Dinámica de Fluidos Computacionales

Formalismo vorticidad - Lineas de corriente

Johan S. Méndez

Departamento de Fisica
Observatorio Astronómico Nacional

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UNIVERSIDAD NACIONAL DE COLOMBIA

FORTRAN

jupyter

Contenido

1. ¿Cómo se construye el modelo ?

$$\frac{\partial \vec{\omega}}{\partial t} + \vec{u} \cdot \nabla \vec{\omega} = \nu \nabla^2 \vec{w}$$

$$\vec{\omega} = \nabla \times \vec{v} \qquad \omega_z = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$$

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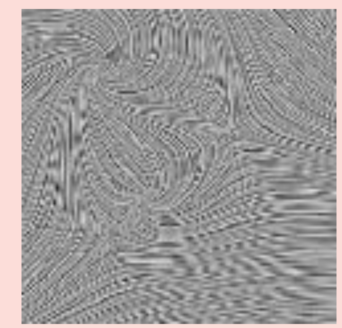
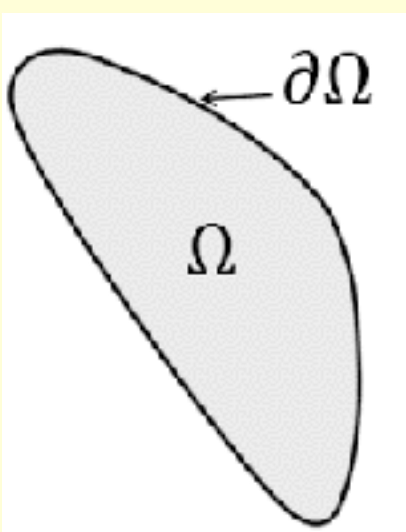
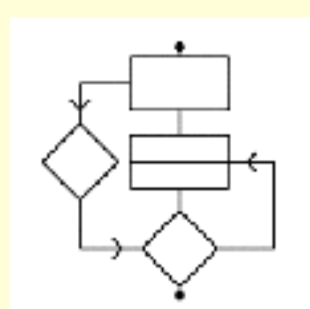


3. Ventajas y desventajas



Condición frontera

Programación



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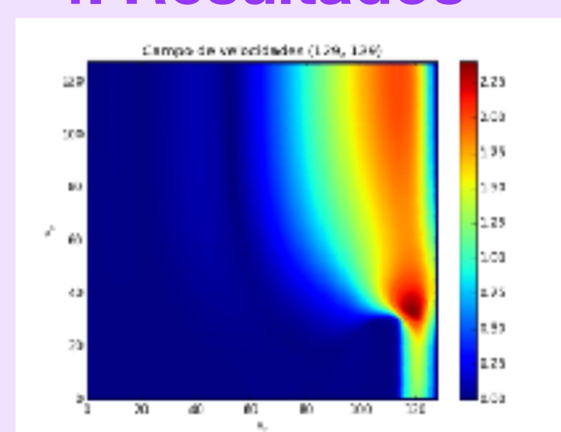

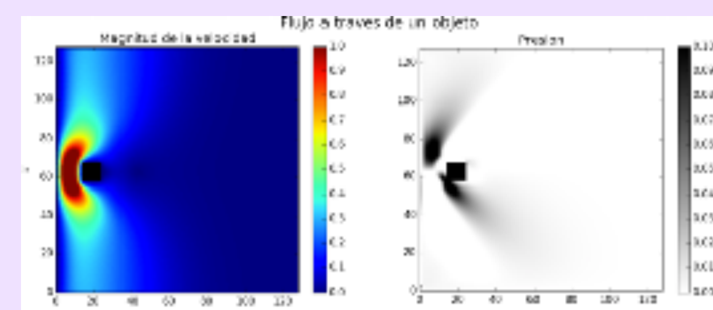

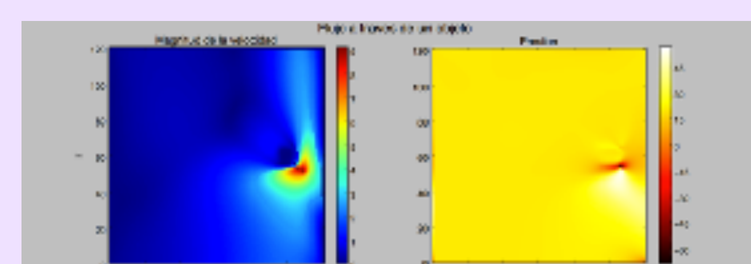

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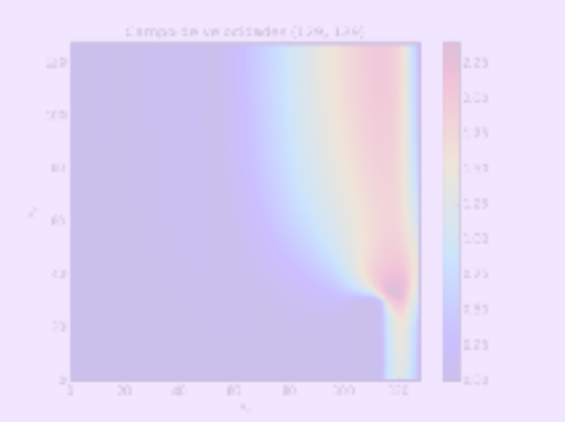
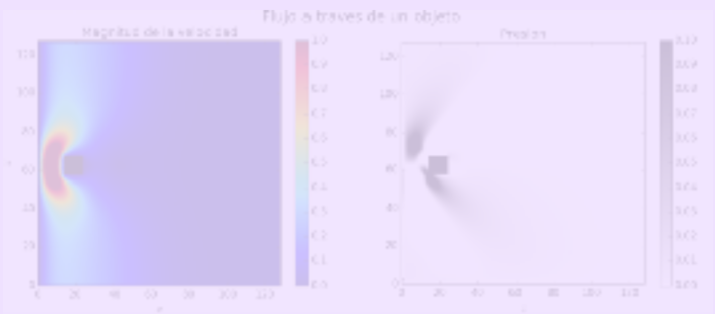
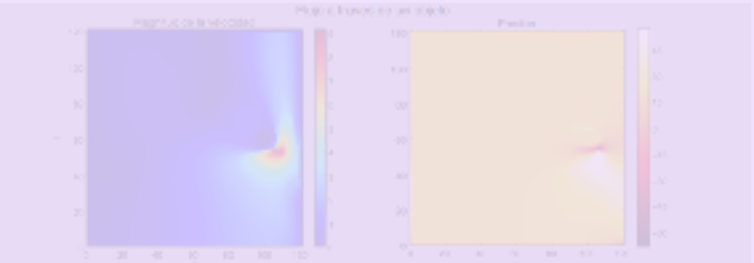






