$$\boxed{C = f^T u}$$

F is not depending on x

$$\frac{\partial C}{\partial x} = \frac{\partial f^T}{\partial x} u + f^T \frac{\partial u}{\partial x} = \phi + f^T \frac{\partial u}{\partial x} \quad (1)$$

starting by expressing the derivative of $K u = f$

it gives $\dfrac{\partial K}{\partial x} u + K \dfrac{\partial u}{\partial x} = \dfrac{\partial f}{\partial x} = 0$

$$\Leftrightarrow \boxed{K \frac{\partial u}{\partial x} = - \frac{\partial K}{\partial x} u}$$

$$(1) \quad \frac{\partial c}{\partial x} = f^T \frac{\partial u}{\partial x} = u^T K \frac{\partial u}{\partial x} = u^T \left(- \frac{\partial K}{\partial x}\right) u = \boxed{-u^T \frac{\partial K}{\partial x} u = \frac{\partial c}{\partial x}}$$

or by expressing $C = f^T u$

as $C = u^T K u$ (quadratic form) (2)

$$dC = \frac{\partial C}{\partial u} du + \frac{\partial C}{\partial x} dx \Rightarrow \boxed{\frac{dC}{dx}} = \frac{\partial C}{\partial u} \frac{du}{dx} + \frac{\partial C}{\partial x}$$

from (2) $\dfrac{\partial C}{\partial u} = 2 u^T K \dfrac{du}{dx}$ $\longrightarrow$ (Recap $\dfrac{\partial u^T K u}{\partial u} = 2 u^T K$)

it comes

$$\frac{dC}{dx} = 2 u^T K \frac{du}{dx} + u^T \frac{\partial K}{\partial x} u$$

$$= -2 u^T \frac{\partial K}{\partial x} u + u^T \frac{\partial K}{\partial x} u = \boxed{-u^T \frac{\partial K}{\partial x} u} = \frac{\partial C}{\partial x}$$