(Artificial) Neural Networks

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Table of Contents

1. Artificial Neural Networks (ANN)

1.1. Recall supervised learning setup

- Input features $x^{(i)} \in \mathbb{R}^n$
- Ouput $y^{(i)}$
- Model parameters $\theta \in \mathbb{R}^k$
- Hypothesis function $h_\theta : \mathbb{R}^n \to y$
- Loss function $\ell : y \times y \to \mathbb{R}_+$

- Machine learning optimization problem

$$\min_{\theta} \sum_{i=1}^{m} \ell\left(h_\theta\left(x^{(i)}\right), y^{(i)}\right)$$

(possibly plus some additional regularization)

- But, many specialized domains required highly engineered special features

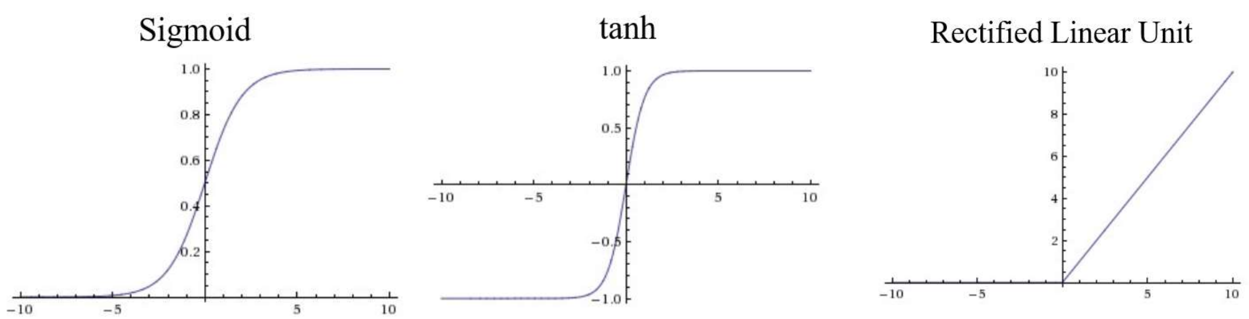
1.2. Neural networks

Neural networks are a simply a machine learning algorithm with a more complex hypothesis class, directly incorporating non-linearity (in the parameters)

Example: neural network with one hidden layer

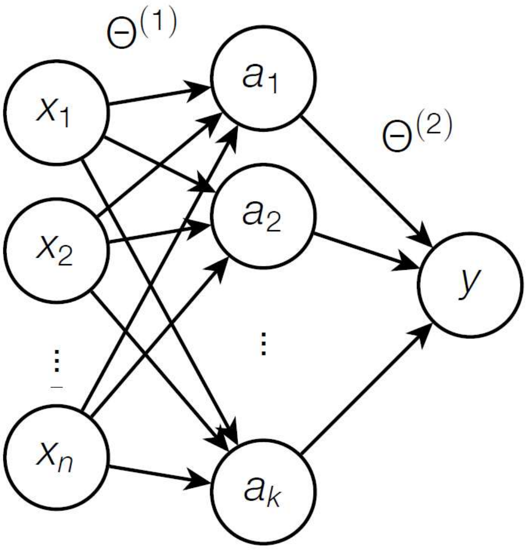
$$h_\theta(x) = \Theta^{(2)} f\left(\Theta^{(1)} x\right)$$

where $\Theta^{(1)} \in \mathbb{R}^{k \times n}$, $\Theta^{(2)} \in \mathbb{R}^{1 \times k}$ and f is some non-linear function applied elementwise to a vector



$$g(x) = \frac{1}{1 + e^{-x}} \qquad g(x) = \tanh(x) \qquad g(x) = \max(0, x)$$

Architectures are often shown graphically

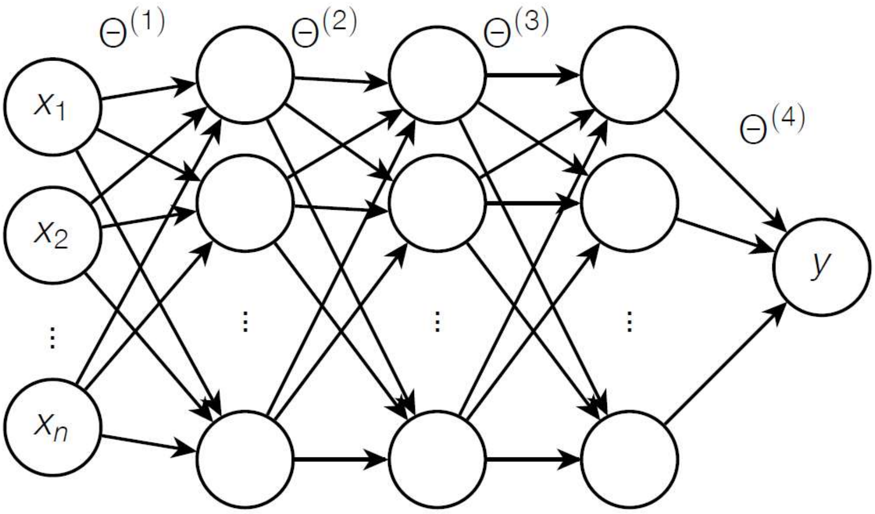


- Middle layer a is referred to as the hidden layer, there is nothing in the data that prescribes what values these should take, left up to the algorithm to decide
- **Viewed another way: neural networks are like classifiers where the features themselves are also learned**

- **Pros**
  - No need to manually engineer good features, just let the neural networks handle this part
- **Cons**
  - Minimizing loss on training data is no longer a convex optimization problem in parameters θ
  - Still need to engineer a good architecture

