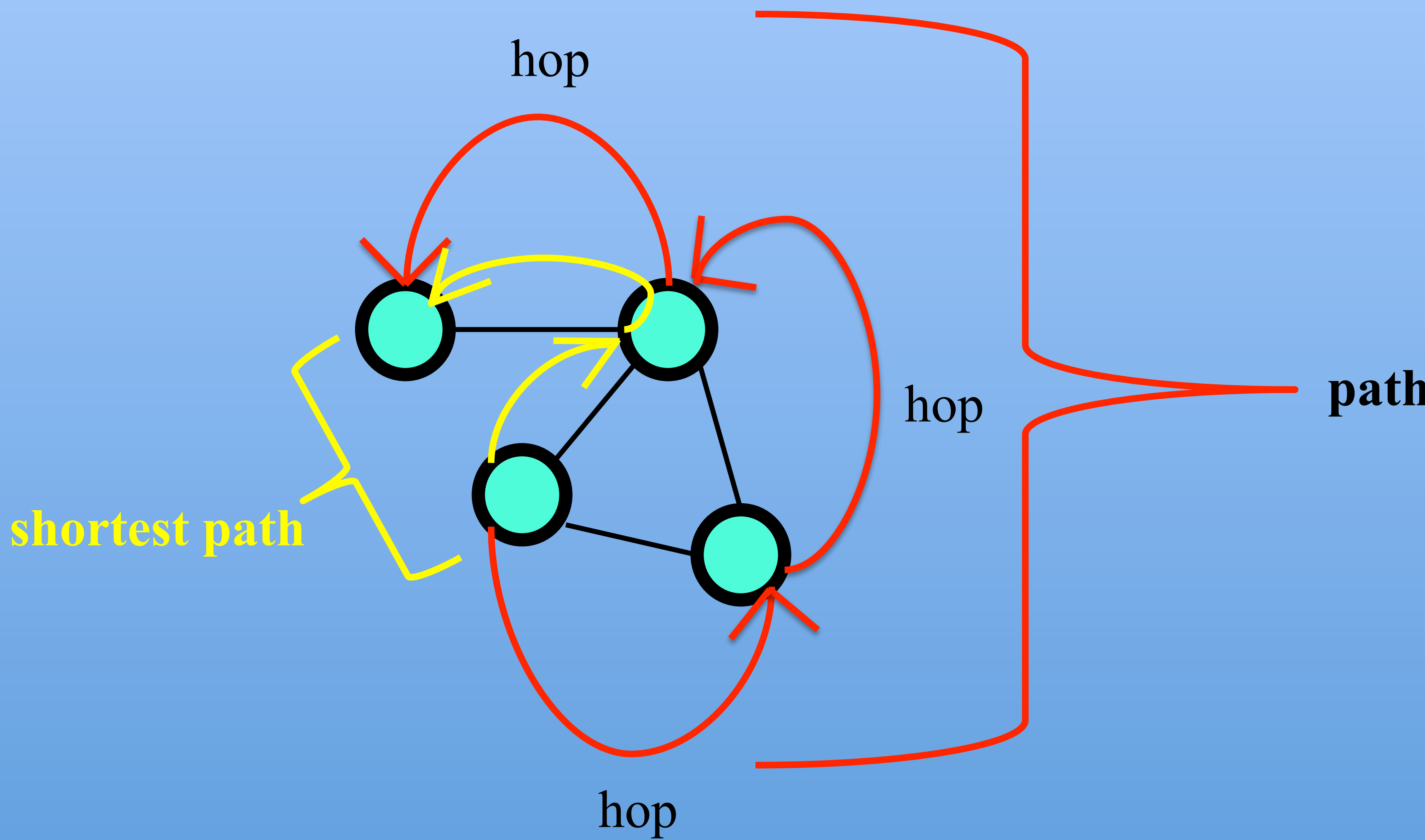


From M. Mitchell,
Complexity: A Guided Tour





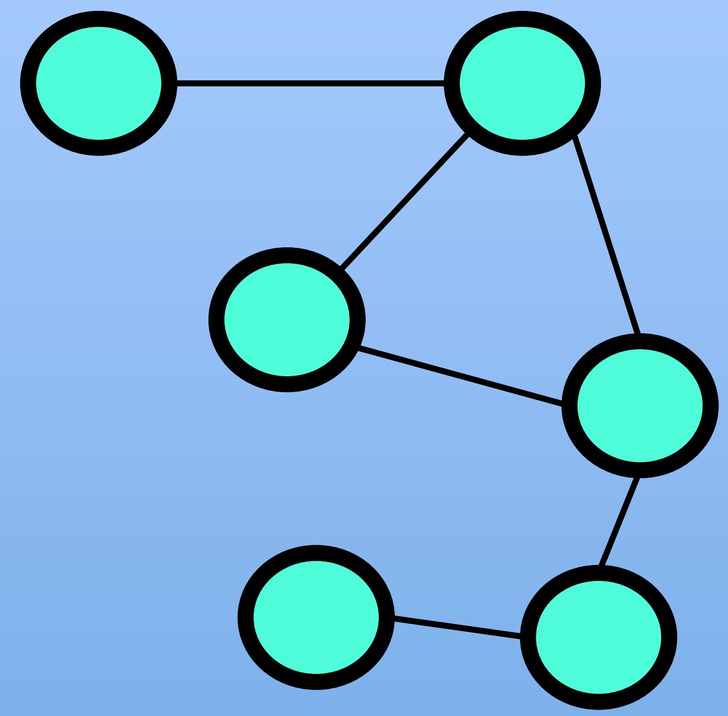
Distance and Paths in Networks



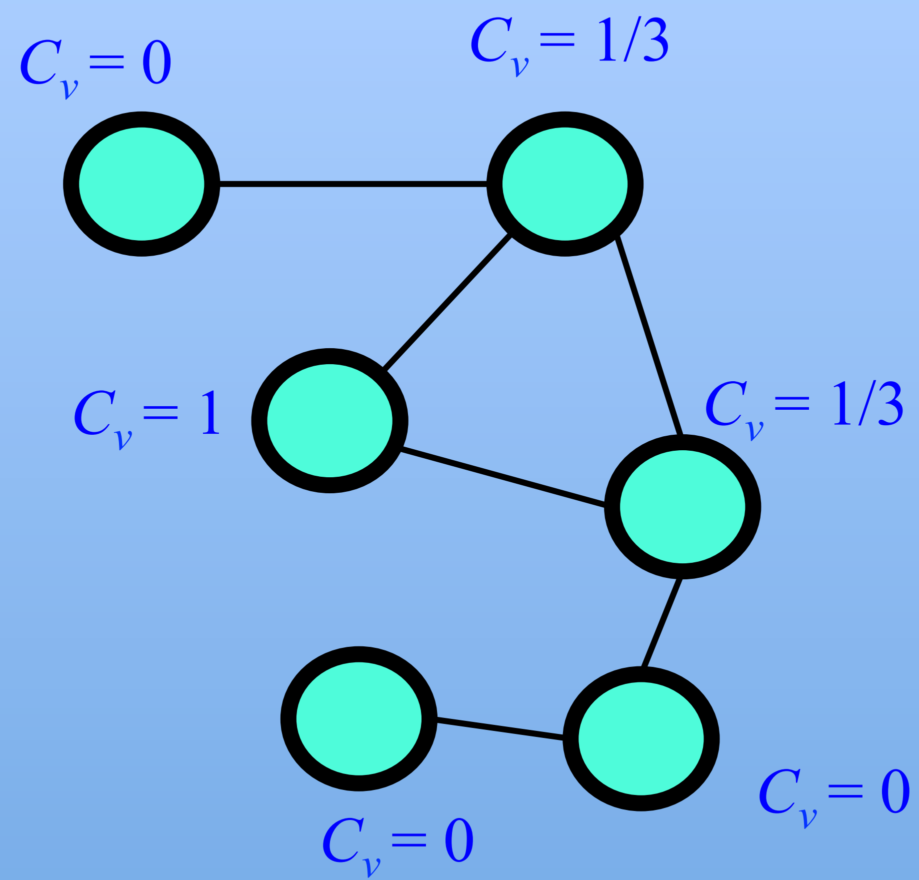
Distance L **between nodes** ***A*** **and** ***B*****:**
Number of hops in shortest path between *A* and *B*

Clustering in Networks

Clustering: To what extent are your friends also friends of one another?



Clustering in Networks



Clustering: To what extent are your friends also friends of one another?

Clustering with respect to a node *v*:

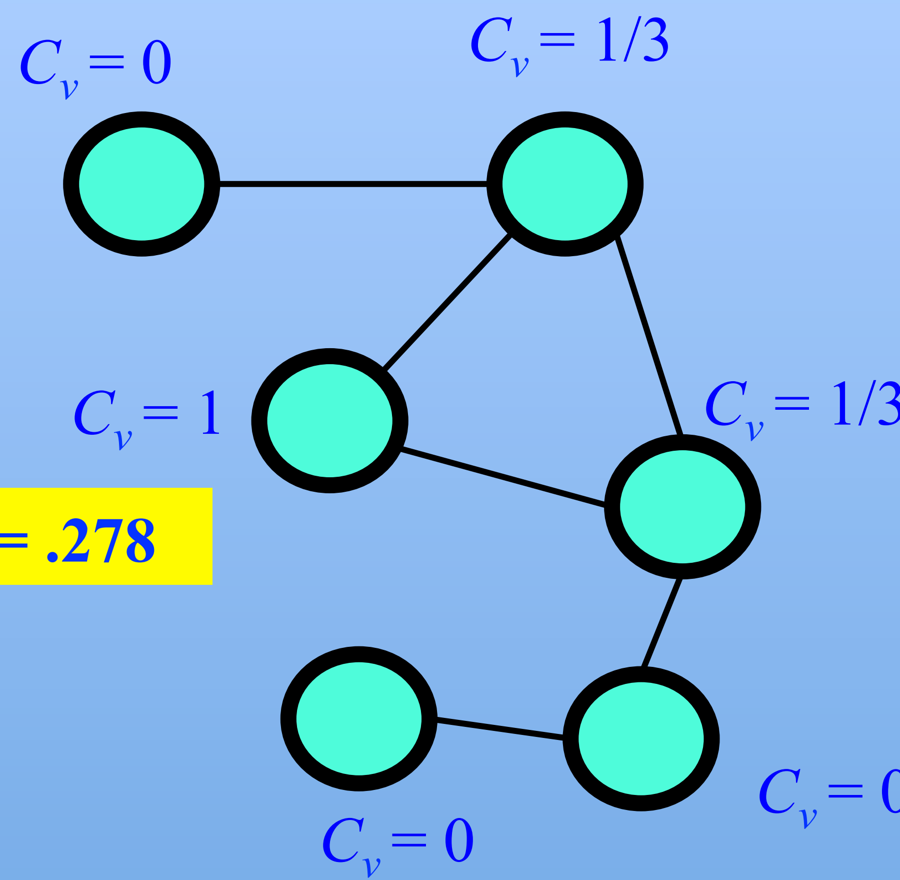
C_v = fraction of pairs of neighbors that are connected to one another

That is, if a node v has k_v neighbors, there are $k_v(k_v - 1)/2$ pairs of neighbors. C_v is the fraction of the $k_v(k_v - 1)/2$ pairs that are linked.

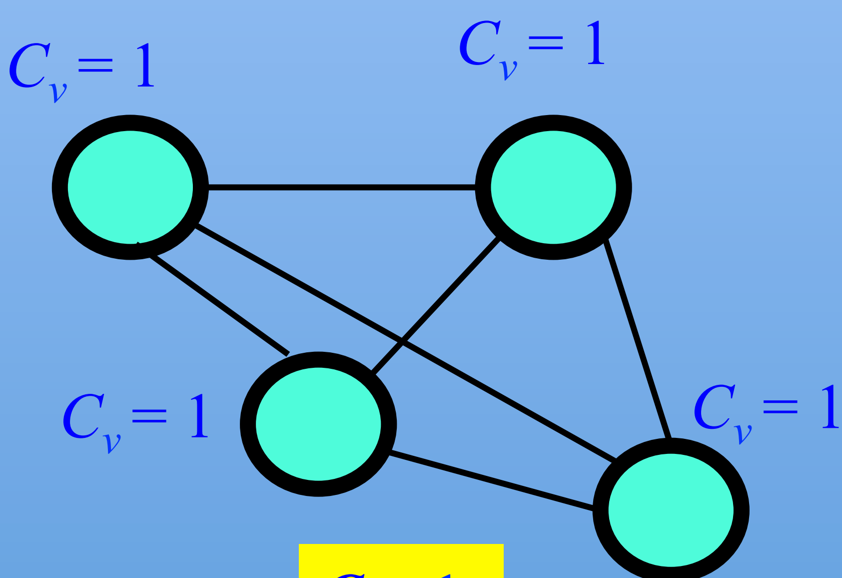
Clustering coefficient:

C = Average C_v over all nodes v. $C = 1.67 / 6 = .278$

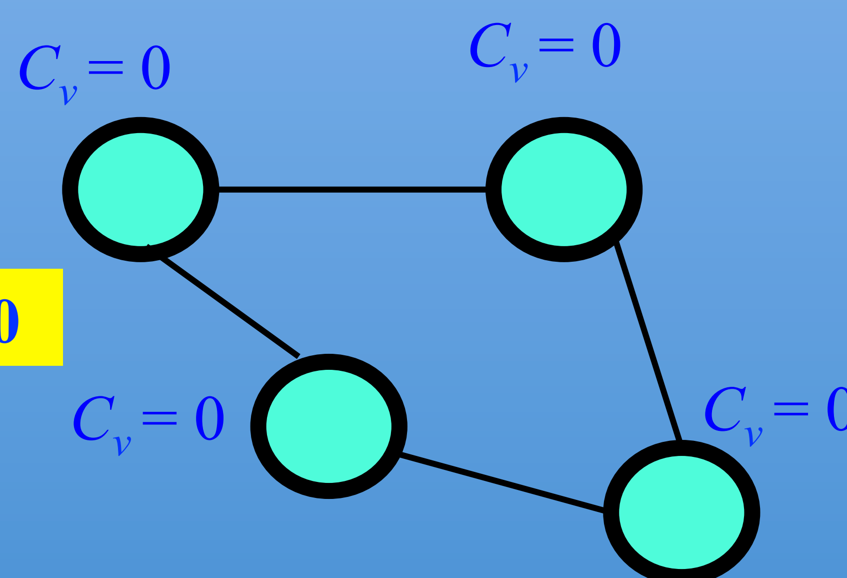
Clustering in Networks



$C = = .278$



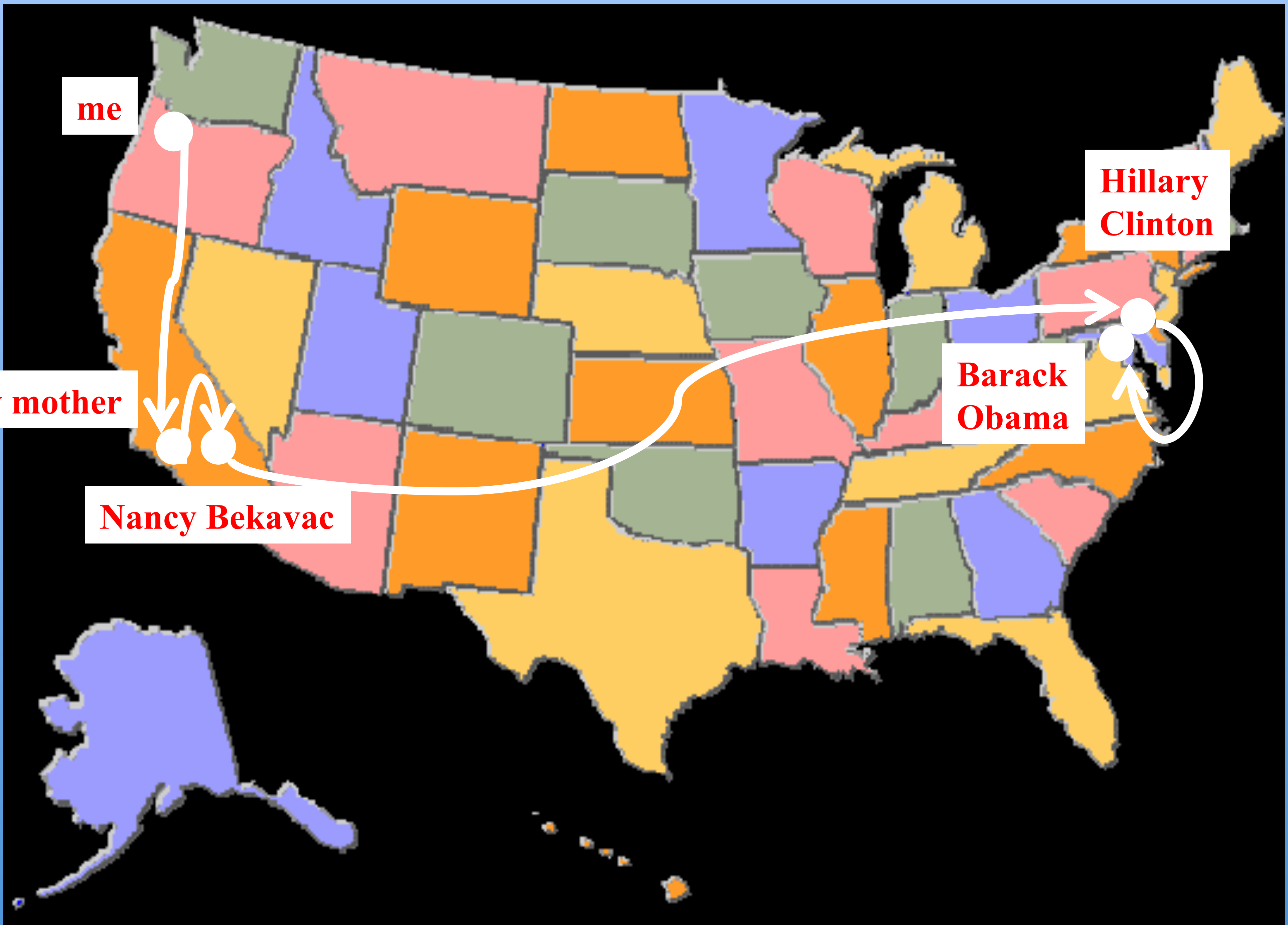
$C = 1$



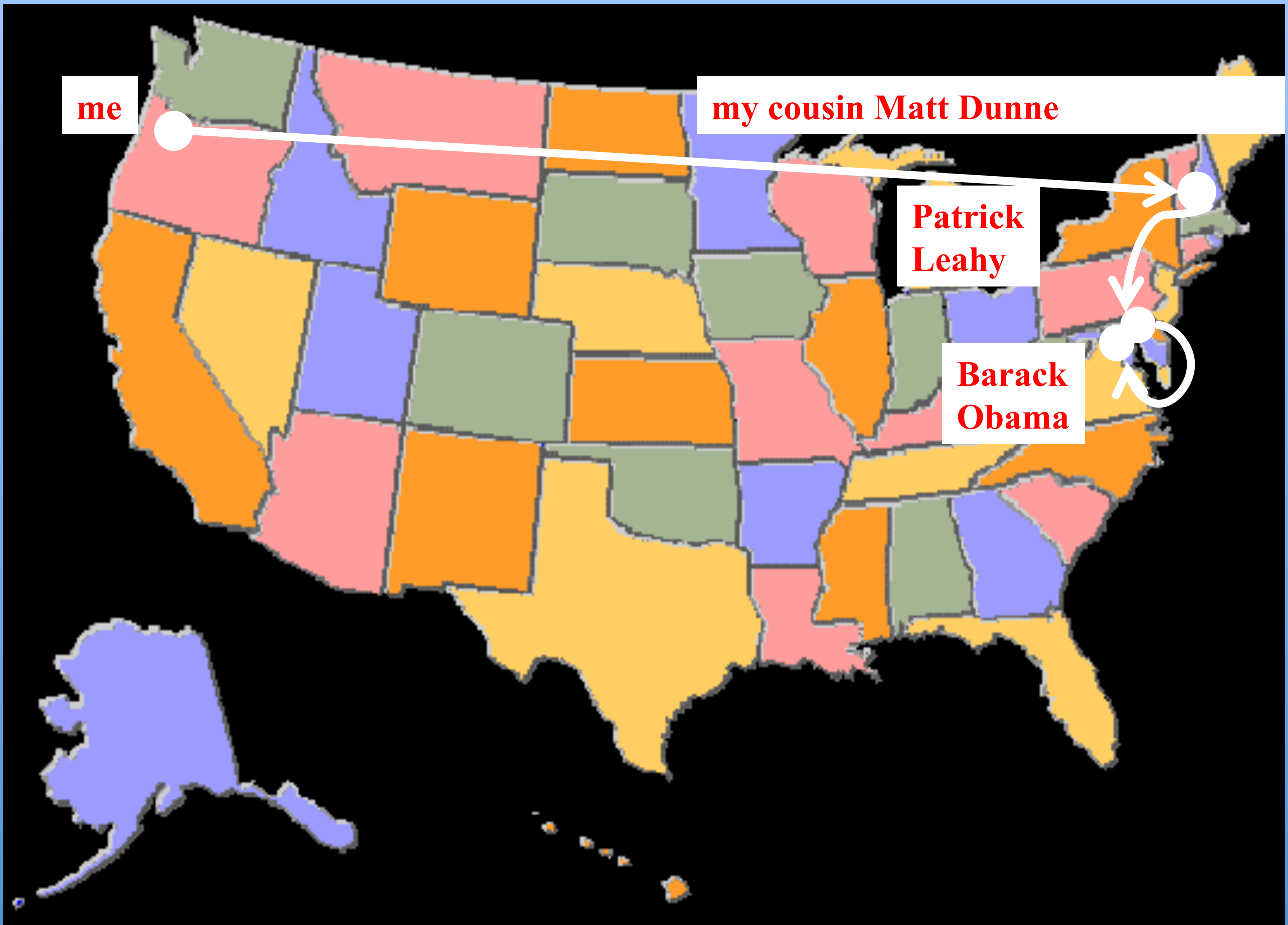
$C = 0$

"It's a small world!"

