Final Exam

HSE 207: Engineering Mechanics

by Seungchul Lee

Problem 1

The spring is compressed by a cord between block A and block B. The masses of blocks A and B are 10 kg and 15 kg, respectively. Assume that spring mass is negligible, and a floor is frictionless.

1) [5pt] If $V_A = 6\,m/s$ when the cord is cut, determine the velocity of block B.

2) [5pt] A spring constant is $k = 15\,kN/m$ and the spring is compressed by $200\,mm$. When the cord is cut, determine the velocity of block A.

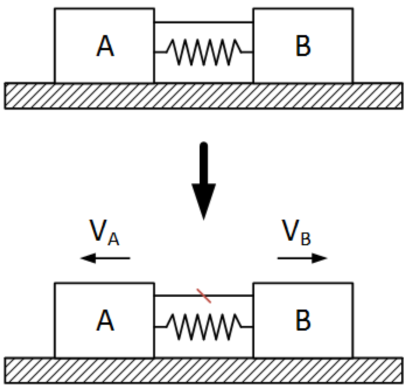

Figure 1

Problem 2

1) [5pt] Explane Figure 2 based on what we learned from the class.

2) [5pt] Explane Figure 3 based on what we learned from the class.

Figure 2

Figure 3

Problem 3

[10pt] The mass m is pushed against the spring at A and released from rest. Neglecting friction, determine the smallest deflection of the spring for which the mass will travel around the loop and remain at all times in contact with the loop. Use the parameters m, g, r, k

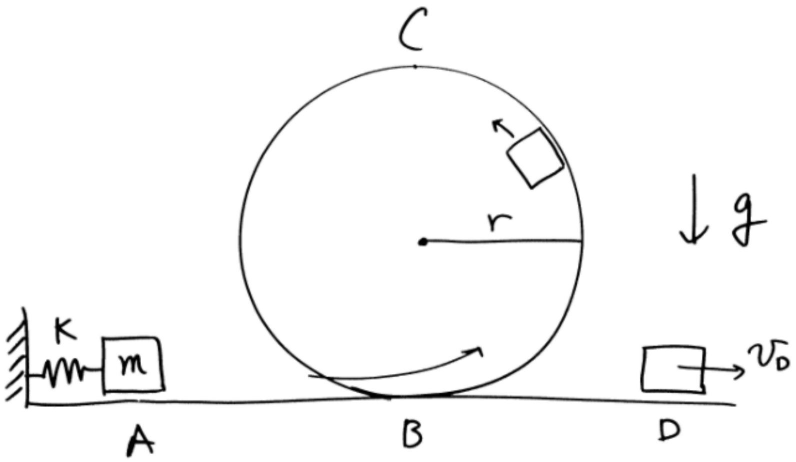

Figure 4

Problem 4

[10pt] A block m is at rest at the top of a smooth cylindrical surface with radius = r. Suddenly, under the force of gravity, it beings to slide down the surface until it leaves the surface. Find the values of θ at which the block leaves the surface.

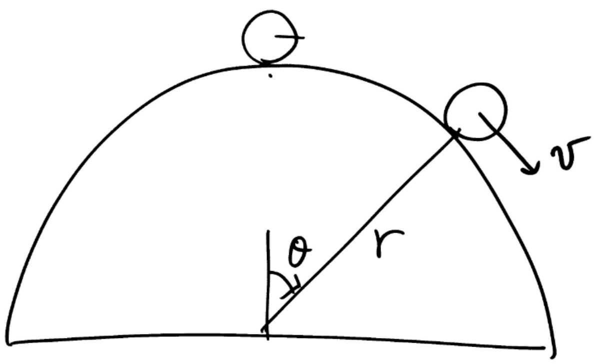

Figure 5

Problem 5

[10pt] The pendulum of mass m is released from rest when it is at A. Determine the *speed* of mass m and the *tension* in the cord when mass m passes through its lowest position B. Use the parameters shown in Figure 6.

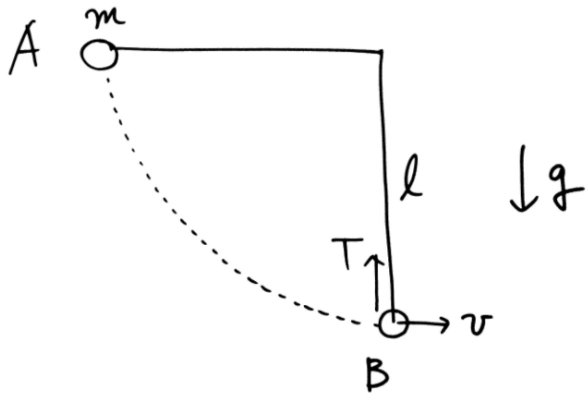

Figure 6

Problem 6

[5pt] Describe the change of room temperature when the door of an refrigerator is open. Clearly define the system in your explanation.

