Recurrent Neural Network (RNN)

Industrial AI Lab.

(Deterministic) Time Series Data

- For example

$$y[0] = 1, \quad y[1] = \frac{1}{2}, \quad y[2] = \frac{1}{4}, \quad \cdots$$

- Closed-form

$$y[n] = \left(\frac{1}{2}\right)^n, \quad n \geq 0$$

- Linear difference equation (LDE) and initial condition

$$y[n] = \frac{1}{2}y[n-1], \quad y[0] = 1$$

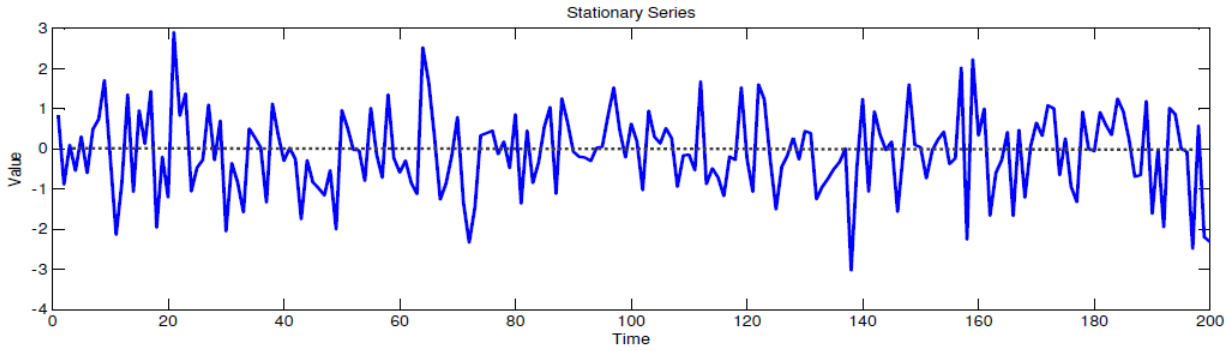
- High order LDEs

$$y[n] = \alpha_1 y[n-1] + \alpha_2 y[n-2]$$

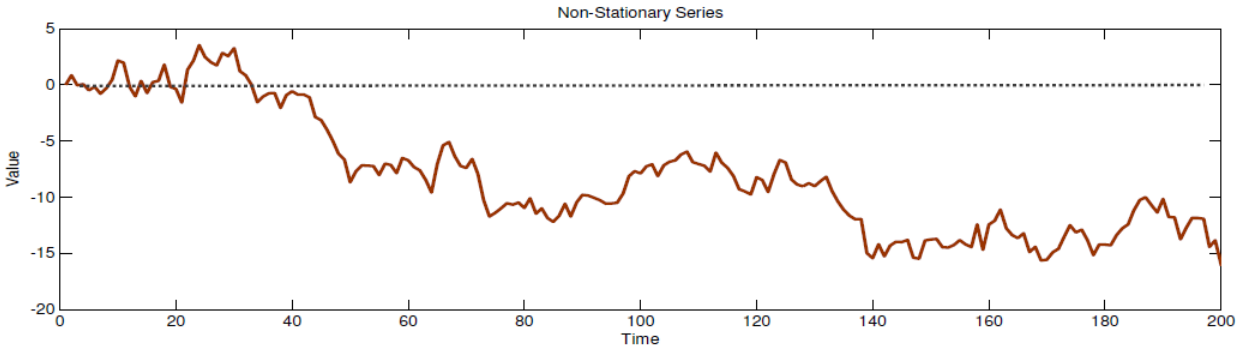
$$y[n] = \alpha_1 y[n-1] + \alpha_2 y[n-2] + \cdots + \alpha_k y[n-k]$$

