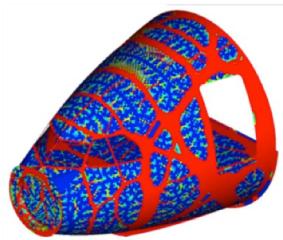


Structural and Multidisciplinary Design Optimization

TopOpt?

Prof. J. Morlier



1

History

- Homogenization of Microstructures was introduced by mathematics in the 1970s.
- First paper by Martin Bendsoe (Technical University of Denmark) and Noboru Kikuchi (University of Michigan) in 1988

A topology optimisation problem can be written in the general form of an optimization problem as

$$\min_{\rho} F = F(\mathbf{u}(\rho), \rho) = \int_{\Omega} f(\mathbf{u}(\rho), \rho) dV$$

subject to

- $\rho \in \{0, 1\}$
- $G_0(\rho) = \int_{\Omega} \rho(\mathbf{u}) dV - V_0 \leq 0$
- $G_j(\mathbf{u}(\rho), \rho) \leq 0$ with $j = 1, \dots, m$

TopOpt

Je choisis un bloc de marbre et j'enlève tout ce dont je n' ai pas besoin..
Auguste Rodin (1840-1917)



Define the design space (marble block, fixed mesh)

Apply charges & BCs

Start optimization with hyper parameters

Interpreting the results

Optimal distribution of material (One can have an idea of the part to be reinforced, in addition to giving an excellent initial design ...)

Why is it so powerful?

→ There is a lot of possible redistribution of FORCES (internal)

3

Stiff structure for your specifications

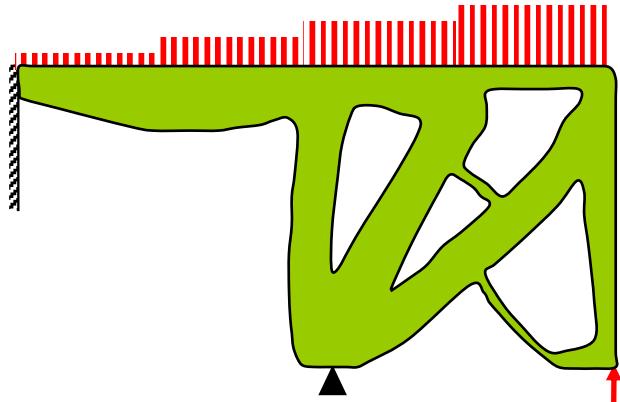
Distributed ramp force

Fixed

Use 40 % material that can fit into this rectangle

Fixed

Point force



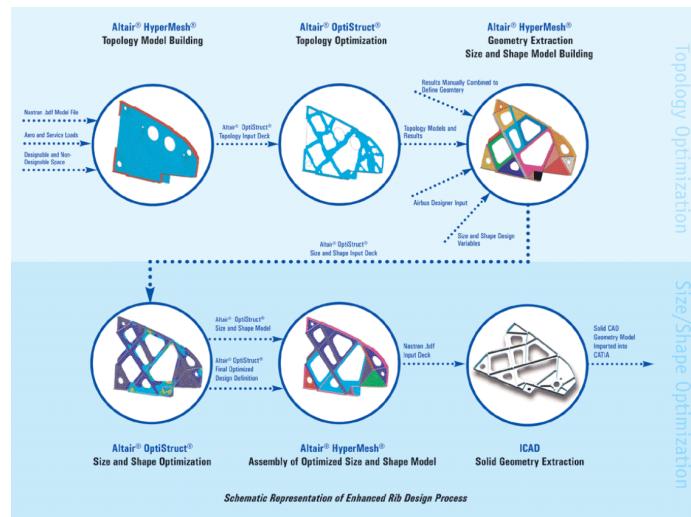
- Wing stiffening ribs
of Airbus A380:

AIRBUS



- Objective: reduce weight
- Constraints: stress, buckling

Topology and shape optimization

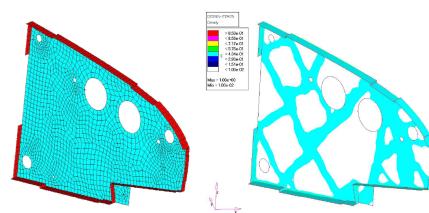


Altair Engineering

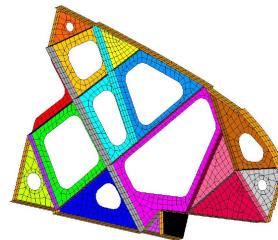
7

Airbus A380 example (cont.)

- Topology optimization:



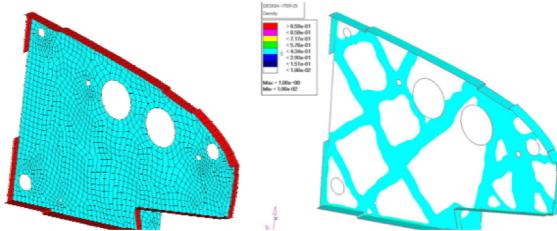
- Sizing / shape optimization:



Altair Engineering

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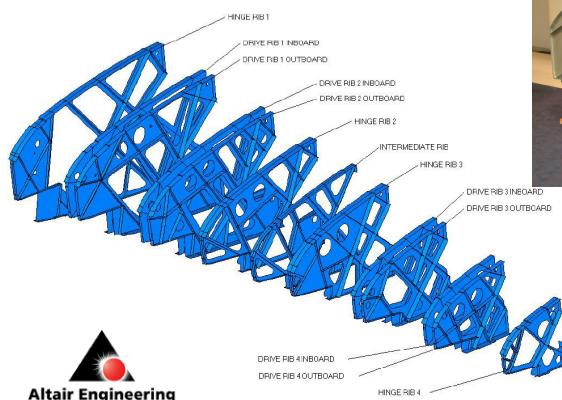
Finally...



9

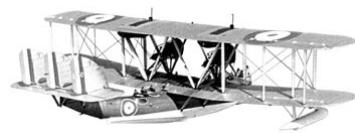
Airbus A380 example (cont.)

- Result: 500 kg weight savings!

