FEA modeling best practice

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Outline

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- FE classification
    - Geometry classification
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    - DOFs per node
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- FE library
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    - 2D beam
    - 2D shell axisymmetric
    - 2D solid planar stress
    - 2D solid planar strain
    - 2D solid axisymmetric
- Industrial examples

FEM review

Elastostatics equations

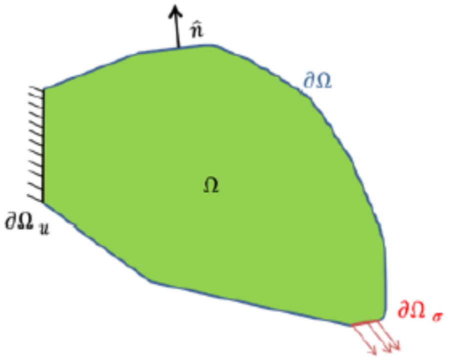


Figure 1.1: linear elastostatics problem definition

Strong formulation

$$
\begin{aligned}
\nabla \cdot [\sigma] + \{b\} &= \{0\} && \forall \{x\} \in \Omega/\partial\Omega \\
\{u\} &= \{\bar{u}\} && \forall \{x\} \in \partial\Omega_u \\
[\sigma] \cdot \{\hat{n}\} &= \{\bar{t}\} && \forall \{x\} \in \partial\Omega_\sigma \\
[\sigma] &= [[E]] : [\varepsilon] && \forall \{x\} \in \Omega
\end{aligned}
$$

Virtual work principle +
Divergence theorem

Weak formulation

$$
\int_{\partial\Omega_\sigma} \{v\}^T \{\bar{t}\} \, d\partial\Omega + \int_\Omega \{v\}^T \{b\} \, d\Omega = \int_\Omega (\nabla_{sym} \{v\})^T E \, [D] \, \nabla_{sym} \{u\} \, d\Omega \quad \forall \{v\} \in \mathbb{V}
$$

FEM review

kinematics

$$\{v\} = [N]^{(h)}\{v\}^{(h)} \qquad \{u\} = [N]^{(h)}\{u\}^{(h)}$$

(a) Reference element in the (ξ, η, ζ) space (b) Corresponding mapped element in the (x, y, z) space

Figure 1.2: Classic Mapping in FEA

$$\{X_g\} = \left\{ \begin{array}{c} x \\ y \\ z \end{array} \right\} = \left[\begin{array}{c} \{x_n\}^T \\ \{y_n\}^T \\ \{z_n\}^T \end{array} \right] \{N_n(\xi, \eta, \zeta)\}$$

Local equilibrium

$$[K]^{(h)}\{u\}^{(h)} = \{f\}^{(h)}$$

$$K_{i,j}^{(h)} = \int_{\Omega} \left([\nabla_{sym} N_i]^{(h)} \right)^T E\,[D]\,[\nabla_{sym} N_j]^{(h)}\,d\Omega$$

$$f_i^{(h)} = \int_{\partial\Omega_\sigma} ([N_i]^{(h)} \cdot \{\bar{t}\})\,d\partial\Omega + \int_{\Omega} ([N_i]^{(h)} \cdot \{b\})\,d\Omega$$

Global equilibrium

$$[K]^{(h)}\{u\}^{(h)} = \{f\}^{(h)}$$

$$K_{i,j}^{(h)} = \sum_{el=1}^{n_{el}} E_{el} \int_{\Omega_{el}} \left([\nabla_{sym} N_i]^{(h)} \right)^T [D]\,[\nabla_{sym} N_j]^{(h)}\,d\Omega_{el}$$

$$f_i^{(h)} = \sum_{el=1}^{n_{el}} \int_{\partial\Omega_\sigma \cap \partial\Omega_{el}} ([N_i]^{(h)} \cdot \{\bar{t}\})\,d\partial\Omega_{el} + \int_{\Omega_{el}} ([N_i]^{(h)} \cdot \{b\})\,d\Omega_{el}$$