(Stochastic) Time Series Data

- Stationary

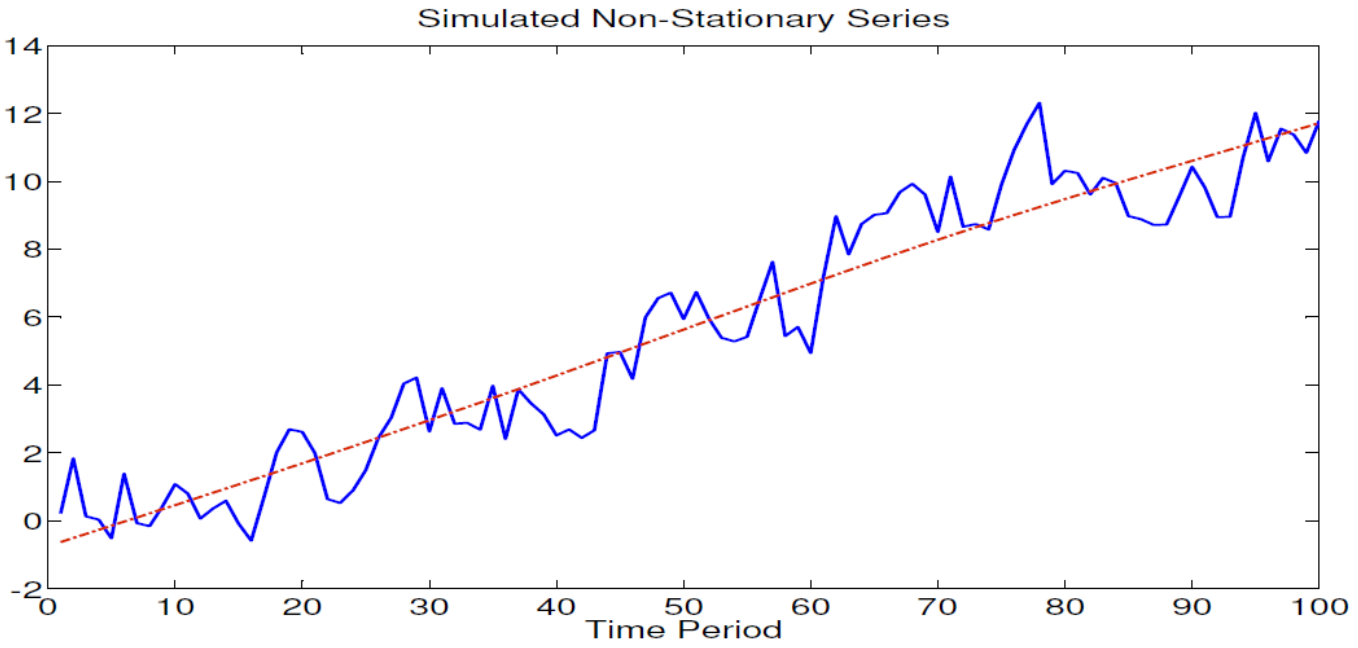


- Non-stationary
  - Mean and variance change over time



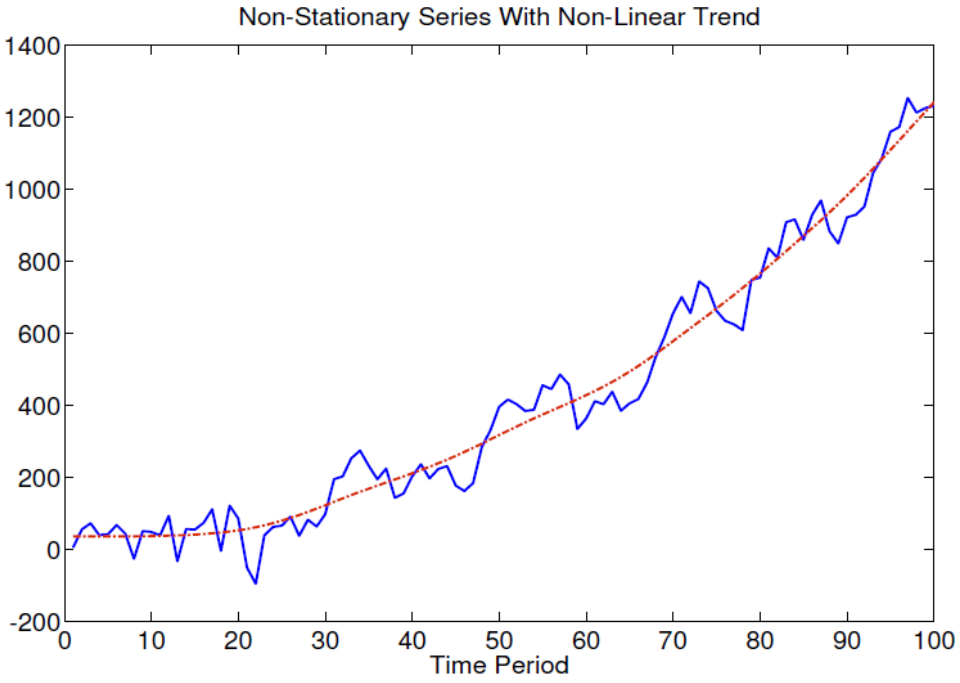
Dealing with Non-Stationarity

- Linear trends



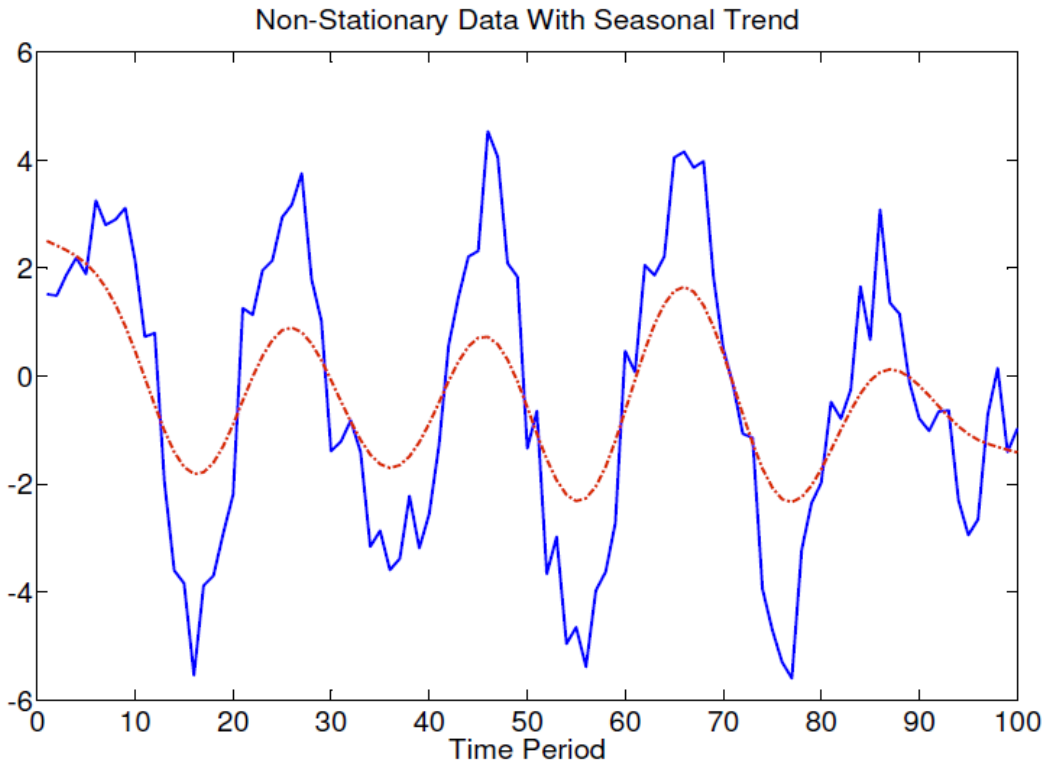
Dealing with Non-Stationarity

- Non-linear trends



Dealing with Non-Stationarity

- Seasonal trends



Dealing with Non-Stationarity

- Model assumption

$$
\begin{aligned}
Y_t = {} & \beta_1 + \beta_2 Y_{t-1} \\
& + \beta_3 t + \beta_4 t^{\beta_5} \\
& + \beta_6 \sin \frac{2\pi}{s} t + \beta_7 \cos \frac{2\pi}{s} t \\
& + u_t
\end{aligned}
$$

Markov Chain

- Joint distribution can be factored into a series of conditional distributions

$$p(q_0, q_1, \cdots, q_T) = p(q_0)\, p(q_1 \mid q_0)\, p(q_2 \mid q_1, q_0) \cdots$$

- Markovian property (assumption)

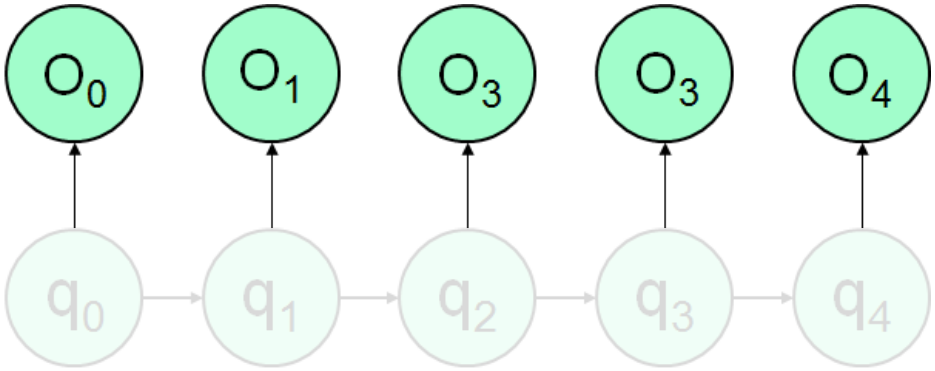
$$p(q_{t+1} \mid q_t, \cdots, q_0) = p(q_{t+1} \mid q_t)$$

- Tractable in computation of joint distribution

$$\begin{aligned} p(q_0, q_1, \cdots, q_T) &= p(q_0)\, p(q_1 \mid q_0)\, p(q_2 \mid q_1, q_0)\, p(q_3 \mid q_2, q_1, q_0) \cdots \\ &= p(q_0)\, p(q_1 \mid q_0)\, p(q_2 \mid q_1)\, p(q_3 \mid q_2) \cdots \end{aligned}$$

Hidden Markov Model (HMM)

- True state (or hidden variable) follows Markov chain
- Observation emitted from state



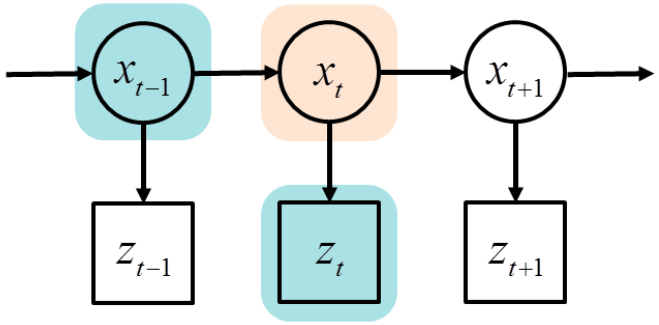
- Question: state estimation

$$\text{What is } p(q_t = s_i \mid O_1, O_2, \cdots, O_T)$$

- HMM can do this, but with many difficulties

Kalman Filter

- Linear dynamical system of motion



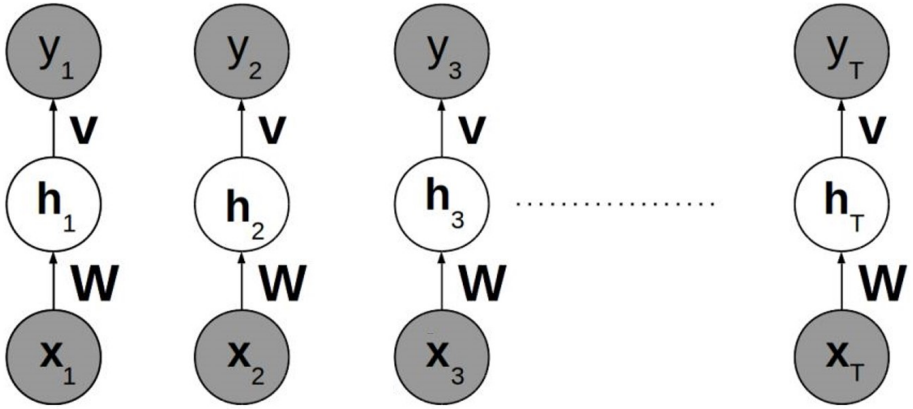
$$x_{t+1} = Ax_t + Bu_t$$
$$z_t = Cx_t$$

- *A, B, C* ?