Potenciales

$$\vec{\omega} = \nabla \times \vec{v} \qquad \omega_z = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$$

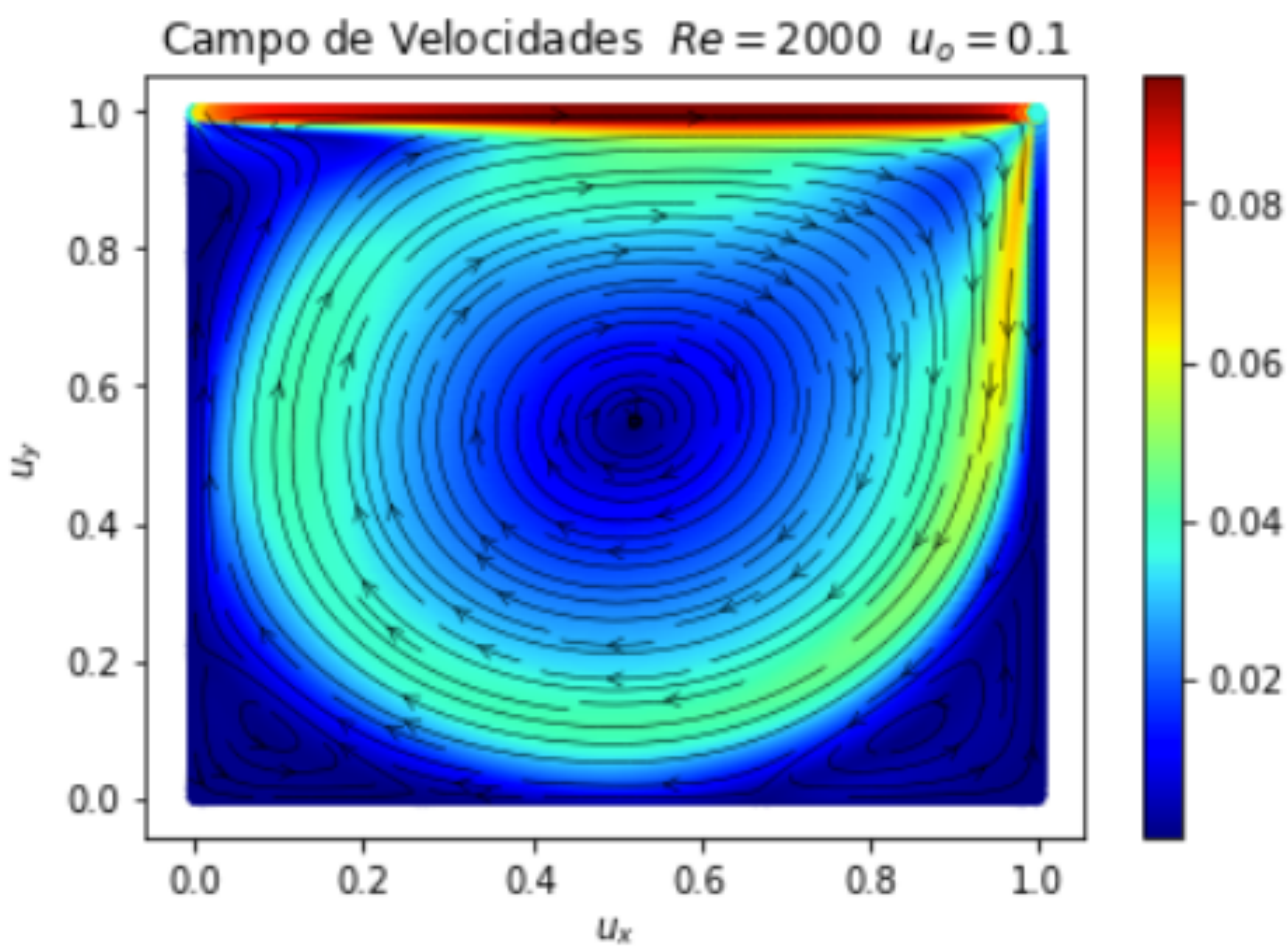
Vorticidad

$$u = \frac{\partial \psi}{\partial y} \qquad v = -\frac{\partial \psi}{\partial x}$$

Lineas de corriente

Exigimos

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$



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Lineas de corriente

Evolución temporal

$$\frac{\partial \vec{\omega}}{\partial t} + \vec{u} \cdot \nabla \vec{\omega} = \nu \nabla^2 \vec{w}$$

Relación

$$\omega = \nabla^2 \psi$$

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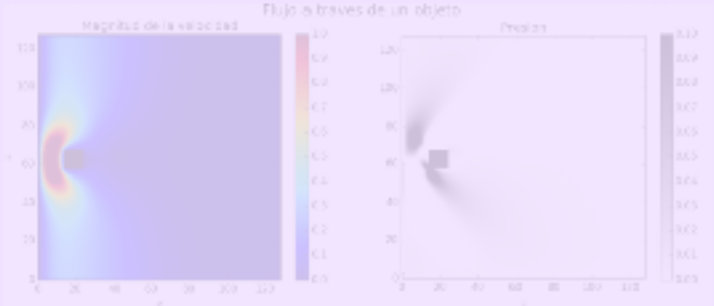

¿Qué se resolvió?

