Lösung zu Catmull-Rom-Splines – Stetigkeitsgrad

C^0:

$$Q_i(t) = \begin{pmatrix} P_{i-3} & P_{i-2} & P_{i-1} & P_i \end{pmatrix} \cdot \frac{1}{2} \begin{pmatrix} -1 & 2 & -1 & 0 \\ 3 & -5 & 0 & 2 \\ -3 & 4 & 1 & 0 \\ 1 & -1 & 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} t^3 \\ t^2 \\ t \\ 1 \end{pmatrix}$$

$$Q_i(1) = \begin{pmatrix} P_{i-3} & P_{i-2} & P_{i-1} & P_i \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} = P_{i-1}$$

$$Q_{i+1}(0) = \begin{pmatrix} P_{i-2} & P_{i-1} & P_i & P_{i+1} \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} = P_{i-1}$$

C^1:

$$Q_i'(t) = \begin{pmatrix} P_{i-3} & P_{i-2} & P_{i-1} & P_i \end{pmatrix} \cdot \frac{1}{2} \begin{pmatrix} -3 & 4 & -1 \\ 9 & -10 & 0 \\ -9 & 8 & 1 \\ 3 & -2 & 0 \end{pmatrix} \cdot \begin{pmatrix} t^2 \\ t \\ 1 \end{pmatrix}$$

$$Q_i'(1) = \begin{pmatrix} P_{i-3} & P_{i-2} & P_{i-1} & P_i \end{pmatrix} \cdot \frac{1}{2} \begin{pmatrix} 0 \\ -1 \\ 0 \\ 1 \end{pmatrix} = \frac{1}{2} \left(P_i - P_{i-2} \right)$$

$$Q_{i+1}'(0) = \begin{pmatrix} P_{i-2} & P_{i-1} & P_i & P_{i+1} \end{pmatrix} \cdot \frac{1}{2} \begin{pmatrix} -1 \\ 0 \\ 1 \\ 0 \end{pmatrix} = \frac{1}{2} \left(P_i - P_{i-2} \right)$$

C^2:

$$Q_i''(t) = \begin{pmatrix} P_{i-3} & P_{i-2} & P_{i-1} & P_i \end{pmatrix} \cdot \frac{1}{2} \begin{pmatrix} -6 & 4 \\ 18 & -10 \\ -18 & 8 \\ 6 & -2 \end{pmatrix} \cdot \begin{pmatrix} t \\ 1 \end{pmatrix}$$

$$Q_i''(1) = \begin{pmatrix} P_{i-3} & P_{i-2} & P_{i-1} & P_i \end{pmatrix} \cdot \begin{pmatrix} -1 \\ 4 \\ -5 \\ 2 \end{pmatrix} = -P_{i-3} + 4P_{i-2} - 5P_{i-1} + 2P_i$$

$$Q_{i+1}''(0) = \begin{pmatrix} P_{i-2} & P_{i-1} & P_i & P_{i+1} \end{pmatrix} \cdot \begin{pmatrix} 2 \\ -5 \\ 4 \\ -1 \end{pmatrix} = 2P_{i-2} - 5P_{i-1} + 4P_i - P_{i+1}$$

C^2-stetig, falls $-P_{i-3} + 2P_{i-2} - 2P_i + P_{i+1} = 0$, also im allgemeinen nicht.