Supervised Learning

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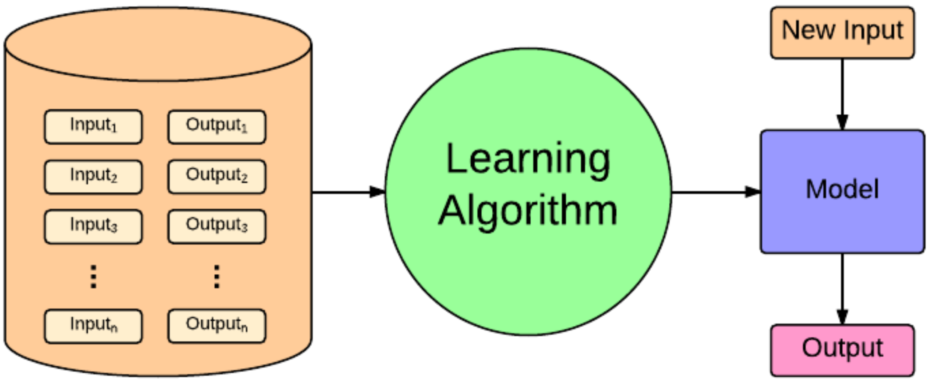
Table of Contents

0. Supervised learning

- Given training set $\left\{\left(x^{(1)}, y^{(1)}\right), \left(x^{(2)}, y^{(2)}\right), \cdots, \left(x^{(m)}, y^{(m)}\right)\right\}$
- Want to find a function g_ω with learning parameter, ω
  - g_ω desired to be as close as possible to y for future (x, y)
  - $i.e., g_\omega(x) \sim y$
- Define a loss function ℓ
- Solve the following optimization problem:

$$\begin{array}{ll} \text{minimize} & f(\omega) = \dfrac{1}{m}\displaystyle\sum_{i=1}^{m} \ell\left(g_\omega\left(x^{(i)}\right), y^{(i)}\right) \\ \text{subject to} & \omega \in \boldsymbol{\omega} \end{array}$$



1. Regression

1.1. k-Nearest Neighbor Regression

The goal is to make quantitative (real valued) predictions on the basis of a (vector of) features or attributes.

We write our model as

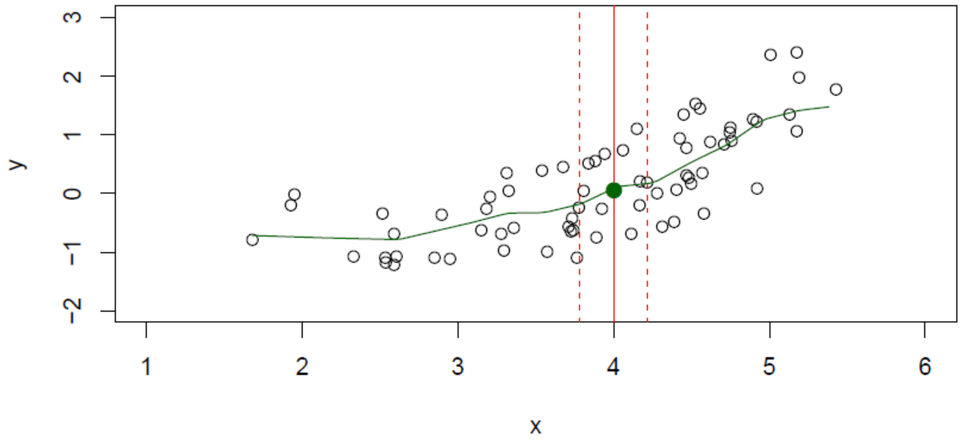
$$Y = f(X) + \epsilon$$

where ϵ captures measurement errors and other discrepancies.

Then, with a good f we can make predictions of Y at new points $X = x$. One possible way so called "nearest neighbor method" is:

$$\hat{f} = \text{Ave}\ (Y \mid X \in \mathcal{N}(x))$$

where $\mathcal{N}(x)$ is some neighborhood of x



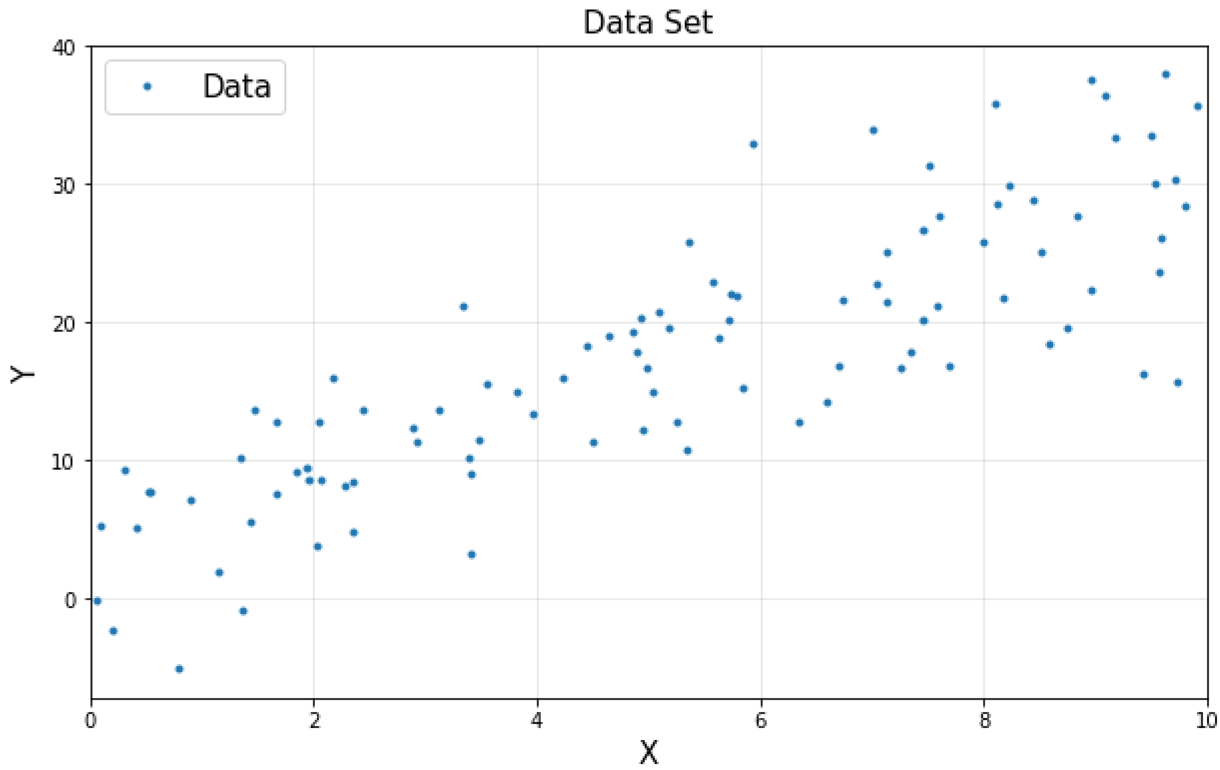
- Regression 에 사용할 데이터 생성

In [1]:
```
import numpy as np
```

In [2]:
```
N = 100
w1 = 3
w0 = 2
x = np.random.uniform(0, 10, N)
y = w1*x + w0 + 5*np.random.normal(0, 1, N)
```

In [3]:
```
import matplotlib.pyplot as plt
% matplotlib inline
```

In [4]:
```
plt.figure(figsize=(10, 6))
plt.title('Data Set', fontsize=15)
plt.plot(x, y, '.', label='Data')
plt.xlabel('X', fontsize=15)
plt.ylabel('Y', fontsize=15)
plt.legend(fontsize=15)
plt.xlim([0, 10])
plt.grid(alpha=0.3)
plt.show()
```



- sklearn.neighbors에 있는 KNeighborsRegressor import

In [5]:
```
from sklearn.neighbors import KNeighborsRegressor
```

In [6]:
```
reg = KNeighborsRegressor(n_neighbors=10)
reg.fit(x.reshape(-1, 1), y)
```

Out[6]:
```
KNeighborsRegressor(algorithm='auto', leaf_size=30, metric='minkowski',
          metric_params=None, n_jobs=1, n_neighbors=10, p=2,
          weights='uniform')
```

In [7]:
```
x_new = np.array([[5]])
```

In [8]:
```
pred = reg.predict(5)
```

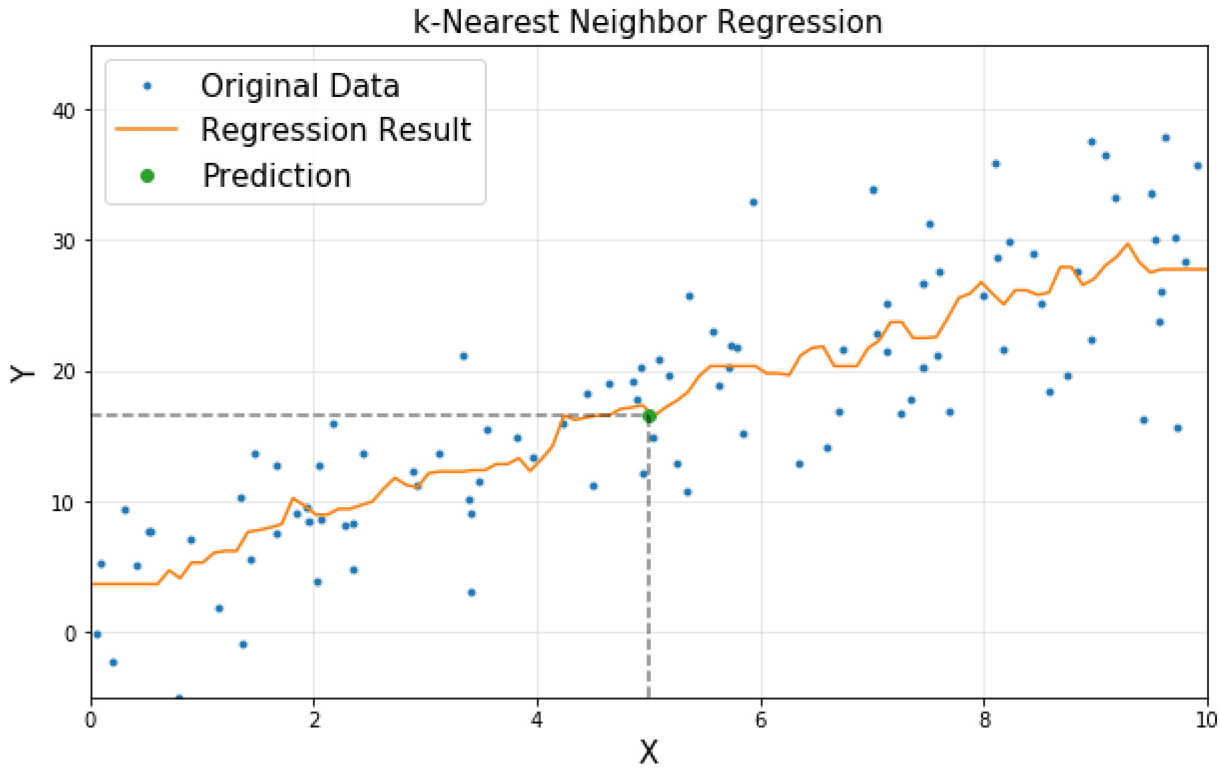
In [9]:
```
print(pred)
```

```
[ 16.5196895]
```

