Naive Utility Calculus

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Table of Contents

Table of Contents

How do we make sense of other people's behavior?

Introduction

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How do we make sense of other people's behavior?

- Why did you sleep late last night?
- Why did you sign up for this course?
- Why did you choose to eat out instead of cooking?

Naive Utility

$$\text{Utility} = \text{Rewards} - \text{Costs}$$

- There is empirical support that humans intuitively use utility-based reasoning to make sense of other people's behavior

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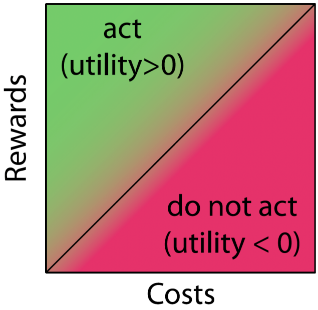

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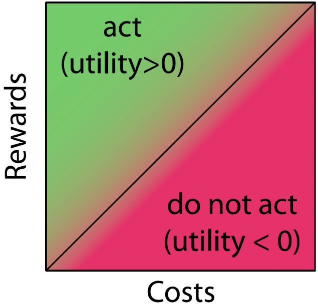

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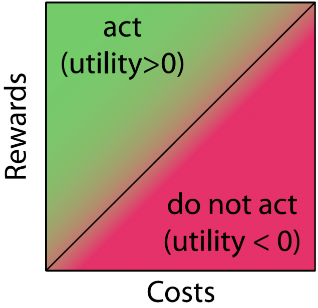


- Pursue/forego high/low-cost implies reward was even higher/lower
- Pursue/forego low/high-cost does not give info about reward

Naive Utility Calculus

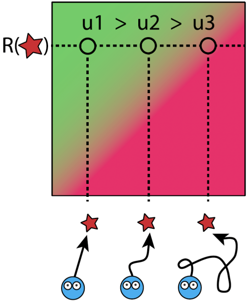
$$U(p, o) = R(o) - C(p)$$

- $U(p, o)$: utility expected from acting according to plan p to reach outcome o
- $R(o)$: subjective reward the agent expects from outcome o
- $C(p)$: subjective cost of executing plan p

Caveats

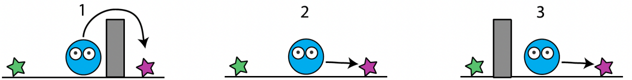
- **Descriptive, not normative:** This is not about how people *should* make decisions (economic utility theory)
- **How we actually operate:** This describes how we *intuitively* make sense of other people's behavior
- People don't explicitly compute utilities when they act - this is the cognitive framework we use to understand others

Utility and Efficiency



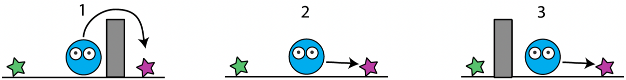
- More efficient paths are less costly and therefore produce higher utilities
- When agents act, they will fulfill their goals as efficiently as possible to maximize utility

Graded Preference Inference



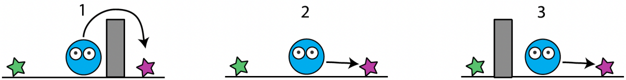
- **Fig.1: Pursue high-cost**: Agent pays high cost (long detour) to get purple star → strong preference inference

Graded Preference Inference



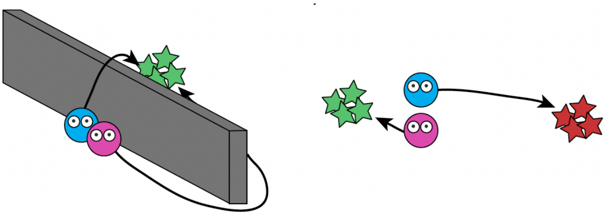
- **Fig.1: Pursue high-cost**: Agent pays high cost (long detour) to get purple star → strong preference inference
- **Fig.2: Pursue low-cost**: Agent pays low cost (short path) to get purple star → weak preference inference

Graded Preference Inference



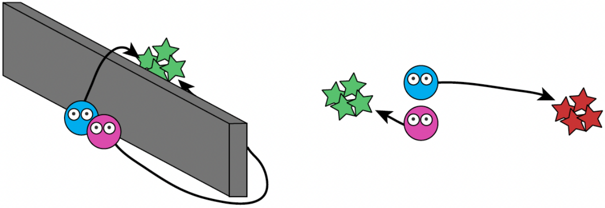
- **Fig.1: Pursue high-cost**: Agent pays high cost (long detour) to get purple star → strong preference inference
- **Fig.2: Pursue low-cost**: Agent pays low cost (short path) to get purple star → weak preference inference
- **Fig.3: Forego high-cost**: Agent foregoes high cost (doesn't climb wall) to get green star, chooses purple instead → no preference inference

