Conception optimale pour l’ingénieur (Aerospace)

C5 by Prof. J. Morlier

2025

Institut Supérieur de l’Aéronautique et de l’Espace
SUPAERO

AU PROGRAMME

Python based

lundi 31 mars 2025			
		09h15 - 12h45	MORLIER Joseph
		14h00 - 16h15	MORLIER Joseph
mardi 01 avril 2025			
		09h15 - 12h45	MORLIER Joseph
		14h00 - 16h15	MORLIER Joseph
mercredi 02 avril 2025			
		09h15 - 12h45	MORLIER Joseph MURADÁS ODRIOZOLA Daniel
		14h00 - 16h15	MAS COLOMER JOAN MURADÁS ODRIOZOLA Daniel
jeudi 03 avril 2025			
		09h15 - 12h45	MAS COLOMER JOAN MURADÁS ODRIOZOLA Daniel

Intro: Sustainable Aviation (Materials) With Both Eyes Open
Design optimization 1: constrained optimization, MOO, Sensibility with examples
Project DO 1 2 3

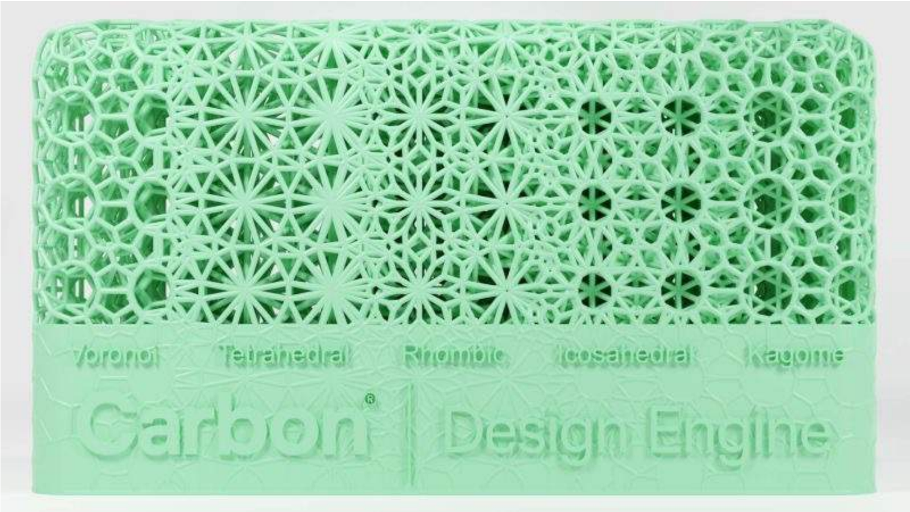
Topology Optimization with examples
Material ecoselection, Ashby Diagram and more

Projet DO 1 2 3
Wrap up and demo from students

Intro to MDAO **Static Aeroelastic problem is a MDAO problem**
Airbus PROJECT by TEAM of 3 (marked*)

vendredi 04 avril 2025		**ORAL MARKED***	
		09h15 - 11h30	MORLIER Joseph MURADÁS ODRIOZOLA Daniel

Eco informed Material Selection?

