Introduction to Time Series Analysis

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Overview

1. Introduction to data with time dimension
2. Stationary and non-stationary time series
3. Auto-correlation function
4. Basic time series models
5. Time series analysis with Eviews

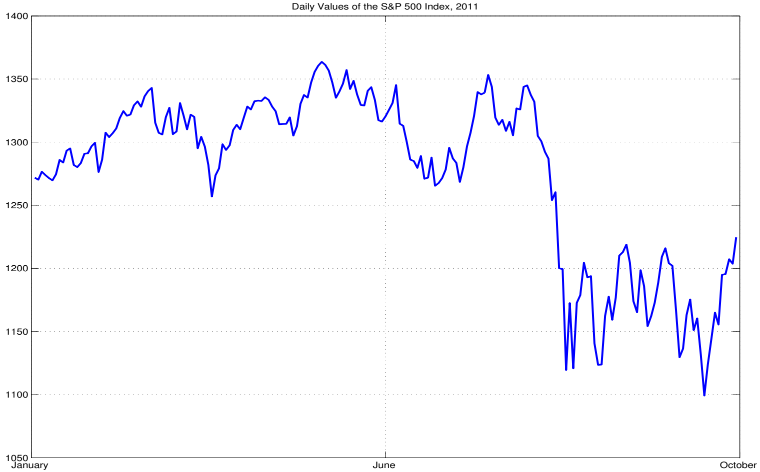
1. Introduction

What is time series?

- Random quantity that evolves over time
- For example, river flow, stock prices or human height
- Sample at intervals to get sequence of observations
- Such sequence is a time series

1. Introduction

Example: Daily Values of the S&P 500 Index



1. Introduction

What makes time series special?

- Consider a sample of ten male students
- If students are of same age, heights are identically distributed
- If selected at random and unrelated, heights are independent
- Independent and identically-distributed (i.i.d.) sample is key
- Law of Large Numbers ensures consistent mean estimate

1. Introduction

What makes time series special?

- Consider observing the height of one student for ten years
- Observations are no longer independent
- Most importantly, observations are not identically distributed
- Averaging out does not give true estimate of mean height

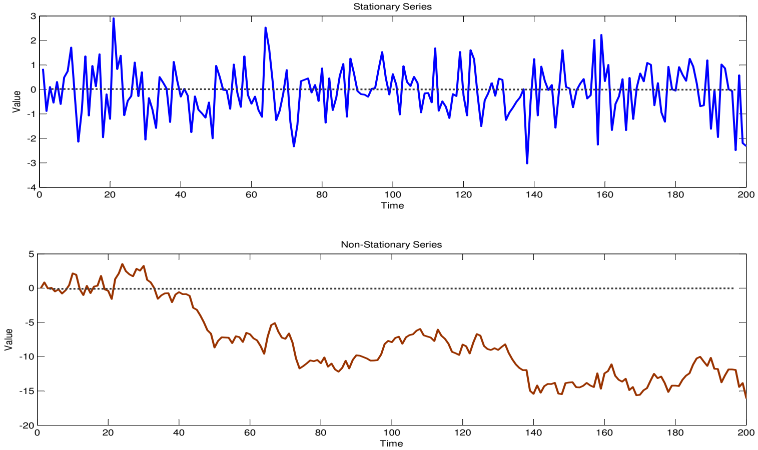
2. Stationarity and Non-Stationary Series

Stationarity

- In general, let $X_t,\ t = 1, 2, ...$ be sequence of random variables
- We get inconsistent estimates if $F_{X_i}(x_i) \neq F_{X_j}(x_j)$
- That is if probability distribution changes over time
- In this case, time-series is **non-stationary**
- Similarly, if $F_{X_i}(x_i) = F_{X_j}(x_j) = F_X(x_k)$, series is **stationary**
- We may use usual inference tools with stationary time-series

2. Stationarity and Non-Stationary Series

Stationary vs Non-Stationary Series



2. Stationarity and Non-Stationary Series

Consequences of Non-Stationary

- Spurious regression results
- Exceptionally high R^2 values and t-ratios
- No economic meaning

2. Stationarity and Non-Stationary Series

Testing for Non-Stationarity

- Many time-series in social sciences are non-stationary
- Several parametric and non-parametric methods are available
- One popular method is the Wald & Wolfowitz "runs" test
- Basic idea is to split time series into several sub-series
- Check if means and standard deviations are different
- Also may use Dickey & Fuller "unit root" test

3. Auto-Correlation Function (ACF)

Correlogram and Auto-Correlations

- Before formal test, a useful "first-pass" is finding the ACF
- Auto-Correlation Function shows correlations of different lags
- For example, X_t may be correlated with X_{t-1}, but not with X_{t-2}
- **Correlogram** shows ACF values at many lags
- Major problem with ACF: it only picks up linear dependence

