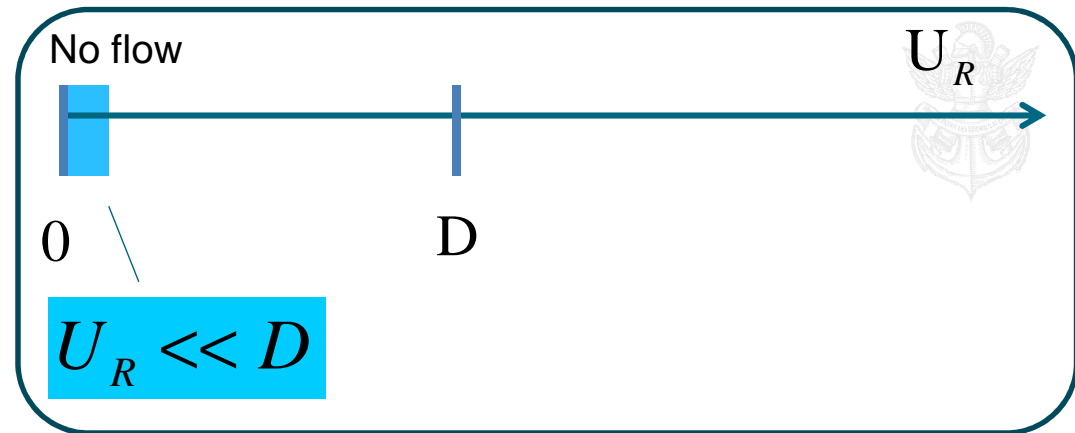


SMALL REDUCED VELOCITY, SMALL MOTION



$D \ll 1$

$$M \int_{\text{Interface}} \underline{\varphi} \cdot [\dots] \cdot \underline{n} \, dS - Mq \int_{\text{Interface}} (\underline{\nabla} P_0 \cdot \underline{\varphi})(\underline{\varphi} \cdot \underline{n}) \, dS = f$$

FLUID-INDUCED STIFFNESS

$$M \int_{\text{Interface}} \underline{\varphi} \cdot [\dots] \underline{n} dS - Mq \int_{\text{Interface}} (\underline{\nabla} P_0 \cdot \underline{\varphi})(\underline{\varphi} \cdot \underline{n}) dS = f$$

Modal
displacement

Mass
number

Hydrostatic
pressure

Modal
shape

Geometry

$$0 = -\frac{1}{F_D^2} \underline{e}_z - \underline{\nabla} P_0$$

$$f = q \frac{M}{F_D^2} \int_{\text{Interface}} (\underline{\varphi} \cdot \underline{e}_z)(\underline{\varphi} \cdot \underline{n}) dS$$

FLUID-INDUCED STIFFNESS

$$f = q \frac{M}{F_D^2} \int_{Interface} (\underline{\varphi} \cdot \underline{e}_z) (\underline{\varphi} \cdot \underline{n}) dS$$

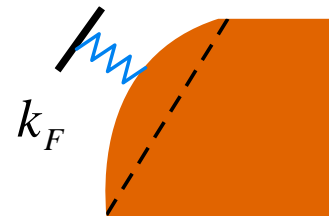
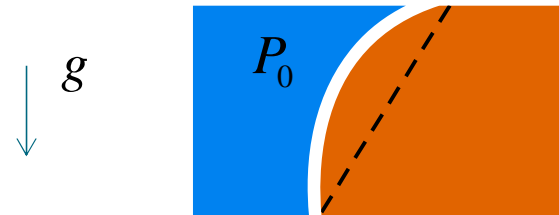


$$f = -q k_F$$

$$k_F = - \frac{M}{F_D^2} \int_{Interface} (\underline{\varphi} \cdot \underline{e}_z) (\underline{\varphi} \cdot \underline{n}) dS$$

