Chapter 4
Analysis of 2D trusses

4.1 Introduction

This chapter deals with the static analysis of two dimensional trusses, which are basically bars oriented in two dimensional cartesian systems. A transformation of coordinate basis is necessary to translate the local element matrices (stiffness matrix, force vector) into the structural (global) coordinate system. Trusses support compressive and tensile forces only, as in bars. All forces are applied at the nodes. After the presentation of the element formulation, some examples are solved by MATLAB codes.

4.2 2D trusses

In figure 4.1 we consider a typical 2D truss in global $x - y$ plane. The local system of coordinates $x' - y'$ defines the local displacements u'_1, u'_2. The element possesses two degrees of freedom in the local setting,

$$\mathbf{u'}^T = [u'_1 \quad u'_2] \tag{4.1}$$

while in the global coordinate system, the element is defined by four degrees of freedom

$$\mathbf{u}^T = [u_1 \quad u_2 \quad u_3 \quad u_4] \tag{4.2}$$

The relation between both local and global displacements is given by

$$u'_1 = u_1 cos(\theta) + u_2 sin(\theta) \tag{4.3}$$

$$u'_2 = u_3 cos(\theta) + u_4 sin(\theta) \tag{4.4}$$

A.J.M. Ferreira, *MATLAB Codes for Finite Element Analysis: Solids and Structures,* Solid Mechanics and Its Applications 157,

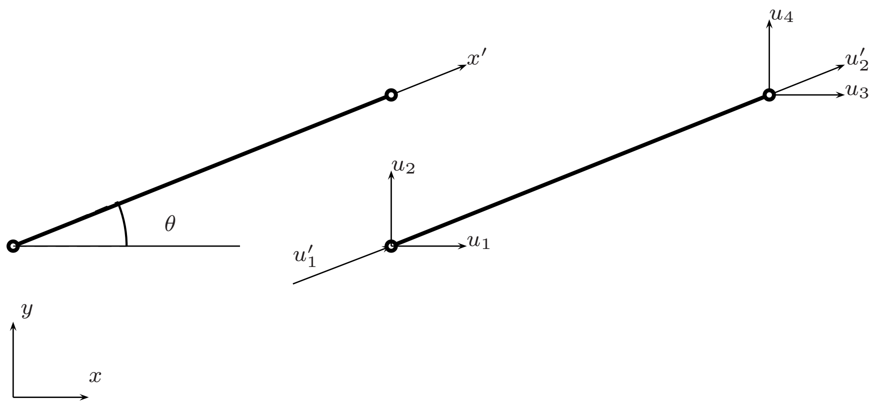

Fig. 4.1 2D truss element: local and global degrees of freedom

where θ is the angle between local axis x' and global axis x, or in matrix form as

$$\mathbf{u}' = \mathbf{L}\mathbf{u} \tag{4.5}$$

being matrix $\mathbf{L}$ defined as

$$\mathbf{L} = \begin{bmatrix} l & m & 0 & 0 \\ 0 & 0 & l & m \end{bmatrix} \tag{4.6}$$

The l, m elements of matrix L can be defined by the nodal coordinates as

$$l = \frac{x_2 - x_1}{L_e}; \quad m = \frac{y_2 - y_1}{L_e} \tag{4.7}$$

being L_e the length of the element,

$$L_e = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \tag{4.8}$$

4.3 Stiffness matrix

In the local coordinate system, the stiffness matrix of the 2D truss element is given by the bar stiffness, as before:

$$\mathbf{K}' = \frac{EA}{L_e} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \tag{4.9}$$

In the local coordinate system, the strain energy of this element is given by

$$U_e = \frac{1}{2}\mathbf{u}'^T\mathbf{K}'\mathbf{u}' \tag{4.10}$$

Replacing $\mathbf{u}' = \mathbf{L}\mathbf{u}$ in (4.10) we obtain

$$U_e = \frac{1}{2}\mathbf{u}^T[\mathbf{L}^T\mathbf{K}'\mathbf{L}]\mathbf{u} \tag{4.11}$$

It is now possible to express the global stiffness matrix as

$$\mathbf{K} = \mathbf{L}^T\mathbf{K}'\mathbf{L} \tag{4.12}$$

or

$$\mathbf{K} = \frac{EA}{L_e}\begin{bmatrix} l^2 & lm & -l^2 & -lm \\ lm & m^2 & -lm & -m^2 \\ -l^2 & -lm & l^2 & lm \\ -lm & -m^2 & lm & m^2 \end{bmatrix} \tag{4.13}$$

4.4 Stresses at the element

In the local coordinate system, the stresses are defined as $\sigma = E\epsilon$. Taking into account the definition of strain in the bar, we obtain

$$\sigma = E\frac{u_2' - u_1'}{L_e} = \frac{E}{L_e}[-1 \quad 1]\begin{Bmatrix} u_1' \\ u_2' \end{Bmatrix} = \frac{E}{L_e}[-1 \quad 1]\mathbf{u}' \tag{4.14}$$

By transformation of local to global coordinates, we obtain stresses as function of the displacements as

$$\sigma = \frac{E}{L_e}[-1 \quad 1]\mathbf{L}\mathbf{u} = \frac{E}{L_e}[-l \quad -m \quad l \quad m]\mathbf{u} \tag{4.15}$$

4.5 First 2D truss problem

In a first 2D truss problem, illustrated in figure 4.2, we consider a downward point force (10,000) applied at node 1. The modulus of elasticity is $E = 30e6$ and all elements are supposed to have constant cross-section area $A = 2$. The supports are located in nodes 2 and 4. The numbering of degrees of freedom is illustrated in figure 4.3.