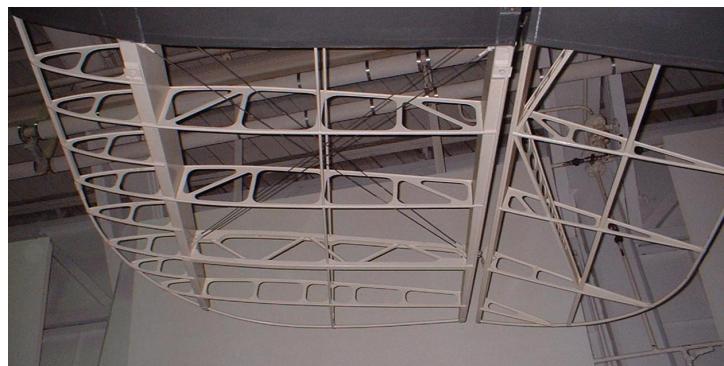


The perforated plates were replaced by reinforced lattice structures
(think of the path of preferential intern forces)

Is this really a discovery?



11



System approach automates the process!

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Industrial problems

TopOpt: Preliminary phases of a project

The idea is to find the best path of stiffening in a given volume of matter.

The mass is only found where it is needed, which is a good starting point for optimization of shape or dimensioning.

Adapted formulation:

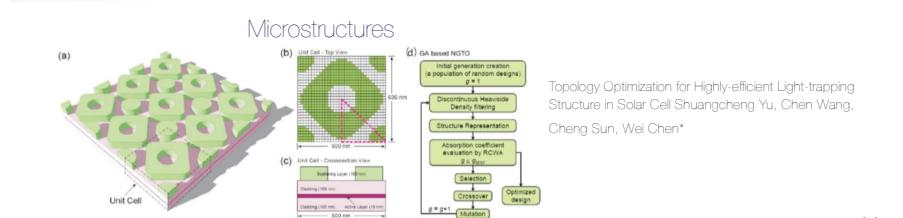
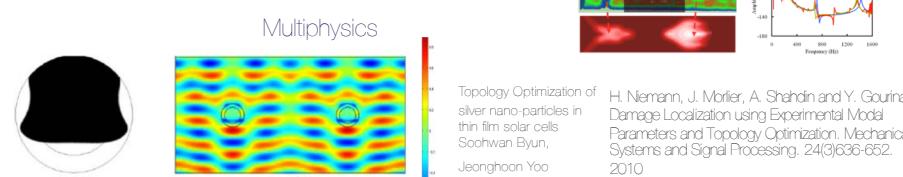
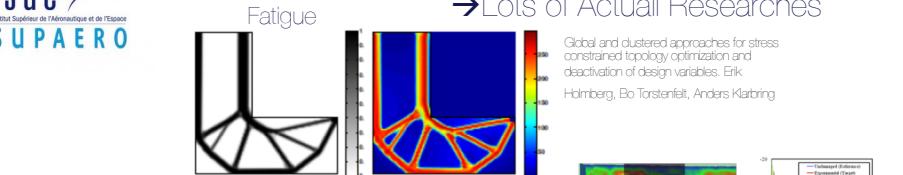
To find the structure with the best static behavior.

The paths of internal forces identified are those which help to rigidify the structure as well as possible

→ The structure will deform less, and stress levels will be possibly limited. But not only...

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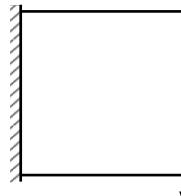
→ Lots of Actual Researches



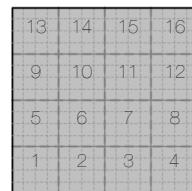
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Discrete Material Optimization:
Prof. Pierre DUYSENX Université de Liège LTAS –
Automotive Engineering

■ Maximum stiffness in the plane of a plate by selecting the best orientations of fibers



Loads and boundary conditions



Design model with 4 X4 patches

Table 4 Material properties

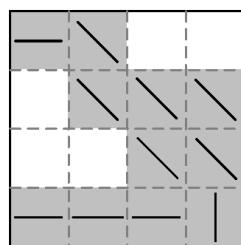
E_x	E_y	G_{xy}	v_{xy}
146.86GPa	10.62GPa	5.45GPa	0.33

Table 3 Orientations

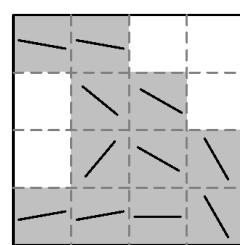
Number of material phases (m)	Number of design variables for each region (m_r)	Discrete orientation angle ($^\circ$)
4	2	90/45/0/-45
9	4	80/60/40/20/0/-20/-40/-60/-80
12	4	90/75/60/45/30/15/0/-15/-30/-45/-60/-75

Discrete Material Optimization: exemple

- Topological optimization: vacuum + composite laminate
□ Volume constraints: $V < 11/16$



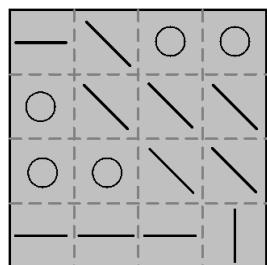
4 orientations
90/45/0/-45



18 orientations
90/80/70/60/50/40/30/20/10/0/
-10/-20/-30/-40/-50/-60/-70/-80

Discrete Material Optimization: exemple

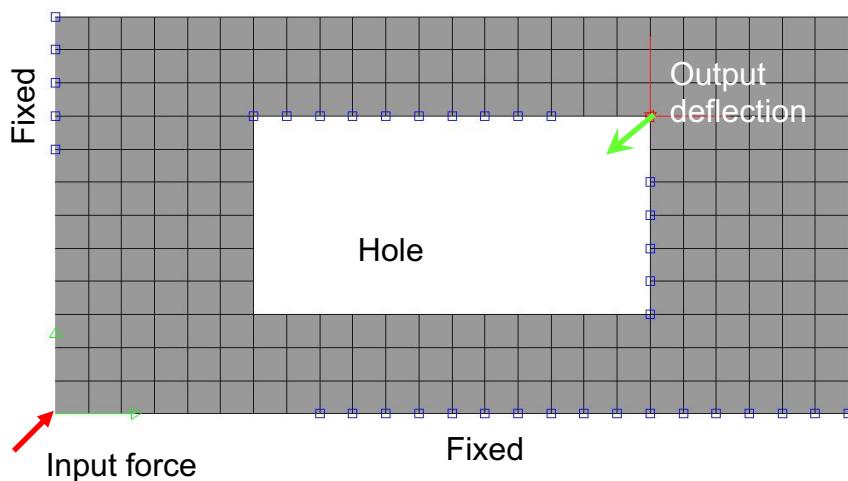
- Use of both glass fibers and foam
- Limitation of the number of domains occupied by the fiber of glass