Small-World Networks

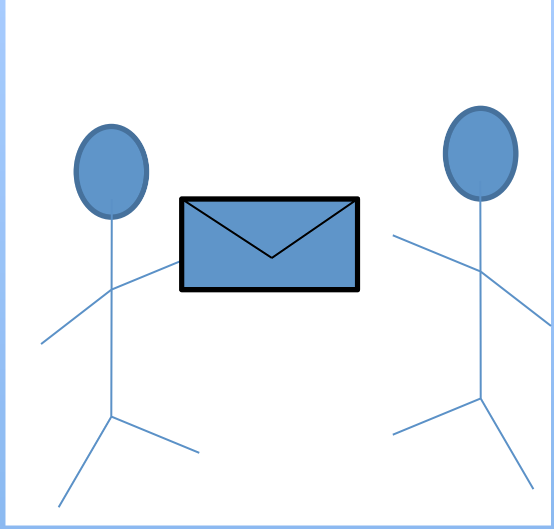


Small-World Networks



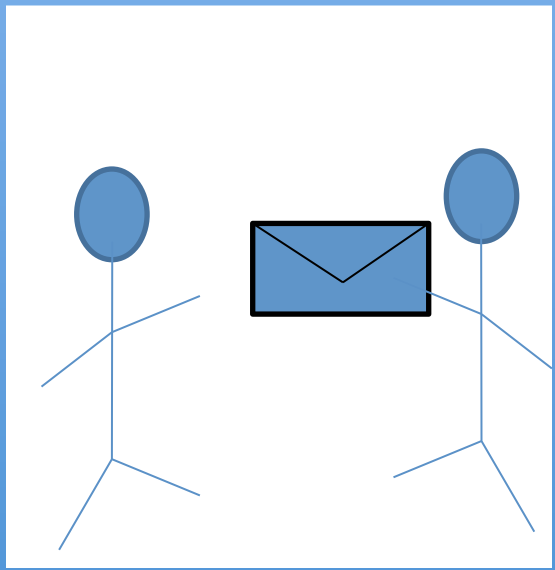
Stanley Milgram

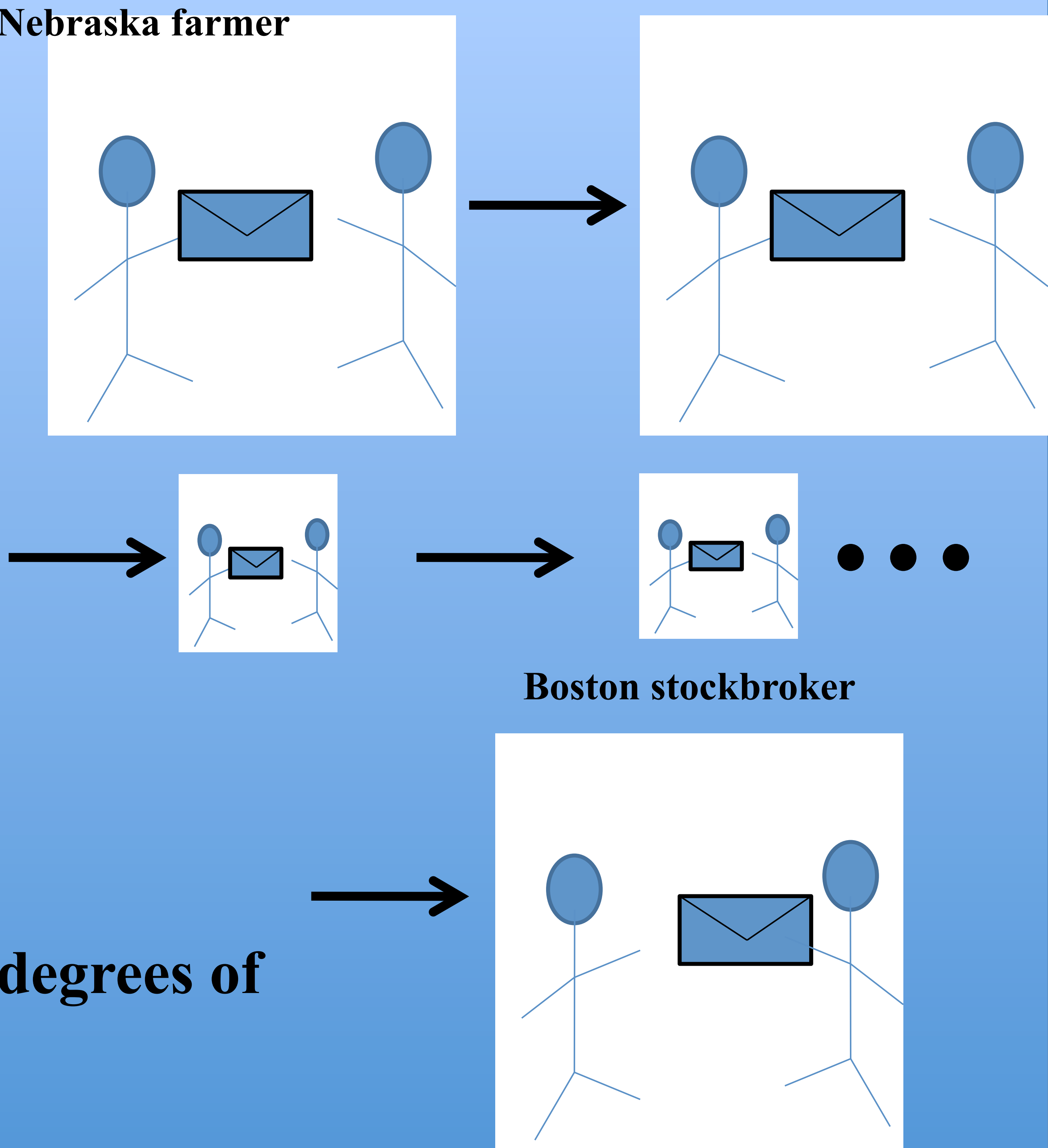
Nebraska farmer



Stanley Milgram

Boston stockbroker

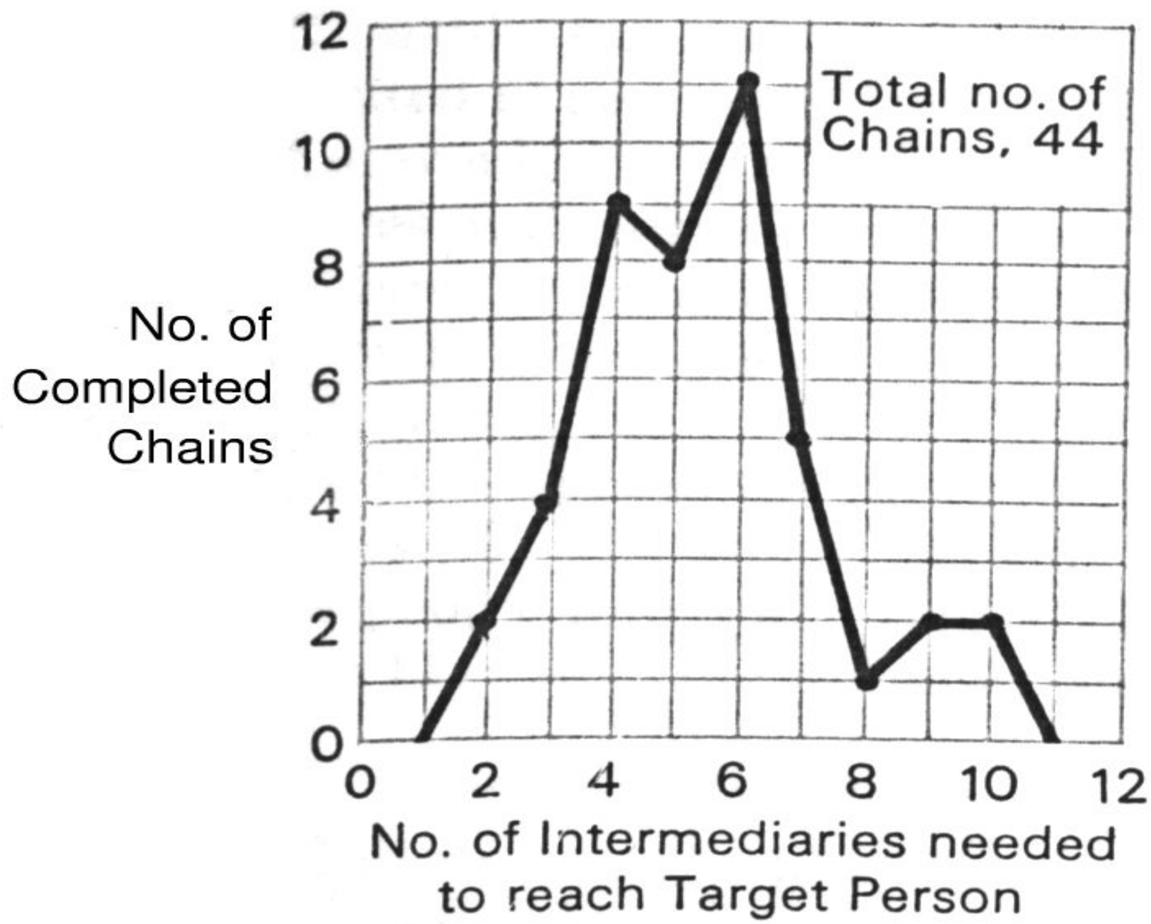




Stanley Milgram

On average: “six degrees of separation”

From S. Milgram, The small-world problem. *Psychology Today*, 1967



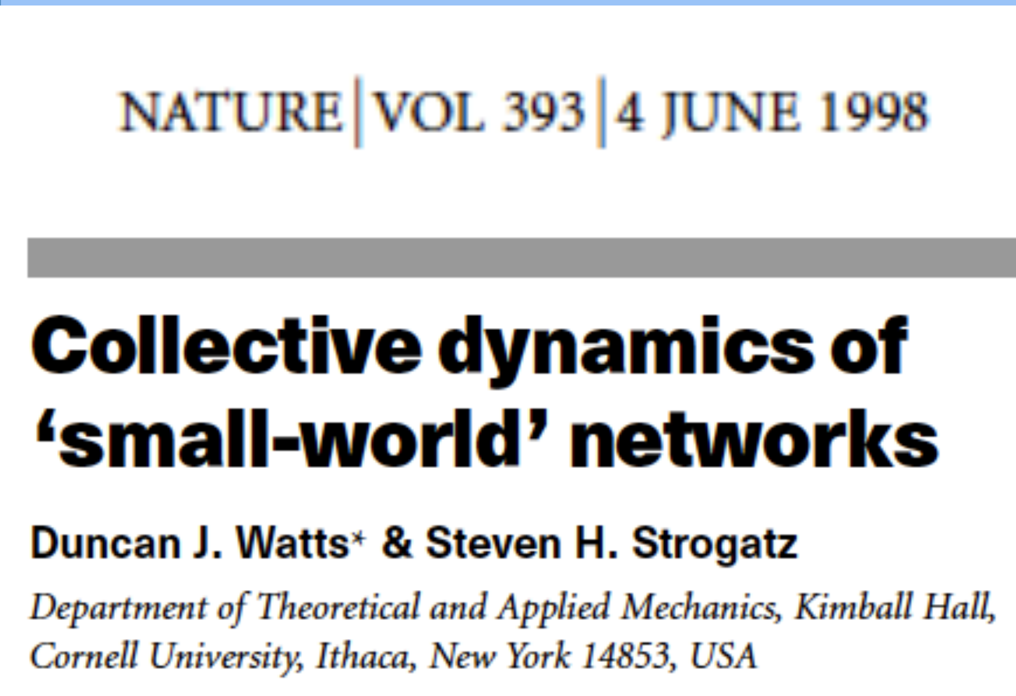
In the Nebraska Study the chains varied from two to 10 intermediate acquaintances with the median at five.

The Small-World Property

The network has relatively few "long-distance" links but there are short paths between most pairs of nodes, usually created by "hubs".

Most real-world complex networks seem to have the small-world property!

Small-World Model
(Watts and Strogatz, 1998)



NATURE | VOL 393 | 4 JUNE 1998

Collective dynamics of 'small-world' networks

Duncan J. Watts* & Steven H. Strogatz

Department of Theoretical and Applied Mechanics, Kimball Hall, Cornell University, Ithaca, New York 14853, USA

Duncan Watts

Steven Strogatz

http://www.columbia.edu/cu/news /01/12/images/duncanWatts.jpg

http://en.wikipedia.org/wiki/ File:Steven_H_Strogatz.jpg

Watts & Strogatz Study

Real-World Networks:

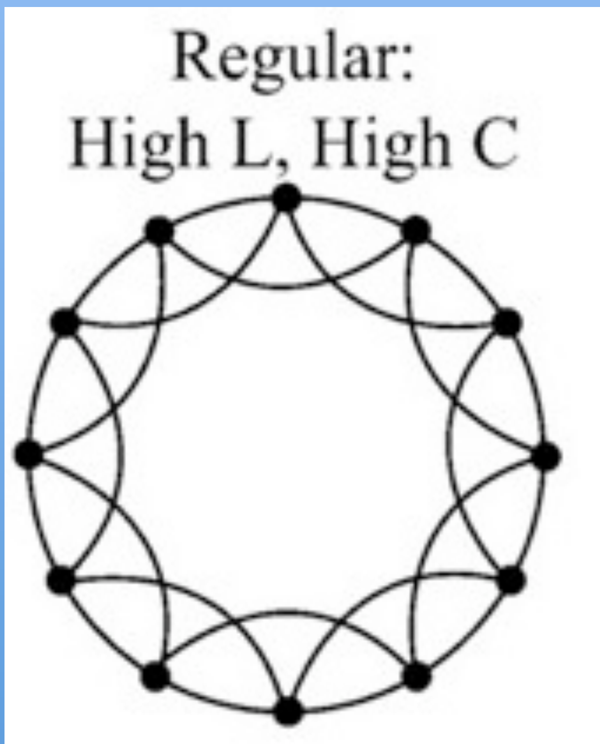
- **Film actors:**
  - Nodes = individual actors;
  - Two nodes are linked if the two actors appeared in the same movie.

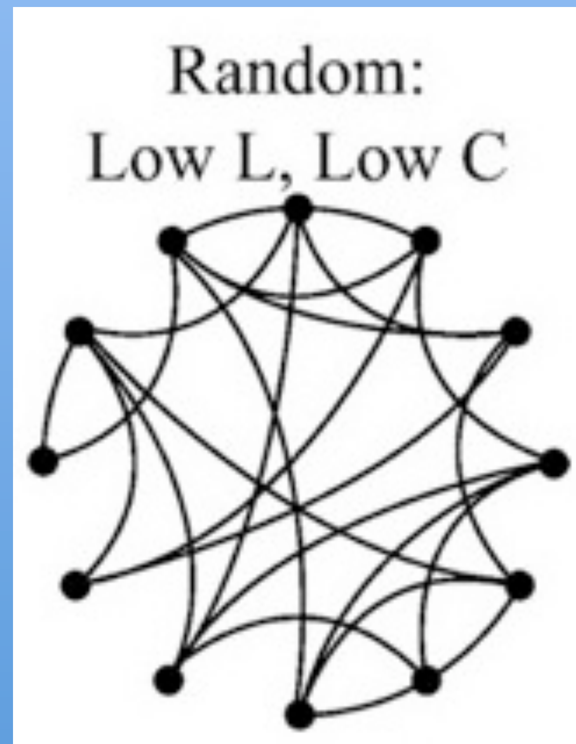
- **Power grid:**
  - Nodes = generators, transformers, and substations;
  - Two nodes are linked if they have a transmission line between them.

- **C. *elegans*:**
  - Nodes = neurons;
  - Two nodes are linked if they are connected by a synapse or a gap junction

Network Structure

Naive guess for network structure of real-world networks would be either completely “regular” or completely random.





High average distance between nodes, high average clustering

Low average distance between nodes Low average clustering

Strogatz and Watts' Empirical Results

	L_{actual}	L_{random}	C_{actual}	C_{random}
Film actors	3.65	2.99	0.79	0.00027
Power grid	18.7	12.4	0.080	0.005
C. elegans	2.65	2.25	0.28	0.05

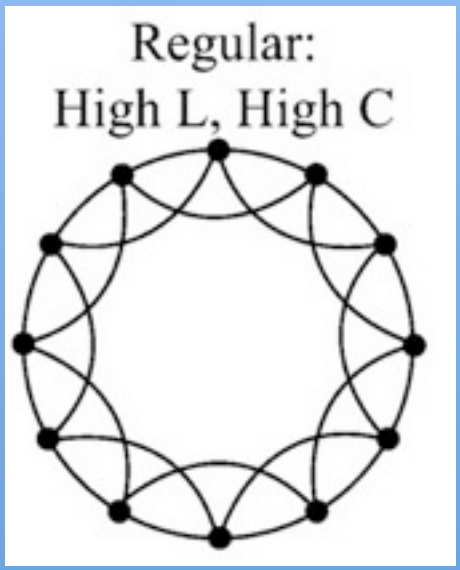
L_{actual} = Average distance (shortest path length) between pairs of nodes in the given network

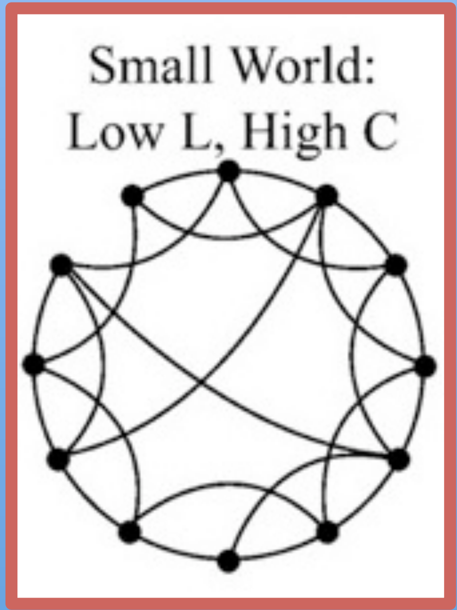
L_{random} = Average distance (shortest path length) between pairs of nodes in a randomly connected network with the same number of nodes and links.

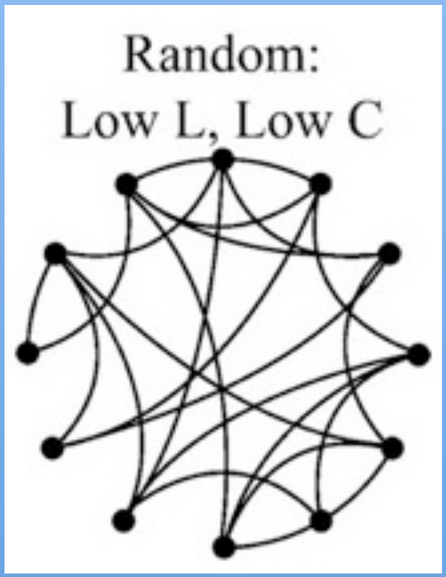
C_{actual} = Clustering coefficient of given network

C_{random} = Clustering coefficient of random network

Network Structure





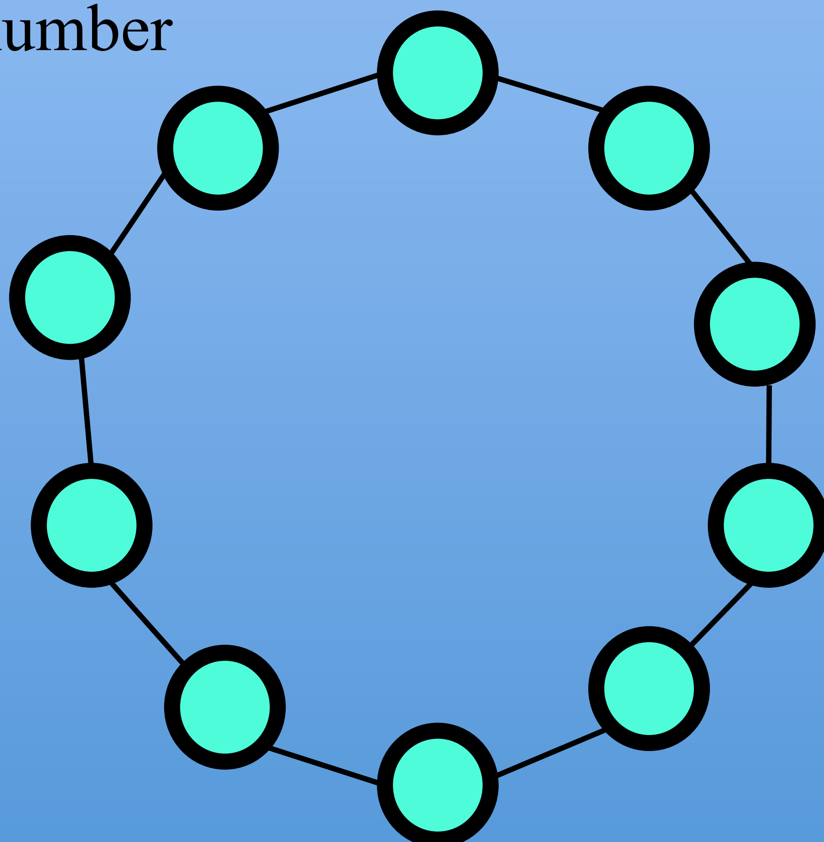


Small-World Network: Has low average distance between nodes, but high clustering

How are small-world networks formed?

