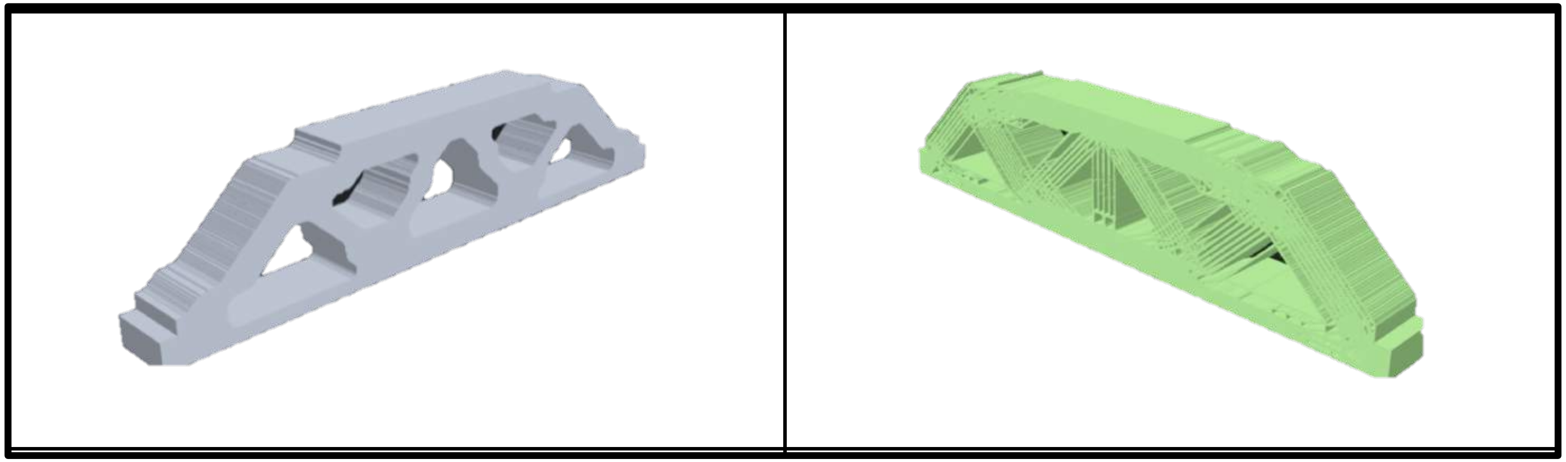
AE4ASM521

- Additive Manufacturing

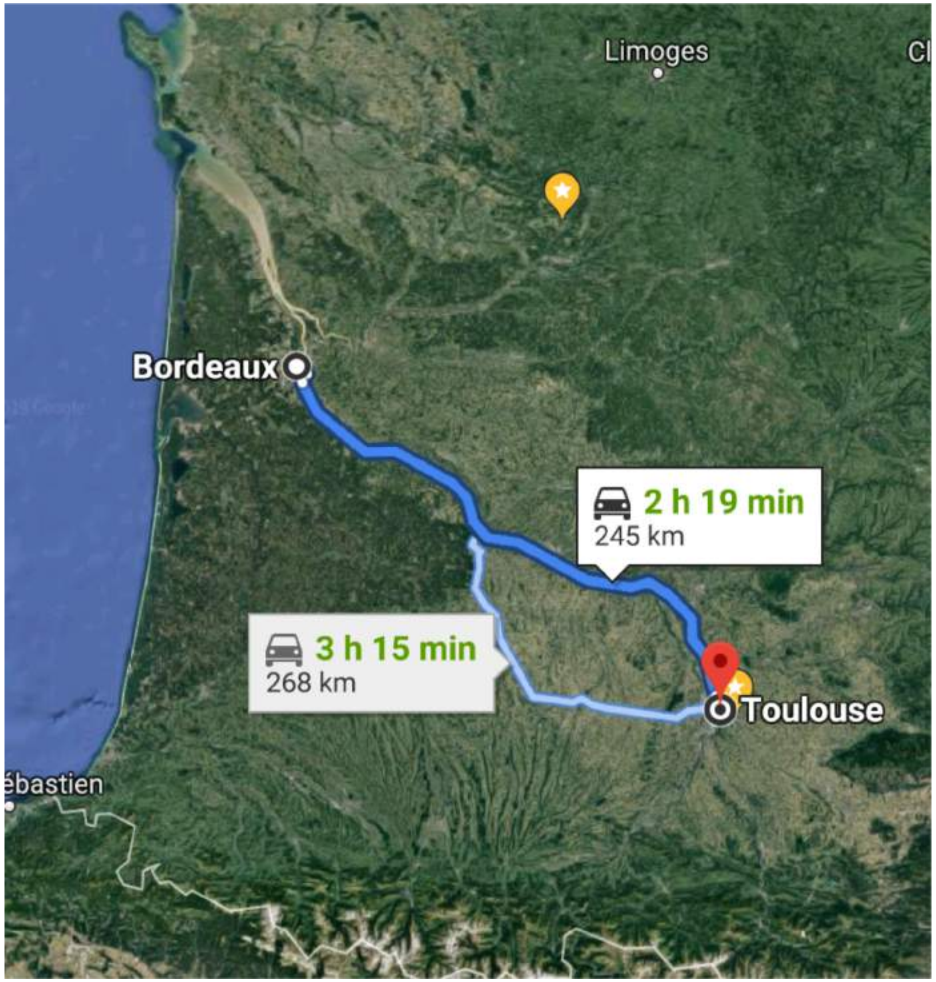


Professor Joseph Morlier

ISAE-SUPAERO, FRANCE

https://ica.cnrs.fr/en/author/jmorlier/

PhD in Bordeaux then… Toulouse



Au programme

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15’	Design Optimization	Refresh
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15’	Current CAD-CAE	Quiz
15’	GGP for ALM	Our research
15’	Ecodesign	That’s new

Fondation ISAE - SUPAERO
Reconnue d’utilité publique

AGENCE NATIONALE DE LA RECHERCHE ANR
PLAN D’ACTION 2021

CONSTRUCTION DE L’ESPACE EUROPEEN DE LA RECHERCHE ET ATTRACTIVITE INTERNATIONALE

Programme : « *Montage de Réseaux Scientifiques Européens ou Internationaux* »
- Edition 2021, Vague 1 -

l’X ÉCOLE POLYTECHNIQUE
IP PARIS

Au programme

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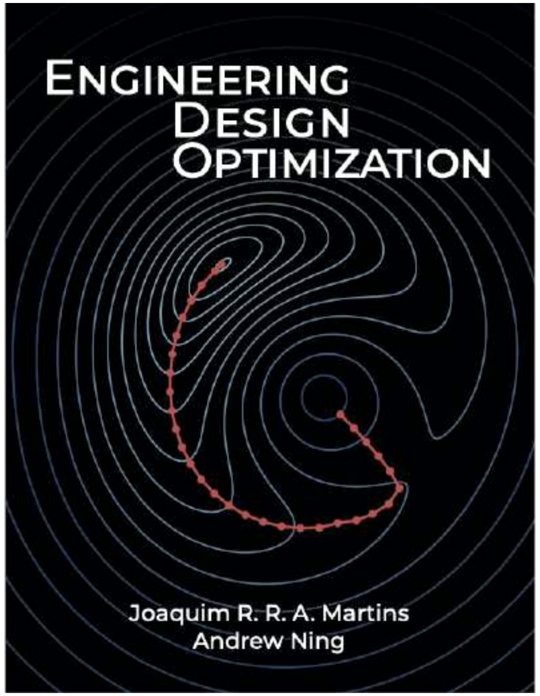
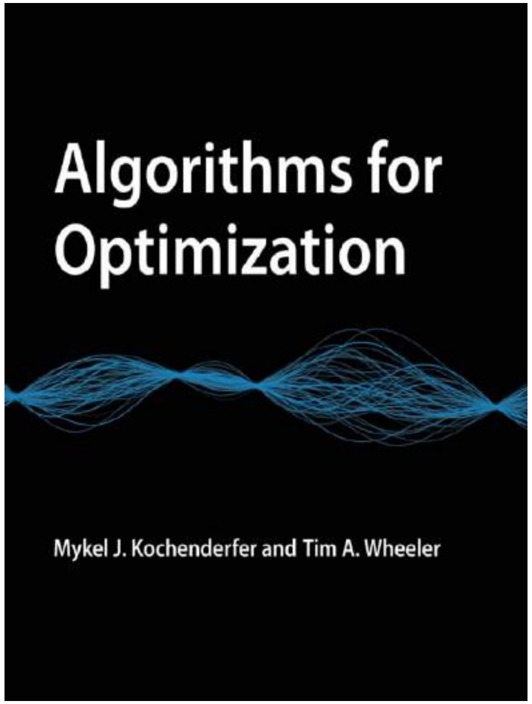
Design Optimization

Good Starting Point (x0)

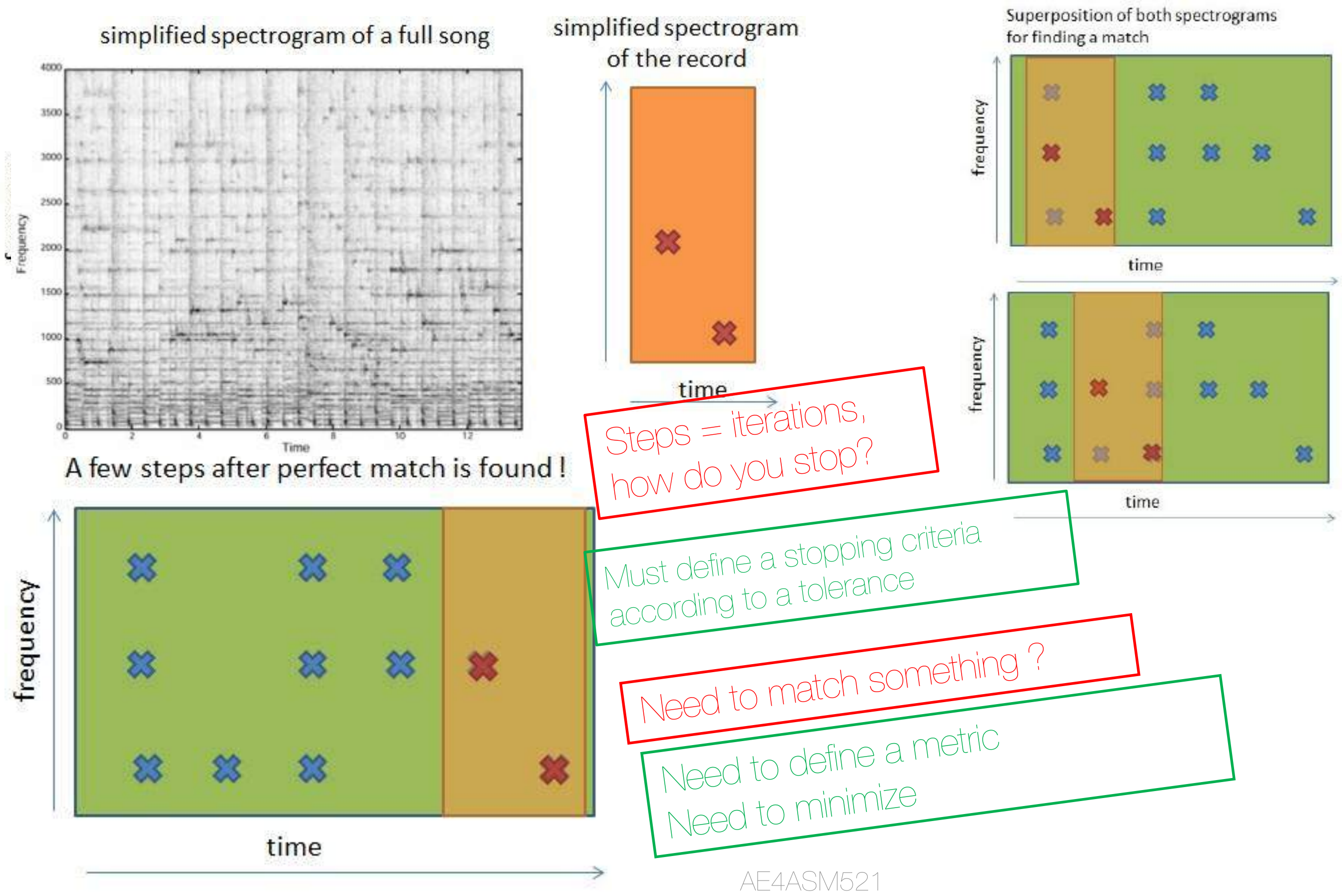




https://github.com/mdobook/resources https://github.com/sisl/algforopt-notebooks

Optimization is everywhere

http://coding-geek.com/how-shazam-works/



Steps = iterations, how do you stop?

