Problem 1.

1) $m_A v_A + m_B v_B = 0$

$$\therefore v_B = -\frac{m_A v_A}{m_B} = -\frac{10\cdot(-6)}{15} = 4\,m/s \longrightarrow$$

2) $$\begin{cases} \frac{1}{2}kx^2 = \frac{1}{2}m_A v_A^2 + \frac{1}{2}m_B v_B^2 \\ m_A v_A = -m_B v_B \end{cases}$$

$k = 15\,kN/m$, $m_A = 10\,kg$, $m_B = 15\,kg$, $x = 200\,mm$.

$\therefore v_A = 6\,m/s \leftarrow$

Problem 2.

1) Conservation of momentum.

2) Conservation of angular momentum.

Problem 3.

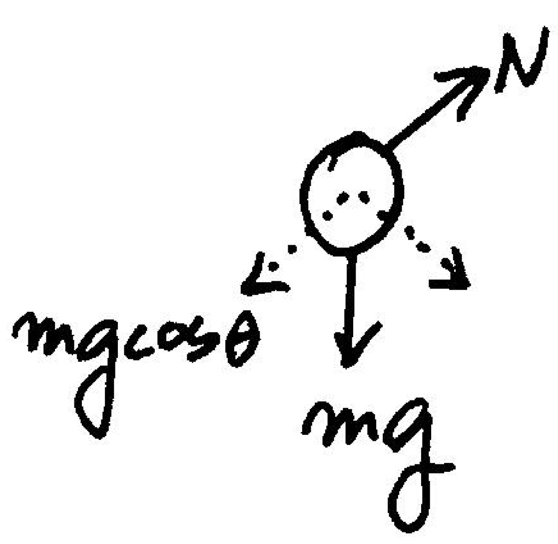
$$\begin{cases} \frac{1}{2}kx^2 = \frac{1}{2}mv_c^2 + mg\cdot 2r \\ mg = \frac{mv_c^2}{r} \end{cases}$$

$$\therefore x = \sqrt{\frac{5mgr}{k}}$$

Problem 4.

$$mgr = \frac{1}{2}mv^2 + mgr\cos\theta$$

$$\Rightarrow v^2 = 2gr(1-\cos\theta)$$



$$mg\cos\theta - N = m\cdot\frac{v^2}{r}$$

$$= m\cdot 2g(1-\cos\theta)$$

It leaves the surface $\Rightarrow N=0$

$$\therefore mg(3\cos\theta - 2) = 0$$

$$\therefore \cos\theta = \frac{2}{3}$$

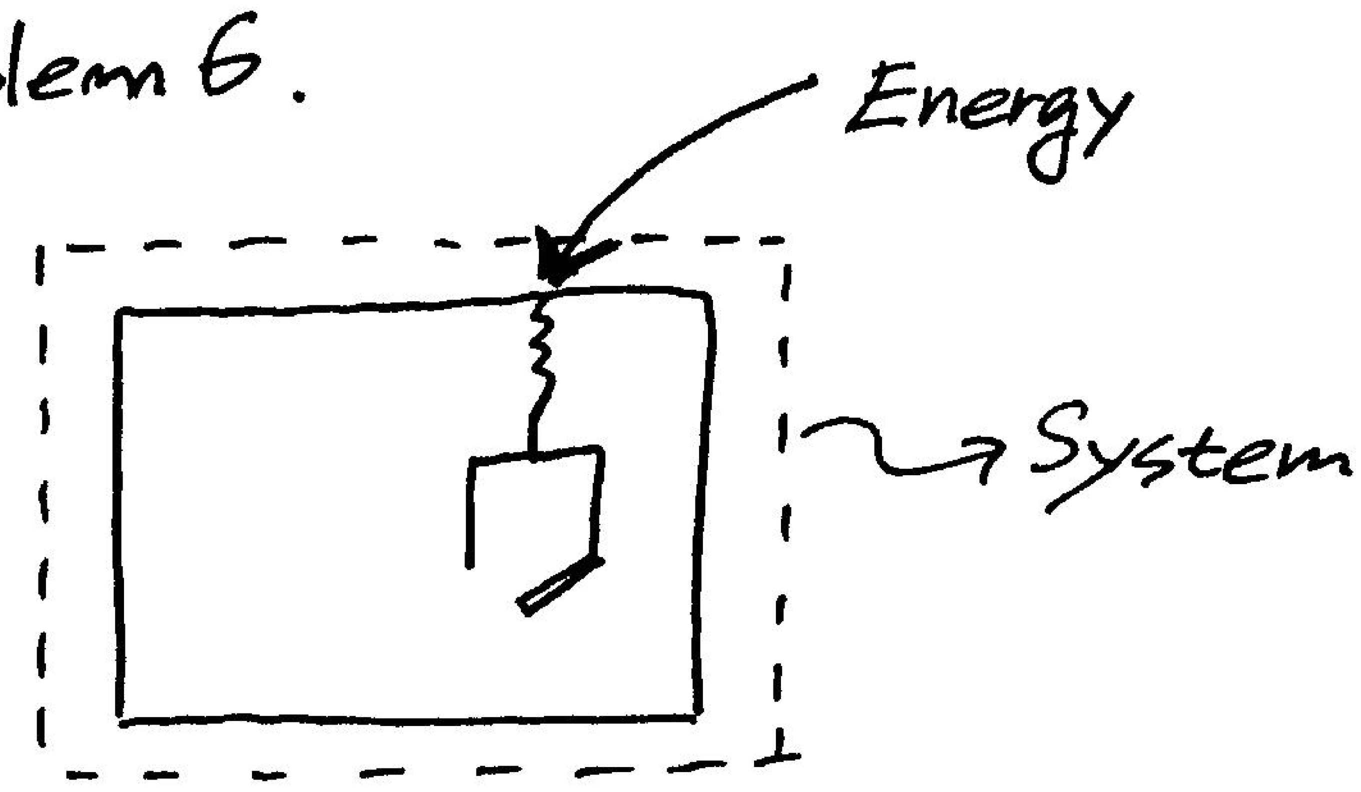
$$\therefore \theta = \cos^{-1}\frac{2}{3}.$$

