

# Unsupervised Learning : K-means Clustering and PCA

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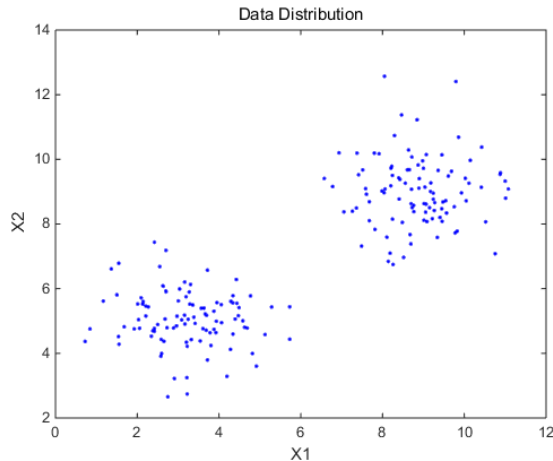
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## 1. K-means Clustering

To see how it works, click [here \(http://isystems.github.io/HSE545/machine%20learning%20all/05%20Clustering/iSystems\\_01\\_K-means\\_Clustering.html\)](http://isystems.github.io/HSE545/machine%20learning%20all/05%20Clustering/iSystems_01_K-means_Clustering.html)

# Unsupervised Learning

- Data clustering is an unsupervised learning problem
- Given:
  - $m$  unlabeled examples  $\{x^{(1)}, x^{(2)} \dots, x^{(m)}\}$
  - the number of partitions  $k$
- Goal: group the examples into  $k$  partitions



$$\{x^{(1)}, x^{(2)}, \dots, x^{(m)}\} \Rightarrow \text{Clustering}$$

- the only information clustering uses is the similarity between examples
- clustering groups examples based of their mutual similarities
- A good clustering is one that achieves:
  - high within-cluster similarity
  - low inter-cluster similarity

## 1.1. (Iterative) Algorithm

Randomly initialize  $k$  cluster centroids  $\mu_1, \mu_2, \dots, \mu_k \in \mathbb{R}^n$

```
Repeat{  
  for  $i = 1$  to  $m$   
     $c_i :=$  index (from 1 to  $k$ ) of cluster centroid closest to  $x^{(i)}$   
  for  $k = 1$  to  $k$   
     $\mu_k :=$  average (mean) of points assigned to cluster  $k$   
}
```

## 1.2. Python code

### Data Load

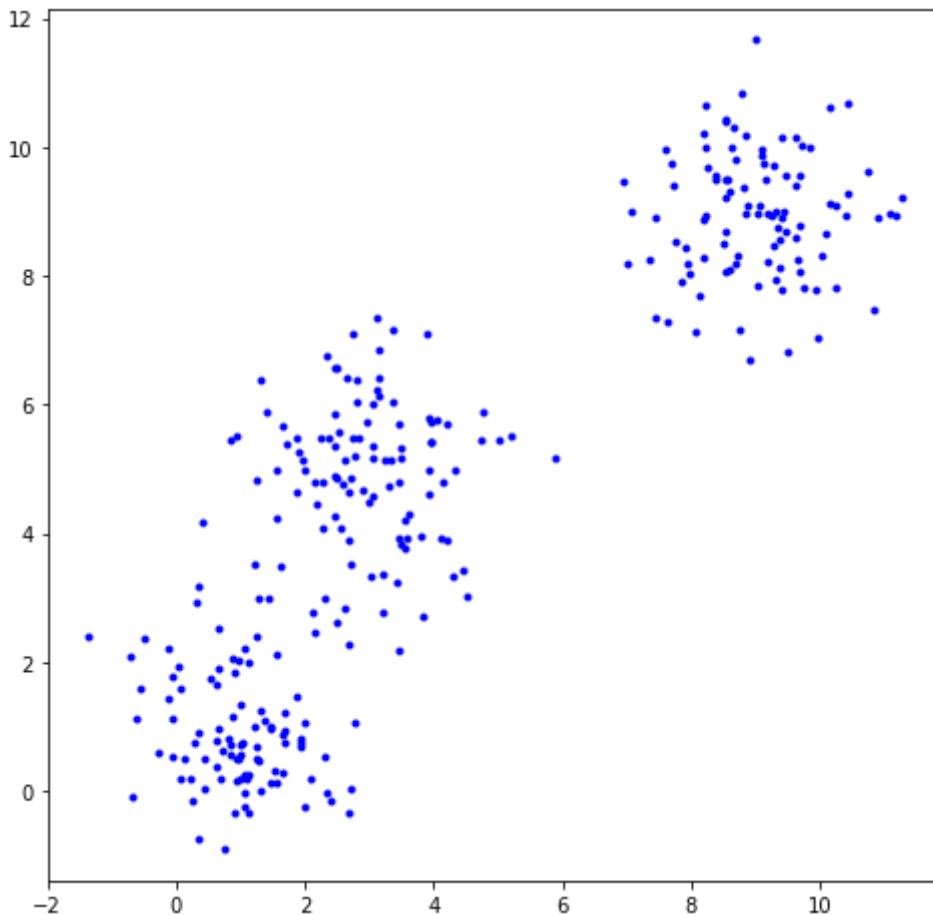
- kmeans\_example 예제 데이터는 아래 링크에서 다운로드
  - [kmeans\\_example.pkl \(data\\_files/kmeans\\_example.pkl\)](#)

