Appendix

A. PROOF OF THE MAIN RESULT: The global classifier can be regarded as a summation of conditional probabilities. Suppose the local classifier for class $y = m$ obtained in node j is c_j^m. The aggregation process of the global model is derived as follows:

- *The global classifier:* $c^m(x) = P(y = m|x)$
- *The local classifier:* $c_j^m(x) = P(y = m|data \in Node_j, x)$
- *The distribution of in j-th node:* $f_x^j = P(x|data \in Node_j)$
- P_N^j can be estimated $P_N^j = \frac{n_j}{\sum_j n_j}$, where n_j is the sample size in j-th client.

$$
\begin{aligned}
c^m(x) &= P(y = m|x) && (1)\\
&= \sum_{j=1}^{N} P(y = m, data \in Node_j|x) && \text{(Total Probability Theorem)}\\
&= \sum_{j=1}^{N} \frac{P(y = m, data \in Node_j, x)}{f_x} && \text{(Bayes' Theorem and Normalisation)}\\
&= \sum_{j=1}^{N} \frac{P(y = m, data \in Node_j, x)}{P(data \in Node_j, x)} * \frac{P(data \in Node_j, x)}{f_x} && \text{(Rewriting using Conditional Probability)}\\
&= \sum_{j=1}^{N} P(y = m|data \in Node_j, x) * \frac{P(data \in Node_j) * P(x|data \in Node_j)}{f_x} && \text{(Expanding Conditional Probability)}\\
&= \sum_{j=1}^{N} c_j^m(x) * \frac{f_x^j P_N^j}{\sum_{j=1}^{N} f_x^j P_N^j} && \text{(Let } \omega_j = \tfrac{f_x^j P_N^j}{\sum_{j=1}^{J} f_x^j P_N^j}\text{. Substituting Local Classifier and Node Distribution)}\\
&= \sum_{j=1}^{J} 1(m \in \mathcal{M}_j)\omega_j \cdot c_j^m(x). && \text{(Weighted sum over local predictions)}
\end{aligned}
$$

Here, $\omega_j = \frac{f_x^j P_N^j}{\sum_{j=1}^{J} f_x^j P_N^j}$ is used to compute the ensemble weight $\omega_{j,m}$, and $\omega_{j,m} = 1(m \in M_j)\omega_j$. The following cross-entropy loss function is minimised when $\omega_j \geq (c_j^m)^{J-1}/J, c^m(x) = \sum_{j=1}^{J} 1(m \in \mathcal{M}_j)\omega_j c_j^m(x)$, which has been proved in work [33]. In this paper, the density estimate f_x^j is obtained using a Gaussian Mixture Model (GMM) [51].

$$
c^0, ..., c^m = \arg\min_{c^0,...,c^m} \left\{ -\sum_{j=1}^{J}\sum_{i=1}^{n_j}\sum_{m=0}^{M} 1(y_{ij} = m) \log c^m(x_{ij}) \right\} \quad s.t. \sum_{m=0}^{M} c^m = 1, \tag{2}
$$

where the function $1(y_{ij} = m)$ is the indicator function that equals 1 when $y_{ij} = m$, otherwise 0.

B. FEATURE IMPORTANCE ANALYSIS:

TABLE I: THE 165 INFLUENTIAL HEART SOUND FEATURES SELECTED THROUGH FEATURE IMPORTANCE ANALYSIS.