Problem 5.

$$mgl = \frac{1}{2}mv^2 \Rightarrow v = \sqrt{2gl}.$$

$$T - mg = m\cdot\frac{v^2}{l} \Rightarrow T = 3mg.$$

Problem 6.



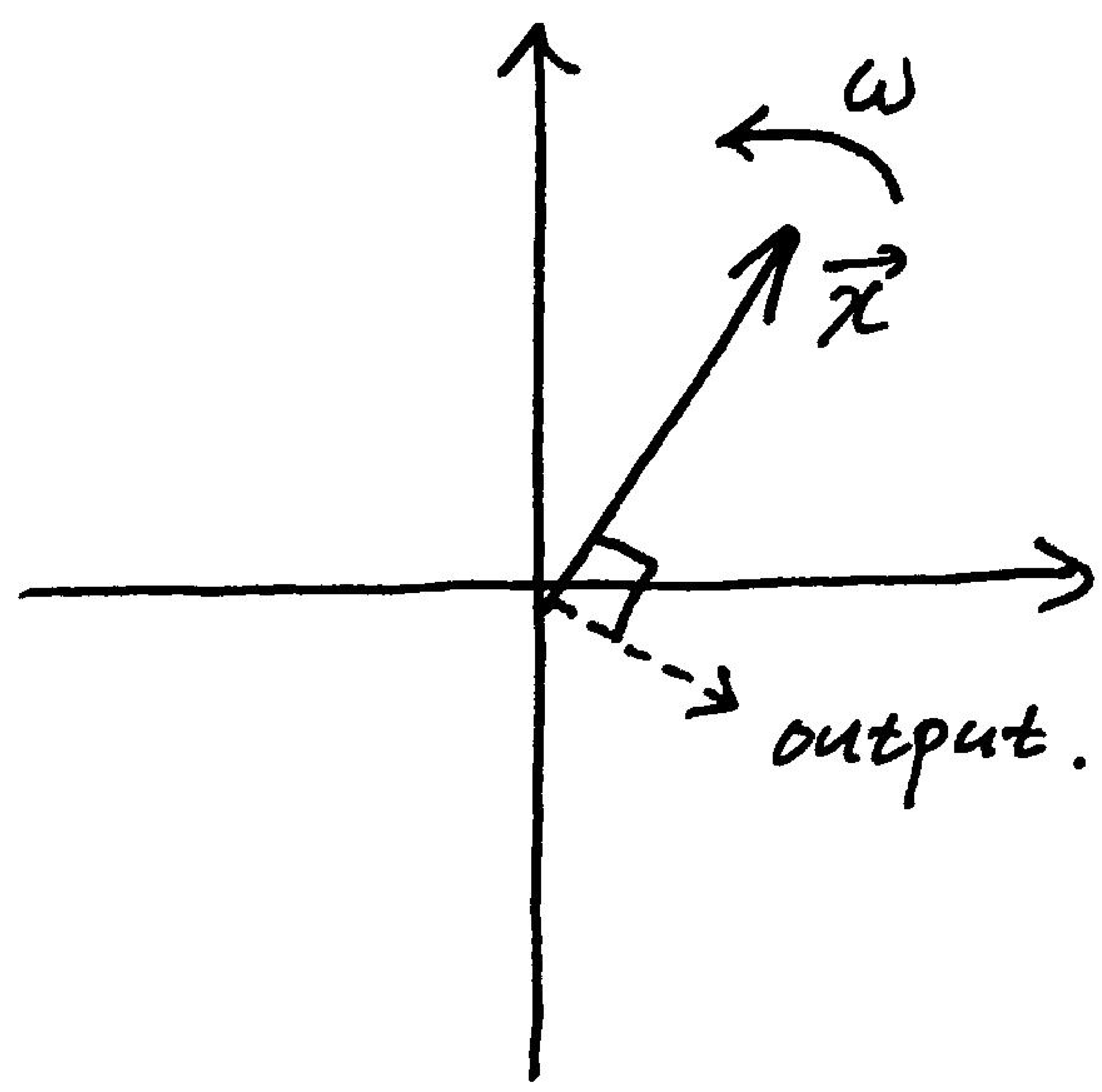
Energy is supplied.