- plot

In [10]:
```
xp = np.linspace(0, 10, 100).reshape(-1, 1)
yp = reg.predict(xp)
```

In [11]:
```
plt.figure(figsize=(10, 6))
plt.title('k-Nearest Neighbor Regression', fontsize=15)
plt.plot(x, y, '.', label='Original Data')
plt.plot(xp, yp, label='Regression Result')
plt.plot(x_new, pred, 'o', label='Prediction')
plt.plot([x_new[0,0], x_new[0,0]], [-5, pred[0]], 'k--', alpha=0.5)
plt.plot([0, x_new[0,0]], [pred[0], pred[0]], 'k--', alpha=0.5)
plt.xlabel('X', fontsize=15)
plt.ylabel('Y', fontsize=15)
plt.legend(fontsize=15)
plt.xlim([0, 10])
plt.ylim([-5, 45])
plt.grid(alpha=0.3)
plt.show()
```



1.2. Linear Regression

선형 회귀 분석 (fitting)

$$\text{Given} \begin{cases} x_i : \text{inputs} \\ y_i : \text{outputs} \end{cases}$$, Find ω_1 and ω_0

$$x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_m \end{bmatrix}, \qquad y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_m \end{bmatrix} \approx \hat{y}_i = \omega_1 x_i + \omega_0$$

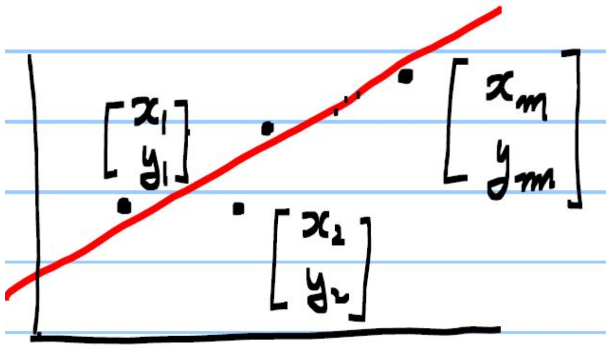
- $\hat{y}_i$: predicted output
- $\omega = \begin{bmatrix} \omega_1 \\ \omega_0 \end{bmatrix}$: Model parameters

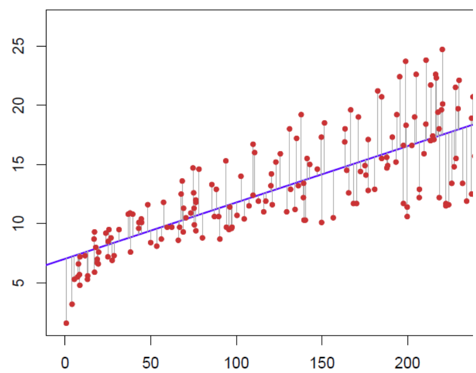
$$\hat{y}_i = f(x_i, \omega) \;\; \text{in general}$$

- in many cases, a linear model to predict y_i used

$$\hat{y}_i = \omega_1 x_i + \omega_0$$

$$\text{such that} \; \min_{\omega_1, \omega_0} \sum_{i=1}^{m} (\hat{y}_i - y_i)^2$$

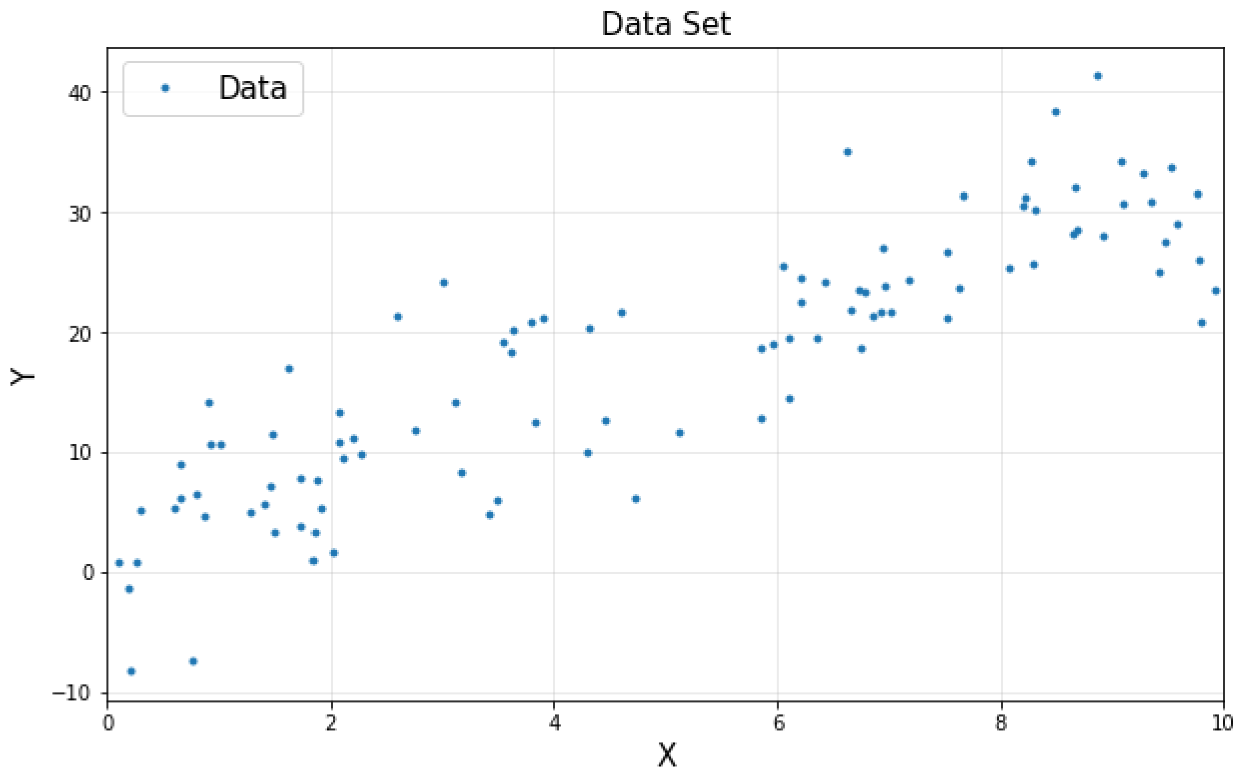




To see how it works, click here (http://i-systems.github.io/HSE545/machine%20learning%20all/03%20Regression/iSystems_01_Regression.html)

- Regression 에 사용할 데이터 생성

In [12]:
```python
import numpy as np

N = 100
w1 = 3
w0 = 2
x = np.random.uniform(0, 10, N)
y = w1*x + w0 + 5*np.random.normal(0, 1, N)

import matplotlib.pyplot as plt
% matplotlib inline

plt.figure(figsize=(10, 6))
plt.title('Data Set', fontsize=15)
plt.plot(x, y, '.', label='Data')
plt.xlabel('X', fontsize=15)
plt.ylabel('Y', fontsize=15)
plt.legend(fontsize=15)
plt.xlim([0, 10])
plt.grid(alpha=0.3)
plt.show()
```



- sklearn.linear_model 에 있는 LinearRegression import

In [13]:
```python
from sklearn.linear_model import LinearRegression
```

In [14]:
```python
reg = LinearRegression()
reg.fit(x.reshape(-1, 1), y)
```

Out[14]: LinearRegression(copy_X=True, fit_intercept=True, n_jobs=1, normalize=False)

- 새로운 데이터에 대하여 predict

In [15]:
```python
x_new = np.array([[6]])
```

In [16]:
```python
pred = reg.predict(x_new)
```

In [17]:
```
print(pred)
```

```
[ 20.78413979]
```

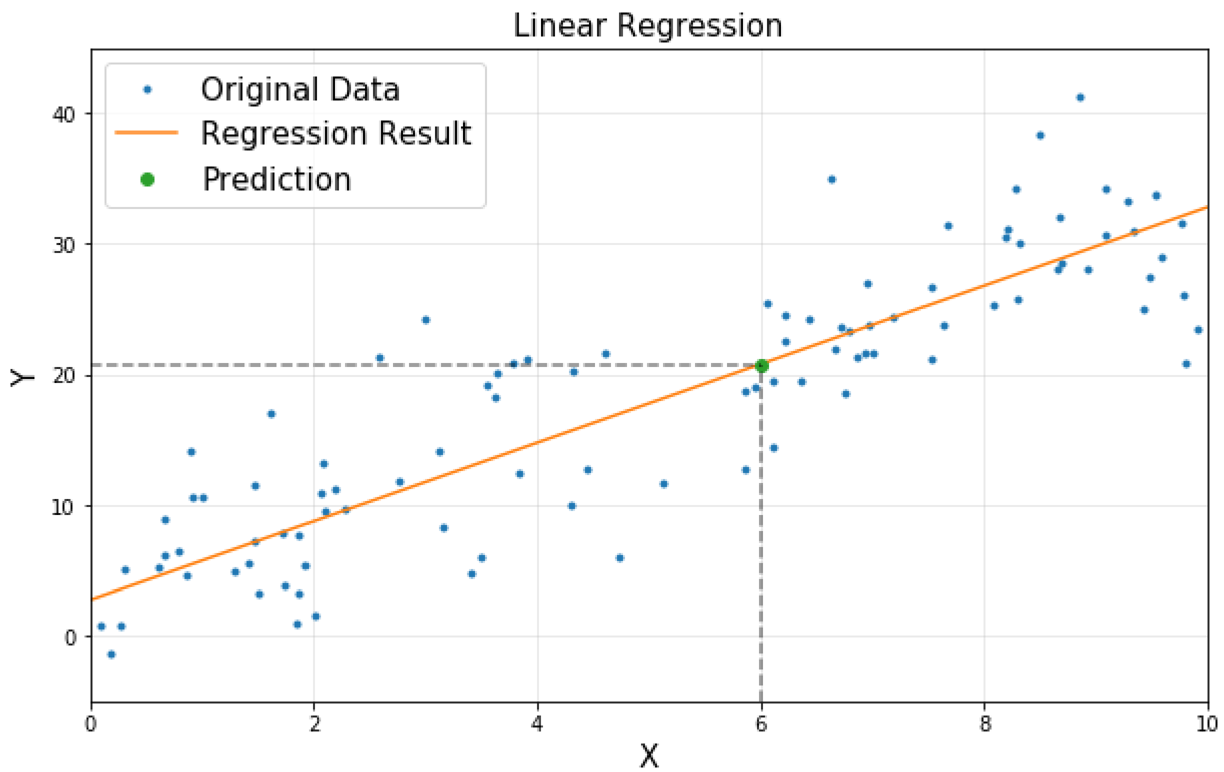
- parameters 확인 및 plot

In [18]:
```
w1_pred = reg.coef_
w0_pred = reg.intercept_
print('w1 pred : ', w1_pred[0])
print('w1 original : ', w1)
print('w0 pred : ', w0_pred)
print('w0 : ', w0)
```

```
w1 pred :  3.00789939237
w1 original :  3
w0 pred :  2.73674343365
w0 :  2
```

