

Midterm Exam
HSE 207: Engineering Mechanics
Solutions

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Problem 1 (5 points)

- Linear translation motion can be considered as a circular motion with the very large radius.
- Linear combination of circular motions can generate many different motions such as linear translation, ellipsoid, etc.

Problem 2 (10 points)

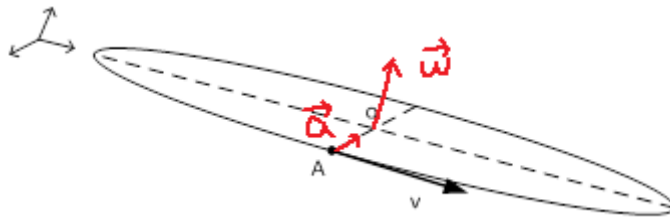
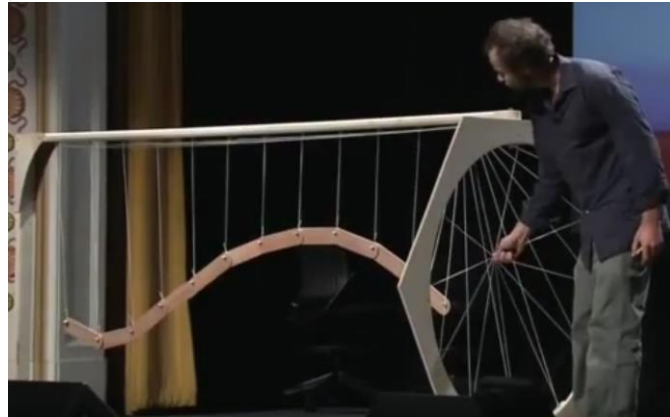
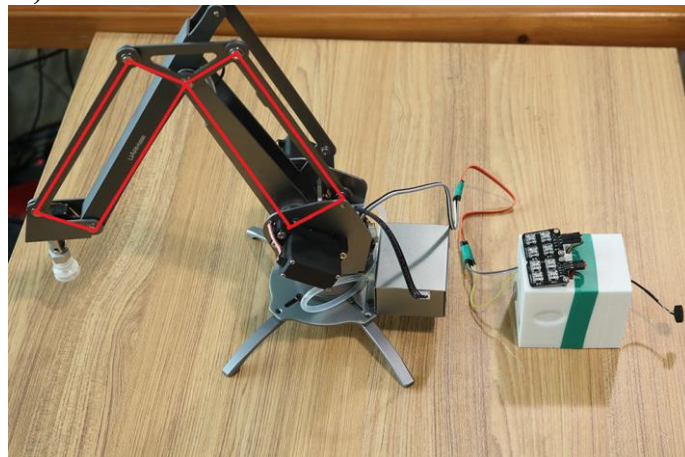


Figure 1

Problem 3 (5 points)

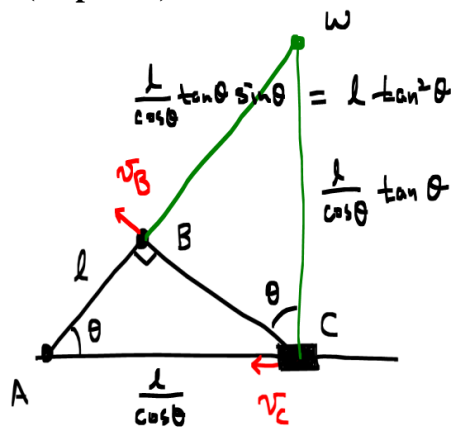


Problem 4 (10 points)



Problem 5 (5 points)

There are many cylinders in a car engine.