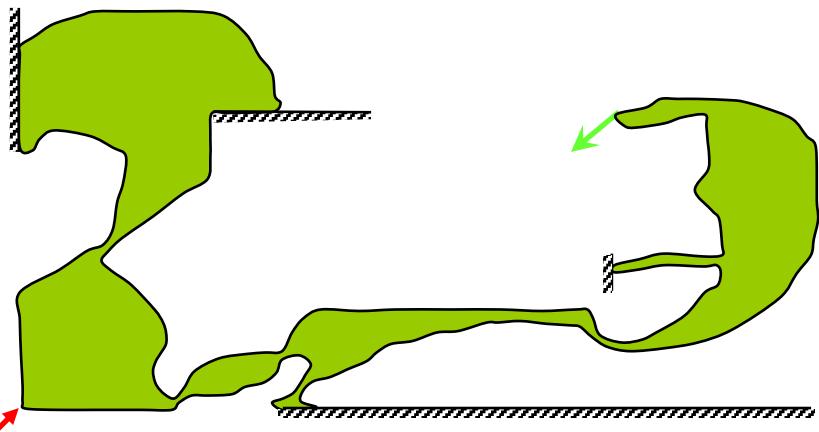
Optimization result of the square plate under vertical force with volume constraint
Glass-epoxy with 4 orientations (90/45/0/-45) and polymer-foam

Compliant mechanism
to your specifications

Use 30 % material



Compliant mechanism
to your specifications

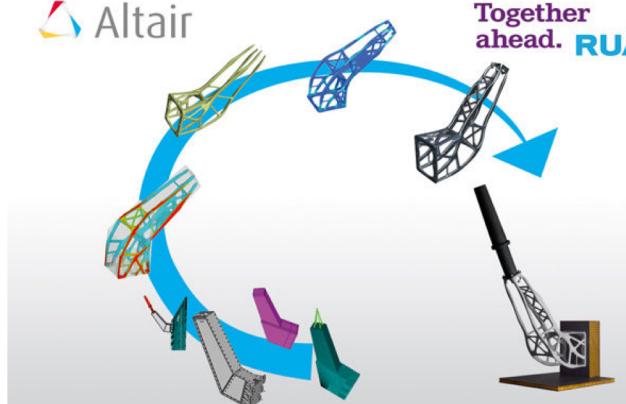


www.mecheng.iisc.ernet.in/~suresh/shortcourse

ALM

Altair

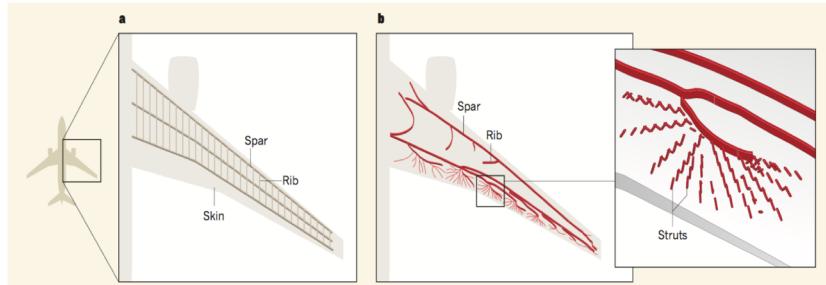
Together
ahead. RUAG



Processus de re-conception pour la fabrication additive par l'optimisation topologique

<http://bcove.me/yg7ogkak>

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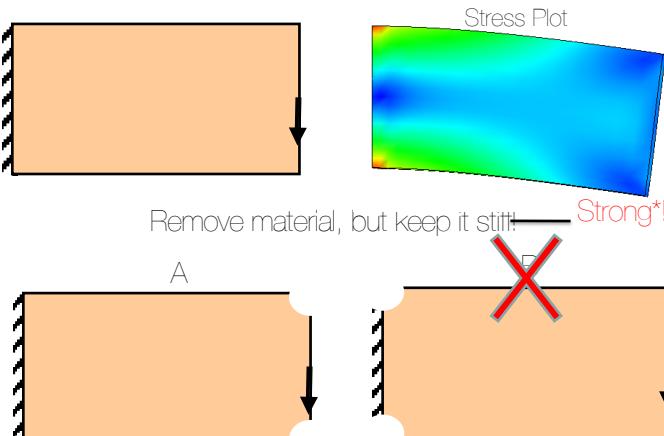


<https://www.nature.com/articles/nature23911>

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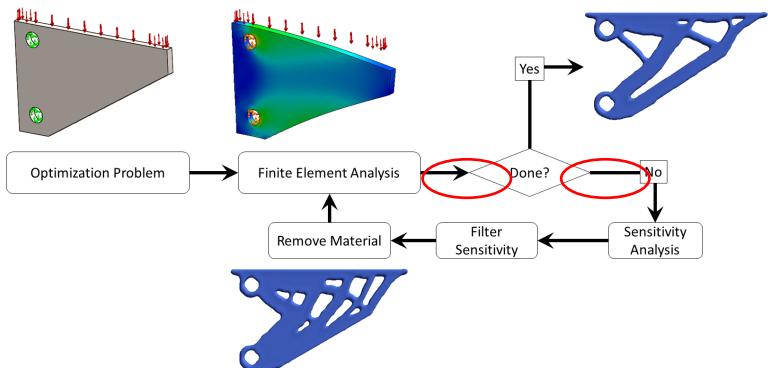
Intuition ...