In [19]:
```
xp = np.linspace(0, 10)
yp = w1_pred*xp + w0_pred
```

In [20]:
```
plt.figure(figsize=(10, 6))
plt.title('Linear Regression', fontsize=15)
plt.plot(x, y, '.', label='Original Data')
plt.plot(xp, yp, label='Regression Result')
plt.plot(x_new, pred, 'o', label='Prediction')
plt.plot([x_new[0,0], x_new[0,0]], [-5, pred[0]], 'k--', alpha=0.5)
plt.plot([0, x_new[0,0]], [pred[0], pred[0]], 'k--', alpha=0.5)
plt.xlabel('X', fontsize=15)
plt.ylabel('Y', fontsize=15)
plt.legend(fontsize=15)
plt.xlim([0, 10])
plt.ylim([-5, 45])
plt.grid(alpha=0.3)
plt.show()
```



2. Classification

2.1. Data Generation for Classification

- Classification에 사용할 데이터 생성

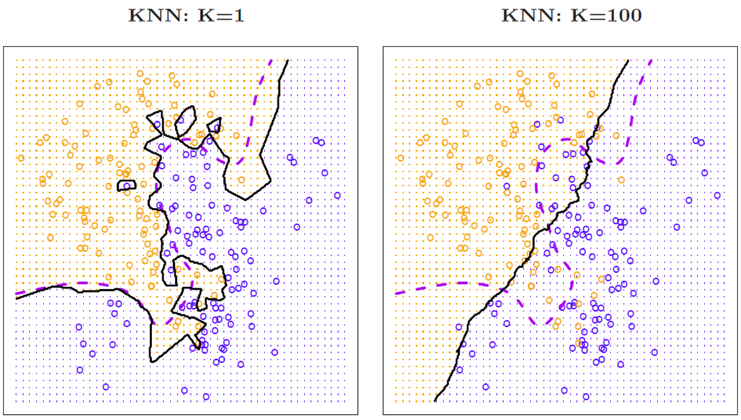
In [21]:
```
import matplotlib.pyplot as plt

C0 = np.random.multivariate_normal([0, 0], np.eye(2), 200)
C1 = np.random.multivariate_normal([10, 10], np.eye(2), 200)
C2 = np.random.multivariate_normal([-5, 5], np.eye(2), 200)

y0 = np.array(C1.shape[0]*[0])
y1 = np.array(C1.shape[0]*[1])
y2 = np.array(C1.shape[0]*[2])
```

- Plot을 통하여 데이터 파악

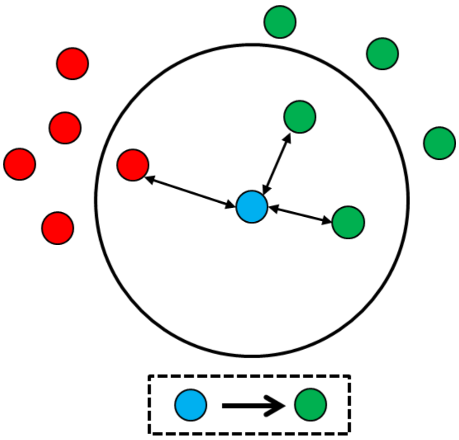
In [22]:
```
plt.figure(figsize=(10, 6))
plt.title('Data Classes', fontsize=15)
plt.plot(C0[:,0], C0[:,1], '.', label='Class 0')
plt.plot(C1[:,0], C1[:,1], '.', label='Class 1')
plt.plot(C2[:,0], C2[:,1], '.', label='Class 2')
plt.legend(loc='lower right', fontsize=15)
plt.xlabel('X1', fontsize=15)
plt.ylabel('X2', fontsize=15)
plt.grid(alpha=0.3)
plt.show()
```

2.2. K-nearest neighbors

- In k-NN classification, an object is assigned to the class most common among its k nearest neighbors (k is a positive integer, typically small).
- If k = 1, then the object is simply assigned to the class of that single nearest neighbor.



- Zoom in,



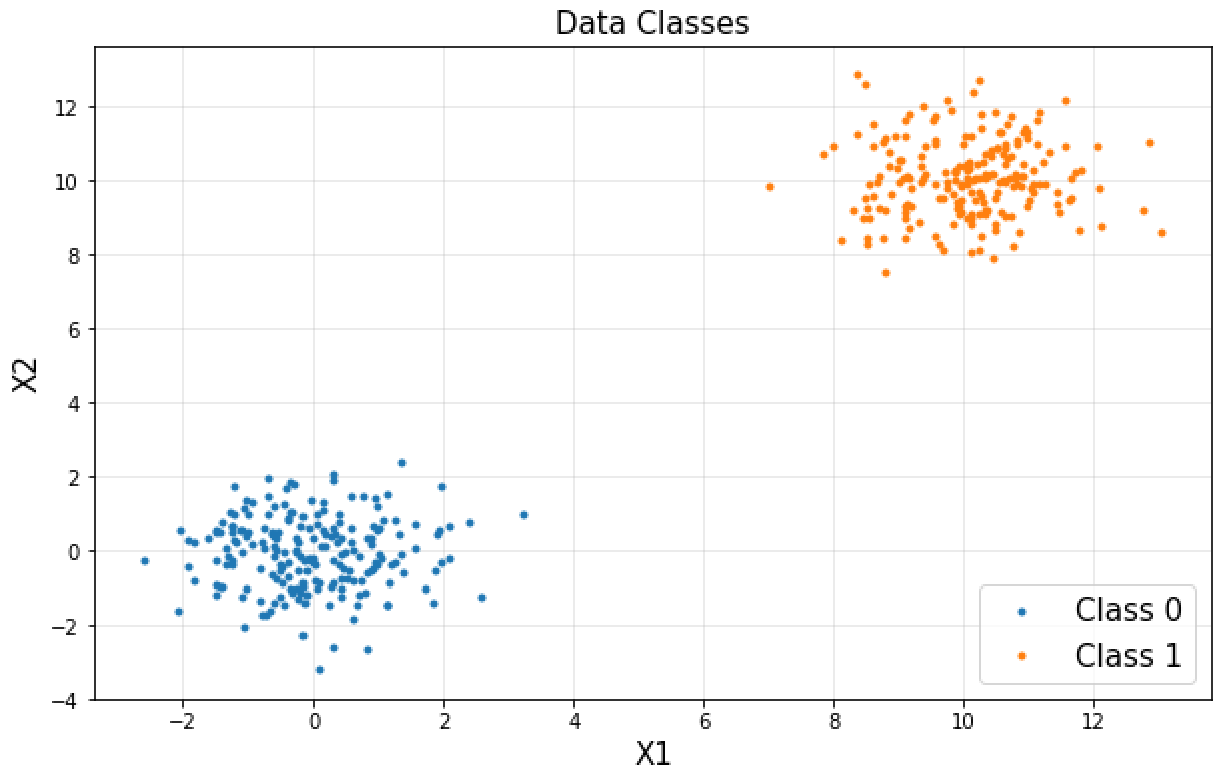
Binary Classification

- C0와 C1 데이터를 분류
- 데이터를 X, y로 병합

In [23]:
```
X = np.vstack([C0, C1])
y = np.hstack([y0, y1])
```

- Plot을 통하여 결과 확인

```
In [24]: plt.figure(figsize=(10, 6))
         plt.title('Data Classes', fontsize=15)
         plt.plot(X[y==0,0], X[y==0,1], '.', label='Class 0')
         plt.plot(X[y==1,0], X[y==1,1], '.', label='Class 1')
         plt.legend(loc='lower right', fontsize=15)
         plt.xlabel('X1', fontsize=15)
         plt.ylabel('X2', fontsize=15)
         plt.grid(alpha=0.3)
         plt.show()
```



- Sklearn neighbors을 import
- KNeighborsClassifier 개체를 선언 후 피팅

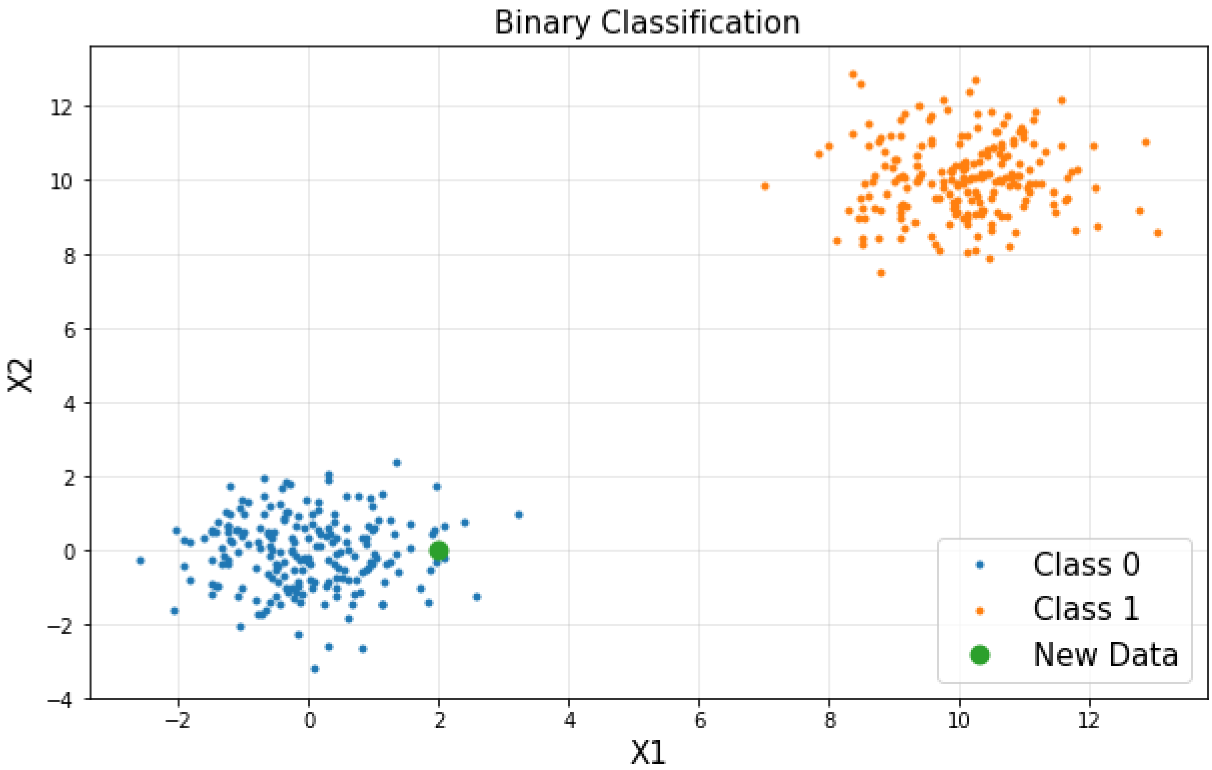
```
In [25]: from sklearn.neighbors import KNeighborsClassifier
```

```
In [26]: clf = KNeighborsClassifier(n_neighbors=2)
         clf.fit(X, y)

Out[26]: KNeighborsClassifier(algorithm='auto', leaf_size=30, metric='minkowski',
                    metric_params=None, n_jobs=1, n_neighbors=2, p=2,
                    weights='uniform')
```

- 새로운 데이터에 대한 결과 확인
- Input shape을 맞추는 것에 주의

```
In [27]: X_new = np.array([2, 0])
         X_new = X_new.reshape(1, -1)
         X_new.shape

Out[27]: (1, 2)
```