Watts and Strogatz's model:

- Start with a *regular* network with N nodes and m neighbors per node. Let M be the number of links.



For this example,

$N = 10$

$m = 2$

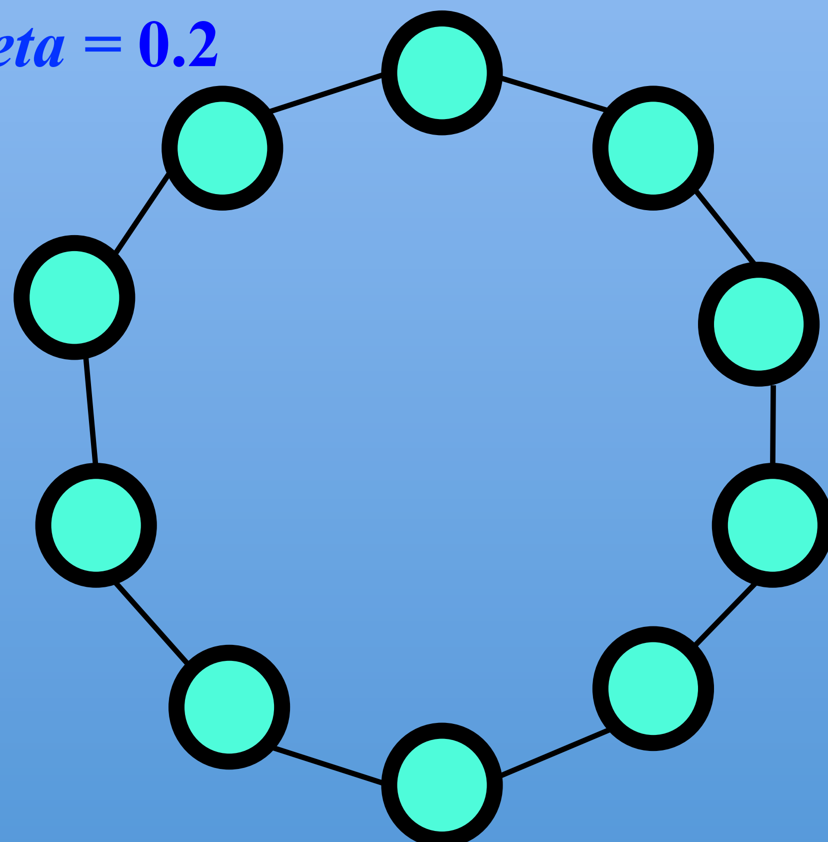
$M = 10$

How are small-world networks formed?

Watts and Strogatz's model:

- Define *beta*, the probability of *rewiring*.

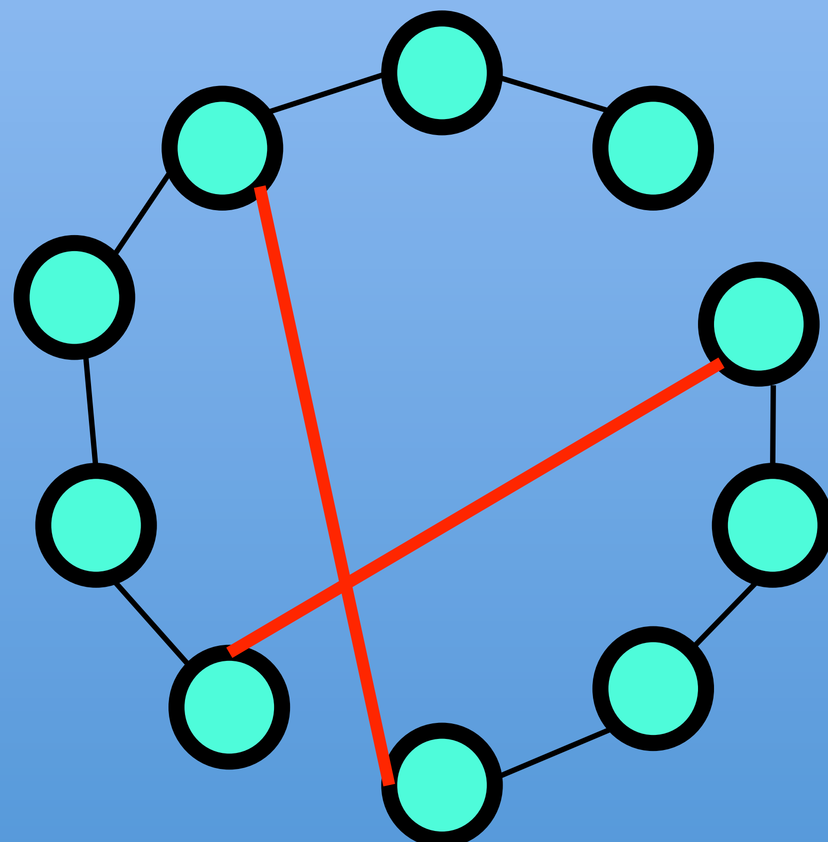
For this example, let *beta* = 0.2



How are small-world networks formed?

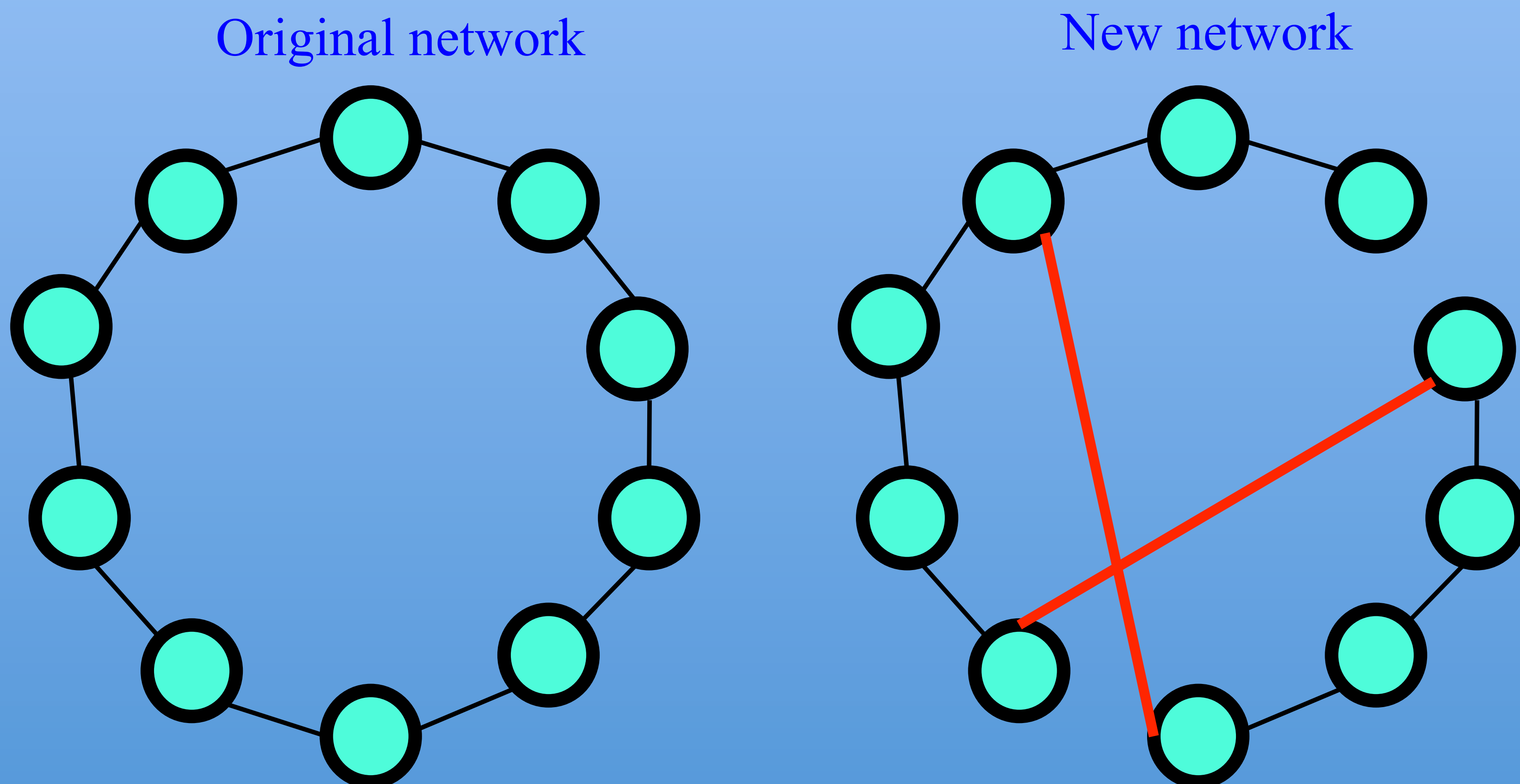
Watts and Strogatz's model:

- Rewire one end of the chosen links to another randomly chosen node.

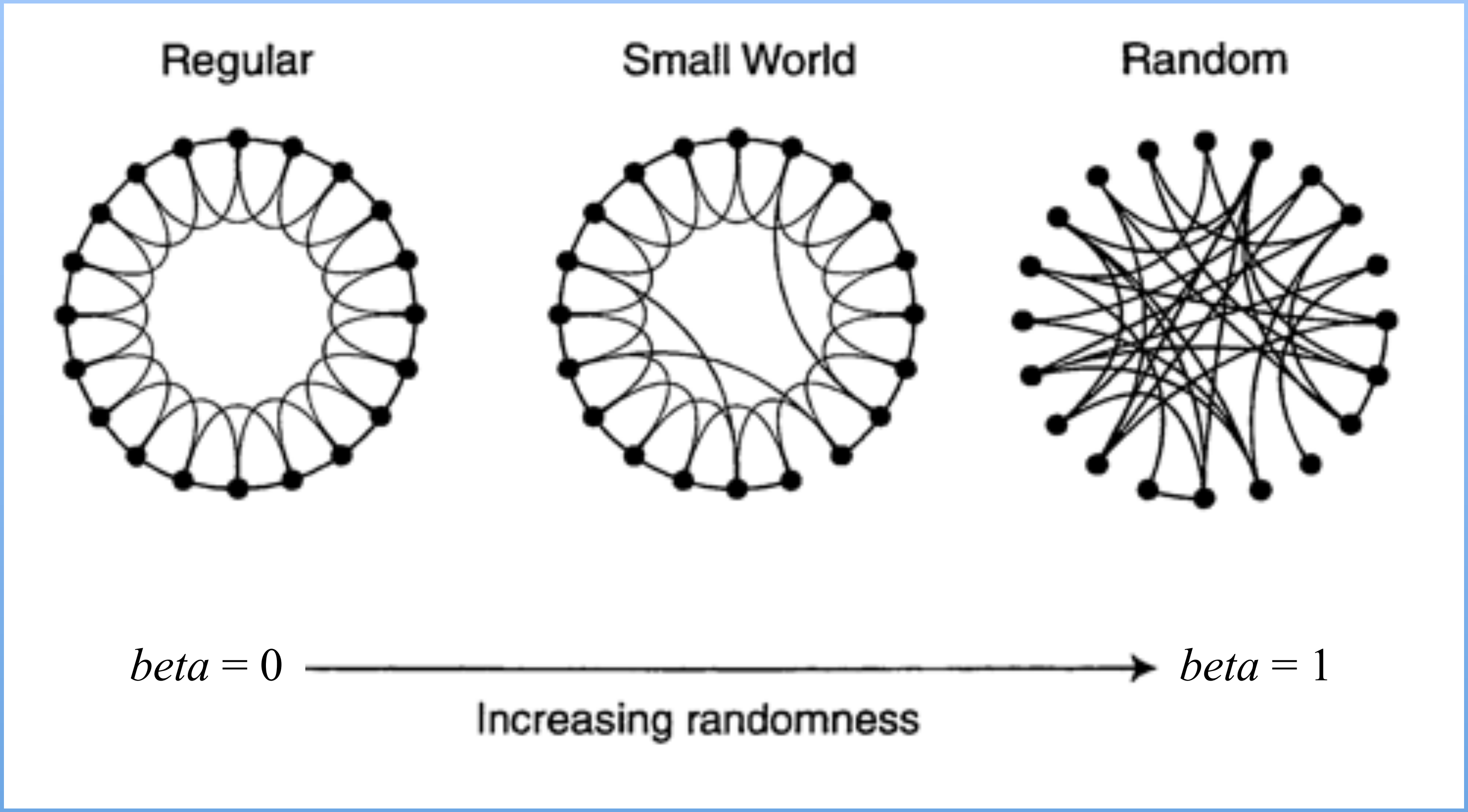


How are small-world networks formed?

Watts and Strogatz's model:



Small-World Network Summary

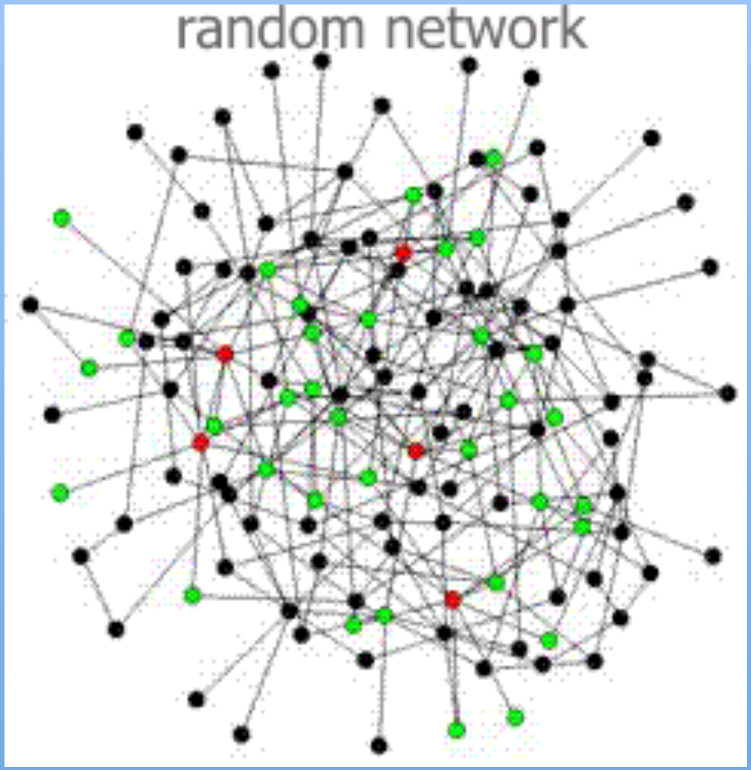


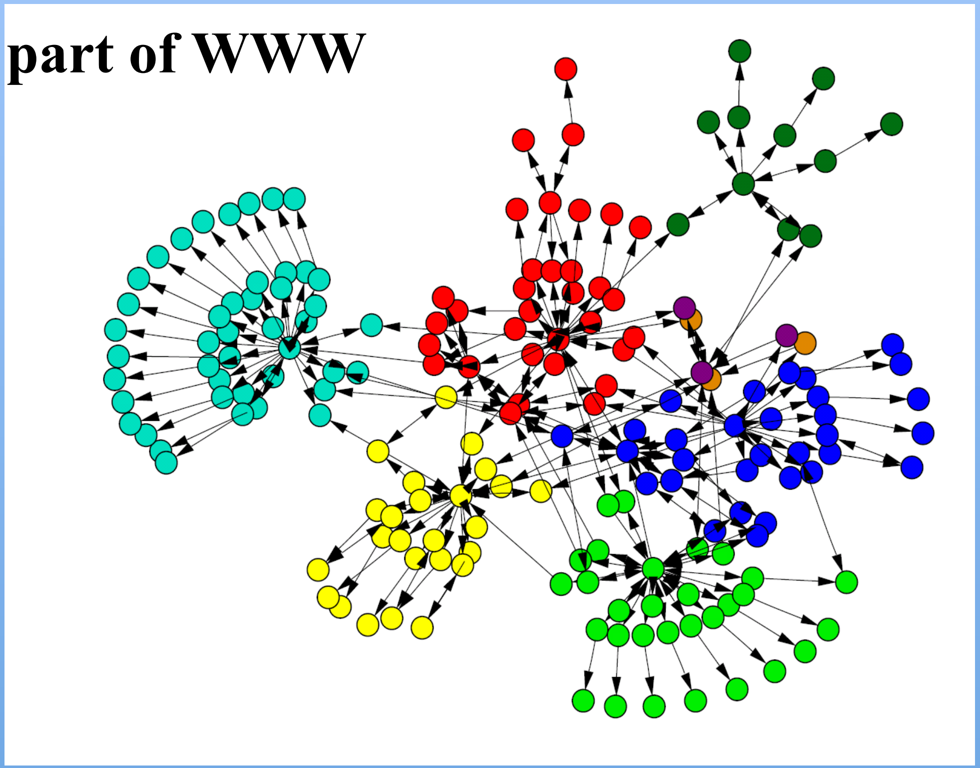
Regular network
- high average distance
- high clustering

Small-world network
- small average distance
- high clustering

Random network
- small average distance
- low clustering

Scale-Free and Long-Tailed Network Structure



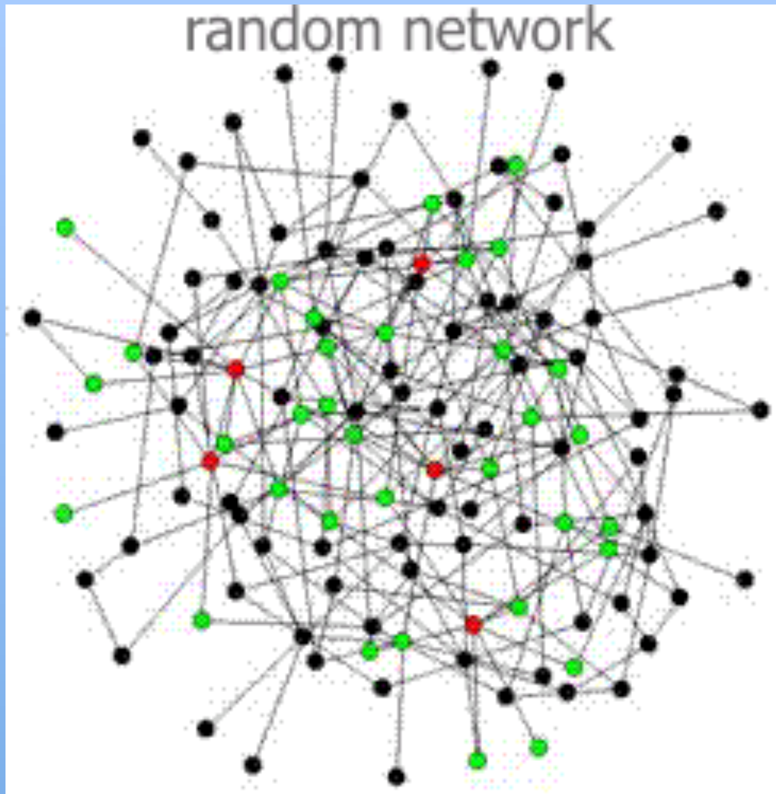


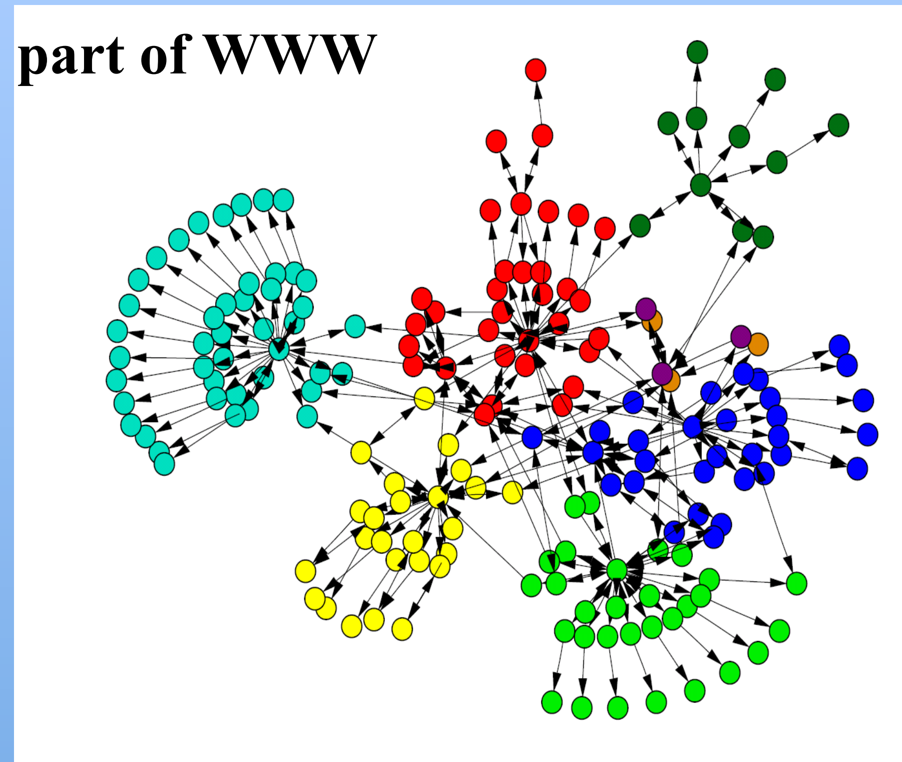
Typical structure of a randomly connected network