1.3. Deep learning

"Deep" neural networks typically refer to networks with multiple hidden layers



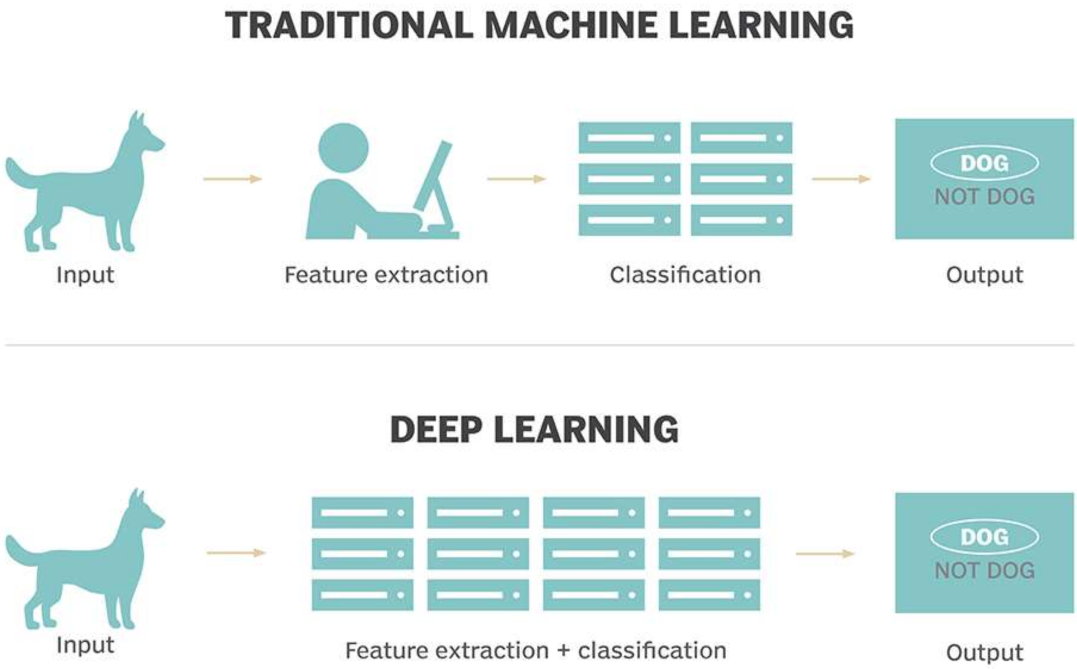
1.4. Machine learning and Neural networks (or Deep learning)

Machine Learning

- Hand-crafted features
- Depends on expertise

Deep Learning

- Automatic feature extraction
- Depends on network structure



2. Structure of Neural Networks

The neuron

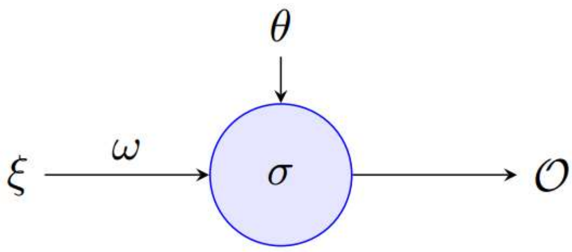
- The sigmoid equation is what is typically used as a transfer function between neurons. It is similar to the step fuction, but is continuous and differentiable.

$$\sigma(x) = \frac{1}{1+e^{-x}}$$

- One useful property of this transfer function is the simplicity of computing its derivative.

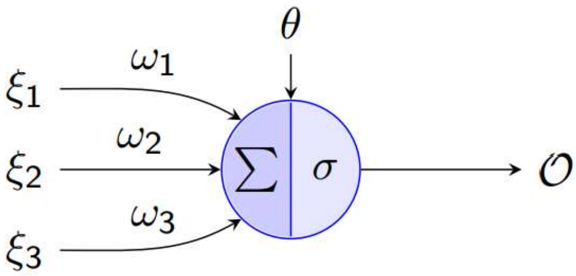
$$\frac{d}{dx}\sigma(x) = \sigma' = \sigma(x)(1-\sigma(x))$$

