Pressure vessel optimization with SymPy, SciPy

joseph.morlier
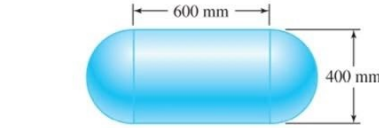
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1   Problem Setup

One of the most famous Aerostructures is here under study and depicted in the next figure. The vessel consists of:

- Cylindrical Section (Length L, Diameter D)

- Two Hemispherical End Caps (each with radius $D/2$)

- A bound constraint for the ratio L/D constrained in the range $[0.1, 10]$

Prob 8.12) The cylindrical pressure vessel with hemispherical end-caps is made of steel. The vessel has a uniform thickness of 18 mm and an outer diameter of 400 mm. When the vessel is pressurized to 3.6 MPa, determine the change in the overall length of the vessel. Use E=200 GPa and v=0.3 for steel. Neglect localized bending.

$$\varepsilon_l = \frac{1}{E}(\sigma_l - v\sigma_c)$$

$$\varepsilon_l = \frac{1}{200000}(18.20 - 0.3 * 36.40)$$

$$\varepsilon_l = 3.64 \times 10^{-5}$$

$$\varepsilon_D = \frac{1}{E}(\sigma - v\sigma_l) \qquad \sigma = \sigma_l \ (for\ spherical\ vessel)$$

$$\varepsilon_D = \frac{18.20}{200000}(1 - 0.3)$$

$$\Delta L = \Delta L_1 + \Delta D$$

$$\sigma_c = \frac{pD_i}{2t}$$

$$D_i = 400 - 2(18)$$

$$D_i = 364\ mm$$

$$\sigma_c = \frac{3.6(364)}{2(18)}$$

$$\sigma_c = 36.40\ MPa$$

$$\sigma_l = \left(\frac{1}{2}\right)\sigma_c = \frac{36.40}{2}$$

$$\sigma_l = 18.20\ MPa$$

THIN-WALLED PRESSURE VESSELS

CIRCUMFERENTIAL STRESS
$$\sigma_t = \frac{\rho D}{2t}$$

LONGITUDINAL STRESS
$$\sigma_l = \frac{\rho D}{4t}$$
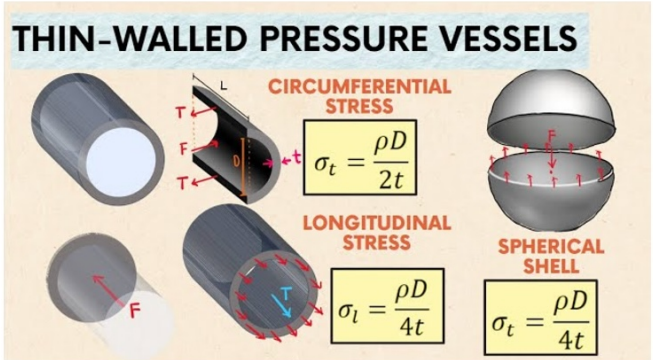
SPHERICAL SHELL
$$\sigma_t = \frac{\rho D}{4t}$$

Figure 1: In this cylindrical pressure vessel, L is 600 mm, and D= 400 mm

We need to minimize the mass M of the vessel, given:

- Internal pressure p,

- Density ρ,

- Yield strength σ_y,

- the two Design Variables (DV or Unknowns) which are cylinder diameter D and length L that describes the internal volume V

2 Theoretical solution

2.1 Expressions for Volume

The internal volume is given by:

$$V = \frac{\pi}{4} D^2 L + \frac{2}{3} \pi \left(\frac{D}{2} \right)^3$$

$$V = \frac{\pi}{4} D^2 L + \frac{\pi}{6} D^3$$

2.2 Expressions for thicknesses

From thin-wall pressure vessel theory, the required thicknesses are:

- Cylindrical section (Hoop stress governs):

$$t_c = \frac{pD}{2\sigma_y}$$

- Hemispherical caps (Membrane stress governs):

$$t_h = \frac{pD}{4\sigma_y}$$

2.3 Mass Expression

The total mass consists of the mass of the cylindrical shell and the two hemispherical end caps. First the Mass of the cylindrical shell is given by:

$$m_c = \rho \cdot Volume = \rho \left(\pi D L t_c \right)$$

Substituting $t_c = \frac{pD}{2\sigma_s}$ it gives:

$$m_c = \rho \pi D L \frac{pD}{2\sigma_y} = \frac{\rho \pi p D^2 L}{2\sigma_y}$$

The Mass of the two hemispherical end caps are given hereafter:

$$m_h = \rho \cdot Volume = \rho \left(2 \cdot 2\pi \left(\frac{D}{2} \right)^2 t_h \right)$$

Substituting $t_h = \frac{pD}{4\sigma_y}$ it leads to:

$$m_h = \rho \cdot 2 \cdot 2\pi \frac{D^2 pD}{4} \frac{p\sigma_y}{4} = \frac{\rho\pi pD^3}{4\sigma_y}$$

Thus, the total mass is:

$$m_{total} = M = m_c + m_h = \frac{\rho\pi pD^2 L}{2\sigma_y} + \frac{\rho\pi pD^3}{4\sigma_y}$$

- Step 1: Express the Dimensionless Mass M^*

  We derived earlier that the total mass of the vessel is:

  $$M = \frac{\rho\pi pD^2 L}{2\sigma_y} + \frac{\rho\pi pD^3}{4\sigma_y}$$

  Dividing by the characteristic mass $V\rho p/\sigma_y$, the dimensionless mass is:

  $$M^* = \frac{M}{V\rho p/\sigma_y}$$

  Using the volume equation:

  $$V = \frac{\pi}{4}D^2 L + \frac{\pi}{6}D^3$$

  we express M^* in terms of $L/D = \lambda$.

- Step 2: Differentiate M w.r.t. λ^*

  We take:

  $$\frac{dM^*}{d\lambda} = 0$$

  to find the critical points.

- Step 3: Solve for λ_{opt} From previous numerical results, we expect L/D very small (i.e. a sphere) to be the optimal shape. This makes sense because: - A sphere minimizes surface area for a given volume. - A spherical pressure vessel distributes stress evenly, reducing material requirements.

3   Possible extension

- Use sciPy minimize with more constraints and real properties from the figure 1

- Add a reserve factor on yield stress.

- Material selection for mass and CO2 minimization

4   Simply with symPy/sciPy

Please check the notebook to start!!