NYC

Credit Derivatives

HW3

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Exercise 1: Given a basket 10 firms with a notional of $1m. Expected recovery rate of 50%. Choose to buy protection through (1) First-to-default swap or (2) First $0.5 m losses? Why

	One default	Two defaults
If $\pi < 50\%$	FtD cover all, but First layer doesn't cover all. ***FtD** is better	Same result as one default. Even though (1)(2) doesn't respond the 2nd default. FtD still hold more coverage that benefit from the 1st default ***FtD** is better
If $\pi > 50\%$	*FtD =First Layer Loss	π is uniform distribution. The chance of $\pi > 50\%$ and $\pi < 50\%$ is about the same. However, the second default might be less likely. * **First Layer Loss** is better.
Conclusion	From the result above, FtD appears to cover more situations. As we mention above, First layer loss protection is only better while $\pi > 50\%$ with two default. The chance is slim based our general assumption. Therefore, if they are priced fairly (1) should be more expensive.(2) First $0.5 m losses will be cheaper	

Exercise 2 (Forward CDS and Option on CDS):

CDS can be traded on either an all-running or an all-upfront basis with same maturity.

Reference Entity	All-running CDS	All-upfront CDS
A	150 bps	6.75%
B	100bps	4.25%
C	75bps	3.00%

What would be the fair all-running spread X on an index credit default swap on A, B, and C

Reference Entity	Protection Leg	Fee Leg
A	100*6.75%	$X \cdot 100\left(\sum_{i=1}^{N} \bar{B}^{A}(0,T_i)\right) = X \cdot 100 \cdot Ann^{A} = X \cdot 100 \frac{675}{150}$
B	100*4.25%	$X \cdot 100\left(\sum_{i=1}^{N} \bar{B}^{B}(0,T_i)\right) = X \cdot 100 \cdot Ann^{B} = X \cdot 100 \cdot \frac{425}{100}$
C	100*3%	$X \cdot 100\left(\sum_{i=1}^{N} \bar{B}^{C}(0,T_i)\right) = X \cdot 100 \cdot Ann^{C} = X \cdot 100 \cdot \frac{300}{75}$
Total	14	1275X
Result	X =109.80 bps	

Exercise 3(Marshall and Olkin's Trick):

1. Use MO trick to generate 1000 sample plot of (U^1, U^2) when

(a) C the Gumbel copula and $\theta = 2, 5, 50$. Use the mixer Y being α -stable distributed with parameters $\alpha := \frac{1}{\theta}$ and $\beta := 1$. check *starnd* function

	Note
Process	(1) Generate $X^1 \ldots X^{(I)}$~iid uniform [0,1] : I Uniform (2) $Y \perp X^{(1)} \ldots . X^{(I)}$: 1 Uniform (3) Y's Laplace transform is $\phi^{-1}(t)$, i.e. $E\left[e^{-tY}\right] = \phi^{-1}(t)$ $Y \sim \alpha$ stable $\left(\alpha = \frac{1}{\theta}, \beta = 1\right)$

	(4)Define $U^{(i)}=\phi^{-1}\left(-\frac{1}{Y}\log(X^{(i)})\right)$, i=1.....N $\because \phi(t)=(-\ln t)^{\theta}\ \therefore \phi^{-1}(s)=e^{-s^{\frac{1}{\theta}}}$
Formula for Gumbel	$C(u^1,u^2)=\exp\left(-\left\{\left(-\log(u^1)\right)^{\theta}+\left(-\log(u^2)\right)^{\theta}\right\}^{\frac{1}{\theta}}\right)$
Result	$Q\left(u^{(1)}\le u^i,....,u^{(I)}\le u^I\right)=C(u^1....u^I)=\phi^{-1}\left(\sum_{i=1}^{I}\phi\left(u^i\right)\right)$

<table>
<tr><th></th><th>Note</th></tr>
<tr><td>

$Y \perp X^{(1)}....X^{(I)}$

$U^{(i)}=\phi^{-1}\left(-\frac{1}{Y}\log(X^{(i)})\right)$

$=e^{\left\{-\frac{1}{Y}\log(X^{(i)})\right\}^{\frac{1}{\theta}}}$

</td><td>

```
%EX3Q1A
clear all;

MC_Loops=1000;

theta=2;
alpha=1/theta;
beta=1;

X1=rand(1,MC_Loops);
X2=rand(2,MC_Loops);
Y=starnd(alpha,beta,MC_Loops);

U1=zeros(1,MC_Loops);
U2=zeros(1,MC_Loops);

for i=1:MC_Loops
    s1=-log(X1(i))/Y(i);
    s2=-log(X2(i))/Y(i);
    U1(i)=exp(-s1^(1/theta));
    U2(i)=exp(-s2^(1/theta));
    plot(U1(i),U2(i))
    hold on
end
```

