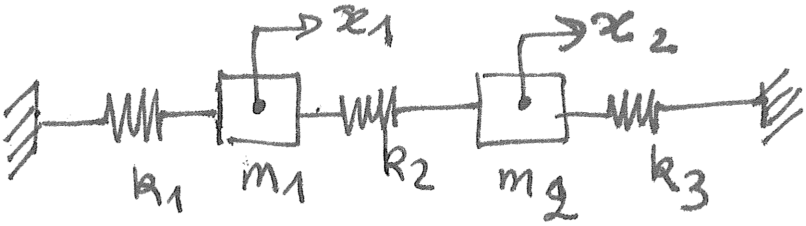
2 DOFs MK



$$\mathcal{L} = T - V \qquad \text{here} \quad \begin{array}{l} q = x \\ \dot{q} = \dot{x} \end{array} \quad \text{generalized coordinates}$$

Lagrange's equation is :

$$\frac{\partial}{\partial t}\left(\frac{\partial \mathcal{L}}{\partial \dot{q}}\right) - \frac{\partial \mathcal{L}}{\partial q} = 0$$

$$\frac{\partial}{\partial t}\left(\frac{\partial T}{\partial \dot{q}}\right) - \underbrace{\cancel{\frac{\partial}{\partial t}\left(\frac{\partial V}{\partial \dot{q}}\right)}}_{\substack{=0 \\ V \text{ independant} \\ \text{of } \dot{q}}} - \underbrace{\cancel{\frac{\partial T}{\partial q}}}_{\substack{=0 \\ T \text{ independant} \\ \text{of } q}} + \frac{\partial V}{\partial q} = 0$$

$$\frac{\partial}{\partial t}\frac{\partial T}{\partial \dot{x}} + \frac{\partial V}{\partial x} = 0 \qquad (1)$$

How to derive quadratic form ?

$$\frac{\partial}{\partial \underline{y}}\left(\underline{y}^T \underline{\underline{A}}\, \underline{x}\right) = \underline{\underline{A}}\, \underline{x}$$

it is an important Reminder

Special case

$$Q(\underline{x}) = \underline{x}^T \underline{\underline{A}}\, \underline{x}$$

$$\left(\frac{\partial Q(\underline{x})}{\partial \underline{x}}\right)^T = 2\, \underline{\underline{A}}\, \underline{x}$$

for Potential Energy (V) $\underline{\underline{A}} \rightarrow \underline{\underline{K}}$

$$\left(\frac{\partial V}{\partial \underline{x}}\right)^T = 2\, \underline{\underline{K}}\, \underline{x}$$

for Kinetic Energy (T) $\underline{x} \rightarrow \underline{\dot{x}}$ & $\underline{\underline{A}} \rightarrow \underline{\underline{M}}$

$$\left(\frac{\partial T}{\partial \underline{\dot{x}}}\right)^T = 2\, \underline{\underline{M}}\, \underline{\dot{x}}$$

thus $\frac{\partial}{\partial t}\left(\frac{\partial T}{\partial \underline{\dot{x}}}\right)^T = \frac{1}{2} \times 2 \times \underline{\underline{M}}\, \underline{\ddot{x}} = \underline{\underline{M}}\, \underline{\ddot{x}}$

$$\left(\frac{\partial V}{\partial \underline{x}}\right)^T = \frac{1}{2} \times 2 \times \underline{\underline{K}}\, \underline{x} = \underline{\underline{K}}\, \underline{x}$$

(1) becomes $\underline{\underline{M}}\, \underline{\ddot{x}} + \underline{\underline{K}}\, \underline{x} = \underline{0}$

Let's write $\underline{\underline{K}}$ & $\underline{\underline{M}}$ from Energies

$$2E_c = m_1\dot{x}_1^{2} + m_2\dot{x}_2^{2} = 2T$$

$$2E_p = k_1x_1^{2} + k_2(x_2 - x_1)^2 + k_3 x_2^{2} = 2V$$

$$= k_1x_1^{2} + k_2(x_2^2 + x_1^2 - 2x_2x_1) + k_3x_2^2$$

$$= x_1^2(k_1+k_2) + x_2^2(k_2+k_3) + -2x_2x_1k_2$$

$$\underset{1\times1}{2T} = \underset{1\times2}{\dot{x}^T}\underset{2\times2}{\begin{bmatrix} \cdot & \cdot \\ \cdot & \cdot \end{bmatrix}}\underset{2\times1}{\dot{x}}$$

quadratic form 1

$$\underset{1\times1}{2V} = \underset{1\times2}{x^T}\underset{2\times2}{\begin{bmatrix} \cdot & \cdot \\ \cdot & \cdot \end{bmatrix}}\underset{2\times1}{x}$$

quadratic form 2

$$\underline{\underline{M}}^{-1} = \begin{bmatrix} 1/m_1 & 0 \\ 0 & 1/m_2 \end{bmatrix}$$

$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \quad \dot{x} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix}$$

$\underline{\underline{M}}$ symetrique & diag (facilement inversible)

$$2T = \begin{bmatrix} \dot{x}_1 & \dot{x}_2 \end{bmatrix} \overbrace{\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}}^{\underline{\underline{M}}} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = m_1\dot{x}_1^{2} + m_2\dot{x}_2^{2}$$

$\underline{\underline{K}}$ symétrique

$$2V = \begin{bmatrix} x_1 & x_2 \end{bmatrix} \overbrace{\begin{bmatrix} k_1+k_2 & -k_2 \\ -k_2 & k_2+k_3 \end{bmatrix}}^{\underline{\underline{K}}} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{array}{c} x_1^2(k_1+k_2) \\ + \\ x_2^2(k_2+k_3) \\ - x_2x_1k_2 \\ - x_1x_2k_2 \end{array}$$