Exploring Complexity MOOC, Spring 2013

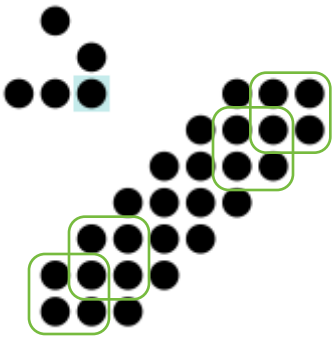
Cellular Automata section advanced exercise:

"In Conway's Game of Life, how many different ways can a *glider* collide with a *block*?"

Solution:

A *block* is the most common 'still life,' consisting of four live cells. It is perfectly symmetrical on the grid, meaning all of its reflections and rotations look exactly the same.

A Life *glider* is a configuration of five 'live' cells which 'moves' diagonally across the grid by reproducing itself in a two-step cycle: after two generations, the glider pattern recurs, but reflected (as if in a mirror) and shifted over (horizontally or vertically) by one cell space. This geometric transformation is called a *glide reflection*, which gives the glider its name. After four generations, the glider will reproduce itself in the exact same orientation, but shifted by one horizontal space and one vertical space.



To find all the ways that a glider can collide with a block in an otherwise 'empty' or 'dead' cell grid, it's enough to consider the glider moving in just one out of the four possible diagonal directions. In the picture, the glider is shown moving 'SE', or down and to the right. Some possible locations of blocks in the glider's path are shown too. Boxes drawn around these potential block locations show that they may overlap by one or two cells.

Life patterns can't interact with each other unless they are within a single cell space apart. A block located even one space further NE or SW than those shown in the picture will be missed by the glider entirely. Fixing the glider's location and following this observation, we can see that there are only six possible locations on a SW–NE diagonal for a block that can be 'hit' by the glider.

Timing is as important as location. Since the glider moves in a two-step cycle (apart from reflections, which we can safely ignore), a second row of six block positions (for a total of twelve), set back one more diagonal space to the SE, will cover all possible cases. The live cells of all twelve potential block positions are shown in the picture. As it happens, out of these twelve collisions, there are only **five** distinct outcomes, disregarding symmetry. Initial configurations for the five distinct collisions are listed on the following page. In the GameOfLife NetLogo model, you can use the command line

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reset-ticks set cells [...] depict cells
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to see these configurations, where "`[...]`" is replaced by the given cell list.

Conditions for the five distinct possible collisions of one glider and one block, with a description of the final (stable) pattern produced:

1: Honey farm (four beehives) after 30 generations.
[[5 0] [5 -1] [4 0] [4 -1] [-2 0] [-1 0] [-1 2] [0 0] [0 1]]

2: Blank (all cells dead) after 31 generations.
[[4 -1] [4 -2] [3 -1] [3 -2] [-2 0] [-1 0] [-1 2] [0 0] [0 1]]

3: Blank after 6 generations.
[[3 -3] [2 -3] [3 -2] [2 -2] [-2 0] [-1 0] [-1 2] [0 0] [0 1]]

4: Five *blinkers*, six *blocks*, and two *ponds* in a mirror-symmetric pattern after 184 generations.
[[1 -4] [1 -5] [0 -5] [0 -4] [-2 0] [-1 0] [-1 2] [0 0] [0 1]]

5: Block ‘moves’ slightly ‘backward’ after 17 generations.
[[0 -6] [-1 -6] [-1 -5] [0 -5] [-2 0] [-1 0] [-1 2] [0 0] [0 1]]