*See Prof Suresh's work:
<https://dl.acm.org/citation.cfm?id=1861606>



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TopOpt relies on FEA
Online computation: <http://www.cloudtopopt.com>



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TopOpt



$$Ku = f$$

$$\text{Compliance } J = f^T u$$

$$\text{Compliance} = 1/\text{Stiffness}$$

1. Objective? Minimize Compliance

2. Constraints? Volume Constraint

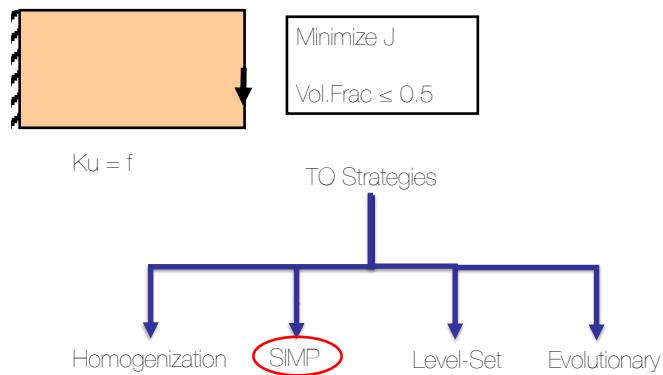
3. Method?

Minimize J
Vol.Frac < 0.5

Method: Gradient based:
Need sensitivities...

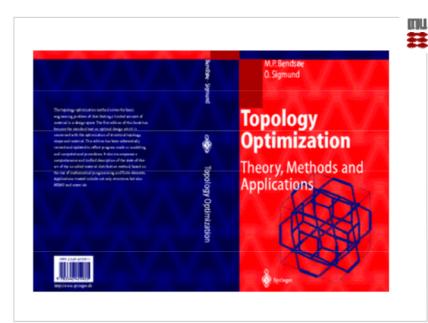
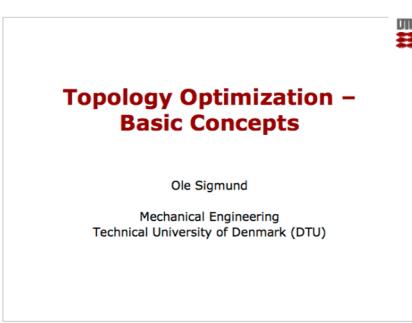
25

Current TO Strategies



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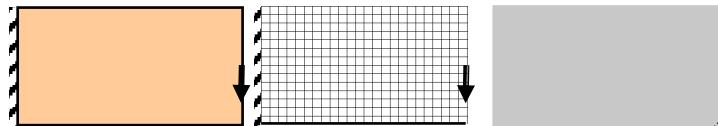
One pioneer, **SIMP** (Solid Isotropic Material with **Penalization**)



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SIMP

SIMP: Solid Isotropic Material with Penalization



$$\text{Min Compliance} \quad 0 < \rho_e \leq 1 : \text{'PseudoDensity'}$$

$$v = 0.5v_0$$

Where do we add holes?

$$\text{Min Compliance}_{\rho_e} \quad \sum \rho_e v_e = 0.5v_0$$

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Intuitive Problem? Quadratic Form

- Objective function; Strain energy

$$\min c(\mathbf{x}) = \mathbf{U}^T \mathbf{F} = \mathbf{U}^T \mathbf{K} \mathbf{U} \quad \text{with} \quad x_e = \frac{\rho_e}{\rho_0} \quad (4)$$

with $\mathbf{K} = \mathbf{K}_0 \sum_{e=1}^N x_e^p$ one can write:

What is p,
(simP)?????????

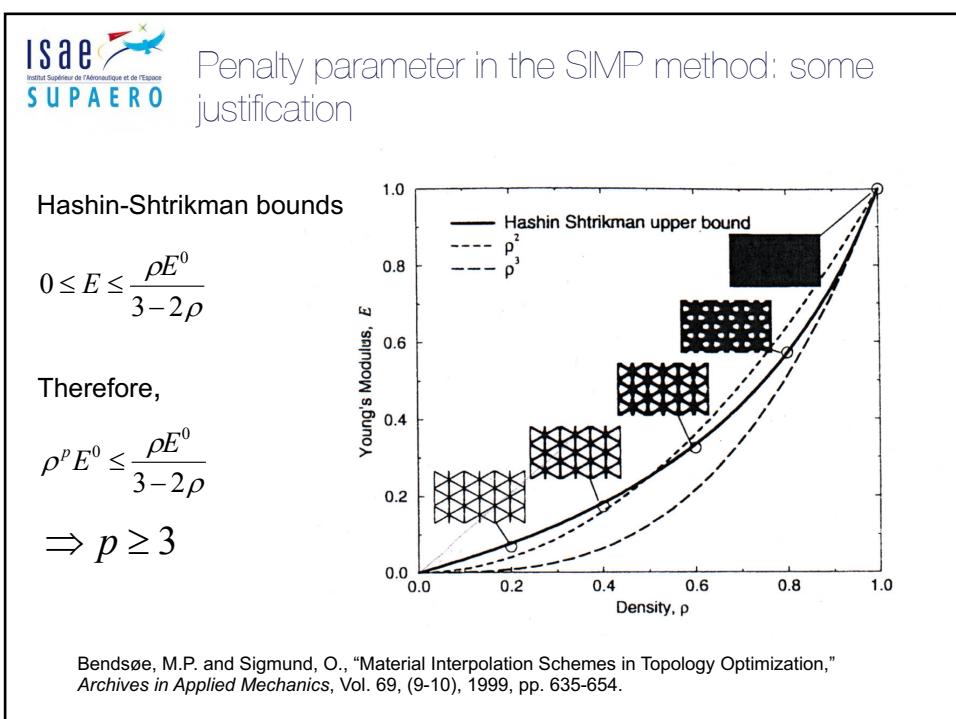
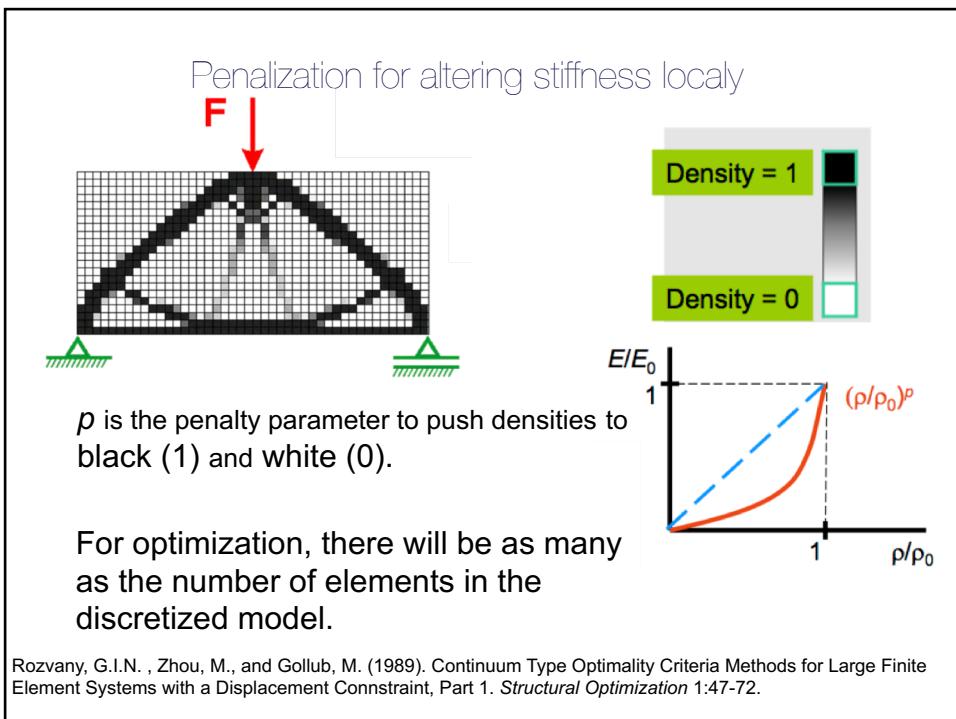
$$\min c(\mathbf{x}) = \sum_{e=1}^N (x_e)^p \mathbf{u}_e^T \mathbf{k}_0 \mathbf{u}_e \quad \text{Scalar} \quad (5)$$