In [1]:

```
import matplotlib.pyplot as plt
from six.moves import cPickle

X = cPickle.load(open('./data_files/kmeans_example.pkl','rb'))

plt.figure(figsize=(8,8))
plt.plot(X[:,0],X[:,1], 'b. ')
plt.axis('equal')
plt.show()
```



## K-means clustering

In [2]:

```
from sklearn.cluster import KMeans

kmeans = KMeans(n_clusters = 3, random_state = 0)
kmeans.fit(X)
```

Out[2]:

```
KMeans(algorithm='auto', copy_x=True, init='k-means++', max_iter=300,
       n_clusters=3, n_init=10, n_jobs=1, precompute_distances='auto',
       random_state=0, tol=0.0001, verbose=0)
```

In [3]:

```
print(kmeans.labels_)
```

[illegible]

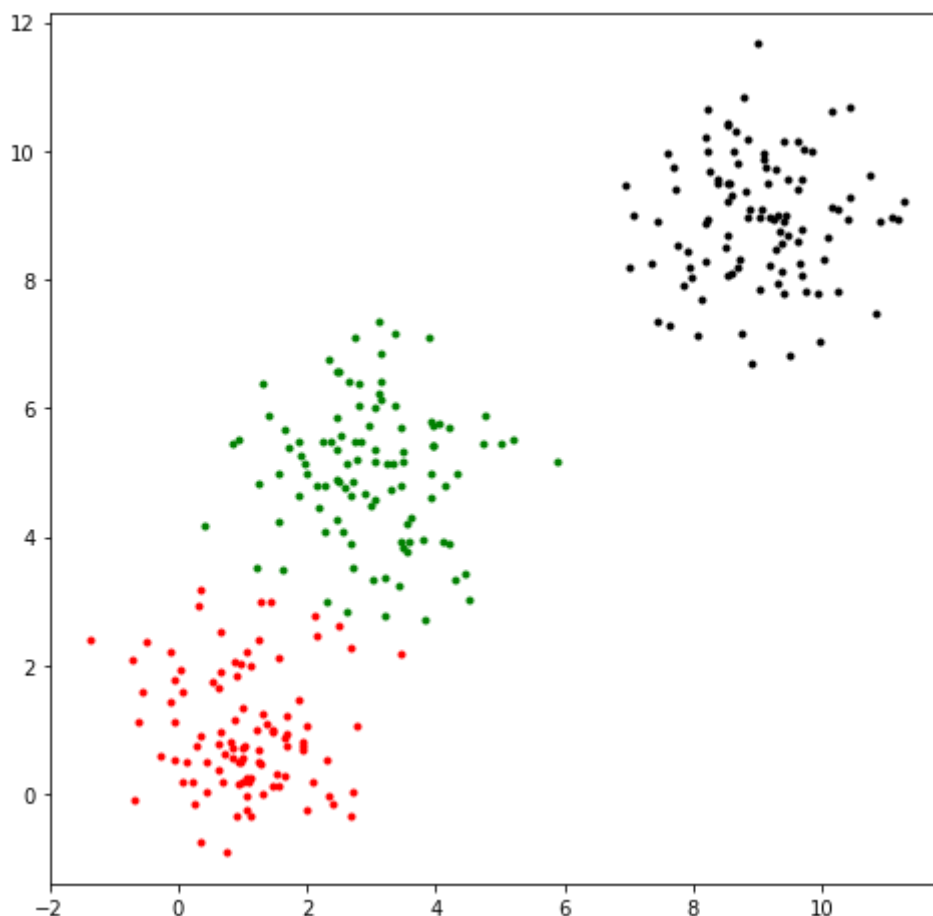
In [4]:

```
plt.figure(figsize=(8,8))  
# plt.hold(True)
```

```
plt.plot(X[kmeans.labels_ == 0,0],X[kmeans.labels_ == 0,1], 'g.')
plt.plot(X[kmeans.labels_ == 1,0],X[kmeans.labels_ == 1,1], 'k.')
plt.plot(X[kmeans.labels_ == 2,0],X[kmeans.labels_ == 2,1], 'r.')