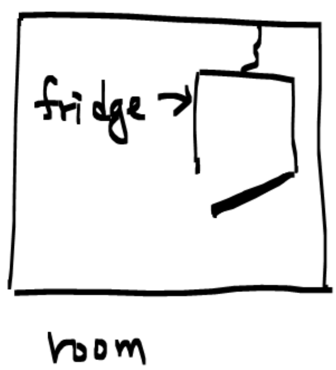

Figure 7

Problem 7

Consider the system which takes the *integral* of input.

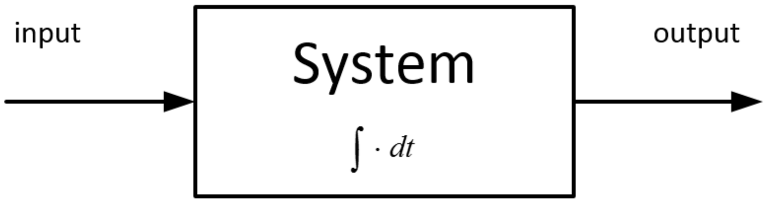

Figure 8

1) [5pt] Draw the ouput of the system in a complex plane of Figure 9. Indicate all the necessary information in your sketch.

2) [5pt] Sketch the ouput of the system in a time domain of Figure 10. Indicate all the necessary information in your sketch.

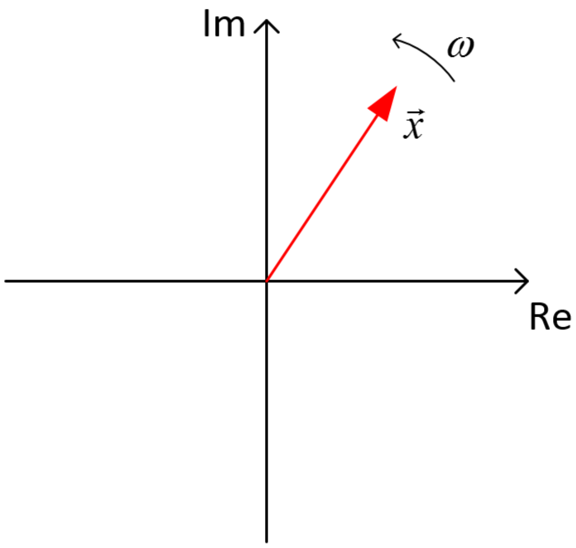

Figure 9: Complex plane

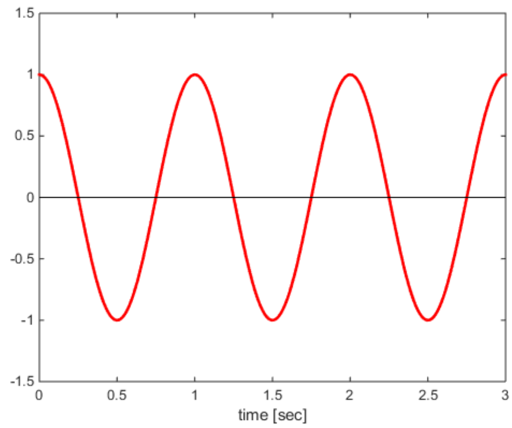

Figure 10: in time domain

Problem 8

We learned in a class that a simple harmonic motion in the spring and mass system can be understood by a circular motion as shown in Figure 11.

