The memo handbook

Overall anatomy of a memo

axes of array
to compute

name

scalar free parameters

```
@memo
def   f[x: X, y: Y](a, b, c):
  alice: …
  bob: …
  return …
```

sequence of statements

expression whose value
to compute for each cell
in returned array

Statements

chooses

Domain of choice (name of Python list/enum or JAX array)

Agent making choice

```
bob: chooses(a in Actions, wpp=exp(β*utility(a)))
```

Name of choice

"With probability proportional to"
(to softmax use exp(…))
wpp=1 creates uniform choice

```
bob: chooses(a in Actions, to_maximize=utility(a))
```

For argmax use to_maximize

thinks

Agent doing the thinking

```
bob: thinks[
   alice: chooses(...),
   charlie: chooses(...),
   ...
]
```

What that agent thinks
(notice the commas!)

observes

Agent observing

Choice being observed (square brackets are a mnemonic for "someone else's choice")

```
bob: observes [alice.x] is y
```

What the choice is observed to actually be. Can create false beliefs this way!

```
bob: observes [alice.x] is charlie.y
```

This value can also be another agent's choice.

knows

Agent who knows

Choices that are known

```
bob: knows(x, alice.y)
```

This utility is useful for the common case of "pushing" a variable into an agent's frame of mind. Roughly shorthand for this:

```
bob: thinks[ alice: chooses(y in Y, wpp=...) ]
bob: observes [alice.y] is alice.y
```

snaps

Agents can remember "snapshots" of their past selves. Useful for counterfactuals and hypotheticals, especially when used with "imagine" expressions (see below...).

"alias" of snapshot ↓

Agent who snapshots ↘

```
alice: snaps(past_alice=self)
```

↑ snapshot of whom?

```
alice: observes [bob.x] is x
return alice[ past_alice[ E[bob.x] ] ]
```

↗ not affected by "observe" statement

Expressions

literals

floating-point numbers only

$\downarrow$

```
3.14
```

also references to declared free parameters

$\downarrow$

```
a, b, c, …
```

operators

memo supports most Python unary/binary ops

$\downarrow$

```
1 + 1
```

also some free bonus functions

$\downarrow$

```
exp(…), log(…), abs(…)
```

can also call any function tagged
@jax.jit (scalar-in-scalar-out)

$\downarrow$

```
@jax.jit
def f(x):
  return np.cos(x)
```

useful for calling deep learning, etc.
JAX is a big ecosystem

choices

```
alice: chooses(x in X, wpp=1)
alice: chooses(y in Y, wpp=f(x, y))
```

↑

can refer to an agent's own choice as if it were simply a variable

probabilistic operators

expectation

$\downarrow$

```
E[alice.x + bob.z]
```

probability

$\downarrow$

```
Pr[alice.y >= 0]
```

(mutual) entropy between choices

$\downarrow$

```
H[alice.x, bob.y, …]
```

variance

$\downarrow$

```
Var[alice.y * 2]
```

queries

```
Var[alice[abs(x) * 2]]
alice[bob.y == 7]
```

can "query" another agent for the value of an expression using square brackets

hypotheticals

set up hypothetical world by
running statements

```
imagine[
  bob: chooses(y in Y, wpp=1),
  alice: observes [bob.y] is bob.y,
  alice[Pr[bob.x == 7]]
]
```

last line = expression to
evaluate in that world

memo calls

```
@memo
def f[x: X](a): …

@memo
def g():
  alice: chooses(x in X, wpp=f[x](3.14))
```

$\uparrow$

can reference one memo from another,
syntax evokes array indexing.
need to pass parameters, too!

cost reflection

```
@memo def f[…](a, b, c): …

cost @ f(3, 4, 5)
```

get number of FLOPs needed
to evaluate f
(note: no axes, params only!)

reference to Python variable

```
N = 5

@memo def f[…](…):
    return {N}
```

↑

use braces for inline reference
to a global Python variable

Things to do with a memo

Running a memo

call it like a function with params
(returns an array w/ prescribed axes)

↓

```
f(a, b)
```

pretty-print table of results

↓

```
f(a, b, print_table=True)
```

```
f(a, b, return_pandas=True)
f(a, b, return_xarray=True)
```

get outputs in other formats

save "comic book" visualization of model via graphviz

↓

```
f(a, b, save_comic="file")
```

Autodiff (useful for fitting)

```
@memo
def f[…](a, b): …
```

returns tuple of value + gradient wrt params a & b

$\downarrow$

```
jax.value_and_grad(f)(a, b)
```