Navier Stokes 2D

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) - \frac{1}{\rho}\frac{\partial P}{\partial y}$$

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Desacople de la presión

No se necesita ecuación de estado

¿Qué se resolvió?

Desacople de la presión

$$\nabla^2 P = 2\rho \left(\frac{\partial^2 \psi}{\partial x^2} \frac{\partial^2 \psi}{\partial y^2} - \left(\frac{\partial^2 \psi}{\partial x \partial y} \right)^2 \right)$$

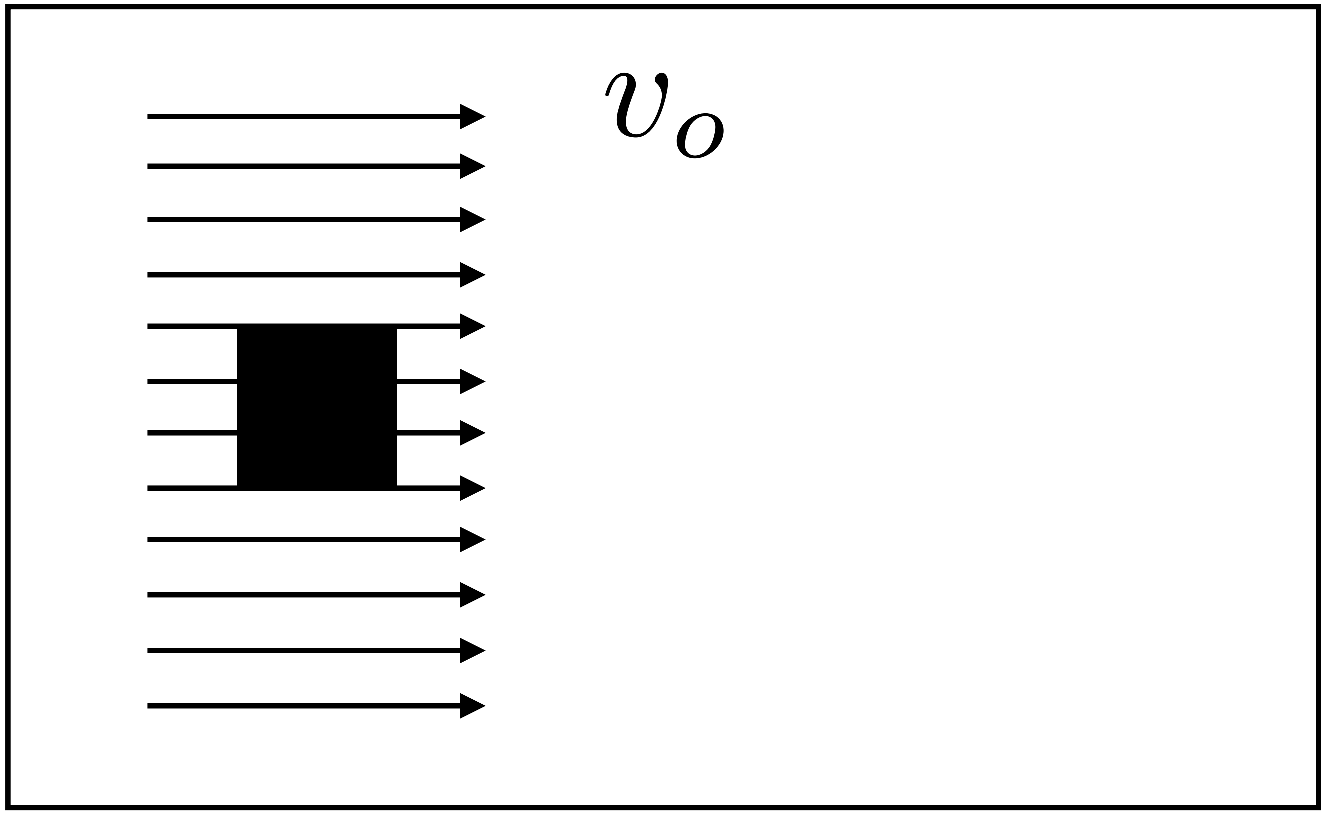
Evolución temporal

$$\frac{\partial \omega}{\partial t} = \nu \nabla^2 \omega - \left(\frac{\partial \psi}{\partial y} \frac{\partial \omega}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial w}{\partial y} \right)$$