- Constraints; mass target

$$\frac{V(\mathbf{x})}{V_0} = f = \text{const} \Leftrightarrow \sum_{e=1}^N V_e x_e = V_0 f = h(\mathbf{x}) \quad \text{Scalar}$$

$$0 < \rho_{\min} \leq \rho_e \leq 1$$

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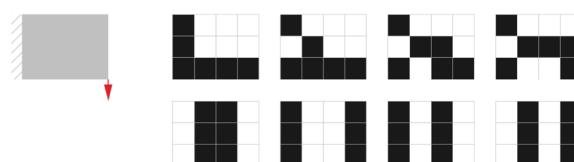
Pixels?



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Pixels

- Finding a solution by checking all the possible combinations IS impossible since the number of topologies nT increases exponentially with the number of finite elements n
- $nT = 2^n$,



The legal (top) and some illegal (bottom) topologies with 4 by 3 elements

Division into elements (pixels or voxels) and binary decision for each
or example 10,000 elements --> 210,000 possible configurations!

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Nice idea !

1. Transform discrete variables continuously (TO USE gradient-based algorithms)
2. Find an objective function with "cheap" derivatives (we will see this later)

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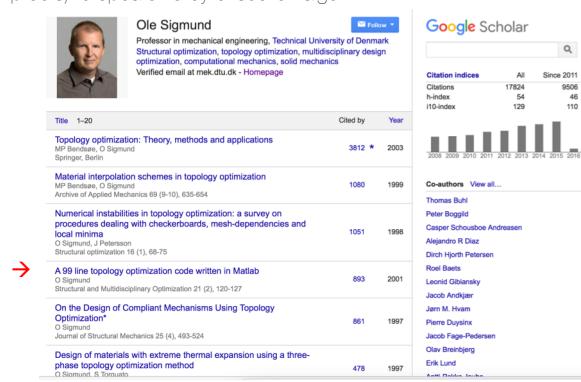


BUT ...IN PRACTICE?

Educational article:

O. Sigmund , A 99 line topology optimization code written in Matlab Struct Multidisc Optim 21, 120–127 Springer-Verlag 2001

Heuristic formulation (intuitive method of optimisation, but with no convergency proofs) to update x_e by bi-section algorithm



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History (1988, Bendsoe)

A topology optimization problem based on the power-law approach, where the objective is to minimize compliance can be written as

$$\left. \begin{array}{l} \min_{\mathbf{x}} : c(\mathbf{x}) = \mathbf{U}^T \mathbf{K} \mathbf{U} = \sum_{e=1}^N (x_e)^p \mathbf{u}_e^T \mathbf{k}_0 \mathbf{u}_e \\ \text{subject to : } \begin{array}{l} \frac{V(\mathbf{x})}{V_0} = f \\ \mathbf{K} \mathbf{U} = \mathbf{F} \\ : \mathbf{0} < \mathbf{x}_{\min} \leq \mathbf{x} \leq \mathbf{1} \end{array} \end{array} \right\}, \quad (1)$$

where \mathbf{U} and \mathbf{F} are the global displacement and force vectors, respectively, \mathbf{K} is the global stiffness matrix, \mathbf{u}_e and \mathbf{k}_e are the element displacement vector and stiffness matrix, respectively, \mathbf{x} is the vector of design variables, \mathbf{x}_{\min} is a vector of minimum relative densities (non-zero to avoid singularity), N ($= \text{nely} \times \text{nely}$) is the number of elements used to discretize the design domain, p is the penalization power (typically $p = 3$), $V(\mathbf{x})$ and V_0 is the material volume and design domain volume, respectively and f (volfrac) is the prescribed volume fraction.

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Compliance minimization

- Compliance is the opposite of stiffness

$$C = \mathbf{f}^T \mathbf{u} = \mathbf{u}^T \mathbf{K} \mathbf{u}$$

- Inexpensive derivatives (use chain rule)

$$\frac{dC}{dx} = 2\mathbf{u}^T \mathbf{K} \frac{d\mathbf{u}}{dx} + \mathbf{u}^T \frac{d\mathbf{K}}{dx} \mathbf{u}$$

But since $\mathbf{K}\mathbf{u} = \mathbf{f}$ if \mathbf{f} does not depend on \mathbf{x}

$$\mathbf{K} \frac{d\mathbf{u}}{dx} = - \frac{d\mathbf{K}}{dx} \mathbf{u}$$

$$\frac{dC}{dx} = -\mathbf{u}^T \frac{d\mathbf{K}}{dx} \mathbf{u}$$

Density design variables

Need a DEMO ?