Must define a stopping criteria according to a tolerance

Need to match something ?

Need to define a metric
Need to minimize

Design Optimization Process

On the road to design optimization

https://medium.com/daptablog/on-the-road-to-design-optimisation-a3c9867f29b6

- **optimization**
  noun [U] (UK usually **optimisation**)
  the act of making something as good as possible
  (Cambridge Dictionary)

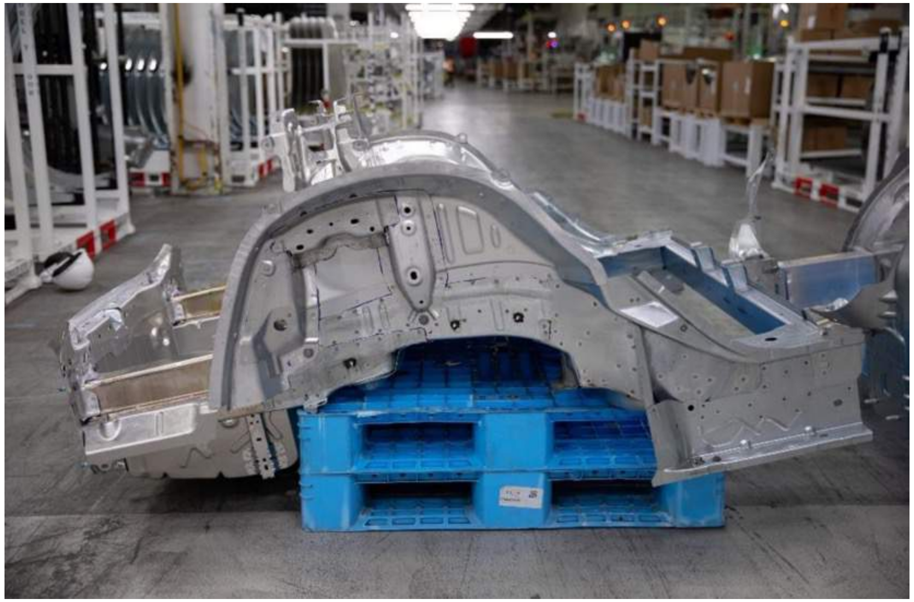
Do Better with Less

- Design optimization is an engineering design methodology **using a mathematical formulation of a design problem** to support selection of the optimal design among many alternatives. (Wikipedia)

Optimization is everywhere

https://www.3dprintingmedia.network/tesla-shows-massive-generatively-designed-3d-printed-part-in-model-y-underbody

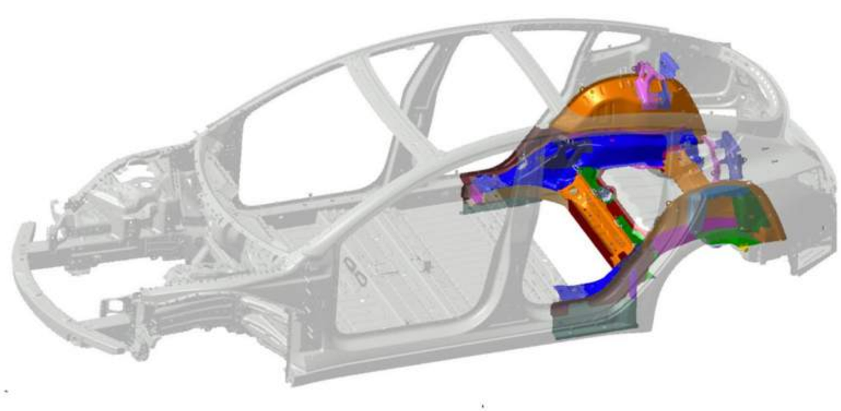
@Tesla



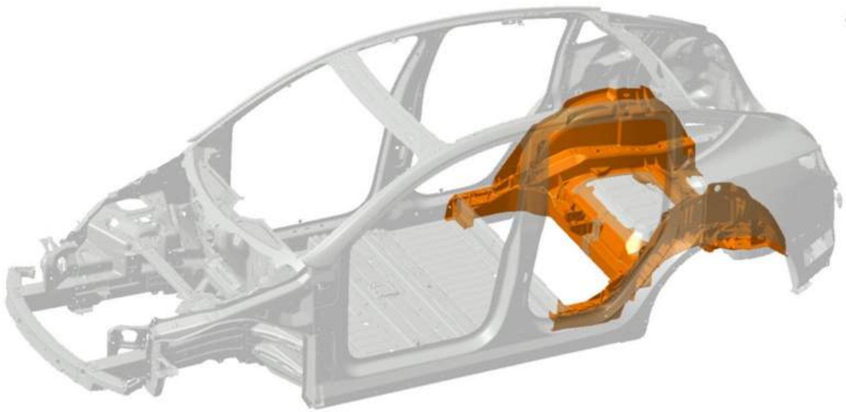
The current underbody part made of 70 different components

The generatively designed underbody, made of 2 and eventually 1 single piece.

Think different!



Model 3 rear underbody
70 pieces of metal



Model Y rear underbody
2 pieces of metal (eventually a single piece)

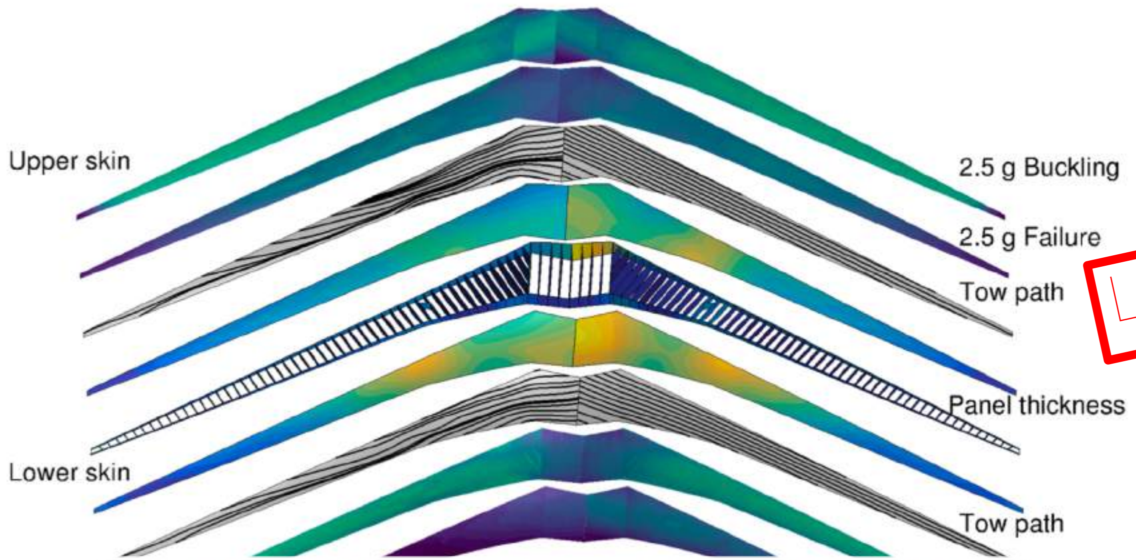
The use of 3D printing for sand casts such as that offered by voxeljet and ExOne for to enable the reduction of subassemblies (form 70 to 1) in a custom cast can bring about a significant transition even before metal AM can be used to produce such large metal parts directly. Producing a complex cast that can reduce the number of parts to this degree needs digital casting technology

Optimization is everywhere

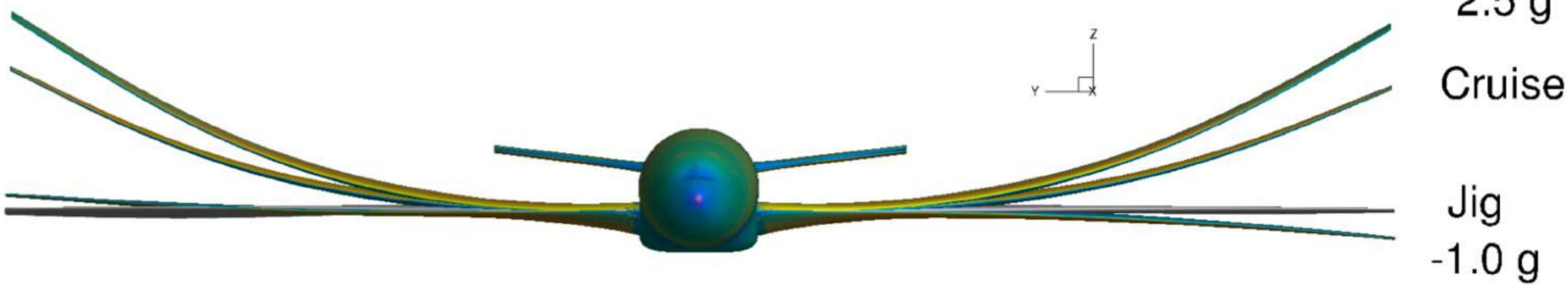
https://www.compositesworld.com/articles/leveraging-motorsports-composites-for-next-gen-rotorcraft

Figure 17: AFP machine manufacturing the tow-steered optimized uCRM-13.5 wingbox (left; courtesy of Aurora Flight Sciences). Static test of the same wing (right; courtesy of NASA)

Prof Martins ‘sTow Steered Wing Structure Design



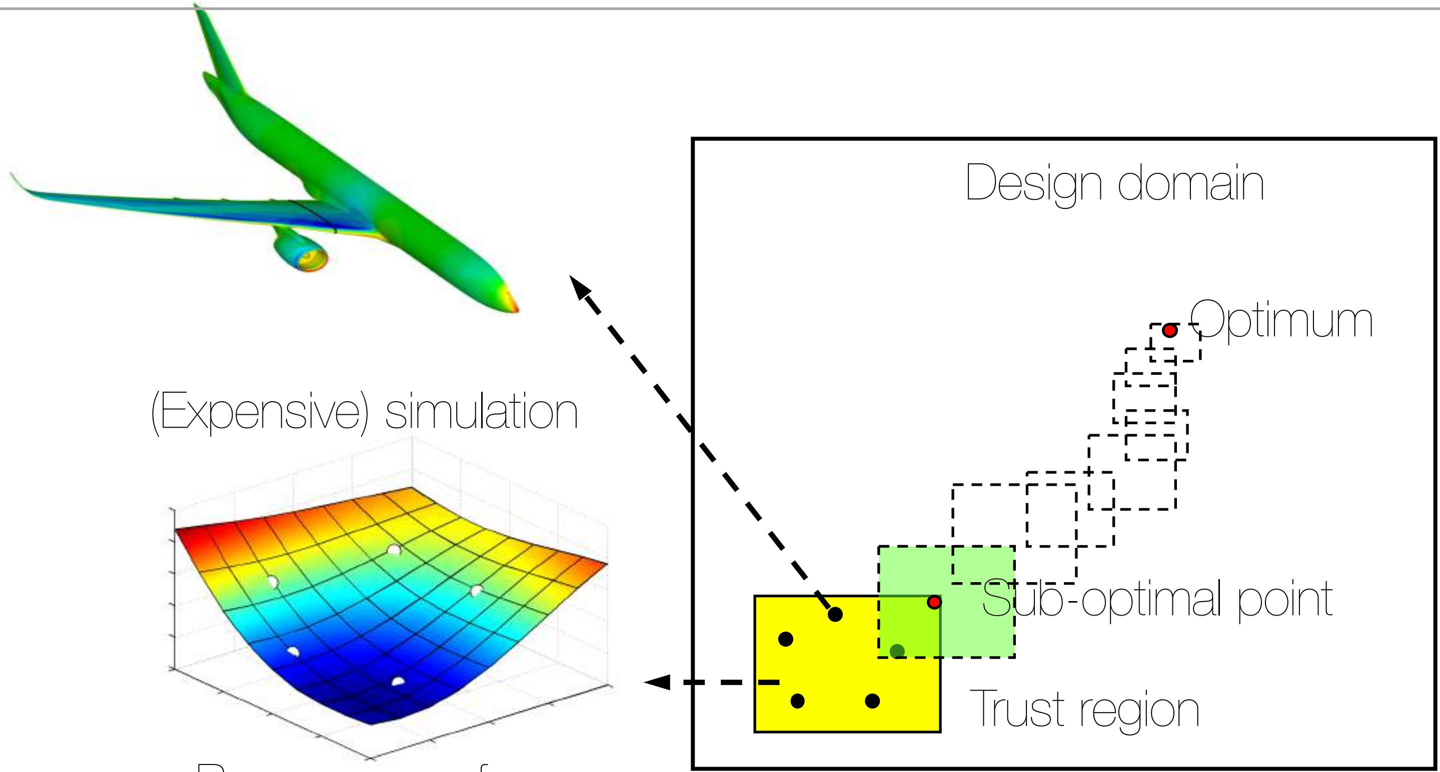
Less fuel burn, lower wing mass



SURROGATE MODELING

learning for Optimizing

Jacobs, J. H., Etman, L. F. P., Van Keulen, F., & Rooda, J. E. (2004). Framework for sequential approximate optimization. Structural and Multidisciplinary Optimization, 27(5), 384-400.



