$$\mathcal{L}\{-e^{-bt}u(-t)\} = \int_{-\infty}^{\infty} -e^{-bt}u(-t)e^{-st}\,dt = \int_{-\infty}^{0} -e^{-bt}e^{-st}\,dt = -\int_{-\infty}^{0} e^{-t(s+b)}\,dt$$

$$u(-t) = \begin{cases} 0, & t>0 \\ 1, & t\leq 0 \end{cases}$$

$$= \frac{-1}{-(s+b)} e^{-t(s+b)} \Big|_{-\infty}^{0} = \frac{1}{s+b}\left[e^{0} - \lim_{t\to-\infty} e^{-t(s+b)}\right]$$

ROC: $Re\{s\}+b<0 \Rightarrow Re\{s\}<-b \Rightarrow \lim_{t\to-\infty} e^{-t(s+b)} = 0$

Si $s+b \geq 0$ no converge

En el ROC: $\mathcal{L}\{-e^{-bt}u(-t)\} = \frac{1}{s+b}[1-0] = \frac{1}{s+b}$

$Re\{s\}=-b$ desde $\mathcal{L}\{e^{-bt}u(t)\} = \frac{1}{s+b}\left[1 - \lim_{t\to\infty} e^{-t(s+b)}\right]$

$s = \sigma + j\omega$; $\sigma \in \mathbb{R}$

$$\mathcal{L}\{e^{-bt}u(t)\} = \frac{1}{s+b}\left[1 - \lim_{t\to\infty} e^{-t(\sigma+j\omega+b)}\right] = \frac{1}{s+b}\left[1 - \lim_{t\to\infty} e^{-tj\omega}e^{-t(\sigma+b)}\right]$$

$$e^{-tj\omega} = \cos(\omega t) - j\,sen(\omega t)$$

$$= \frac{1}{s+b}\left\{1 - \lim_{t\to\infty}[\cos(\omega t) - j\,sen(\omega t)]e^{-t(\sigma+b)}\right\}$$ Si $Re\{s\}=-b \Rightarrow \sigma=-b$

$$= \frac{1}{s+b}\left\{1 - \lim_{t\to\infty}[\cos(\omega t) - j\,sen(\omega t)]e^{-t(-b+b)}\right\} = \frac{1}{s+b}\left\{1 - \lim_{t\to\infty}[\cos(\omega t) - j\,sen(\omega t)]e^{0}\right\}$$

$$= \frac{1}{s+b}\left[1 - \lim_{t\to\infty} \cos(\omega t) - j\,sen(\omega t)\right]$$

Ahora $t = \frac{2\pi n}{\omega} \Rightarrow t \to \infty \Rightarrow n \to \infty$

$$\lim_{t \to \infty} \cos(\omega t) - j\,\text{sen}(\omega t) = \lim_{n \to \infty} \overbrace{\cos\left(\cancel{\omega}\frac{2\pi n}{\cancel{\omega}}\right)}^{1} - j\,\overbrace{\text{sen}\left(\cancel{\omega}\frac{2\pi n}{\cancel{\omega}}\right)}^{0} = 1$$

Ahora con $t = \frac{2\pi n + \pi}{\omega} \Rightarrow t \to \infty \Rightarrow n \to \infty$

$$\lim_{t \to \infty} \cos(\omega t) - j\,\text{sen}(\omega t) = \lim_{n \to \infty} \overbrace{\cos\left(\cancel{\omega}\frac{2\pi n + \pi}{\cancel{\omega}}\right)}^{-1} - j\,\overbrace{\text{sen}\left(\cancel{\omega}\frac{2\pi n + \pi}{\cancel{\omega}}\right)}^{0} = -1$$

Como los límites no son iguales ante cualquier t, entonces el límite no existe, haciendo que la transformada diverja en el punto $Re\{s\} = -6$