</td></tr>
<tr><td>Function</td><td>

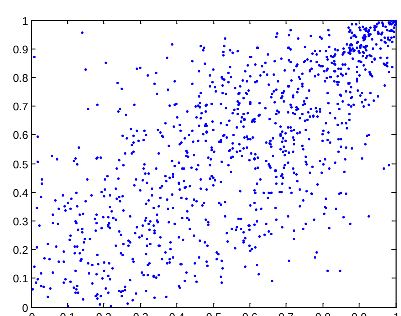
```
function X=starnd(alpha,beta,N)

if nargin==2;
    N=1;
end

V=rand(1,N)*pi-pi/2;
W=gamrnd(ones(1,N),1);

if alpha==1
    X=(2/pi)*((pi/2+beta*V).*tan(V)-beta*log(W.*cos(V)./(pi/2+beta*V)));
else
   B=atan(beta*tan(pi*alpha/2))/alpha;
   S=(1+beta^2*tan(pi*alpha/2)^2)^(1/(2*alpha));
   X=S.*sin(alpha*(V+B))./(cos(V).^(1/alpha)).*(cos(V-alpha*(V+B))./W).^((1-
alpha)/alpha);
end
```

</td></tr>
<tr><td>$\theta = 2$</td><td>



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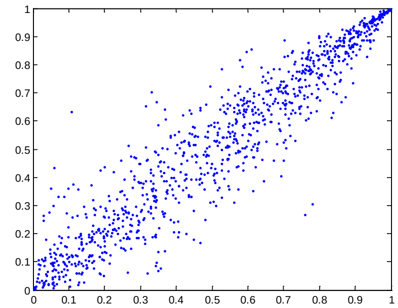
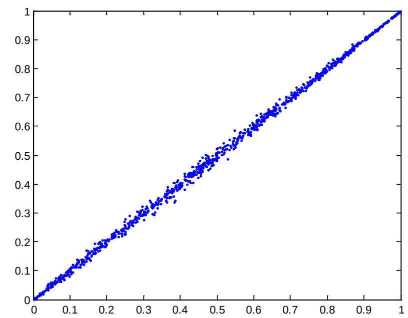
$\theta = 5$	
$\theta = 50$	

(b) C is the Clayton Copula and $\theta = 2, 5, 50$. Use the mixer Y being α -stable distributed with parameters $A := \frac{1}{\theta}$ and $B := 1$. check *gamrand* function

	Note
Process	(1) Generate $X^1 \ldots X^{(I)}$~iid uniform [0,1] : I Uniform (2) $Y \perp X^{(1)} \ldots . X^{(I)}$: 1 Uniform (3) Y's Laplace transform is $\phi^{-1}(t)$, i.e. $E\left[e^{-tY}\right] = \phi^{-1}(t)$ $Y \sim \text{Gamma}\left(A = \frac{1}{\theta}, B = 1\right)$ (4)Define $U^{(i)} = \phi^{-1}\left(-\frac{1}{Y}\log(X^{(i)})\right)$, i=1.....N $\because \phi(t) = t^{-\theta} - 1 \therefore \phi^{-1}(s) = (1+S)^{-\frac{1}{\theta}}$
Formula for Gumbel	$C(u^1, u^2) = \left\{\left(u^1\right)^{-\theta} + \left(u^2\right)^{-\theta} - 1\right\}^{-\frac{1}{\theta}}$
Result	$Q\left(u^{(1)} \le u^i, \ldots, u^{(I)} \le u^I\right) = C(u^1 \ldots u^I) = \phi^{-1}\left(\sum_{i=1}^{I} \phi\left(u^i\right)\right)$