Recurrent Neural Network (RNN)

- RNNs are a family of neural networks for processing sequential data

- Feedforward Network and Sequential Data
  - Separate parameters for each value of the time index
  - Cannot share statistical strength across different time indices

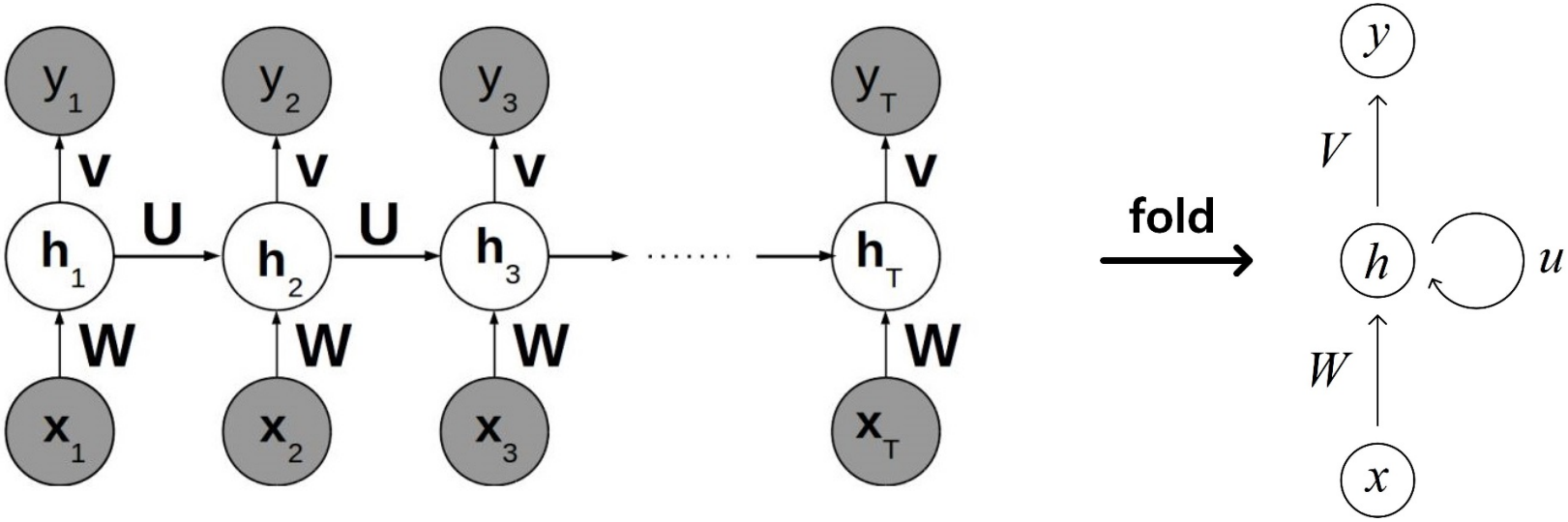


Recurrent Neural Network (RNN)

Recurrent Neural Network (RNN)

Structure of RNN

- Order matters
- Recurrence
  - It is possible to use the same transition function f with the same parameters at every time step

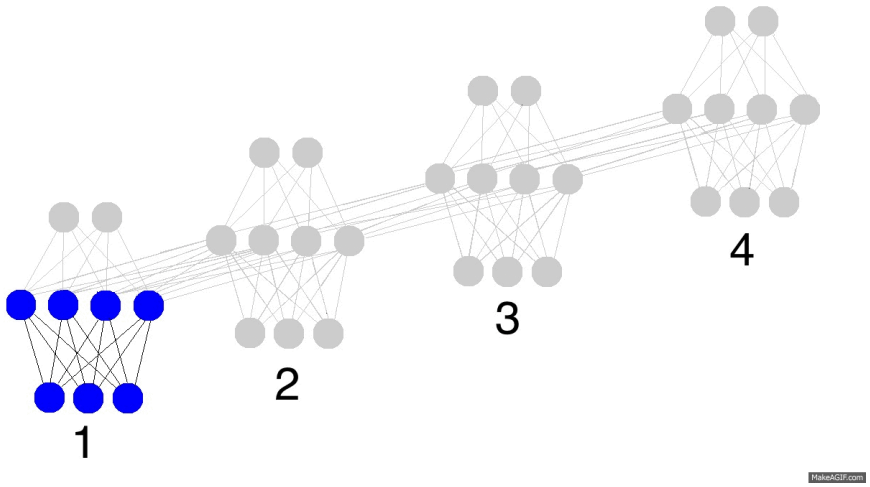


Hidden State

- Summary of the past sequence of inputs up to t
- Keep some aspects of the past sequence with more precision than other aspects
- Network learns the function f

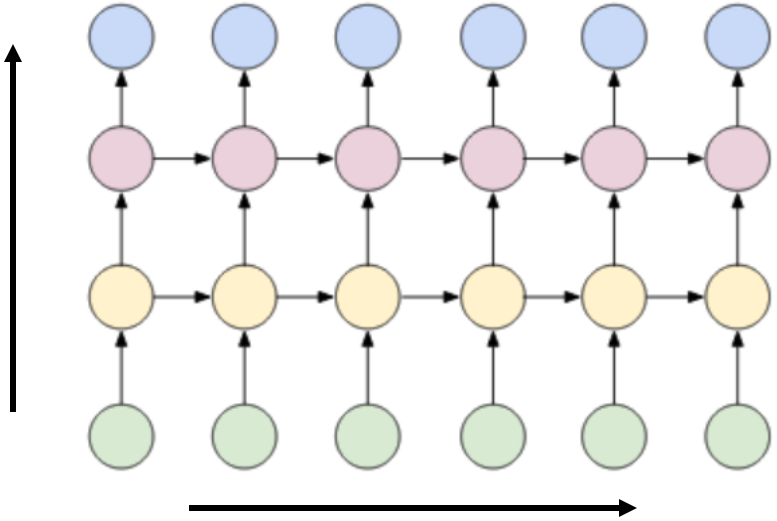
$$h^{(t)} = f\left(h^{(t-1)}, x^{(t)}\right)$$

$$f\left(h^{(t-1)}, x^{(t)}\right) = g\left(Wx_t + Uh_{t-1}\right)$$



Deep Recurrent Networks

- Three blocks of parameters and associated transformation
  - From the input to the hidden state (from green to yellow)
  - From the previous hidden state to the next hidden state (from yellow to red)
  - From the hidden state to the output (from red to blue)



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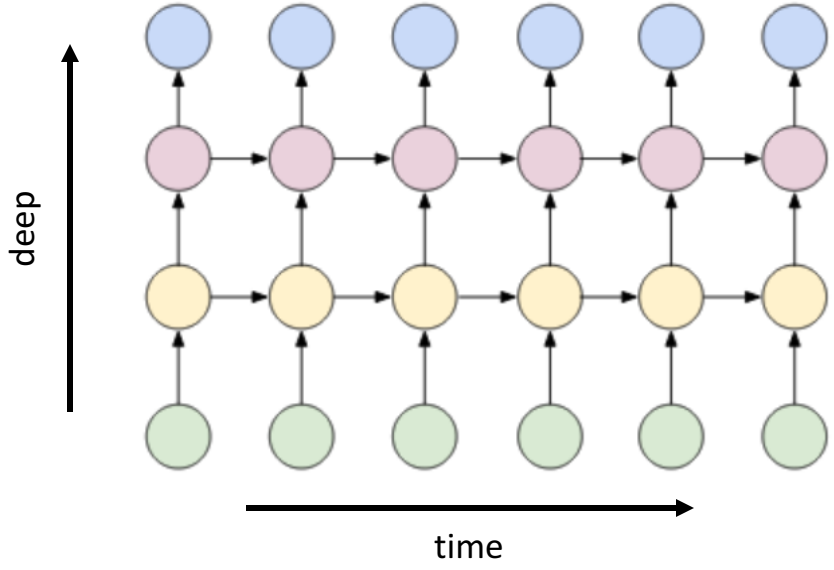
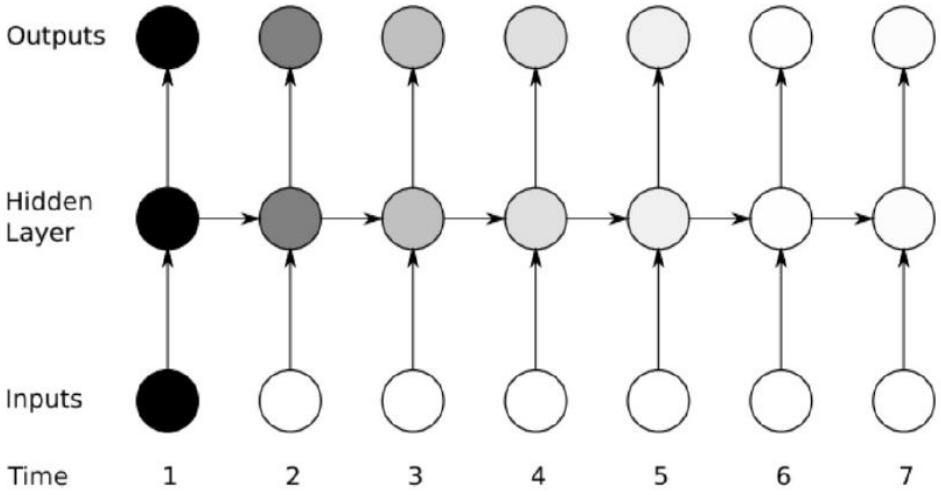