Response surfaces

Or metamodels, surrogate models etc…

Bayesian Optimization in **1-slide** ?

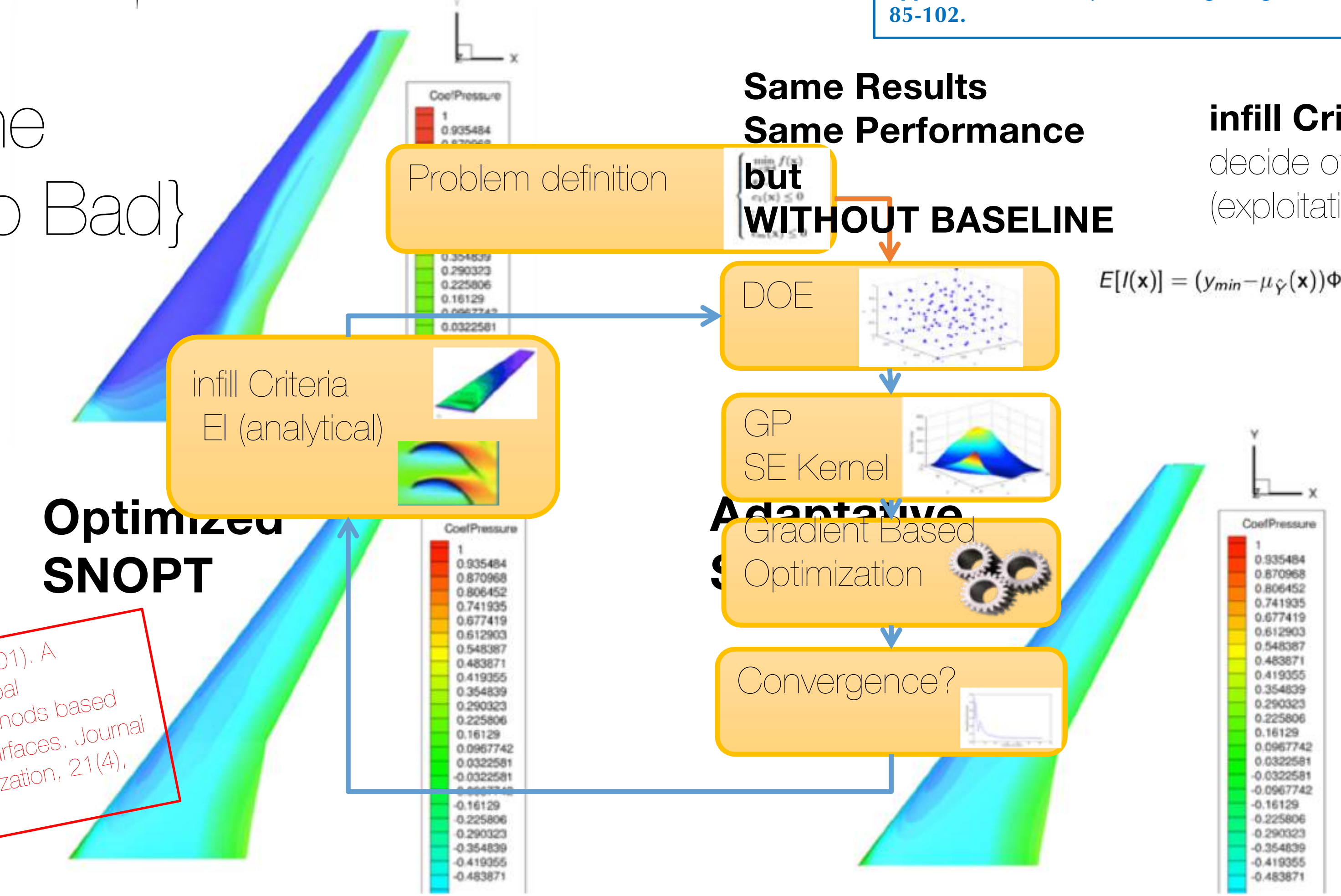
Bartoli, N., Lefebvre, T., Dubreuil, S., Olivanti, R., Priem, R., Bons, N., ... & Morlier, J. (2019). Adaptive modeling strategy for constrained global optimization with application to aerodynamic wing design. Aerospace Science and technology, 90, 85-102.

Baseline
{Not so Bad}

Same Results
Same Performance
but
WITHOUT BASELINE

infill Criteria??
decide of the new point
(exploitation-exploration tradeoff)

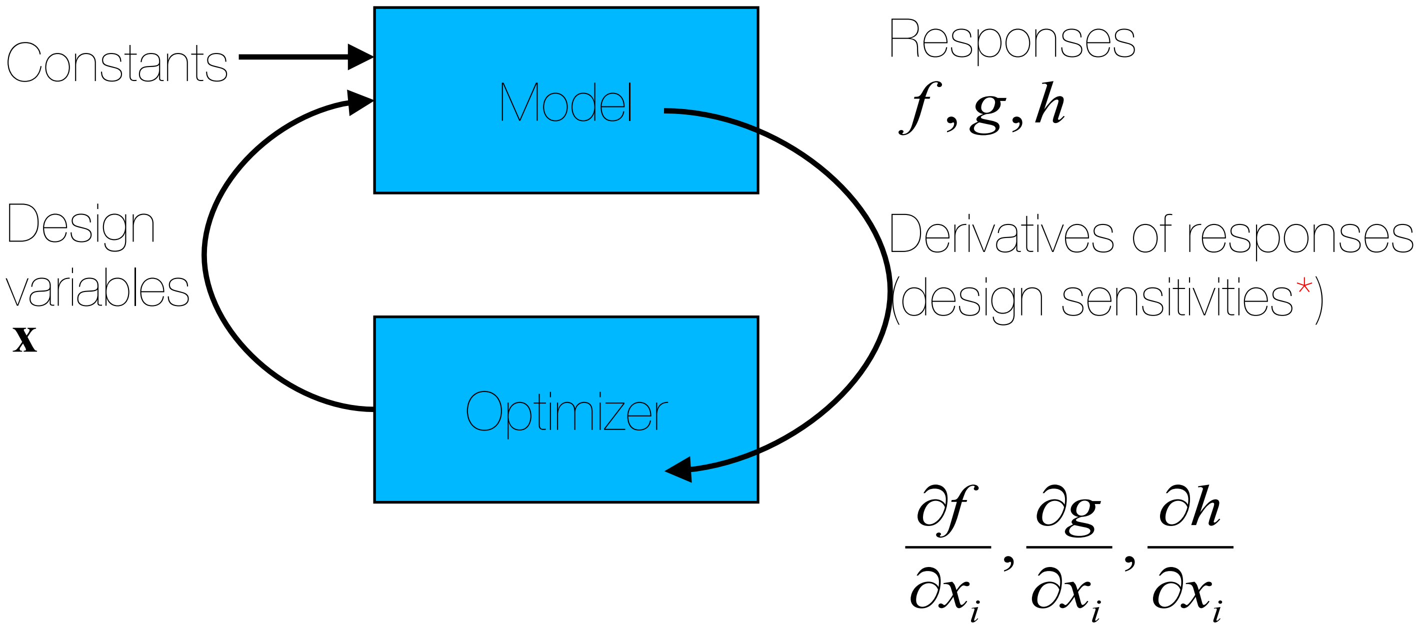
$$E[I(\mathbf{x})] = (y_{min} - \mu_{\hat{Y}}(\mathbf{x}))\Phi\left(\frac{y_{min} - \mu_{\hat{Y}}(\mathbf{x})}{\sigma_{\hat{Y}}(\mathbf{x})}\right) + \sigma_{\hat{Y}}(\mathbf{x})\varphi\left(\frac{y_{min} - \mu_{\hat{Y}}(\mathbf{x})}{\sigma_{\hat{Y}}(\mathbf{x})}\right)$$



Jones, D. R. (2001). A taxonomy of global optimization methods based on response surfaces. Journal of global optimization, 21(4), 345-383.

Gradient Based Optimization

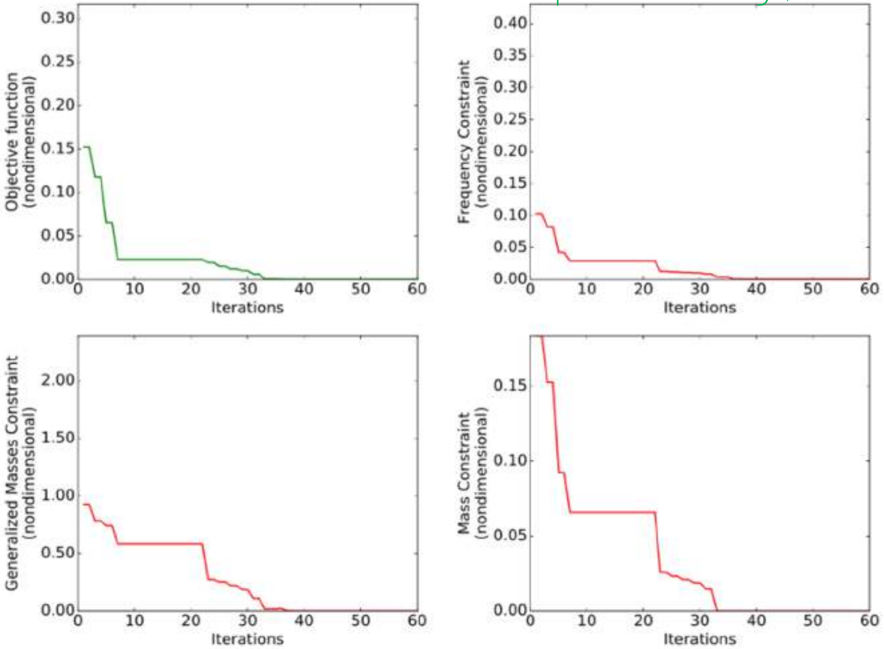
- Costly if Finite Differences is used for sensitivities
- Difficult to implement Adjoint in industrial code
- Sensitive to discontinuity
- Sensitive to X_0



*SOL200 in MSC Nastran for example

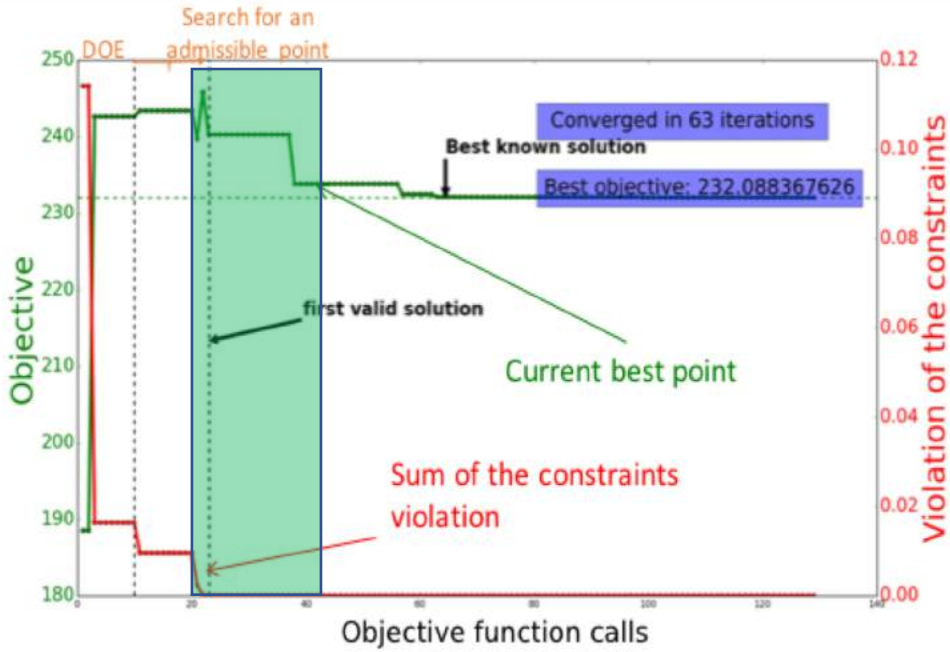
New Graphs for BO

Gradient based Optimality, Feasibility



Stopping criteria: tolfun, tolx, maxiter

SBO Exploration, Exploitation



Stopping criteria: Max Budget (Function calls)

X_0

https://smt.readthedocs.io/en/latest

https://github.com/SMTorg/smt

SMT 0.8.0 documentation »

AEROSPACE ENGINEERING
UNIVERSITY of MICHIGAN

NASA

isae SUPAERO

ONERA
THE FRENCH AEROSPACE LAB

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Getting started

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SMT: Surrogate Modeling Toolbox

The surrogate modeling toolbox (SMT) is an open-source Python package consisting of libraries of surrogate modeling methods (e.g., radial basis functions, kriging), sampling methods, and benchmarking problems. SMT is designed to make it easy for developers to implement new surrogate models in a well-tested and well-document platform, and for users to have a library of surrogate modeling methods with which to use and compare methods.