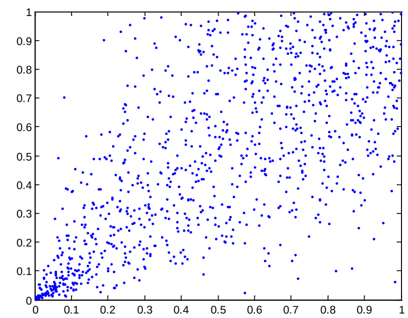
	Note

$Y \perp X^{(1)} \ldots . X^{(I)}$	`%HW3.1.1`
	`clear all;`
	
	`MC_Loops=1000;`
	
	`theta=2;`
	`A=1/theta;`
	`B=1;`
	`X1=rand(1,MC_Loops);`
	`X2=rand(1,MC_Loops);`
	
	`Y=gamrnd(A,B,MC_Loops,1);`

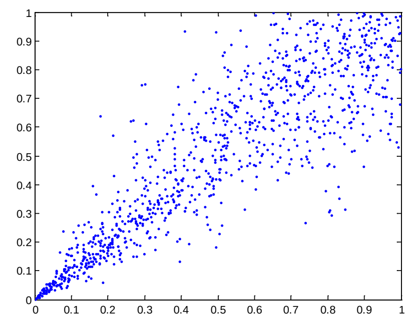
$$U^{(i)} = \phi^{-1}\left(-\tfrac{1}{Y}\log(X^{(i)})\right)$$
$$= e^{\left\{-\frac{1}{Y}\log(X^{(i)})\right\}^{\frac{1}{\theta}}}$$
$$Q\left(u^{(1)} \le u^{i}, \dots, u^{(I)} \le u^{I}\right)$$
$$= C(u^{1} \dots u^{I})$$
$$= \phi^{-1}\left(\sum_{i=1}^{I} \phi\left(u^{i}\right)\right)$$

```
U1=zeros(1,MC_Loops);
U2=zeros(1,MC_Loops);

for i=1:MC_Loops
    s1=-log(X1(i))/Y(i);
    s2=-log(X2(i))/Y(i);

    U1(i)=(1+s1)^(-1/theta);
    U2(i)=(1+s2)^(-1/theta);

    plot(U1(i),U2(i))
    hold on
end
```

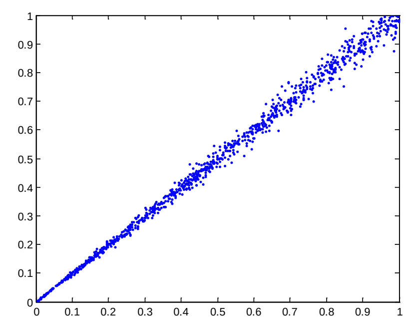
$\theta = 2$



$\theta = 5$



$\theta = 50$



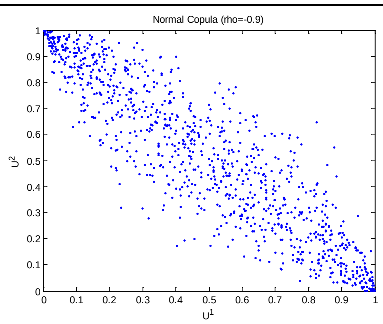
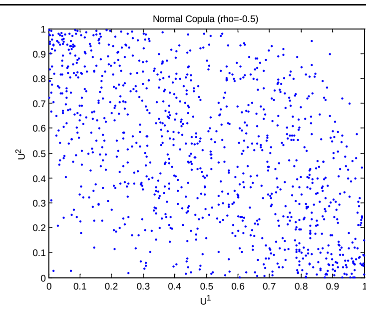
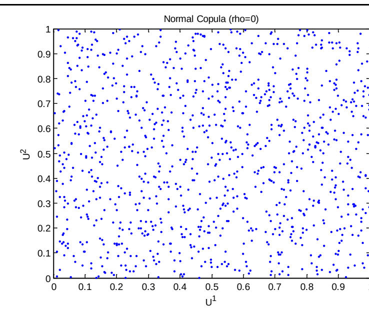
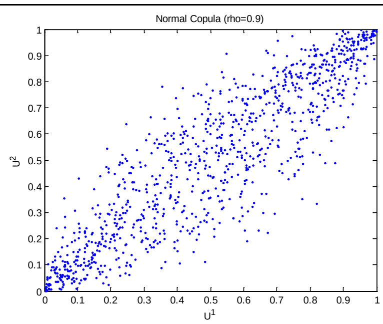
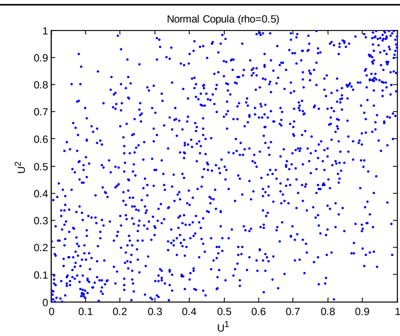
2. Use Cholesky factorization to generate sample plots of $\left(U^{1}, U^{2}\right)$ when C is the Normal copula with correlation $\rho \in \{-0.9, -0.5 - 0, 0.5, 0.9\}$