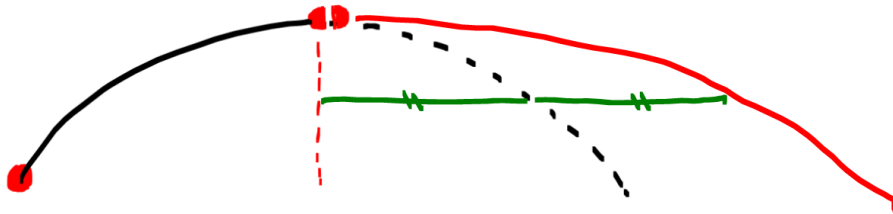
Problem 6 (10 points)

$$v_B = l \dot{\theta} = l \tan^2 \theta \omega$$

$$\therefore \omega = \frac{\dot{\theta}}{\tan^2 \theta}$$

$$\Rightarrow v_C = \frac{l}{\cos \theta} \tan \theta \cdot \omega$$

$$= \frac{l}{\sin \theta} \dot{\theta}$$

Problem 7 (10 points)**Problem 8 (10 points)**

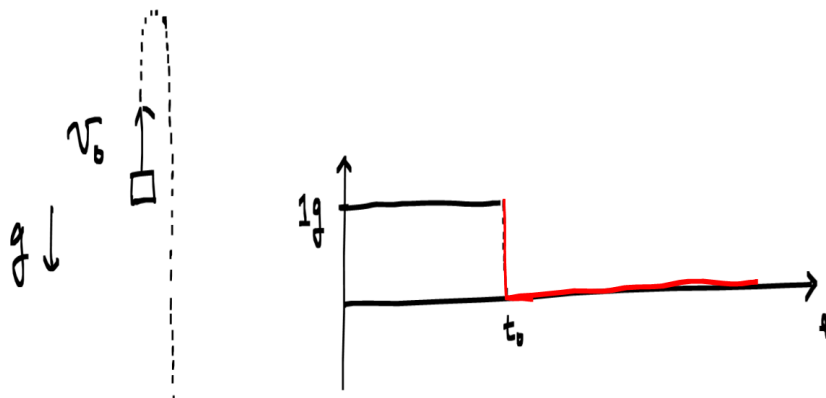
- 1) Inertia reference frame
- 2) Non-inertia reference frame

Problem 9 (10 points)

The same length = 0.1 m



Problem 10 (5 points)

Zero

Problem 11 (10 points)

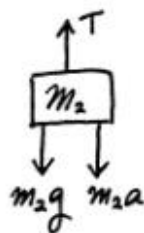
Problem 12 (20 points)

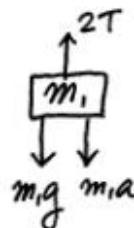
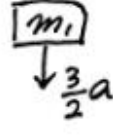
Problem 12.

(1)  =  $T - m_2 g = m_2 a$

 =  $m_1 g - 2T = m_1 \cdot \frac{a}{2}$

$$\therefore a = \frac{2m_1 - 4m_2}{m_1 + 4m_2} g$$

(2)  = 0 $T - m_2 g - m_2 a = 0$

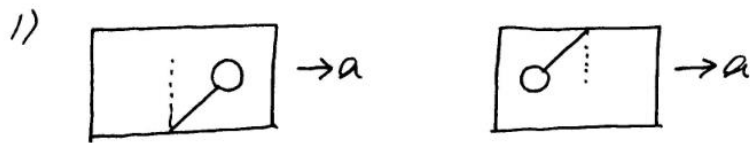
 =  $m_1 g + m_1 a - 2T = \frac{3}{2} m_1 a$

$$\therefore a = \frac{2m_1 - 4m_2}{m_1 + 4m_2} g$$

(3) weight of mass $m_1 = m_1 g - m_1 \cdot \frac{a}{2}$

weight of mass $m_2 = m_2 g + m_2 a$.

Problem 13 (20 points)
 problem 13.