The code is available open-source on GitHub.

Cite us

To cite SMT: M. A. Bouhlel and J. T. Hwang and N. Bartoli and R. Lafage and J. Morlier and J. R. R. A. Martins.

A Python surrogate modeling framework with derivatives. Advances in Engineering Software, 2019.

```
@article{SMT2019,
        Author = {Mohamed Amine Bouhlel and John T. Hwang and Nathalie Bartoli and Rémi Lafage
        Journal = {Advances in Engineering Software},
        Title = {A Python surrogate modeling framework with derivatives},
        pages = {102662},
        year = {2019},
        issn = {0965-9978},
        doi = {https://doi.org/10.1016/j.advengsoft.2019.03.005},
        Year = {2019}}
```

Focus on derivatives

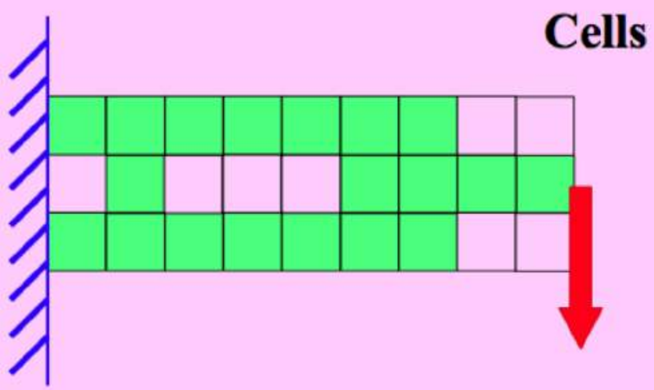
SMT is meant to be a general library for surrogate modeling (also known as metamodeling, interpolation, and regression), but its distinguishing characteristic is its focus on derivatives, e.g., to be used for gradient-based optimization. A surrogate model can be represented mathematically as

$$y = f(\mathbf{x}, \mathbf{xt}, \mathbf{yt}),$$

Topology Optimization

MIT Course Number ESD.77 / 16.888

$$\begin{aligned} \text{minimize} \quad & f(\mathbf{x}) \\ \text{subject to} \quad & g(\mathbf{x}) \leq 0 \\ & h(\mathbf{x}) = 0 \end{aligned}$$



Design variables (x)

x : density of each cell

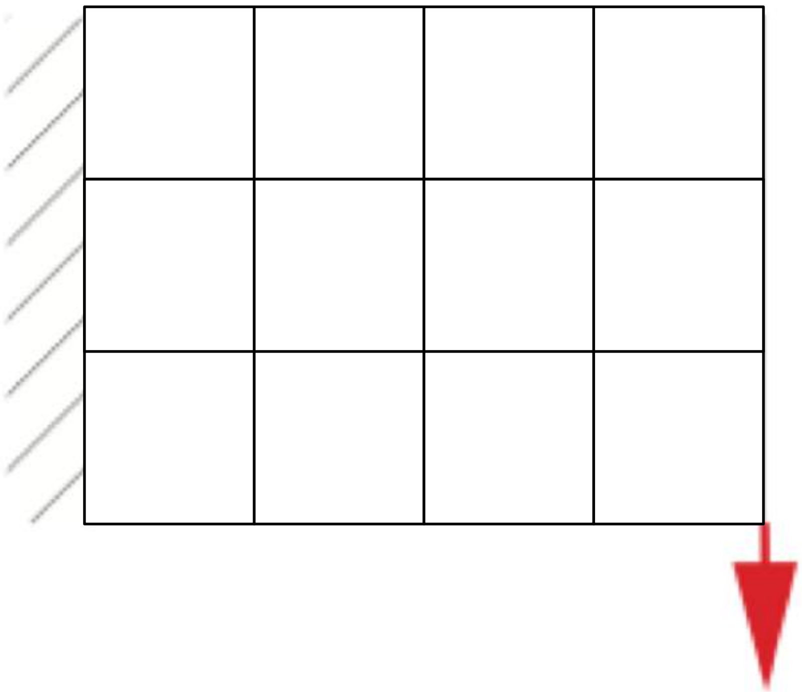
Number of design variables (ndv)

ndv = 27

***f*(x) : compliance**

***g*(x) : mass**

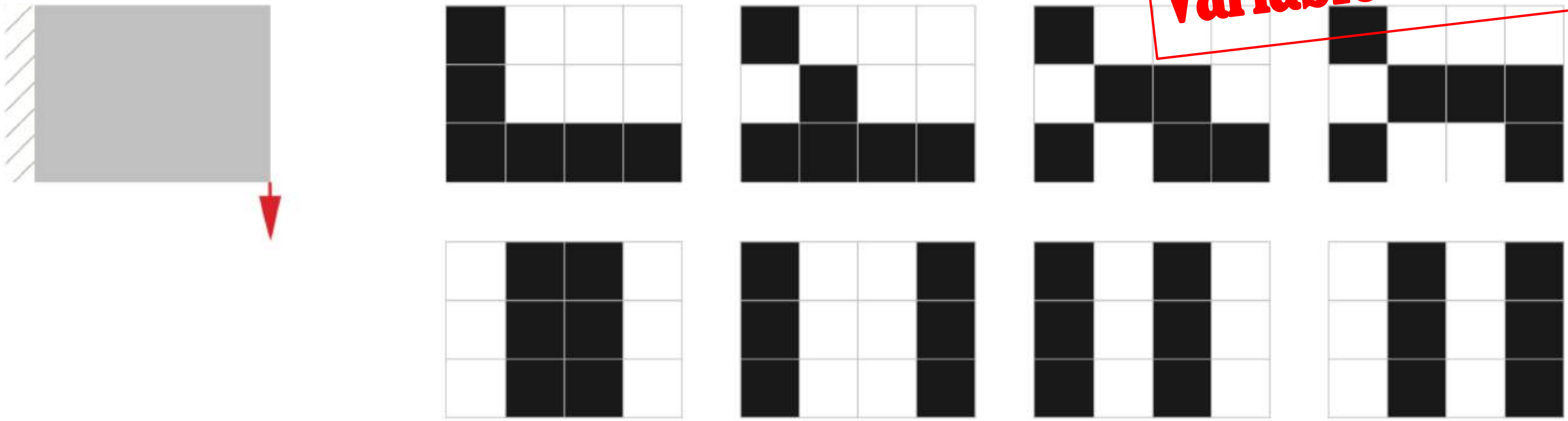
Quiz ! draw material (black) or void (white)



Goal: 50% gain in mass

Results

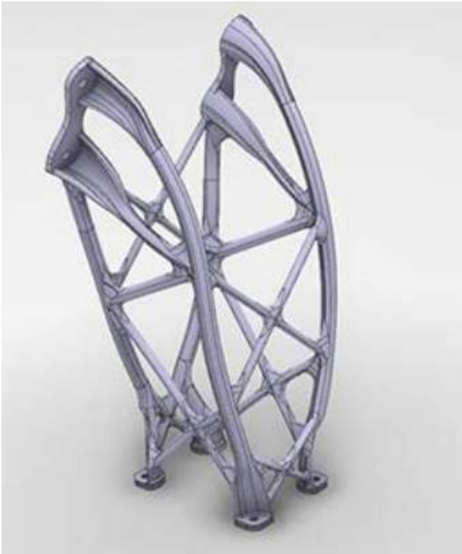
discrete or continuous variables ?



The legal (top) and some illegal (bottom) topologies with 4 by 3 elements

Some Aerospace Parts

Airbus A320 nacelle hinge bracket by EADS and Airbus



A structural bracket for Eurostar E3000 satellites by Airbus Defense and Space in UK

An antenna bracket for a Sentinel-1-Satellite by RUAG

Coupling
TopOpt & ALM
offer new possible feasible designs (see Jun Wu's course)

Au programme

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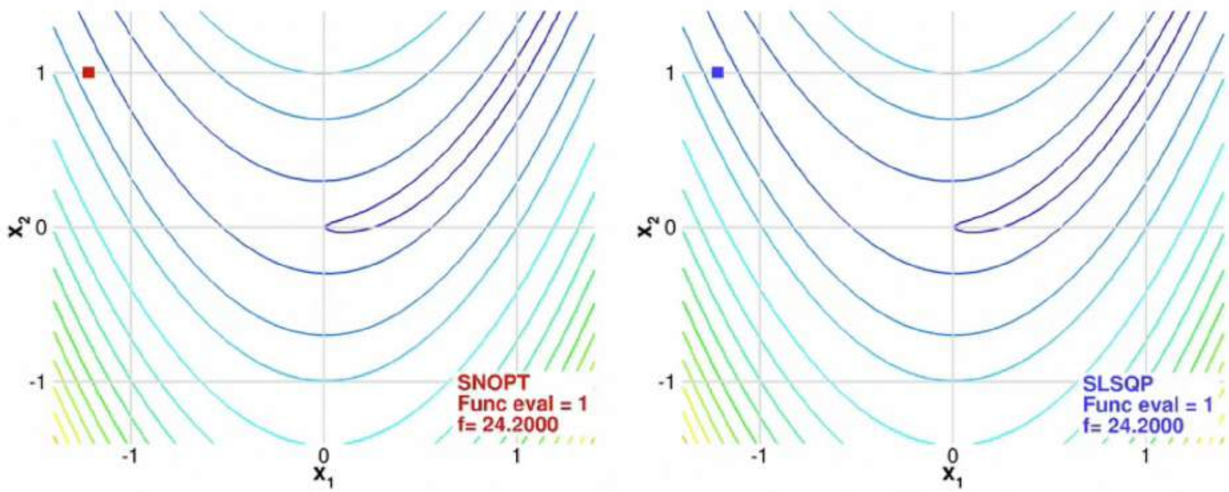
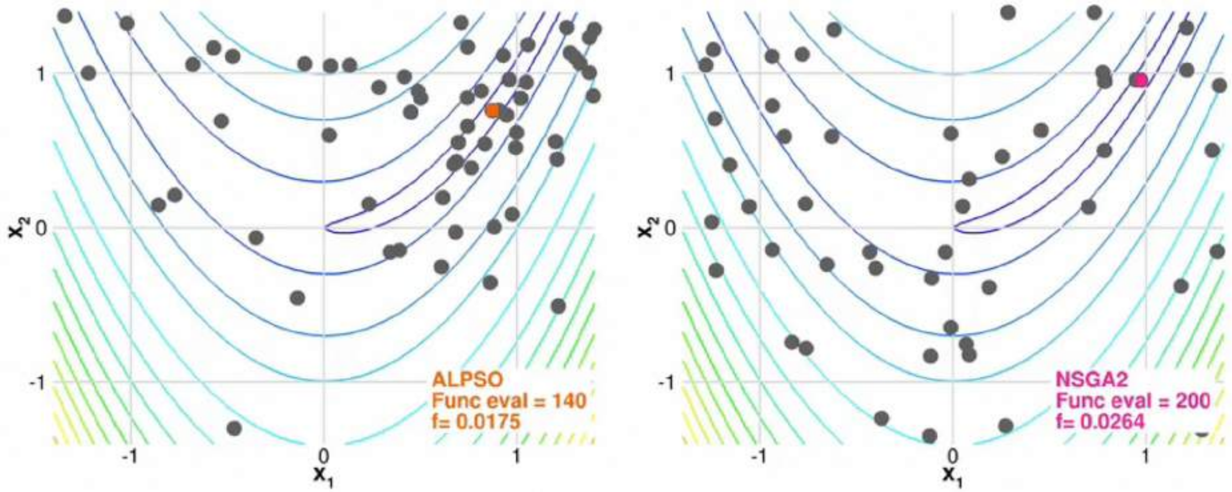
Computing Derivatives