	Note
	`%HW3.1.2` `clear all;` `nPath=1000;` `rho = 0;`

$$X = A \cdot Z$$

$$U^{(i)} = \overline{F}\left(X^{(i)}\right) : \overline{F} \sim N(0,1)$$

```
x=zeros(2,nPath);
row1=zeros(1,nPath);
row2=zeros(1,nPath);

sig=[1,rho;rho,1];
A = chol(sig,'lower');

Z = randn(2,nPath);
x=A*Z;

result=normcdf(x,0,1);

row1=result(1,:);
row2=result(2,:);

for i=1:nPath
    plot(row1(i),row2(i));
    hold on
end

hold off

title('Normal Copula (rho=0)');
xlabel('U^1');
ylabel('U^2');
```

$\rho = -0.9$	$\rho = -0.5$	$\rho = 0$
		
$\rho = 0.9$	$\rho = 0.5$	
		

Exercise 4:

Given constant recoveries π, interest rate r. The thresholds, $\left(U^1, \ldots U^I\right)$, are uniform random variables distributed with a copula C. Assume that the default default times are of the Cox type with constant pre-default intensities $\lambda^i > 0$. That is $\tau^i \triangleq \inf\left\{t \mid e^{-\lambda^i t} \leq U^i\right\}$. Denote each individual name spread by $\overline{s}^i$ (CDS-spread) whereas the spread of the corresponding Nth-to-default is denoted by $\overline{s}^{NtD}$

1. Show $\overline{s}^i = (1-\pi)\lambda^i$

	Note
Price of Fee leg (1): method 1	$\overline{s}^i \cdot \int_0^{T_N} \overline{B}(0,t)dt = \overline{s}^i \cdot \int_0^{T_N} e^{-(r+\lambda^i)t}dt$
Method 2	$\overline{s}^i \cdot E^Q\left[\int_0^{T_N} D(t)\cdot 1_{\{\tau>T\}}dt\right] = \overline{s}^i \cdot E^Q\left[\int_0^{T_N\wedge\tau} e^{-rt}dt\right]$
Method 3	$\overline{s}^i \cdot E^Q\left[\int_0^{T_N} D(t)\cdot 1_{\{\tau>T\}}dt\right] = \overline{s}^i \cdot E^Q\left[\int_0^{T_N} e^{-(r+\lambda)t}dt\right] = \overline{s}^i \cdot \int_0^{T_N} e^{-(r+\lambda)t}dt$
Price of Recovery Leg (2)	$E^Q\left[(1-\pi)e^{-r\tau}\cdot 1_{\{\tau\le T\}}\right] = (1-\pi)\cdot\int_0^{T_N} e^{-rt}\cdot P(t,G_T)dt = (1-\pi)\cdot\int_0^{T_N} e^{-rt}\cdot\lambda\cdot e^{-\lambda^i t}dt$ $= (1-\pi)\cdot\lambda^i\int_0^{T_N} e^{-rt}\cdot e^{-\lambda^i t}dt$
From Aven's result	$P(t\mid G_T) = -\frac{\partial}{\partial t}Q(\tau_1>t\mid G_T)\mid_{T=t} = -\frac{\partial}{\partial T}e^{-\lambda T}\mid_{T=t} = \lambda^i\cdot e^{-\lambda^i T}\mid_{T=t} = \lambda^i\cdot e^{-\lambda^i t}$
Combine (1)(2)	Combine (1)(2) we can have $S^i = (1-\pi)\lambda^i$

- **Discrete Case**

	Note
Price of Fee leg	If CDS buyer pay $S\Delta$ at T_n, then the value of fee leg will be: $S^i\Delta\sum_{k=1}^{n}\tilde{E}\left[D(T_n)\cdot 1_{\{\tau>T_n\}}\right] = S^i\Delta\sum_{k=1}^{n}B(0,T_n)P(0,T_n) = S^i\Delta\sum_{k=1}^{n}\overline{B}(0,T_n)$ (1)
Price of Recovery Leg	$(1-\pi)\sum_{k=1}^{n}\tilde{E}\left[D(T_n)\cdot 1_{\tau\in[T_{n-1},T_n]}\right] = (1-\pi)\sum_{k=1}^{n}B(0,T_n)\left(Q(\tau\le T_{n-1})-Q(\tau\le T_n)\right)$ $=(1-\pi)\sum_{k=1}^{n}B(0,T_n)\left(P(0,T_{n-1})-P(0,T_n)\right)$ $=(1-\pi)\sum_{k=1}^{n}B(0,T_n)\left(\frac{P(0,T_{n-1})-P(0,T_n)}{\Delta}\right)\Delta$ $=(1-\pi)\sum_{k=1}^{n}B(0,T_n)\left(\frac{e^{-\lambda(n-1)\Delta}-e^{-\lambda(n)\Delta}}{\Delta}\right)\Delta$ $=(1-\pi)\sum_{k=1}^{n}B(0,T_n)\left(\frac{e^{-\lambda\Delta}}{\Delta}\right)\Delta$ $\approx(1-\pi)\cdot\lambda\cdot\Delta\cdot\sum_{k=1}^{n}B(0,T_n)$ (2)
Combine (1)(2)	$S\Delta\sum_{k=1}^{n}\overline{B}(0,T_n)\approx(1-\pi)\cdot\lambda\cdot\Delta\cdot\sum_{k=1}^{n}B(0,T_n)$ $S=(1-\pi)\lambda$. Therefore for ith contract, we can have $S^i=(1-\pi)\lambda^i$