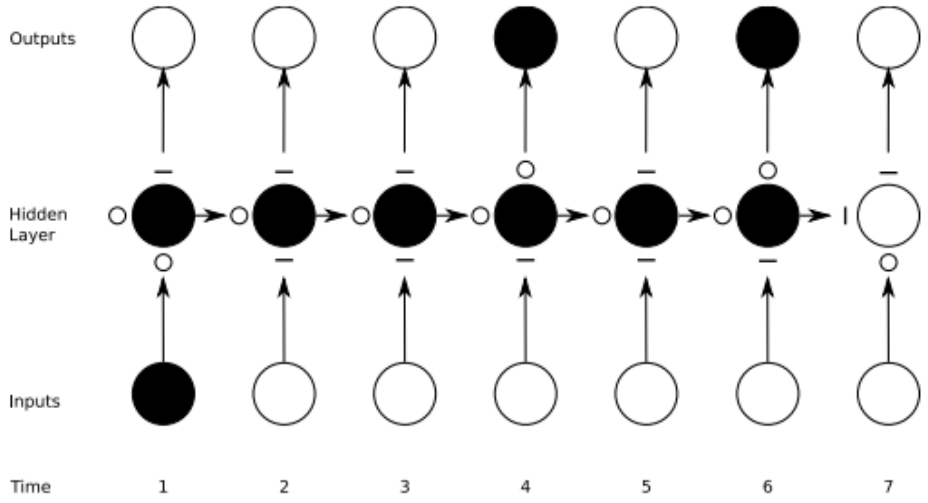
RNN with LSTM

- Long-Term Dependencies
  - Gradients propagated over many stages tend to either **vanish** or **explode**
  - Difficulty with long-term dependencies arises from the exponentially smaller weights given to long-term interactions

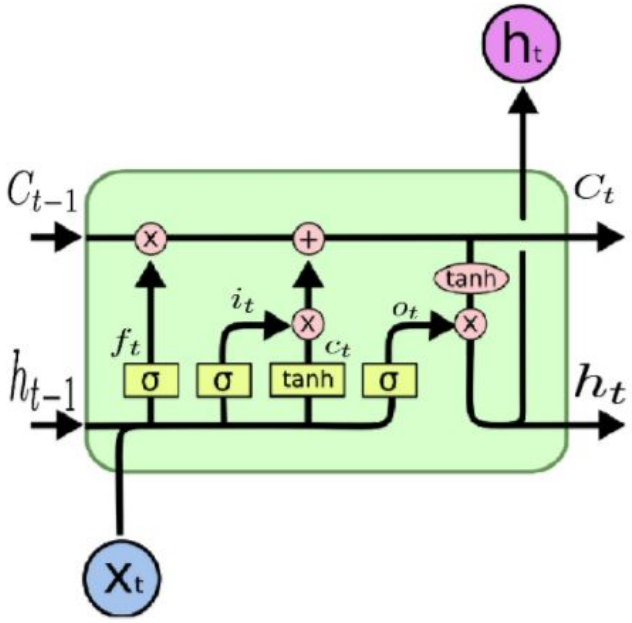


Long Short-Term Memory (LSTM)

- Allow the network to **accumulate** information over a long duration
- Once that information has been used, it might be used for the neural network to **forget** the old state



Long Short-Term Memory (LSTM)



$$f_t = \sigma\left(W_f \cdot [h_{t-1}, x_t] \; + \; b_f\right)$$

$$i_t = \sigma\left(W_i \cdot [h_{t-1}, x_t] \; + \; b_i\right)$$

$$\tilde{C}_t = \tanh(W_C \cdot [h_{t-1}, x_t] \; + \; b_C)$$

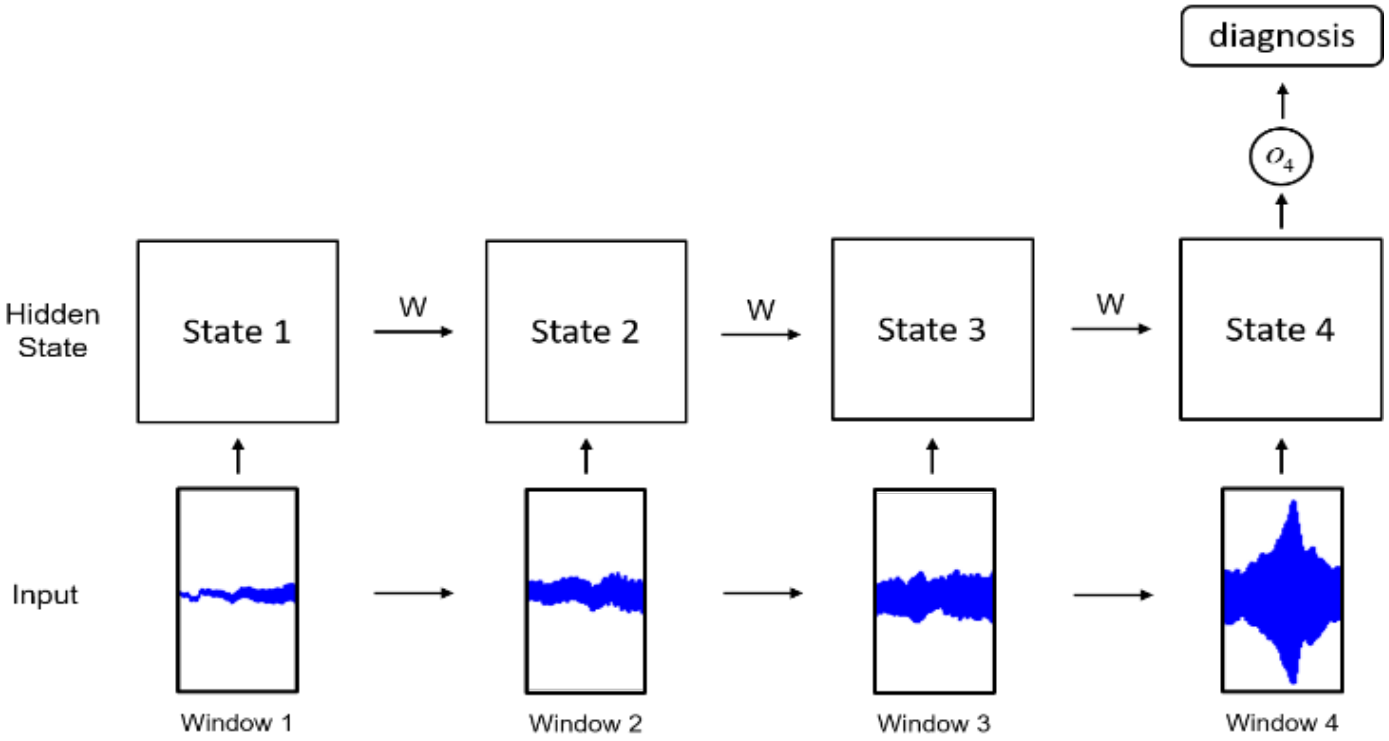
$$C_t = f_t * C_{t-1} + i_t * \tilde{C}_t$$