Numerical integration

$$K_{i,j}^{(h)} = \sum_{el=1}^{n_{el}} E_{el} \sum_{k=1}^{N_{GP}} \left([\nabla_{sym} N_{ik}]^{(h)} \right)^T [D]\,[\nabla_{sym} N_{jk}]^{(h)}\,||[J_k]||\,\omega_k$$

$$f_i^{(h)} = \sum_{el=1}^{n_{el}} \sum_{l=1}^{n_{GP}} ([N_{il}]^{(h)} \cdot \{\bar{t}_l\})\,||[J_l]||\,\omega_l + \sum_{k=1}^{N_{GP}} ([N_{ik}]^{(h)} \cdot \{b_k\})\,||[J_k]||\,\omega_k$$

FEM review

Solution and post processing

$$[\mathbf{K_{bb}}]\{u_b\} + [\mathbf{K_{bf}}]\{u_f\} = \{R_b\} + \{F_b\}$$
$$[\mathbf{K_{fb}}]\{u_b\} + [\mathbf{K_{ff}}]\{u_f\} = \{R_f\} + \{F_f\}$$

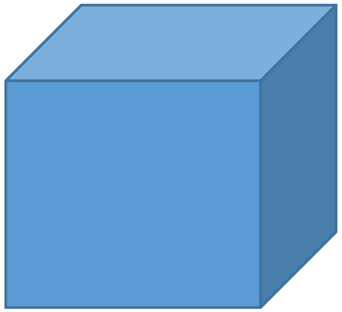
$$\{u_f\} = [\mathbf{K_{ff}}]^{-1}\left(-[\mathbf{K_{fb}}]\{u_b\} + \{F_f\}\right)$$
$$\{R_b\} = [\mathbf{K_{bb}}]\{u_b\} + [\mathbf{K_{bf}}]\{u_f\} - \{F_b\}$$

$$\{\varepsilon\} = [B_{gp}]\{u_{el}\} \quad \{\sigma\} = [D][B_{gp}]\{u_{el}\}$$

