기계인공지능 HW※05 Sol

1.

X, Y=aX+b (for nonzero a; if a is zero, Y become constant)

covariance coefficient γ_{XY} is

$$\gamma_{XY} = \frac{Cov(X,Y)}{\sqrt{Var(X)}\sqrt{Var(Y)}} = \frac{E[(X-\bar{X})(Y-\bar{Y})]}{\sqrt{E[(X-\bar{X})^2}\sqrt{E[(Y-\bar{Y})^2}}$$

$$= \frac{E[(x-\bar{x})(ax-b-a\bar{x}+b)]}{\sqrt{E[(x-\bar{x})^2]}\sqrt{E[(ax-b-a\bar{x}+b)^2]}}$$

$$= \frac{E[(x-\bar{x})(ax-a\bar{x})]}{\sqrt{E[(x-\bar{x})^2]}\sqrt{E[ax-a\bar{x})^2]}}$$

$$= \frac{aE[(x-\bar{x})^2]}{|a|\sqrt{E[(x-\bar{x})^2]}\sqrt{E[(x-\bar{x})^2]}} = \frac{a}{|a|} = \begin{cases} 1 & \text{if } a>0 \\ -1 & \text{if } a<0 \end{cases}$$

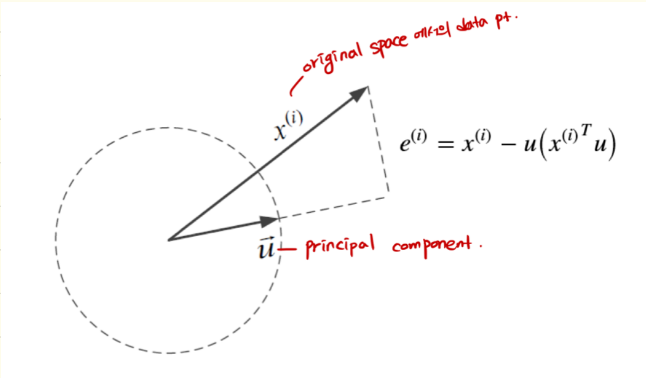
It means X and Y are highly correlated
and it is linear line.

4 (3)

For same data set, the result of linear regression and PCA are different

we do linear regression to find expression of tendancy of output y for each input x
Hence, we define error as difference between data point and projected (parallel to y axis) data point.
And minimize it.

On the other hands, PCA is a dimension reduction method.
So, we want to reduce dimension with minimum information loss.
Thus, data have to projected on the principal direction. (max variance)
So, define the error as follows.



we can see the definition of error (want to minimize) for two methods are different.
Since objects to minimize are different, results also different, ovbious.

5

(2)

Sample covariance matrix S can be decomposed as follows

$$S = \begin{bmatrix} | & | \\ x_1 & x_2 \\ | & | \end{bmatrix} \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} \begin{bmatrix} | & | \\ x_1 & x_2 \\ | & | \end{bmatrix}^T$$

where x_1, x_2 are eigenvectors & λ_1, λ_2 are eigenvalues of S

For arbitrary unit vector $u = a\vec{x}_1 + b\vec{x}_2$

multiply S

$$\Rightarrow S \cdot u = S(a\vec{x}_1 + b\vec{x}_2) = aS\vec{x}_1 + bS\vec{x}_2$$

$$= a\lambda_1\vec{x}_1 + b\lambda_2\vec{x}_2 \quad \text{(eigen analysis)}$$

Since the absolute value of λ_1 is greater than λ_2,

the vector u will close to x_1 vector

when we repeat process of multiplying S and normalizing u.