Feature Name	Shap value	weight	gain	cover	total_gain	total_cover
udspecRasta_lengthL1norm_sma_de_stddevRisingSlope numeric	0.1749088	4	59.07317352	1330	236.2926941	5320
fcc_sma[5]_peakMeanRel numeric	0.066608705	2	33.78628922	908	67.57257843	1816
fcc_sma[4]_percentile99.0 numeric	0.027021766	1	9.735995293	1330	9.735995293	1330
cm_fftMag_spectralSkewness_sma_meanFallingSlope numeric	0.01917158	1	5.615310669	1330	5.615310669	1330
cm_fftMag_spectralSlope_sma_risetime numeric	0.016092975	1	3.445723057	1330	3.445723057	1330
udSpec_Rfilt_sma_de[22]_quartile3 numeric	0.015471268	2	6.0129776	374	12.0259552	748
cm_fftMag_spectralFlux_sma_de_quartile2 numeric	0.014841603	1	6.25514555	747	6.25514555	747
udspec_lengthL1norm_sma_de_lpc0 numeric	0.014638417	1	13.98928738	328	13.98928738	328
udSpec_Rfilt_sma[11]_risetime numeric	0.014299023	3	1.715965867	611	5.14789772	1833
udSpec_Rfilt_sma_de[23]_quartile3 numeric	0.014214776	1	9.673725128	679	9.673725128	679
fcc_sma[4]_iqr1-3 numeric	0.013853458	2	7.019974709	430	14.03994942	860
oicingFinalUnclipped_sma_flatness numeric	0.013512484	1	8.57629776	498	8.57629776	498
udSpec_Rfilt_sma[0]_quartile2 numeric	0.013094406	1	2.308807135	1330	2.308807135	1330
oicingFinalUnclipped_sma_lpc0 numeric	0.01280389	1	7.025602341	596	7.025602341	596
cm_fftMag_spectralHarmonicity_sma_percentile1.0 numeric	0.012099205	1	11.92963409	258	11.92963409	258
cm_fftMag_spectralCentroid_sma_skewness numeric	0.011240978	2	0.81099081	942	1.621981621	1884
fcc_sma[3]_peakMeanAbs numeric	0.011181817	1	14.97934723	451	14.97934723	451
udSpec_Rfilt_sma_de[13]_stddevRisingSlope numeric	0.010769798	1	1.756378412	1330	1.756378412	1330
fcc_sma[3]_iqr2-3 numeric	0.010016562	1	5.267727852	738	5.267727852	738
cm_fftMag_spectralSkewness_sma_de_lpgain numeric	0.009126852	1	1.210110903	1330	1.210110903	1330
oicingFinalUnclipped_sma_lpgain numeric	0.007829903	1	2.564095974	611	2.564095974	611
udSpec_Rfilt_sma_de[2]_risetime numeric	0.007798587	1	1.246006727	1330	1.246006727	1330
cm_RMSenergy_sma_peakRangeAbs numeric	0.007496908	1	7.285607815	734	7.285607815	734
cm_fftMag_spectralCentroid_sma_minRangeRel numeric	0.007475868	1	0.660296619	1330	0.660296619	1330
cm_fftMag_spectralVariance_sma_flatness numeric	0.007420569	1	2.622623205	775	2.622623205	775
fcc_sma[3]_amean numeric	0.006865831	1	3.782421112	393	3.782421112	393
udSpec_Rfilt_sma[2]_linregerrQ numeric	0.006835031	1	0.912325859	1326	0.912325859	1326
cm_RMSenergy_sma_flatness numeric	0.006573637	1	6.422821045	749	6.422821045	749
udSpec_Rfilt_sma[5]_quartile3 numeric	0.006294543	1	0.478050798	1321	0.478050798	1321
udSpec_Rfilt_sma[5]_iqr1-2 numeric	0.005954946	1	1.598445177	1111	1.598445177	1111
fcc_sma[12]_iqr2-3 numeric	0.005753407	1	6.725850105	583	6.725850105	583
fcc_sma[13]_lpgain numeric	0.005692956	1	0.502746999	1298	0.502746999	1298
cm_fftMag_fband250-650_sma_de_peakDistStddev numeric	0.005576141	1	0.389642864	1330	0.389642864	1330
udSpec_Rfilt_sma_de[24]_quartile2 numeric	0.005573068	2	1.020026922	1330	2.040053844	2660
udSpec_Rfilt_sma[6]_quartile3 numeric	0.005310398	1	5.443786621	434	5.443786621	434

Table I (continued)