FE classification

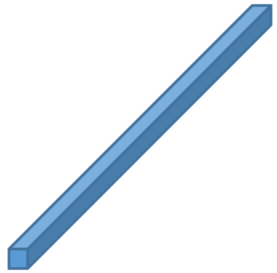
Geometry classification

No main dimension



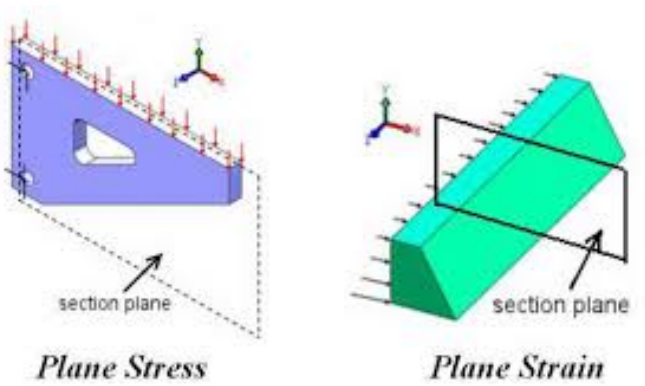
2 main dimensions

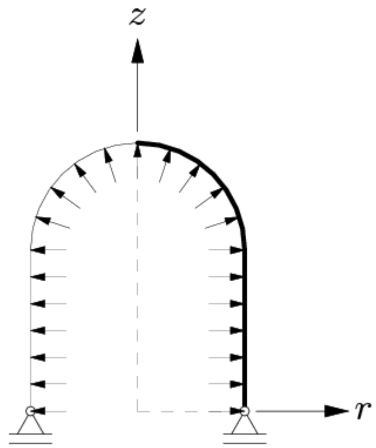
1 main dimensions



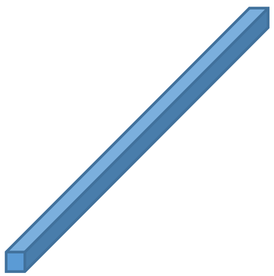
FE classification

Kinematic hypothesis





Axial symmetry

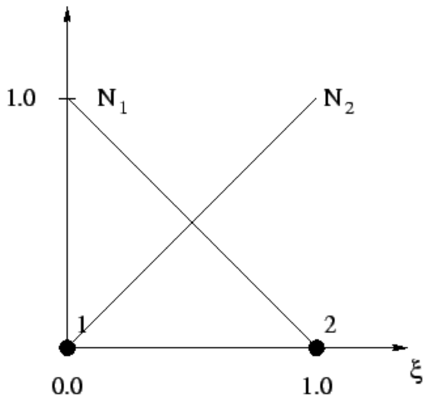


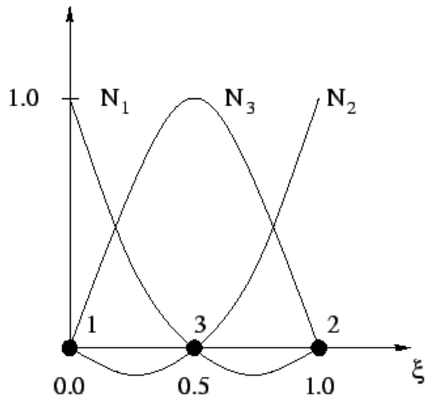
Rigid section

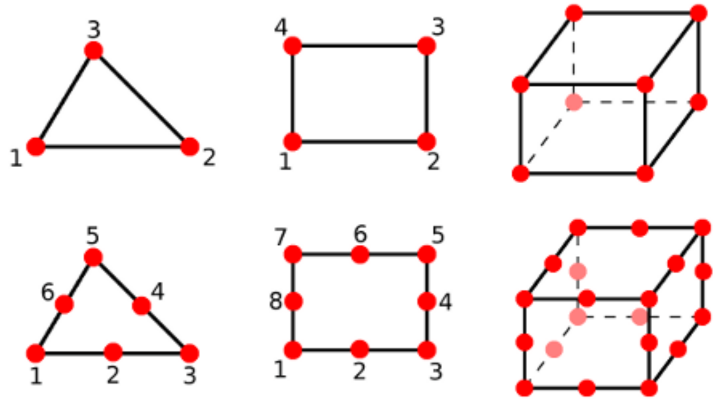
Rigid thickness

FE classification

Shape function polynomial basis







Triangle element with 3 and 6 nodes

Rectangle element with 4 and 8 nodes

Box element (for 3D) with 8 and 20 nodes

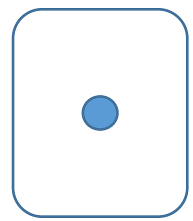
Sample of some simple element shapes and standard node placement. By convention nodes are numbered anti-clockwise.

FE classification

DOFs per node

Solid point:
Material point kinematics
(u, v, w)

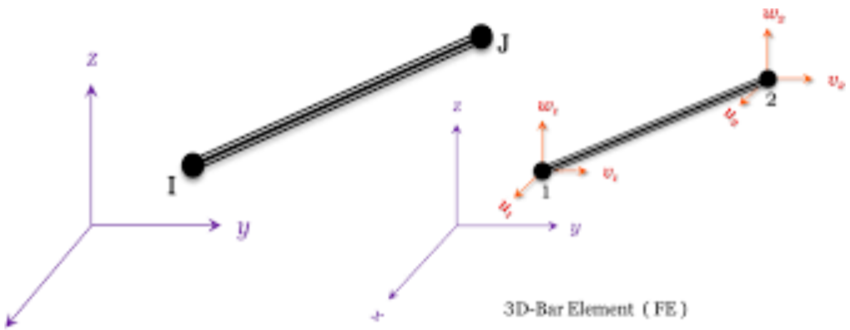
Shell thickness:
Rigid thickness kinematics
$(u, v, w, \phi_x, \phi_y)$



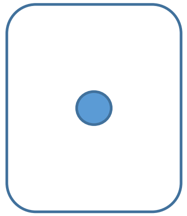
Beam cross section:
Rigid section
$(u, v, w, \phi_x, \phi_y, \phi_z)$