In [28]:
```
plt.figure(figsize=(10, 6))
plt.title('Binary Classification', fontsize=15)
plt.plot(X[y==0,0], X[y==0,1], '.', label='Class 0')
plt.plot(X[y==1,0], X[y==1,1], '.', label='Class 1')
plt.plot(X_new[0,0], X_new[0,1], 'o', label='New Data', ms=5, mew=5)
plt.legend(loc='lower right', fontsize=15)
plt.xlabel('X1', fontsize=15)
plt.ylabel('X2', fontsize=15)
plt.grid(alpha=0.3)
plt.show()
```



- Class 0에 속함

In [29]:
```
pred = clf.predict(X_new)
print(pred)
```

```
[0]
```

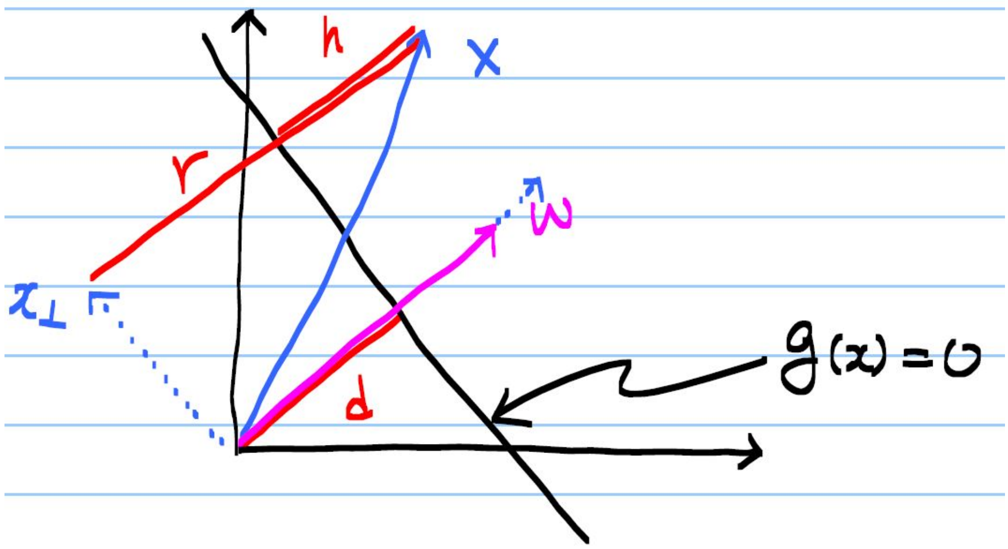
3. Support Vector Machine (SVM)

To see how it works, click [here (http://i-systems.github.io/HSE545/machine%20learning%20all/04%20Classification/iSystems_02_SVM.html)](http://i-systems.github.io/HSE545/machine%20learning%20all/04%20Classification/iSystems_02_SVM.html)

- 가장 많이 쓰이는 모델
- 경계선과 데이터 사이의 거리 (margin) 을 최대화 하는 모델

3.0. Distance from a line

$$\omega = \begin{bmatrix} \omega_1 \\ \omega_2 \end{bmatrix}, x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \implies g(x) = \omega^T x + \omega_0 = \omega_1 x_1 + \omega_2 x_2 + \omega_0$$



- If $\vec{p}$ and $\vec{q}$ are on the decision line

$$\begin{aligned} g(\vec{p}) = g(\vec{q}) = 0 &\implies \omega^T \vec{p} + \omega_0 = \omega^T \vec{q} + \omega_0 = 0 \\ &\implies \omega^T (\vec{p} - \vec{q}) = 0 \end{aligned}$$

$$\therefore\ \omega : \text{normal to the line (orthogonal)} \implies \text{tells the direction of the line}$$

- If x is on the line and $x = d\frac{\omega}{||\omega||}$ (where d is a normal distance from the origin to the line)

$$\begin{aligned} g(x) &= \omega^T x + \omega_0 = 0 \\ &\implies \omega^T d\frac{\omega}{||\omega||} + \omega_0 = d\frac{\omega^T \omega}{||\omega||} + \omega_0 = d||\omega|| + \omega_0 = 0 \\ &\therefore d = -\frac{\omega_0}{||\omega||} \end{aligned}$$

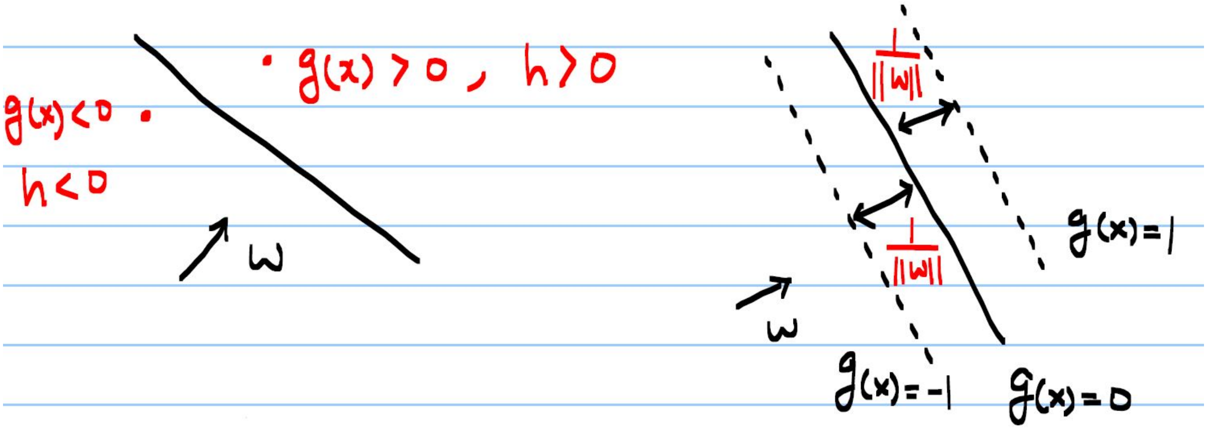
- for any vector of x

$$x = x_\perp + r\frac{\omega}{||\omega||}$$

$$\omega^T x = \omega^T \left(x_\perp + r\frac{\omega}{||\omega||} \right) = r\frac{\omega^T \omega}{||\omega||} = r||\omega||$$

$$\begin{aligned} g(x) &= \omega^T x + \omega_0 \\ &= r||\omega|| + \omega_0 \qquad (r = d + h) \\ &= (d + h)||\omega|| + \omega_0 \\ &= \left(-\frac{\omega_0}{||\omega||} + h \right) ||\omega|| + \omega_0 \\ &= h||\omega|| \end{aligned}$$

$$\therefore\ h = \frac{g(x)}{||\omega||} \implies \textbf{orthogonal distance from the line}$$

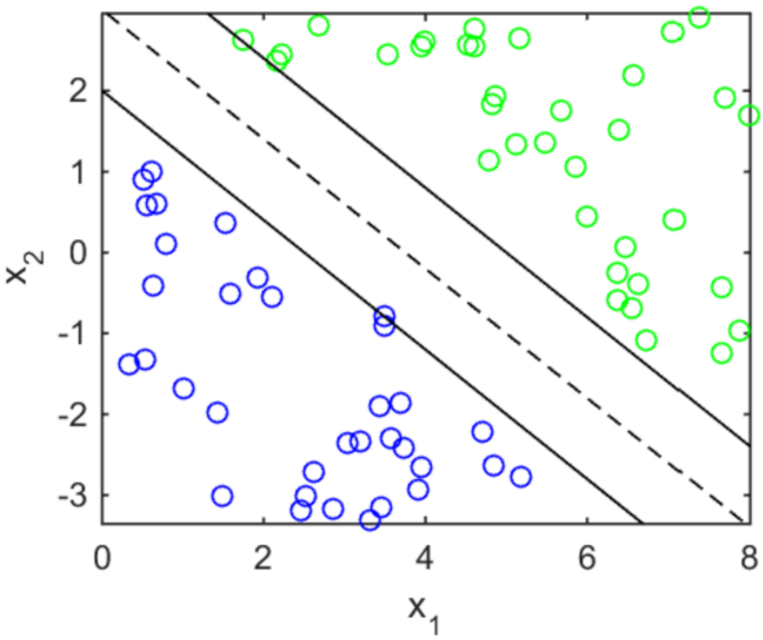


- Distance (= margin)

$$\text{margin} = \frac{2}{\|\omega\|_2}$$

- Minimize $\|\omega\|_2$ to maximize the margin

$$\begin{aligned} \text{minimize} \quad & \|\omega\|_2 \\ \text{subject to} \quad & C_1\omega + \omega_0 \geq 1 \\ & C_2\omega + \omega_0 \leq -1 \end{aligned}$$



3.1. Binary Classification

- C0와 C1 데이터를 분류
- 데이터를 X, y로 병합

In [30]:
```
X = np.vstack([C0, C1])
y = np.concatenate([y0, y1])
```

- sklearn.svm 모듈에서 SVC import
- svc 개체를 선언 후 피팅

In [31]:
```
from sklearn.svm import SVC
```

In [32]:
```
clf = SVC()
clf.fit(X, y)
```

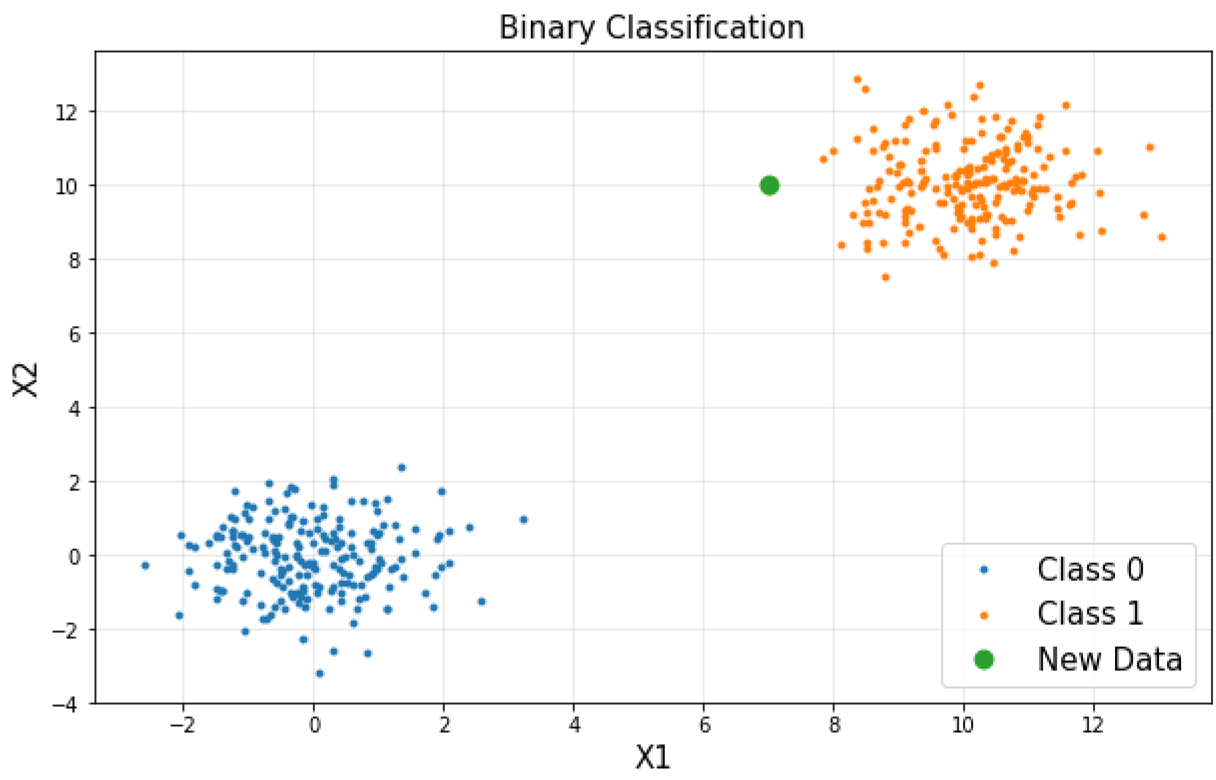
Out[32]:
```
SVC(C=1.0, cache_size=200, class_weight=None, coef0=0.0,
  decision_function_shape=None, degree=3, gamma='auto', kernel='rbf',
  max_iter=-1, probability=False, random_state=None, shrinking=True,
  tol=0.001, verbose=False)
```