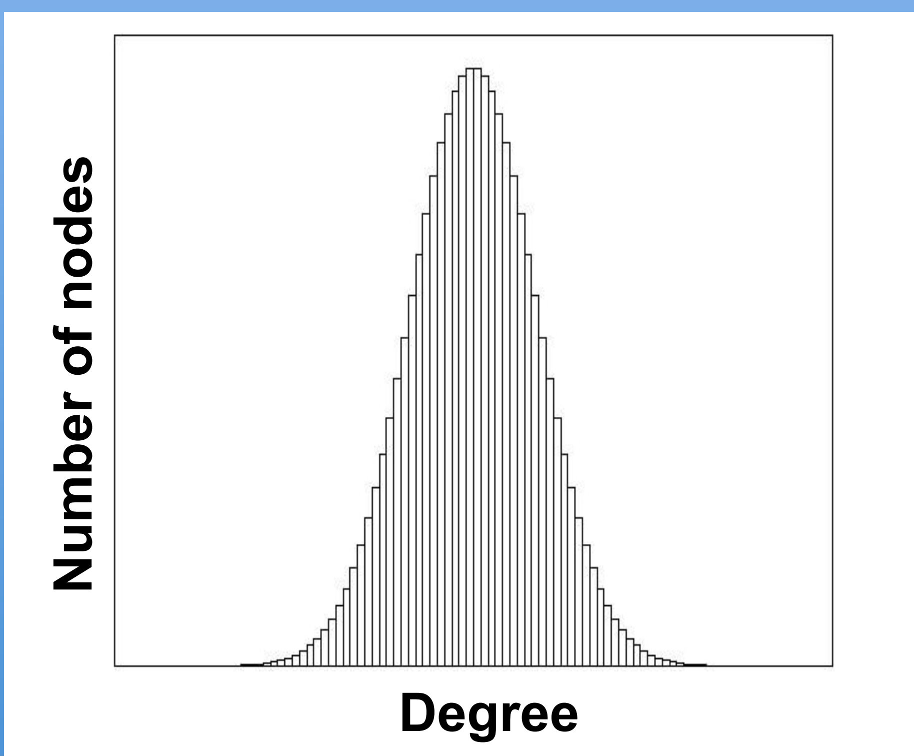
Typical structure of World Wide Web (nodes = web pages, links = links between pages)

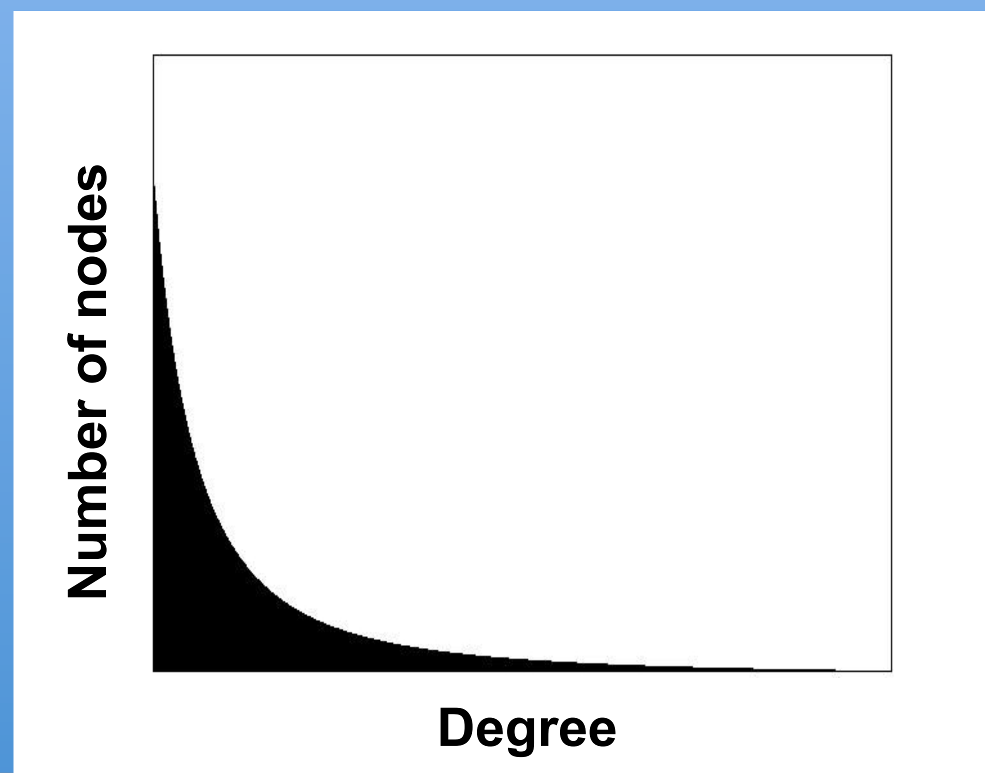
http://www.dichotomistic.com/images/random%20network.gif

M. E. J. Newman and M. Girvan, Finding and evaluating community structure in networks", *Physical Review E* **69** (2003): 026113

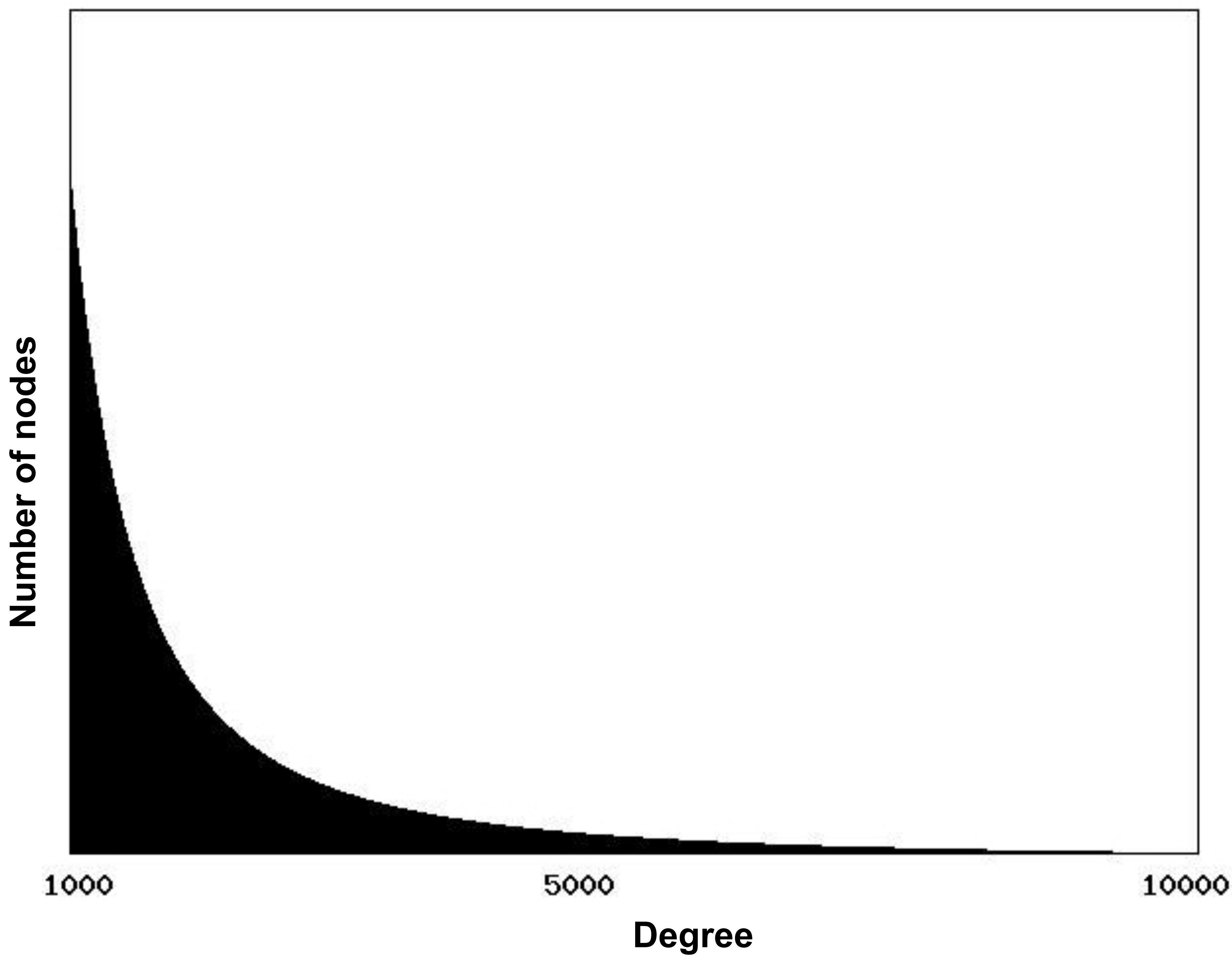

random network


part of WWW

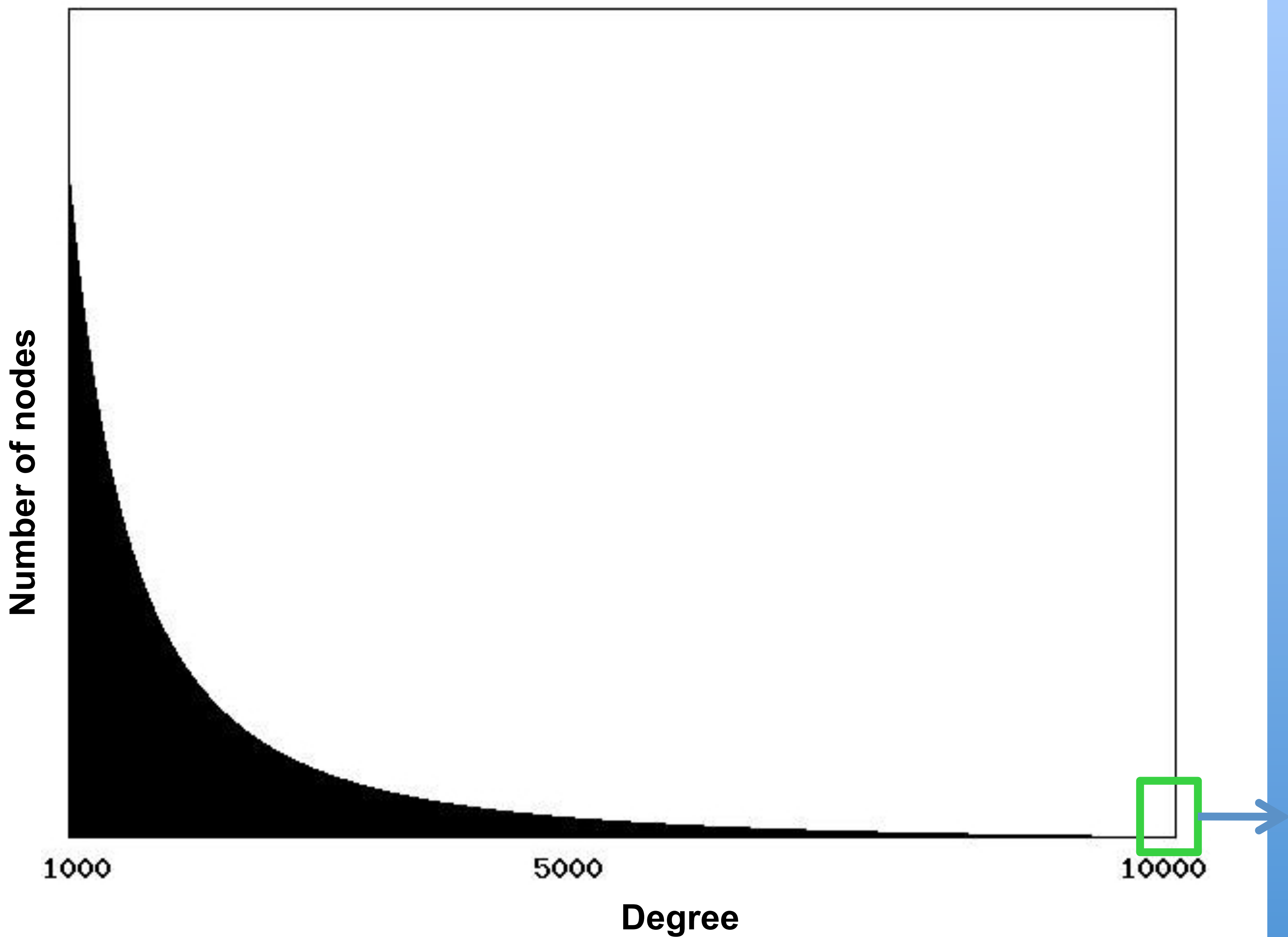

Number of nodes
Degree


Number of nodes
Degree

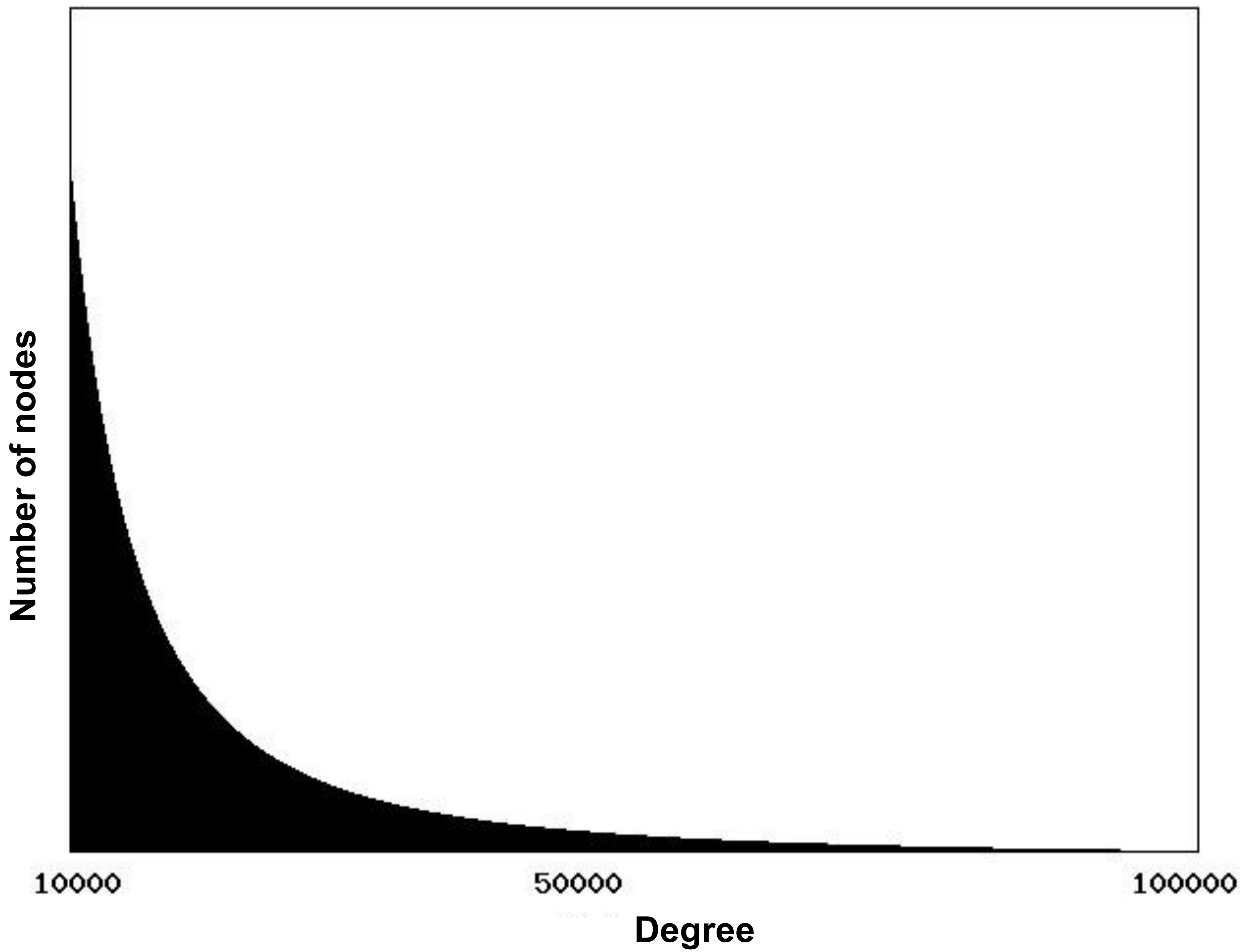
The Web's Approximate Degree Distribution



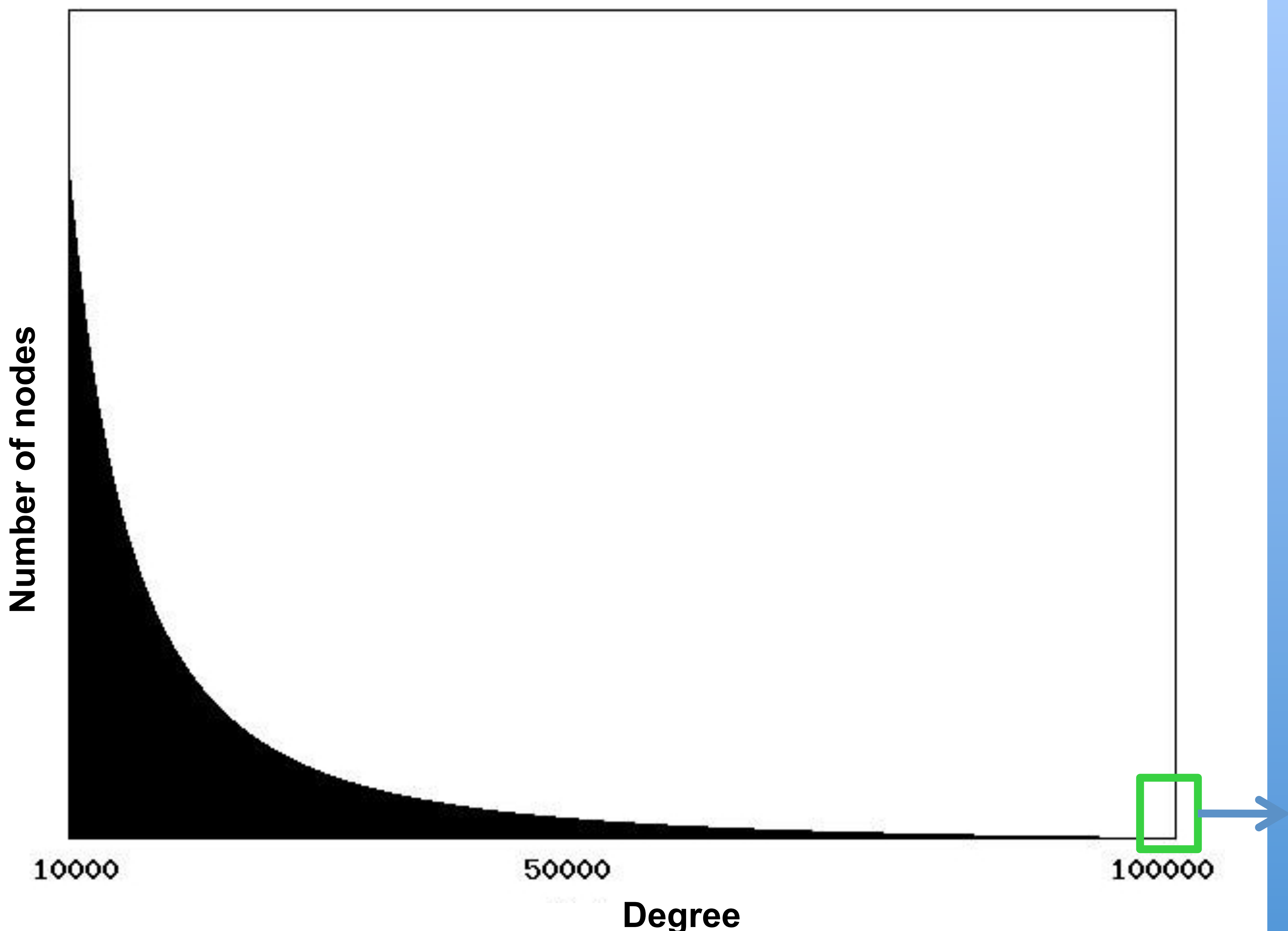
The Web's Approximate Degree Distribution