Feature Name	Shap value	weight	gain	cover	total_gain	total_cover
udspecRasta_lengthL1norm_sma_de_iqr1-2 numeric	0.005278943	1	1.099442482	1018	1.099442482	1018
fcc_sma[5]_lpgain numeric	0.005124319	1	1.139160156	424	1.139160156	424
udSpec_Rfilt_sma_de[13]_meanRisingSlope numeric	0.005056385	1	0.394382507	1280	0.394382507	1280
fcc_sma_de[3]_kurtosis numeric	0.004974036	1	2.690096855	819	2.690096855	819
fcc_sma_de[2]_percentile1.0 numeric	0.00495234	1	0.621264398	790	0.621264398	790
cm_fftMag_fband250-650_sma_linregc1 numeric	0.004844865	1	0.281745851	1312	0.281745851	1312
fcc_sma_de[2]_skewness numeric	0.004828318	1	0.945549786	770	0.945549786	770
udspec_lengthL1norm_sma_meanSegLen numeric	0.004793007	1	1.342338324	979	1.342338324	979
udSpec_Rfilt_sma[6]_meanSegLen numeric	0.004744658	1	0.997637093	1169	0.997637093	1169
udSpec_Rfilt_sma[0]_risetime numeric	0.004732204	1	3.225561857	74	3.225561857	74
fcc_sma[2]_maxSegLen numeric	0.004728207	1	0.519239247	1327	0.519239247	1327
udSpec_Rfilt_sma_de[14]_stddevRisingSlope numeric	0.00470226	1	1.475333691	391	1.475333691	391
fcc_sma_de[2]_quartile2 numeric	0.004379132	1	1.170669794	1303	1.170669794	1303
fcc_sma_de[11]_peakDistStddev numeric	0.004351366	1	0.770152211	1330	0.770152211	1330
fcc_sma_de[9]_peakDistStddev numeric	0.004338105	1	0.388319731	1325	0.388319731	1325
cm_fftMag_spectralVariance_sma_linregc2 numeric	0.004324389	1	2.174813986	879	2.174813986	879
fcc_sma[1]_quartile1 numeric	0.004088204	2	2.74508667	351.5	5.49017334	703
udSpec_Rfilt_sma[6]_iqr2-3 numeric	0.004022839	1	1.191879869	1294	1.191879869	1294
cm_fftMag_psySharpness_sma_minRangeRel numeric	0.003896863	1	0.401606768	1062	0.401606768	1062
udspecRasta_lengthL1norm_sma_meanSegLen numeric	0.003853667	1	0.445445478	1295	0.445445478	1295
udSpec_Rfilt_sma_de[3]_leftctime numeric	0.003800792	1	2.882632017	592	2.882632017	592
fcc_sma[10]_peakRangeRel numeric	0.003772016	2	1.125457048	1041.5	2.250914097	2083
fcc_sma_de[5]_lpc1 numeric	0.003659269	1	2.361129761	923	2.361129761	923
cm_fftMag_spectralSkewness_sma_lpc0 numeric	0.003499466	1	3.718276978	523	3.718276978	523
udSpec_Rfilt_sma_de[12]_stddevFallingSlope numeric	0.003484988	1	1.881630421	407	1.881630421	407
fcc_sma[2]_upleveltime50 numeric	0.003321341	1	0.834810019	983	0.834810019	983
cm_fftMag_spectralSlope_sma_de_quartile3 numeric	0.003269716	1	2.736748695	569	2.736748695	569
fcc_sma_de[4]_pctlrange0-1 numeric	0.003264664	1	1.344154358	555	1.344154358	555
udSpec_Rfilt_sma[10]_meanSegLen numeric	0.003213854	1	0.999613822	1074	0.999613822	1074
cm_fftMag_spectralSkewness_sma_de_flatness numeric	0.003093224	1	3.515030384	127	3.515030384	127
cm_fftMag_spectralSkewness_sma_de_percentile99.0 numeric	0.00309266	1	1.360512137	950	1.360512137	950
udSpec_Rfilt_sma[10]_lpc1 numeric	0.003090039	1	0.669033051	888	0.669033051	888
fcc_sma[7]_linregerrQ numeric	0.003059623	1	0.362798691	1302	0.362798691	1302
fcc_sma[13]_iqr2-3 numeric	0.00305221	1	2.921410799	89	2.921410799	89
fcc_sma_de[9]_quartile2 numeric	0.002958984	1	0.74704951	901	0.74704951	901
fcc_sma[8]_range numeric	0.002835295	1	0.767194033	1327	0.767194033	1327