- 새로운 데이터에 대한 결과 확인
- Input shape을 맞추는 것에 주의

In [33]:
```
X_new = np.array([7, 10])
X_new = X_new.reshape(1, -1)
X_new.shape
```

Out[33]:
```
(1, 2)
```

In [34]:
```
plt.figure(figsize=(10, 6))
plt.title('Binary Classification', fontsize=15)
plt.plot(X[y==0,0], X[y==0,1], '.', label='Class 0')
plt.plot(X[y==1,0], X[y==1,1], '.', label='Class 1')
plt.plot(X_new[0,0], X_new[0,1], 'o', label='New Data', ms=5, mew=5)
plt.legend(loc='lower right', fontsize=15)
plt.xlabel('X1', fontsize=15)
plt.ylabel('X2', fontsize=15)
plt.grid(alpha=0.3)
plt.show()
```



- 새로운 데이터는 Class 1에 속함

In [35]:
```
clf.predict(X_new)
```

Out[35]:
```
array([1])
```

3.2. Multi Classification

- C0, C1, C2 데이터를 분류
- Binary classification 에 이용된 코드와 동일
- X, y로 병합

In [36]:
```
X = np.vstack([C0, C1, C2])
y = np.concatenate([y0, y1, y2])
```

- sklearn.svm 모듈에서 SVC import
- svc 개체를 선언 후 피팅

In [37]:
```
from sklearn.svm import SVC
```

In [38]:
```
clf = SVC()
clf.fit(X, y)
```

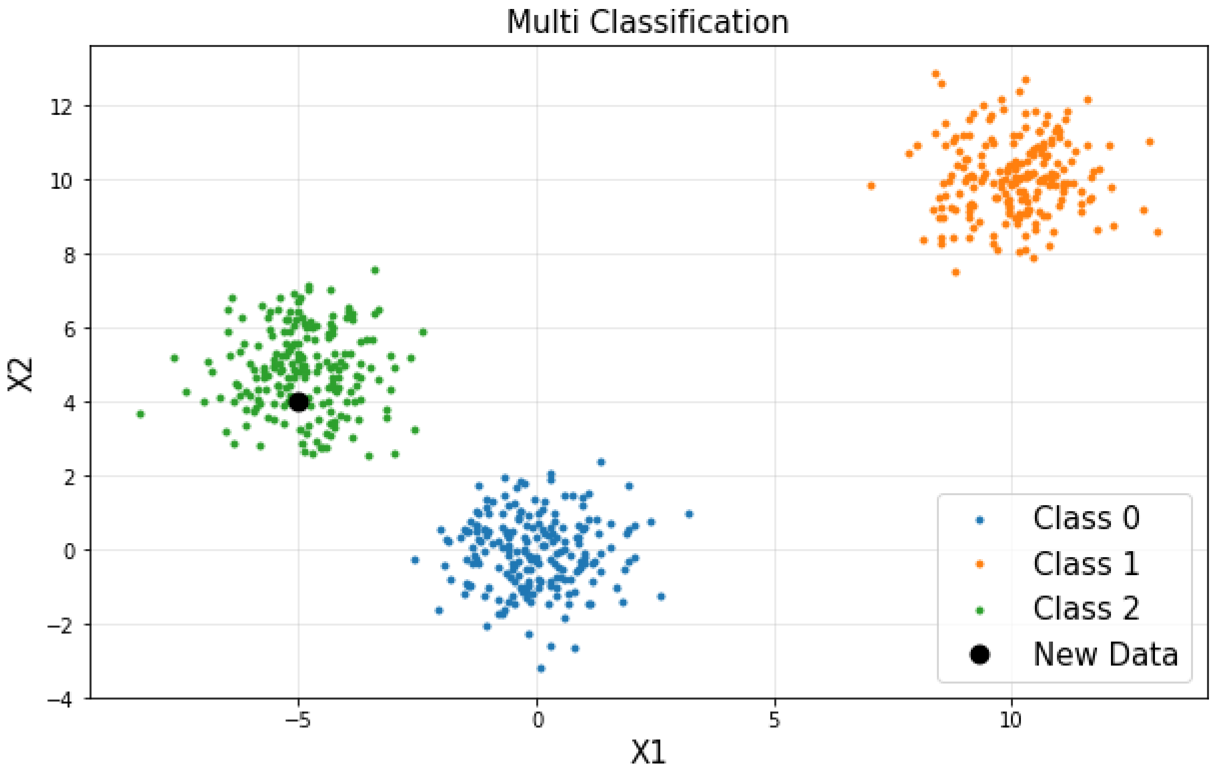
Out[38]:
```
SVC(C=1.0, cache_size=200, class_weight=None, coef0=0.0,
  decision_function_shape=None, degree=3, gamma='auto', kernel='rbf',
  max_iter=-1, probability=False, random_state=None, shrinking=True,
  tol=0.001, verbose=False)
```

- 새로운 데이터에 대한 결과 확인
- Input shape을 맞추는 것에 주의

In [39]:
```
X_new = np.array([-5, 4])
X_new = X_new.reshape(1, -1)
X_new.shape
```

Out[39]:
```
(1, 2)
```

In [40]:
```
plt.figure(figsize=(10, 6))
plt.title('Multi Classification', fontsize=15)
plt.plot(X[y==0,0], X[y==0,1], '.', label='Class 0')
plt.plot(X[y==1,0], X[y==1,1], '.', label='Class 1')
plt.plot(X[y==2,0], X[y==2,1], '.', label='Class 2')
plt.plot(X_new[0,0], X_new[0,1], 'ko', label='New Data', ms=5, mew=5)
plt.legend(loc='lower right', fontsize=15)
plt.xlabel('X1', fontsize=15)
plt.ylabel('X2', fontsize=15)
plt.grid(alpha=0.3)
plt.show()
```



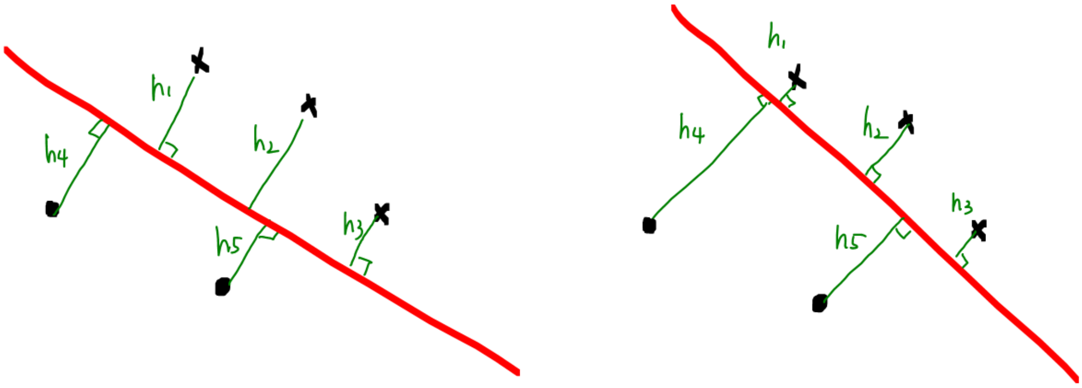
- 새로운 데이터는 Class1에 속함

In [41]:
```
clf.predict(X_new)
```

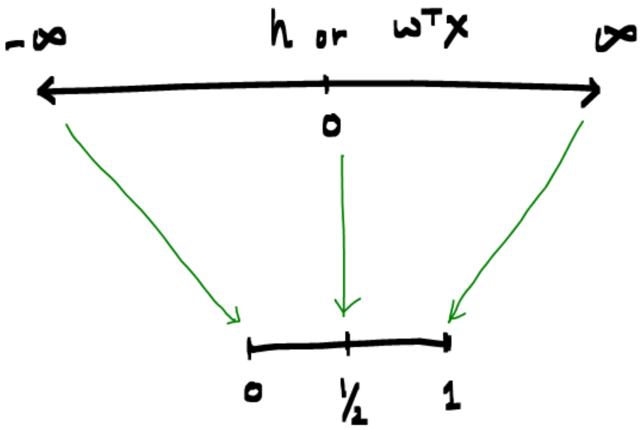
Out[41]: array([2])

4. Logistic Regression

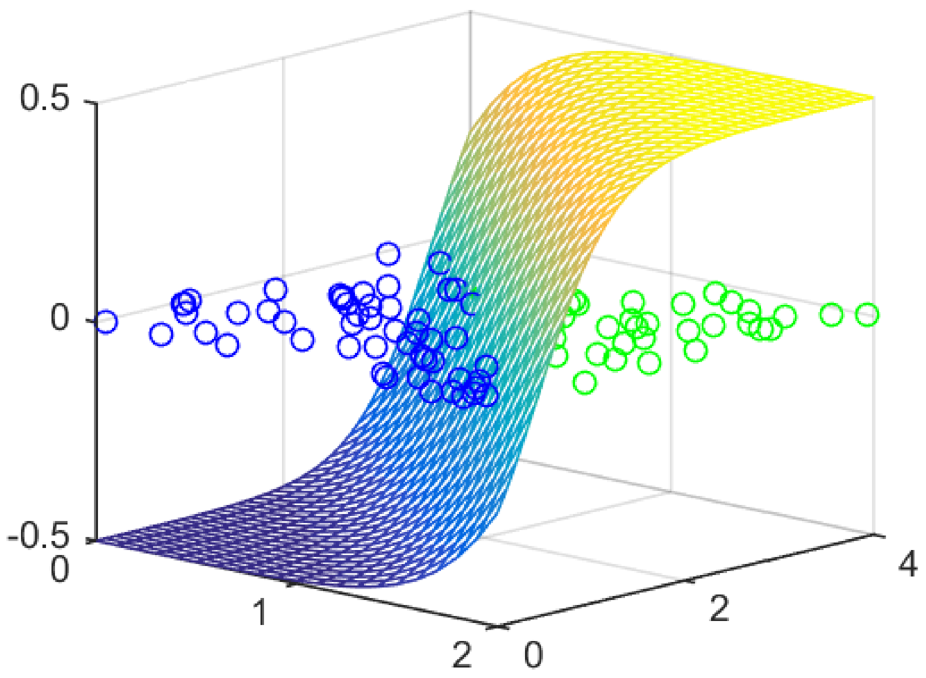
- Logistic regression is a classification algorithm - don't be confused
- We want to use *distance information of all data points* $\rightarrow$ logistic regression