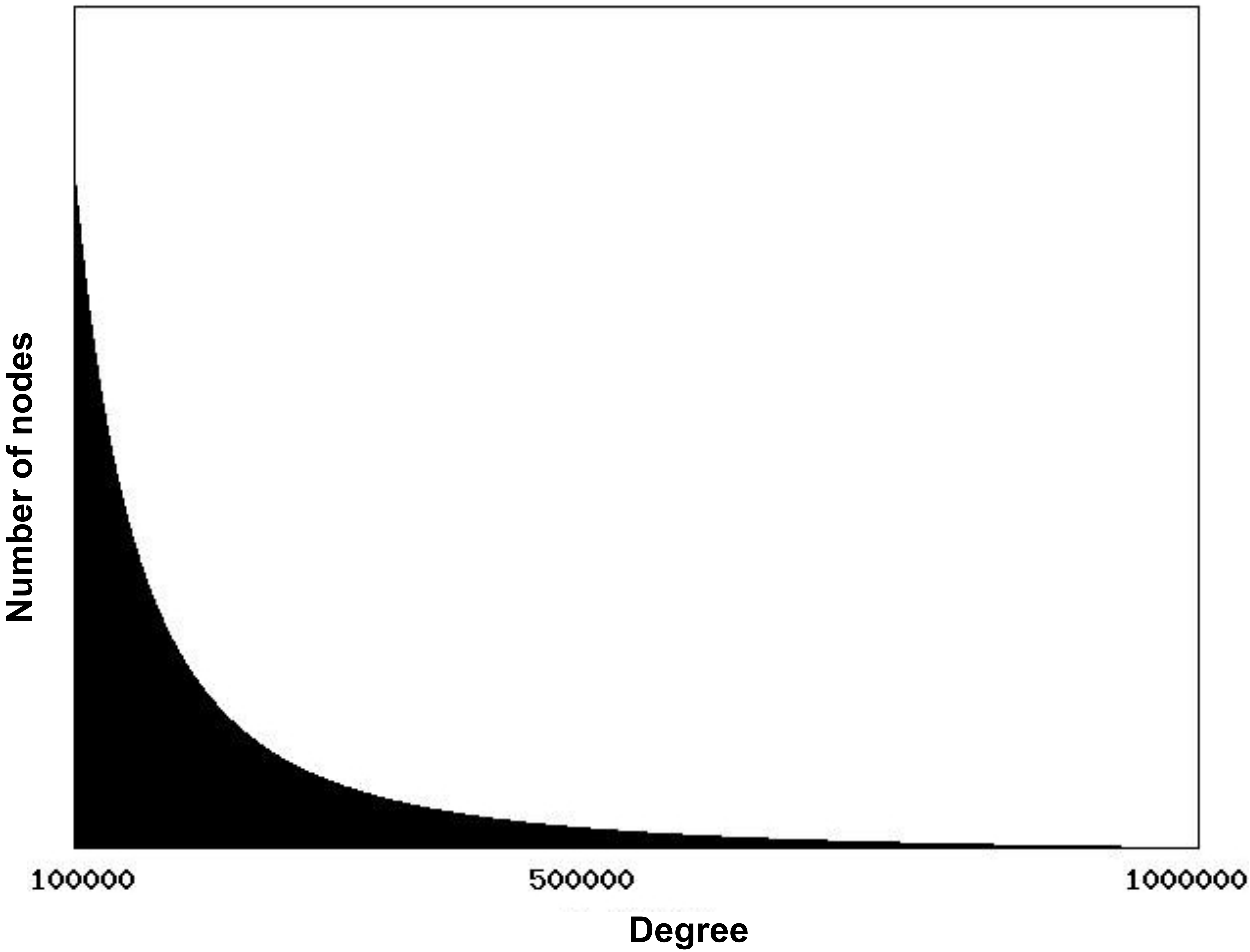
The Web's Approximate Degree Distribution



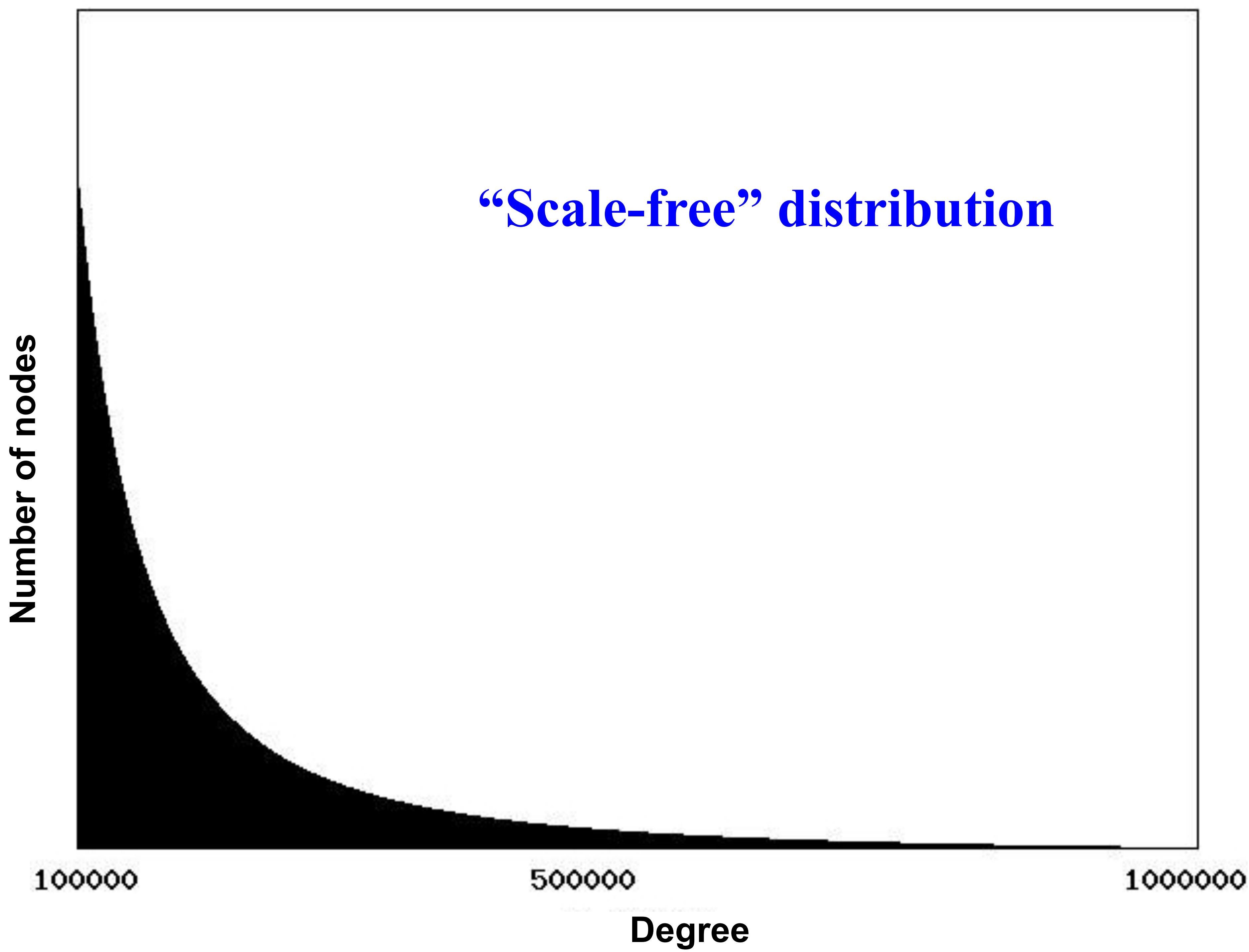
The Web’s Approximate Degree Distribution



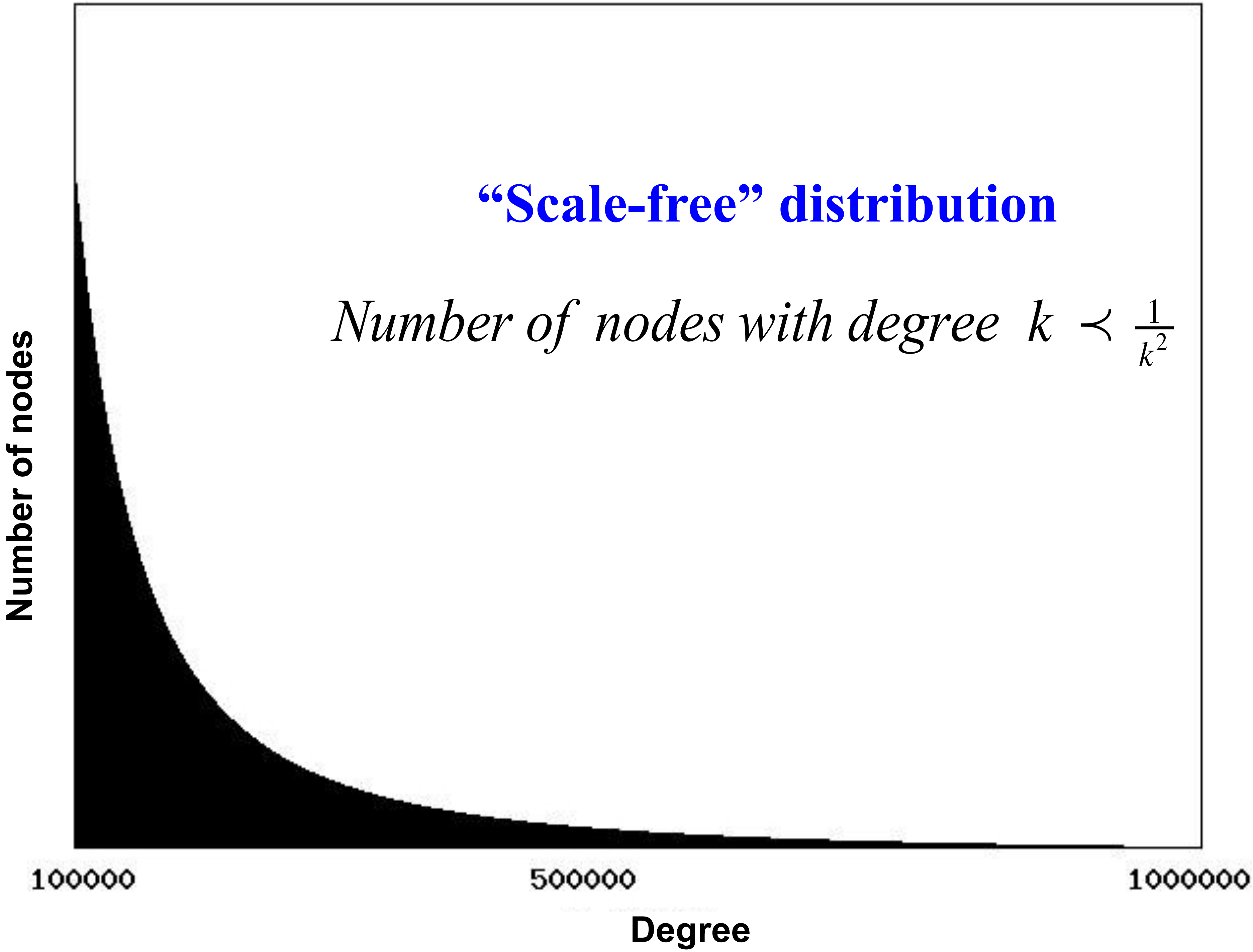
The Web's Approximate Degree Distribution



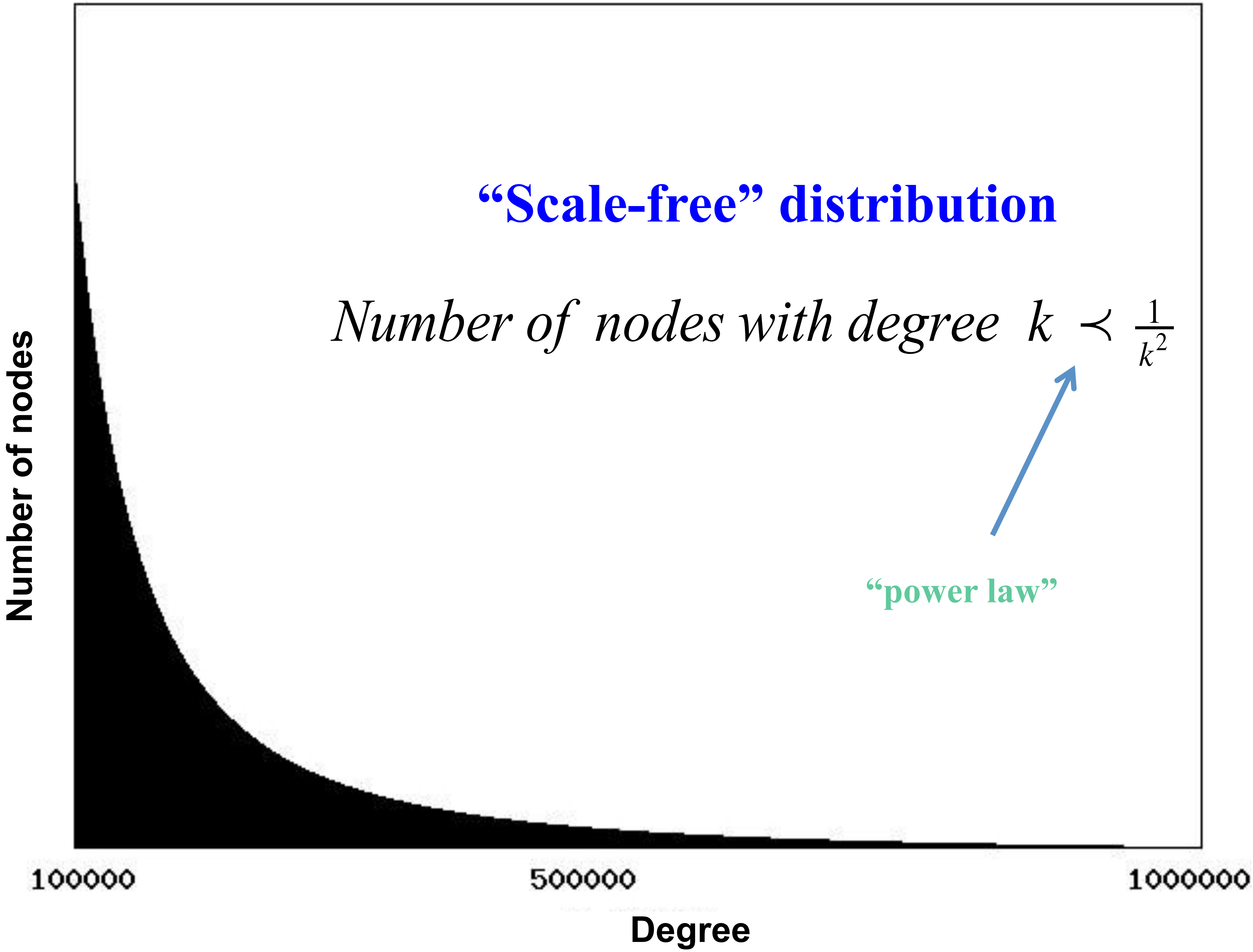
The Web's Approximate Degree Distribution



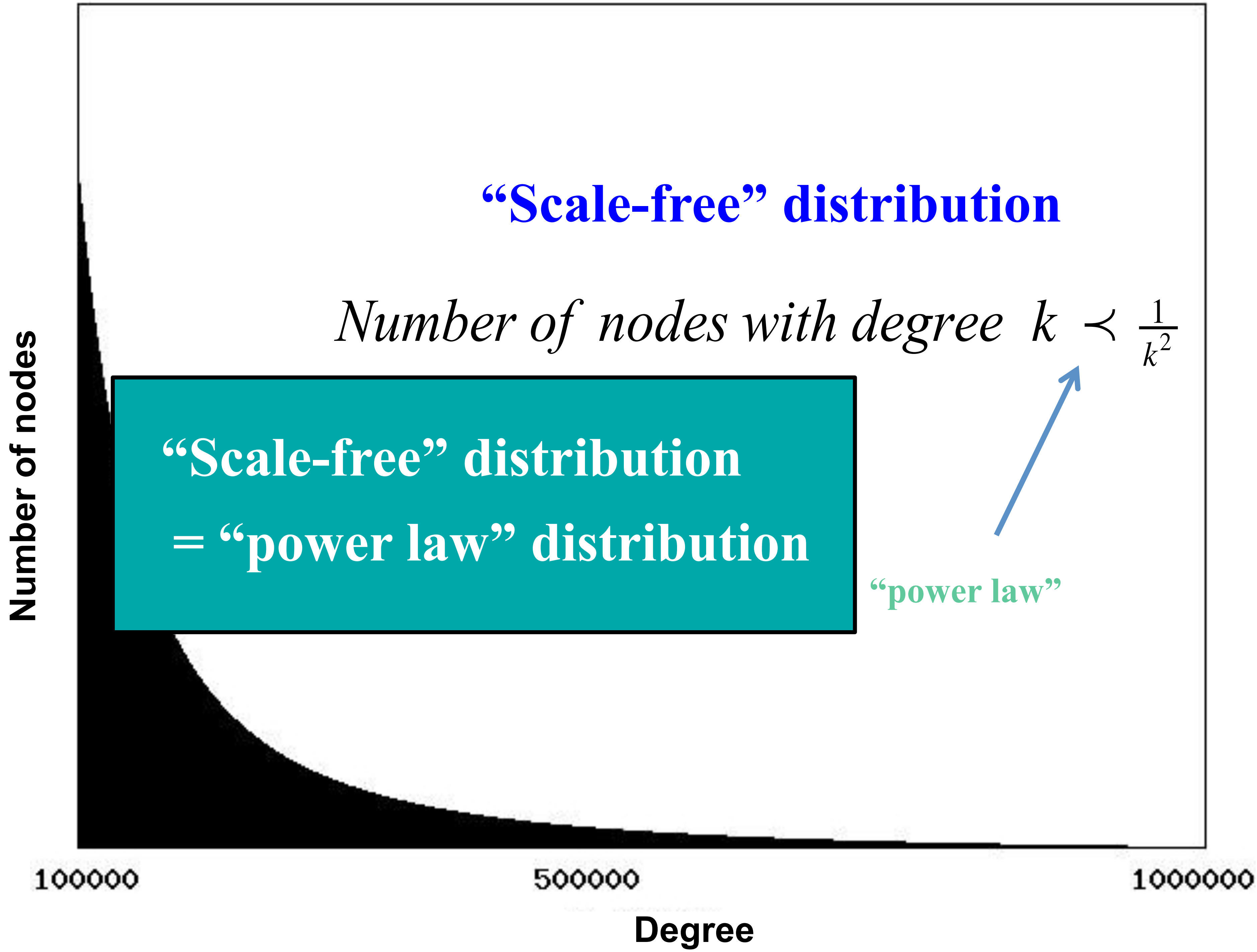
The Web's Approximate Degree Distribution



The Web's Approximate Degree Distribution



The Web's Approximate Degree Distribution



"Scale-free" distribution

$$\textit{Number of nodes with degree } k \prec \frac{1}{k^2}$$

"power law"

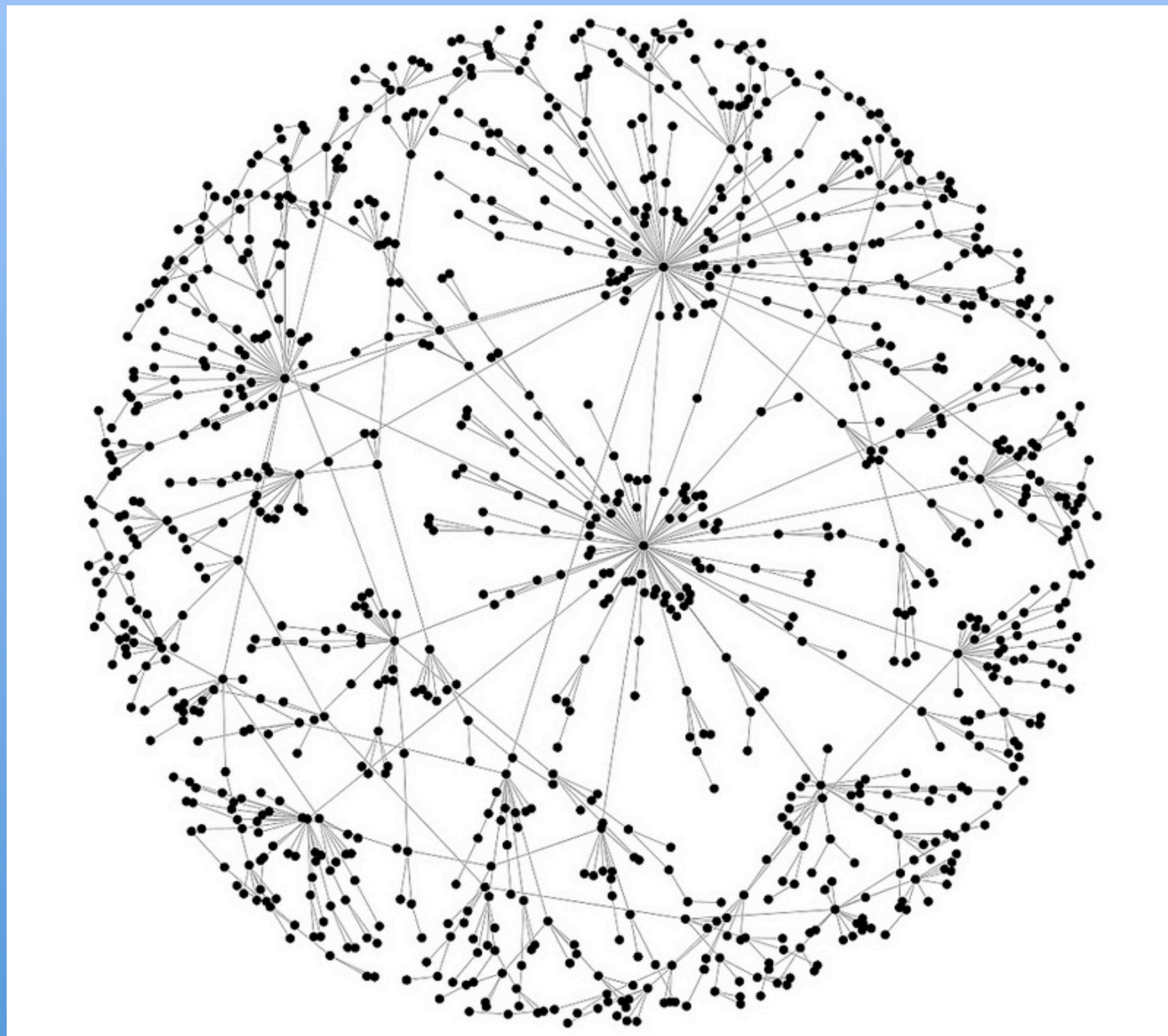
"Scale-free" distribution

= "power law" distribution

Definition of Scale-Free Network:

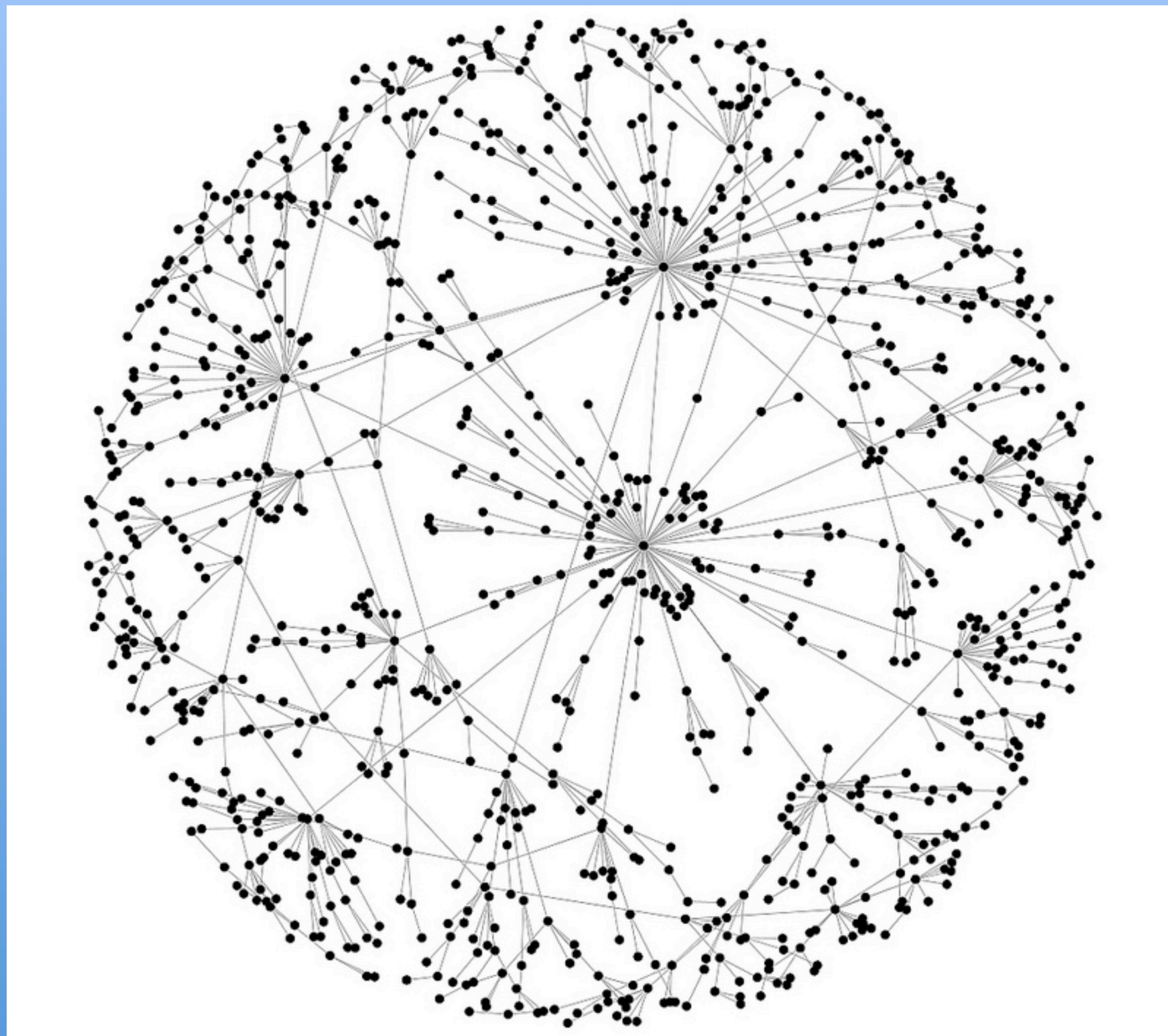
Network with scale-free (i.e., power-law) degree distribution

Scale-Free Networks are "fractal-like"



http://www.flickr.com/photos/sjcockell/8425835703/in/set-72157622439955072/lightbox/

Scale-Free Networks have the small-world property: low average distances between nodes and high clustering



Albert-Lázló Barabási

Réka Albert

SCIENCE VOL 286 15 OCTOBER 1999

Emergence of Scaling in Random Networks

Albert-László Barabási* and Réka Albert

Systems as diverse as genetic networks or the World Wide Web are best described as networks with complex topology. A common property of many large networks is that the vertex connectivities follow a scale-free power-law distribution. This feature was found to be a consequence of two generic mechanisms: (i) networks expand continuously by the addition of new vertices, and (ii) new vertices attach preferentially to sites that are already well connected.

Example of a more skeptical paper on power-law distributions in networks and other real-world data.

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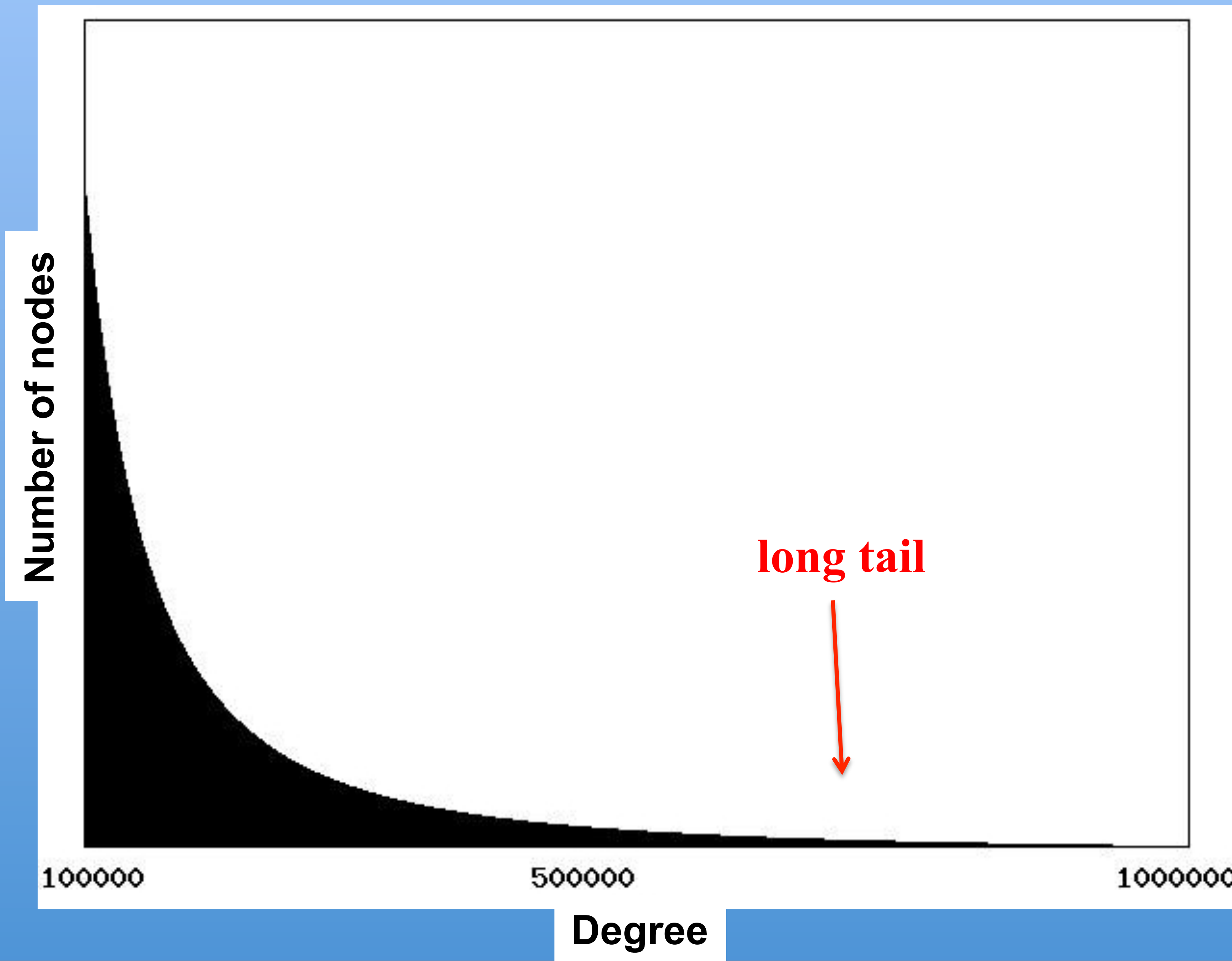
Power-Law Distributions in Empirical Data*

Aaron Clauset[†]
Cosma Rohilla Shalizi[‡]
M. E. J. Newman[§]

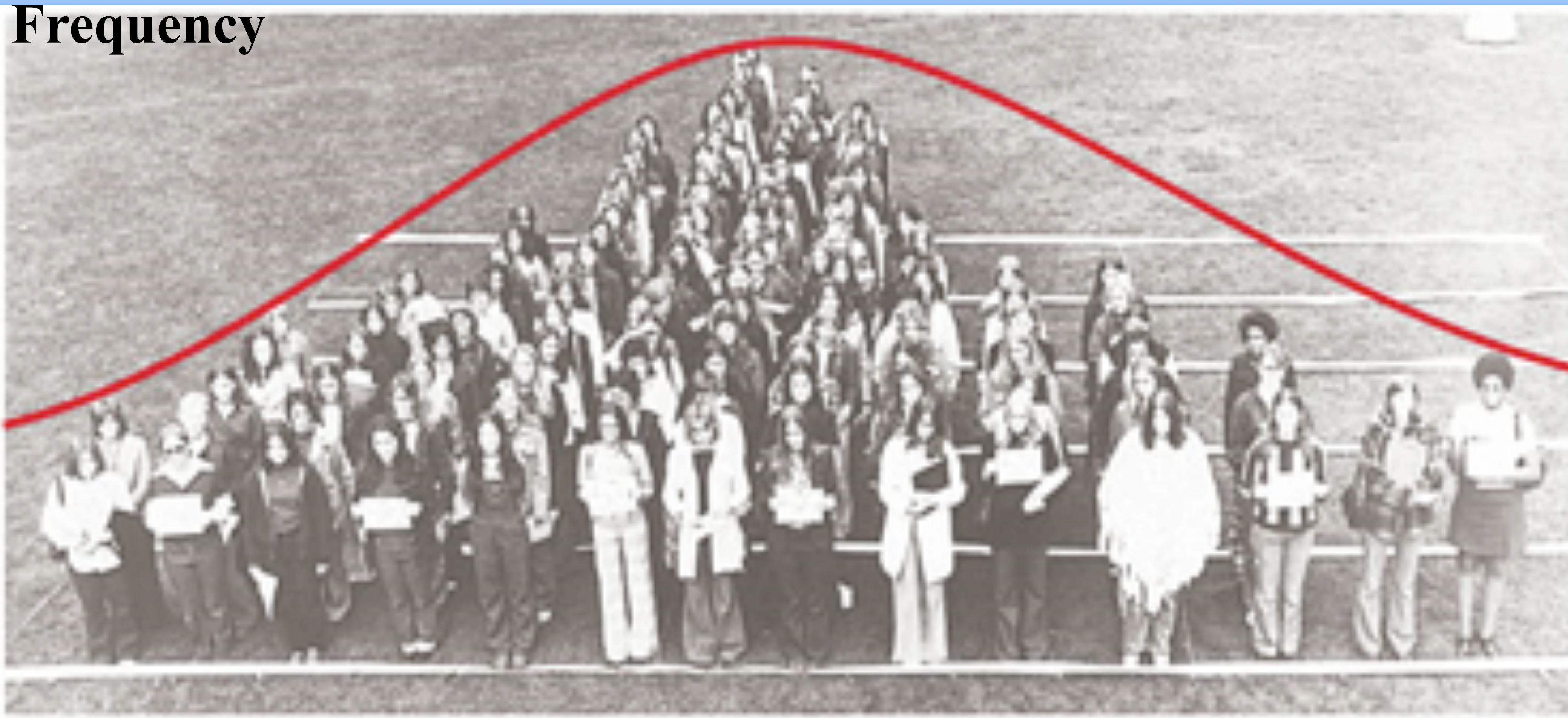
Evelyn Fox Keller in 2005:

“Current assessments of the commonality of power laws are probably overestimates.”

More generally accepted statement: Many real-world networks have *long-tailed degree distributions*

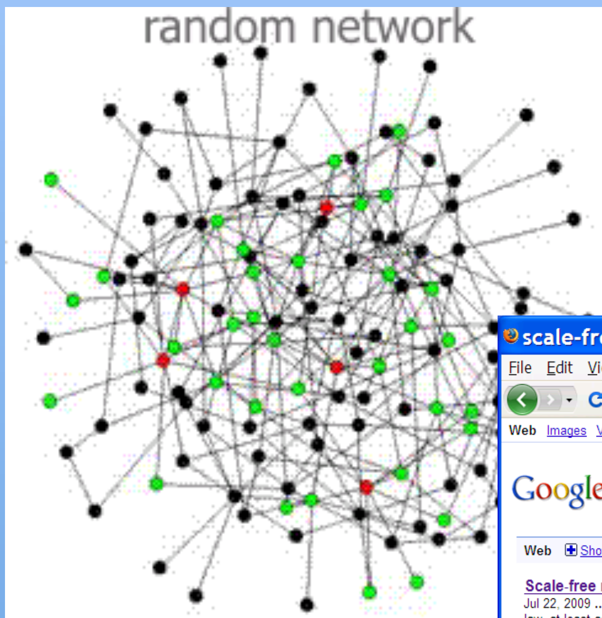


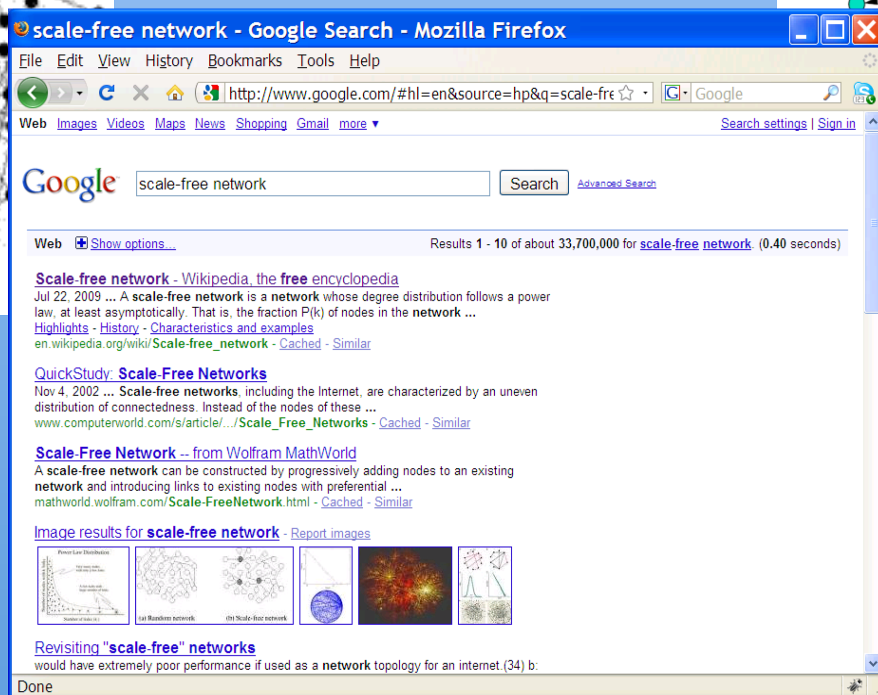
Contrast long-tailed distributions with “Normal” (bell-curve) distributions.

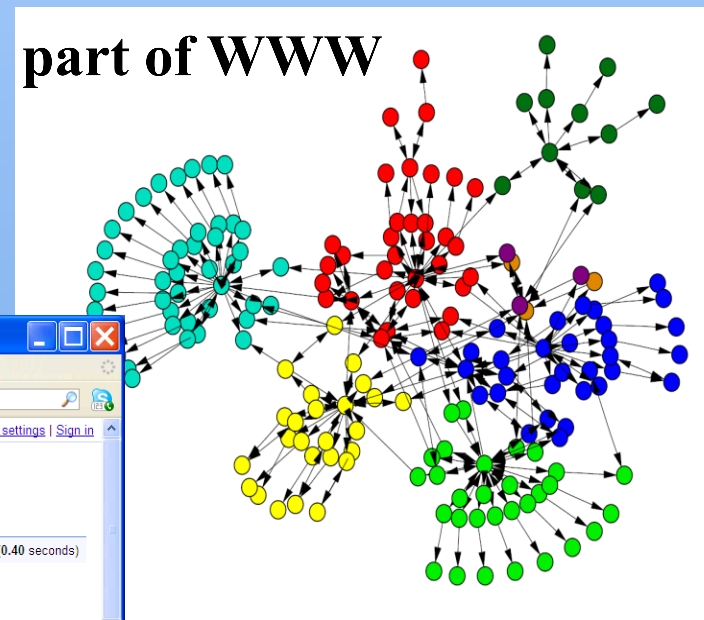


http://scienceblogs.com/builtonfacts/2009/02/the_central_limit_theorem_made.php

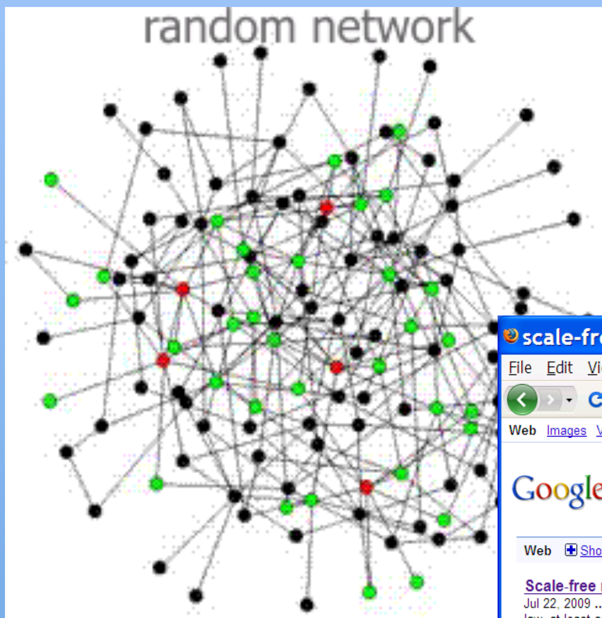
The long-tailed degree-distribution of the Web helps to explain why Google works so well

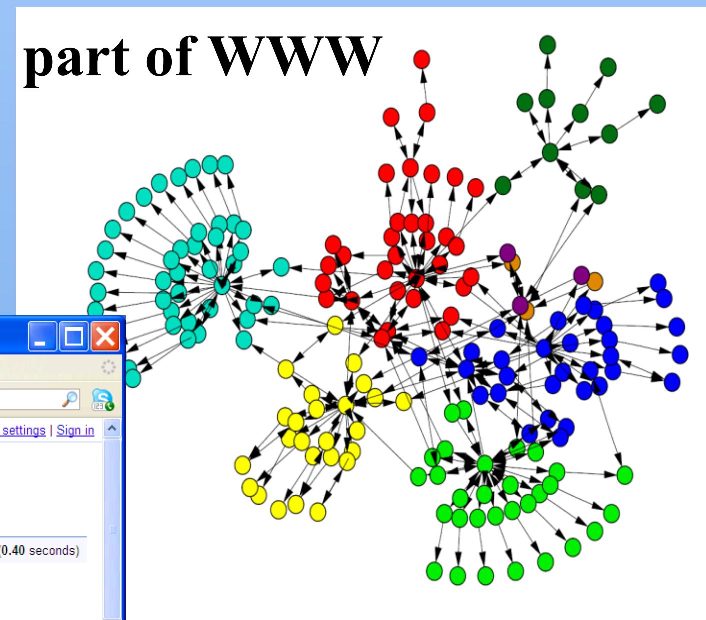


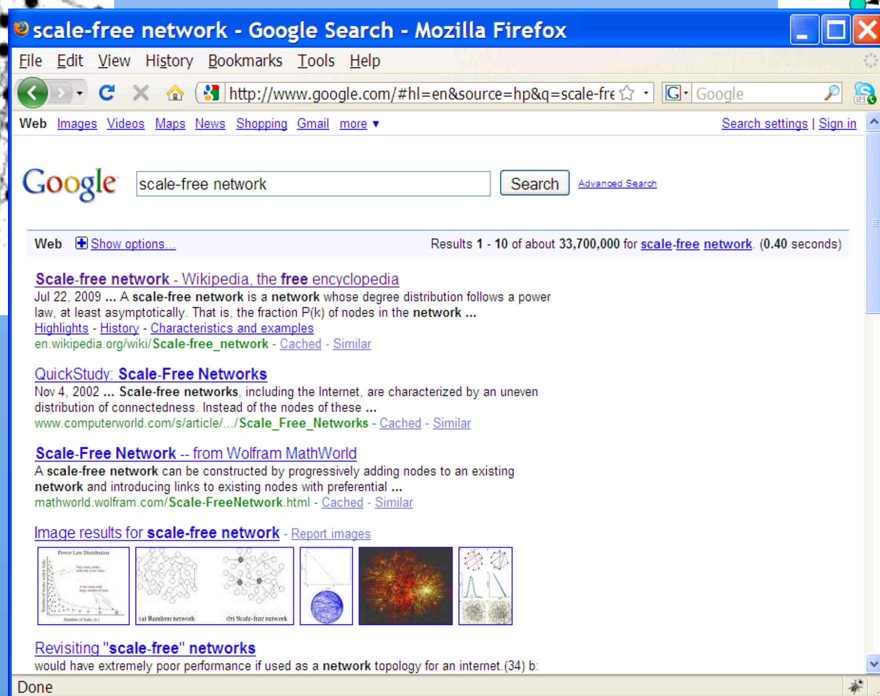




The long-tailed degree-distribution of the Web helps to explain why Google works so well







It also explains some of the success of other networks in nature!