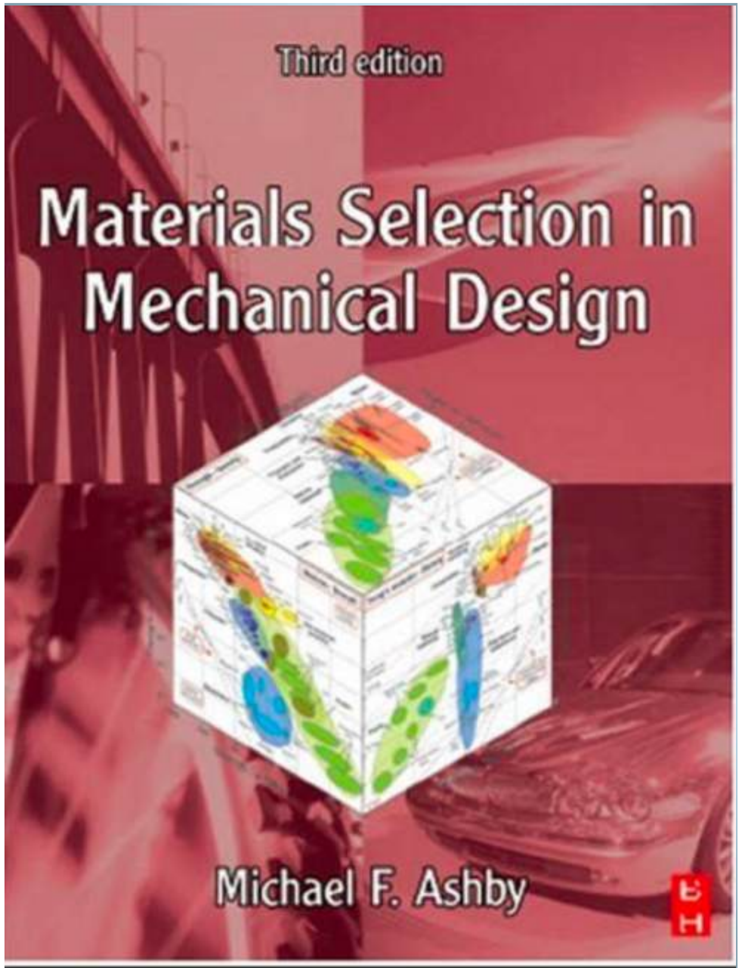


Au programme (14H-17H15)

- 1/ Eco Material selection course
- 2/ Exercices using Ahsby diagram (pen&paper)

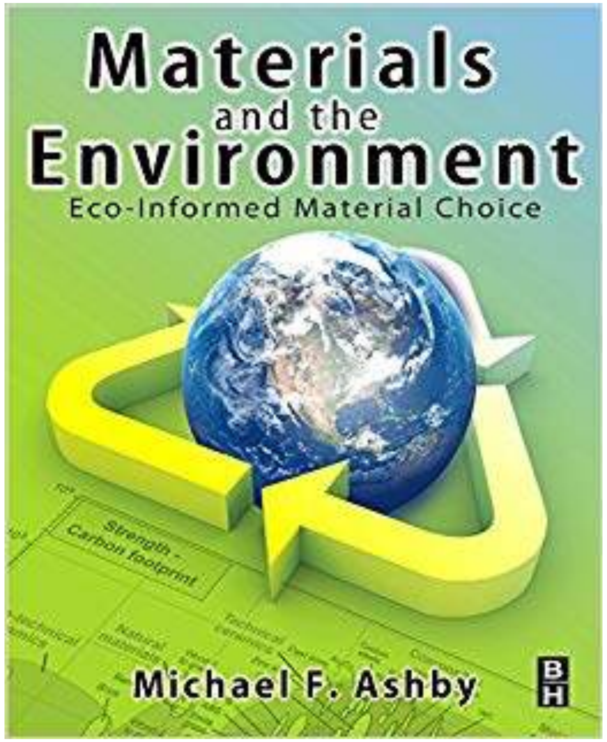
Learning outcomes

- 1/ Discovering Material profiles
- 2/ Use Ashby's diagram
- 3/ Select material (mono& multiobjective)
- 4/ Discovering Eco Properties