$\Rightarrow$ Room temperature is increasing.

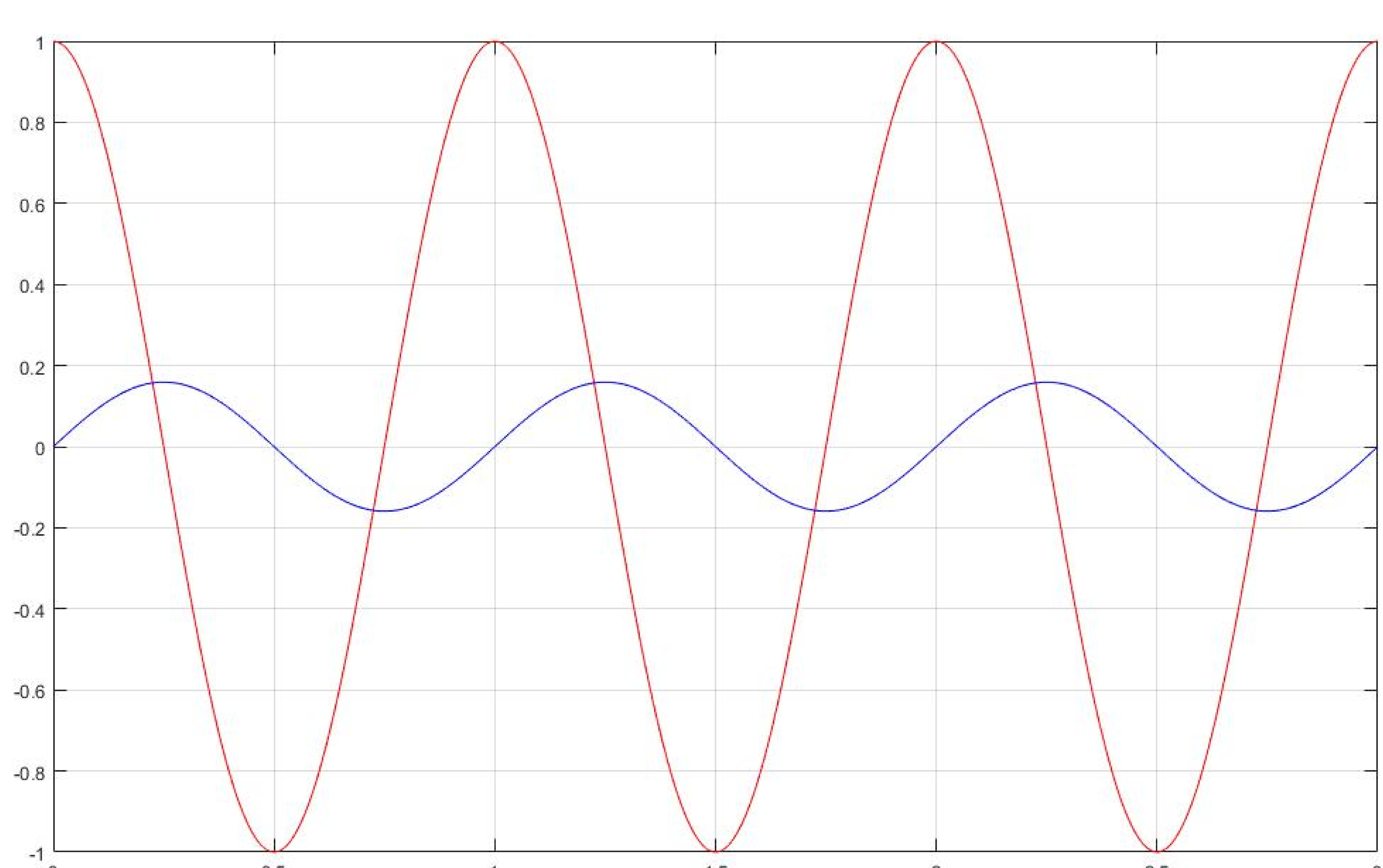
Problem 7.