```

```
plt.axis('equal')
plt.show()
```



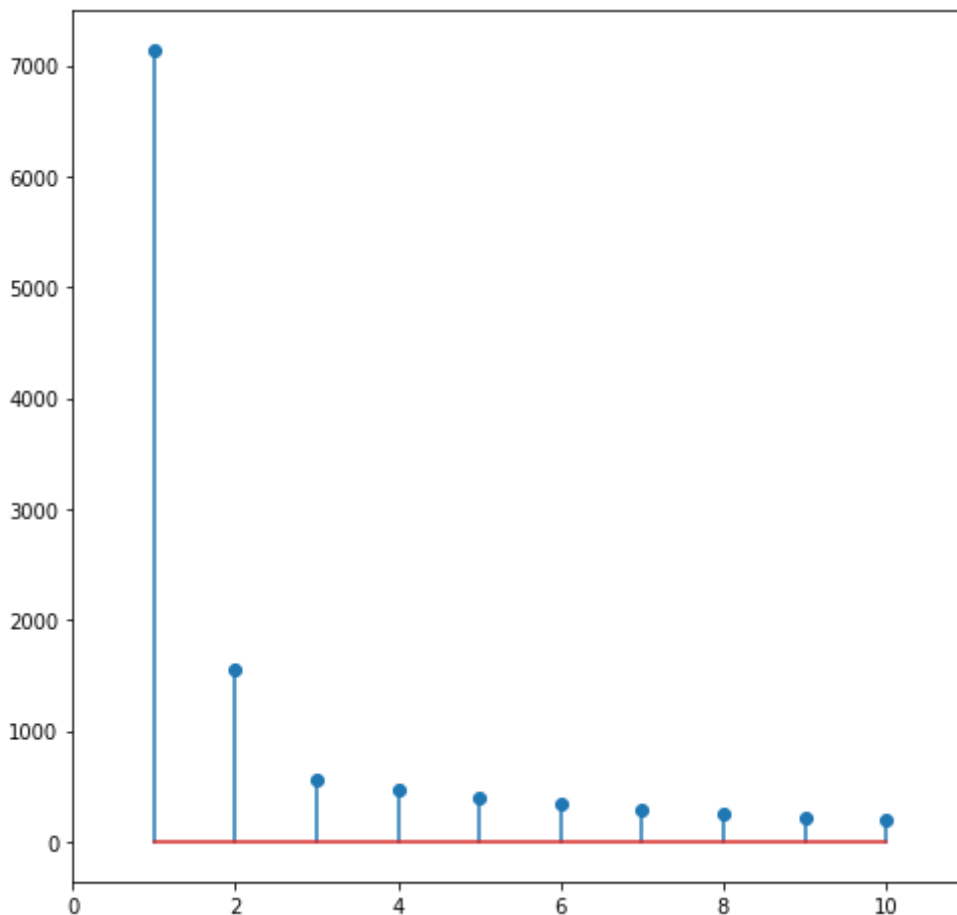
### 1.3. Choosing the Number of Clusters

- Idea: when adding another cluster does not give much better modeling of the data
- One way to select  $k$  for the K-means algorithm is to try different values of  $k$ , plot the K-means objective versus  $k$ , and look at the 'elbow-point' in the plot

In [5]:

```
cost = []
for i in range(1,11):
    kmeans = KMeans(n_clusters=i, random_state=0).fit(X)
    cost.append(abs(kmeans.score(X)))

plt.figure(figsize=(8,8))
plt.stem(range(1,11),cost)
plt.xlim([0, 11])
plt.show()
```