$$o_t = \sigma\left(W_o \; [h_{t-1}, x_t] \; + \; b_o\right)$$

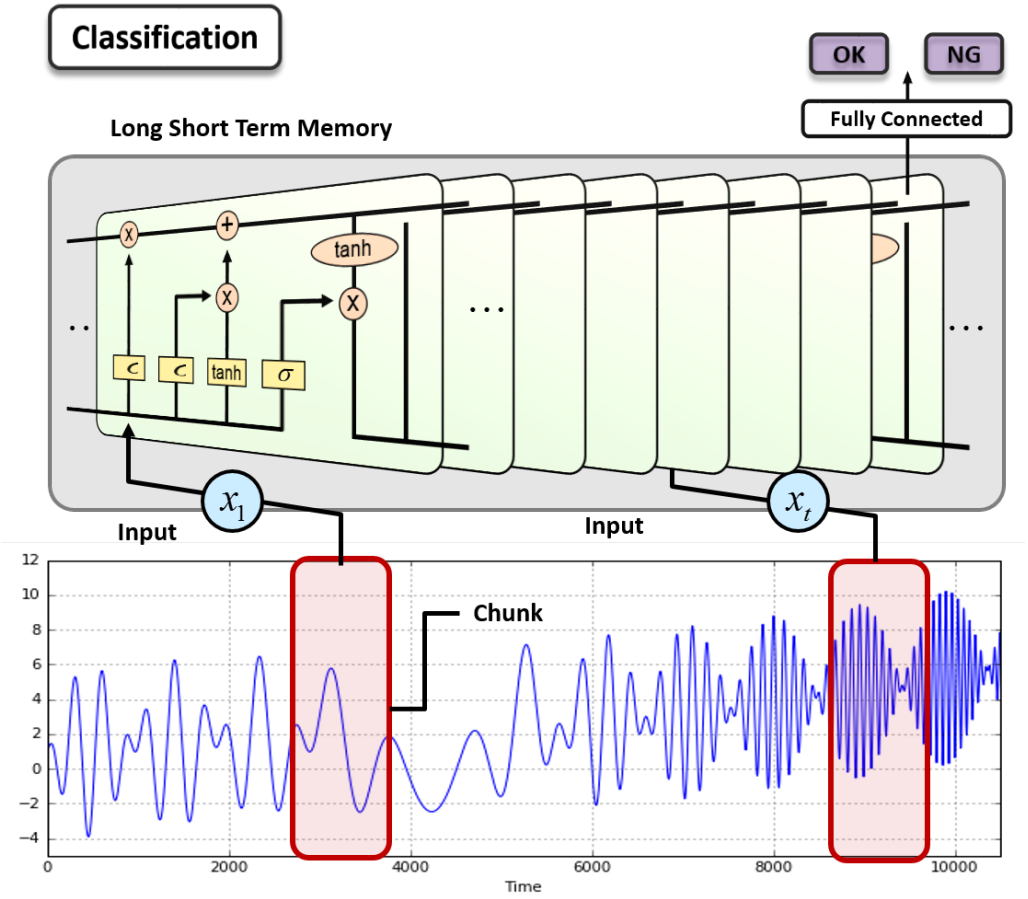
$$h_t = o_t * \tanh\left(C_t\right)$$

- Summary
  - Connect LSTM cells in a recurrent manner
  - Train parameters in LSTM cells