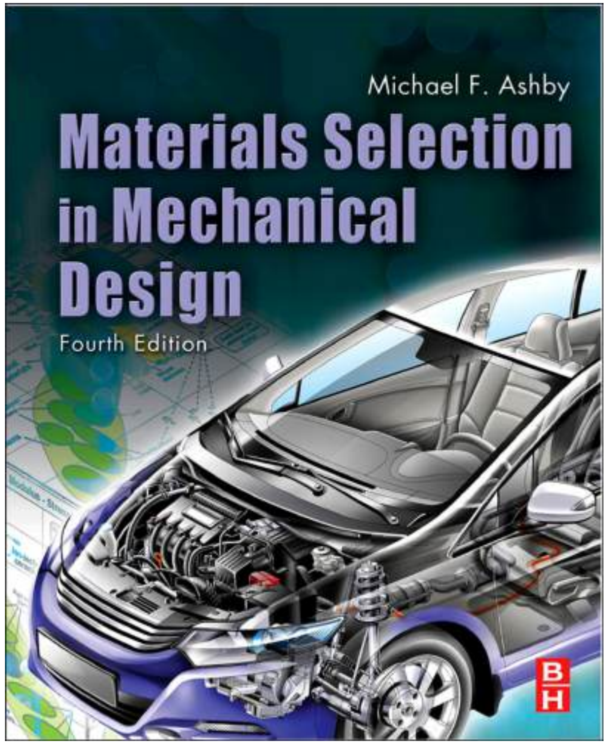


Prof. Ashby

To Start



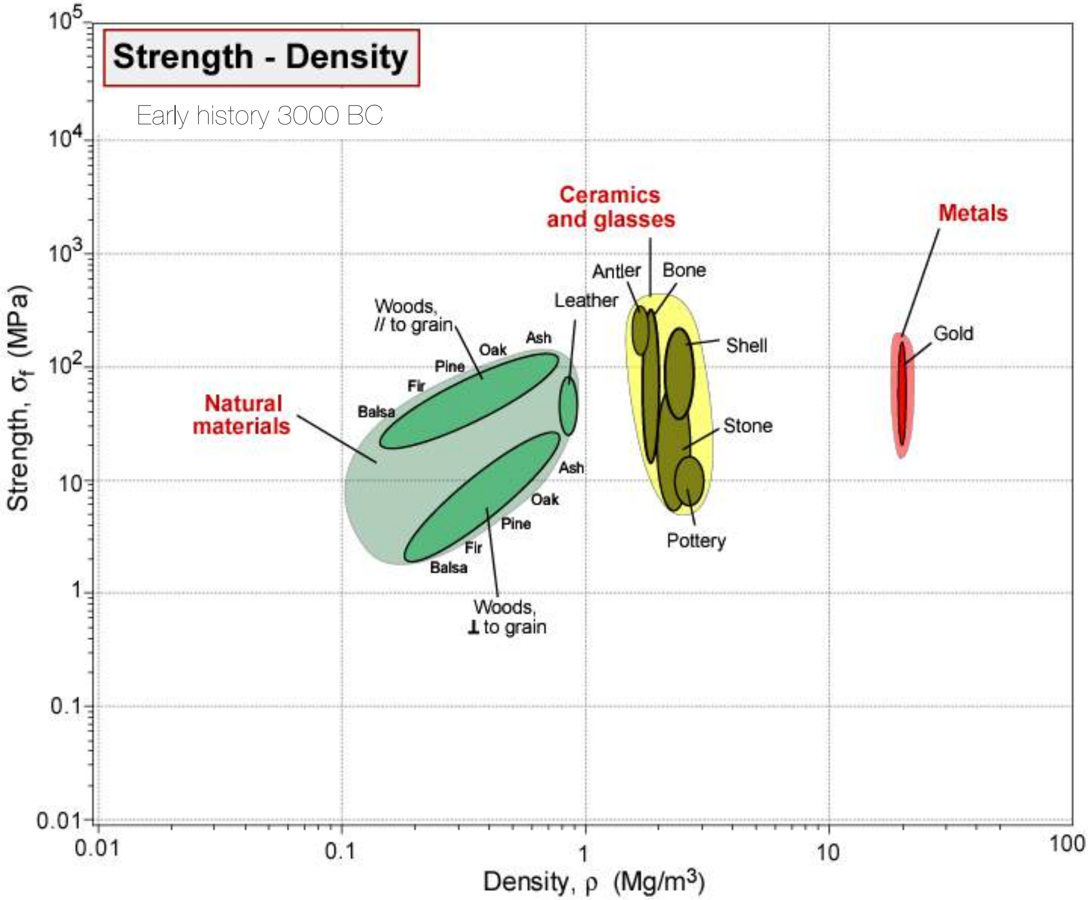
Materials
and the
Environment
Eco-Informed Material Choice
Michael F. Ashby