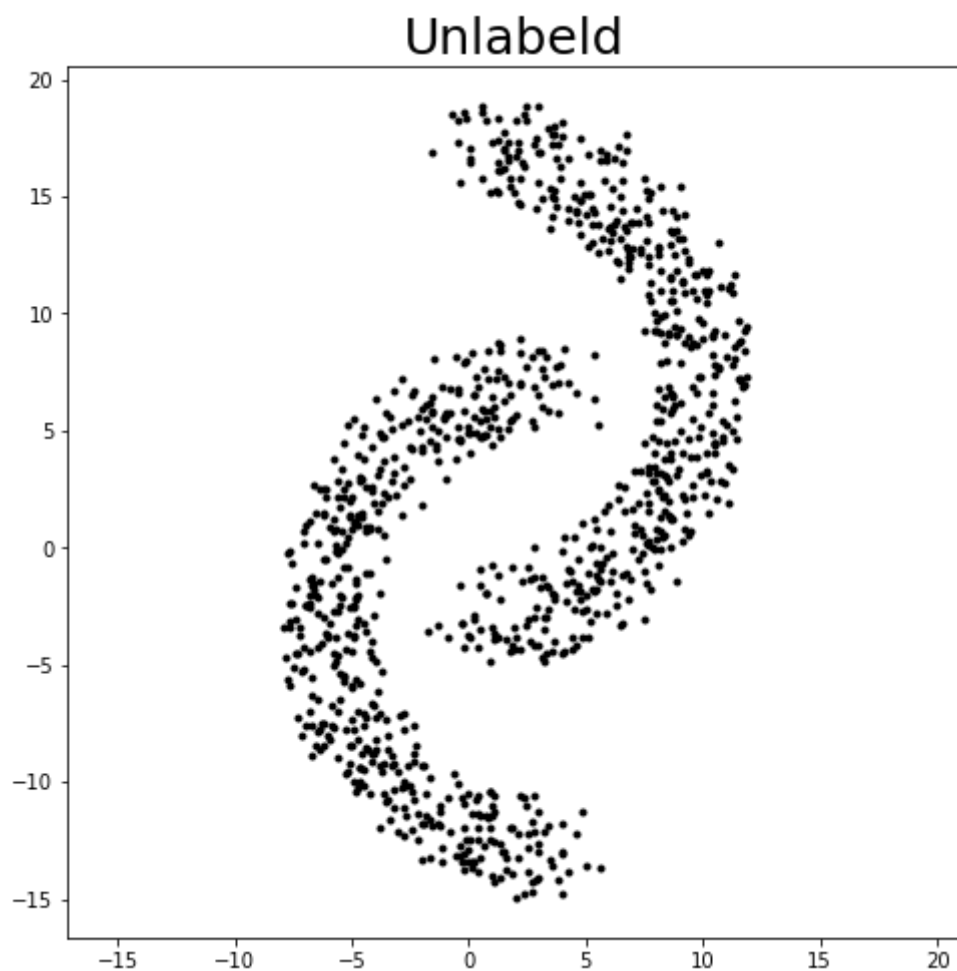
## 1.4. K-means: Limitations

- kmeans\_lim 예제 데이터는 아래 링크에서 다운로드
  - [kmeans\\_lim.pkl \(data\\_files/kmeans\\_lim.pkl\)](#)

In [6]:

```
from six.moves import cPickle
X = cPickle.load(open('./data_files/kmeans_lim.pkl','rb'))

plt.figure(figsize=(8,8))
plt.axis('equal')
plt.plot(X[:,0], X[:,1], 'k.')
plt.title('Unlabeled', fontsize='25')
plt.show()
```



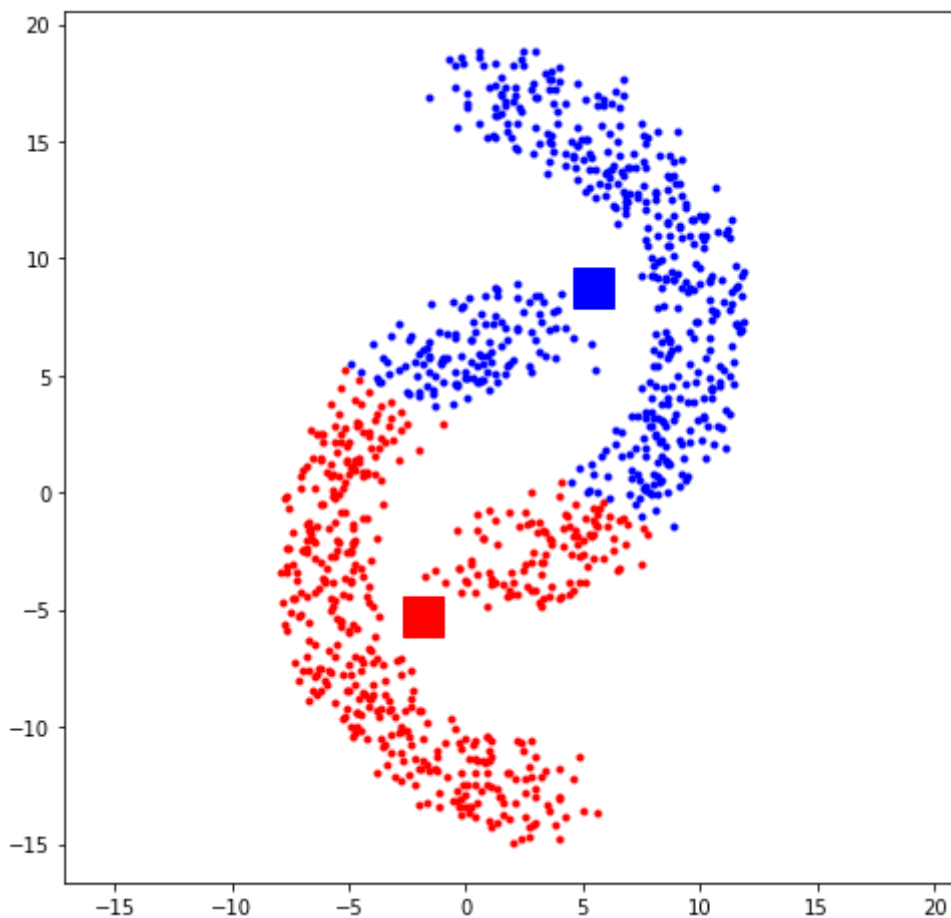
In [7]:

```
