# MHD

Modelo matemático

# Consideraciones

**Ecuacion de continuidad**

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{v}) = 0$$

**Ecuación de Euler**

$$\frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \nabla) = - \left( \frac{1}{\rho} \right) \nabla p + \vec{F}_{ext}$$

**Ecuaciones de Maxwell**

$$\nabla \cdot \vec{B} = 0 \qquad \nabla \cdot \vec{E} = \frac{q}{V}$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{E}}{\partial t} \qquad \nabla \times \vec{B} = \vec{J}$$

**No se consideran corrientes de desplazamiento, ademas :**

$$\mu = 1, \quad \epsilon = 1, \quad c = 1$$

**Unidades Lorentz-Heaviside**

# Evolution temporal del campo Magnético

## Ley de Ohm

$$\vec{J} = \sigma(\vec{E} + \vec{u} \times \vec{B}) \longrightarrow \nabla \times \vec{B} = \vec{J}$$

$$\frac{\partial \vec{B}}{\partial t} = \nabla \times (\vec{u} \times \vec{B}) + \eta_\rho \nabla^2 \vec{B}$$

$\nabla \times (\vec{u} \times \vec{B})$ **Como se transporta el campo magnético en el fluido**

$\eta_\rho \nabla^2 \vec{B}$ **Como se difunden las líneas de campo en el tiempo**

**Se calcula el campo magnético a través del vector potencial**

$$\nabla \times \vec{A} \longrightarrow \frac{\partial \vec{A}}{\partial t} = (\vec{u} \cdot \nabla)\vec{A} = \eta_\rho \nabla^2 \vec{A}$$

**Ecuación de Advección con difusión**

# Discretización de las ecuaciones

## Rotacional del Vector potencial

$$\vec{B} = \nabla \times \vec{A} \qquad B_x = \frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}$$

$$B^n_{x_{i,j,k}} = \frac{A^n_{z_{i,j+1,k}} - A^n_{z_{i,j-1,k}}}{2\Delta y} - \frac{A^n_{z_{i,j,k+1}} - A^n_{z_{i,j,k-1}}}{2\Delta y}$$

**Utilizando la discretización central de primer orden**

## Advección y difusión

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial x} = \eta_\nu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial x} = \eta_\nu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right)$$

$$\frac{\partial u}{\partial t} = \frac{u^{n+1}_{i,j} - u^n_{i,j}}{\Delta t}$$

$$\frac{\partial u}{\partial x} = \frac{u^n_{i,j} - u^n_{i-1,j}}{\Delta x} = \alpha_{u,x}$$

$$\frac{\partial^2 u}{\partial x^2} = \frac{u^n_{i+1,j} - 2u^n_{i,j} + u^n_{i-1,j}}{\Delta x^2} = D_{u,x}$$

$$u^{n+1}_{i,j} = u^n_{i,j} - u^n_{i,j}\alpha_{u,x}\Delta t - v^n_{i,j}\alpha_{u,x}\Delta t + \eta_\nu(D_{u,x}\Delta t + D_{u,y}\Delta t)$$

$$v^{n+1}_{i,j} = v^n_{i,j} - u^n_{i,j}\alpha_{v,x}\Delta t - v^n_{i,j}\alpha_{v,x}\Delta t + \eta_\nu(D_{v,x}\Delta t + D_{v,y}\Delta t)$$