- Recall

$$\frac{dC}{dx} = -\mathbf{u}^T \frac{dK}{dx} \mathbf{u}$$

- For density variables

$$\frac{dC}{d\rho^e} \propto -\mathbf{u}^T \rho^{p-1} K^e \mathbf{u}$$

- Want to increase density of elements with high strain energy and vice versa
- To minimize compliance for given weight can use an optimality criterion method.



Matlab Code

```

x(1:nely,1:nelx) = volfrac;
% INITIALIZE

loop = 0; change = 1;
while change > 0.01
    loop = loop + 1;
    xold = x;
    [U]=FE(nelx,nely,x,penal); % FE-ANALYSIS
    [K]=Ik;
    c = 0.;

    for ely = 1:nely
        for elx = 1:nelx
            n1 = (nely+1)*(elx-1)+ely;
            n2 = (nely+1)* elx +ely;
            Ue = U([2*n1-1;2*n1; 2*n2-1;2*n2; 2*n2+1;2*n2+2; 2*n1+1;2*n1+2],1);
            c = c + x(ely,elx)^penal*Ue^KE*Ue; % OBJECTIVE FUNCTION
            dc(ely,elx) = -penal*x(ely,elx)^(penal-1)*Ue^KE*Ue; % SENSITIVITY ANALYSIS
        end
    end

```

Sensitivity $\frac{\partial c}{\partial x_e} = -p(x_e)^{p-1} \mathbf{u}_e^T \mathbf{k}_0 \mathbf{u}_e$

Update rule

- OPTIMALITY CRITERIA METHOD

$$B_e = \begin{cases} \max(x_{\min}, x_e - m) & \text{if } x_e B_e^\eta \leq \max(x_{\min}, x_e - m), \\ x_e B_e^\eta & \text{if } \max(x_{\min}, x_e - m) < x_e B_e^\eta < \min(1, x_e + m) \\ \min(1, x_e + m) & \text{if } \min(1, x_e + m) \leq x_e B_e^\eta, \end{cases}$$

$$B_e = \frac{-\frac{\partial c}{\partial x_e}}{\lambda \frac{\partial V}{\partial x_e}}$$

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Element Stiffness Matrix

```

function [KE]=lk
%Element Stiffness Matrix

E = 1. ;
nu =1/3. ;
k=[ 1/2*nu/6 1/8+nu/8 -1/4+nu/12 -1/8-nu/8
-1/4-nu/12 -1/8+3*nu/8 ... nu/6 1/8-3*nu/8];
KE = E/(1-nu^2)* ...
[ k(1) k(2) k(3) k(4) k(5) k(6) k(7) k(8)
k(2) k(1) k(8) k(7) k(6) k(5) k(4) k(3) k(3) k(8) k(1) k(6)
k(7) k(4) k(5) k(2) k(4) k(7) k(6) k(1) k(8) k(3) k(2) k(5)
k(5) k(6) k(7) k(8) k(1) k(2) k(3) k(4) k(6) k(5) k(4) k(3)
k(2) k(1) k(8) k(7) k(4) k(5) k(2) k(3) k(8) k(1) k(6)
k(8) k(3) k(2) k(5) k(4) k(7) k(6) k(1)];

```

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FEM Analysis

```

function [U]=FE(nelx,nely,x,penal)
[KE] = lk;
K = sparse(2*(nelx+1)*(nely+1), 2*(nelx+1)*(nely+1));
F = sparse(2*(nely+1)*(nelx+1),1); U = zeros(2*(nely+1)*(nelx+1),1);
for elx = 1:nelx
    for ely = 1:nely
        n1 = (nely+1)*(elx-1)+ely;
        n2 = (nely+1)* elx +ely;
        edof = [2*n1-1; 2*n1; 2*n2-1; 2*n2; 2*n2+1; 2*n2+2; 2*n1+1; 2*n1+2];
        K(edof,edof) = K(edof,edof) + x(ely,elx)^penal*KE;
    end
    end
F(2*(nelx+1)*(nely+1),1)=-1;
fixeddofs=union([1,2],[2*nely+1:2*(nely+1)]);
alldofs = [1:2*(nely+1)*(nelx+1)];
freedofs = setdiff(alldofs,fixeddofs);
% SOLVING
U(freedofs,:)=K(freedofs, freedofs)\ F(freedofs,:);
U(fixeddofs,:)= 0;

```

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```

function [xnew]=OC(nelx,nely,x,volfrac,dc)
l1 = 0; l2 = 100000; move = 0.2;
while (l2-l1 > 1e-4)
    lmid = 0.5*(l2+l1);
    xnew = max(0.001,max(x-move,min(1.,min(x+move,x.*sqrt(-dc./lmid)))));
    if sum(sum(xnew)) - volfrac*nelx*nely > 0;
        l1 = lmid;
    else
        l2 = lmid;
    end
end
    