2. If the thresholds are independent (i.e., independent U^i's) show that : $\overline{s}^{FtD} = \sum_{i=1}^{I}\overline{s}^i$

Note	Derivation Process
$\because$ independent U^i's $\therefore$ default time are independent	$Q\left(\tau^{FtD} > T\right) = Q\left(\tau^1,.....\tau^N > T\right) = \prod_{i=1}^{N} Q\left(\tau^i > T\right) = \prod_{i=1}^{N} e^{-\lambda^i T} = e^{-\sum_{i=1}^{N} \lambda^i T}$
Fee leg with the result we got above (1)	$\overline{s}^{FtD} \cdot \tilde{E}\left[D(T) \cdot 1_{\{\tau^{FtD} > T\}}\right] = \overline{s}^{FtD} \int_0^T e^{-rt} e^{-\sum_{i=1}^{N} \lambda^i T} dt$
Protection Leg (2)	$\tilde{E}\left[(1-\pi)\left(e^{-r\tau_1} \cdot 1_{\{\tau_1 \le T\}} ... e^{-r\tau_N} \cdot 1_{\{\tau_N \le T\}}\right)\right] = (1-\pi)\int_0^T e^{-r\tau_1}...e^{-r\tau_N} P\left(t, G_T\right) dt$ $= (1-\pi)\int_0^T e^{-r\tau_1}...e^{-r\tau_N} \sum_{i=1}^{N} \lambda^i \cdot e^{-\sum_{i=1}^{N} \lambda^i t} dt$ $= (1-\pi)\sum_{i=1}^{N} \lambda^i \int_0^T e^{-r\tau_1}...e^{-r\tau_N} \cdot e^{-\sum_{i=1}^{N} \lambda^i t} dt$
From Aven's result	$P\left(t, G_T\right) = -\frac{\partial}{\partial t} Q\left(\tau_1 > t.....\tau_N > t \mid G_T\right)\|_{T=t} = -\frac{\partial}{\partial T} e^{-\sum_{i=1}^{N} \lambda^i T}\|_{T=t} = -\frac{\partial}{\partial T} e^{-\sum_{i=1}^{N} \lambda^i T}\|_{T=t}$ $= \sum_{i=1}^{N} \lambda^i \cdot e^{-\sum_{i=1}^{N} \lambda^i T}\|_{T=t} = \sum_{i=1}^{N} \lambda^i \cdot e^{-\sum_{i=1}^{N} \lambda^i t}$
Combine (1) (2)	$\overline{s}^{FtD} = (1-\pi)\sum_{i=1}^{N} \lambda^i = \sum_{i=1}^{N} \lambda^i (1-\pi)$ we also know that $\overline{s}^i = (1-\pi)\lambda^i$. Therefore $\overline{s}^{FtD} = \sum_{i=1}^{N} \overline{s}^i$ (Here we may change N to I) to fit the question.