- basic idea: find the decision boundary (hyperplane) of $g(x) = \omega^T x = 0$ such that maximizes $\prod_i |h_i|$
  - Inequality of arithmetic and geometric means
  $$\frac{h_1 + h_2}{2} \geq \sqrt{h_1 h_2}$$
  and that equality holds if and only if $h_1 = h_2$
- Roughly speaking, this optimization of $\max \prod_i |h_i|$ tends to position a hyperplane in the middle of two classes
$$h = \frac{g(x)}{\|\omega\|} = \frac{\omega^T x}{\|\omega\|} \approx \omega^T x$$
- We link or squeeze $(-\infty, +\infty)$ to $(0, 1)$ for several reasons:



- If $\sigma(z)$ is the sigmoid function, or the logistic function
$$\sigma(z) = \frac{1}{1 + e^{-z}} \implies \sigma(\omega^T x) = \frac{1}{1 + e^{-\omega^T x}}$$
  - logistic function generates a value where is always either 0 or 1
  - Crosses 0.5 at the origin, then flattens out
- Classified based on probability



To see how it works, click [here (http://i-systems.github.io/HSE545/machine%20learning%20all/04%20Classification/iSystems_03_logistic_regression.html)](http://i-systems.github.io/HSE545/machine%20learning%20all/04%20Classification/iSystems_03_logistic_regression.html)

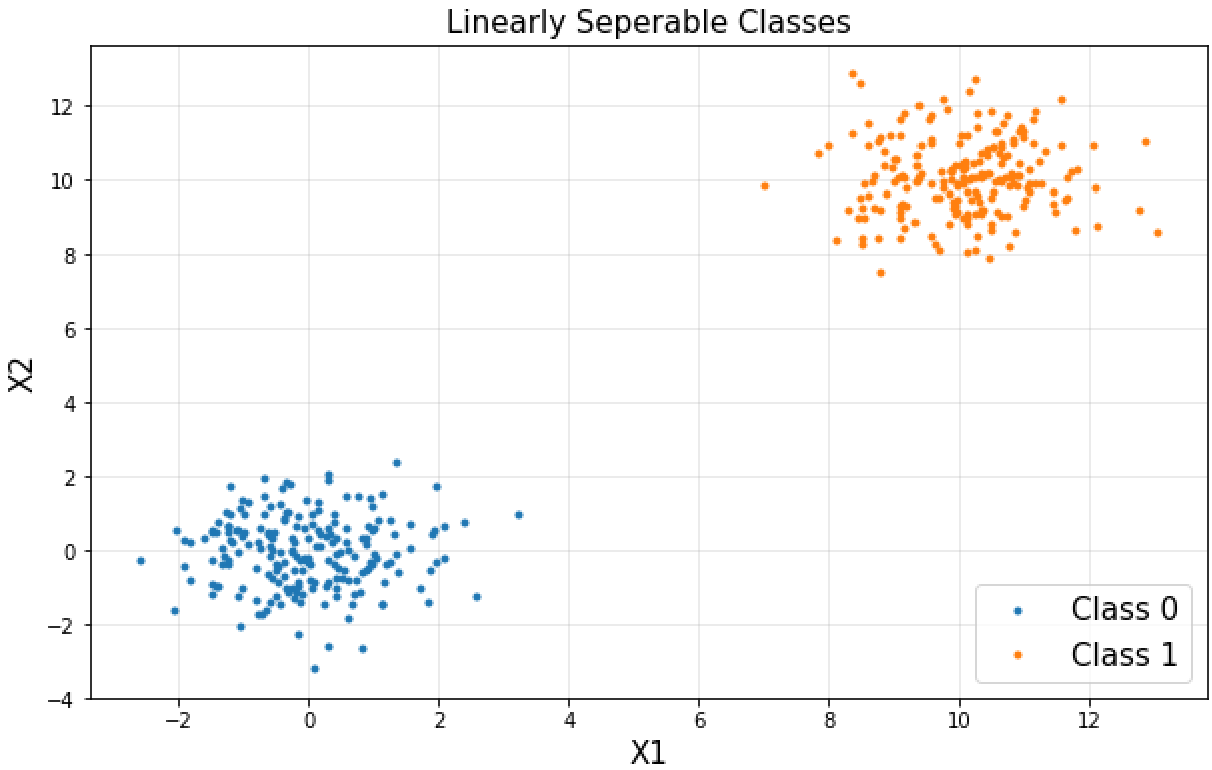
4.1. Binary Classification

- C0와 C1 데이터를 분류
- 데이터를 X, y로 병합

In [42]:
```
X = np.vstack([C0, C1])
y = np.hstack([y0, y1])
```

- Plot을 통하여 결과 확인

In [43]:
```
plt.figure(figsize=(10, 6))
plt.title('Linearly Seperable Classes', fontsize=15)
plt.plot(X[y==0,0], X[y==0,1], '.', label='Class 0')
plt.plot(X[y==1,0], X[y==1,1], '.', label='Class 1')
plt.legend(loc='lower right', fontsize=15)
plt.xlabel('X1', fontsize=15)
plt.ylabel('X2', fontsize=15)
plt.grid(alpha=0.3)
plt.show()
```



- Sklearn linear_model을 import
- LogisticRegression 개체를 선언 후 피팅

In [44]:
```
from sklearn import linear_model
```

In [45]:
```
clf = linear_model.LogisticRegression()
clf.fit(X, y)
```

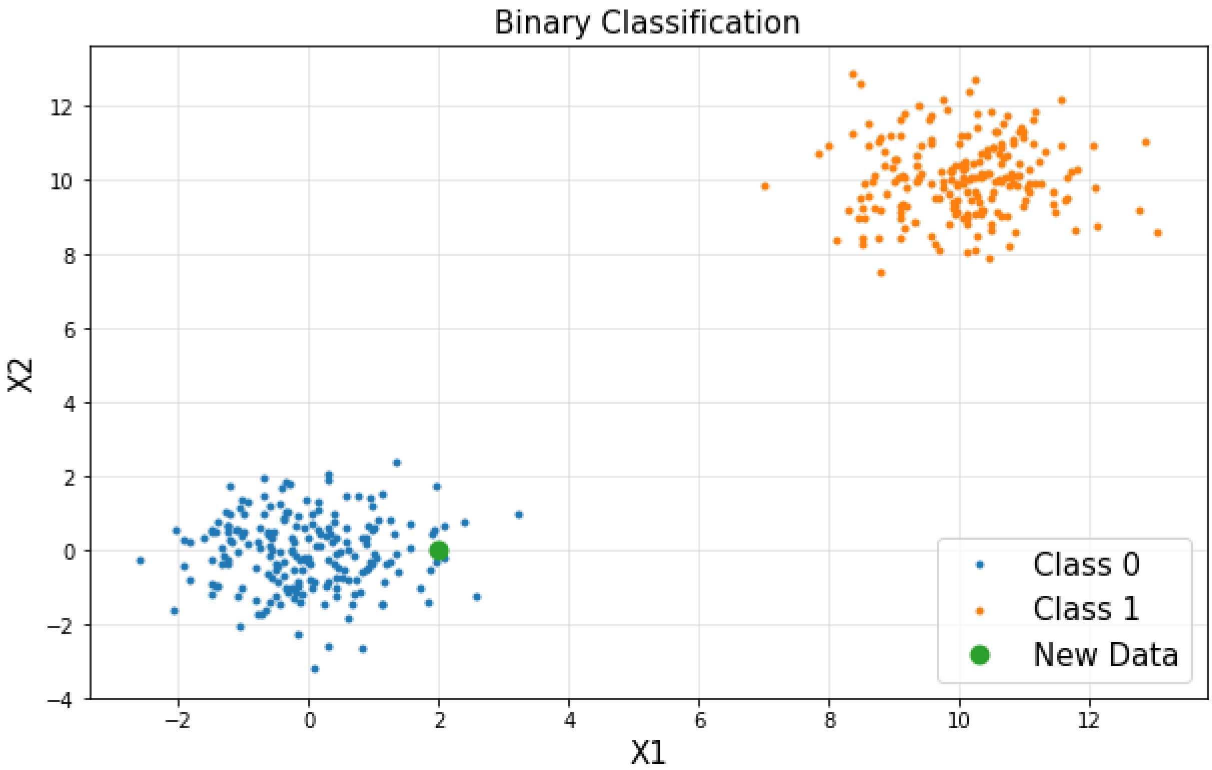
Out[45]:
```
LogisticRegression(C=1.0, class_weight=None, dual=False, fit_intercept=True,
          intercept_scaling=1, max_iter=100, multi_class='ovr', n_jobs=1,
          penalty='l2', random_state=None, solver='liblinear', tol=0.0001,
          verbose=0, warm_start=False)
```

- 새로운 데이터에 대한 결과 확인
- Input shape을 맞추는 것에 주의

In [46]:
```
X_new = np.array([2, 0])
X_new = X_new.reshape(1, -1)
X_new.shape
```

Out[46]:
```
(1, 2)
```

In [47]:
```
plt.figure(figsize=(10, 6))
plt.title('Binary Classification', fontsize=15)
plt.plot(X[y==0,0], X[y==0,1], '.', label='Class 0')
plt.plot(X[y==1,0], X[y==1,1], '.', label='Class 1')
plt.plot(X_new[0,0], X_new[0,1], 'o', label='New Data', ms=5, mew=5)
plt.legend(loc='lower right', fontsize=15)
plt.xlabel('X1', fontsize=15)
plt.ylabel('X2', fontsize=15)
plt.grid(alpha=0.3)
plt.show()
```



- Class 0에 속함

In [48]:
```
pred = clf.predict(X_new)
print(pred)
```

```
[0]
```

In [49]:
```
pred = clf.predict_proba(X_new)
print(pred)
```

```
[[ 0.9538944  0.0461056]]
```

4.2. Multi Classification

- C0, C1, C2 데이터를 분류
- Binary classification 에 이용된 코드와 동일
- X, y로 병합

In [50]:
```
X = np.vstack([C0, C1, C2])
y = np.hstack([y0, y1, y2])
```

- Plot을 통하여 결과 확인

In [51]:
```
plt.figure(figsize=(10, 6))
plt.title('Linearly Seperable Classes', fontsize=15)
plt.plot(X[y==0,0], X[y==0,1], '.', label='Class0')
plt.plot(X[y==1,0], X[y==1,1], '.', label='Class1')
plt.plot(X[y==2,0], X[y==2,1], '.', label='Class2')
plt.legend(loc='lower right', fontsize=15)
plt.xlabel('X1', fontsize=15)
plt.ylabel('X2', fontsize=15)
plt.grid(alpha=0.3)
plt.show()
```