Single input neuron



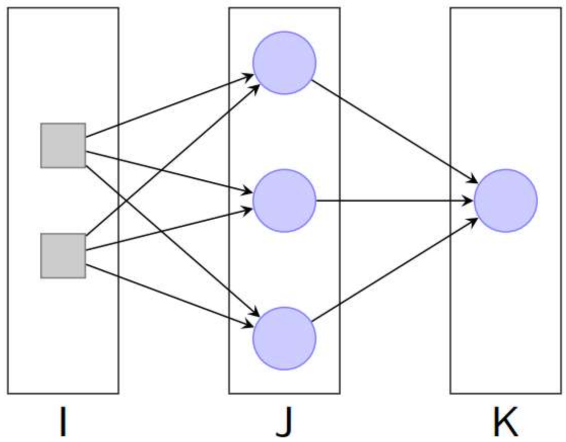
$$O = \sigma(\xi\omega + \theta)$$

Multiple input neuron



$$O = \sigma(\xi_1\omega_1 + \xi_2\omega_2 + \xi_3\omega_3 + \theta)$$

A neural network



3. Learning: Backpropagation Algorithm (Optional)

Notation

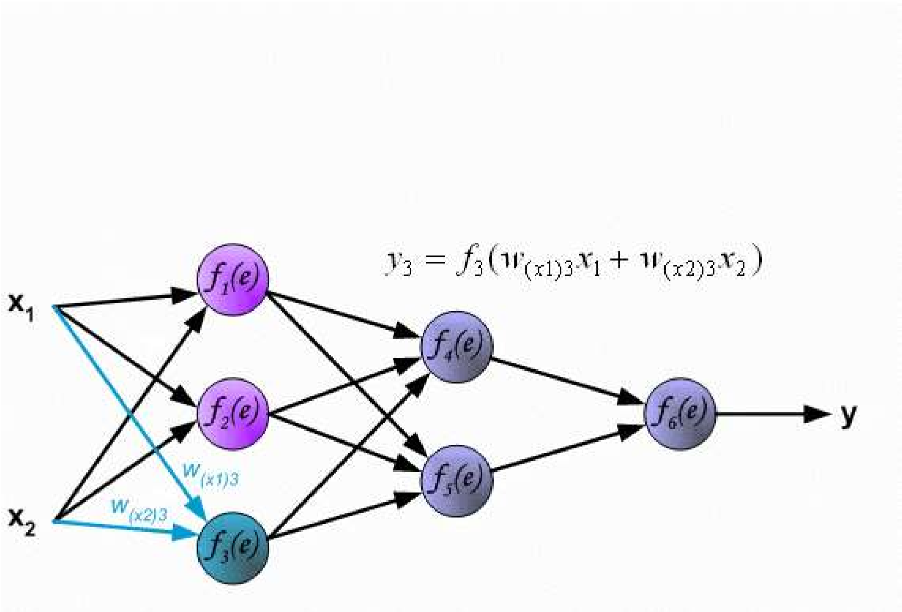
- x_j^ℓ: Input to node j of layer ℓ
- W_{ij}^ℓ: Weight from layer $\ell - 1$ node i to layer ℓ node j
- $\sigma(x) = \frac{1}{1+e^{-x}}$: Sigmoid transfer function
- θ_j^ℓ: Bias of node j of layer ℓ
- O_j^ℓ: Output of node j in layer ℓ
- t_j: Target value of node j of the output layer

The error calculation

Given a set of training data points t_k and output layer output O_k we can write the error as

$$E = \frac{1}{2} \sum_{k \in K} (O_k - t_k)^2$$

- Forward propagation
  - the initial information propagates up to the hidden units at each layer and finally produces output
- Backpropagation
  - allows the information from the cost to flow backwards through the network in order to compute the gradients



We want to calculate $\frac{\partial E}{\partial W_{jk}^\ell}$, the rate of change of the error with respect to the given connective weight, so we can minimize it.

Now we consider two cases: the node is an output node, or it is in a hidden layer

1) Output layer node

$$
\begin{aligned}
\frac{\partial E}{\partial W_{jk}} &= \frac{\partial}{\partial W_{jk}}\frac{1}{2}(O_k - t_k)^2 = (O_k - t_k)\frac{\partial}{\partial W_{jk}}O_k = (O_k - t_k)\frac{\partial}{\partial W_{jk}}\sigma(x_k) \\
&= (O_k - t_k)\sigma(x_k)(1-\sigma(x_k))\frac{\partial}{\partial W_{jk}}x_k \\
&= (O_k - t_k)O_k(1-O_k)O_j
\end{aligned}
$$

For notation purposes, I will define δ_k to be the expression $(O_k - t_k)O_k(1-O_k)$, so we can rewrite the equation above as

$$\frac{\partial E}{\partial W_{jk}} = O_j\delta_k$$

2) Hidden layer node

$$
\begin{aligned}
\frac{\partial E}{\partial W_{ij}} &= \frac{\partial}{\partial W_{ij}}\frac{1}{2}\sum_{k\in K}(O_k - t_k)^2 = \sum_{k\in K}(O_k - t_k)\frac{\partial}{\partial W_{ij}}O_k = \sum_{k\in K}(O_k - t_k)\frac{\partial}{\partial W_{ij}}\sigma(x_k) \\
&= \sum_{k\in K}(O_k - t_k)\sigma(x_k)(1-\sigma(x_k))\frac{\partial}{\partial W_{ij}}x_k \\
&= \sum_{k\in K}(O_k - t_k)O_k(1-O_k)\frac{\partial x_k}{\partial O_j}\cdot\frac{\partial O_j}{\partial W_{ij}} = \sum_{k\in K}(O_k - t_k)O_k(1-O_k)W_{jk}\cdot\frac{\partial O_j}{\partial W_{ij}} \\
&= \frac{\partial O_j}{\partial W_{ij}}\cdot\sum_{k\in K}(O_k - t_k)O_k(1-O_k)W_{jk} \\
&= O_j(1-O_j)\frac{\partial x_j}{\partial W_{ij}}\cdot\sum_{k\in K}(O_k - t_k)O_k(1-O_k)W_{jk} \\
&= O_j(1-O_j)O_i\cdot\sum_{k\in K}(O_k - t_k)O_k(1-O_k)W_{jk} \\
&= O_iO_j(1-O_j)\sum_{k\in K}\delta_kW_{jk}
\end{aligned}
$$

Similar to before we will now define all terms besides O_i to be $\delta_j = O_j(1-O_j)\sum_{k\in K}\delta_kW_{jk}$, so we have

$$\frac{\partial E}{\partial W_{ij}} = O_i\delta_j$$

How weights affect errors

- For an output layer node $k \in K$

$$\frac{\partial E}{\partial W_{jk}} = O_j\delta_k$$

  where

$$\delta_k = (O_k - t_k)O_k(1-O_k)$$

- For a hidden layer node $j \in J$

$$\frac{\partial E}{\partial W_{ij}} = O_i\delta_j$$

  where

$$\delta_j = O_j(1-O_j)\sum_{k\in K}\delta_kW_{jk}$$

What about the bias?