kmeans = KMeans(n_clusters=2, random_state=0).fit(X)

plt.figure(figsize=(8,8))
# plt.hold(True)

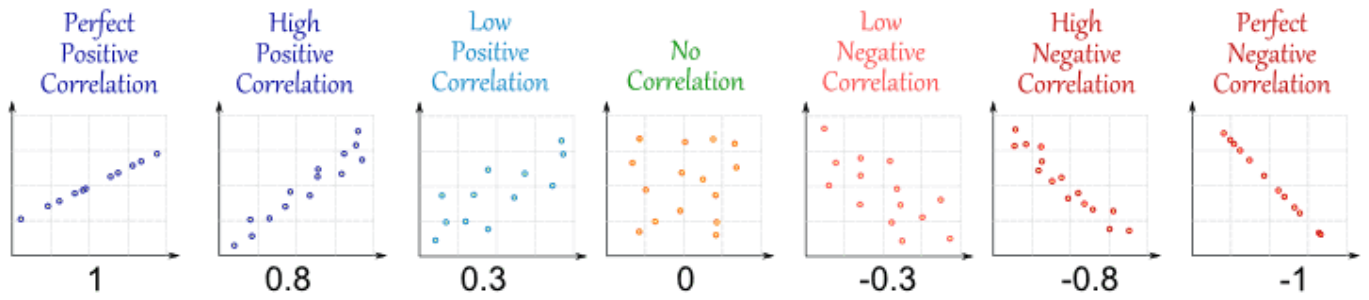
plt.axis('equal')
plt.plot(X[kmeans.labels_ == 0,0],X[kmeans.labels_ == 0,1],'r.')
plt.plot(kmeans.cluster_centers_[0][0], kmeans.cluster_centers_[0][1],'rs',markersize=20)

plt.plot(X[kmeans.labels_ == 1,0],X[kmeans.labels_ == 1,1],'b.')
plt.plot(kmeans.cluster_centers_[1][0], kmeans.cluster_centers_[1][1],'bs',markersize=20)
plt.show()
```