3. Show analytically that if $U^i = U^1$ for i=2,3…I. $\overline{s}^{FtD} = \max_{i \in \{1....I\}} \overline{s}^i$

Note	Derivation Process
If they are not independent	$Q\left(\tau^{FtD} > T\right) = Q\left(\tau^1 > T,.....,\tau^N > T\right) = Q\left(e^{-\lambda_1 t} > u_1,.....,e^{-\lambda_N t} > u_N\right)$ $= Q\left(e^{-\lambda_{<N>} t} > u_1\right) = e^{-\lambda_{<N>} t}$
Fee leg with the result we got above (1)	$\overline{s}^{FtD} \cdot \tilde{E}\left[D(T) \cdot 1_{\{\tau^{FtD} > T\}}\right] = \overline{s}^{FtD} \int_0^T e^{-rt} e^{-\lambda_{<N>} t} dt$
Protection Leg (2)	$\tilde{E}\left[(1-\pi)\left(e^{-r\tau_1} \cdot 1_{\{\tau_1 \le T\}} ... e^{-r\tau_N} \cdot 1_{\{\tau_N \le T\}}\right)\right] = (1-\pi)\int_0^T e^{-r\tau_1}...e^{-r\tau_N} P\left(t, G_T\right) dt$ $= (1-\pi)\int_0^T e^{-r\tau_1}...e^{-r\tau_N} \cdot \lambda_{<N>} e^{-\lambda_{<N>} t} dt = (1-\pi)\lambda_{<N>} \int_0^T e^{-r\tau_1}...e^{-r\tau_N} \cdot e^{-\lambda_{<N>} t} dt$
From Aven's result	$P\left(t, G_T\right) = -\frac{\partial}{\partial t} Q\left(\tau_1 > t.....\tau_N > t \mid G_T\right)\|_{T=t} = -\frac{\partial}{\partial T} e^{-\lambda_{<N>} T}\|_{T=t} = \lambda_{<N>} e^{-\lambda_{<N>} T}\|_{T=t}$ $= \lambda_{<N>} e^{-\lambda_{<N>} t}$
Combine (1) (2)	$\overline{s}^{FtD} = (1-\pi)\lambda_{<N>}$ we know that $\lambda_{<N>} = \max_{i \in \{i....N\}} \{\lambda_i\}$. Therefore $\overline{s}^{FtD} = \max_{i \in \{i....I\}} \overline{s}^i$ (Here we may change N to I) to fit the question.

4. What would be the prices for all other NtD swaps, $\overline{s}^{NtD}$ in the setting of the previous question.

Comment	Based on the logic from previous questions, $\overline{s}^{NtD}$ would be the Nth largest spread of the all firms in the portfolio.

5. Compute $\overline{S}^{NtD}$ and $E^Q\left[\tau^{NtD}\right]$ for the normal copula with $\sum_{ii} = 1 \sum_{ij} = \rho$

Individual Spread	**100 bps**
Maturity	**5 years**
Interest rate R	**4%**
Recovery rate	**35%**

Note	Derivation Process
$Q^{FtD}\left(\tau^{FtD} >\mid G_T\right) = C\left(Y_T^1,\ldots,Y_T^N \mid G_T\right)$	(code below)
Numerical Result	(output below)

```
clear all;

%define underlying parameters
nPath=100000;T=5;pi=0.35;
N=5;
rho=0.2;

r=0.04;
spread=0.01;
lambda=spread/(1-pi); % Intensity

x=zeros(5,nPath);

sig=[1,rho,rho,rho,rho;
    rho,1,rho,rho,rho;
    rho,rho,1,rho,rho;
    rho,rho,rho,1,rho;
    rho,rho,rho,rho,1];

A = chol(sig,'lower');

ZZ=normrnd(0,1,5,nPath);
xVal=A*ZZ;

UU=normcdf(xVal,0,1);
TAU=-log(UU)/lambda;
TAU=sort(TAU,'ascend');

NtdProtPV=(1-pi).*exp(-r.*TAU).*(TAU<T);
NtdRAPV=(1-exp(-r.*min(TAU,T)))./r;

meanNtdProtPV=mean(NtdProtPV,2);
meanNtdRAPV=mean(NtdRAPV,2);

NtdSpread= meanNtdProtPV./meanNtdRAPV*10000;
NtdTau=mean(TAU,2);

series=1:1:5;
disp('EX4.Q5:');
disp(['NtD                 :',num2str(series)]);
disp(['NtD Spread(bps)    :',num2str(NtdSpread')]);
disp(['NtD Survival Time :',num2str(NtdTau')]);
```

Numerical Result:

```
EX4.Q5:
NtD           :  1          2            3            4              5
Spread(bps)   :442.3732     89.59785     17.34494     2.520427       0.2376288
Survival Time : 16.86096    33.47403     53.8091     82.69703      138.2433
```

NtD	1	2	3	4	5
Spread	442.3732	89.59785	17.34494	2.520427	0.2376288
Survival Time	16.86096	33.47403	53.8091	82.69703	138.2433

***A:* basket of FtD, 2td,...5tD and Portfolio *B:* Basket of the corresponding 5 CDS's.**

6. Are the initial time value of two portfolio the same?

Comment	Yes, they are the same. Since they are all-running, it costs buyer nothing to enter the contract. Therefore, they have the same initial value.

