Chapter 7
2D frames

7.1 Introduction

In this chapter bidimensional frames under static loading are analysed. In figure 7.1, we show the 2D frame two-noded element. Each node has three global degrees of freedom, two displacements in global axes and one rotation.

The vector of global displacements is given by

$$\mathbf{u}^T = [u_1, \quad u_2, \quad u_3, \quad u_4, \quad u_5, \quad u_6] \tag{7.1}$$

We define a local basis with cosines l, m, with respect to θ, the angle between x' and x. In this local coordinate set the displacements are detailed as

$$\mathbf{u'}^T = [u'_1, \quad u'_2, \quad u'_3, \quad u'_4, \quad u'_5, \quad u'_6] \tag{7.2}$$

Noting that $u'_3 = u_3, u'_6 = u_6$, we derive a local-global transformation matrix in the form

$$\mathbf{u}' = \mathbf{L}\mathbf{u} \tag{7.3}$$

where

$$\mathbf{L} = \begin{bmatrix} l & m & 0 & 0 & 0 & 0 \\ -m & l & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & l & m & 0 \\ 0 & 0 & 0 & -m & l & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{7.4}$$

In local coordinates, the stiffness matrix of the frame element is obtained by combination of the stiffness of the bar element and the Bernoulli beam element, in the form

A.J.M. Ferreira, *MATLAB Codes for Finite Element Analysis: Solids and Structures,* Solid Mechanics and Its Applications 157,

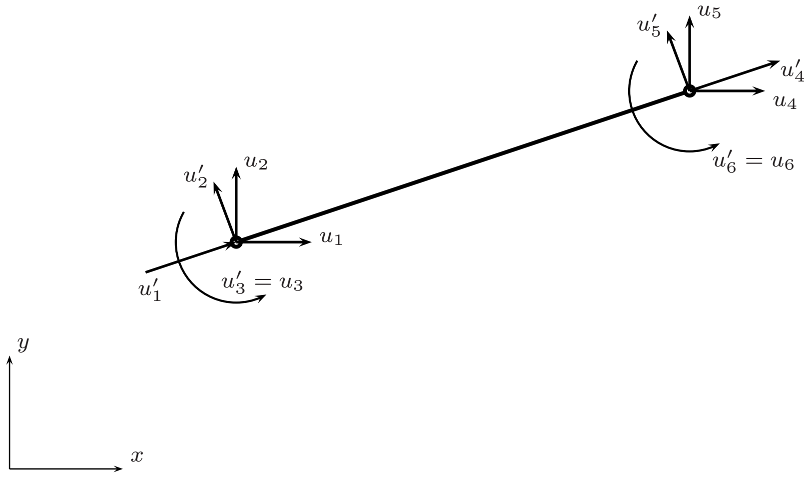

Fig. 7.1 A 2D frame element

$$\mathbf{K}'^e = \begin{bmatrix} \frac{EA}{L} & 0 & 0 & -\frac{EA}{L} & 0 & 0 \\ & \frac{12EI}{L^3} & \frac{6EI}{L^2} & 0 & \frac{12EI}{L^3} & \frac{6EI}{L^2} \\ & & \frac{4EI}{L} & 0 & -\frac{6EI}{L^2} & \frac{2EI}{L} \\ & & & \frac{EA}{L} & 0 & 0 \\ & & & & \frac{12EI}{L^3} & -\frac{6EI}{L^2} \\ sim. & & & & & \frac{4EI}{L} \end{bmatrix} \tag{7.5}$$

In global coordinates, the strain energy is given by

$$U_e = \frac{1}{2}\mathbf{u}'^T\mathbf{K}'\mathbf{u}' = \frac{1}{2}\mathbf{u}^T\mathbf{L}^T\mathbf{K}'\mathbf{L}\mathbf{u} = \mathbf{u}^T\mathbf{K}\mathbf{u} \tag{7.6}$$

where

$$\mathbf{K} = \mathbf{L}^T\mathbf{K}'\mathbf{L} \tag{7.7}$$