fcc_sma_de[13]_risetime numeric	0.002822805	1	1.911473036	60	1.911473036	60
cm_fftMag_spectralSlope_sma_linregc1 numeric	0.002801212	1	1.542387009	347	1.542387009	347
udSpec_Rfilt_sma[11]_segLenStddev numeric	0.002742412	1	0.933002472	385	0.933002472	385
fcc_sma_de[7]_pctlrange0-1 numeric	0.002739604	1	1.344755292	506	1.344755292	506
udSpec_Rfilt_sma_de[6]_percentile1.0 numeric	0.002661523	1	1.024646759	1330	1.024646759	1330
udSpec_Rfilt_sma[15]_peakRangeRel numeric	0.001648004	1	0.53542912	263	0.53542912	263
fcc_sma[2]_linregc1 numeric	0.001599217	1	0.394384265	600	0.394384265	600
cm_fftMag_psySharpness_sma_linregc1 numeric	0.001595959	1	0.334819168	1324	0.334819168	1324
udSpec_Rfilt_sma[7]_leftctime numeric	0.001591305	1	1.373440266	44	1.373440266	44
fcc_sma[3]_upleveltime90 numeric	0.001587785	1	2.038755417	760	2.038755417	760
fcc_sma[5]_rqmean numeric	0.001532889	2	0.726613462	1266.5	1.453226924	2533
fcc_sma[10]_skewness numeric	0.001477398	1	0.527558625	324	0.527558625	324
cm_fftMag_spectralKurtosis_sma_de_flatness numeric	0.001469919	1	0.467760682	1241	0.467760682	1241
udSpec_Rfilt_sma[25]_upleveltime50 numeric	0.001460258	1	0.302553505	1326	0.302553505	1326
fcc_sma_de[5]_lpc4 numeric	0.001403587	1	0.539424777	1291	0.539424777	1291
udSpec_Rfilt_sma_de[10]_upleveltime90 numeric	0.001380694	1	1.193989992	23	1.193989992	23
cm_fftMag_spectralRollOff75.0_sma_de_stddevFallingSlope numeric	0.001334538	1	0.547998667	798	0.547998667	798
oicingFinalUnclipped_sma_range numeric	0.00132699	1	1.335298538	31	1.335298538	31
udSpec_Rfilt_sma[6]_pctlrange0-1 numeric	0.001314231	1	0.401833385	1185	0.401833385	1185
cm_fftMag_fband250-650_sma_de_range numeric	0.001303879	1	0.647293746	507	0.647293746	507
oicingFinalUnclipped_sma_de_quartile3 numeric	0.001280073	1	0.825291157	49	0.825291157	49
udSpec_Rfilt_sma[17]_lpc3 numeric	0.001216713	1	0.705320001	55	0.705320001	55
fcc_sma[6]_qregc1 numeric	0.001205926	1	0.807898402	100	0.807898402	100
cm_fftMag_fband1000-4000_sma_de_minPos numeric	0.00111533	1	0.484175861	522	0.484175861	522
udSpec_Rfilt_sma_de[25]_minSegLen numeric	0.001072463	1	1.136362314	33	1.136362314	33
udSpec_Rfilt_sma_de[6]_lpc4 numeric	0.001038906	1	0.51261425	532	0.51261425	532
cm_fftMag_spectralSlope_sma_minPos numeric	0.001018498	1	1.17227602	775	1.17227602	775
udSpec_Rfilt_sma_de[22]_skewness numeric	0.000986683	1	0.63786608	796	0.63786608	796
udspec_lengthL1norm_sma_de_pctlrange0-1 numeric	0.00096522	1	0.421422035	44	0.421422035	44
udSpec_Rfilt_sma_de[3]_quartile2 numeric	0.000914101	1	0.789074838	27	0.789074838	27
cm_fftMag_fband1000-4000_sma_qregc3 numeric	0.000913065	1	0.399933308	712	0.399933308	712
udSpec_Rfilt_sma_de[7]_upleveltime75 numeric	0.000890098	1	0.495113492	1330	0.495113492	1330
fcc_sma[12]_stddevFallingSlope numeric	0.000889177	1	0.514449418	41	0.514449418	41
cm_fftMag_spectralFlux_sma_lpc0 numeric	0.000887822	1	0.672494352	83	0.672494352	83
udSpec_Rfilt_sma_de[17]_peakRangeRel numeric	0.000886993	1	0.509827614	34	0.509827614	34
udSpec_Rfilt_sma_de[0]_maxPos numeric	0.000876428	1	0.861174822	16	0.861174822	16