If we incorporate the bias term θ into the equation you will find that

$$\frac{\partial O}{\partial \theta} = 1$$

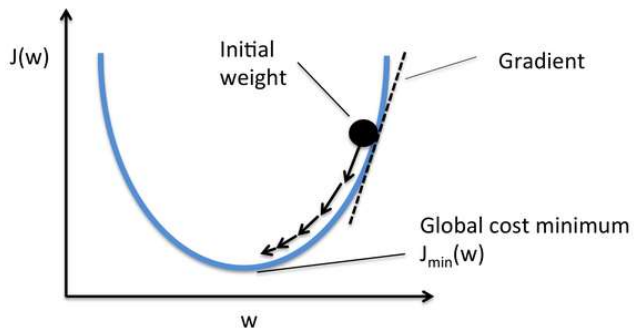
This is why we view the bias term as output from a node which is always one. This holds for any layer ℓ, a substitution into the previous equations gives us that

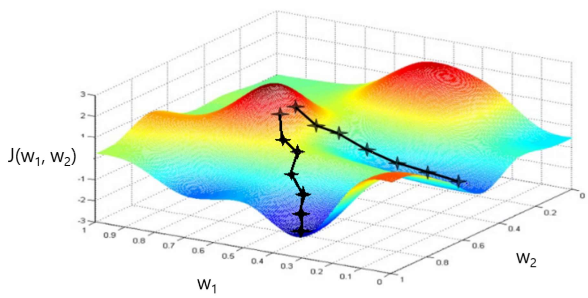
$$\frac{\partial E}{\partial \theta} = \delta_\ell$$

(Stochastic) Gradient Descent

- Negative gradients points directly downhill of cost function
- We can decrease cost by moving in the direction of the negative gradient (η is a learning rate)

$$W := W - \eta \nabla_W \left(h_W \left(x^{(i)} \right), y^{(i)} \right)$$





The backpropagation algorithm using gradient descent

1. Run the network forward with your input data to get the netwrok output
2. For each output node compute

$$\delta_k = (O_k - t_k) O_k (1 - O_k)$$

3. For eatch hidden node calculate

$$\delta_j = O_j (1 - O_j) \sum_{k \in K} \delta_k W_{jk}$$

4. Update the weights and biases as follows

   Given

$$\Delta W = -\eta \delta_\ell O_{\ell - 1}$$
$$\Delta \theta = -\eta \delta_\ell$$

   apply

$$W \leftarrow W + \Delta W$$
$$\theta \leftarrow \theta + \Delta \theta$$

4. Deep Learning Libraries

Caffe

- Platform: Linux, Mac OS, Windows
- Written in: C++
- Interface: Python, MATLAB

Theano

- Platform: Cross-platform
- Written in: Python
- Interface: Python

Tensorflow

- Platform: Linux, Mac OS, Windows
- Written in: C++, Python
- Interface: Python, C/C++, Java, Go, R

5. TensorFlow

- `tensorflow` is an open-source software library for deep learning.

5.1. Computational Graph

- `tf.constant`
- `tf.Variable`
- `tf.placeholder`

In [1]:

```
import tensorflow as tf

a = tf.constant([1, 2, 3])
b = tf.constant([4, 5, 6])

A = a + b
B = a * b
```