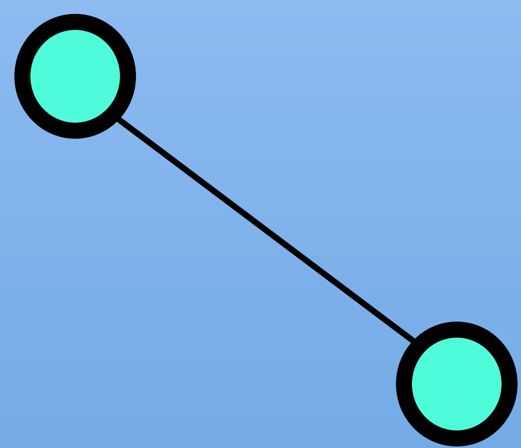
How does the structure of scale-free networks arise?

- Multiple hypotheses
- One prominent hypothesis: preferential attachment

("The rich get richer.")

Preferential Attachment Model (Netlogo)

Start with two linked nodes.



At each time step, create a new node.

Preferential Attachment Model (Netlogo)

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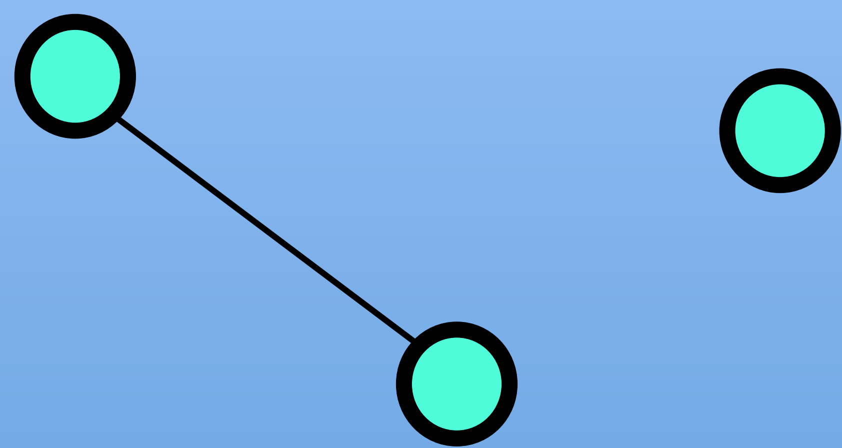

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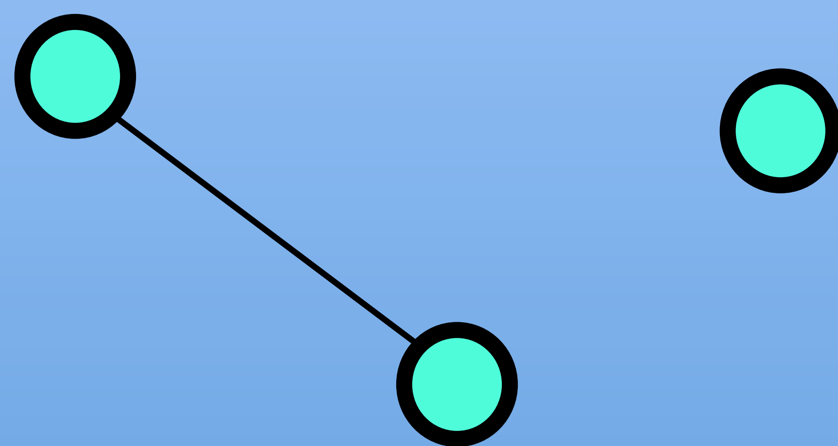

At each time step, create a new node.

This new node randomly chooses an existing node to link to, **but with a bias**: the more links an existing node has, the more likely it is to be chosen.

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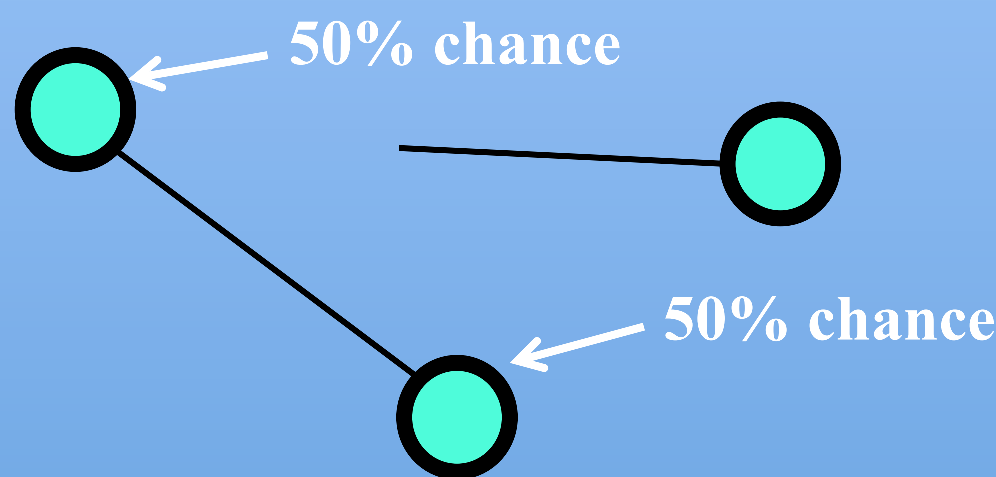

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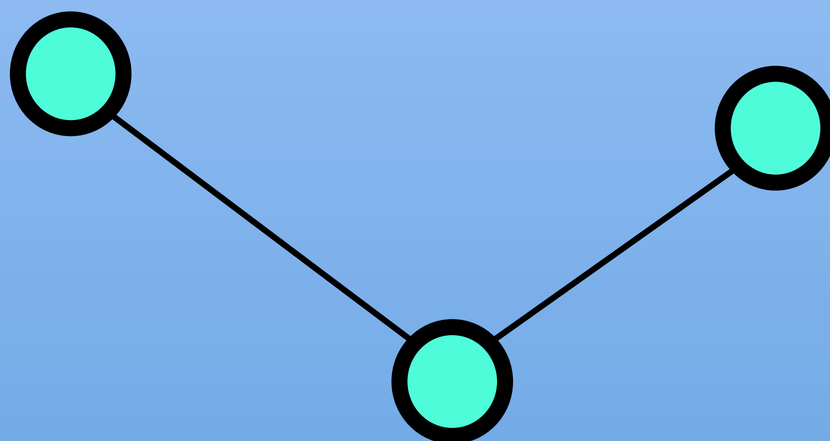

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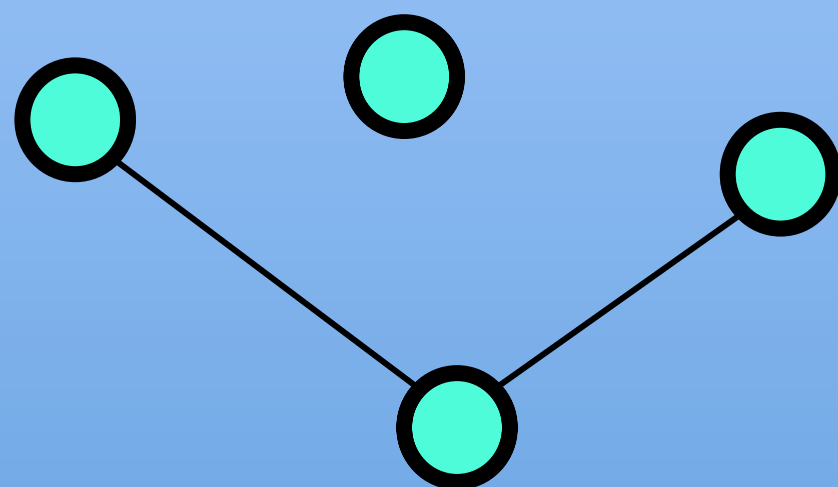

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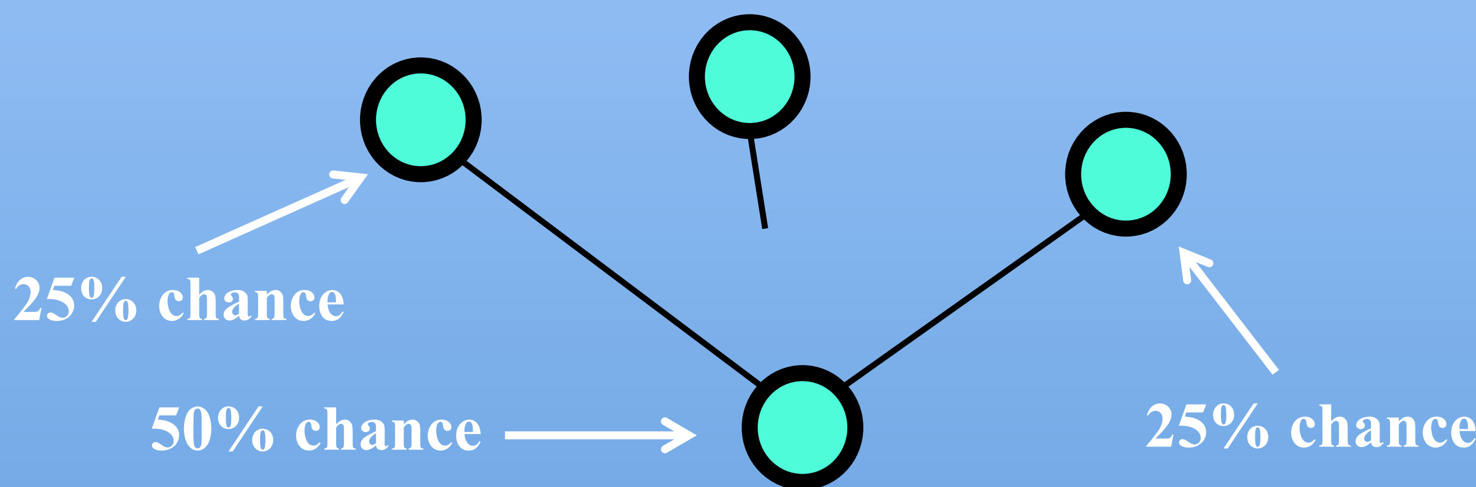

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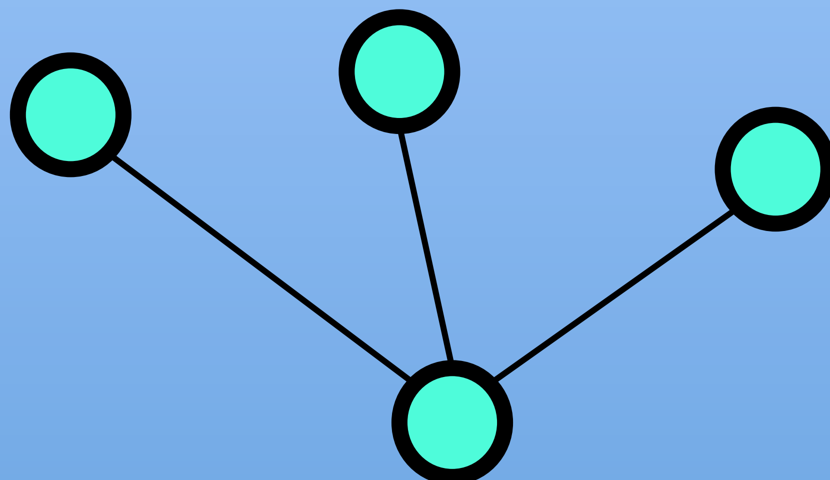

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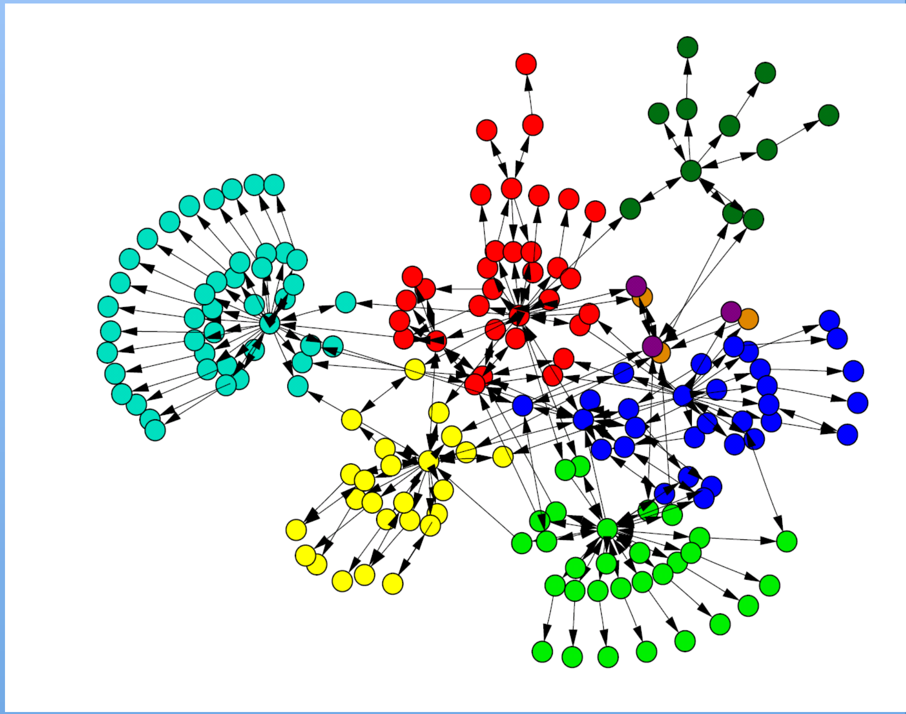
Robustness of Scale-Free (or Long-Tailed) Networks



- **Vulnerable** to targeted “hub” failure
- **Robust** to random node failure

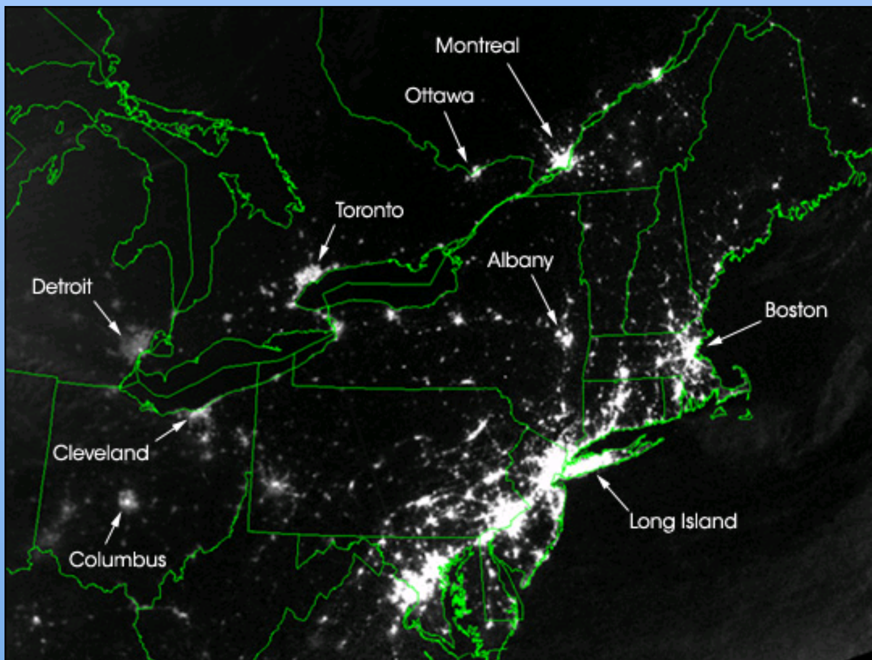
unless....

nodes can **cause** other nodes to fail

Can result in ***cascading failure***

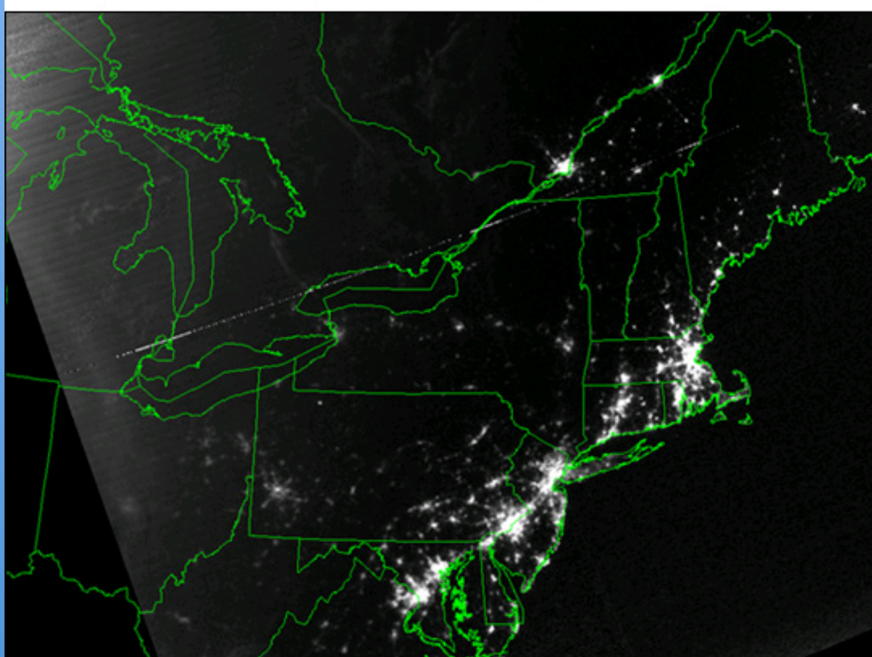
August, 2003 electrical blackout in northeast US and Canada

http://earthobservatory.nasa.gov/images/imagerecords/3000/3719/NE_US_OLS2003227.jpg



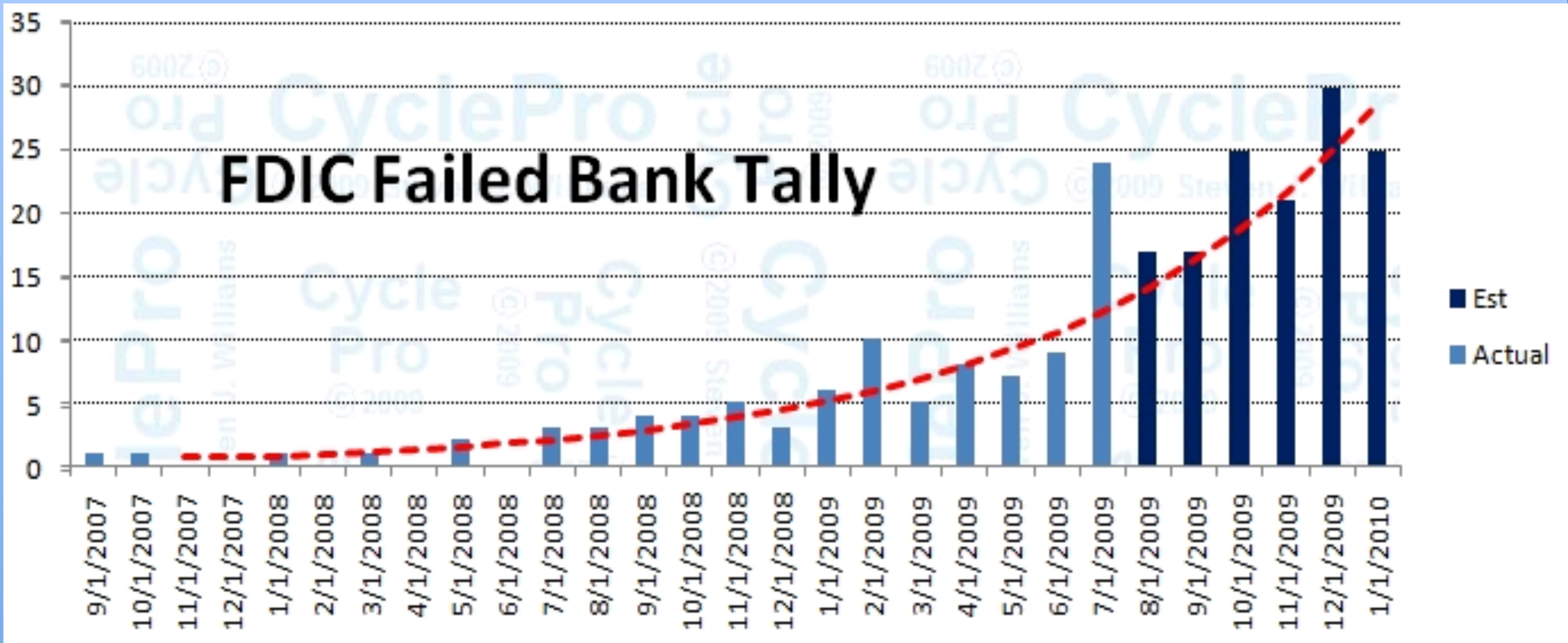
August 14, 2003 • 9:29 p.m. EDT • About 20 hours before blackout