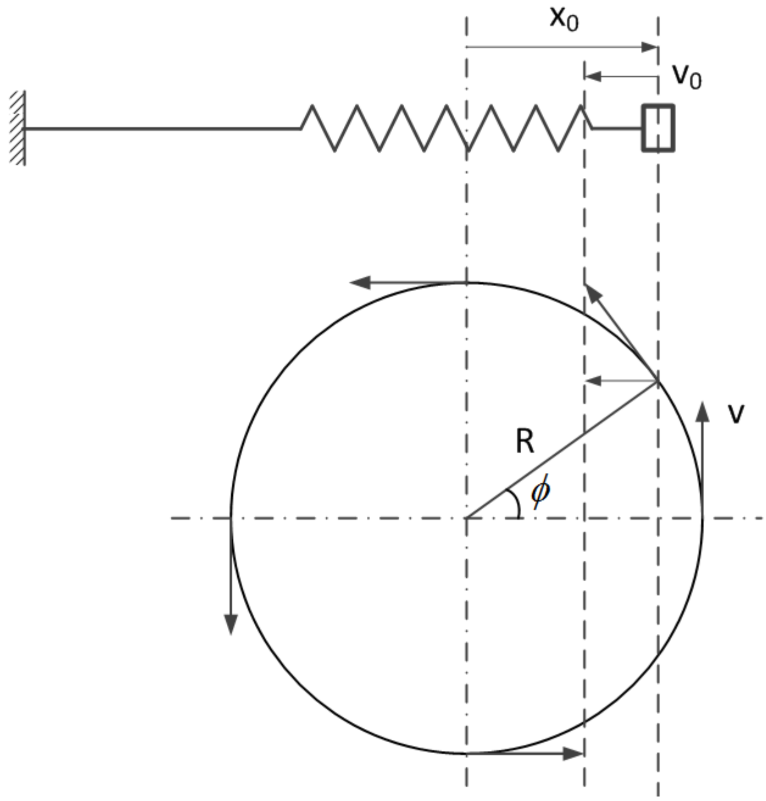

Figure 11

1) [2pt] Show that the equation of motion is

$$\ddot{x} + \omega_n^2 x = \ddot{x} + \frac{k}{m}x = 0$$

2) [2pt] Show that $x(t) = R\cos(\omega_n t + \phi)$ satisfies the above equation of motion

3) [2pt] Show that $z(t) = Re^{j(\omega_n t + \phi)}$ satisfies the above equation of motion

4) [5pt] Given the initial conditions of $x(0) = x_0$, and $\dot{x}(0) = v_0$, find the values of R and ϕ using geometrical relationship shown in Figure 11.

5) [5pt] Use the mechanical energy conservation to find the maximum displacement R and show its value is the same as the one derived from 4)

6) [5pt] Use the mechanical energy conservation to find the maxium speed V at $x = 0$ and show its value is the same as the one derived from 4)

Problem 9

[5pt] Explain why a circular motion can be a solution of equation of $m\ddot{z} + kz = 0$ using Figure 12.

Hint: $p(t) = z$ and think about directions

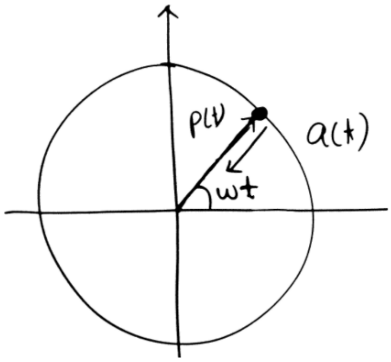

Figure 12

Problem 10

[5pt] Two figures show the free oscillation of two different systems. What makes two systems respond differently?

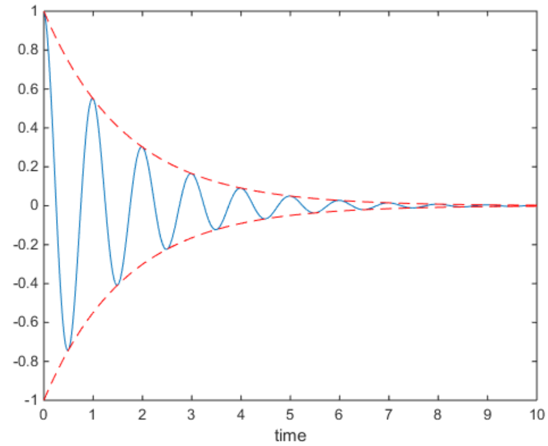

Figure 13: System A

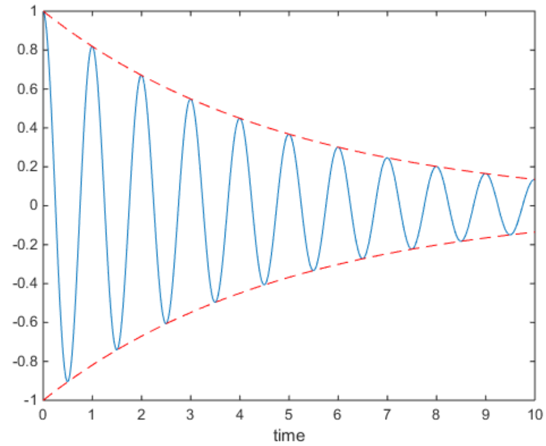

Figure 14: System B

Problem 11

1) [2pt] Compute the natural frequency ω_n (angular velocity) for the system (a)

2) [2pt] Compute the oscillating frequency ω (angular velocity) for the system (b)

3) [5pt] Explain why the system (b) is oscillating with a slower ω than $\omega_n = \sqrt{\frac{k}{m}}$.