Costs and Rewards Vary Across Agents



- **Costs vary**: Blue agent can climb walls easily, pink agent cannot $\rightarrow$ different action costs

Costs and Rewards Vary Across Agents



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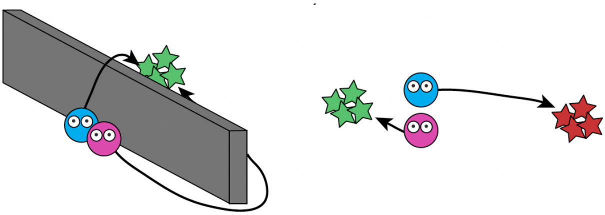
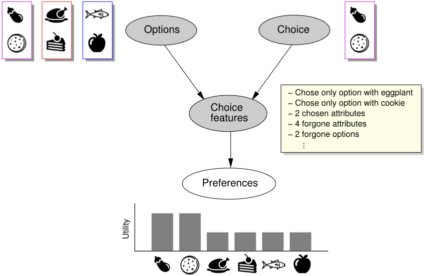


- **Costs vary**: Blue agent can climb walls easily, pink agent cannot → different action costs
- **Rewards vary**: Blue agent prefers red stars, pink agent prefers green stars → different goal values
- **Same action, different inference**: Identical behavior can imply different preferences based on agent capabilities and values

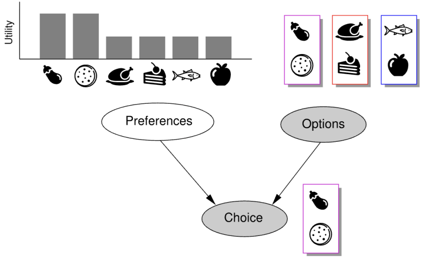
Table of Contents

Feature-Based Approach



- **Method**: Look at simple features of the choice and apply rules
- **Example**: Alice chooses {eggplant sandwich} over {turkey, tuna, ham}
- **Feature**: "She chose the only option with eggplant"
- **Rule**: "When someone picks the unique option, they probably like that thing"
- **Conclusion**: "Alice likes eggplant"

Inverse Decision-Making Approach



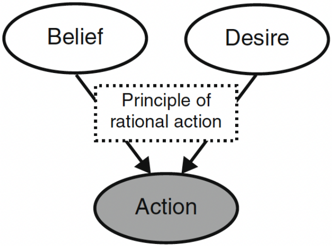
- **Method**: Use a model of how people actually make decisions, then work backwards
- **Example**: Same choice - Alice chooses {eggplant sandwich} over {turkey, tuna, ham}
- **Model thinking**: "If Alice really liked eggplant, how likely would she be to make this choice? What if she liked turkey instead? What if she just wanted any sandwich?"
- **Calculation**: Uses math to figure out which preference scenario makes this choice most probable

NUC vs. Inverse Decision-Making

- **Similarity**: NUC is similar to inverse decision-making as it works on assumption that agents maximize utilities
- **Key Differences**:
  - Uses events with complex spatiotemporal structures and not just isolated discrete choices
  - Computes both variables costs and rewards, whereas inverse decision-making only infers rewards without costs

Inverse Planning

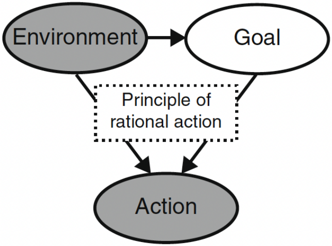
(a)



Belief-Desire Psychology ("Forward-thinking")

- Belief + Desire → Action
- "Sarah believes store is open + wants coffee → walks to coffee shop"

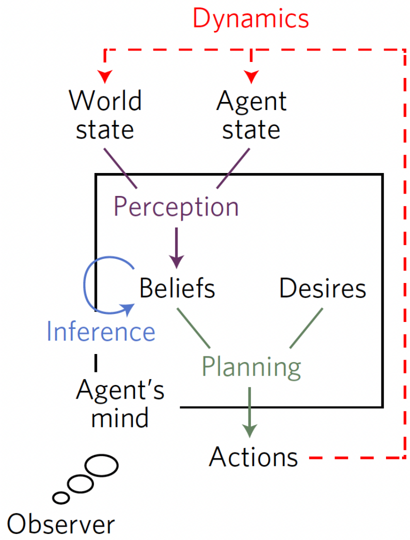
(b)