- Sklearn linear_model을 import
- LogisticRegression 개체를 선언 후 피팅

In [52]:
```
from sklearn import linear_model
```

In [53]:
```
clf = linear_model.LogisticRegression()
clf.fit(X, y)
```

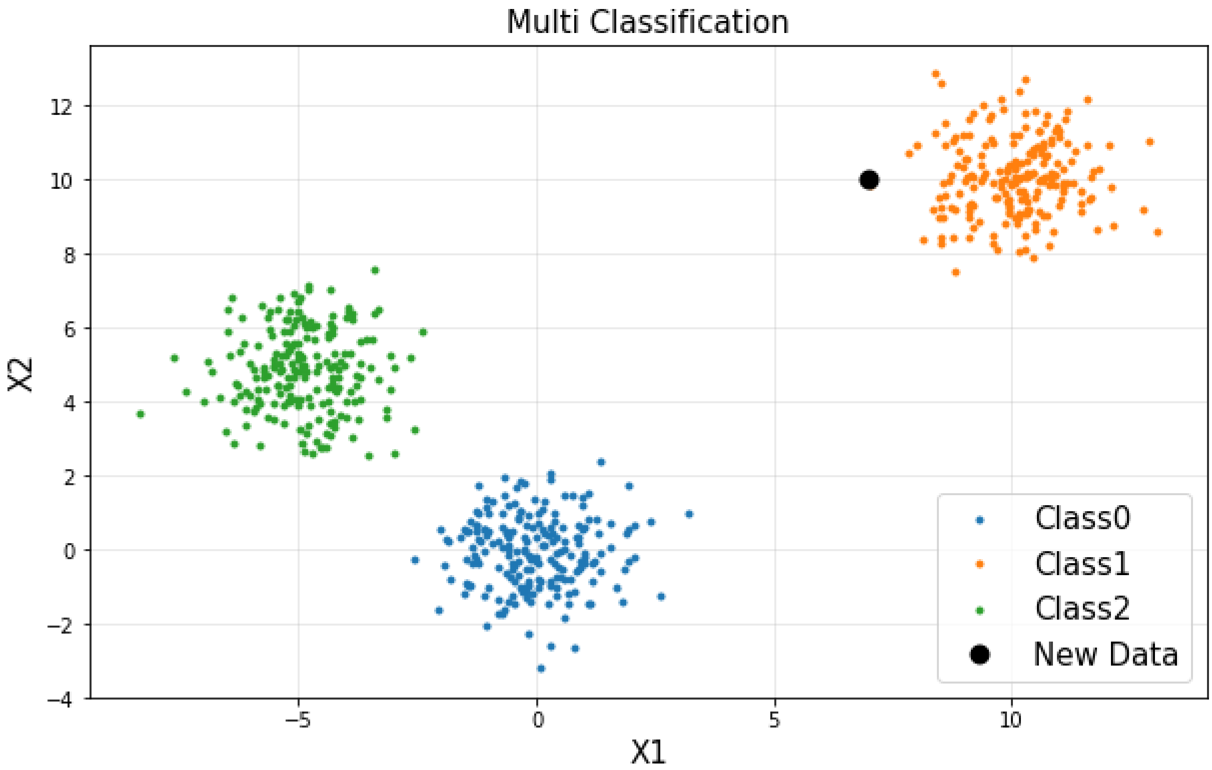
Out[53]:
```
LogisticRegression(C=1.0, class_weight=None, dual=False, fit_intercept=True,
          intercept_scaling=1, max_iter=100, multi_class='ovr', n_jobs=1,
          penalty='l2', random_state=None, solver='liblinear', tol=0.0001,
          verbose=0, warm_start=False)
```

- 새로운 데이터에 대한 결과 확인
- Input shape을 맞추는 것에 주의

In [54]:
```
X_new = np.array([7, 10])
X_new = X_new.reshape(1, -1)
X_new.shape
```

Out[54]:
```
(1, 2)
```

In [55]:
```
plt.figure(figsize=(10, 6))
plt.title('Multi Classification', fontsize=15)
plt.plot(X[y==0,0], X[y==0,1], '.', label='Class0')
plt.plot(X[y==1,0], X[y==1,1], '.', label='Class1')
plt.plot(X[y==2,0], X[y==2,1], '.', label='Class2')
plt.plot(X_new[0,0], X_new[0,1], 'ko', label='New Data', ms=5, mew=5)
plt.legend(loc='lower right', fontsize=15)
plt.xlabel('X1', fontsize=15)
plt.ylabel('X2', fontsize=15)
plt.grid(alpha=0.3)
plt.show()
```



- Predict로 예측

In [56]:
```
prob = clf.predict(X_new)
print(prob)
```

```
[1]
```

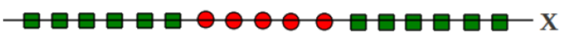
In [57]:
```
prob = clf.predict_proba(X_new)
print(prob)
```

```
[[  1.15846006e-04   9.90478147e-01   9.40600702e-03]]
```

5. Nonlinear Classification

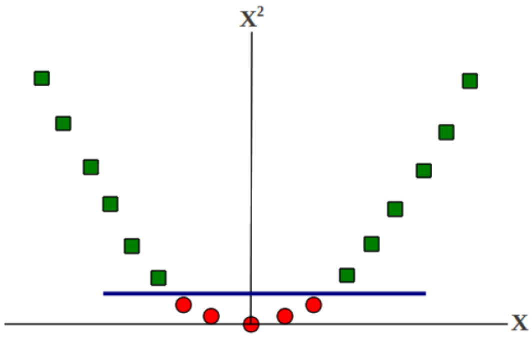
Classifying non-linear separable data

- Consider the binary classification problem
  - each example represented by a single feature x
  - No linear separator exists for this data

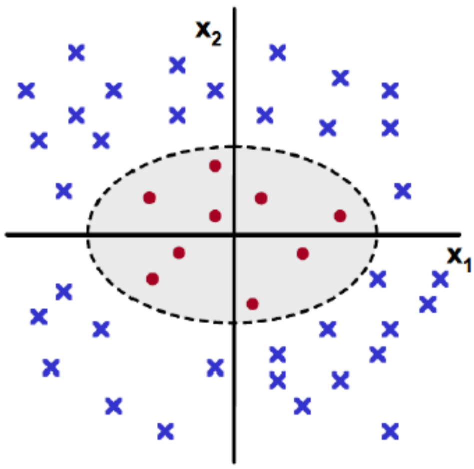


- Now map each example as $x \rightarrow \{x, x^2\}$

- Data now becomes linearly separable in the new representation

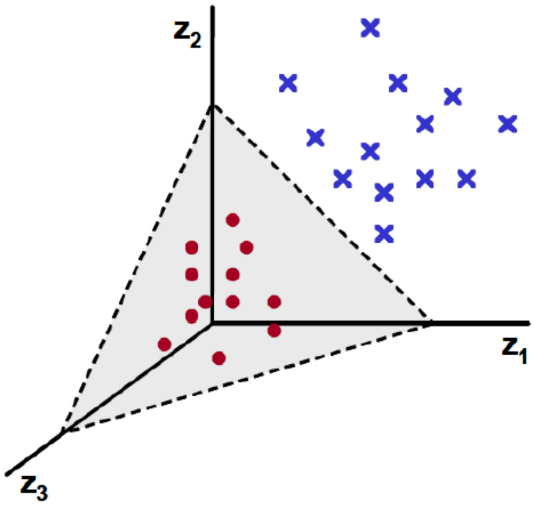


- Linear in the new representation $=$ nonlinear in the old representation
- Let's look at another example
  - Each example defined by a two features $x = \{x_1, x_2\}$
  - No linear separator exists for this data



- Now map each example as $x = \{x_1, x_2\} \rightarrow z = \{x_1^2, \sqrt{2}x_1x_2, x_2^2\}$
  - Each example now has three features (derived from the old represenation)

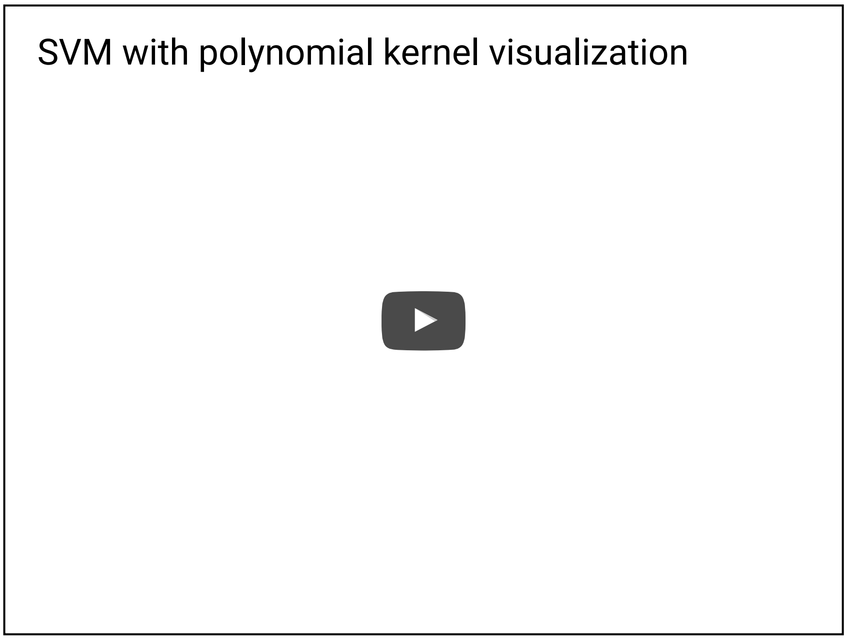
- Data now becomes linear separable in the new representation



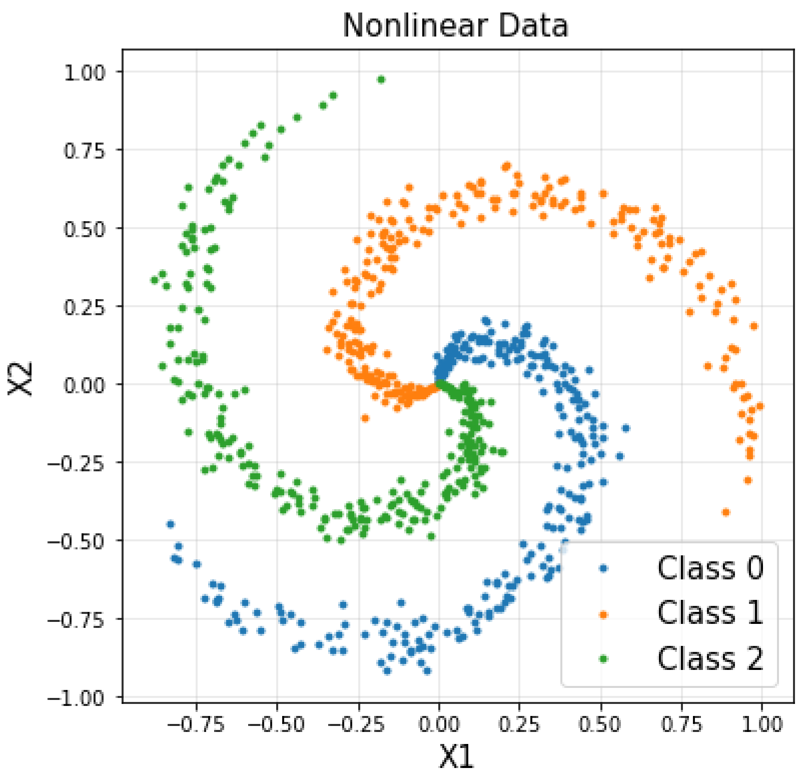
To see how it works, click [here (http://i-systems.github.io/HSE545/machine%20learning%20all/04%20Classification/iSystems_02_SVM.html#4.-Nonlinear-Support-Vector-Machine)](http://i-systems.github.io/HSE545/machine%20learning%20all/04%20Classification/iSystems_02_SVM.html#4.-Nonlinear-Support-Vector-Machine)