Gradient-based methods take a more direct path to … the optimum

Gradient-based



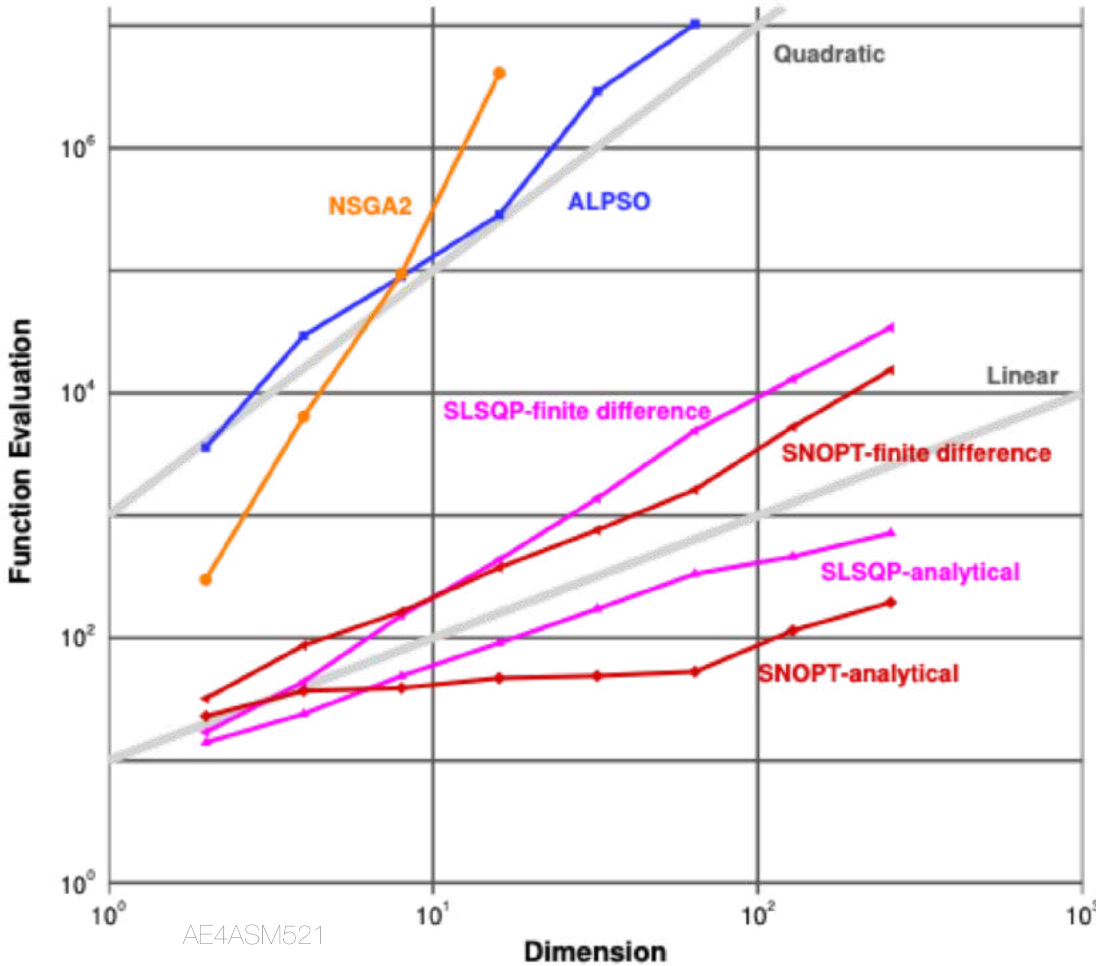
Gradient-free



Gradient-based optimization is the only hope

for large numbers of design variables

Need accurate derivative



[Lyu et al. ICCFD8-2014-0203]

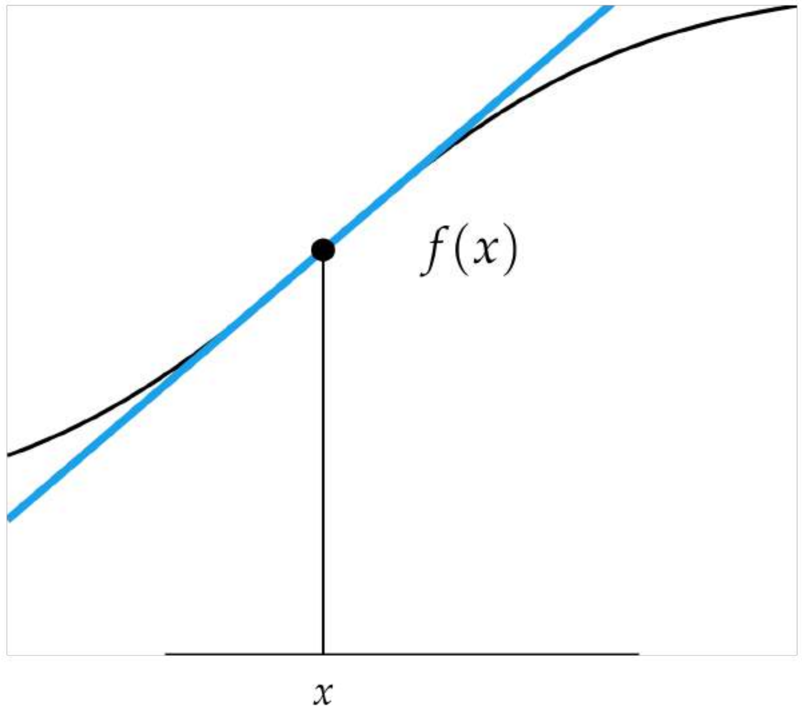
Derivatives

- Derivatives tell us which direction to search for a solution

Derivatives

- Slope of Tangent Line

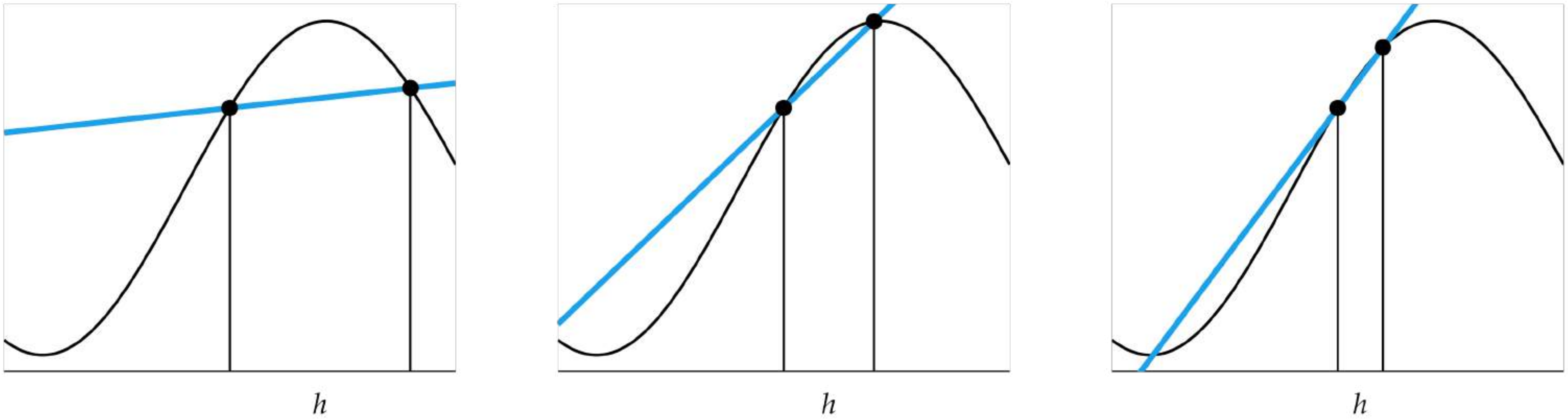
$$f'(x) \equiv \frac{df(x)}{dx}$$



Derivatives

$$f(x + \Delta x) \approx f(x) + f'(x)\Delta x$$

$$f'(x) = \frac{\Delta f(x)}{\Delta x}$$



Derivatives in Multiple Dimensions

- Directional Derivative

$$\nabla_{\mathbf{s}} f(\mathbf{x}) \equiv \underbrace{\lim_{h \to 0} \frac{f(\mathbf{x} + h\mathbf{s}) - f(\mathbf{x})}{h}}_{\text{forward difference}}$$

$$= \underbrace{\lim_{h \to 0} \frac{f(\mathbf{x} + h\mathbf{s}/2) - f(\mathbf{x} - h\mathbf{s}/2)}{h}}_{\text{central difference}}$$

$$= \underbrace{\lim_{h \to 0} \frac{f(\mathbf{x}) - f(\mathbf{x} - h\mathbf{s})}{h}}_{\text{backward difference}}$$

Numerical Differentiation: Finite Difference

- Error Analysis
  - Forward Difference: $O(h)$
  - Central Difference: $O(h^2)$

Numerical Differentiation: Complex Step

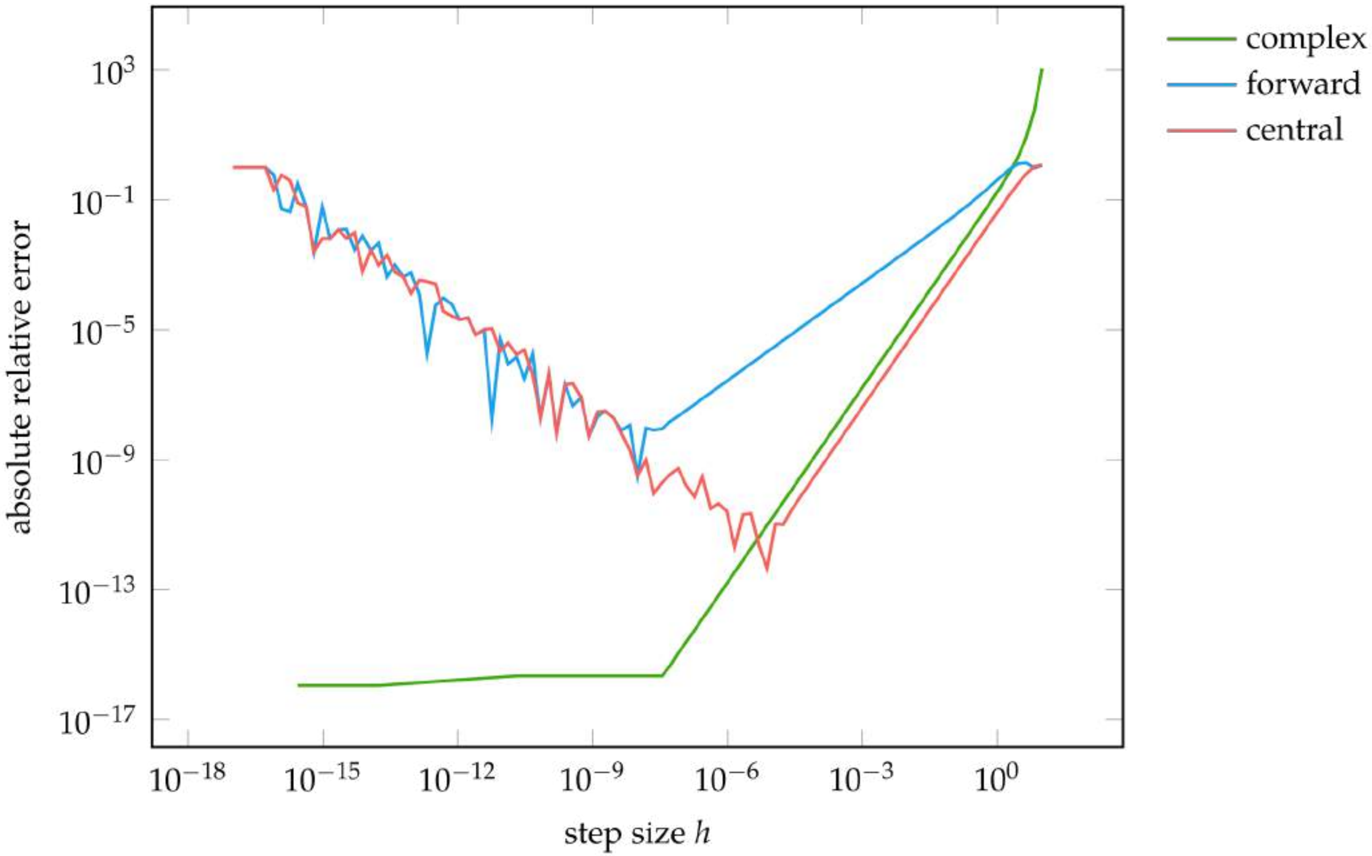
- Taylor series expansion using imaginary step

$$f(x+ih) = f(x) + ihf'(x) - h^2\frac{f''(x)}{2!} - ih^3\frac{f'''(x)}{3!} + \cdots$$

$$f'(x) = \frac{\mathrm{Im}(f(x+ih))}{h} + O(h^2) \text{ as } h \to 0$$

$$f(x) = \mathrm{Re}(f(x+ih)) + O(h^2)$$

Numerical Differentiation Error Comparison



Automatic Differentiation

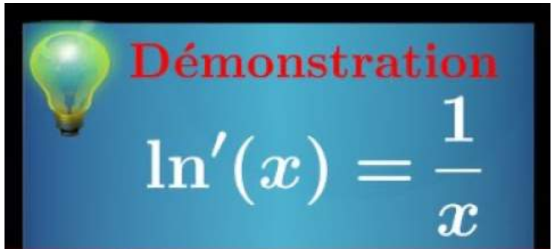
- Evaluate a function and compute partial derivatives simultaneously using the chain rule of differentiation

$$\frac{d}{dx} f(g(x)) = \frac{d}{dx}(f \circ g)(x) = \frac{df}{dg}\frac{dg}{dx}$$

AD. . . is Computer Sciences

A program is composed of elementary operations like addition, subtraction, multiplication, and division.