In [2]:

```
A
```

Out[2]:

```
<tf.Tensor 'add:0' shape=(3,) dtype=int32>
```

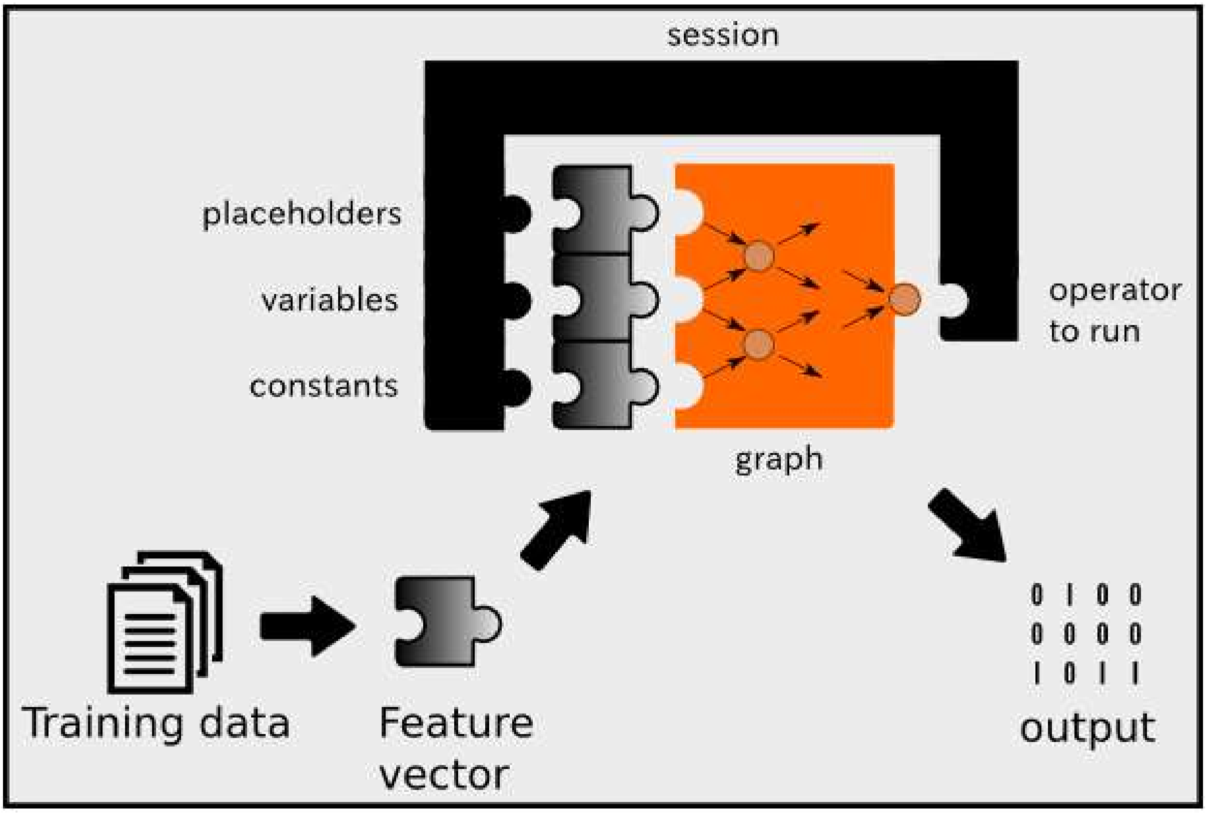
In [3]:

```
B
```

Out[3]:

```
<tf.Tensor 'mul:0' shape=(3,) dtype=int32>
```

To run any of the three defined operations, we need to create a session for that graph. The session will also allocate memory to store the current value of the variable.



In [4]:

```
sess = tf.Session()
sess.run(A)
```

Out[4]:

```
array([5, 7, 9], dtype=int32)
```

In [5]:

```
sess.run(B)
```

Out[5]:

```
array([ 4, 10, 18], dtype=int32)
```

`tf.Variable` is regarded as the decision variable in optimization. We should initialize variables to use `tf.Variable`.

In [6]:

```
w = tf.Variable([1, 1])
```

In [7]:

```
init = tf.global_variables_initializer()
sess.run(init)
```

In [8]:

```
sess.run(w)
```

Out[8]:

```
array([1, 1], dtype=int32)
```

The value of `tf.placeholder` must be fed using the `feed_dict` optional argument to `Session.run()`.

In [9]:

```
x = tf.placeholder(tf.float32, [2, 2])
```

In [10]:

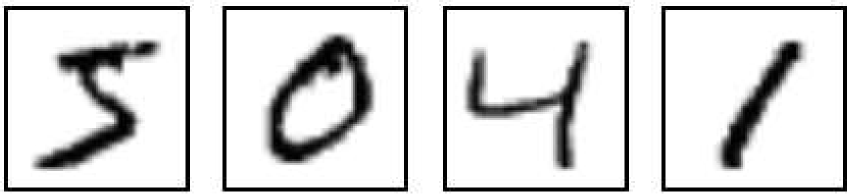
```
sess.run(x, feed_dict={x : [[1,2],[3,4]]})
```

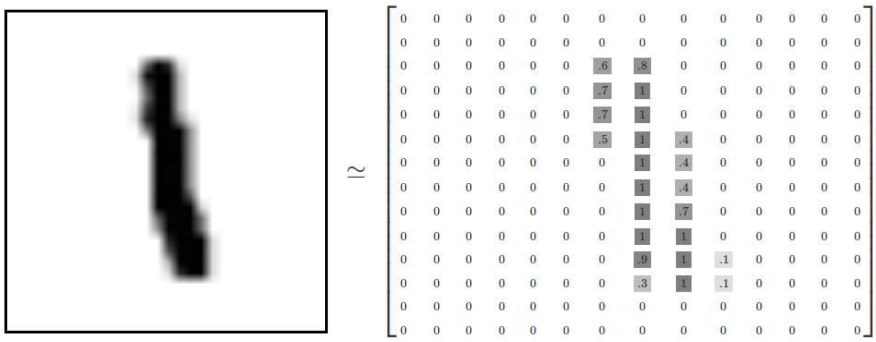
Out[10]:

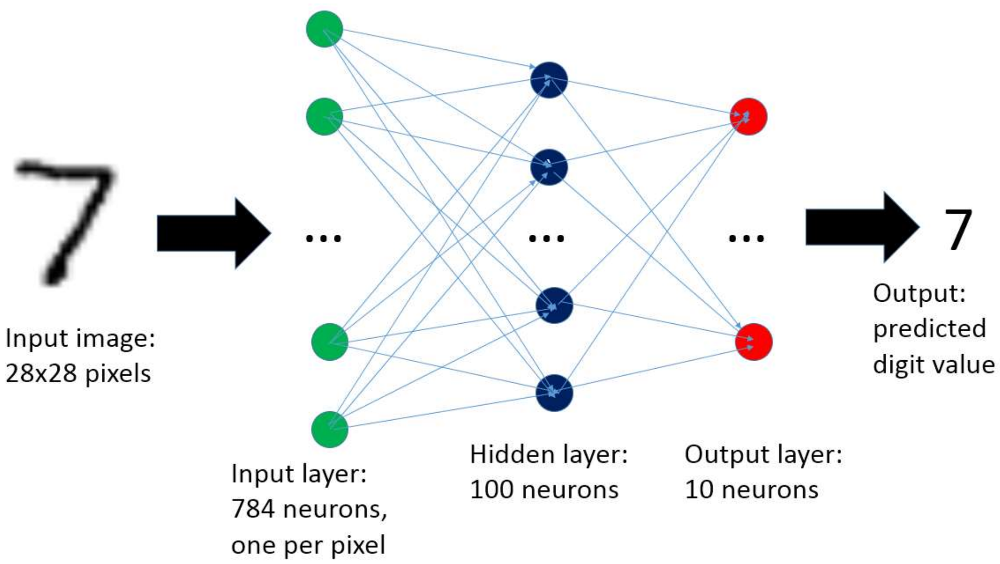
```
array([[ 1.,  2.],
       [ 3.,  4.]], dtype=float32)
```

