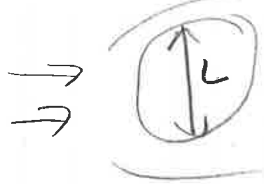
Fluid-Solid Interactions

Week 1

Dimensional Analysis



U, ρ, μ, L

$Re = \frac{\rho U L}{\rho}$ Reynolds Number

$f(x_1, x_2, \dots, x_N) = 0$, x_i are dimensional quantities

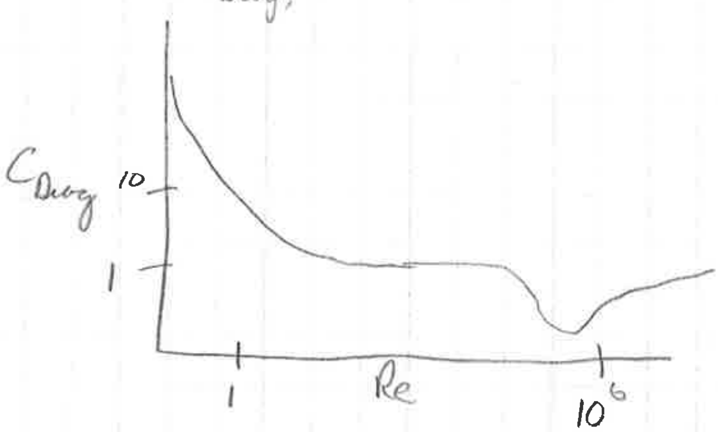
A physical law must relate only dimensionless quantities

$F(X_1, X_2, \dots, X_p) = 0$, X_i are dimensionless

Ex: Drag on a sphere

$$f(D, U, \rho, \mu, L) = 0 \iff F\left(\frac{D}{\rho U^2 L^2}, \frac{\rho U L}{\mu}\right) = 0$$

$$\iff F(C_{Drag}, Re) = 0$$



Week 1 cont'd

$$[x] = L^{\alpha} M^{\beta} T^{\gamma}$$

dimensions length mass time

Ex: gravity

$$g = 9.81 \ m/s^2$$

$$[g] = L^1 M^0 T^{-2}$$

Buckingham Π-theorem

$$f(x_1, x_2, \dots, x_N) = 0 \longrightarrow F(X_1, X_2, \dots, X_p) = 0$$

Dimensional Dimensionless

$$R = \text{rank} \begin{bmatrix} \alpha_1 & \cdots & \alpha_N \\ \beta_1 & \cdots & \beta_N \\ \gamma_1 & \cdots & \gamma_N \end{bmatrix} \Rightarrow p = N - R$$

Drag on a sphere

$$\begin{array}{c c} & \begin{matrix} D & U & \rho & \mu & L \end{matrix} \\ \begin{matrix} L \\ M \\ T \end{matrix} & R = \text{rank} \begin{bmatrix} 1 & 1 & -3 & -1 & 1 \\ 1 & 0 & 1 & 1 & 0 \\ -2 & -1 & 0 & -1 & 0 \end{bmatrix} = 3 \end{array} \Rightarrow p = 5 - 3 = 2$$

$$\Rightarrow F\left(\frac{D}{\rho U^2 L^2}, \frac{\rho U L}{\mu}\right) = 0$$

$$[X] = L^0 M^0 T^0$$

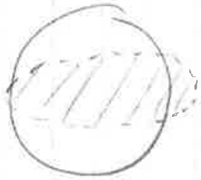
$$X = \frac{\text{time}}{\text{time}} = \frac{\text{length}}{\text{length}} = \frac{\text{force}}{\text{force}} = \cdots$$

$$C_{drag} = \frac{D}{\rho U^2 L^2} = \frac{\text{Drag force}}{\text{Scm of dynamic pressure}}$$

Week 1 cont'd

Fluid			Solid
Coordinates	$\underline{x}$	$\underline{x}$	Coordinates
Time	t	t	Time
Velocity Field	$\underline{U}$	$\underline{\xi}$	Displacement field
Viscosity	μ	E	Stiffness
Size	L	L	Size
Gravity	g	g	Gravity
Density	ρ	ρ_s	Density
Velocity Data	U_0	ξ_0	Displacement Data

Continuum



E Young's Modulus

ρ_s Density

Mass-Spring System

$$E = K/L$$

$$\rho_s = M/L^3$$

Fluid Alone

$$f(\underline{x}, t, \underline{U}, \mu, L, g, \rho, U_0) = 0$$

	$\underline{x}$	t	$\underline{U}$	μ	L	g	ρ	U_0
L	1	0	1	-1	1	1	-3	1
M	0	0	0	1	0	0	1	0
T	0	1	-1	-1	0	-2	0	-1

Independent

$R = 3$

$P = N - R = 8 - 3 = 5$

$$F\left(\frac{U}{U_0}, \frac{x}{L}, \frac{U_0 t}{L}, \frac{\rho U_0 L}{\mu}, \frac{U_0}{\sqrt{gL}}\right) = 0$$

Froude Number

$$F_R = \frac{U_0}{\sqrt{gL}}$$

Week 1 cont'd

$$\frac{U_o t}{L} = \frac{t}{T_{Fluid}} \quad \text{where} \quad T_{Fluid} = \frac{L}{U_o}$$

Solid Alone

$$f(\underline{x}, t, \underline{\xi}, E, L, g, \rho_s, \xi_o) = 0$$

	$\underline{x}$	t	$\underline{\xi}$	E	L	g	ρ_s	ξ_o
L	1	0	1	-1	1	1	-3	1
M	0	0	0	1	0	0	1	0
T	0	1	0	-2	0	-2	0	0

Indep

$$p = N - R = 8 - 3 = 5$$

$$F\left(\frac{\xi}{L}, \frac{x}{L}, \frac{t\sqrt{E/\rho_s}}{L}, \frac{\xi_o}{L}, \frac{\rho_s g L}{E}\right) = 0$$

D, displacement number

G, elastogravity number

$$\frac{t\sqrt{E/\rho_s}}{L} = \frac{t}{T_{solid}}, \quad T_{solid} = \frac{L}{c}, \quad c = \sqrt{E/\rho_s}$$

time elastic wave traverses a solid

These give decoupled FSI equations

Fluid and Solid

$$g(\underline{U}; \underline{x}, t; \mu, \rho, U_o, L, g; E, \rho_s, \xi_o) = 0$$

$N = 11$

	$\underline{U}$	$\underline{x}$	t	μ	ρ	U_o	L	g	E	ρ_s	ξ_o
L	1	1	0	-1	-3	1	1	1	-1	-3	1
M	0	0	0	1	1	0	0	0	1	1	0
T	-1	0	1	-1	0	-1	0	-2	-2	0	0