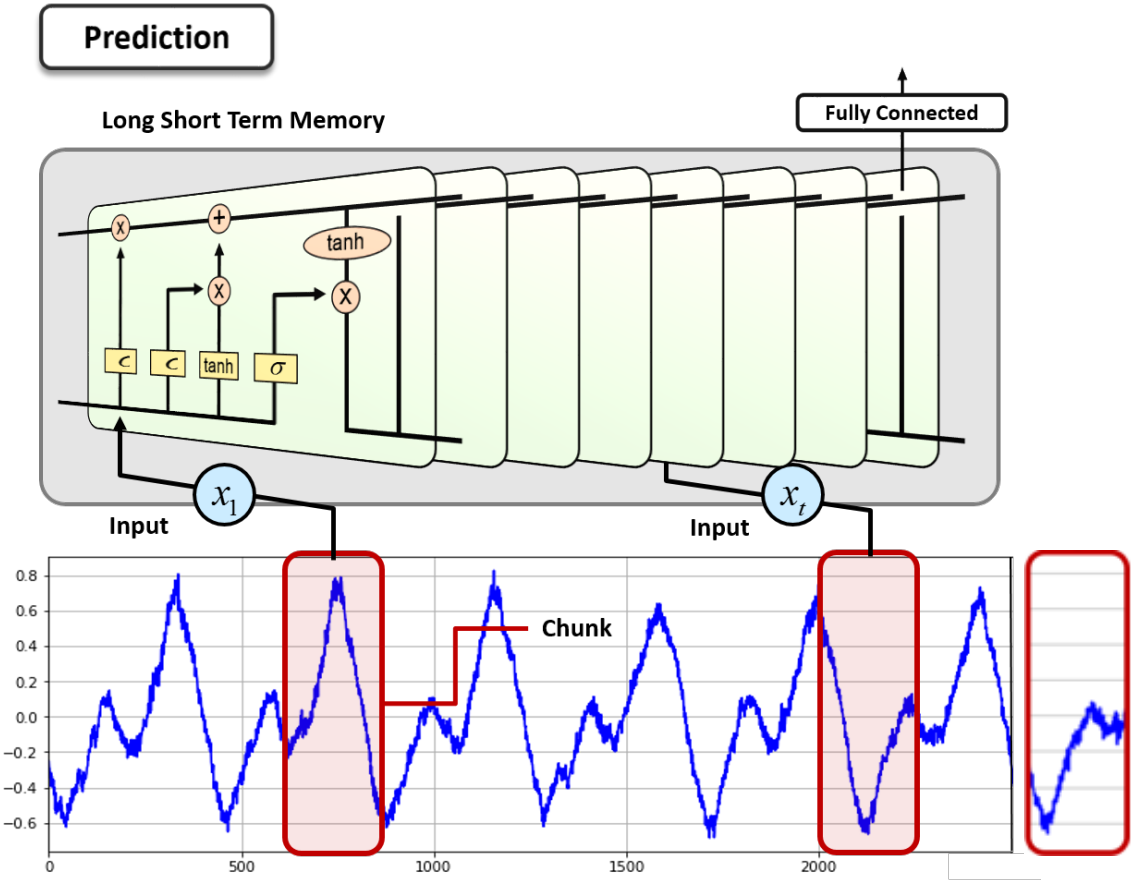
Time Series Data and RNN



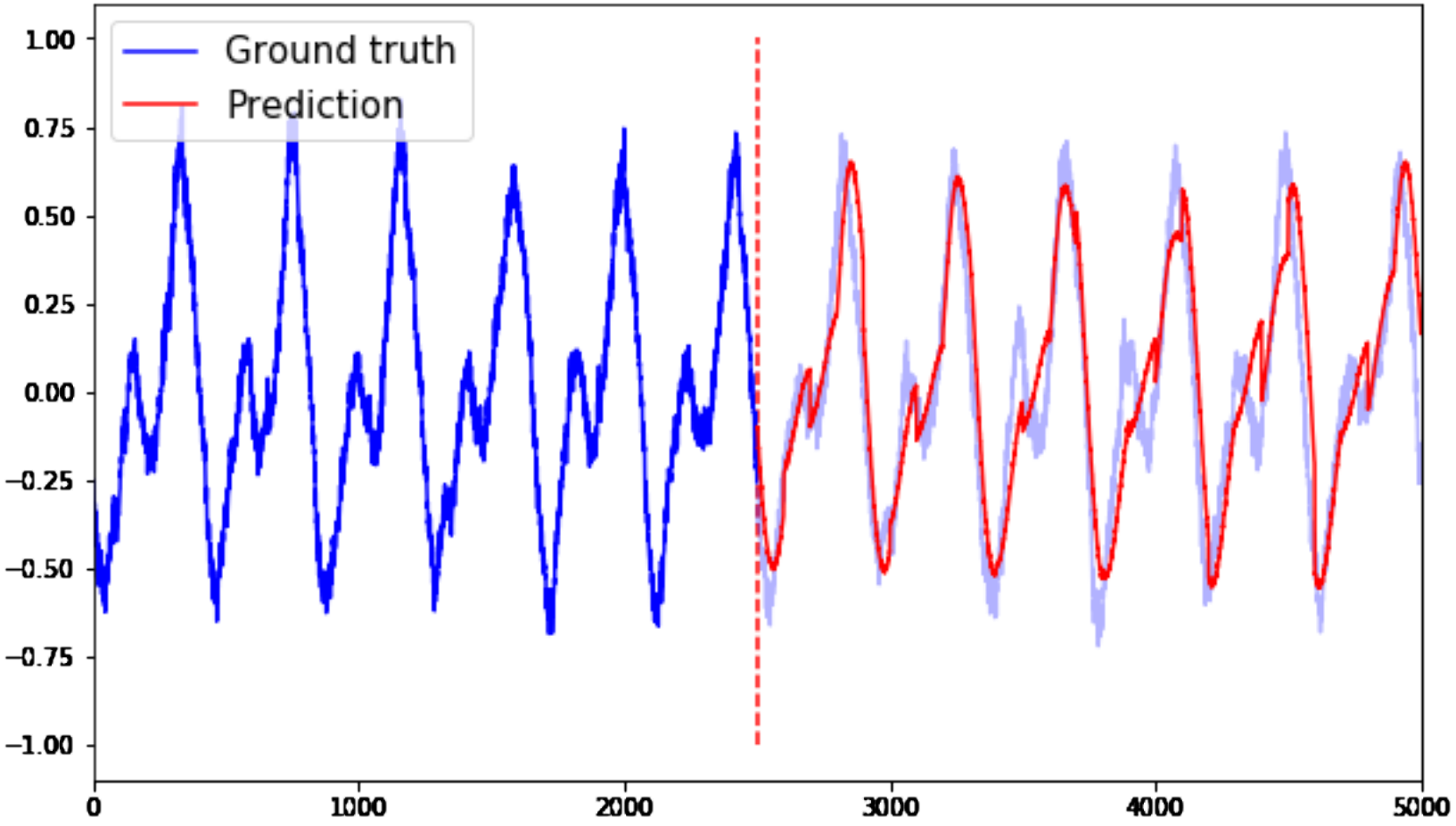
RNN for Classification



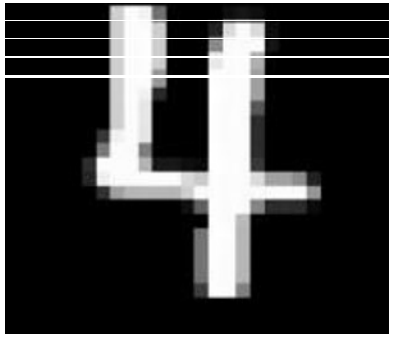
RNN for Prediction

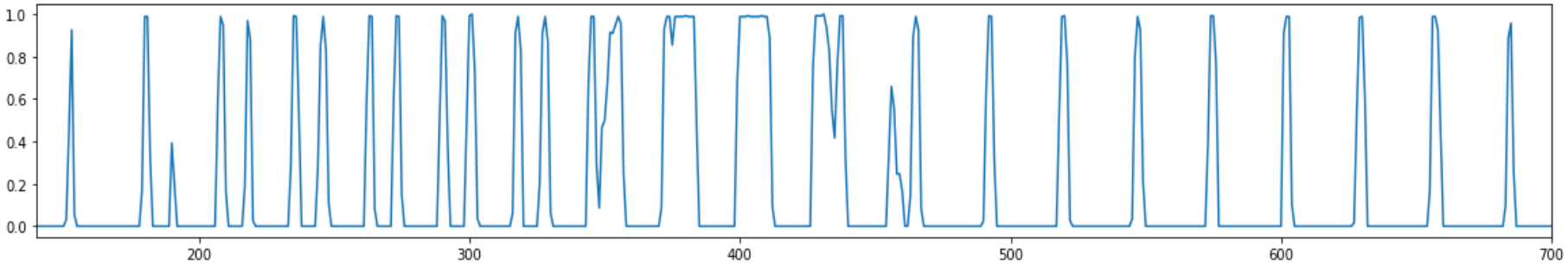


Prediction Example

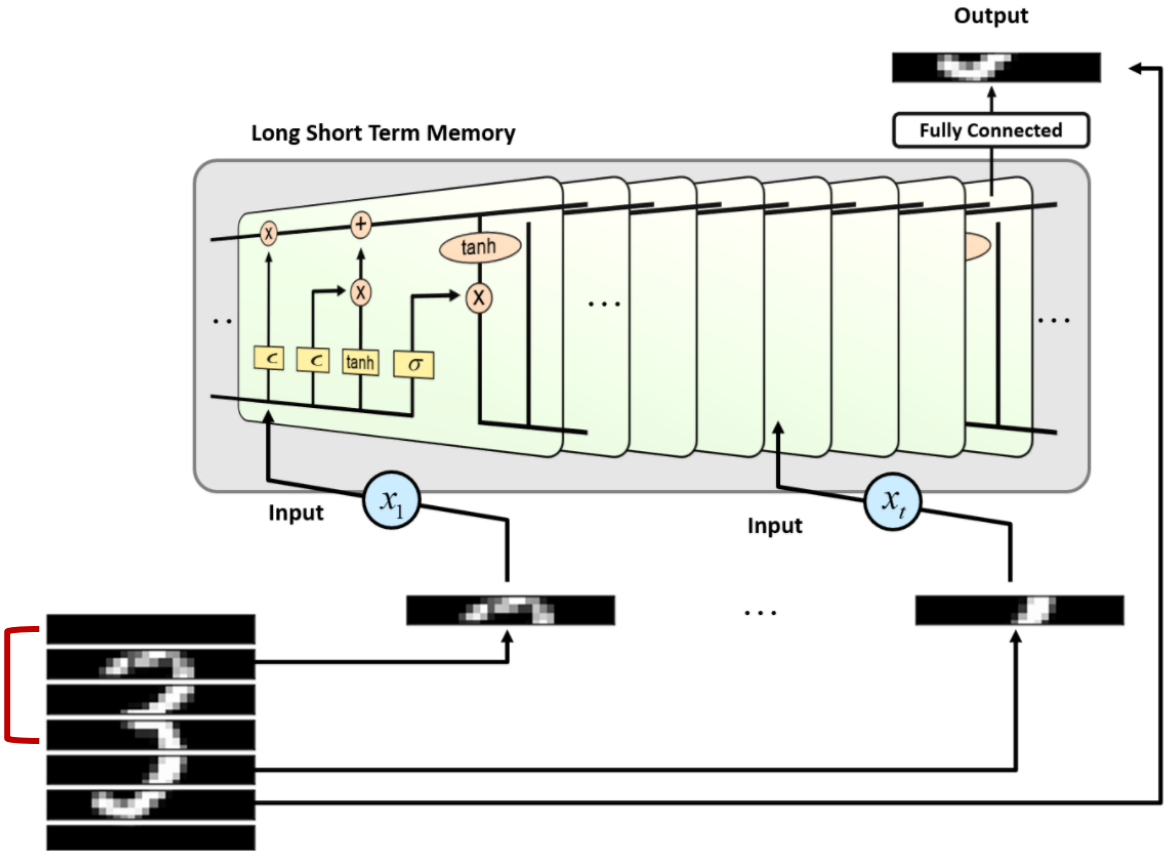


From Image to Time Series Data

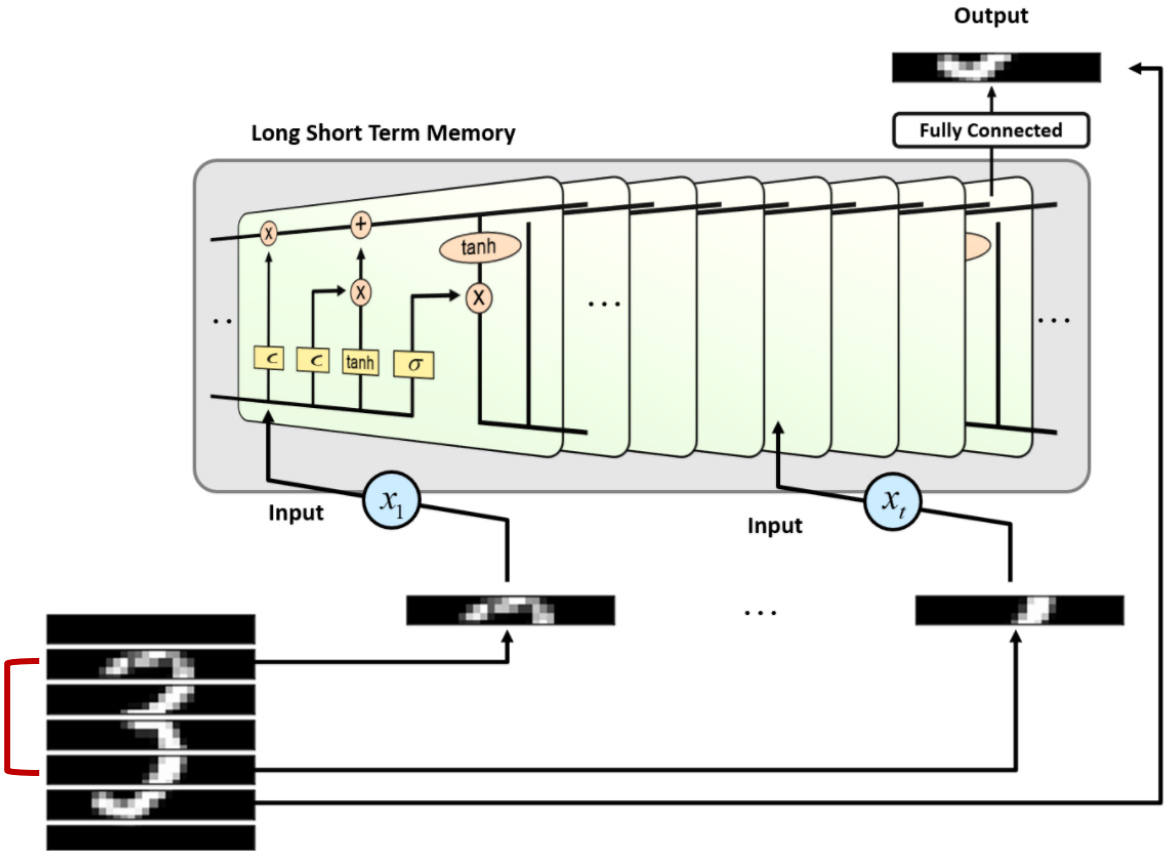




Prediction of MNIST



Prediction of MNIST



RNN with TensorFlow

- An example for predicting a next piece of an image
- Regression problem

- Import Library

```
import tensorflow as tf
from six.moves import cPickle
import numpy as np
import matplotlib.pyplot as plt
```