6. ANN with TensorFlow

- MNIST (Mixed National Institute of Standards and Technology database) database
  - Handwritten digit database
  - 28×28 gray scaled image
  - Flattened array into a vector of $28 \times 28 = 784$

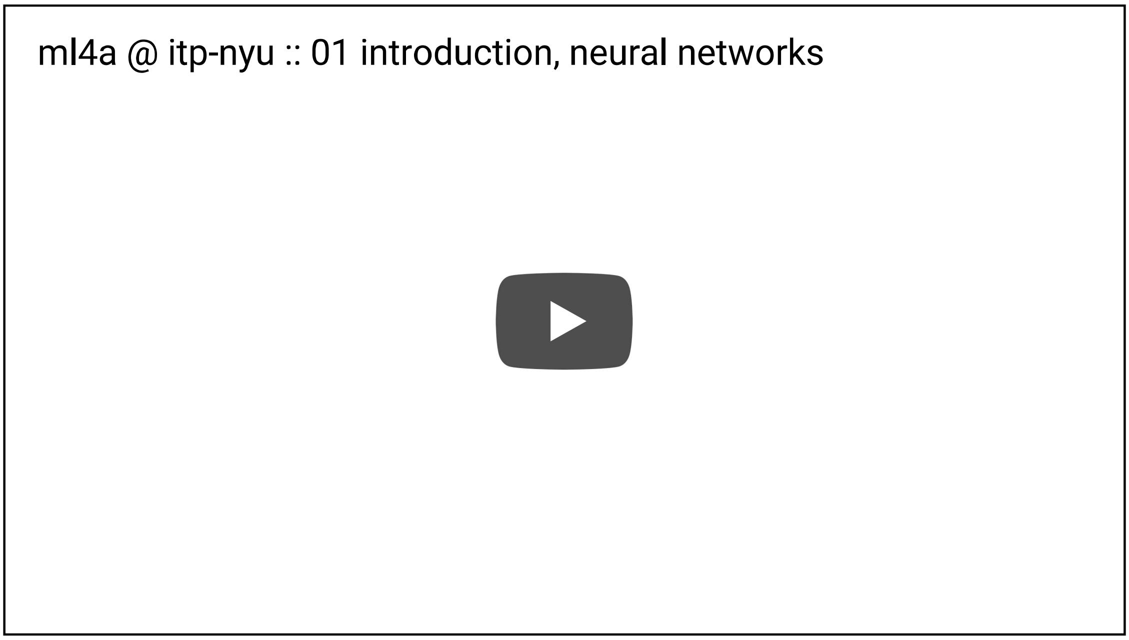






In [1]:

```
%%html
<center><iframe src="https://www.youtube.com/embed/z0bynQjEpll?start=2088&end=3137"
width="560" height="315" frameborder="0" allowfullscreen></iframe></center>
```



6.1. Import Library

In [22]:

```
# Import Library
import numpy as np
import matplotlib.pyplot as plt
import tensorflow as tf
```

6.2. Load MNIST Data

- Download MNIST data from tensorflow tutorial example

In [23]:

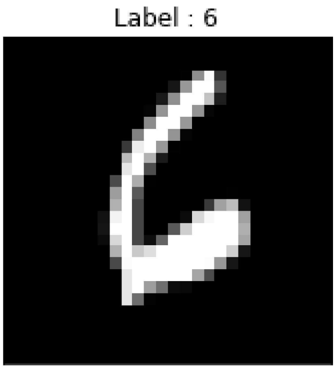
```
from tensorflow.examples.tutorials.mnist import input_data
mnist = input_data.read_data_sets("MNIST_data/", one_hot=True)
```

```
Extracting MNIST_data/train-images-idx3-ubyte.gz
Extracting MNIST_data/train-labels-idx1-ubyte.gz
Extracting MNIST_data/t10k-images-idx3-ubyte.gz
Extracting MNIST_data/t10k-labels-idx1-ubyte.gz
```

In [24]:

```
train_x, train_y = mnist.train.next_batch(10)
img = train_x[3,:].reshape(28,28)

plt.figure(figsize=(5,3))
plt.imshow(img,'gray')
plt.title("Label : {}".format(np.argmax(train_y[3])))
plt.xticks([])
plt.yticks([])
plt.show()
```



One hot encoding

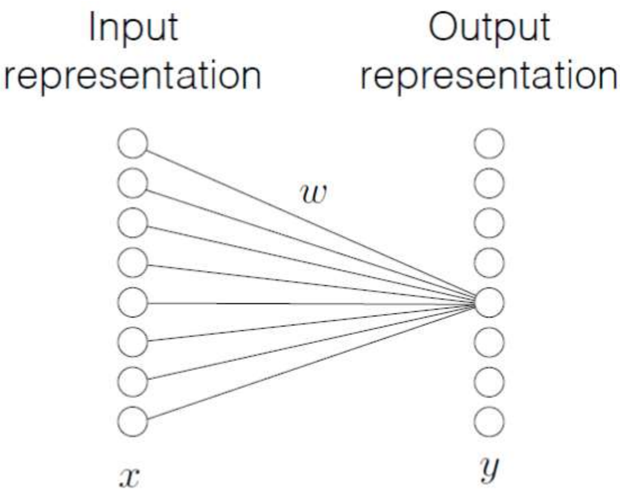
In [25]:

```
print ('Train labels : {}'.format(train_y[3, :]))
```

```
Train labels : [ 0.  0.  0.  0.  0.  0.  1.  0.  0.  0.]
```

