Advent of Code 2024 - Day 13

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1 Problem

You are given a claw machine with 2 buttons to push. It has a claw that can move in 2 dimensions, and a single prize at position (x_P, y_P). The claw starts at position $(0, 0)$. Button A moves the claw Δx_A in the x direction and Δy_A in the y direction, and costs 3 units to push. Button A moves the claw Δx_B in the x direction and Δy_B in the y direction, and costs 1 unit to push. What is the smallest number of units required to reach the prize.

2 Solution

Call the number of pushes of button A n_A and the number of pushes of button B n_B. The final position of the claw becomes

$$(\Delta x_A \cdot n_A + \Delta x_B \cdot n_B, \Delta y_A \cdot n_A + \Delta y_B \cdot n_B)\,. \tag{1}$$

To reach the prize the x and y position of the claw has to equal the x and y position of the prize. This gives us

$$\begin{cases} \Delta x_A \cdot n_A + \Delta x_B \cdot n_B = x_P \\ \Delta y_A \cdot n_A + \Delta y_B \cdot n_B = y_P. \end{cases} \tag{2}$$

First we solve for n_B.

$$\Delta x_A \cdot n_A + \Delta x_B \cdot n_B = x_P \tag{3}$$

$$\Delta x_B \cdot n_B = x_P - \Delta x_A \cdot n_A \tag{4}$$

$$\cdot n_B = \frac{x_P - \Delta x_A \cdot n_A}{\Delta x_B} \tag{5}$$

We substitute n_B in equation 2 and solve for n_A.

$$\Delta y_A \cdot n_A + \Delta y_B \cdot \frac{x_P - \Delta x_A \cdot n_A}{\Delta x_B} = y_P \tag{6}$$

$$\Delta y_A \cdot \Delta x_B \cdot n_A + \Delta y_B \cdot (x_P - \Delta x_A \cdot n_A) = y_P \cdot \Delta x_B \tag{7}$$

$$\Delta y_A \cdot \Delta x_B \cdot n_A + \Delta y_B \cdot x_P - \Delta x_A \cdot \Delta y_B \cdot n_A = y_P \cdot \Delta x_B \tag{8}$$

$$n_A \cdot (\Delta y_A \cdot \Delta x_B - \Delta x_A \cdot \Delta y_B) = y_P \cdot \Delta x_B - \Delta y_B \cdot x_P \tag{9}$$

$$n_A = \frac{y_P \cdot \Delta x_B - \Delta y_B \cdot x_P}{\Delta y_A \cdot \Delta x_B - \Delta x_A \cdot \Delta y_B} \tag{10}$$

To calculate n_B, calculate n_A using equation 10 and substitute the result into equation 5. Note that for the solution to be valid it must be an integer solution, as the buttons can only be pressed an integer amount of times.

Because there is only one possible solution, the minimum cost is given by

$$3 \cdot n_A + n_B \tag{11}$$