7. Does the recovery leg of Portfolio A replicate the recovery leg of portfolio B?

Comment	Yes. When the first bond default, FtD in Portfolio A will be executed with the same level of payment of particular bond in Portfolio B. Same case for 2tD 5tD

8. Does the premium leg of Portfolio A replicate the premium leg of portfolio B?

Comment	No. The premium leg of portfolio can't replicate each other. For Portfolio A, more is paid earlier in the time frame. But for portfolio B, spread decrease more steadily. But the PV of the payment streams should be equal to each other.

9. Will a basket of all 5 NtD protection contract be a perfect hedge for the portfolio of corresponding individual credit default swap?

Comment	For a perfect hedge we will need to exactly replicate the cash flows. From fee leg point, the spread payment streams for 2 portfolios are very different cash flow. It won't be a perfect hedge. They occur at different times with different sizes though their PV should be the same If defaults occur, the protection payment received from A and B are the same.

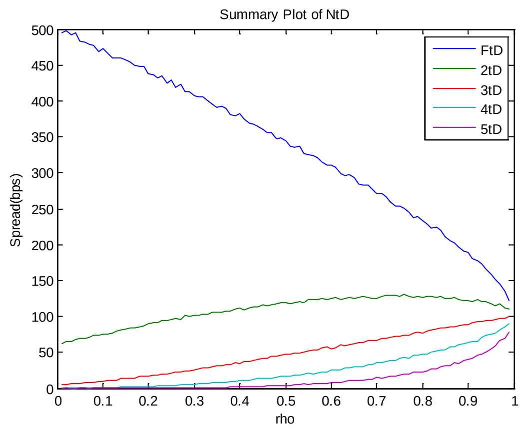
10. Compute and plot NtD prices as a function of correlation. $\rho \in (0,1)$

Code

```
clear all;

%define underlying parameters
nPath=10;pi=0.35;T=5;r=0.04;
NtD=1;
nfirm=5
rhos =0.01:0.01:0.99;
size=99;
spread=0.01;

lambda=spread/(1-pi);%Intensity

for NtD=1:5
    for i=1:size
        rho=rhos(i);

    sig=rho*(ones(nfirm,nfirm)-eye(nfirm))+eye(nfirm);

    A=chol(sig,'lower');

    ZZ=normrnd(0,1,nfirm,nPath);
    x=A*ZZ;

    UU=normcdf(x,0,1);
    UU=sort(UU,1,'descend');
    TAU=-log(UU)/lambda;

    nTau=TAU(NtD,:);
    ntdTaus=(nTau<nfirm).*nTau;

    NtdProtPV=(1-pi).*(ntdTaus>0).*exp(-r.*ntdTaus);
```

```
    NtdRAPV=(1-exp(-(r.*ntdTaus)))./r+(ntdTaus==0).*(1-exp(-r.*T))./r;

    meanNtdProtPV=mean(NtdProtPV);
    meanNtdRAPV=mean(NtdRAPV);

    NtDspread(1,NtD)= meanNtdProtPV/meanNtdRAPV*10000;
    spread(NtD,i)=NtDspread(1,NtD);

    end
end

plot(rhos,spread(1,:),rhos,spread(2,:),rhos,spread(3,:),rhos,spread(4,:),rhos,spread(5,:));
title(['Summary Plot of NtD']);
xlabel('rho');
ylabel('Spread(bps)');
Legend('FtD','2tD','3tD','4tD','5tD','Location','NorthEast');

figure;
XX=cumsum(spread);
plot(rhos,XX(5,:));
```

Summary Plot



11. Explain why $\overline{s}^{FtD}$ is decreasing with correlation, where $\overline{s}^{NtD}$ is increasing for large N

Comment	About $\overline{s}^{FtD}$: when correlation is low and approaching independence, $\overline{s}^{FtD}$ can be high because first default take the max of U (of 5). As correlation increases, first default and last default are pulled forward the same point of 2tD …5tD.
	About $\overline{s}^{NtD}$: when correlation is low and approaching independence, $\overline{s}^{NtD}$ can be low because last default take the min of U (of 5). As correlation increases, first default and last default are pulled forward the point of 2tD…5tD.
	Intuitively, when correlation is high, defaults happen one after another in a short period of time, , the FtD is not as good a credit protection tool since the second default will soon follow. The FtD decrease with correlation increase. But for large N, the NtD default will occur a lot sooner when correlation is high. Therefore the NtD contract will be more expensive as correlation increase.