6.3. Build a Model

First, the layer performs several matrix multiplication to produce a set of linear activations

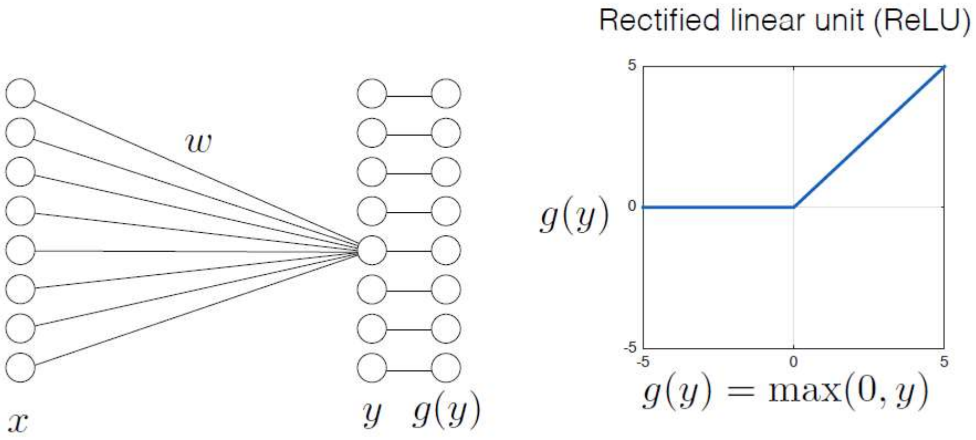


$$y_j = \left(\sum_i \omega_{ij} x_i\right) + b_j$$

$$y = \omega^T x + b$$

```
# hidden1 = tf.matmul(x, weights['hidden1']) + biases['hidden1']
hidden1 = tf.add(tf.matmul(x, weights['hidden1']), biases['hidden1'])
```

Second, each linear activation is running through a nonlinear activation function



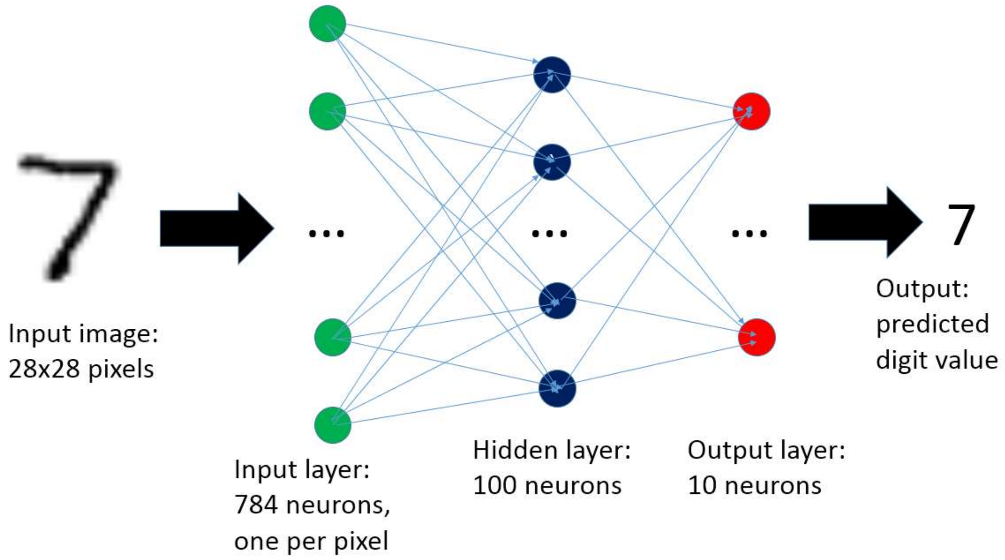

```
hidden1 = tf.nn.relu(hidden1)
```

Third, predict values with an affine transformation


```
# output = tf.matmul(hidden1, weights['output']) + biases['output']
output = tf.add(tf.matmul(hidden1, weights['output']), biases['output'])
```

6.4. Define the ANN's Shape

- Input size
- Hidden layer size
- The number of classes



In [26]:

```
n_input = 28*28
n_hidden1 = 100
n_output = 10
```

6.5. Define Weights, Biases and Network

- Define parameters based on predefined layer size
- Initialize with normal distribution with $\mu = 0$ and $\sigma = 0.1$

In [27]:

```
weights = {
    'hidden1' : tf.Variable(tf.random_normal([n_input, n_hidden1], stddev = 0.1)),
    'output' : tf.Variable(tf.random_normal([n_hidden1, n_output], stddev = 0.1)),
}

biases = {
    'hidden1' : tf.Variable(tf.random_normal([n_hidden1], stddev = 0.1)),
    'output' : tf.Variable(tf.random_normal([n_output], stddev = 0.1)),
}

x = tf.placeholder(tf.float32, [None, n_input])
y = tf.placeholder(tf.float32, [None, n_output])
```

In [28]:

```
# Define Network
def build_model(x, weights, biases):
    # first hidden layer
    hidden1 = tf.add(tf.matmul(x, weights['hidden1']), biases['hidden1'])
    # non linear activate function
    hidden1 = tf.nn.relu(hidden1)

    # Output layer with linear activation
    output = tf.add(tf.matmul(hidden1, weights['output']), biases['output'])
    return output
```