Michael F. Ashby
Materials Selection in Mechanical Design
Fourth Edition

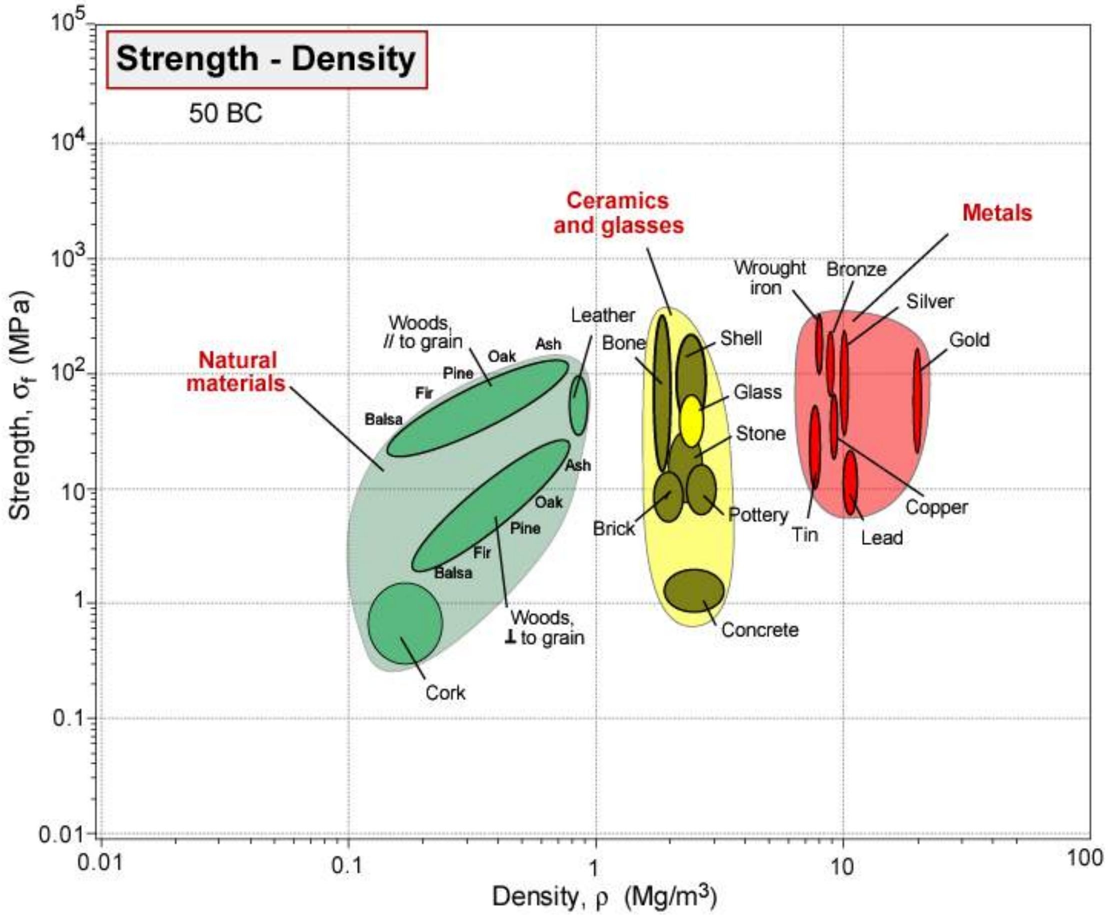