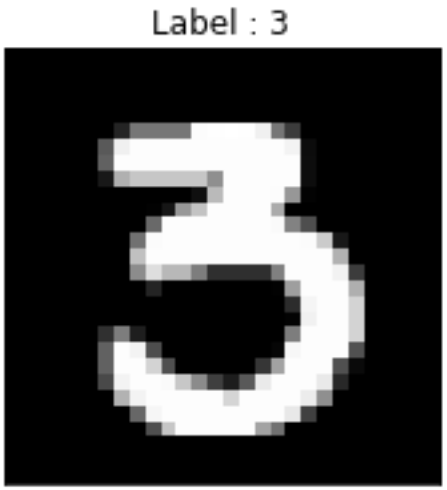
- Load MNIST Data
  - Download MNIST data from the TensorFlow tutorial example

```
from tensorflow.examples.tutorials.mnist import input_data
mnist = input_data.read_data_sets("MNIST_data/", one_hot=True)
```

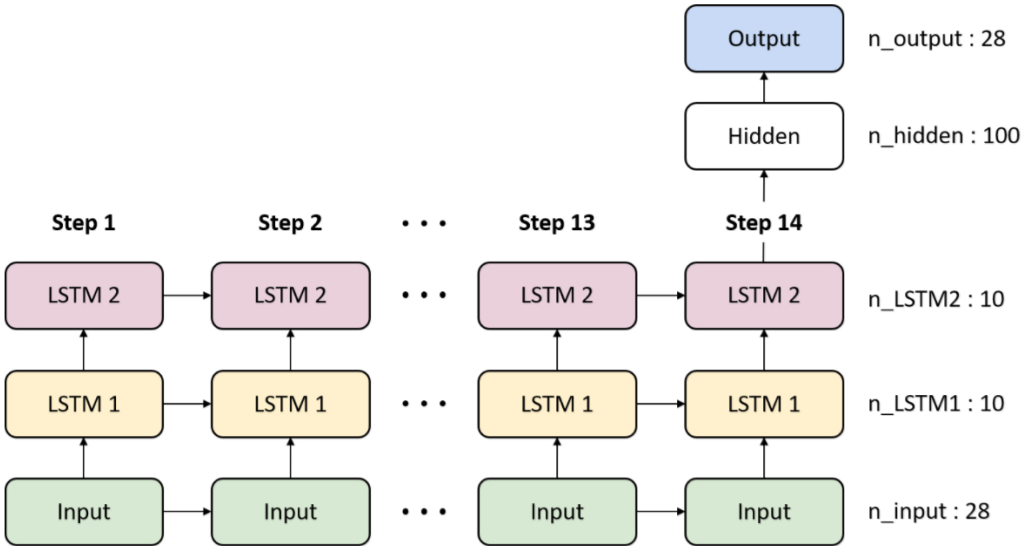
```
Extracting MNIST_data/train-images-idx3-ubyte.gz
Extracting MNIST_data/train-labels-idx1-ubyte.gz
Extracting MNIST_data/t10k-images-idx3-ubyte.gz
Extracting MNIST_data/t10k-labels-idx1-ubyte.gz
```

RNN with TensorFlow

```
# Check data
train_x, train_y = mnist.train.next_batch(10)
img = train_x[9,:].reshape(28, 28)

plt.figure(figsize=(5, 3))
plt.imshow(img,'gray')
plt.title("Label : {}".format(np.argmax(train_y[9])))
plt.xticks([])
plt.yticks([])
plt.show()
```



RNN Structure




```
n_step = 14
n_input = 28

## LSTM shape
n_lstm1 = 10
n_lstm2 = 10

## Fully connected
n_hidden = 100
n_output = 28
```

LSTM, Weights and Biases

- LSTM Cell
  - Do not need to define weights and biases of LSTM cells

- Fully connected
  - Define parameters based on the predefined layer size
  - Initialize with a normal distribution with $\mu = 0$ and $\sigma = 0.01$

```
weights = {
    'hidden' : tf.Variable(tf.random_normal([n_lstm2, n_hidden], stddev=0.01)),
    'output' : tf.Variable(tf.random_normal([n_hidden, n_output], stddev=0.01))
}

biases = {
    'hidden' : tf.Variable(tf.random_normal([n_hidden], stddev=0.01)),
    'output' : tf.Variable(tf.random_normal([n_output], stddev=0.01))
}

x = tf.placeholder(tf.float32, [None, n_step, n_input])
y = tf.placeholder(tf.float32, [None, n_output])
```

Build a Model

- First, define the LSTM cells

```
lstm = tf.contrib.rnn.BasicLSTMCell(n_lstm)
```

- Second, compute hidden state (h) and LSTM cell (c) with the predefined LSTM cell and input

```
h, c = tf.nn.dynamic_rnn(lstm, input_tensor, dtype=tf.float32)
```

```
def build_model(x, weights, biases):
    with tf.variable_scope('rnn'):
        # Build RNN network
        with tf.variable_scope('lstm1'):
            lstm1 = tf.contrib.rnn.BasicLSTMCell(n_lstm1)
            h1, c1 = tf.nn.dynamic_rnn(lstm1, x, dtype=tf.float32)
        with tf.variable_scope('lstm2'):
            lstm2 = tf.contrib.rnn.BasicLSTMCell(n_lstm2)
            h2, c2 = tf.nn.dynamic_rnn(lstm2, h1, dtype=tf.float32)

        # Build classifier
        hidden = tf.add(tf.matmul(h2[:,-1,:], weights['hidden']), biases['hidden'])
        hidden = tf.nn.relu(hidden)
        output = tf.add(tf.matmul(hidden, weights['output']), biases['output'])
        return output
```

Cost, Initializer and Optimizer

- Loss
  - Regression: Squared loss

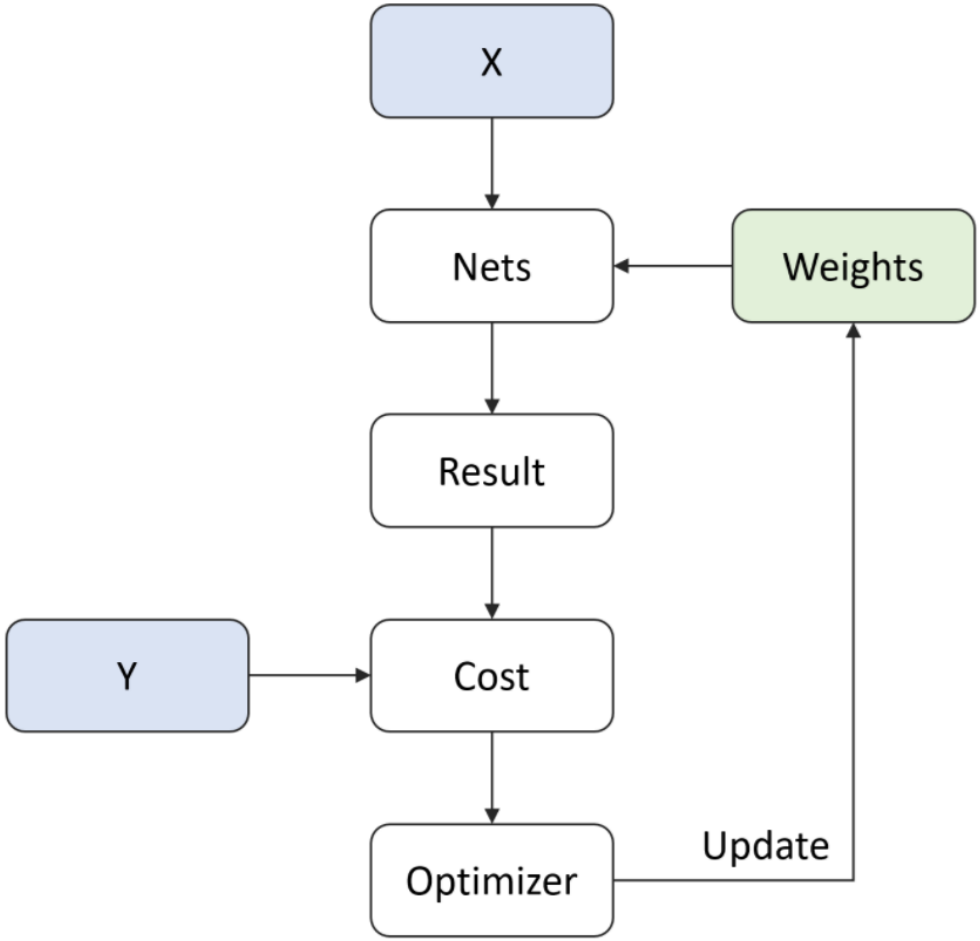
$$\frac{1}{N}\sum_{i=1}^{N}(\hat{y}^{(i)} - y^{(i)})^2$$

- Initializer
  - Initialize all the empty variables

- Optimizer
  - AdamOptimizer: the most popular optimize

```
LR = 0.0005

pred = build_model(x, weights, biases)
loss = tf.square(tf.subtract(y, pred))
loss = tf.reduce_mean(loss)

optm = tf.train.AdamOptimizer(LR).minimize(loss)

init = tf.global_variables_initializer()
```

Summary of Optimization Process



Iteration Configuration

- Define parameters for training RNN
  - n_iter: the number of training steps
  - n_prt: check loss for every n_prt iteration

```
n_iter = 2500
n_prt = 100
```

Optimization

- Do not run on CPU. It will take quite a while.