2)

$$\Sigma F_x = ma ; T \sin \theta = ma .$$

$$\Sigma F_y = 0 ; T \cos \theta - mg = 0 .$$

3)

$$\Sigma F_x = 0 ; T \sin \theta - ma = 0 .$$

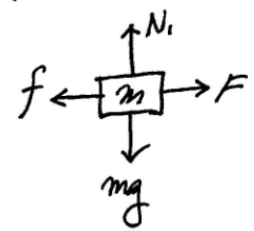
$$\Sigma F_y = 0 ; T \cos \theta - mg = 0 .$$

4) weight of a ball $= T = \frac{ma}{\sin \theta} = \frac{ma}{\sin(\tan^{-1}(\frac{a}{g}))}$

Problem 14 (10 points)

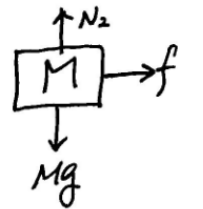
Problem 14.

1)



A free-body diagram for mass m . It is a square box labeled m . Four force vectors are shown: N_1 pointing vertically upwards, mg pointing vertically downwards, f pointing horizontally to the left, and F pointing horizontally to the right.

$$f \leftarrow \boxed{m} \rightarrow F = \boxed{m} \rightarrow a \quad F - f = ma.$$

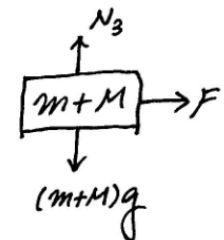


A free-body diagram for mass M . It is a square box labeled M . Four force vectors are shown: N_2 pointing vertically upwards, Mg pointing vertically downwards, f pointing horizontally to the right, and an unlabeled force pointing horizontally to the left.

$$\boxed{M} \rightarrow f = \boxed{M} \rightarrow a \quad f = Ma.$$

$$\therefore a = \frac{F}{m+M}.$$

2)

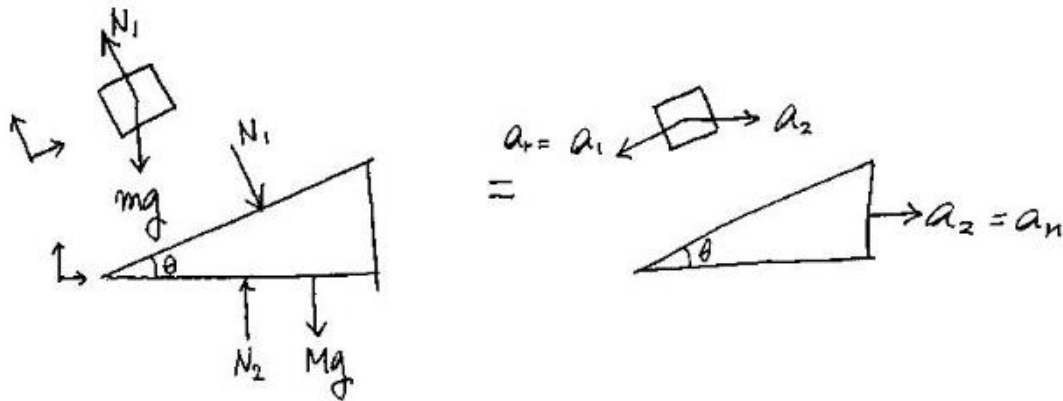


A free-body diagram for the combined mass $m+M$. It is a square box labeled $m+M$. Four force vectors are shown: N_3 pointing vertically upwards, $(m+M)g$ pointing vertically downwards, F pointing horizontally to the right, and an unlabeled force pointing horizontally to the left.

$$\boxed{m+M} \rightarrow F = \boxed{m+M} \rightarrow a \quad F = (m+M)a$$

$$\therefore a = \frac{F}{m+M}.$$

Problem 15 (20 points)



$$\text{For } M : \quad N_2 - Mg - N_1 \cos \theta = 0$$

$$N_1 \sin \theta = M a_2$$

$$\text{For } m : \quad N_1 - mg \cos \theta = -m a_1 \sin \theta$$

$$-mg \sin \theta = m(-a_1 + a_2 \cos \theta)$$

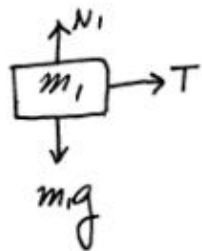
$$\therefore a_n = \frac{mg \sin \theta \cos \theta}{M + m \sin^2 \theta}, \quad a_1 = \frac{mg \sin \theta \cos^2 \theta}{M + m \sin^2 \theta} + g \sin \theta.$$

$$N_1 = \frac{M}{\sin \theta} a_2 = \frac{M mg \cos \theta}{M + m \sin^2 \theta}$$

$$N_2 = Mg + N_1 \cos \theta = Mg + \frac{M mg \cos^2 \theta}{M + m \sin^2 \theta}.$$

Problem 16 (20 points)

Problem 16.