3. Auto-Correlation Function (ACF)

Example: Correlogram

Date: 10/17/11 Time: 15:28
Sample: 1 200
Included observations: 200

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.970	0.970	191.11	0.000
		2	0.945	0.055	373.16	0.000
		3	0.918	-0.020	546.07	0.000
		4	0.901	0.133	713.24	0.000
		5	0.883	0.015	874.84	0.000
		6	0.866	-0.000	1031.1	0.000
		7	0.846	-0.053	1180.8	0.000
		8	0.826	0.001	1324.3	0.000
		9	0.806	-0.005	1461.7	0.000
		10	0.785	-0.040	1592.9	0.000
		11	0.769	0.054	1719.2	0.000
		12	0.753	0.027	1841.2	0.000

3. Auto-Correlation Function (ACF)

Interpreting Correlogram

- ACF may help identify the nature of the series
- For example, long-decaying ACF may indicate non-stationarity
- Zero auto-correlations may signal random noise
- Other patters may suggest certain types of processes

4. Common Time-Series Models

Dealing with Non-Stationarity

- Many non-stationary series may have stationary differences
- Even if we can't work with levels, can use stationary differences
- Given that now stationary series, what are the common models?

4. Auto-Regressive Model

Auto-Regressive Models

- AR(p) model is the "workhorse" of time-series econometrics
- We model variable at time t as function of its p lags
- For example, let Y_t, $t = 1, 2...$ be stationary time series
- Basic Auto-Regressive model of order p is given by:

$$Y_t = \alpha_0 + \alpha_1 Y_{t-1} + \alpha_2 Y_{t-2} + ... + \alpha_p Y_{t-p} + \epsilon_t$$

4. Auto-Regressive Model

AR(1) Model

- Simplest model of this form is $AR(1)$:

$$Y_t = \alpha_0 + \alpha_1 Y_{t-1} + \epsilon_t$$

- A process of this form is stationary as long as $|\alpha_1| < 1$
- Then we can use usual least squares techniques to estimate $\hat{\alpha}_1$
- If $|\alpha_1| \geq 1$, process has a **unit root** and is non-stationary
- Note that we can't use the usual t-test on $\hat{\alpha}_1$ for $H_0 : |\alpha_1| \geq 1$
- Under H_0 $\hat{\alpha}_1$ has unknown distribution!

4. Auto-Regressive Model

Testing for Unit Root in AR(1) Model

- Instead use the Dickey & Fuller test
- Rewrite model in terms of differences:

$$Y_t - Y_{t-1} = (\alpha_0 - 1)Y_{t-1} + \epsilon_t$$

- Or alternatively as

$$\Delta Y_t = \delta_0 Y_{t-1} + \epsilon_t$$

- Under H_0, $\delta_0 = 0$

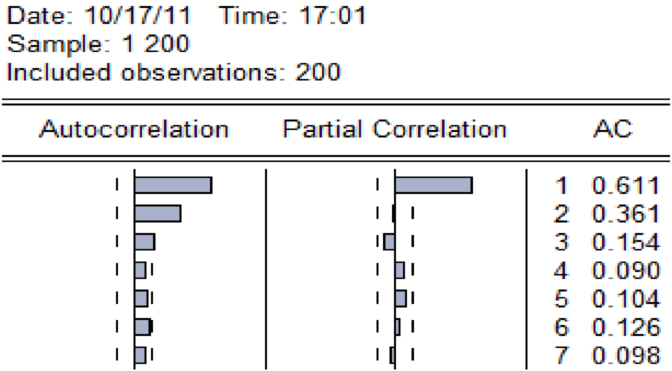
4. Auto-Regressive Model

Identifying AR(1) Process

- When to use AR(1) model?
- For AR(1) process, can show that $E[X_t X_{t+n}] = \frac{\sigma_\epsilon^2}{1-\alpha_1^2} \times \alpha_1^{|n|}$
- Exponential decay of auto-covariance for AR(1) process
- ACF that decays quickly may signal AR(1) model

4. Auto-Regressive Model

AR(1) ACF



4. Conditional Heteroscedasticity Models

General Auto-Regressive Conditional Heteroscedasticity (GARCH) Model

- In many applications, it is useful to model variance of series
- For example, stock volatility tends to cluster
- GARCH model fits and predicts volatility clusters well

4. Conditional Heteroscedasticity Models

GARCH Model

- Let $\epsilon_t = Y_t - \alpha_0 - \alpha_1 Y_{t-1}$ be AR(1) errors
- We suspect conditional heteroscedasticity (clustering) of ϵ_t's
- Most basic GARCH model for ϵ_t's is $\epsilon_t = \sigma_t z_t$, there $z_t \sim N(0, 1)$, and σ_t is given by:

$$\sigma_t^2 = \alpha_0 + \alpha_1 \sigma_{t-1}^2 + \alpha_2 \sigma_{t-2}^2 + ... + \alpha_p \sigma_{t-p}^2$$

- We can estimate the model with least squares
- GARCH predicts financial market volatility very well

5. Time-series with Eviews

Eviews

- One of the most popular tools for time-series analysis is Eviews
- Packages also available in STATA
- Many free tools are implemented in R statistical computing suite