## 2. Correlation Analysis

- Statistical relationship between two sets of data
- <http://rpsychologist.com/d3/correlation/> (<http://rpsychologist.com/d3/correlation/>)

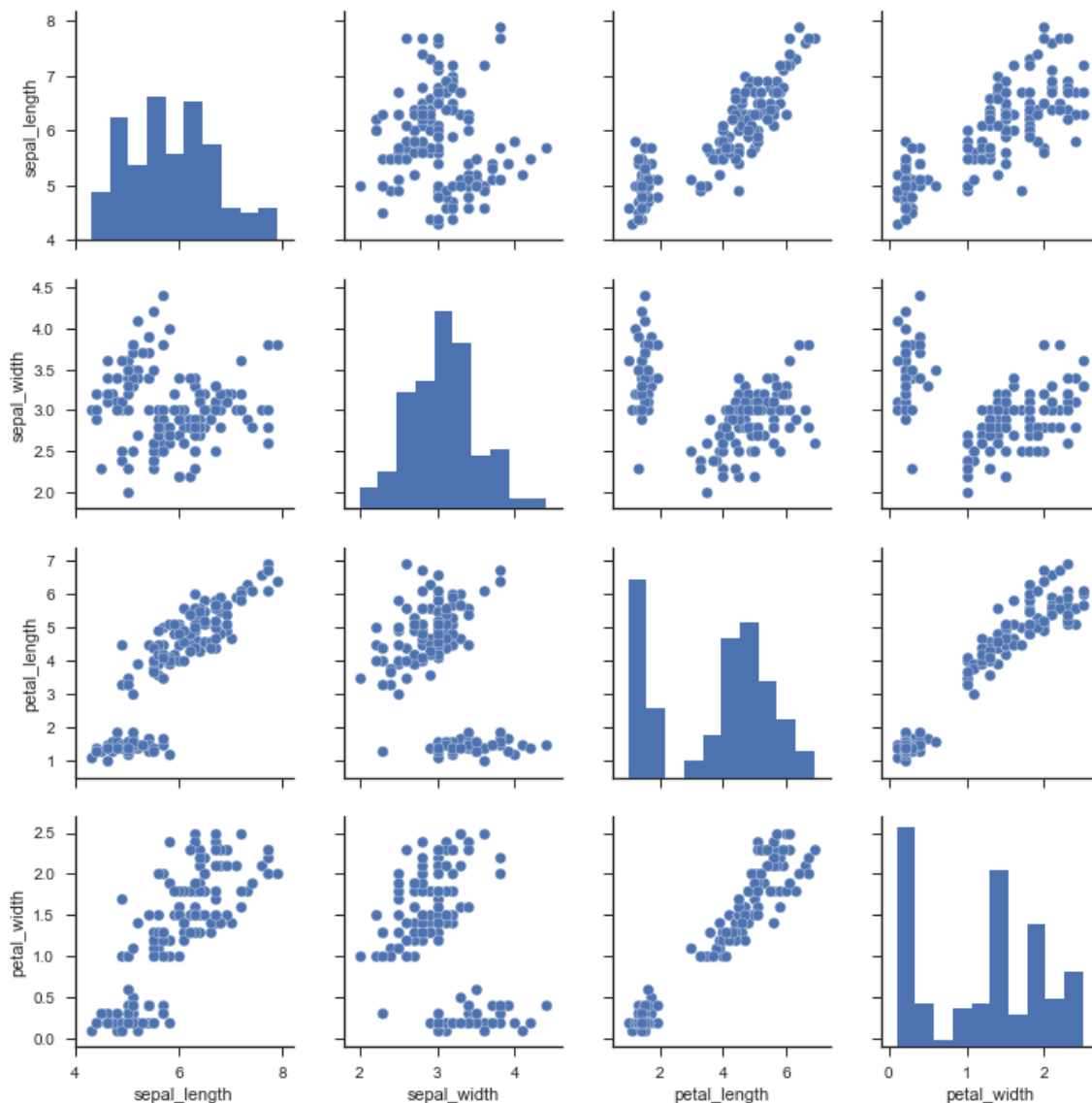


In [8]:

```
import numpy as np
import seaborn as sns
%matplotlib inline

sns.set(style="ticks", color_codes=True)
iris = sns.load_dataset("iris")

g = sns.pairplot(iris)
# sns.plt.show()
```





### 3. Principal Component Analysis (PCA)

**Motivation:** Can we describe high-dimensional data in a "simpler" way?

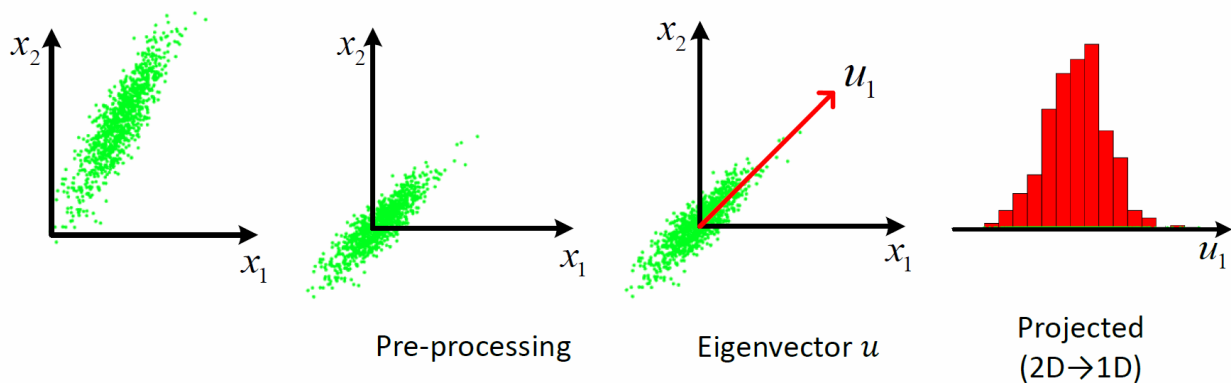
- Dimension reduction without losing too much information
- Find a low-dimensional, yet useful representation of the data

#### 3.1. Dimension Reduction method ( $n \rightarrow k$ )

1. Choose top  $k$  (orthonormal) eigenvectors,  $U = [u_1, u_2, \dots, u_k]$
2. Project  $x_i$  onto span  $\{u_1, u_2, \dots, u_k\}$

$$z^{(i)} = \begin{bmatrix} u_1^T x^{(i)} \\ u_2^T x^{(i)} \\ \vdots \\ u_k^T x^{(i)} \end{bmatrix} \quad \text{or} \quad z = U^T x$$

- Pictorial summary of PCA



$x^{(i)} \rightarrow$  projection onto unit vector  $u \implies u^T x^{(i)} =$  distance from the origin along  $u$