```

lmid: 50.0000	l1: 0.0000	l2: 50.0000
lmid: 25.0000	l1: 0.0000	l2: 25.0000
lmid: 12.5000	l1: 0.0000	l2: 12.5000
lmid: 6.2500	l1: 6.2500	l2: 12.5000
lmid: 9.3750	l1: 6.2500	l2: 9.3750
lmid: 7.8125	l1: 6.2500	l2: 7.8125
lmid: 7.0313	l1: 7.0313	l2: 7.8125
lmid: 7.4219	l1: 7.0313	l2: 7.4219
lmid: 7.2266	l1: 7.2266	l2: 7.4219
lmid: 7.3242	l1: 7.3242	l2: 7.4219
lmid: 7.3730	l1: 7.3242	l2: 7.3730
lmid: 7.3486	l1: 7.3242	l2: 7.3486
lmid: 7.3364	l1: 7.3364	l2: 7.3486
lmid: 7.3425	l1: 7.3425	l2: 7.3486
lmid: 7.3456	l1: 7.3425	l2: 7.3456
lmid: 7.3441	l1: 7.3441	l2: 7.3456
lmid: 7.3448	l1: 7.3448	l2: 7.3456
lmid: 7.3452	l1: 7.3448	l2: 7.3452
lmid: 7.3450	l1: 7.3450	l2: 7.3452
lmid: 7.3451	l1: 7.3450	l2: 7.3451

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- fmincon
- MMA...

The MMA approach, which was initially proposed by Svanberg (see Mini Project) is based on the first-order Taylor series expansion of the objective and constraint functions.

With this method, an explicit convex subproblem is generated to approximate the implicit nonlinear problem.

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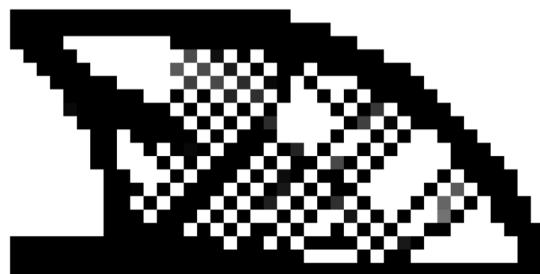
```
top(nelx, nely, volfrac, penal, rmin)
```

- nelx and nely: number of elements in the horizontal and vertical directions,
- volfrac: volume fraction,
- penal: penalization power,
- rmin: filter size (divided by element size).

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Numerical instability

- top(40, 20, 0.5, 3, 1.0)



effect → Checkerboard Pattern

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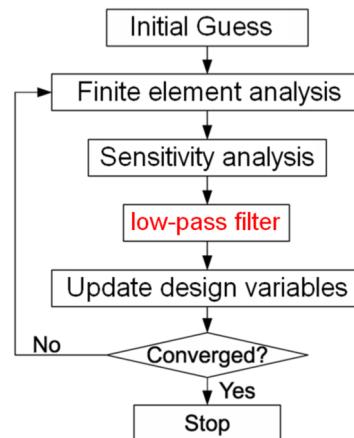
Example 1 -- Checkerboard Pattern Problem

→ Solution: LOW PASS Filter

$$\widehat{\frac{\partial c}{\partial x_e}} = \frac{1}{x_e \sum_{f=1}^N \hat{H}_f} \sum_{f=1}^N \hat{H}_f x_f \frac{\partial c}{\partial x_f}.$$

$$\hat{H}_f = r_{\min} - \text{dist}(e, f),$$

$$\{f \in N \mid \text{dist}(e, f) \leq r_{\min}\}, \quad e = 1, \dots, N$$



[dc] = check(nelx, nely, rmin, x, dc);
% FILTERING OF SENSITIVITIES

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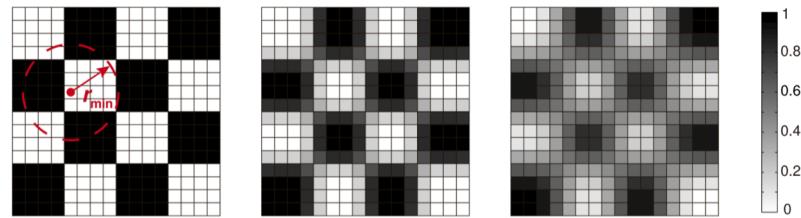
Example 2

- top(40, 20, 0.5, 3, 1.5)



- top(40, 20, 0.5, 3, 3) effect?

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A checkerboard field and filtered fields ($r_{\min} = 1.5l_e$ and $3l_e$)

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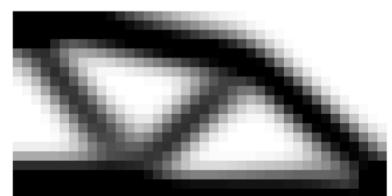
Example 2

→ Size of the filter makes it possible to obtain a more physical representation (Unblur?)

- $r_{\min}=1.5$
 $\text{Obj}=82.7562;$



- $r_{\min}=3$
 $\text{Obj}=99.1929;$



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Example 3: Change mesh !

- $\text{top}(60, 30, 0.5, 3, 1.0)$

Obj: 83.0834



- $\text{top}(40, 20, 0.5, 3, 1.0)$

Obj: 80.4086;



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Example 3: mixing



- $\text{top}(60, 30, 0.5, 3, 1.5)$

Obj: 81.3491

→ Filter size

- $\text{top}(60, 30, 0.5, 3, 2.25)$

Obj: 83.5963

→ +mesh dependency, combined effect It's complicated

- $\text{top}(40, 20, 0.5, 3, 1.5)$

Obj=82.7562;



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Optimisation topologique avec Matlab
Ce n'est pas moi!
J. Morlier^{**}

Le maillage EF est composé de quadrangle de taille 1*1. La densité appelée x , est définie sur $0.001 \leq x \leq 1$, la borne inf est non nulle, pour que K ne soit pas singulière. La correspondance entre les éléments de la matrice des densités et le maillage EF est donnée sur la figure 1.1.

x(1)			x(10)
.			.
x(90)			x(100)

Figure 1: Indice de la matrice densité

Nous allons dans cette exercice essayer (en lisant l'article associé au code top.m) d'optimiser la conception d'un cadre de bicyclette.

Le domaine de conception est donné ci dessous (figure 3)

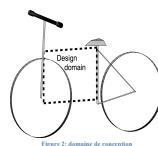
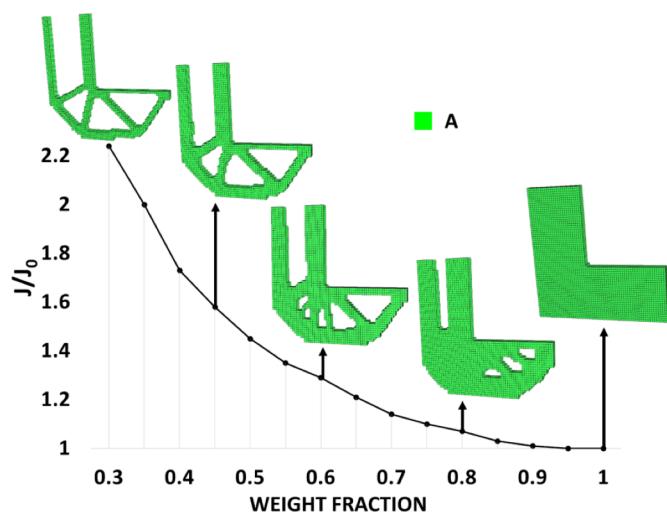


Figure 2: domaine de conception

^{**}d'après le site:
<http://www.topopt.dtu.dk>

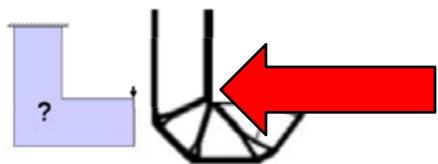
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The engineer's way: Pareto



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At this time the structure is rigid... but feasible?