The evolution of structural materials

from Mike Ashby, 2018



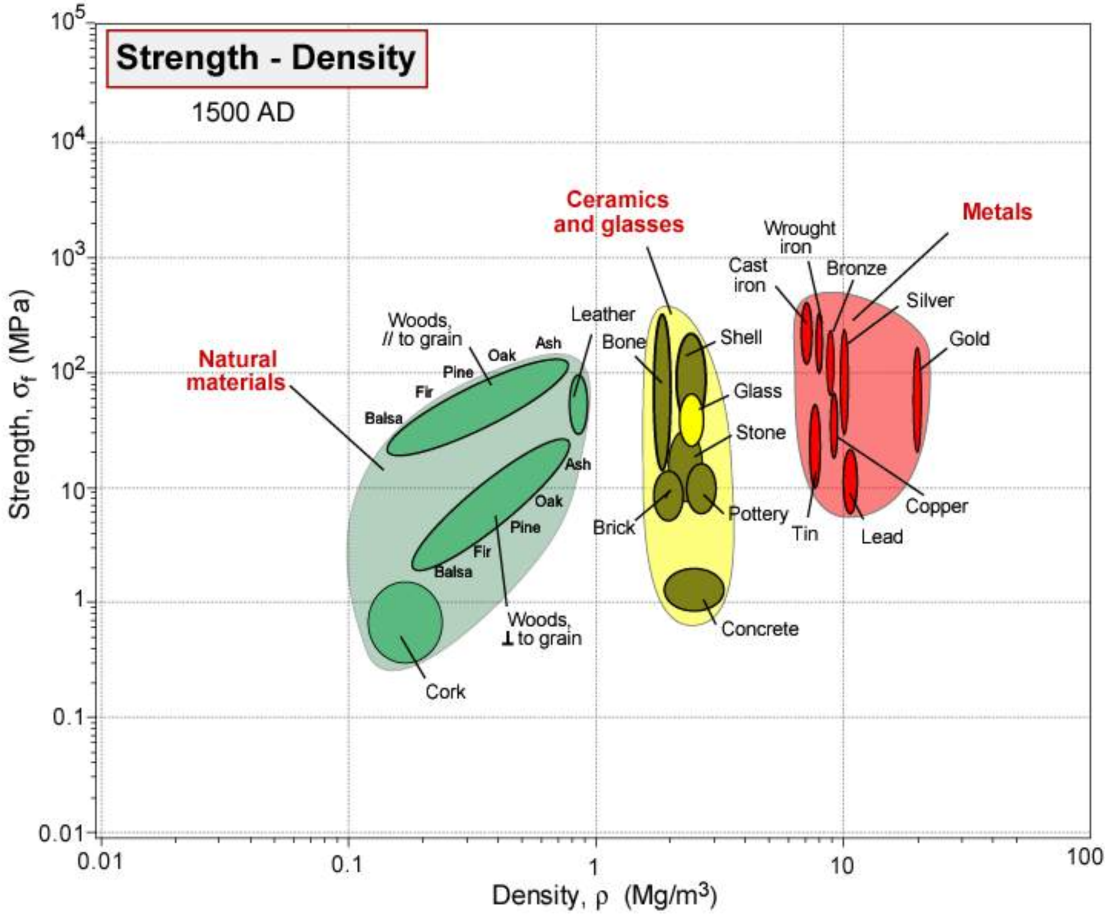
Egyptian Pyramids

50BC