9:29pm
1 day before



August 15, 2003 • 9:14 p.m. EDT • About 7 hours after blackout

9:14pm
Day of blackout

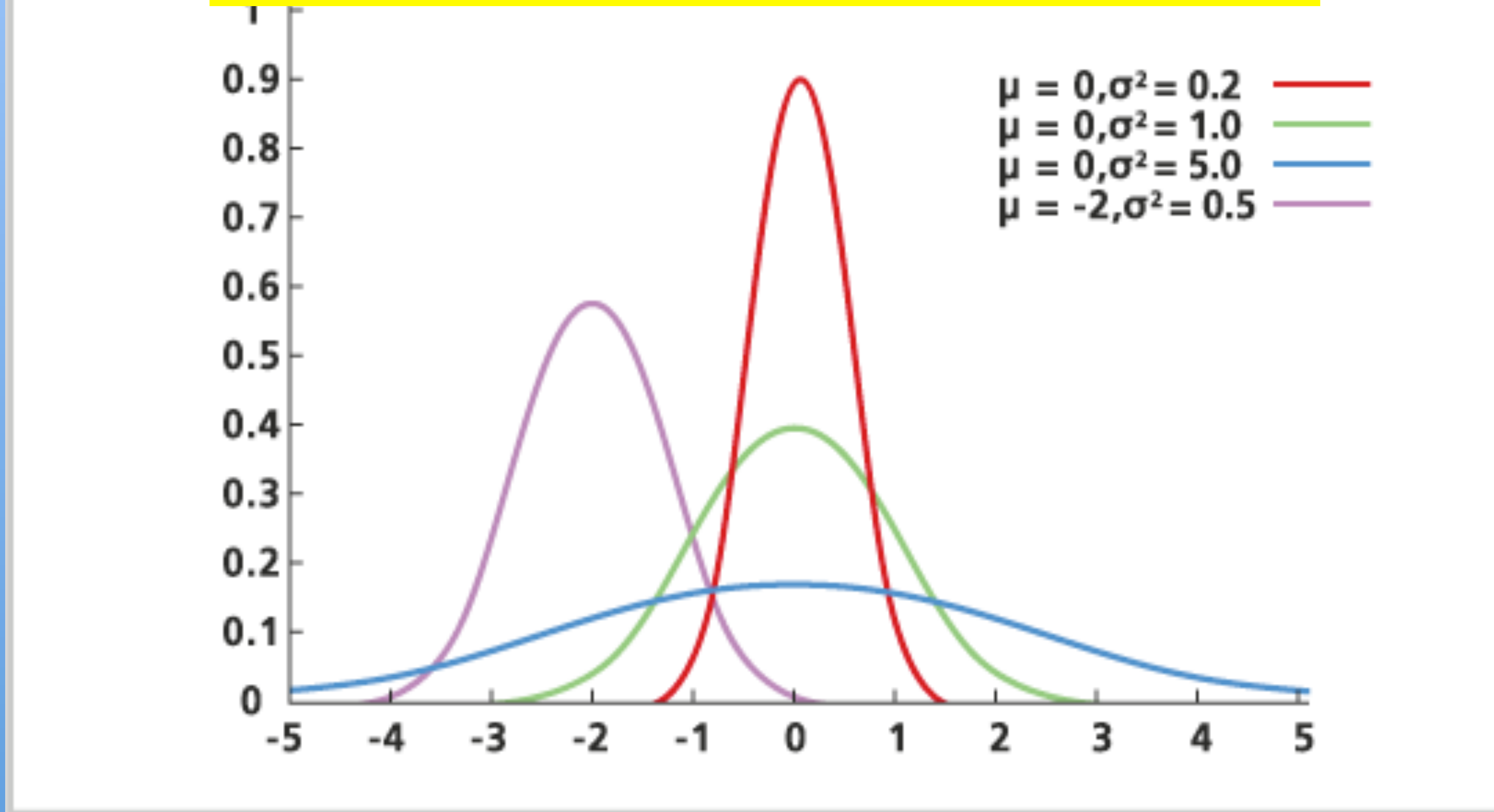


http://www.geocities.com/WallStreet/Exchange/9807/Charts/SP500/fdicfail_0907.jpg

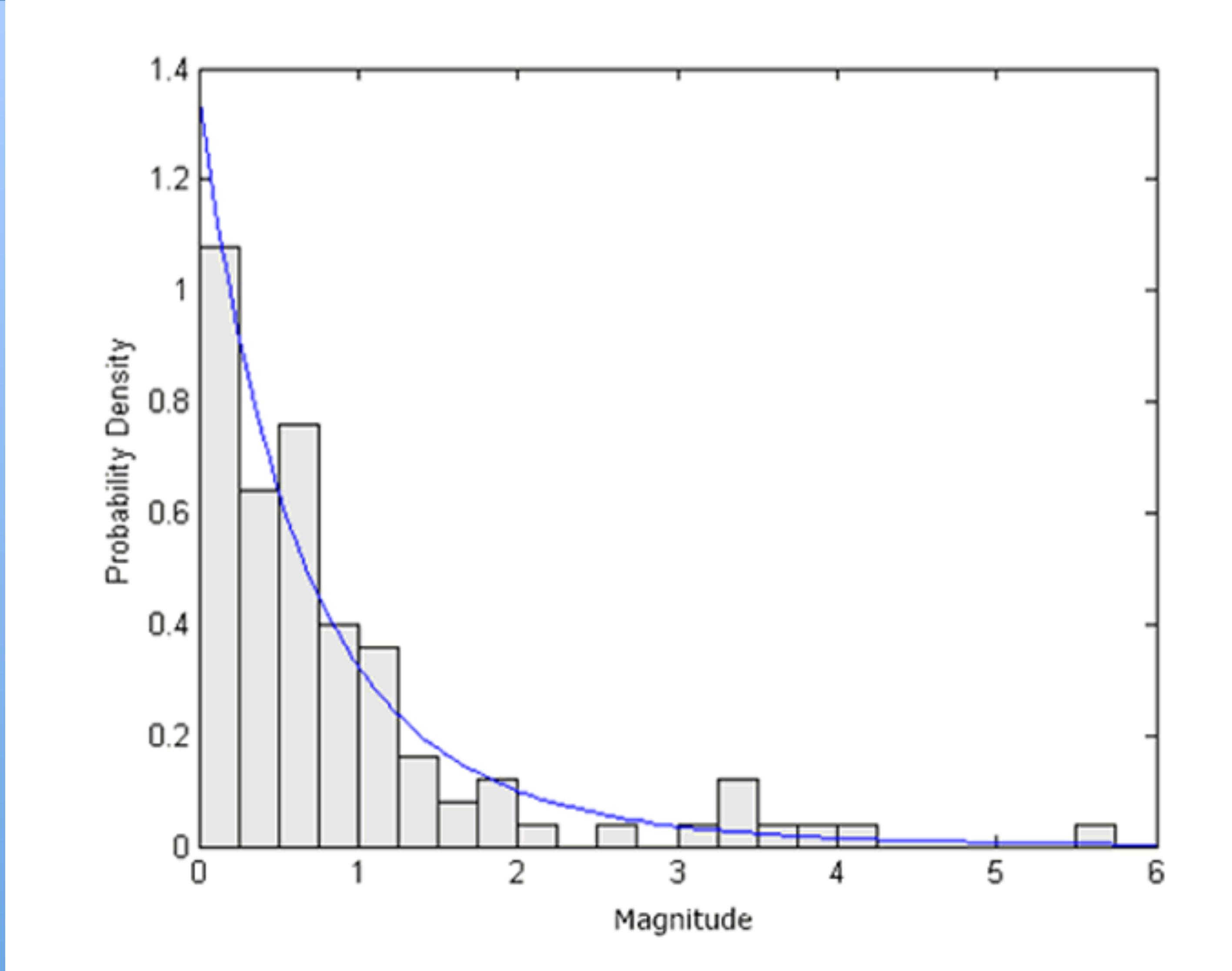
We see similar patterns of cascading failure in biological systems, ecological systems, computer and communication networks, wars, etc.

Normal (“bell-curve) distribution

“Events in ‘tail’ are highly unlikely”

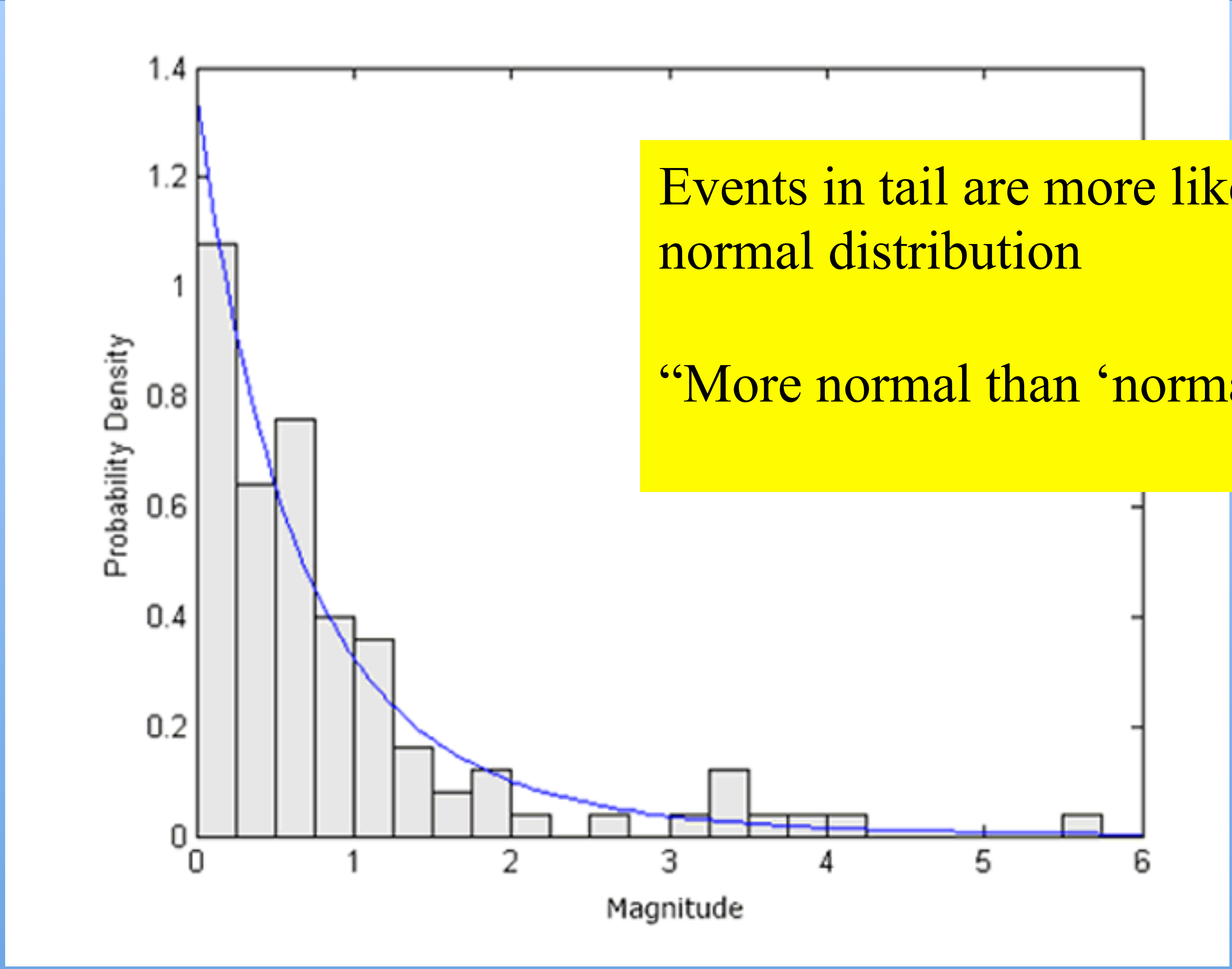


http://resources.esri.com/help/9.3/arcgisdesktop/com/gp_toolref/process_simulations_sensitivity_analysis_and_error_analysis_modeling/Random_Normal_Distribution.gif



Long-tailed distribution

http://www.marketoracle.co.uk/images/mauldin_16_10_07image003.gif

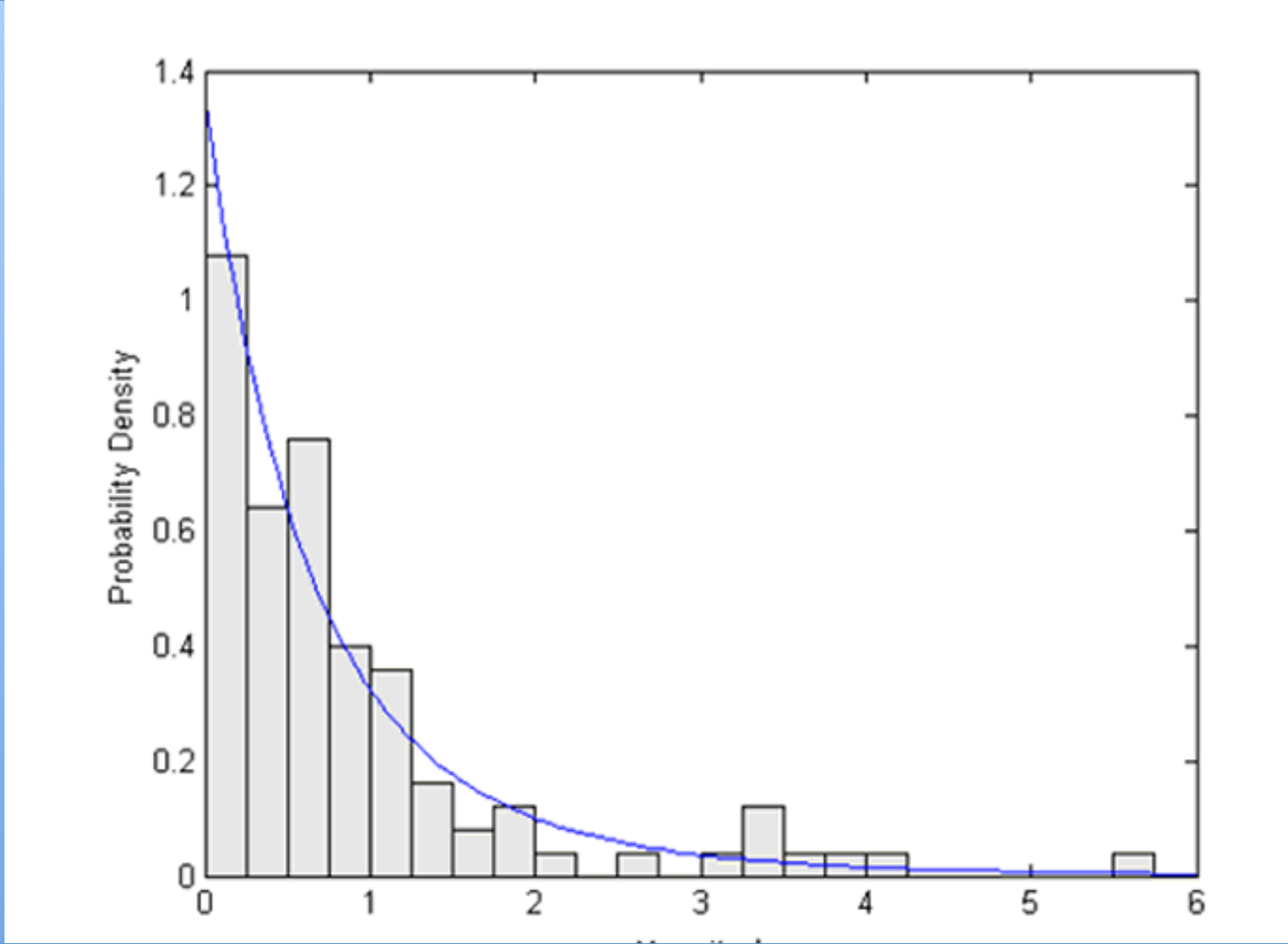


Events in tail are more likely than in normal distribution

“More normal than ‘normal’ ” ?

Long-tailed distribution

http://www.marketoracle.co.uk/images/mauldin_16_10_07image003.gif



"Few economists saw our current crisis coming, but this predictive failure was the least of the field's problems. More important was the profession's blindness to the very possibility of catastrophic failures in a market economy.

-- Paul Krugman, *New York Times*, September 6, 2009

The New York Times

July 20, 2008

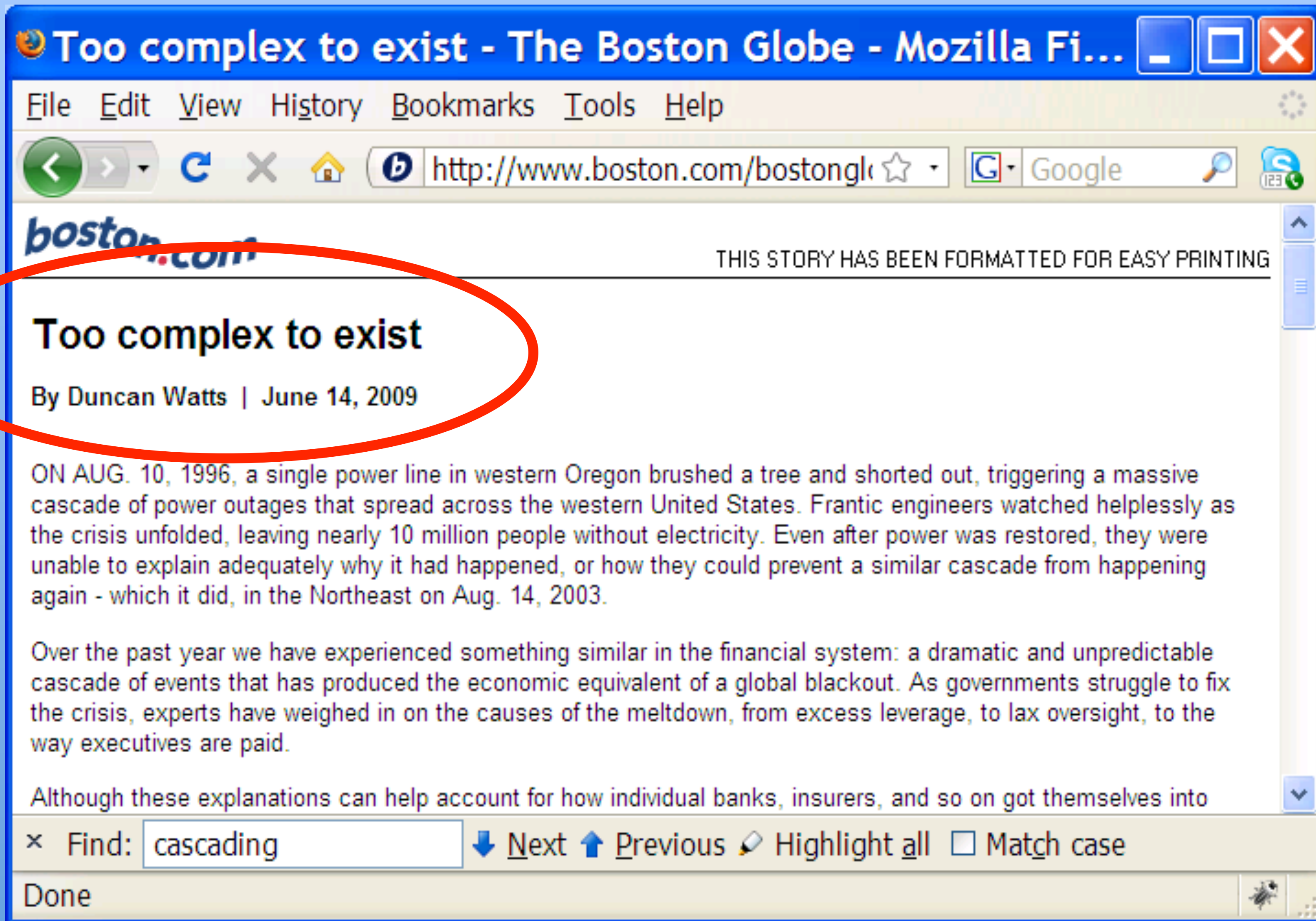
THE NATION

Too Big to Fail?

By PETER S. GOODMAN

IN the narrative that has governed American commercial life for the last quarter-century, saving companies from their own mistakes was not supposed to be part of the government's job description. Economic policy makers in the United States took swaggering pride in the cutthroat but lucrative form of capitalism that was supposedly indigenous to their frontier nation.

Through this uniquely American lens, saving businesses from collapse was the sort of



boston.com

THIS STORY HAS BEEN FORMATTED FOR EASY PRINTING

Too complex to exist

By Duncan Watts | June 14, 2009

ON AUG. 10, 1996, a single power line in western Oregon brushed a tree and shorted out, triggering a massive cascade of power outages that spread across the western United States. Frantic engineers watched helplessly as the crisis unfolded, leaving nearly 10 million people without electricity. Even after power was restored, they were unable to explain adequately why it had happened, or how they could prevent a similar cascade from happening again - which it did, in the Northeast on Aug. 14, 2003.

Over the past year we have experienced something similar in the financial system: a dramatic and unpredictable cascade of events that has produced the economic equivalent of a global blackout. As governments struggle to fix the crisis, experts have weighed in on the causes of the meltdown, from excess leverage, to lax oversight, to the way executives are paid.

Although these explanations can help account for how individual banks, insurers, and so on got themselves into

"Next to the mysteries of dynamics on a network—whether it be epidemics of disease, cascading failures in power systems, or the outbreak of revolutions—the problems of networks that we have encountered up to now are just pebbles on the seashore."

— Duncan Watts, 2003