Consider the function $f(a,b) = \ln(ab + \max(a,2))$. If we want to compute the partial derivative with respect to a at a point, we need to apply the chain rule several times:[9]



$$\begin{aligned}
\frac{\partial f}{\partial a} &= \frac{\partial}{\partial a}\ln(ab + \max(a,2)) \\
&= \frac{1}{ab + \max(a,2)}\frac{\partial}{\partial a}(ab + \max(a,2)) \\
&= \frac{1}{ab + \max(a,2)}\left[\frac{\partial(ab)}{\partial a} + \frac{\partial \max(a,2)}{\partial a}\right] \\
&= \frac{1}{ab + \max(a,2)}\left[\left(b\frac{\partial a}{\partial a} + a\frac{\partial b}{\partial a}\right) + \left((2 > a)\frac{\partial 2}{\partial a} + (2 < a)\frac{\partial a}{\partial a}\right)\right] \\
&= \frac{1}{ab + \max(a,2)}[b + (2 < a)]
\end{aligned}$$

One example

- Forward Accumulation is equivalent to expanding a function using the chain rule and computing the derivatives inside-out
- Requires n-passes to compute n-dimensional gradient

$$f(a,b) = \underline{\ln}(\underline{ab} \underline{+} \underline{\max}(a, 2))$$

$$\frac{\partial f}{\partial a}(3,2)$$

AD computational graphs

$$\frac{\partial f(3,2)}{\partial a}$$

- Forward Accumulation

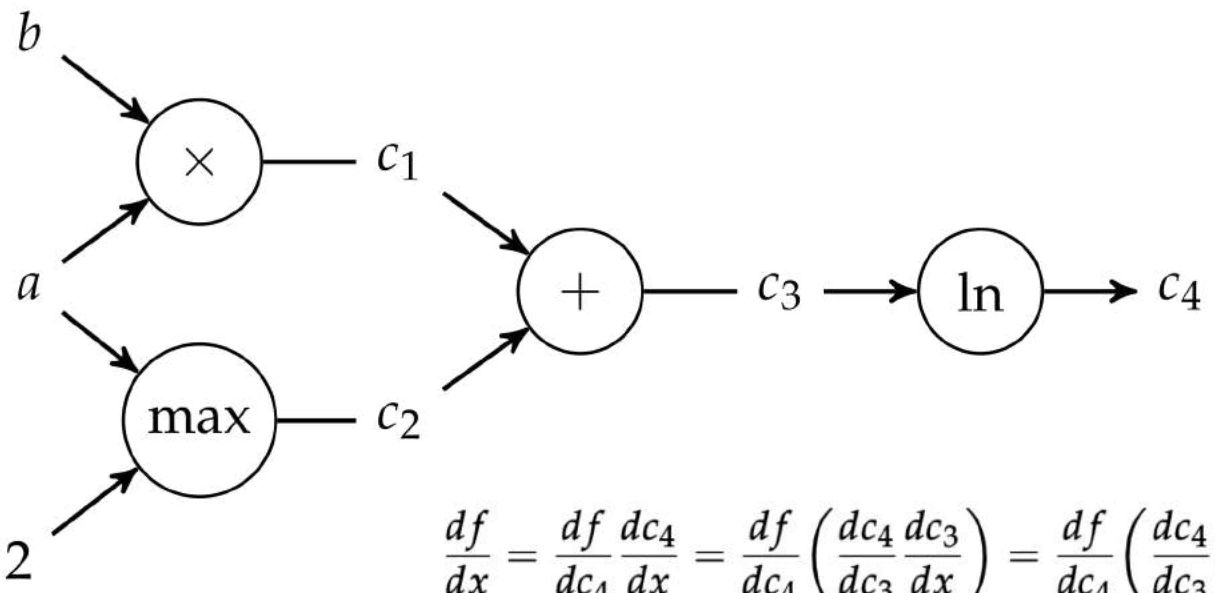
$$f(a,b) = \ln(ab + \max(a,2))$$

$b = 2$

$\frac{\partial b}{\partial a} = \dot{b} = 0$

$a = 3$

$\dot{a} = 1$



$$\frac{df}{dx} = \frac{df}{dc_4}\frac{dc_4}{dx} = \frac{df}{dc_4}\left(\frac{dc_4}{dc_3}\frac{dc_3}{dx}\right) = \frac{df}{dc_4}\left(\frac{dc_4}{dc_3}\left(\frac{dc_3}{dc_2}\frac{dc_2}{dx} + \frac{dc_3}{dc_1}\frac{dc_1}{dx}\right)\right)$$

Automatic Differentiation

- Forward Accumulation

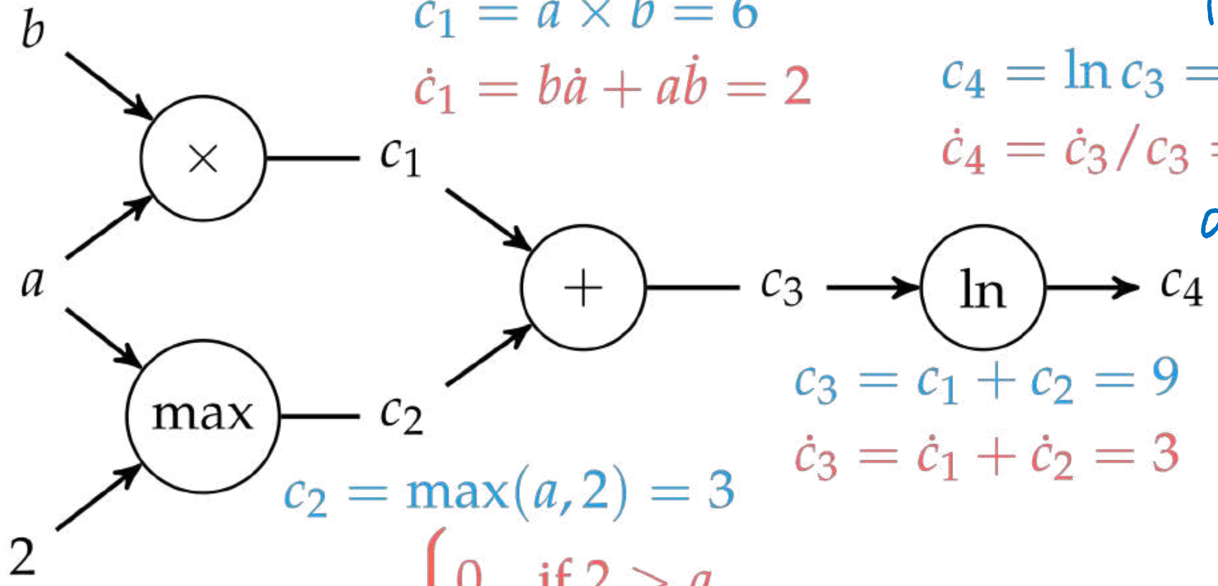
$$f(a,b) = \ln(\, ab + \max(a, 2))$$

$b = 2$

$\dot{b} = 0$

$a = 3$

$\dot{a} = 1$



$c_1 = a \times b = 6$

$\dot{c}_1 = b\dot{a} + a\dot{b} = 2$

$c_2 = \max(a, 2) = 3$

$$\dot{c}_2 = \begin{cases} 0 & \text{if } 2 > a \\ \dot{a} & \text{if } 2 < a \end{cases} = 1$$

$c_3 = c_1 + c_2 = 9$

$\dot{c}_3 = \dot{c}_1 + \dot{c}_2 = 3$

$c_4 = \ln c_3 = \ln 9$

$\dot{c}_4 = \dot{c}_3 / c_3 = \frac{1}{3}$

$f(3,2) = \ln 9$

$df(3,2)/da = 1/3$

In Julia

The `ForwardDiff.jl` package supports an extensive set of mathematical operations and additionally provides gradients and Hessians.

```
julia> using ForwardDiff
julia> a = ForwardDiff.Dual(3,1);
julia> b = ForwardDiff.Dual(2,0);
julia> log(a*b + max(a,2))
Dual{Nothing}(2.1972245773362196,0.3333333333333333)
```

Au programme

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Topopt review

History

- Homogenization of Microstructures was introduced by mathematics in the 1970s.
- First paper by Martin Bendsoe (Technical University of Denmark) and Noboru Kikuchi (University of Michigan) in 1988

A topology optimisation problem can be written in the general form of an optimization problem as

$$\min_{\rho} \; F = F(\mathbf{u}(\rho), \rho) = \int_{\Omega} f(\mathbf{u}(\rho), \rho) \mathrm{d}V$$

subject to

- $\rho \in \{0, 1\}$
- $G_0(\rho) = \int_{\Omega} \rho(\mathbf{u}) \mathrm{d}V - V_0 \leq 0$
- $G_j(\mathbf{u}(\rho), \rho) \leq 0$ with $j = 1, \ldots, m$

TopOpt

Je choisis un bloc de marbre et j'enlève tout ce dont je n' ai pas besoin..
Auguste Rodin (1840-1917)

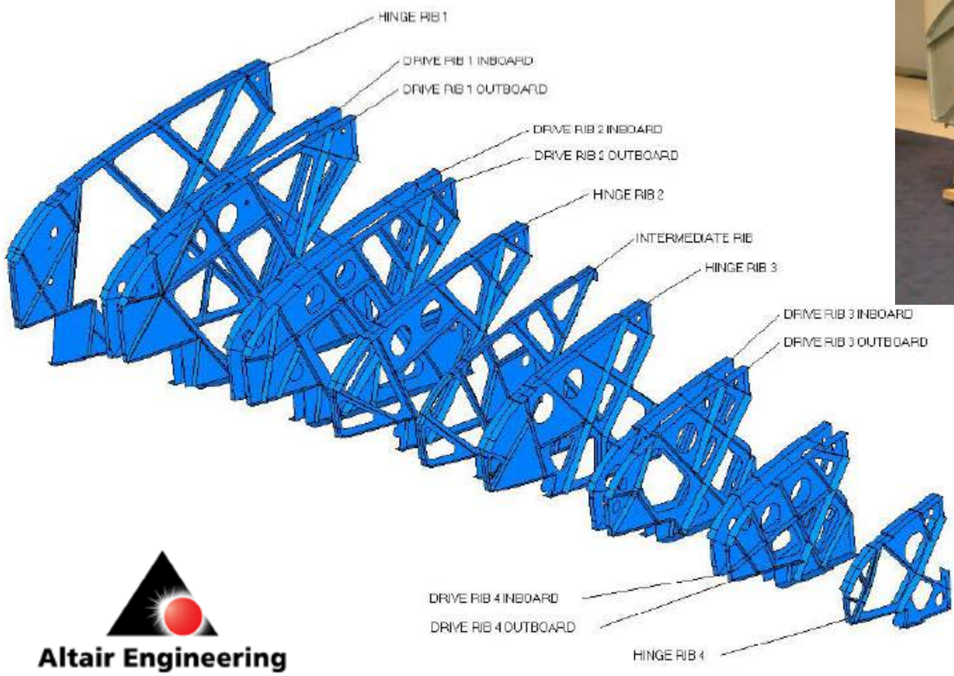
- Define the design space (marble block, fixed mesh)
- Apply loads & BCs
- Start optimization with hyper parameters
- Interpreting the results
- Optimal distribution of material (One can have an idea of the part to be reinforced, in addition to giving an excellent initial design …)

Why is it so powerful?

→There is a lot of possible redistribution of **INTERNAL FORCES**

Airbus A380 example (cont.)

- Result: 500 kg weight savings!



