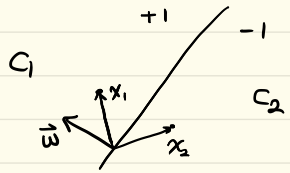
기계인공지능 HW ※4 Sol

prob 4.

(a) Let $f(x) = \omega^T x$

$y \in \{-1, 1\}$



If the data point is in the C_1

The angle between ω and x is less than $\angle 90°$

so the value of inner product is positive. ($\omega^T x > 0$)

① when the decision boundary classify this point as +1 (well classified)

Then, $y(\omega^T x) > 0$

② when the decision boundary classify this point as -1 (misclassified)

Then, $y(\omega^T x) < 0$

If the data point is in the C_2

The angle between ω and x is greater than $\angle 90°$

so the value of inner product is negative ($\omega^T x < 0$)

① when the decision boundary classify this point as -1 (well classified)

Then, $y(\omega^T x) > 0$

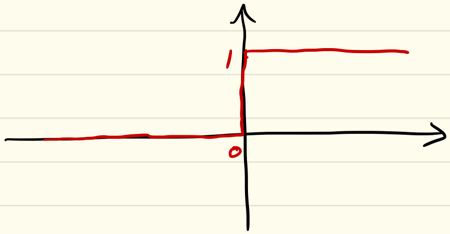
② when the decision boundary classify this point as +1 (misclassified)

Then, $y(\omega^T x) < 0$

(b) From the problem 4-(a)
we show $y(w^T x) < 0$ when it is misclassified and $y(w^T x) > 0$ when it is well classified.

$$J = \Sigma u(-y(w^T x))$$

J is sum of unit step function value



$u(x) = 0$ when $x < 0$
so, $u(-y(w^T x)) = 0$ when it is well classified.

And $u(x) = 1$ when $x > 0$
so, $u(-y(w^T x)) = 1$ when it is misclassified.

Thus, $J = \#$ of misclassified point

(c) Since the function is not differentible, we can't use gradient decent algorithm.

(d)

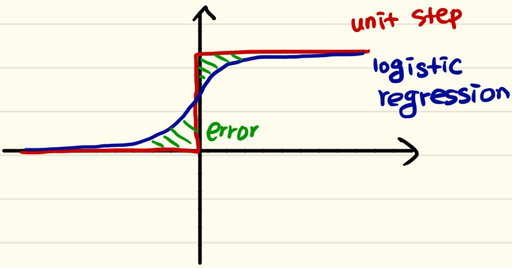
$$\sigma(z) = \frac{1}{1+e^{-z}}$$

$$\frac{d\sigma}{dz} = \frac{-1\cdot(-e^{-z})}{(1+e^{-z})^2} = \frac{e^{-z}}{1+e^{-z}} \cdot \frac{1}{1+e^{-z}}$$

$$\frac{1}{1+e^{-z}} = \sigma(z) \quad \text{and} \quad \frac{e^{-z}}{1+e^{-z}} = 1-\sigma(z)$$

Thus, $\frac{d\sigma}{dz} = \sigma(z)(1-\sigma(z))$

(e)



Since the sigmoid function is monotonic, classification boundary does not changed from ideal Classification.

However, there are some error

So, the logistic regression can approximately solve the classification problem //

done