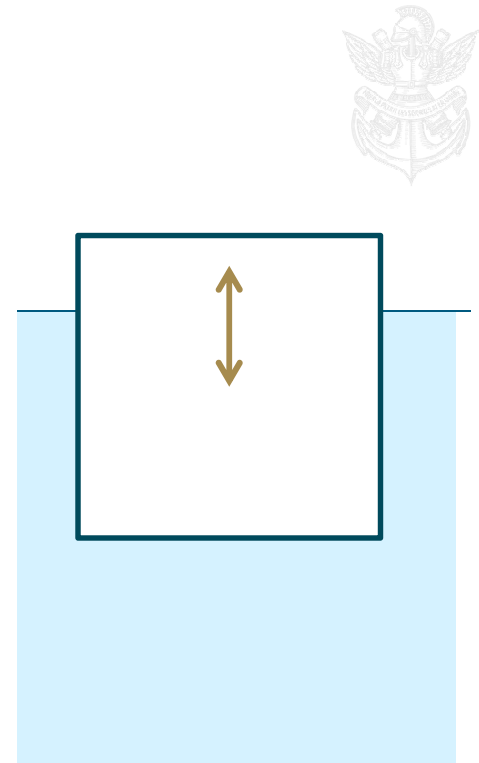
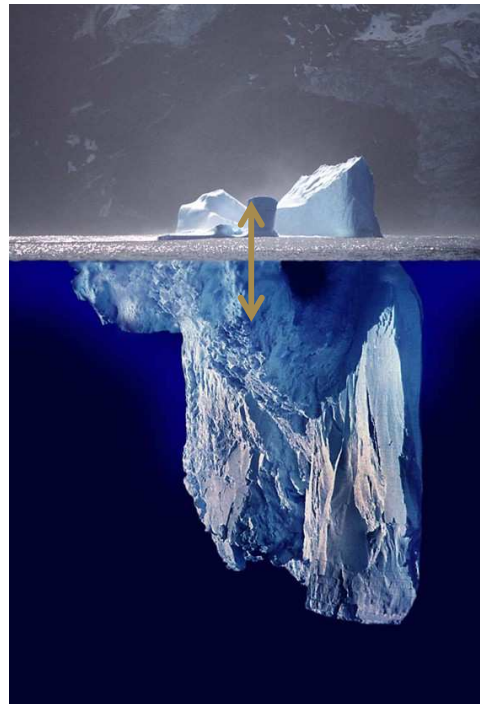
$$M = \frac{\rho}{\rho_s} \quad F_D^2 = \frac{L}{T_{Solid}^2 g}$$

FLUID-INDUCED STIFFNESS



$$k_F = - \frac{M}{F_D^2} \int_{Interface} (\underline{\varphi} \cdot \underline{e}_z) (\underline{\varphi} \cdot \underline{n}) dS$$

EXAMPLE : ICEBERGS AND ICE CUBES



FROM ICE CUBES TO ICEBERGS

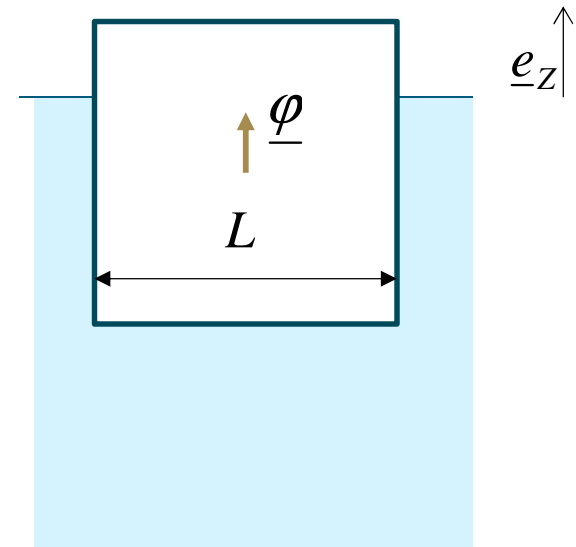
$$f = q \frac{M}{F_D^2} \int_{Interface} (\underline{\varphi} \cdot \underline{e}_z) (\underline{\varphi} \cdot \underline{n}) dS$$