Fuente de lineas de corriente

$$\omega = \nabla^2 \psi$$

Sistema



Diferencias finitas

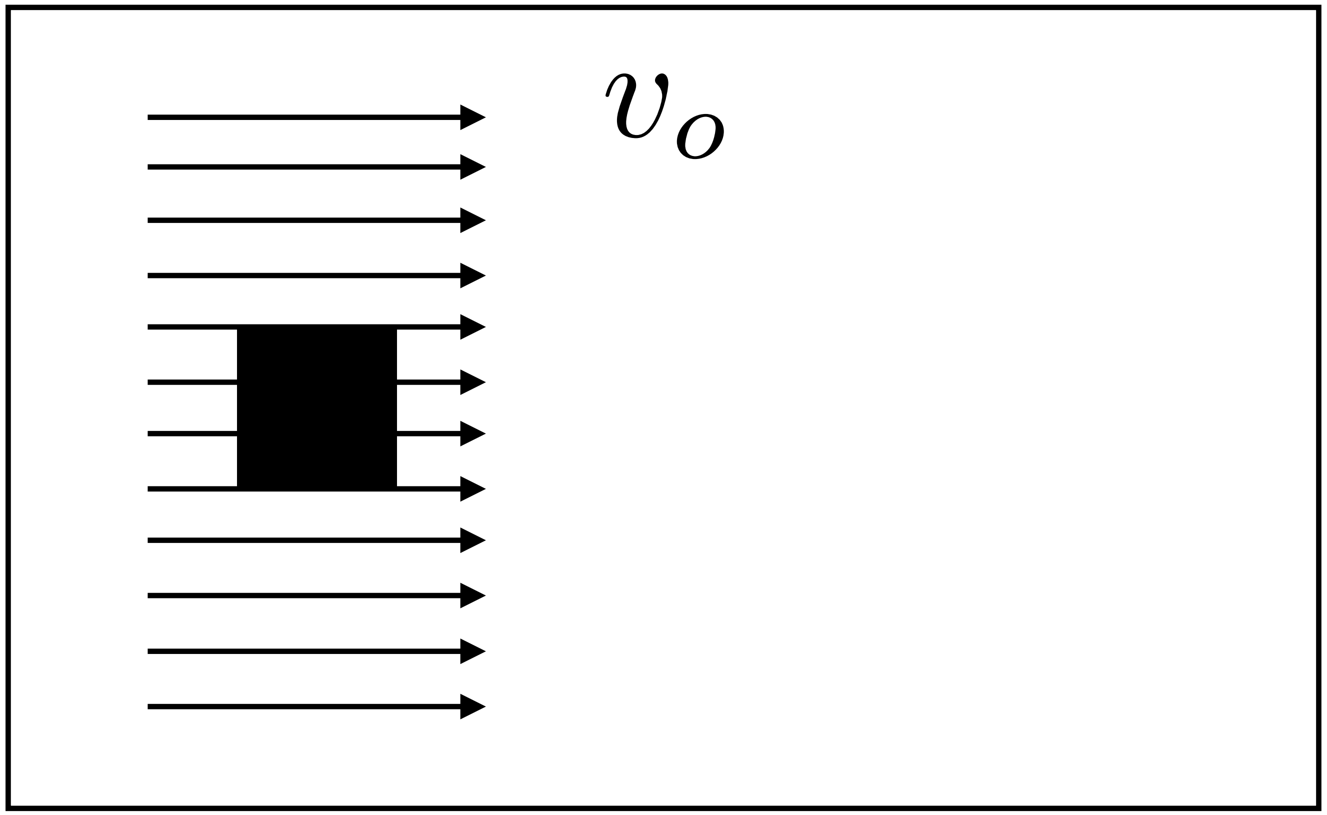
Malla regular

Euler adelante

Laplaciano lineas de corriente

$$\psi_{i,j}^{n+1} = \frac{\Delta x^2 \Delta y^2 w_{i,j}^{n+1} + \Delta y^2 (\psi_{i+1,j}^{n+1} + \psi_{i-1,j}^{n+1}) + \Delta x^2 (\psi_{i,j+1}^{n+1} + \psi_{i,j-1}^{n+1})}{2(\Delta x^2 + \Delta y^2)}$$

Sistema



$$\alpha_1 = 1 - 2\nu\Delta t\left(\frac{1}{\Delta x^2} + \frac{1}{\Delta y^2}\right)$$

$$\alpha_2 = \frac{\Delta t}{\Delta x}\left(\frac{\nu}{\Delta x} - \frac{\psi^n_{i,j+1} - \psi^n_{i,j-1}}{4\Delta y}\right)$$

$$\alpha_3 = \frac{\Delta t}{\Delta x}\left(\frac{\nu}{\Delta x} + \frac{\psi^n_{i,j+1} - \psi^n_{i,j-1}}{4\Delta y}\right)$$

$$\alpha_4 = \frac{\Delta t}{\Delta y}\left(\frac{\nu}{\Delta y} - \frac{\psi^n_{i+1,j} - \psi^n_{i-1,j}}{4\Delta x}\right) \quad \alpha_5 = \frac{\Delta t}{\Delta y}\left(\frac{\nu}{\Delta y} + \frac{\psi^n_{i+1,j} - \psi^n_{i-1,j}}{4\Delta x}\right)$$

Evolución temporal

$$\omega^{n+1}_{i,j} = \alpha_1\omega^n_{i,j} + \alpha_2\omega^n_{i+1,j} + \alpha_3\omega^n_{i-1,j} + \alpha_4\omega^n_{i,j+} + \alpha_5\omega^n_{i,j-1}$$

Sistema

Calculo de la presión

$$P_{i,j}^{n+1} = \frac{\Delta y^2 (P_{i+1,j}^{n+1} + P_{i-1,j}^{n+1}) + \Delta x^2 (P_{i,j+1}^{n+1} + P_{i,j-1}^{n+1})}{2(\Delta x^2 + \Delta y^2)}$$

$$+ \frac{\rho}{\Delta x^2 + \Delta y^2} \left[\frac{1}{16} (\psi_{i+1,j+1}^{n+1} - \psi_{i+1,j-1}^{n+1} - \psi_{i-1,j+1}^{n+1} + \psi_{i-1,j-1}^{n+1}) \right]$$

$$+ \frac{\rho}{\Delta x^2 + \Delta y^2} \left[(\psi_{i+1,j}^{n+1} - 2\psi_{i,j}^{n+1} + \psi_{i-1,j}^{n+1})(\psi_{i,j+1}^{n+1} - 2\psi_{i,j}^{n+1} + \psi_{i,j-1}^{n+1}) \right]$$