Exercise 5 (Relative Trading):

Given 5 years index and its traches.

Product	Upfront Payment	Running Spread (bps)	Fixed Spread DV01
Index	-0.5	275	4.22
0-10% Tranche	71		2.9
10-15% Tranche	35		4.1
15-25 Tranche		325	4.4
25-35 Tranche		80	4.5
35-100 Tranche		11	4.2

1. PV of Protection Leg for index

	Notional	Upfront Payment (%)	Running Spread	Fixed Spread	%	Protection Leg	PV of	Total
Index	$100	-0.5%	275	4.22	11.105%	For Index	$11.105	$11.105

2. PV of Protection Leg for Each Tranches

	Upfront Payment (%)	Running Spread	Fixed Spread:DV01	%	Notion	PV	Protection Leg
0-10% Tranche	71.0%		2.9	71.000%	100	$71.00	0-10% Tranche
10-15% Tranche	35.0%		4.1	35.000%	100	$35.00	10-15% Tranche
15-25 Tranche		325	4.4	14.300%	100	$14.30	15-25 Tranche
25-35 Tranche		80	4.5	3.600%	100	$3.60	25-35 Tranche
35-100 Tranche		11	4.2	0.462%	100	$0.46	35-100 Tranche

Comment	As per the response in BB, we assume this question is about buying $100 worth of contract on 'each' tranches. This is a little bit different from what we will get in the following question. (PV is $10.94)

3. Construct a relative value trade and quantify its net PV.

	Upfront Payment (%)	Running Spread	Fixed Spread	%	Notional				PV of	Total
Index	-0.5%	275	4.22	11.105%	$100		Protection Leg	For Index	$11.105	$11.105
	Upfront Payment (%)	Running Spread	Fixed Spread:DV01	%	Notion	PV	Protection Leg	Weight	Total	
0-10% Tranche	71.0%		2.9	71.000%	100	$71.00	0-10% Tranche	0.1	$7.10	
10-15% Tranche	35.0%		4.1	35.000%	100	$35.00	10-15% Tranche	0.05	$1.75	
15-25 Tranche		325	4.4	14.300%	100	$14.30	15-25 Tranche	0.1	$1.43	
25-35 Tranche		80	4.5	3.600%	100	$3.60	25-35 Tranche	0.1	$0.36	
35-100 Tranche		11	4.2	0.462%	100	$0.46	35-100 Tranche	0.65	$0.30	$10.940
									Profit	**$0.1647**

Trade Strategy	We can buy protection leg of each tranches and short index and grasp profit of $0.1647

4. What the spread of 35%-100% tranche need to be in order eliminate relative value opportunity ?

Trade Strategy	We need 65*X*4.2=0.3+0.1647 X=17.03 bps

5. What is the initial net cash flow from relative value trade?

Initial Cash Flow

			Initial Cash Flow	Total
Q5	Sell Index	Get	($0.50)	
	Buy Tranches	Pay	$7.10	
			$1.75	($9.35)

Conclusion: Initial Payment is Net PV is -$9.35

6. What is the on-going annual cash flow assuming no defaults?

On-going Cash Flow

			Running Cash Flow	Total
Q6	Sell Index	Receive	$2.75	
	Buy Tranches	0-10% Tranche	$0.00	
		10-15% Tranche	$0.00	
		15-25 Tranche	$0.33	
		25-35 Tranche	$0.08	
		35-100 Tranche	$0.07	$2.2735

Conclusion: Running Cash Flow is net inflow of $2.2735

7. Once the position is established, will a parallel increase in the level of risk-free interest rates result in a gain or loss? Why?

Comment	We will lose money. Our receivables on selling index will be reduced. We are paying running payment on the last three tranches. The biggest one is in the middle. We might not be able to enjoy huge deduction of our liability. Initial payment won't be impacted. Therefore. We will lose money.

8. Once the position is established, will a parallel increase in the level of CDS spreads result in a gain or loss? Why?

Comment	We sell index and buy the individual tranches, therefore we pay the CDS spread on the tranches and receive the CDS spreads on the index. However, the incase of the CDS will cause DV01 value to come down. We indeed pay less for the last three tranches, with the reduced DV01. Our income from index would be calculated through smaller DV01. The fact that that increase 100 names. Therefore the total effect would be a loss.

9. If no defaults are realized, will this position have resulted in a gain or loss on initial investment relative to the risk-free rate?

Comment	If no default occurs, we would continue to receive a net cash flow from running spread payments without reduction of notional. We should have a gain.