1) $x = A e^{j\omega t}$

$$\int x\,dt = \frac{1}{j\omega} A e^{j\omega t} = \frac{1}{\omega} A e^{j(\omega t - \frac{\pi}{2})}$$



2) $T = 1\,s \Rightarrow 2\pi = \omega.$



problem 8.

1)

$$kx \leftarrow \boxed{} = \boxed{m} \rightarrow \ddot{x}$$

$$-kx = m\ddot{x}$$

$$\Rightarrow \quad \ddot{x} + \frac{k}{m} = 0$$

$$\ddot{x} + \omega_n^2 x = 0 \quad , \quad \omega_n^2 = \frac{k}{m} .$$

2)

$$x(t) = R\cos(\omega_n t + \phi)$$

$$\dot{x}(t) = -R\omega_n \sin(\omega_n t + \phi)$$

$$\ddot{x}(t) = -R\omega_n^2 \cos(\omega_n t + \phi)$$

$$\Rightarrow \quad \ddot{x} + \omega_n^2 x = 0 .$$

3)

$$z(t) = R e^{j(\omega_n t + \phi)}$$

$$\dot{z}(t) = j\omega_n R e^{j(\omega_n t + \phi)}$$

$$\ddot{z}(t) = -\omega_n^2 R e^{j(\omega_n t + \phi)}$$

$$\Rightarrow \quad \ddot{z} + \omega_n^2 z = 0 .$$

4)

$$x_0 = R\cos\phi \quad \Rightarrow \quad \cos\phi = \frac{x_0}{R}$$

$$v_0 = R\omega_n \sin\phi \quad \Rightarrow \quad \sin\phi = \frac{v_0}{R\omega_n}$$

$$\left(\frac{x_0}{R}\right)^2 + \left(\frac{v_0}{R\omega_n}\right)^2 = 1$$

$$\Rightarrow \quad R = \sqrt{x_0^2 + \left(\frac{v_0}{\omega_n}\right)^2}$$

$$\phi = \tan^{-1}\left(\frac{v_0}{x_0\omega_n}\right)$$

5) $\frac{1}{2}kx_0^2 + \frac{1}{2}mv_0^2 = \frac{1}{2}kR^2$

$$\Rightarrow R^2 = x_0^2 + \frac{m}{k}v_0^2 = x_0^2 + \left(\frac{v_0}{\omega_n}\right)^2$$

$$\therefore R = \sqrt{x_0^2 + \left(\frac{v_0}{\omega_n}\right)^2}$$

6) $\frac{1}{2}kx_0^2 + \frac{1}{2}mv_0^2 = \frac{1}{2}mV^2$

$$\Rightarrow V^2 = v_0^2 + \omega_n^2 x_0^2$$

$$\therefore V = \sqrt{v_0^2 + (\omega_n x_0)^2}$$

$$R = \frac{V}{\omega_n} = \sqrt{x_0^2 + \left(\frac{v_0}{\omega_n}\right)^2}.$$

Problem 9.

$z = p(t) = r \cdot e^{j\omega t}$

$\dot{z} = v(t) = j\omega r e^{j\omega t}$

$\ddot{z} = a(t) = -\omega^2 \cdot p(t).$

$\omega^2 = \frac{k}{m} \quad \Rightarrow m\ddot{z} + kz = 0.$

Problem 10.

ξ (Damping ratio)

Problem 11.

1) $\omega_n = \sqrt{\frac{k}{m}}$

2) $\omega = \sqrt{\omega_n^2 - \gamma^2} = \sqrt{\frac{k}{m} - \frac{c^2}{4m^2}}$

3) Damper. $\left(\frac{\omega}{\omega_n} < 1 \right)$

Problem 12.

i) $\gamma = 1$

$M \approx 1.7$

$\phi \approx -90^\circ$

ii) $\gamma = 1.5$

$M \approx 0.7$

$\phi \approx -140^\circ$

$\therefore x_1(\infty) = 8.5\cos(\omega_n t - 90^\circ)$

$x_2(\infty) = 0.7\cos(1.5\omega_n t - 140^\circ)$

$\therefore \quad 8.5\cos(\omega_n t - 90^\circ) + 0.7\cos(1.5\omega_n t - 140^\circ).$

Problem 13.

- Increase amplitude of the input signal by increasing the speaker volume.
- Tune the frequency of the input signal to match resonance (ω_n) of the given glass.