Inverse Planning ("Backward-thinking")

- Observe: Environment + Action → Infer: Goal
- "See maze layout + this path → agent wants point A"

Goal-Directed Action Understanding



Limitations

- Model does not explain multiple causes behind other people's goals
- Treats cost as constant, observable and uniform across agents

Research Questions

- **RQ1**: Can NUC support joint inference of costs and rewards when we know neither, using a coherent generative model?
- **RQ2**: Does NUC drive fine-grained quantitative inferences or only coarse qualitative ones?
- **RQ3**: Is NUC a unified generative model supporting probabilistic inference, or a collection of simple heuristics?

Table of Contents

Hierarchical Mind Model

Level	**Component**	**Observable?**
4	Desires (Reward functions)	No
3	Goals (World states)	No
2	Intentions (Goal sequences)	No
1	Actions (Behaviors)	Yes

$\downarrow$ **Inference Direction** $\uparrow$

Generative Process

Desires (Rewards)

$\downarrow$ $+$ Costs

Goals

$\downarrow$ Utility Max

Intentions

$\downarrow$ Observable

Actions

MDP Planning per Goal

Goal: Reach Object A

$$V^*(s) = \max_a \sum_{s'} P(s'|s,a)[R(a,s) - C(a,s) + \gamma V^*(s')]$$

Policy: $p(a|s) \propto \exp(\sum_{s'} P(s'|s,a) V^*(s')/\alpha)$

Each goal → Separate MDP → Efficient path

The Inference Problem

Hidden
Costs &
Rewards

$\longrightarrow$

Observed
Actions

Forward

$\longleftarrow$

Bayesian Inference

$$p(C, R|A) \propto p(A|C, R) \cdot p(C, R)$$

Two Types of Rationality

Type	What	Formula
Rational Choice	Intention selection	$p(I \mid C, R) \propto \exp(U(I)/\beta)$
Rational Action	Efficient execution	$p(A \mid I)$ via MDP policy

Likelihood: $p(A \mid C, R) = \sum_I p(A \mid I) \cdot p(I \mid C, R)$

Table of Contents

Experimental Setup

- **Domain**: Gridworld navigation tasks with agents moving toward goals
- **Participants**: Online studies using Amazon Mechanical Turk
- **Method**: Participants watch agent behaviors and make preference judgments
- **Key Variables**: Action costs (walls, detours), goal rewards (different objects), agent capabilities
- **Measures**: Preference strength ratings, choice predictions, confidence judgments

Experiment 1: Graded Preference Inference

- **Question**: Do people make graded preference inferences based on action efficiency?
- **Design**: Agents choose between two goals with varying path costs
- **Manipulation**: High cost vs. low cost paths to preferred goal
- **Results**:
  - Higher costs → stronger preference inferences
  - Lower costs → weaker preference inferences
  - Linear relationship between cost and inferred preference strength
- **Conclusion**: People make graded preference inferences consistent with NUC

Experiment 2: Individual Differences in Costs

- **Question**: Do people consider individual differences in action costs?
- **Design**: Two agents with different capabilities (can/cannot climb walls)
- **Manipulation**: Same action, different costs for each agent
- **Results**:
  - People adjusted preference inferences based on agent capabilities
  - Wall-climbing agent: weaker preference for taking long route
  - Non-climbing agent: stronger preference for taking long route
- **Conclusion**: People consider individual cost differences when inferring preferences

Experiment 5: Joint Cost-Reward Inference

- **Question**: Can people jointly infer costs and rewards from agent behavior?
- **Design**: Agents with unknown capabilities and unknown goal preferences
- **Manipulation**: Multiple trials revealing different cost-reward tradeoffs
- **Results**:
  - People successfully inferred both costs and rewards simultaneously
  - Judgments consistent with Bayesian model predictions
  - Performance improved with more observations
- **Conclusion**: NUC supports complex joint inference in realistic scenarios

Table of Contents

Discussion

- **Key Findings**: People use Naive Utility Calculus for preference inference
  - Make graded inferences based on action efficiency
  - Consider individual differences in costs and rewards
  - Perform complex joint inference of multiple variables
- **Theoretical Contribution**: NUC as a unified computational framework
  - Goes beyond simple heuristics
  - Supports probabilistic inference mechanisms
  - Handles spatiotemporal action sequences
- **Future Directions**:
  - Real-world applications beyond gridworlds
  - Neural mechanisms underlying utility-based reasoning
  - Cross-cultural and developmental studies