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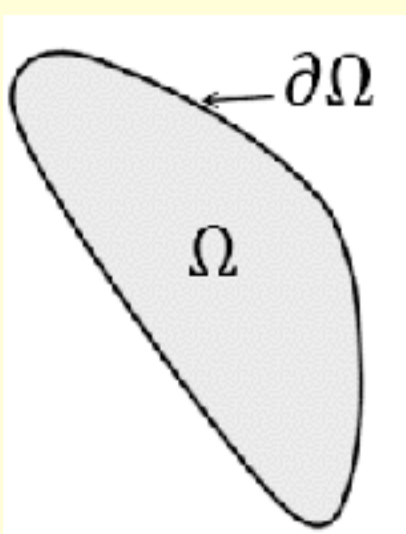

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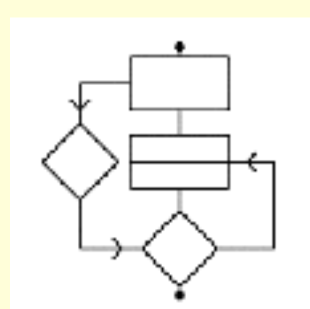

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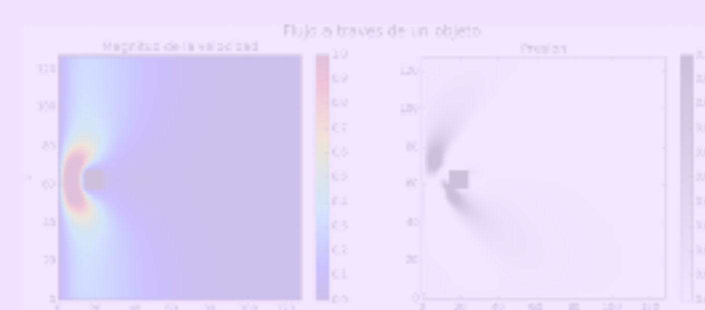


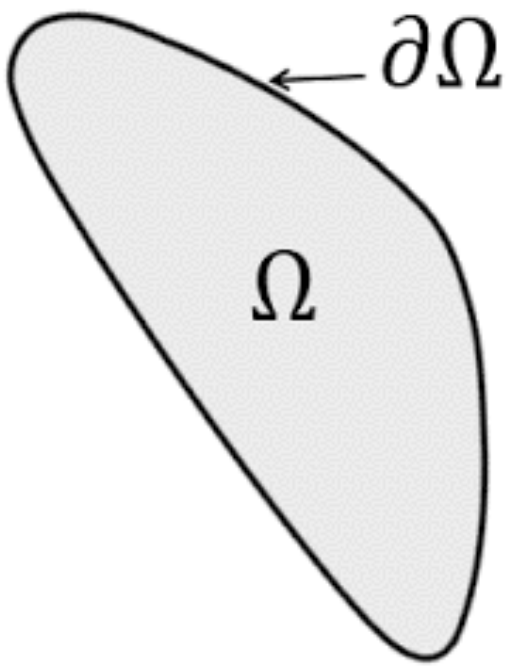
Ventajas

No se necesita ecuación de estado

Se puede calcular el perfil presión

Manejar potenciales es más fácil

Desventajas



Condiciones de frontera

Lineas de corriente nulas en la fronteras físicas

$$\psi = 0$$

Consideramos la entrada de fluido $\psi = u_o y$

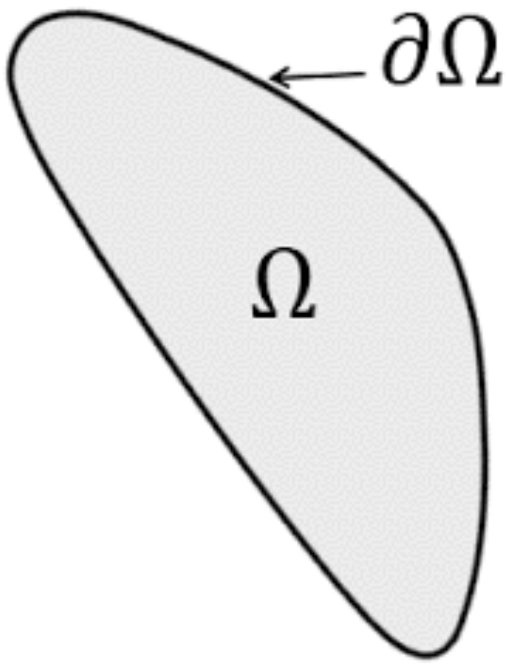
Que cumple

$$\frac{\partial \psi}{\partial x} = 0 \longrightarrow \psi_{i,j} = \frac{4\psi_{i-1,j} - \psi_{i-2,j}}{3}$$

También consideramos que nada cambia al lado derecho y arriba

$$\frac{\partial \psi}{\partial y} = 0 \longrightarrow \psi_{i,j} = \frac{4\psi_{i,j-1} + \psi_{i,j-2}}{3}$$

Desventajas



Condiciones de frontera

Físicamente no es intuitivo el valor de la Vorticidad en el objeto

$$\psi(x,y+h)=\psi(x,y)+h\cancel{\frac{\partial\psi}{\partial y}}+\frac{h^2}{2}\frac{\partial^2\psi}{\partial y^2}$$

$$\frac{\partial}{\partial y}\left(\frac{\partial\psi}{\partial y}\right)=\frac{\partial u}{\partial y}=\frac{2(\psi(x,y+h)-\psi(x,y))}{h^2}$$

$$\cancel{\frac{\partial v}{\partial x}}-\omega=\frac{2(\psi(x,y)-\psi(x,y+h))}{h^2}$$

$$\boxed{\omega_{i,j}=\frac{2(\psi_{i,j}-\psi_{i,j+1})}{\Delta y^2}}$$

Condición sobre las paredes físicas

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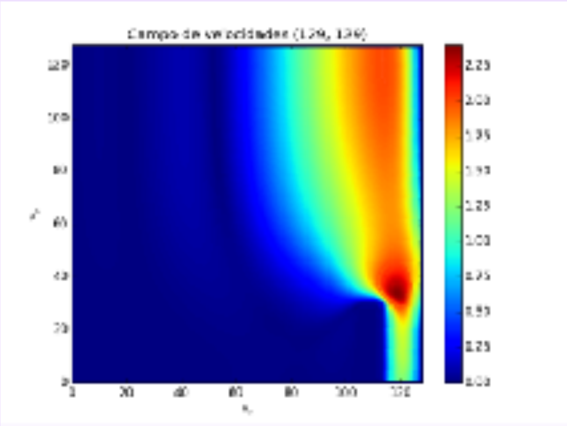