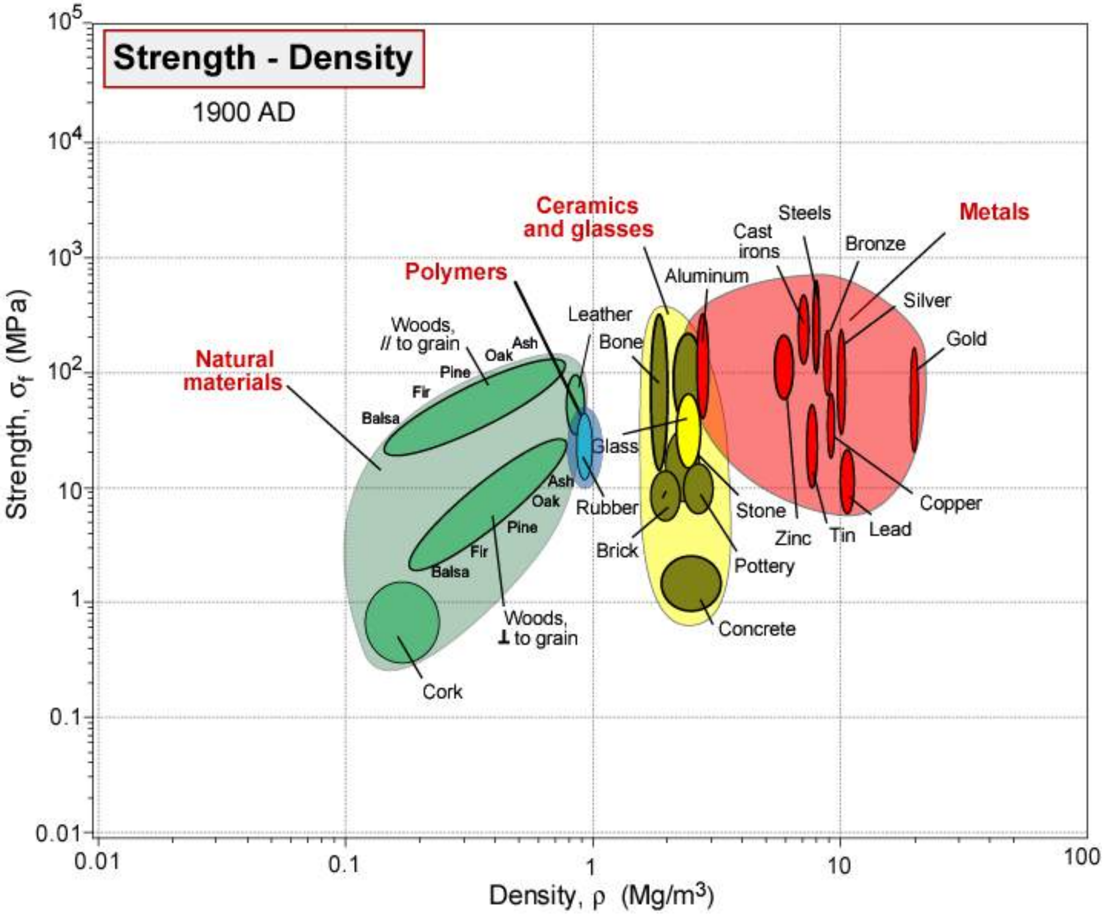
Roman Temples

1500 AD

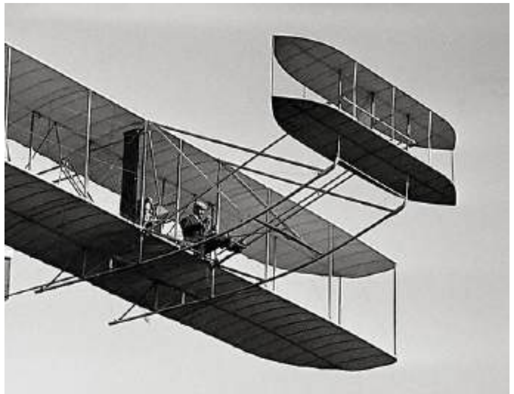


Medieval Castles

