



En t :

$$v_L(t) = L \frac{di_L(t)}{dt} \quad i_L(t) = \frac{1}{L} \int_0^t v_L(\tau) d\tau + i_L(0^+)$$

$$v_C(t) = \frac{1}{C} \int_0^t i_C(\tau) d\tau + v_C(0^+) \quad i_C(t) = C \frac{dv_C(t)}{dt}$$

$$v_R(t) = R \cdot i_R(t) \quad i_R(t) = \frac{v_R(t)}{R} \quad i_R(t) = i_C(t) = i_L(t) = i(t)$$

Ahora en s (Transformada de Laplace):

$$V_R(s) = R \cdot I(s)$$

$$V_L(s) = sL I(s) - L i(0^+)$$

$$V_C(s) = \frac{1}{sC} I(s) + \frac{1}{s} v_C(0^+)$$

$$\mathcal{L}\{v_i(t)\} = V_i(s)$$

Por LKV: $V_i(s) = V_R(s) + V_L(s) + V_C(s)$

$$V_i(s) = R \cdot I(s) + (sL I(s) - L i(0^+)) + \left(\frac{1}{sC} I(s) + \frac{1}{s} v_C(0^+) \right)$$

$$V_i(s) = \left(R + sL + \frac{1}{sC} \right) I(s) - L i(0^+) + \frac{1}{s} v_C(0^+)$$

Se despeja $I(s)$:

$$I(s) = \frac{sC}{LCs^2 + RCs + 1} \left[V_i(s) + L i(0^+) - \frac{v_C(0^+)}{s} \right]$$

$$I(s) = \frac{sC V_i(s)}{LCs^2 + RCs + 1} + \frac{sC L i(0^+)}{LCs^2 + RCs + 1} - \frac{C v_C(0^+)}{LCs^2 + RCs + 1}$$

Se tiene que $V_C(s) = Y(s)$, $v_C(t) = y(t)$, $X(s) = V_i(s)$, $x(t) = v_i(t)$

$$I(s) = \frac{sC X(s)}{LCs^2 + RCs + 1} + \frac{sC L i(0^+)}{LCs^2 + RCs + 1} - \frac{C v_C(0^+)}{LCs^2 + RCs + 1}$$

Se sabe que $Y(s) = V_C(s) = \frac{1}{sC} I(s) + \frac{1}{s} v_C(0^+)$

$$Y(s) = \frac{1}{sL} \left[\frac{sX(s)}{LCs^2 + RCs + 1} + \frac{s(Li(0^+))}{LCs^2 + RCs + 1} - \frac{(V_c(0^+))}{LCs^2 + RCs + 1} \right] + \frac{V_c(0^+)}{s}$$

$$Y(s) = \frac{X(s)}{LCs^2 + RCs + 1} + \frac{Li(0^+)}{LCs^2 + RCs + 1} + \frac{V_c'(0^+)}{s(LCs^2 + RCs + 1)} + \frac{V_c(0^+)}{s}$$

Se combinan los términos con $V_c(0^+) = y(0^+)$

$$Y(s) = \frac{X(s)}{LCs^2 + RCs + 1} + \frac{Li(0^+)}{LCs^2 + RCs + 1} + V_c(0^+) \left[\frac{1}{s} - \frac{1}{s(LCs^2 + RCs + 1)} \right]$$

$$Y(s) = \frac{X(s)}{LCs^2 + RCs + 1} + \frac{Li(0^+)}{LCs^2 + RCs + 1} + \frac{LCs + RC}{LCs^2 + RCs + 1} y(0^+)$$

$$Y(s) = \frac{X(s)}{LCs^2 + RCs + 1} + \frac{RC + LCs}{LCs^2 + RCs + 1} y(0^+) + \frac{L}{LCs^2 + RCs + 1} \dot{y}(0^+)$$