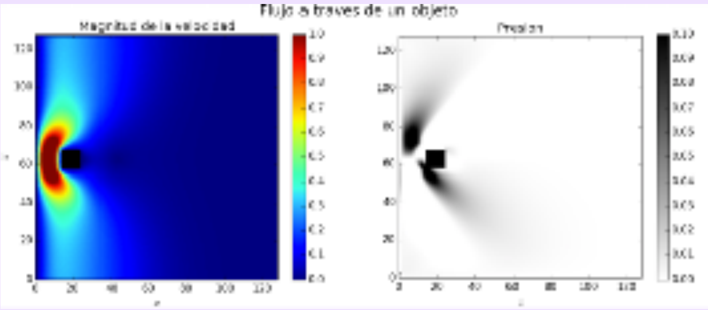

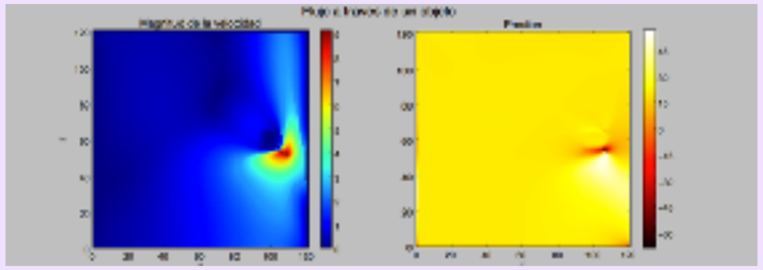

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