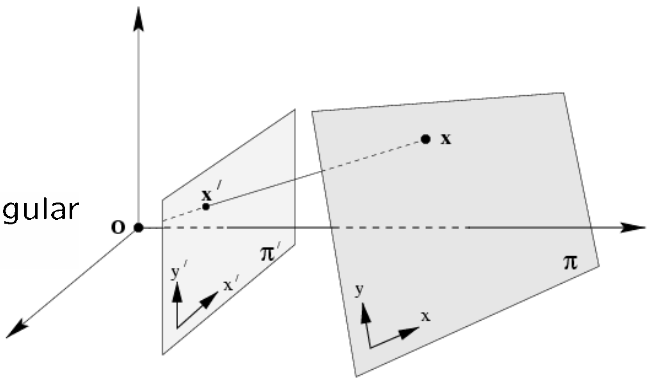
Reminder: Plane Projective transformation

$$\begin{pmatrix} x'_1 \\ x'_2 \\ x'_3 \end{pmatrix} = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

or $\mathbf{x}' = \mathtt{H}\mathbf{x}$, where $\mathtt{H}$ is a 3×3 non-singular homogeneous matrix.

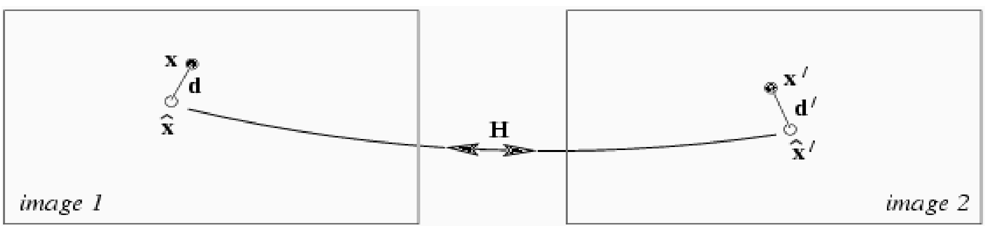


- A projective transformation is also called a ``homography'' and a ``collineation''.
- H has 8 degrees of freedom.
- It can be estimated from 4 or more point correpondences

Maximum Likelihood estimation of homography H

If the measurement error is Gaussian, then the ML estimate of H and the corrected correspondences $\{\hat{\mathbf{x}}_i \leftrightarrow \hat{\mathbf{x}}'_i\}$ is given by minimizing

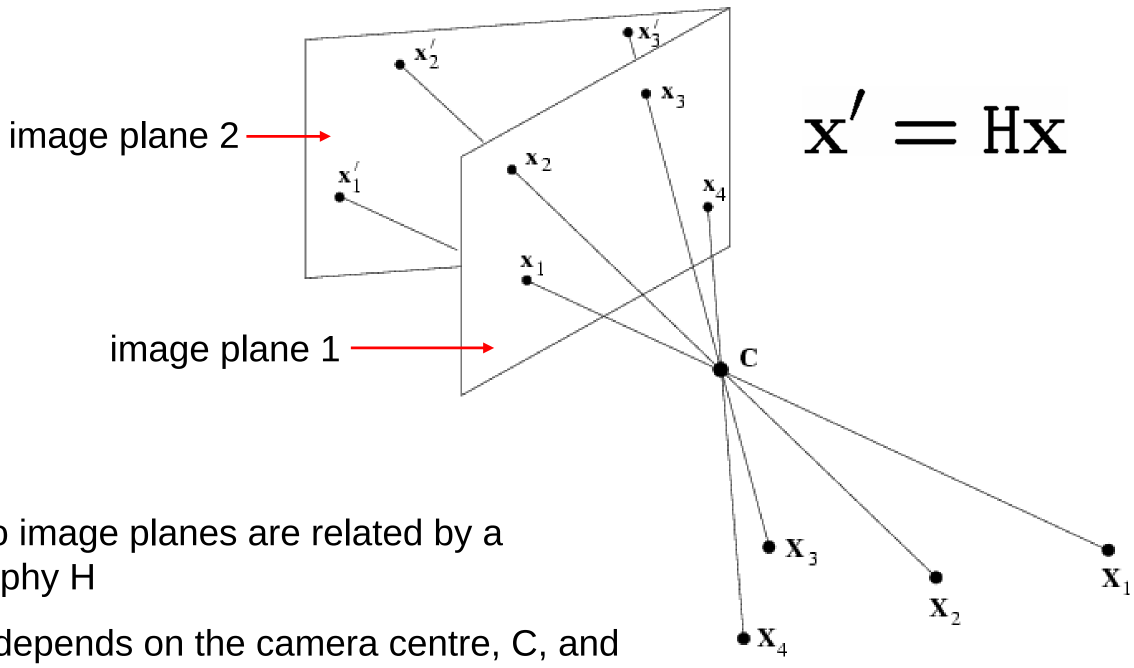
$$\mathcal{C} = \sum_i d^2(\hat{\mathbf{x}}_i, \mathbf{x}_i) + d^2(\hat{\mathbf{x}}'_i, \mathbf{x}'_i) \quad \text{subject to} \quad \hat{\mathbf{x}}_i = \hat{\mathtt{H}}\hat{\mathbf{x}}'_i, \forall i$$



Cost function minimization:

- for 2D affine transformations there is a matrix solution (non-examinable)
- for 2D projective transformation, minimize numerically using e.g. non-linear gradient descent

e.g. Camera rotating about its centre



$$\mathbf{x}' = \mathrm{H}\mathbf{x}$$

- The two image planes are related by a homography H
- H only depends on the camera centre, C, and the planes, not on the 3D structure

Example : Building panoramic mosaics

4 frames from a sequence of 30

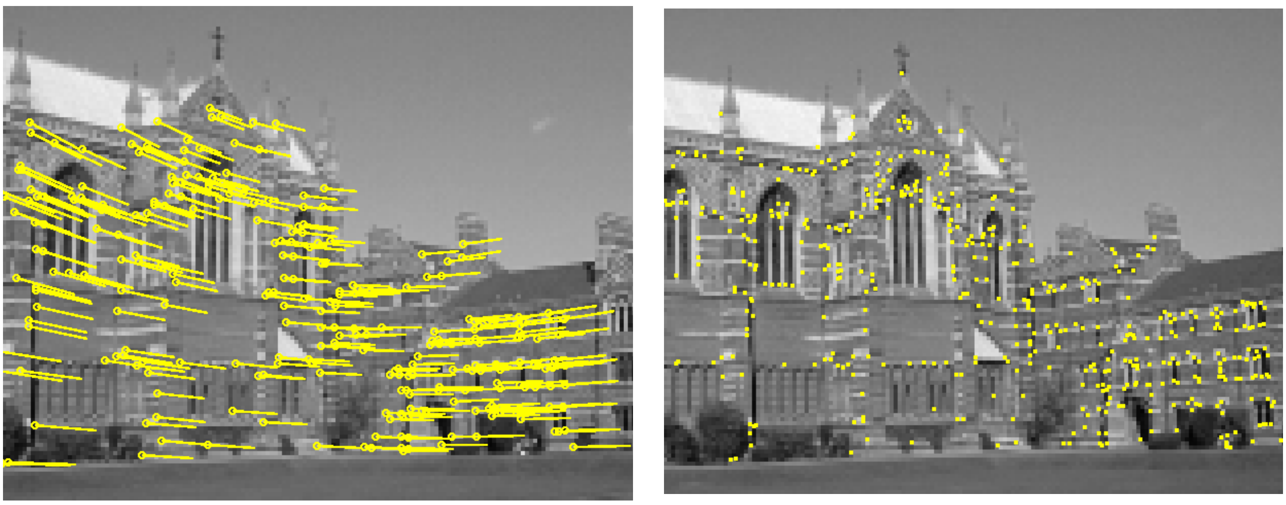
The camera rotates (and zooms) with fixed centre

Example : Building panoramic mosaics

30 frames

The camera rotates (and zooms) with fixed centre

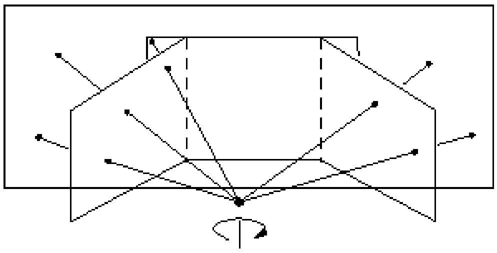
Homography between two frames



ML estimate of homography from these 100s of point correspondences

Choice of mosaic frame

Choose central image as reference



This produces the classic "bow-tie" mosaic.

General Linear Least Squares

$$\min_{a,b} \sum_i (y_i - a x_i - b)^2$$

Write the residuals as an n-vector

$$\begin{pmatrix} w_1 \\ w_2 \\ . \\ . \\ w_n \end{pmatrix} = \begin{pmatrix} y_1 \\ y_2 \\ . \\ . \\ y_n \end{pmatrix} - \begin{bmatrix} x_1 & 1 \\ x_2 & 1 \\ . & \\ . & \\ x_n & 1 \end{bmatrix} \begin{pmatrix} a \\ b \end{pmatrix}$$

$$\mathbf{w} = \mathbf{y} - H\theta$$

H is a n x 2 matrix

Then the sum of squared residuals becomes

$$\mathbf{w}^\top \mathbf{w} = (\mathbf{y} - H\theta)^\top (\mathbf{y} - H\theta)$$

We want to minimize this w.r.t. θ

$$\frac{d}{d\boldsymbol{\theta}} (\mathbf{y} - \mathsf{H}\boldsymbol{\theta})^\top (\mathbf{y} - \mathsf{H}\boldsymbol{\theta}) = 2\mathsf{H}^\top (\mathbf{y} - \mathsf{H}\boldsymbol{\theta}) = \mathbf{0}$$

$$\mathsf{H}^\top \mathsf{H}\boldsymbol{\theta} = \mathsf{H}^\top \mathbf{y}$$

$$\boldsymbol{\theta} = (\mathsf{H}^\top \mathsf{H})^{-1} \mathsf{H}^\top \mathbf{y} = \mathsf{H}^+ \mathbf{y}$$

=

Summary

- If the generative model is linear $\mathbf{y} = \mathsf{H}\boldsymbol{\theta}$
- then the ML solution for Gaussian noise is $\boldsymbol{\theta} = \mathsf{H}^+ \mathbf{y}$
- where the matrix $\mathsf{H}^+ = (\mathsf{H}^\top \mathsf{H})^{-1} \mathsf{H}^\top$ is the pseudo-inverse of H