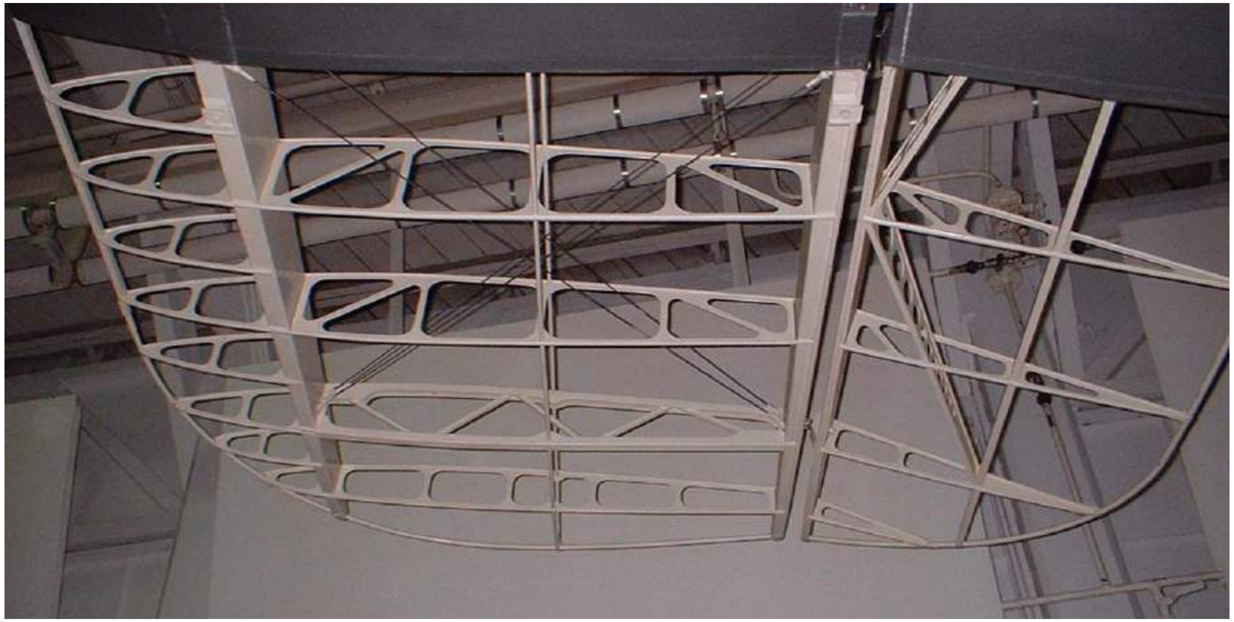
Altair Engineering

Wing rib designs

The perforated plates were replaced by reinforced lattice structures
(think of the path of preferential intern foces)

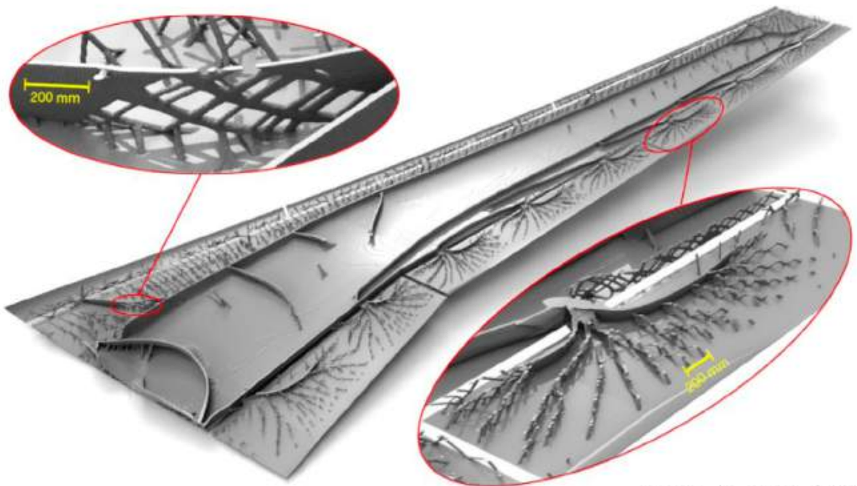
Is this totally new?

Supermarine Southampton, 1925

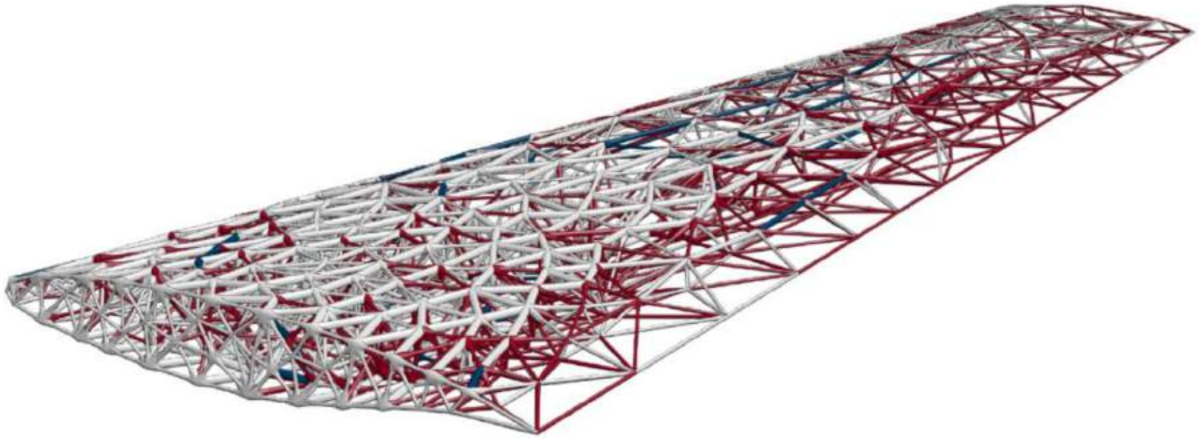


System approach automates the process!

Concurrent method TOPOPT vs LAYOPT (continuous vs discrete)

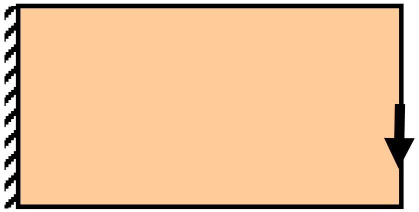


Aage, N. et al. (2017)



Opgenoord, M. M. and Willcox, K. E. (2018)

TopOpt



$Ku = f$

1. Objective? Minimize Compliance
2. Constraints? Volume Constraint
3. Method?

Minimize J

Vol.Frac ≤ 0.5

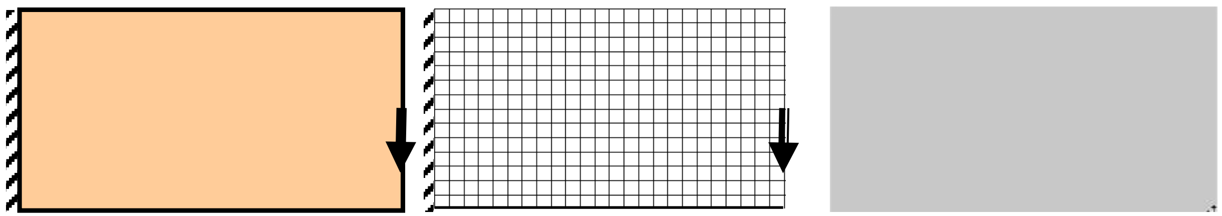
Compliance $J = f^T u$

Method: Gradient based:
Need sensitivities…

Compliance = 1/Stiffness

SIMP

SIMP: Solid Isotropic Material with Penalization



$$\mathit{Min}\ \text{Compliance}$$
$$v = 0.5v_0$$

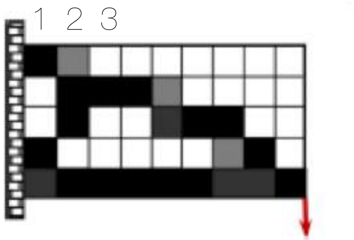
Where do we add holes?

$$0 < \rho_e \leq 1 : \text{'PseudoDensity'}$$

$$\mathop{\mathit{Min}}_{\rho_e}\ \text{Compliance}$$
$$\sum \rho_e v_e = 0.5v_0$$

Pixels?

Intuitive Problem? Quadratic Form



$x_1 = 1$
$x_2 = 0.5$
$x_3 = 0$
$\dots$

- Objective function; Strain energy

$$\min c(\mathbf{x}) = \mathbf{U}^T \mathbf{F} = \mathbf{U}^T \mathbf{K}\mathbf{U} \qquad \text{with} \quad x_e = \frac{\rho_e}{\rho_0} \qquad (4)$$

with $\mathbf{K} = \mathbf{K}_0 \sum_{e=1}^{N} x_e^p$ one can write:

$$\min c(\mathbf{x}) = \sum_{e=1}^{N} (x_e)^p \mathbf{u}_e^T \mathbf{k}_0 \mathbf{u}_e \qquad (5)$$

Scalar

- Contraints: mass target

$$\frac{V(\mathbf{x})}{V_0} = f = const \Leftrightarrow \sum_{e=1}^{N} V_e x_e - V_0 f = 0 = h(\mathbf{x})$$

Scalar

$$0 < \rho_{\min} \leq \rho_e \leq 1$$

$$\min c(\mathbf{x}) = \sum_{e=1}^{N} (x_e)^p \mathbf{u}_e^T \mathbf{k}_0 \mathbf{u}_e$$

Quadratic Form

$x \in \mathbb{R}^{m \times 1}$, $\mathbb{A} \in \mathbb{R}^{m \times m}$

$$\boxed{\text{Quadratic form} : \quad x^T \mathbb{A} x}$$

$x^T \mathbb{A} x$ is a scalar value.

$$(1 \times m) \times (m \times m) \times (m \times 1) \rightarrow 1 \times 1$$

K is linked through E and xe

Rozvany, G.I.N. , Zhou, M., and Gollub, M. (1989). Continuum Type Optimality Criteria Methods for Large Finite Element Systems with a Displacement Connstraint, Part 1. Structural Optimization 1:47-72.

$$\mathbf{K} = \mathbf{K}_0 \sum_{e=1}^{N} x_e^p \qquad x_e = \frac{\rho_e}{\rho_0}$$

What is **p**, (sim**P**)?????????

- **But HOW ??**

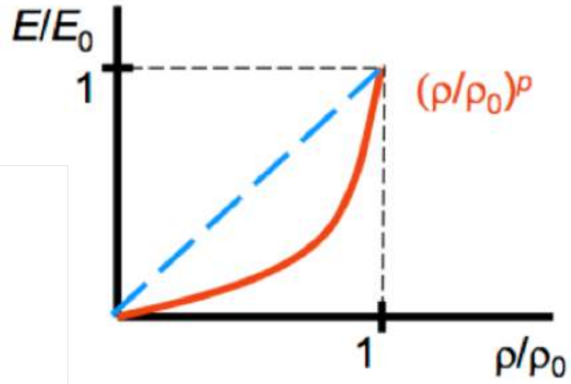
Avoid intermediate densities !

Solid Isotropic Material with Penalization (SIMP)

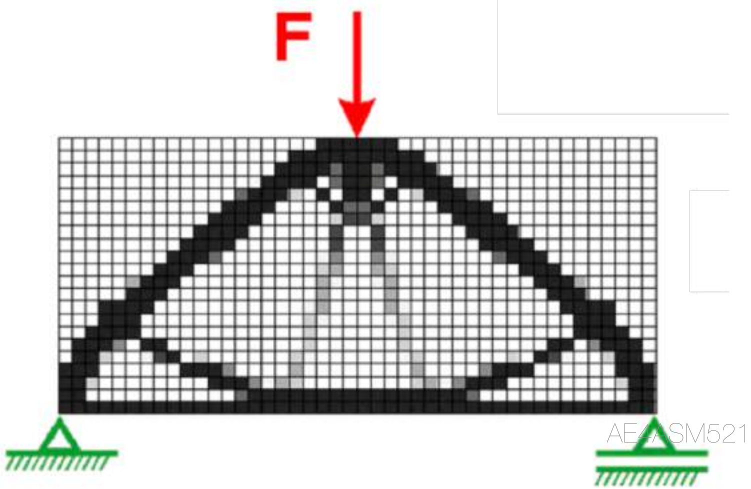
$$E(x) = E_{min} + (E_0 - E_{min})x^p$$

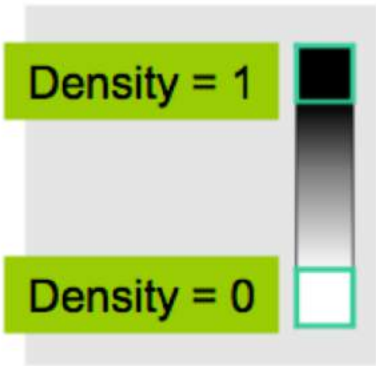
p is the penalty parameter to push densities to black (1) and white (0).