FE library

3D bar element



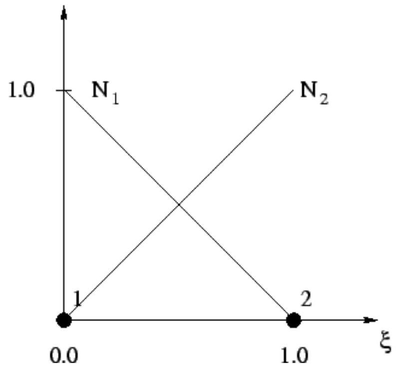
Source: ANSYS (2007)



Rod cross section:
Rigid section
(u, v, w)

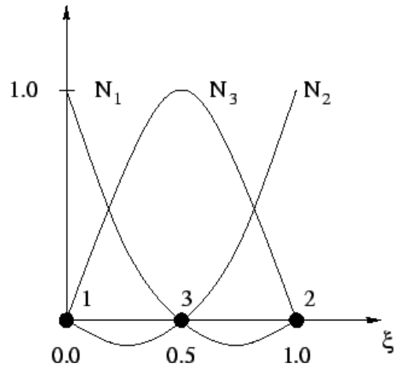
Shape functions

Linear 2 node element



$< 1, \xi >$

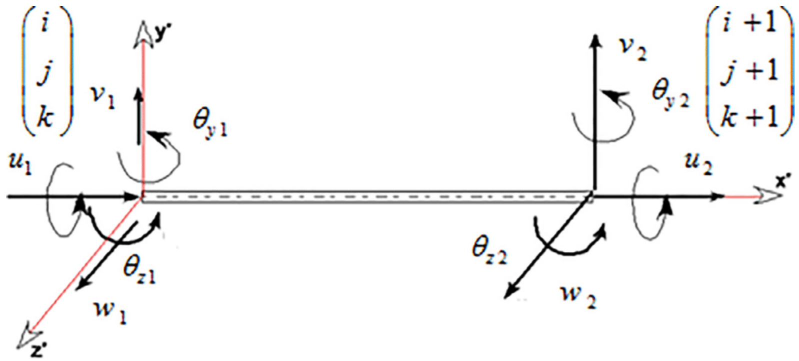
Quadratic 3 node element



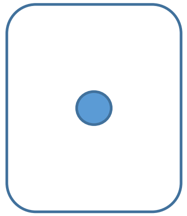
$< 1, \xi, \xi^2 >$

FE library

3D beam element



2 node element



Beam cross section:
Rigid section
$(u, v, w, \phi_x, \phi_y, \phi_z)$

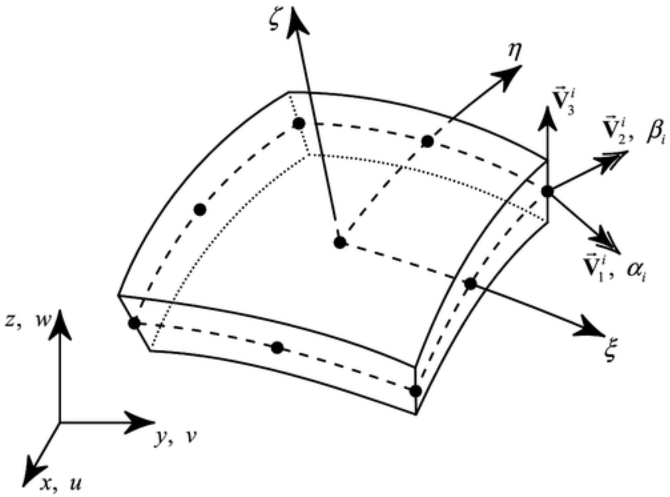
Shape functions

Traction, Torsion $< 1, \xi >$

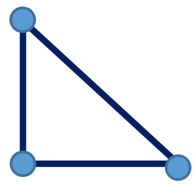
Bending $< 1, \xi, \xi^2, \xi^3 >$

FE library

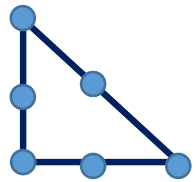
3D shell element



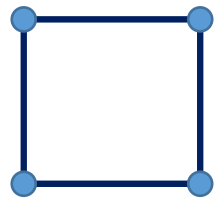
Shell thickness:
Rigid thickness kinematics
$(u, v, w, \phi_x, \phi_y)$