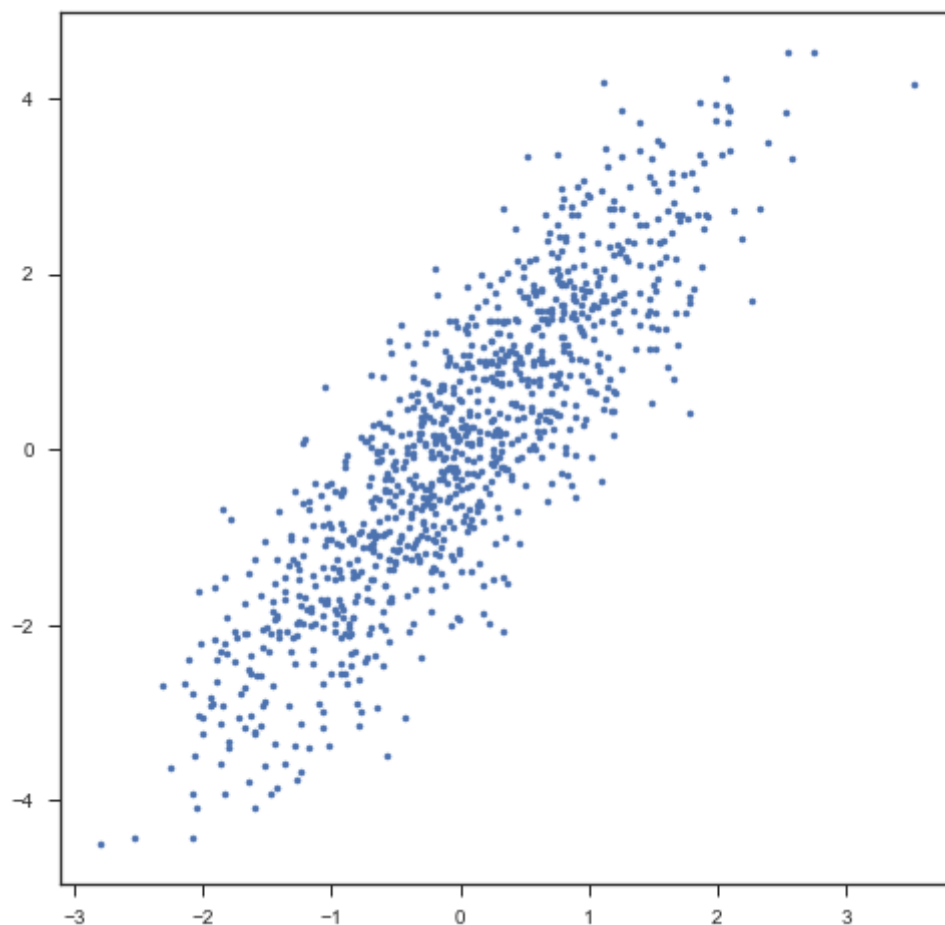
#### 3.2. Python code

- `pca_example` 예제 데이터는 아래 링크를 통해 받을 수 있습니다.
  - [pca\\_example.pkl \(data\\_files/pca\\_example.pkl\)](#)

In [9]:

```
import numpy as np
import matplotlib.pyplot as plt
from six.moves import cPickle
%matplotlib inline

X = cPickle.load(open('./data_files/pca_example.pkl','rb'))
plt.figure(figsize=(8, 8))
plt.plot(X[:, 0], X[:, 1], 'b.')
plt.show()
```