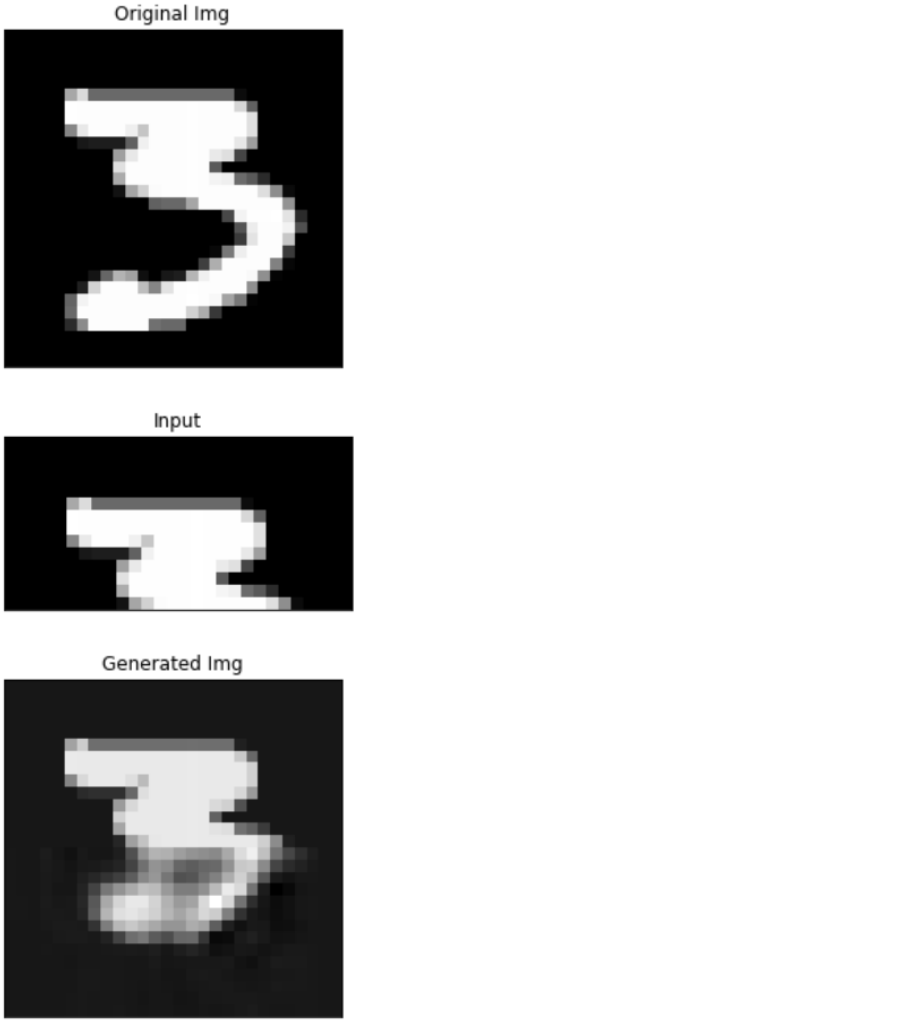
```
# Run initialize
# config = tf.ConfigProto(allow_soft_placement=True)  # GPU Allocating policy
# sess = tf.Session(config=config)
sess = tf.Session()
sess.run(init)

for i in range(n_iter):
    train_x, train_y = mnist.train.next_batch(50)
    train_x = train_x.reshape(-1, 28, 28)

    for j in range(n_step):
        sess.run(optm, feed_dict={x: train_x[:,j:j+n_step,:],  y: train_x[:,j+n_step]})
    if i % n_prt == 0:
        c = sess.run(loss, feed_dict={x: train_x[:,13:13+n_step,:],  y: train_x[:,13+n_step]})
        print ("Iter : {}".format(i))
        print ("Cost : {}".format(c))
```

Test or Evaluation

- Do not run on CPU. It will take quite a while.
- Predict the MNIST image
- MNIST is 28 x 28 image.
  - The model predicts a piece of 1 x 28 image.
  - First, 14 x 28 image will be fed into a model, then the model predict the last 14 x 28 image, recursively.

```
test_x, test_y = mnist.test.next_batch(10)
test_x = test_x.reshape(-1, 28, 28)

idx = 0
gen_img = []

sample = test_x[idx, 0:14, :]
input_img = sample.copy()

feeding_img = test_x[idx, 0:0+n_step, :]

for i in range(n_step):
    test_pred = sess.run(pred, feed_dict={x: feeding_img.reshape(1, 14, 28)})
    feeding_img = np.delete(feeding_img, 0, 0)
    feeding_img = np.vstack([feeding_img, test_pred])
    gen_img.append(test_pred)
```

Test or Evaluation



Load pre-trained Model

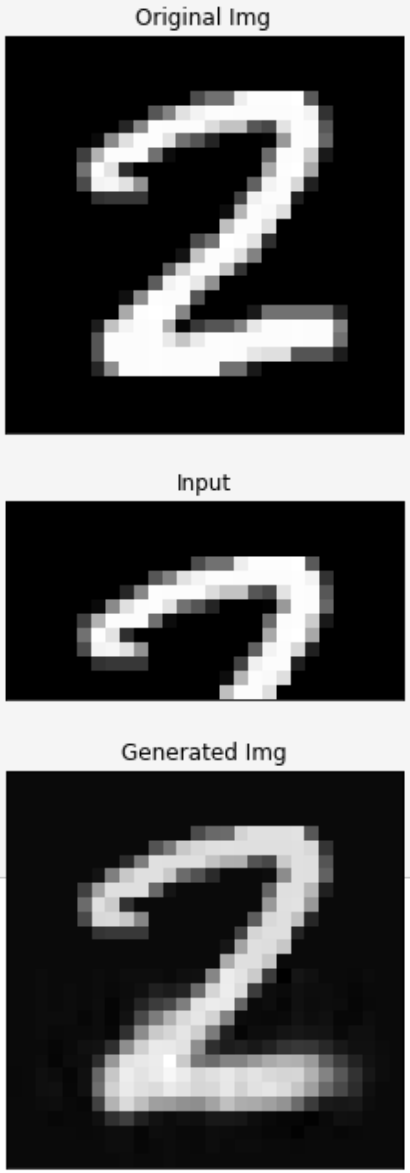
- We trained the model on GPU for you.
- You can load the pre-trained model to see RNN MNIST results
- LSTM size
  - n_lstm1 = 128
  - n_lstm2 = 256

```
from RNN import RNN
my_rnn = RNN()
my_rnn.load('./data_files/RNN_mnist/checkpoint/RNN_5000')
```

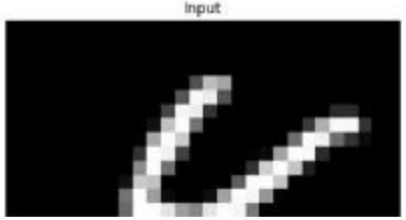
```
INFO:tensorflow:Restoring parameters from ./data_files/RNN_mnist/checkpoint/RNN_5000
Model loaded from file : ./data_files/RNN_mnist/checkpoint/RNN_5000
```

Test with pre-trained Model

```
test_x, test_y = mnist.test.next_batch(10)
test_x = test_x.reshape(-1, 28, 28)

sample = test_x[0, 0:14,:]

gen_img = my_rnn.predict(sample)

plt.imshow(test_x[0], 'gray')
plt.title('Original Img')
plt.xticks([])
plt.yticks([])
plt.show()

plt.figure(figsize=(4,3))
plt.imshow(sample, 'gray')
plt.title('Input')
plt.xticks([])
plt.yticks([])
plt.show()

plt.imshow(gen_img, 'gray')
plt.title('Generated Img')
plt.xticks([])
plt.yticks([])
plt.show()
```



Input



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