- 이 부분 코드는 이해할 필요가 없으며, 개념적인 것만 이해하시면 됩니다
- Nonlinear Example

In [58]:
```
%%html
<center><iframe src="https://www.youtube.com/embed/3liCbRZPrZA"
width="420" height="315" frameborder="0" allowfullscreen></iframe></center>
```



In [59]:
```
N = 250 # number of points per class
D = 2 # dimensionality
K = 3 # number of classes
X = np.zeros([N*K, D]) # data matrix (each row = single example)
y = np.zeros(N*K) # class labels
for j in range(K):
    ix = range(N*j,N*(j+1))
    r = np.linspace(0.0, 1, N) # radius
    t = np.linspace(j*4, (j+1)*4, N) + np.random.randn(N)*0.2 # theta
    X[ix] = np.c_[r*np.sin(t), r*np.cos(t)]
    y[ix] = j

plt.figure(figsize=(6, 6))
plt.title('Nonlinear Data', fontsize=15)
plt.plot(X[y==0,0], X[y==0,1], '.', label='Class 0')
plt.plot(X[y==1,0], X[y==1,1], '.', label='Class 1')
plt.plot(X[y==2,0], X[y==2,1], '.', label='Class 2')
plt.xlim(min(X[:,0]) - 0.1, max(X[:,0]) + 0.1)
plt.ylim(min(X[:,1]) - 0.1, max(X[:,1]) + 0.1)
plt.legend(loc='lower right', fontsize=15)
plt.xlabel('X1', fontsize=15)
plt.ylabel('X2', fontsize=15)
plt.grid(alpha=0.3)
plt.show()
```

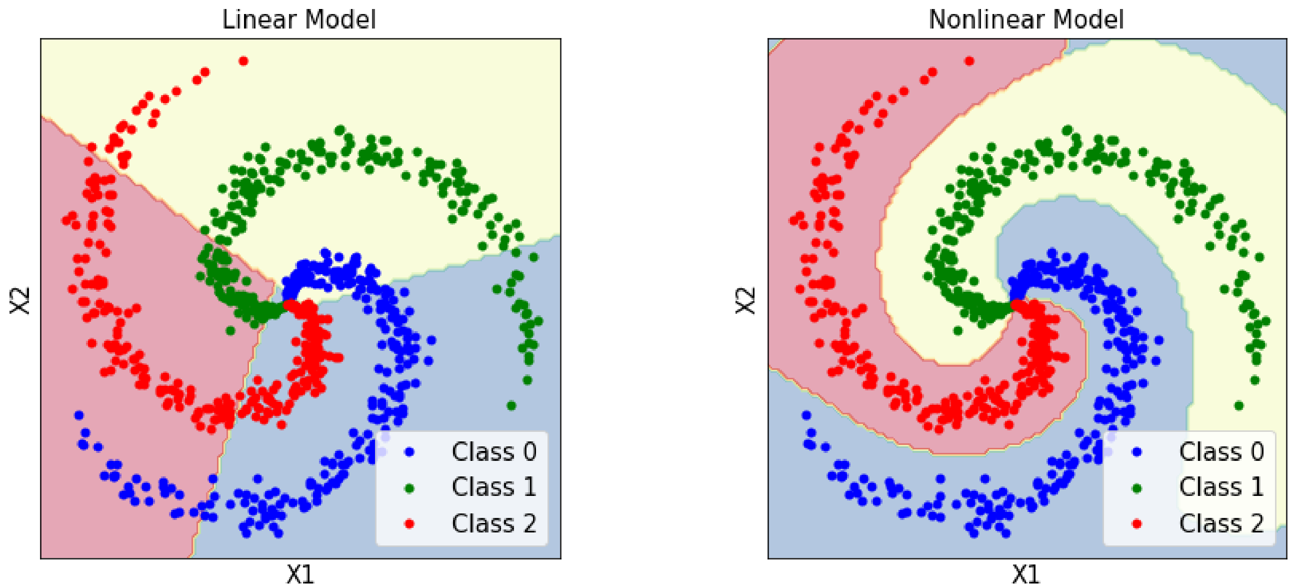


In [60]:
```
from sklearn.svm import SVC
```

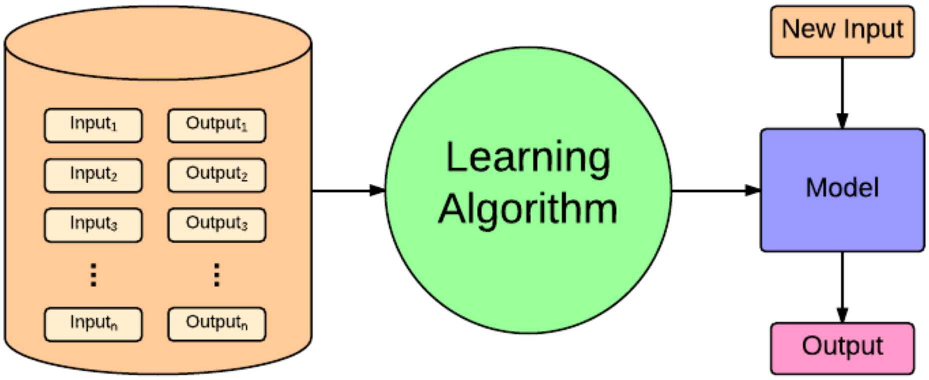
In [61]:
```
svc = SVC(kernel='linear', C=1).fit(X, y)
rbf_svc = SVC(kernel='rbf', C=1, gamma=5).fit(X, y)
```

In [62]:
```
# create a mesh to plot in
h = .02  # step size in the mesh
x_min, x_max = X[:,0].min() - 0.1, X[:,0].max() + 0.1
y_min, y_max = X[:,1].min() - 0.1, X[:,1].max() + 0.1
xx, yy = np.meshgrid(np.arange(x_min, x_max, h),
                     np.arange(y_min, y_max, h))
```

In [63]:
```
# title for the plots
titles = ['Linear Model', 'Nonlinear Model']

fig = plt.figure(figsize=(14, 6))
for i, clf in enumerate((svc, rbf_svc)):
    plt.subplot(1, 2, i+1)
    plt.subplots_adjust(wspace=0.4, hspace=0.4)

    Z = clf.predict(np.c_[xx.ravel(), yy.ravel()])

    # Put the result into a color plot
    Z = Z.reshape(xx.shape)
    plt.contourf(xx, yy, Z, cmap=plt.cm.Spectral_r, alpha=0.4)

    # Plot also the training points
    plt.plot(X[y==0,0], X[y==0,1], 'b.', label='Class 0', mew=3)
    plt.plot(X[y==1,0], X[y==1,1], 'g.', label='Class 1', mew=3)
    plt.plot(X[y==2,0], X[y==2,1], 'r.', label='Class 2', mew=3)
    plt.legend(loc='lower right', fontsize=15)
    plt.xlabel('X1', fontsize=15)
    plt.ylabel('X2', fontsize=15)
    plt.xlim(xx.min(), xx.max())
    plt.ylim(yy.min(), yy.max())
    plt.xticks([])
    plt.yticks([])
    plt.title(titles[i], fontsize=15)

plt.show()
```



6. Save Model



- cPickle을 이용하여 학습된 모델 저장
  - 5.3. Nonlinear SVM 예제의 모델

In [64]:
```
from six.moves import import cPickle
```

In [65]:
```
cPickle.dump(svc, open('./data_files/svc_model.pkl', 'wb'))
```

- 학습된 모델 불러오기

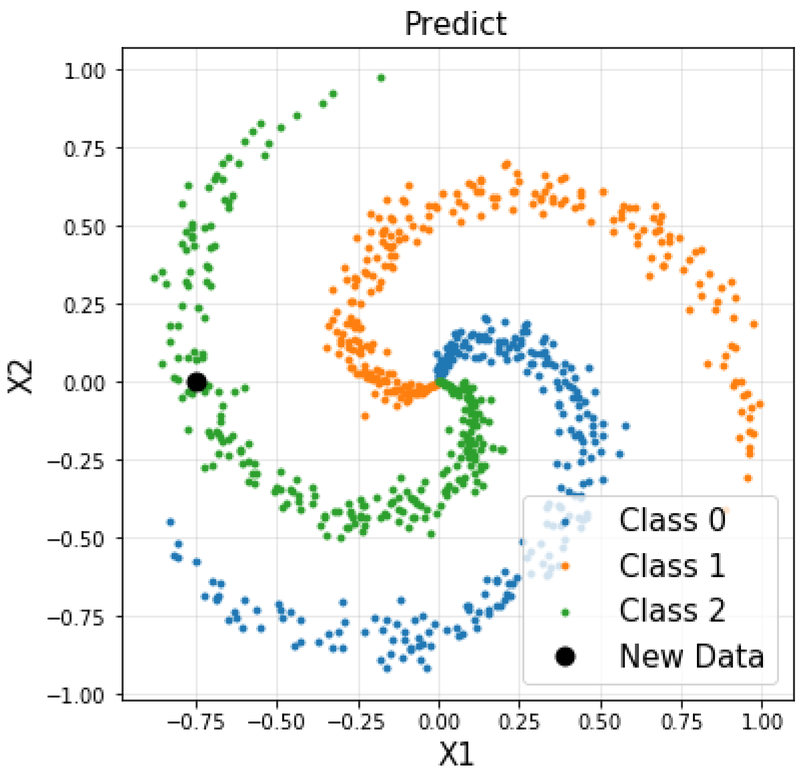
In [66]:
```
svc_restore = cPickle.load(open('./data_files/svc_model.pkl', 'rb'))
```

- 새로운 데이터로 테스트해보기

In [67]:
```
X_new = np.array([-0.75, 0])
X_new = X_new.reshape(1, -1)
X_new.shape
```

Out[67]: (1, 2)

In [68]:
```
plt.figure(figsize=(6, 6))
plt.title('Predict', fontsize=15)
plt.plot(X[y==0,0], X[y==0,1], '.', label='Class 0')
plt.plot(X[y==1,0], X[y==1,1], '.', label='Class 1')
plt.plot(X[y==2,0], X[y==2,1], '.', label='Class 2')
plt.plot(X_new[0,0], X_new[0,1], 'ko', label='New Data', ms=5, mew=5)
plt.xlim(min(X[:,0]) - 0.1, max(X[:,0]) + 0.1)
plt.ylim(min(X[:,1]) - 0.1, max(X[:,1]) + 0.1)
plt.legend(loc='lower right', fontsize=15)
plt.xlabel('X1', fontsize=15)
plt.ylabel('X2', fontsize=15)
plt.grid(alpha=0.3)
plt.show()
```



- 저장된 모델을 이용한 새 데이터 예측

In [69]:
```
svc_restore.predict(X_new)
```

Out[69]:
```
array([ 2.])
```

In [70]:
```
%%javascript
$.getScript('https://kmahelona.github.io/ipython_notebook_goodies/ipython_notebook_to
c.js')
```