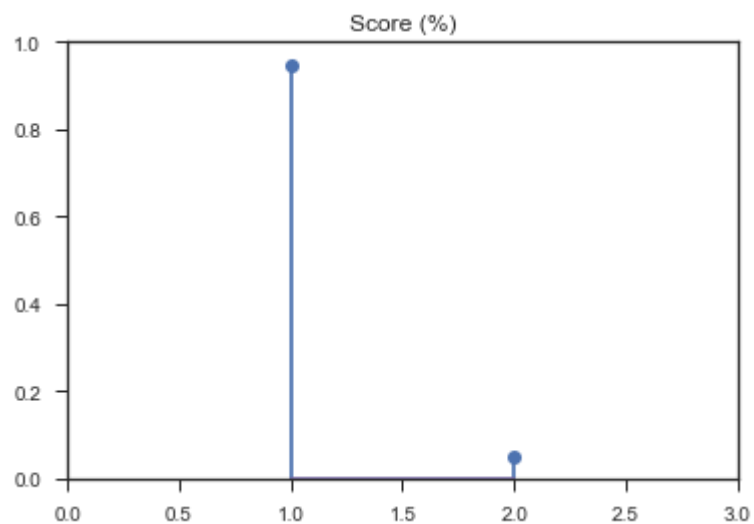


In [10]:

```
from sklearn.decomposition import PCA

# Apply PCA
pca = PCA(n_components=2)
pca.fit(X)

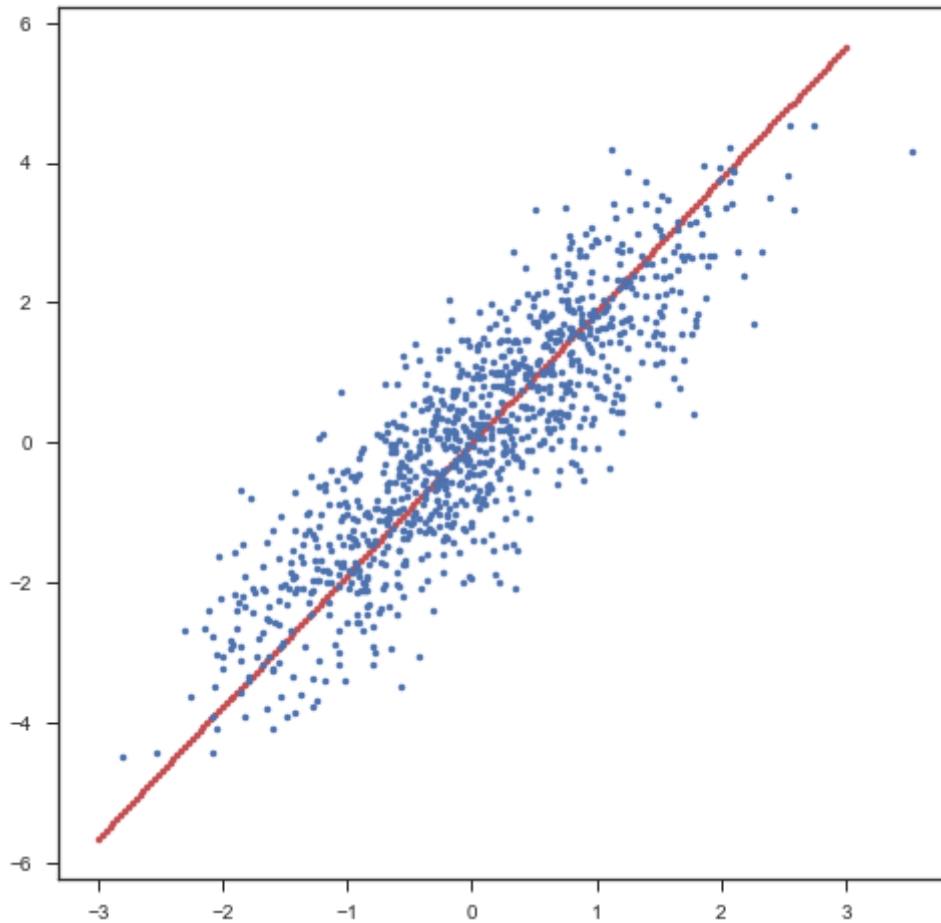
plt.figure()
plt.stem(range(1,3), pca.explained_variance_ratio_)
plt.xlim([0, 3])
plt.ylim([0, 1])
plt.title('Score (%)')
plt.show()
```



In [11]:

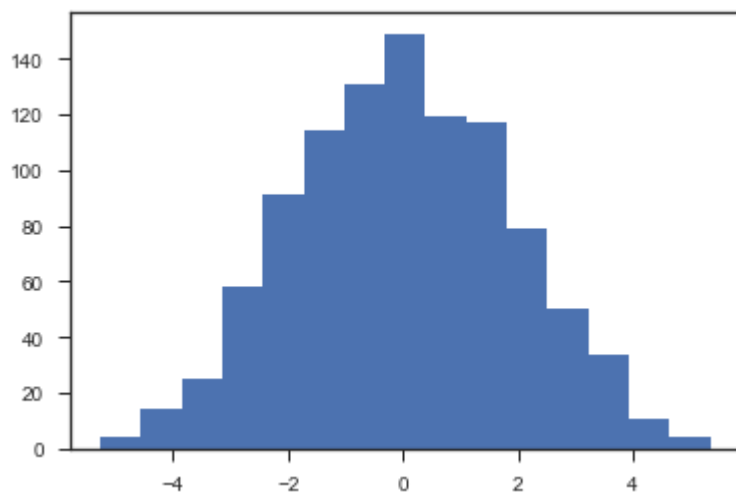
```
# Nomalization and calculate gradient
principal_axis = pca.components_[0, :]
u1 = principal_axis/(np.linalg.norm(principal_axis))
h = u1[1]/u1[0]

x = np.linspace(-3,3,200)
plt.figure(figsize=(8, 8))
# plt.hold(True)
plt.plot(x,x.dot(h),'r.')
plt.plot(X[:, 0], X[:, 1],'b.')
plt.show()
```



In [12]:

```
pca = PCA(n_components=1)
pca.fit(X)
reduced_coordinate = pca.fit_transform(X)
plt.hist(reduced_coordinate, 15)
plt.show()
```



In [13]:

```
%%javascript
$.getScript('https://kmahelona.github.io/ipython_notebook_goodies/ipython_notebook_toc.
js')
```