$$M = \frac{\rho_{water}}{\rho_{ice}}$$

$$F_D = \frac{L}{T_{solid} \sqrt{gL}}$$

$$\underline{\varphi} = \underline{e}_z$$



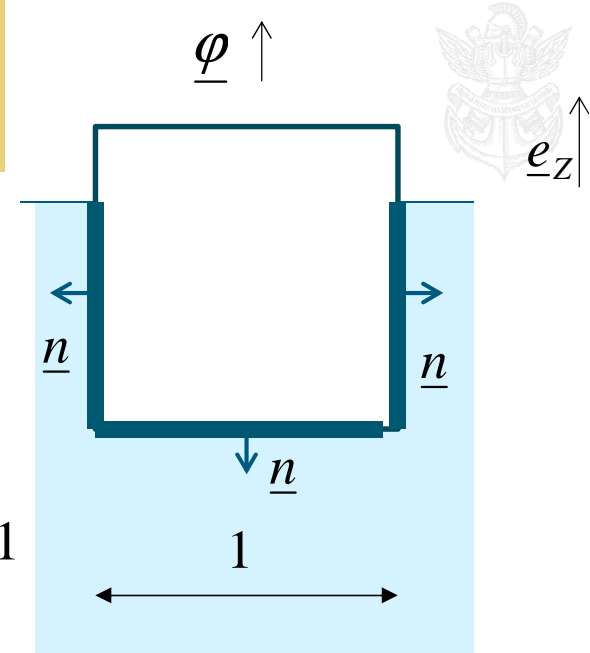
FROM ICE CUBES TO ICEBERGS

$$\int_{\text{Interface}} (\underline{\varphi} \cdot \underline{e}_z) (\underline{\varphi} \cdot \underline{n}) dS$$

$$\int_{\text{Sides}} (\underline{\varphi} \cdot \underline{e}_z) (\underline{\varphi} \cdot \underline{n}) dS = 0$$

$$\int_{\text{Bottom}} (\underline{\varphi} \cdot \underline{e}_z) (\underline{\varphi} \cdot \underline{n}) dS = -1$$

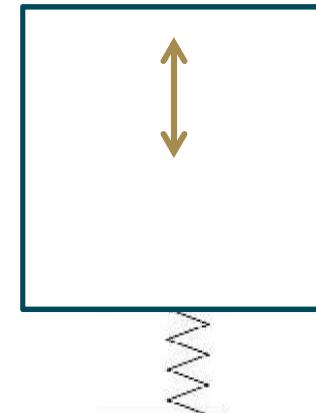
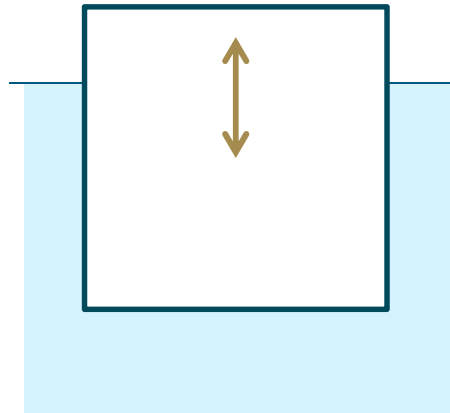
$$\int_{\text{Interface}} (\underline{\varphi} \cdot \underline{e}_z) (\underline{\varphi} \cdot \underline{n}) dS = -1$$



FROM ICE CUBES TO ICEBERGS

$$M = \frac{\rho_{\text{water}}}{\rho_{\text{ice}}} \quad F_D = \frac{1}{T_{\text{solid}}} \sqrt{\frac{L}{g}} \quad \int_{\text{Interface}} (\underline{\varphi} \cdot \underline{e}_z) (\underline{\varphi} \cdot \underline{n}) dS = -1$$

$$f = - \left[\frac{\rho_{\text{water}}}{\rho_{\text{ice}}} \frac{g}{LT_{\text{solid}}^2} \right] q$$



FROM ICE CUBES TO ICEBERGS

$$f = - \left[\frac{\rho_{\text{water}}}{\rho_{\text{ice}}} \frac{g}{LT_{\text{solid}}^2} \right] q$$



$$\frac{d^2 q}{dt^2} = - \left[\frac{\rho_{\text{water}}}{\rho_{\text{ice}}} \frac{g}{LT_{\text{solid}}^2} \right] q$$

$$\frac{d^2 q}{dt^2} + \left[\frac{\rho_{\text{water}}}{\rho_{\text{ice}}} \frac{g}{LT_{\text{solid}}^2} \right] q = 0$$

$$\frac{d^2 q}{dt^2} + \left[\frac{\rho_{\text{water}}}{\rho_{\text{ice}}} \frac{g}{LT_{\text{solid}}^2} \right] q = 0$$