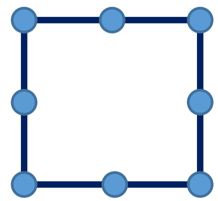
3-node Triangular
$< 1, \xi, \eta >$



6-node Triangular
$< 1, \xi, \eta, \xi^2, \xi\eta, \eta^2 >$



4-node Quadrilateral
$< 1, \xi, \eta, \xi\eta >$



8-node Quadrilateral
$< 1, \xi, \eta, \xi\eta, \xi^2, \eta^2, \xi^2\eta, \xi\eta^2 >$

FE library

3D solid element

Solid point:
Material point kinematics
(u, v, w)

Hexahedral

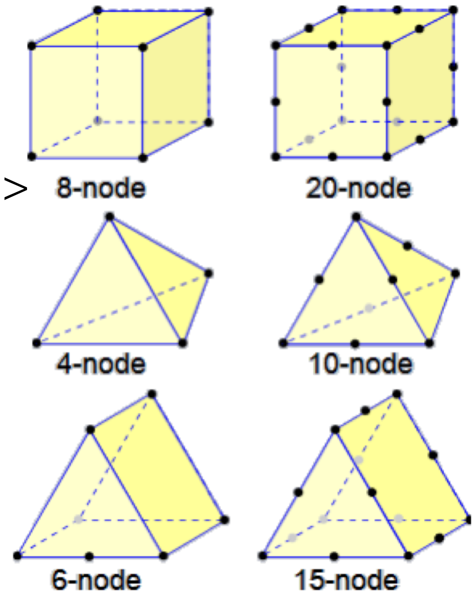
$< 1, \xi, \eta, \zeta, \xi\eta, \xi\zeta, \eta\zeta, \xi\eta\zeta >$

Tetrahedral

$< 1, \xi, \eta, \zeta >$

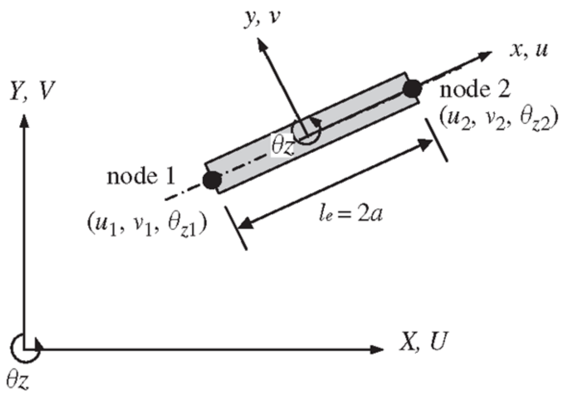
Pentahedral

$< 1, \xi, \eta, \zeta, \xi\zeta, \eta\zeta >$

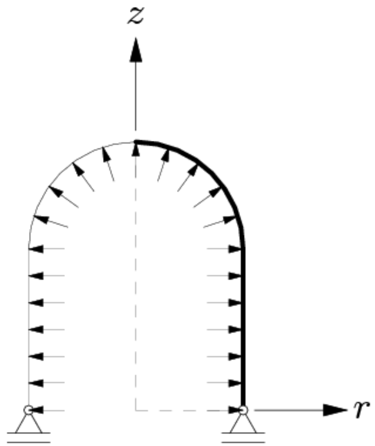


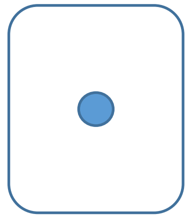
FE library

2D beam element \ 2D shell axisymmetric



2 node element





Beam cross section:
Rigid section
(u, v, ϕ_z)