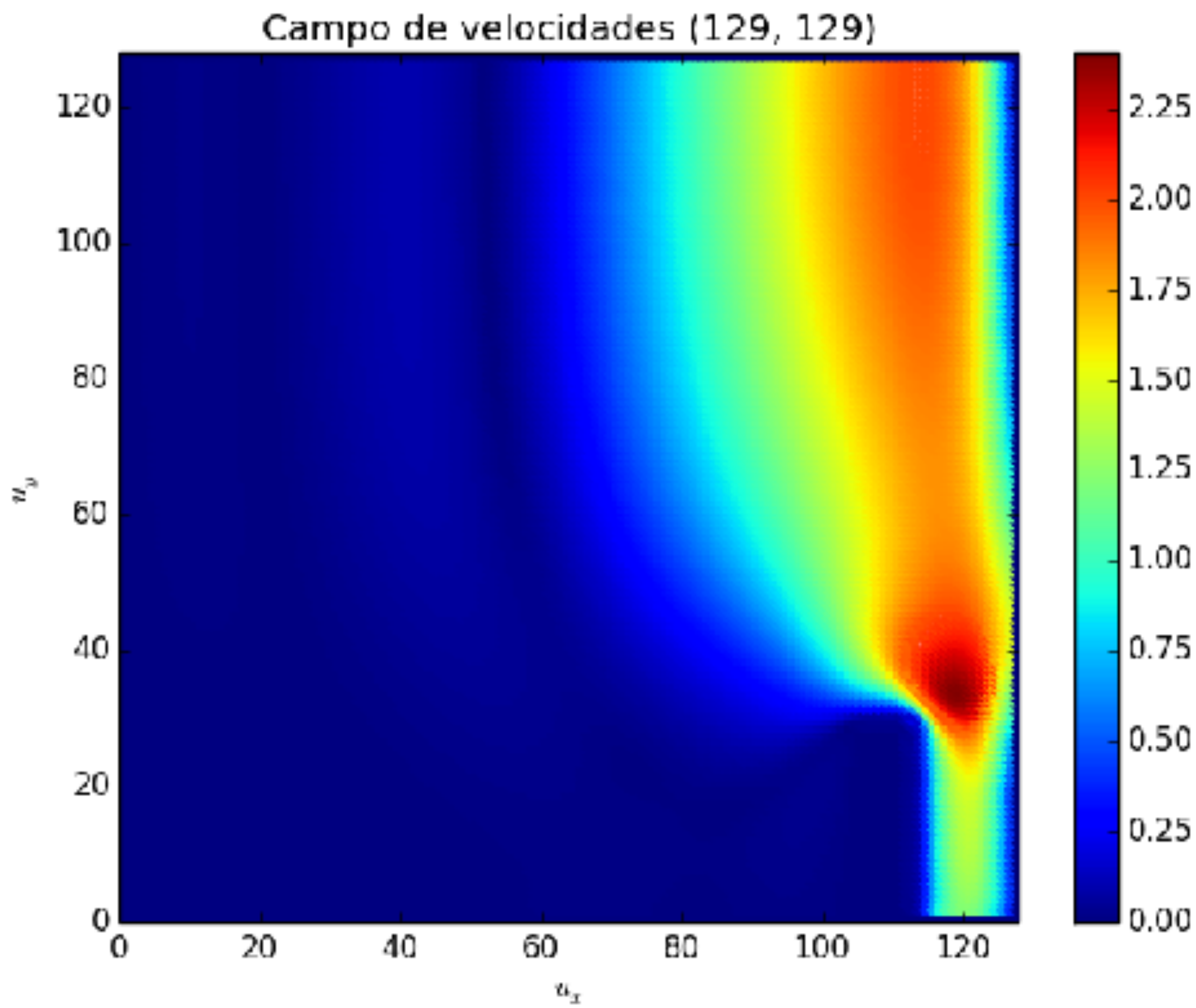
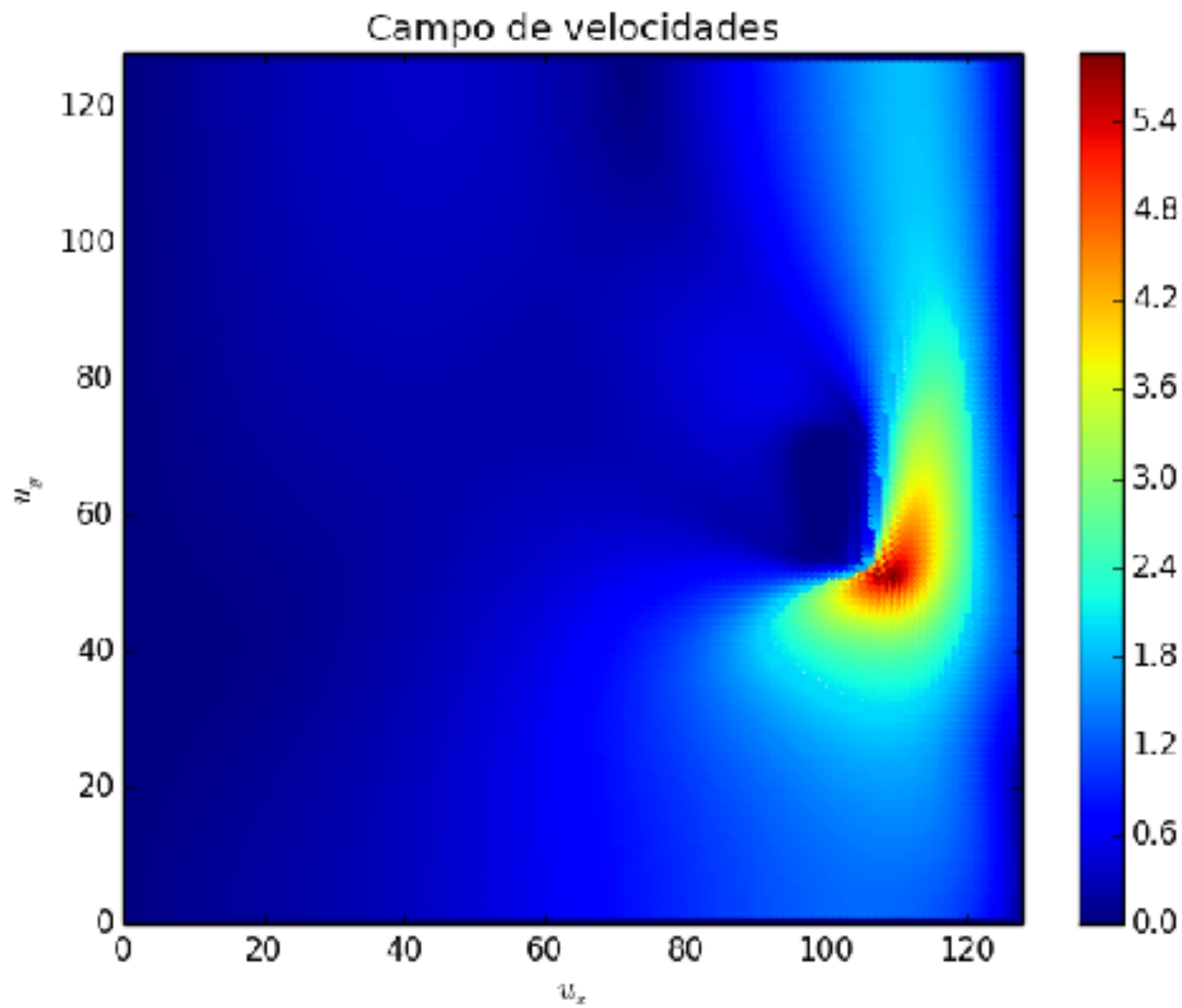
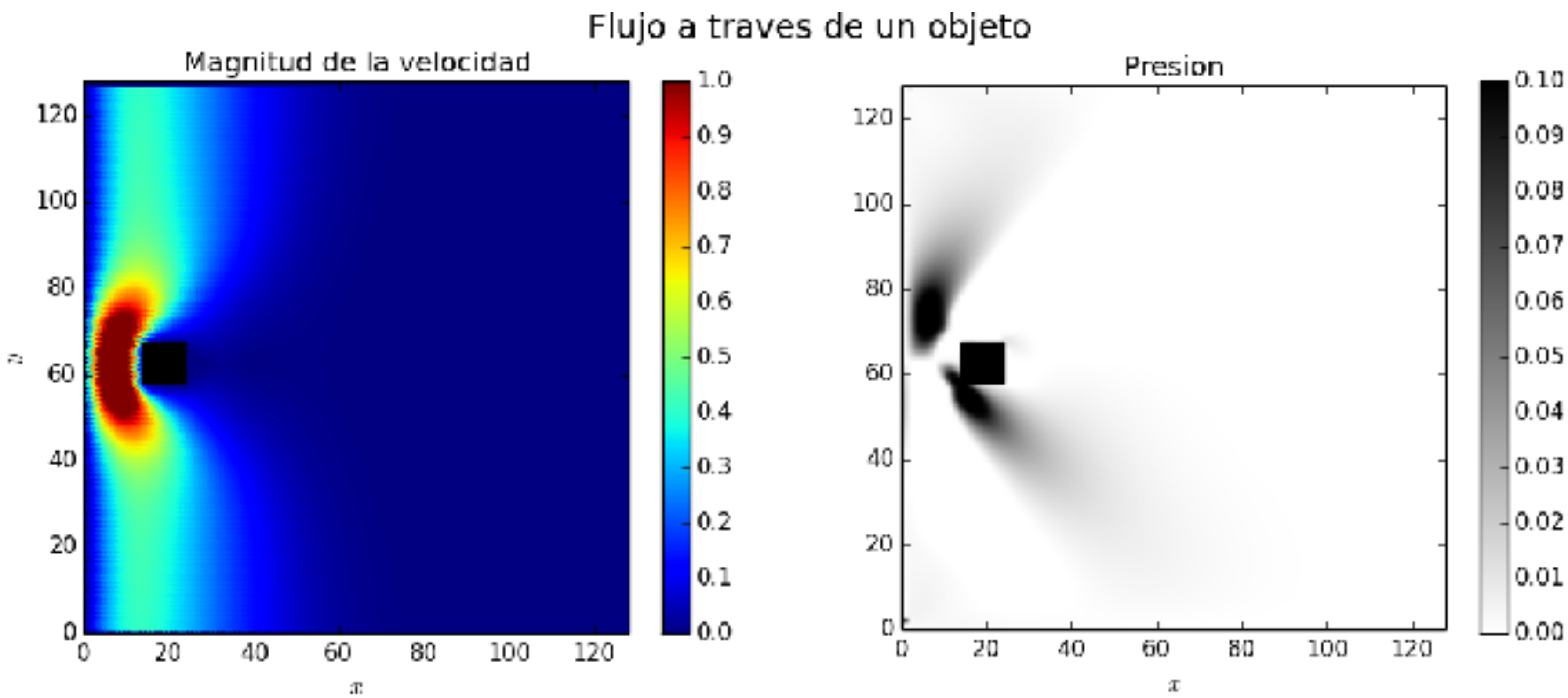