FROM ICE CUBES TO ICEBERGS

$$\frac{d^2 q}{dt^2} + \left[\frac{\rho_{\text{water}}}{\rho_{\text{ice}}} \frac{g}{LT_{\text{solid}}^2} \right] q = 0$$

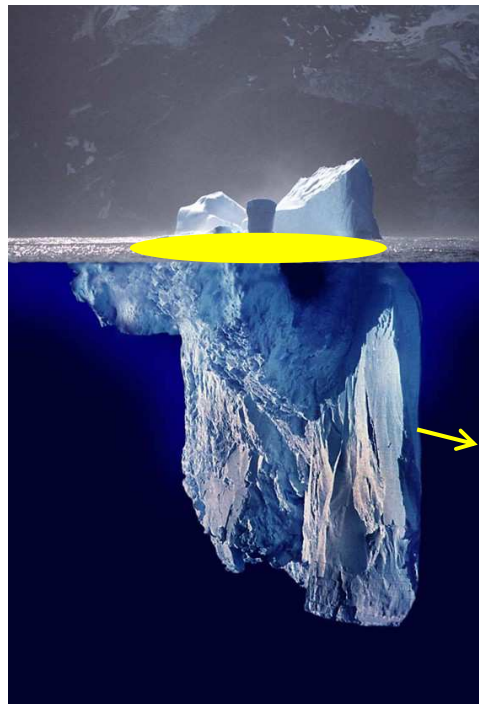
$$\bar{t} = \frac{t}{T_{\text{solid}}}$$

$$\frac{d^2 q}{dt^2} + \left[\frac{\rho_{\text{water}}}{\rho_{\text{ice}}} \frac{g}{L} \right] q = 0 \quad \Rightarrow \quad \omega = \sqrt{\frac{\rho_{\text{water}}}{\rho_{\text{ice}}} \frac{g}{L}}$$

$$T = 2\pi / \sqrt{\frac{\rho_{\text{water}}}{\rho_{\text{ice}}} \frac{g}{L}}$$

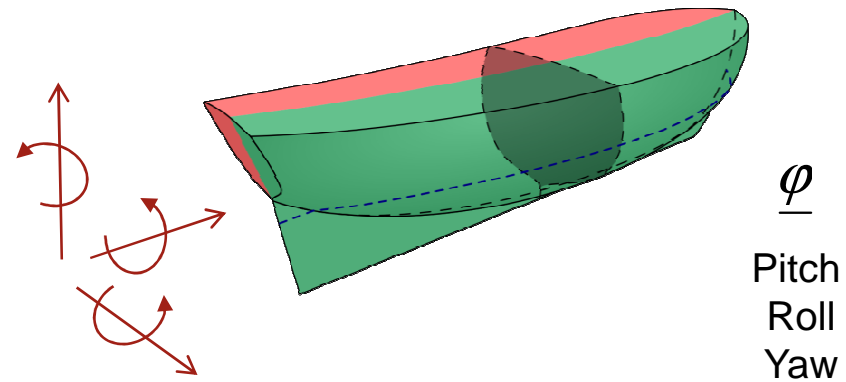
FROM ICE CUBES TO ICEBERGS

$$f = q \frac{M}{F_D^2} \int_{\text{Interface}} (\underline{\varphi} \cdot \underline{e}_z) (\underline{\varphi} \cdot \underline{n}) dS$$



$$\int_{\text{Interface}} (\underline{e}_z \cdot \underline{n}) dS = S_0$$

MORE GENERAL CASE



$$k_F = - \frac{M}{F_D^2} \int_{Interface} (\underline{\varphi} \cdot \underline{e}_Z) (\underline{\varphi} \cdot \underline{n}) dS$$

Stiffness in each type of motion

$$f = -q k_F$$

FLUID INDUCED FORCE



$$M \int_{Interface} \underline{\varphi} \cdot [\dots] \underline{n} dS - Mq \int_{Interface} (\underline{\nabla} P_0 \cdot \underline{\varphi})(\underline{\varphi} \cdot \underline{n}) dS = f$$



$$\left[-p \underline{\underline{I}} + \frac{1}{S_T} (\underline{\nabla} \underline{u} + \underline{\nabla}^t \underline{u}) \right]$$

Linearized fluid dynamics



Hydrostatics

SMALL REDUCED VELOCITY, SMALL MOTION