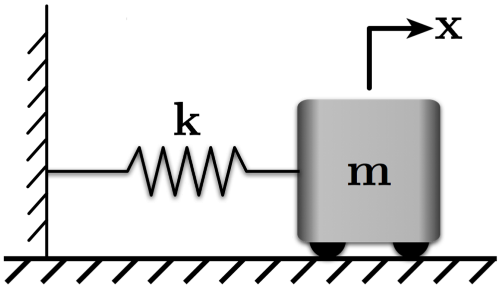

Figure 15: a

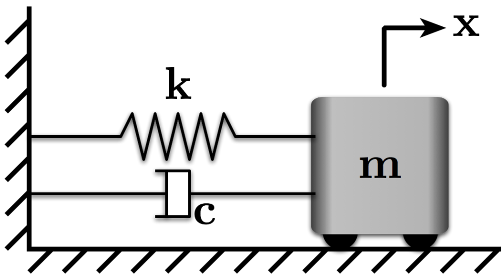

Figure 16: b

Problem 12

[10pt] For a given linear time invariant second order system (for instance, a mass-spring-damper system), we obtained the following plots showing amplitude and phase of forced damped oscillation (amplitude plot on the left, and phase plot on the right) with respect to frequency of external excitation force with $\gamma = \frac{\Omega}{\omega_n}$ and $\zeta = 0.3$. Find the output steady state solution of amplitude and phase with an input force of $f(t) = \mathrm{Re}\left(5 \cdot e^{j\omega_n t} + 1 \cdot e^{j1.5\omega_n t}\right)$

(For simplicity, assume $k = 1$)

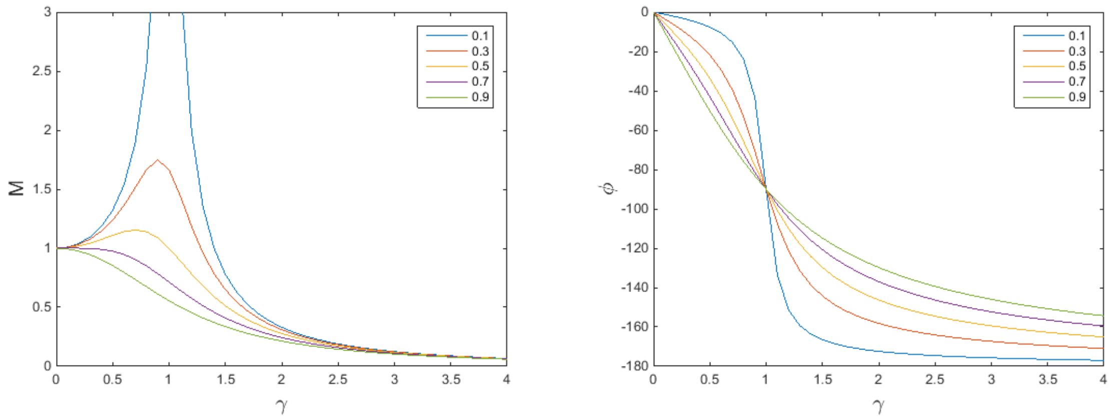

Figure 17

Problem 13

[5pt] A snapshot of a glass broken by the sound of a speaker is captured in Figure 18. Explain how it is possible.

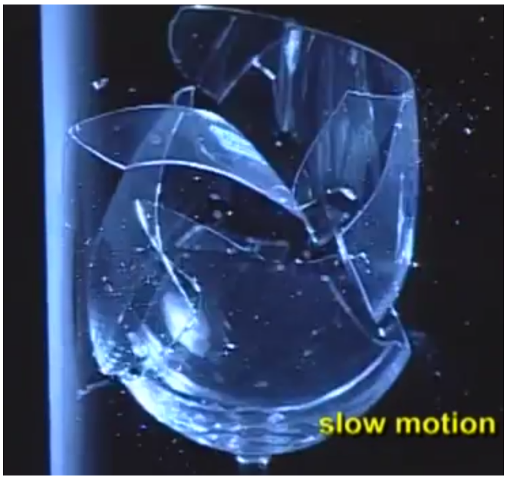

Figure 18