기계인공지능 HW※03 Sol

From

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} \sim \begin{bmatrix} wx' \\ wy' \\ w \end{bmatrix} = \begin{bmatrix} \theta_1 & \theta_2 & \theta_3 \\ \theta_4 & \theta_5 & \theta_6 \\ \theta_7 & \theta_8 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$\Rightarrow \frac{x'}{1} = \frac{\theta_1 x + \theta_2 y + \theta_3}{\theta_7 x + \theta_8 y + 1} \quad \text{and} \quad \frac{y'}{1} = \frac{\theta_4 x + \theta_5 y + \theta_6}{\theta_7 x + \theta_8 y + 1}$$

$$-x'(\theta_7 x + \theta_8 y + 1) + (\theta_1 x + \theta_2 y + \theta_3) = 0$$
$$-y'(\theta_7 x + \theta_8 y + 1) + (\theta_4 x + \theta_5 y + \theta_6) = 0$$

Then,
$$\theta_1 x + \theta_2 y + \theta_3 - \theta_7 x' x - \theta_8 x' y - x' = 0 \quad \cdots \text{①}$$

$$\theta_4 x + \theta_5 y + \theta_6 - \theta_7 y' x - \theta_8 y' y - y' = 0 \quad \cdots \text{②}$$

// done

---

we can rewrite eq ① & ② as following

$$\begin{bmatrix} x & y & 1 & 0 & 0 & 0 & -x'x & -x'y \\ 0 & 0 & 0 & x & y & 1 & -y'x & -y'y \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_2 \\ \vdots \\ \theta_8 \end{bmatrix} = \begin{bmatrix} x' \\ y' \end{bmatrix}$$

Hence, for m pairs of matching points,

$$\begin{bmatrix} x_1 & y_1 & 1 & 0 & 0 & 0 & -x_1'x_1 & -x_1'y_1 \\ 0 & 0 & 0 & x_1 & y_1 & 1 & -y_1'x_1 & -y_1'y_1 \\ \vdots & & & & & & & \\ x_m & y_m & 1 & 0 & 0 & 0 & -x_m'x_m & -x_m'y_m \\ 0 & 0 & 0 & x_m & y_m & 1 & -y_m'x_m & -y_m'y_m \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_2 \\ \vdots \\ \theta_8 \end{bmatrix} = \begin{bmatrix} x_1' \\ y_1' \\ \vdots \\ x_m' \\ y_m' \end{bmatrix}$$

$$\underbrace{\hphantom{xxxxxxxxxxxxxxxx}}_{\Phi} \qquad \underbrace{\theta} \qquad \underbrace{b} \quad \text{// done}$$