A free-body diagram for mass m_1 . It shows a rectangular box labeled m_1 . Four force vectors are drawn from the center: an upward arrow labeled N_1 , a downward arrow labeled $m_1 g$, a rightward arrow labeled T , and a leftward arrow labeled $m_1 a$.

$$T = m_1 a.$$



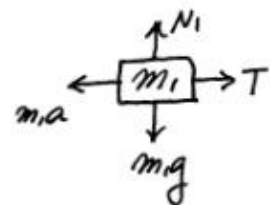
A free-body diagram for mass m_2 . It shows a rectangular box labeled m_2 . Three force vectors are drawn from the center: an upward arrow labeled $2T$, a downward arrow labeled $m_2 g$, and a downward arrow labeled $\frac{a}{2}$.

$$m_2 g - 2T = m_2 \cdot \frac{a}{2}$$

$$1) \quad a_1 = \frac{2m_2 g}{4m_1 + m_2}, \quad a_2 = \frac{m_2 g}{4m_1 + m_2}.$$

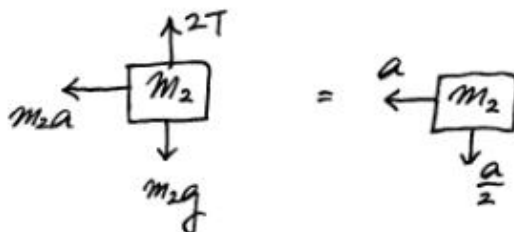
$$2) \quad T_1 = \frac{2m_1 m_2 g}{4m_1 + m_2}, \quad T_2 = \frac{4m_1 m_2 g}{4m_1 + m_2}.$$

4)



A free-body diagram for mass m_1 . It shows a rectangular box labeled m_1 . Four force vectors are drawn from the center: an upward arrow labeled N_1 , a downward arrow labeled $m_1 g$, a rightward arrow labeled T , and a leftward arrow labeled $m_1 a$.

$$= 0$$

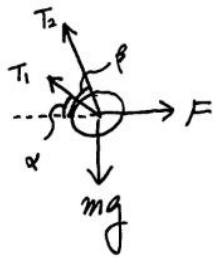


A free-body diagram for mass m_2 . It shows a rectangular box labeled m_2 . Three force vectors are drawn from the center: an upward arrow labeled $2T$, a downward arrow labeled $m_2 g$, and a leftward arrow labeled $m_2 a$.

$$= \frac{a}{2}$$

Problem 17 (10 points)

Problem 17.



$$\Sigma F_x = 0; F - T_1 \cos \alpha - T_2 \cos \beta = 0.$$

$$\Sigma F_y = 0; T_1 \sin \alpha + T_2 \sin \beta - mg = 0.$$

The force in cable AC is zero

$$\Rightarrow T_1 = 0$$

$$\Rightarrow \begin{cases} F - T_2 \cos \beta = 0 \\ T_2 \sin \beta - mg = 0 \end{cases}$$

$$\Rightarrow \tan \beta = \frac{mg}{F}$$

$$\Rightarrow \frac{d + l_1}{l_2} = \frac{mg}{F}$$

$$\therefore d = \frac{mg}{F} l_2 - l_1$$

Problem 18 (10 points)

- 1) Bicycle transition between public bicycle stations
- 2) Daejeon
- 3) Bicycle stations which have higher utilization, bicycle path that are often used by users, etc.
- 4) Which station has higher priority when more bicycles are allocated?