cm_RMSenergy_sma_de_stddevFallingSlope numeric	0.000875804	1	0.281979769	1330	0.281979769	1330
udSpec_Rfilt_sma[9]_percentile1.0 numeric	0.000866589	1	0.848887801	93	0.848887801	93
cm_fftMag_spectralFlux_sma_lpc4 numeric	0.000853217	1	0.259635895	117	0.259635895	117
cm_fftMag_spectralFlux_sma_peakMeanRel numeric	0.000839713	1	0.686322689	433	0.686322689	433
udSpec_Rfilt_sma_de[4]_quartile2 numeric	0.000838053	1	0.496914715	32	0.496914715	32
cm_fftMag_spectralEntropy_sma_peakDistStddev numeric	0.000832948	1	0.411569834	208	0.411569834	208
udSpec_Rfilt_sma_de[2]_quartile2 numeric	0.000829766	1	0.520026982	24	0.520026982	24
udSpec_Rfilt_sma[12]_upleveltime90 numeric	0.000786718	1	0.829185367	27	0.829185367	27
cm_zcr_sma_de_peakRangeRel numeric	0.00076202	1	0.883337021	1330	0.883337021	1330
udspecRasta_lengthL1norm_sma_quartile1 numeric	0.000733587	1	0.555594325	16	0.555594325	16
cm_fftMag_spectralSkewness_sma_qregc3 numeric	0.000690544	1	0.295283973	27	0.295283973	27
udSpec_Rfilt_sma_de[12]_peakRangeRel numeric	0.00067913	1	0.487163782	35	0.487163782	35
udSpec_Rfilt_sma[13]_lpgain numeric	0.000065719	1	0.308226794	1295	0.308226794	1295
cm_fftMag_spectralVariance_sma_range numeric	0.000638415	1	0.73640269	1119	0.73640269	1119
cm_fftMag_spectralKurtosis_sma_peakMeanMeanDist numeric	0.000616125	1	0.413334399	1330	0.413334399	1330
udSpec_Rfilt_sma[19]_minPos numeric	0.0005893	1	0.760017276	1330	0.760017276	1330
udspec_lengthL1norm_sma_de_meanRisingSlope numeric	0.000573997	1	0.358050197	1330	0.358050197	1330
cm_fftMag_spectralKurtosis_sma_linregc1 numeric	0.00056523	1	0.284590483	1330	0.284590483	1330
udspecRasta_lengthL1norm_sma_de_lpc0 numeric	0.000546952	1	0.879844904	12	0.879844904	12
fcc_sma[2]_lpc2 numeric	0.000519563	1	0.439446568	23	0.439446568	23
udSpec_Rfilt_sma[0]_maxPos numeric	0.000489432	1	0.514279604	1330	0.514279604	1330
udspec_lengthL1norm_sma_leftctime numeric	0.000487373	1	0.960110188	23	0.960110188	23
udSpec_Rfilt_sma[8]_minRangeRel numeric	0.000476519	1	0.255015016	26	0.255015016	26
cm_RMSenergy_sma_iqr1-2 numeric	0.000443091	1	0.301709265	1330	0.301709265	1330
cm_RMSenergy_sma_upleveltime90 numeric	0.000413161	1	0.188143015	19	0.188143015	19
cm_fftMag_spectralRollOff75.0_sma_upleveltime75 numeric	0.000366831	1	0.074845433	31	0.074845433	31
udspecRasta_lengthL1norm_sma_maxPos numeric	0.000324066	1	0.175449252	10	0.175449252	10
fcc_sma[6]_minPos numeric	0.000319397	1	0.101224005	18	0.101224005	18
udspec_lengthL1norm_sma_lpc0 numeric	0.000275772	2	0.0727164	6.5	0.1454328	13
udspec_lengthL1norm_sma_percentile99.0 numeric	0.000275314	1	0.221500084	8	0.221500084	8
udSpec_Rfilt_sma[7]_lpc4 numeric	0.000273154	1	0.122567415	14	0.122567415	14
udSpec_Rfilt_sma[4]_lpc0 numeric	0.000215668	1	0.021040797	14	0.021040797	14
udspec_lengthL1norm_sma_de_risetime numeric	0.000187452	1	0.08378467	9	0.08378467	9
udSpec_Rfilt_sma[16]_percentile1.0 numeric	0.000168175	1	0.068053588	9	0.068053588	9
udspec_lengthL1norm_sma_maxPos numeric	9.77E-05	1	0.018804565	8	0.018804565	8
udspec_lengthL1norm_sma_risetime numeric	8.27E-05	1	0.017634902	8	0.017634902	8