6.6. Define Cost, Initializer and Optimizer

Loss

- Classification: Cross entropy
  - Equivalent to apply logistic regression

$$-\frac{1}{N}\sum_{i=1}^{N} y^{(i)} \log(h_\theta\left(x^{(i)}\right)) + (1 - y^{(i)}) \log(1 - h_\theta\left(x^{(i)}\right))$$

Initializer

- Initialize all the empty variables

Optimizer

- AdamOptimizer: the most popular optimizer

In [29]:

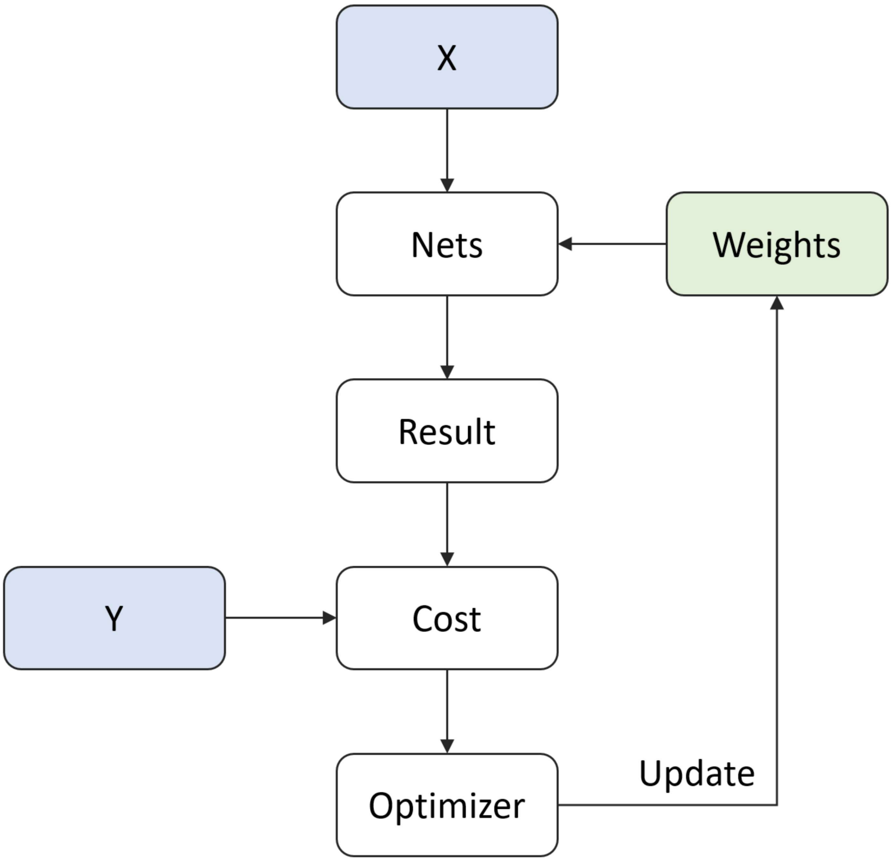
```
# Define Cost
LR = 0.0001

pred = build_model(x, weights, biases)
loss = tf.nn.softmax_cross_entropy_with_logits(logits=pred, labels=y)
loss = tf.reduce_mean(loss)

# optimizer = tf.train.GradientDescentOptimizer(learning_rate).minimize(cost)
optm = tf.train.AdamOptimizer(LR).minimize(loss)

init = tf.global_variables_initializer()
```

6.7. Summary of Model



6.8. Define Configuration

- Define parameters for training ANN
  - `n_batch`: batch size for stochastic gradient descent
  - `n_iter`: the number of learning steps
  - `n_prt`: check loss for every `n_prt` iteration

In [30]:

```
n_batch = 50      # Batch Size
n_iter = 2500     # Learning Iteration
n_prt = 250       # Print Cycle
```

6.9. Optimization

In [31]:

```
# Run initialize
# config = tf.ConfigProto(allow_soft_placement=True)  # GPU Allocating policy
# sess = tf.Session(config=config)
sess = tf.Session()
sess.run(init)

# Training cycle
for epoch in range(n_iter):
    train_x, train_y = mnist.train.next_batch(n_batch)
    sess.run(optm, feed_dict={x: train_x,  y: train_y})

    if epoch % n_prt == 0:
        c = sess.run(loss, feed_dict={x : train_x,  y : train_y})
        print ("Iter : {}".format(epoch))
        print ("Cost : {}".format(c))
```

```
Iter : 0
Cost : 2.4568586349487305
Iter : 250
Cost : 1.4568665027618408
Iter : 500
Cost : 0.7992963194847107
Iter : 750
Cost : 0.6279309988021851
Iter : 1000
Cost : 0.4135037958621979
Iter : 1250
Cost : 0.4587893784046173
Iter : 1500
Cost : 0.3380467891693115
Iter : 1750
Cost : 0.4487552344799042
Iter : 2000
Cost : 0.39212024211883545
Iter : 2250
Cost : 0.3634752631187439
```

6.10. Test

In [32]:

```
test_x, test_y = mnist.test.next_batch(100)

my_pred = sess.run(pred, feed_dict={x : test_x})
my_pred = np.argmax(my_pred, axis=1)

labels = np.argmax(test_y, axis=1)

accr = np.mean(np.equal(my_pred, labels))
print("Accuracy : {}%".format(accr*100))
```

```
Accuracy : 92.0%
```