Check the
stress?

60

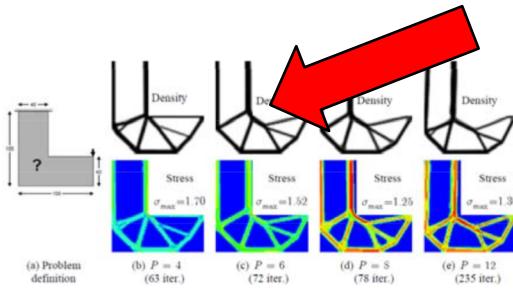
Stress based TOPOPT
That's the subject of the marked
miniproject...Just after the pause

$$\begin{aligned}
 & \min_{\rho} : \max_e (\sigma_e)_{VM} \\
 & \text{s.t.} : \sum_{e=1}^N v_e \rho_e \leq V^* \\
 & \quad : 0 \leq \rho \leq 1 \\
 & \quad : \mathbf{K}(\rho) \mathbf{U} = \mathbf{F}
 \end{aligned}$$

$$\begin{aligned}
 & \min_{\rho} : \sum_{e=1}^N v_e \rho_e \\
 & \text{s.t.} : (\sigma_e)_{VM} \leq \rho_e^p \sigma^*, \quad \text{if } \rho_e > 0, \quad e = 1, \dots, N \\
 & \quad : 0 \leq \rho \leq 1 \\
 & \quad : \mathbf{K}(\rho) \mathbf{U} = \mathbf{F}
 \end{aligned}$$

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Results

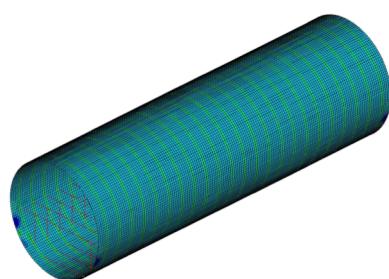


Le et al., SMO (2010)

Its it better?

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Topopt on barrel (Fuselage) using Optistruct



Engineering Optimization
Vol. 41, No. 12, December 2009, 1103–1118

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Structural topology optimization for multiple load cases using a dynamic aggregation technique

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(Received 1 December 2008; final version received 16 March 2009)

A series of techniques is presented for overcoming some of the numerical instabilities associated with SIMP. These techniques are used to create a robust topology optimization algorithm designed to be able to successfully solve problems of interest in the field of structural optimization and topology applications. A variant of the Kresselmeier-Steinbauer (KS) function in which the aggregation parameter α is varied during the optimization process is used to overcome the numerical instabilities. Results from this method are compared with those obtained using the standard formulation. It is shown that the KS function is more stable than the standard function for those problems for which the latter is highly susceptible to local minima. Adaptive mesh-refinement is presented as a means of addressing the meshing issues associated with the use of the KS function. The use of adaptive mesh-refinement is also demonstrated, and when used in combination with nine-node elements, readily eliminate checkerboarding

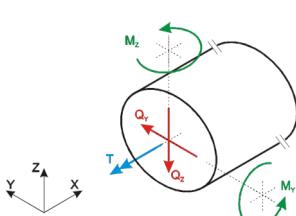
Keywords: topology optimization; Kresselmeier-Steinbauer function; adaptive mesh refinement; multiple load cases

1. Introduction

Since the introduction more than two decades ago (Bendsoe and Kikuchi 1988), topology optimization has emerged as an important discipline of engineering design. This discipline, as well as the types of problems to which it is applied, researchers from a wide range of disciplines and industries have adopted the method due to its ability to produce highly efficient, light-weight structures. A classic example of the use of topology optimization is the design of aircraft wings in which topology optimization is used in the preliminary structural layout design of the leading edge and wingbox ribs. Examples such as this highlight the need for increased robustness in topology optimization algorithms. The desire to generate optimized topologies for feasible real-world structures has led to the development of various methods that attempt to incorporate this into account factors such as multiple load cases, material failure constraints, and buckling effects.

These considerations often give rise to optimization constraints that are difficult to implement due to numerical instabilities inherent in most topology optimization procedures. This article

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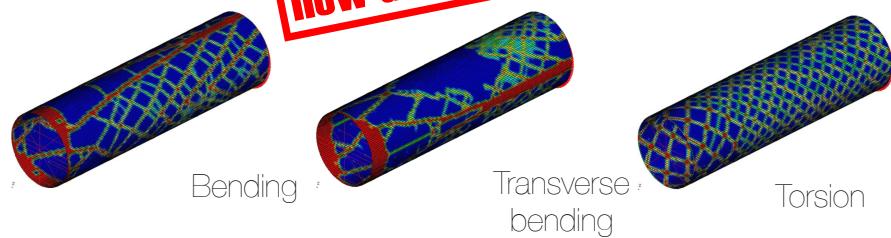


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Topology Optimisation –Altair Optistruct

- Results for 3 load cases

How do you combine the results?

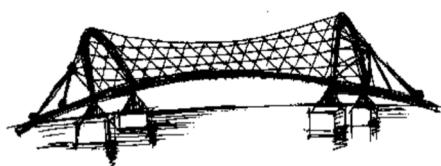


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Conclusion

"The art of structure is where to put the holes"

Robert Le Ricolais
French-American engineer and philosopher
(1894-1977)



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