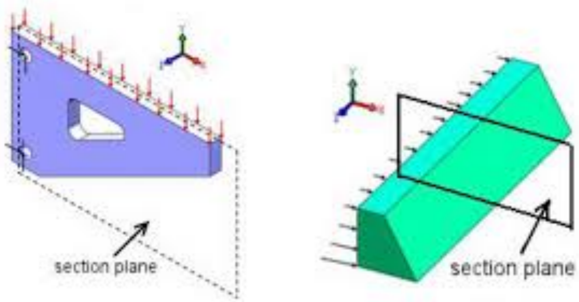
Shape functions

Traction $< 1, \xi >$

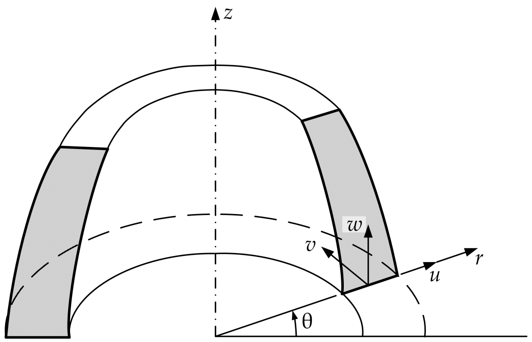
Bending $< 1, \xi, \xi^2, \xi^3 >$

FE library

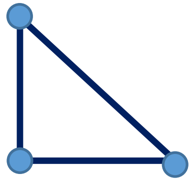
2D solid plane stress\ 2D solid plane strain\ 2D solid axisymmetric



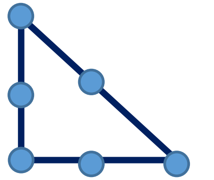
Plane Stress *Plane Strain*



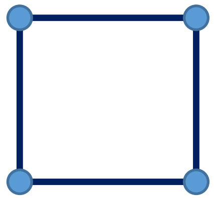
Solid point:
Material point kinematics
(u, v)