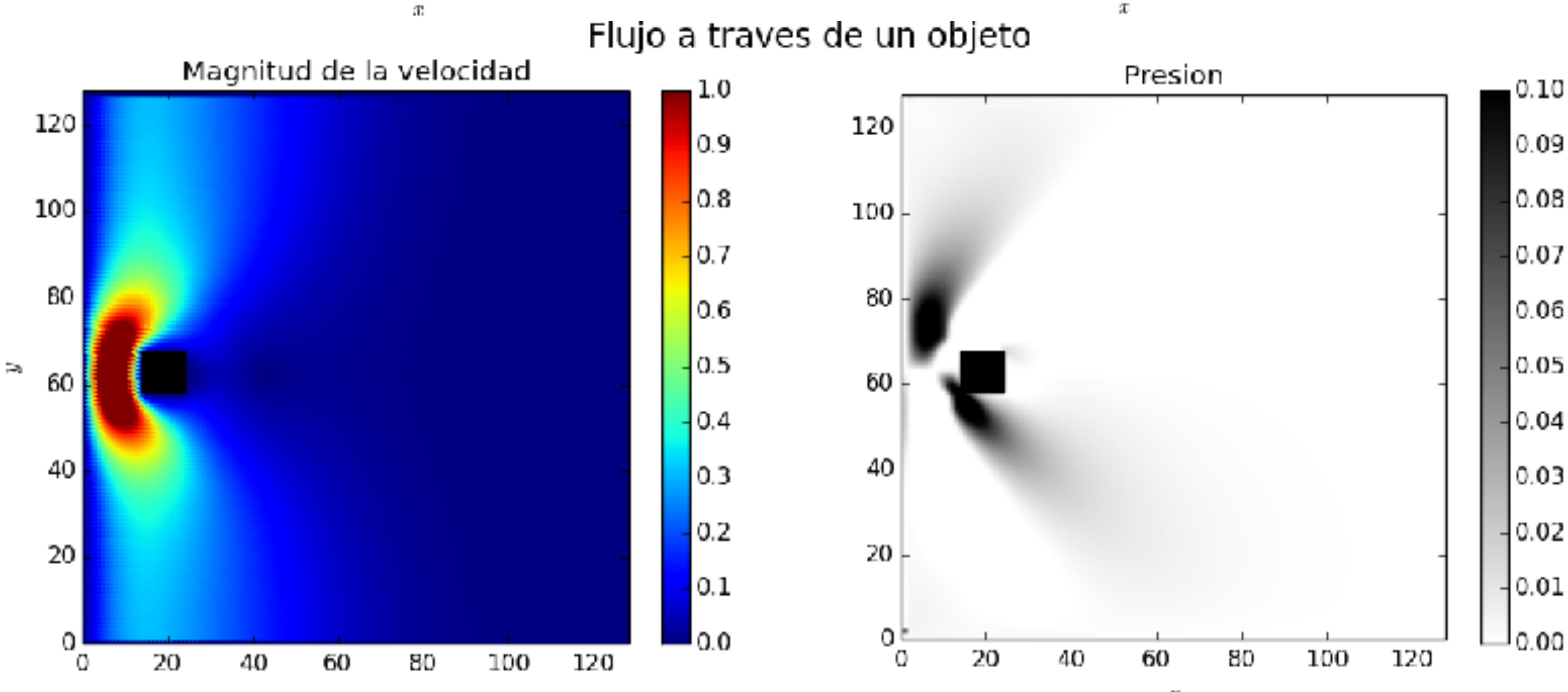
Flujo por la izquierda

Otra condición
Vorticidad

Ventajas



Flujo a traves de un objeto
Magnitud de la velocidad
Presion
x
y
0 20 40 60 80 100 120
1.0 0.9 0.8 0.7 0.6 0.5 0.4 0.3 0.2 0.1 0.0
0.10 0.09 0.08 0.07 0.06 0.05 0.04 0.03 0.02 0.01 0.00



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