E_{min} is a small value that avoid stiffness matrix singularity



Penalization for altering stiffness localy

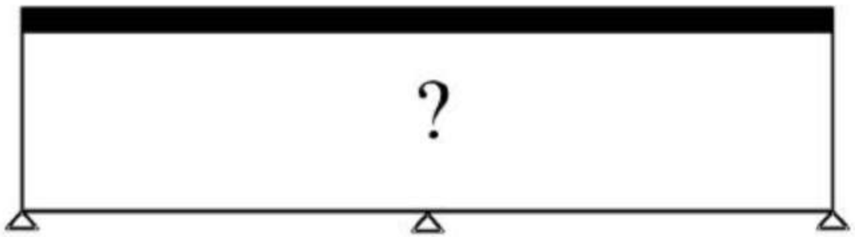


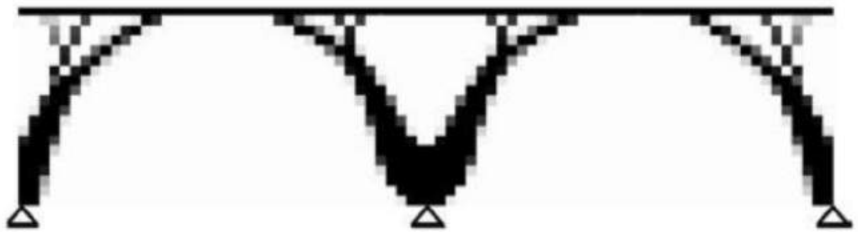


Nice idea !

1. Transform discrete variables **continuously (TO USE gradient-based algorithms)**
2. Find an objective function with "**cheap**" derivatives (we will see this later but you can also **use AD**)

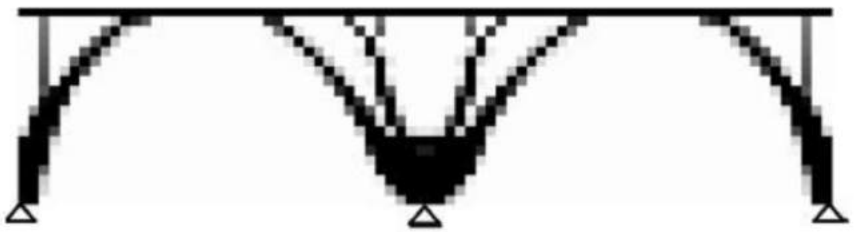
Can you comment this ?
1 problem; 3 optimizers





MMA

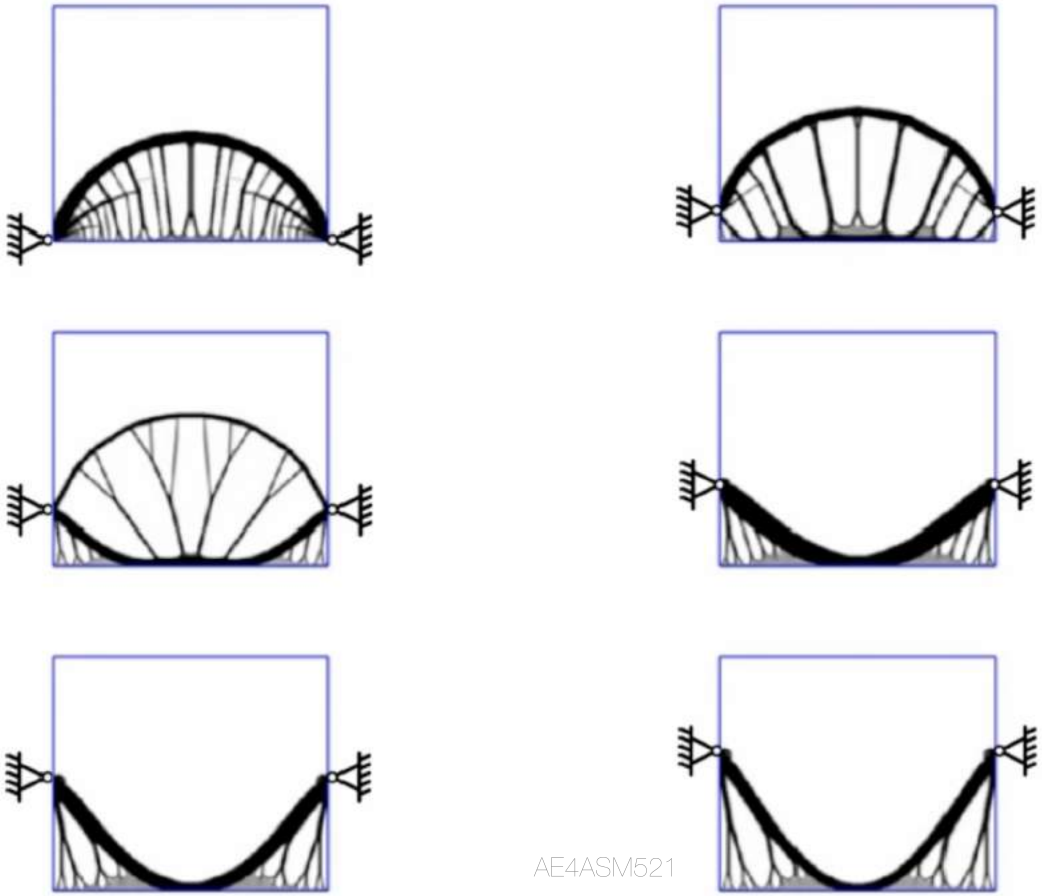
- **Local Optimum**



Conlin & GCMMA

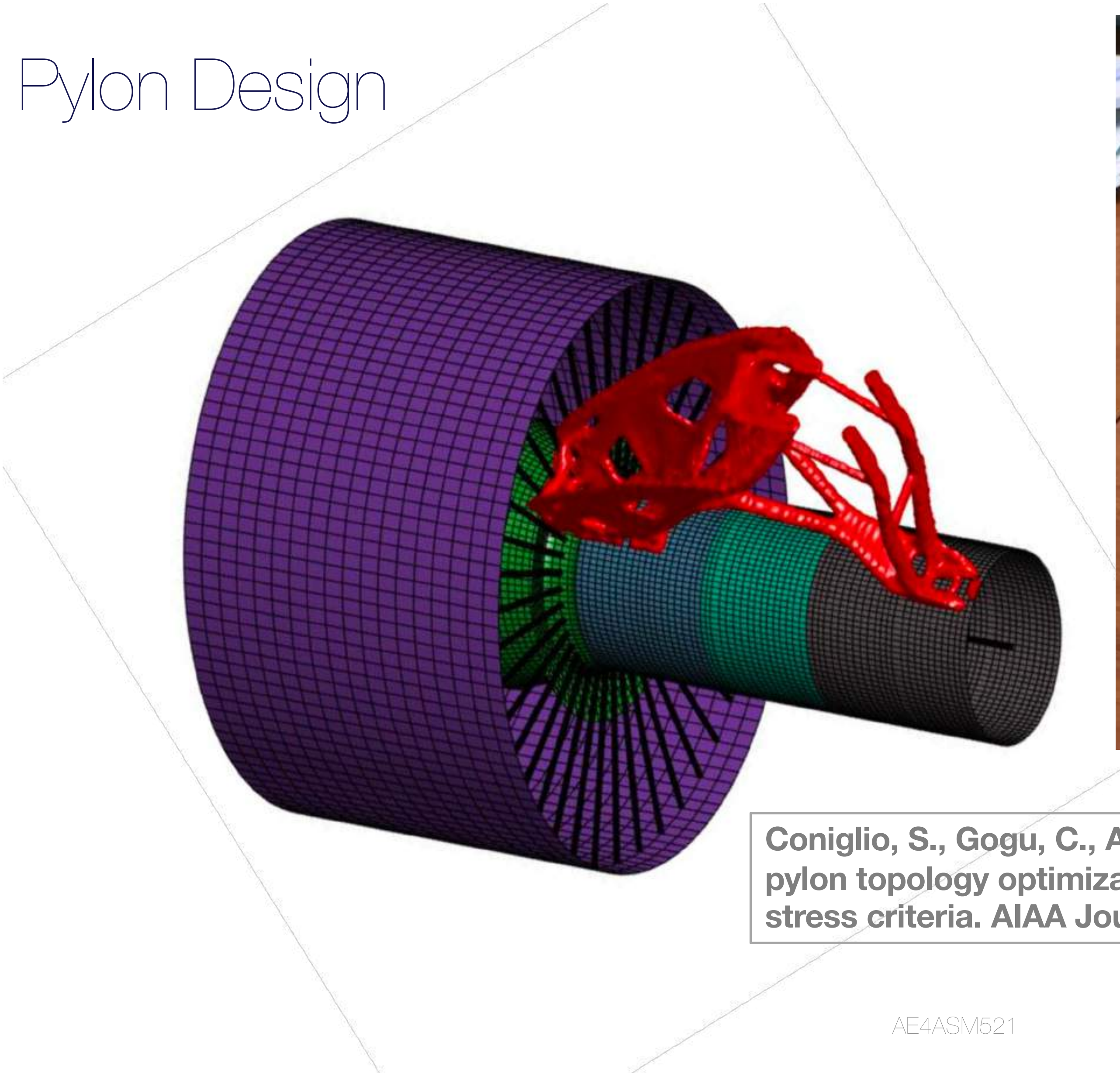
Can you comment this ?

Small changes in BCs



...link to Uncertainty of modeling

Pylon Design



Coniglio, S., Gogu, C., Amargier, R., & Morlier, J. (2019). Engine pylon topology optimization framework based on performance and stress criteria. AIAA Journal, 57(12), 5514-5526.

History (1988, Bendsoe)

A topology optimization problem based on the power-law approach, where the objective is to minimize compliance can be written as

$$\left.\begin{aligned} \min_{\mathbf{x}}: &\quad c(\mathbf{x}) = \mathbf{U}^T\mathbf{K}\mathbf{U} = \sum_{e=1}^{N}(x_e)^p\ \mathbf{u}_e^T\ \mathbf{k}_0\ \mathbf{u}_e \\ \text{subject to}: &\quad \frac{V(\mathbf{x})}{V_0} = f \\ : &\quad \mathbf{K}\mathbf{U} = \mathbf{F} \\ : &\quad \mathbf{0} < \mathbf{x}_{\min} \le \mathbf{x} \le \mathbf{1} \end{aligned}\right\}, \tag{1}$$

where $\mathbf{U}$ and $\mathbf{F}$ are the global displacement and force vectors, respectively, $\mathbf{K}$ is the global stiffness matrix, $\mathbf{u}_e$ and $\mathbf{k}_e$ are the element displacement vector and stiffness matrix, respectively, $\mathbf{x}$ is the vector of design variables, $\mathbf{x}_{\min}$ is a vector of minimum relative densities (non-zero to avoid singularity), N $(= \texttt{nelx} \times \texttt{nely})$ is the number of elements used to discretize the design domain, p is the penalization power (typically $p = 3$), $V(\mathbf{x})$ and V_0 is the material volume and design domain volume, respectively and f (`volfrac`) is the prescribed volume fraction.

Compliance minimization   self adjoint

- Compliance is the opposite of stiffness

$$C = \mathbf{f}^T \mathbf{u} = \mathbf{u}^T K \mathbf{u}$$

- Inexpensive derivatives (use chain rule)

$$\frac{dC}{dx} = 2\mathbf{u}^T K \frac{d\mathbf{u}}{dx} + \mathbf{u}^T \frac{dK}{dx}\mathbf{u}$$

But since $K\mathbf{u} = \mathbf{f}$ if $\mathbf{f}$ does not depend on x

$$K\frac{d\mathbf{u}}{dx} = -\frac{dK}{dx}\mathbf{u}$$

$$\frac{dC}{dx} = -\mathbf{u}^T \frac{dK}{dx}\mathbf{u}$$

Knowing displacements you also know gradients

Density design variables

Need a DEMO ?

- Recall $$\frac{dC}{dx} = -\mathbf{u}^T \frac{dK}{dx}\mathbf{u}$$
- For density variables

$$\frac{dC}{d\rho^e} \propto -\mathbf{u}^T \rho^{p-1} K^e \mathbf{u}$$

- Want to increase density of elements with high strain energy and vice versa
- To minimize compliance for given weight can use an optimality criterion method.

And for other responses?

$$O = f(x, U)$$

$$\frac{\partial O}{\partial x} = \frac{\partial f}{\partial U}^T \frac{\partial U}{\partial x}$$

$$KU = F$$

$$\frac{\partial K}{\partial x} U + K \frac{\partial U}{\partial x} = 0$$

$$\frac{\partial O}{\partial x} = \frac{\partial f}{\partial U}^T \frac{\partial U}{\partial x} = -\frac{\partial f}{\partial U}^T K^{-1} \frac{\partial K}{\partial x} U = -\frac{\partial f}{\partial U}^T \delta$$

$K\lambda = \frac{\partial f}{\partial U}$ Adjoint Method

$K\delta = \frac{\partial K}{\partial x} U$ Direct Method

Either one solution per response
Either one solution per design variables
That's why Compliance!