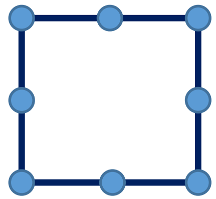
3-node Triangular
$< 1, \xi, \eta >$



6-node Triangular
$< 1, \xi, \eta, \xi^2, \xi\eta, \eta^2 >$



$< 1, \xi, \eta, \xi\eta \ >$



$< 1, \xi, \eta, \xi\eta, \xi^2, \eta^2, \xi^2\eta, \xi\eta^2 >$

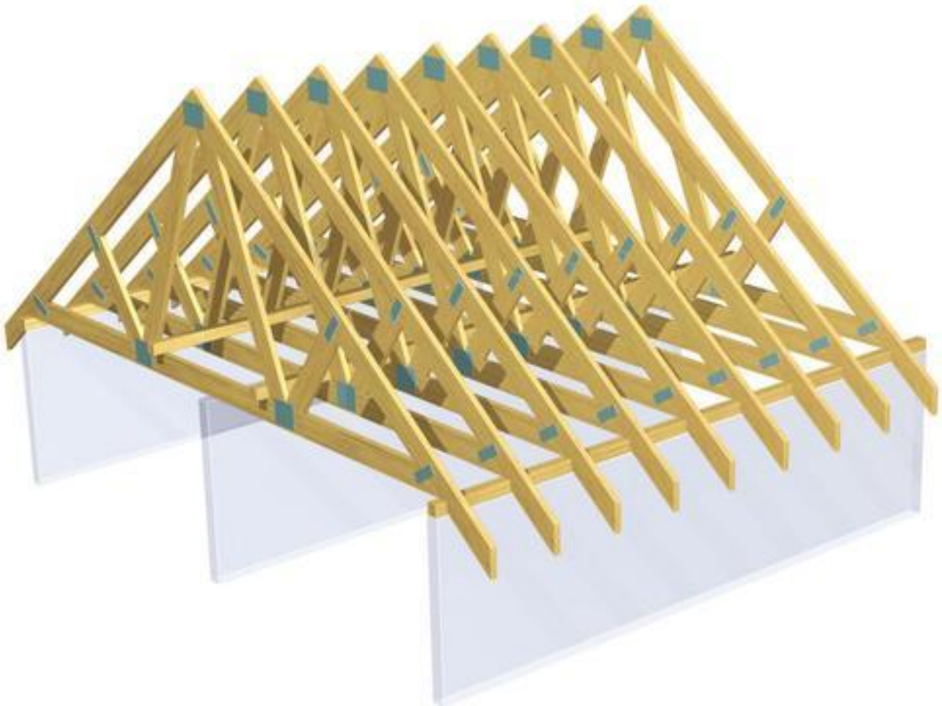
Industrial Examples

Bridge piers
What kind of FE can you use?

Industrial Examples

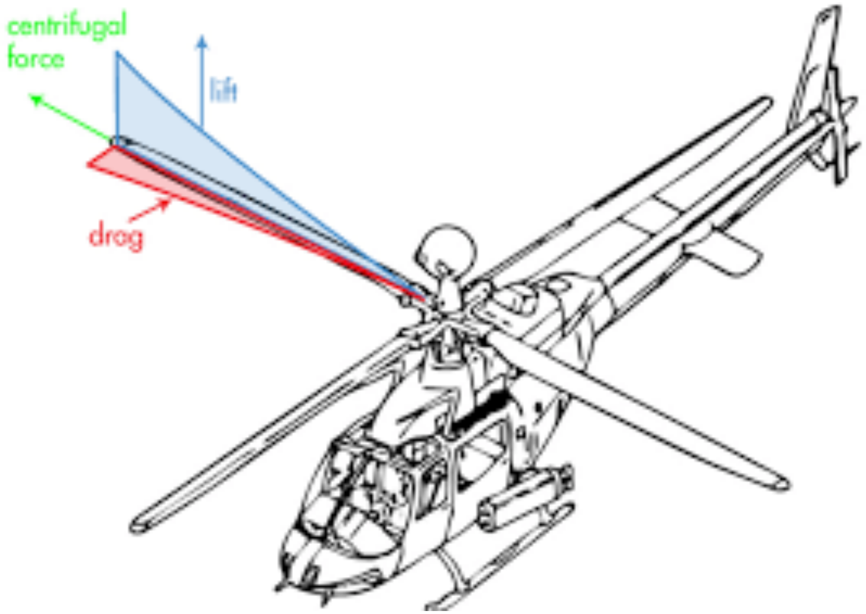
Bridge cables
What kind of FE can you use?

Industrial Examples



Roof frames
What kind of FE can you use?

Industrial Examples



Helicopter blades
What kind of FE can you use?

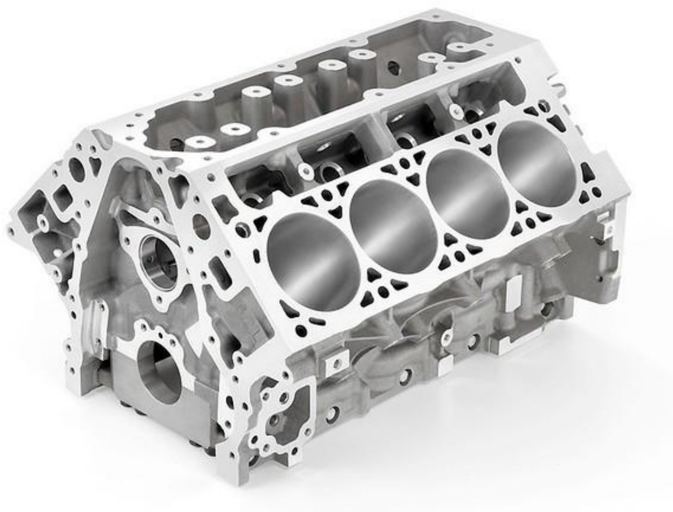
Industrial examples

Wing box
What kind of FE can you use?

Industrial examples

Dam
What kind of FE can you use?

Industrial examples



Engine block
What kind of FE can you use?

Industrial examples

Rocket Fairings
What kind of FE can you use?

Industrial Examples

Water reservoir
What kind of FE can you use?