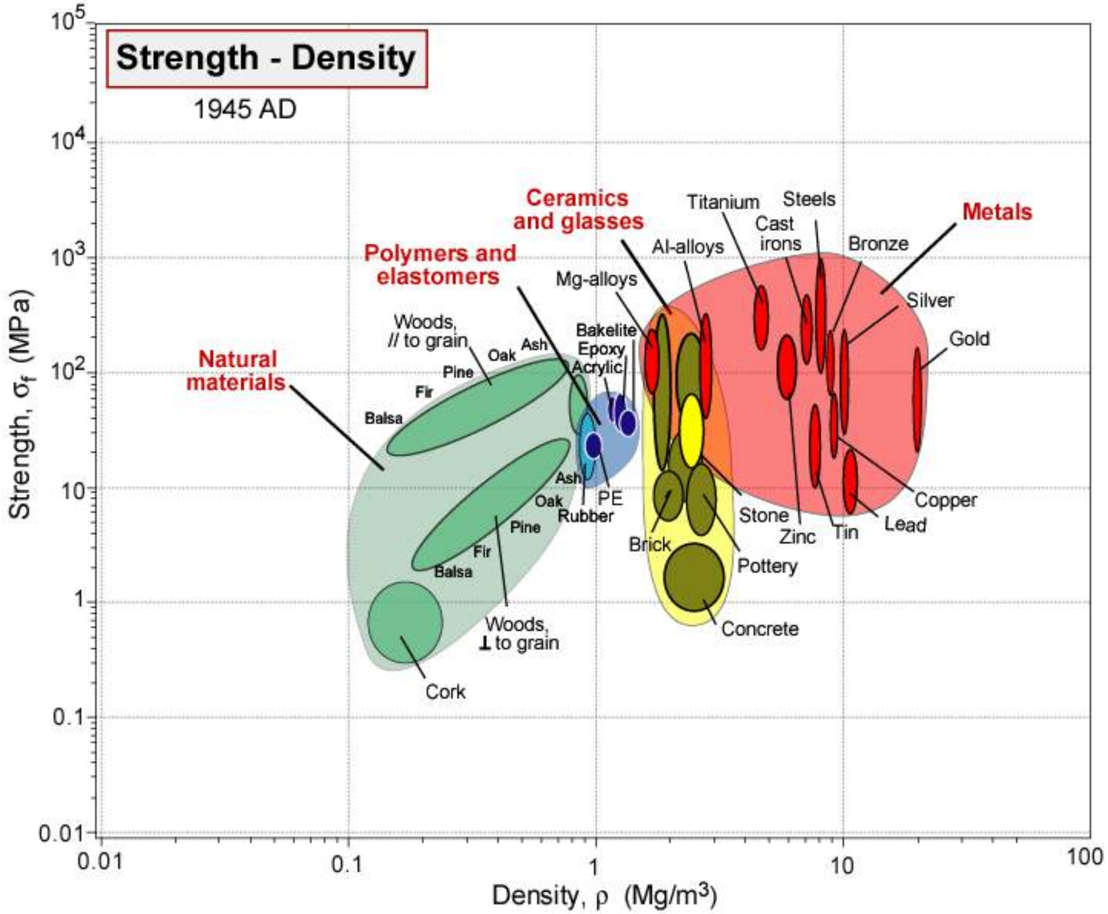
1900 AD



Art Nouveaux

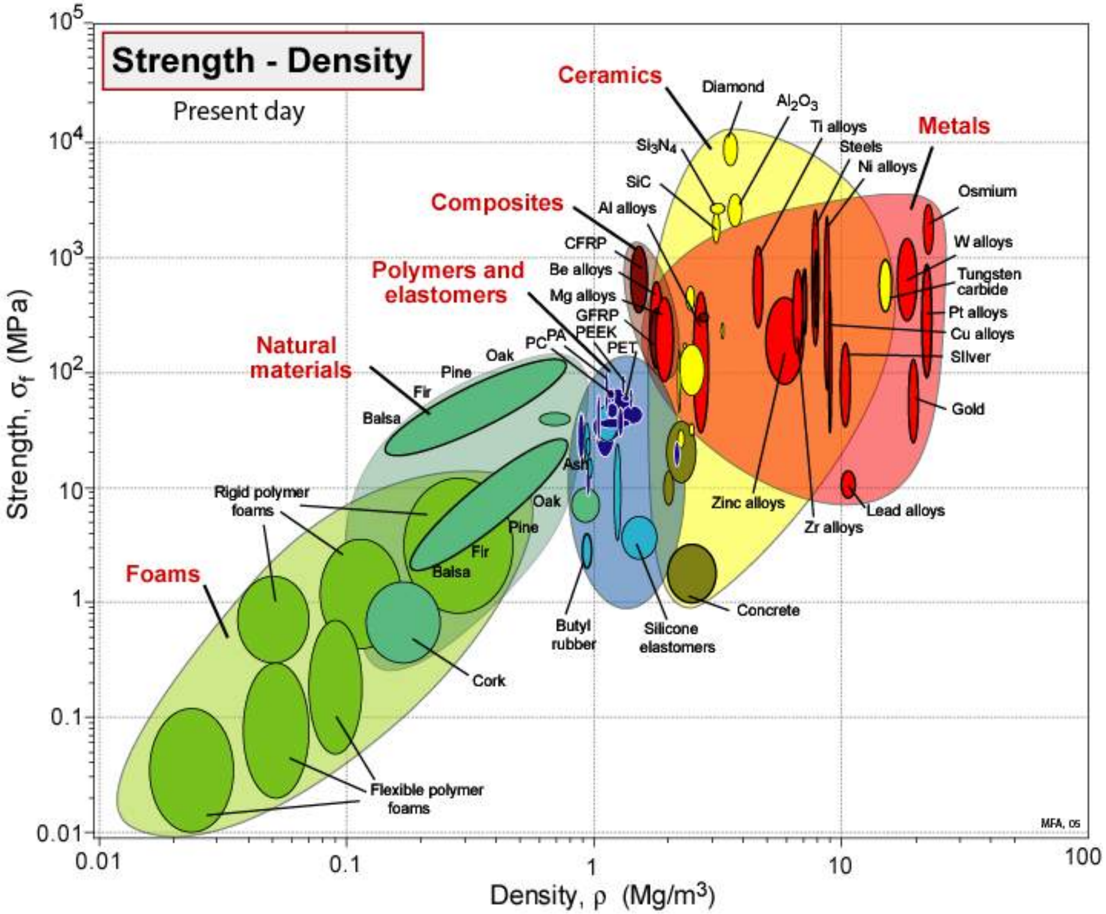


1945 AD



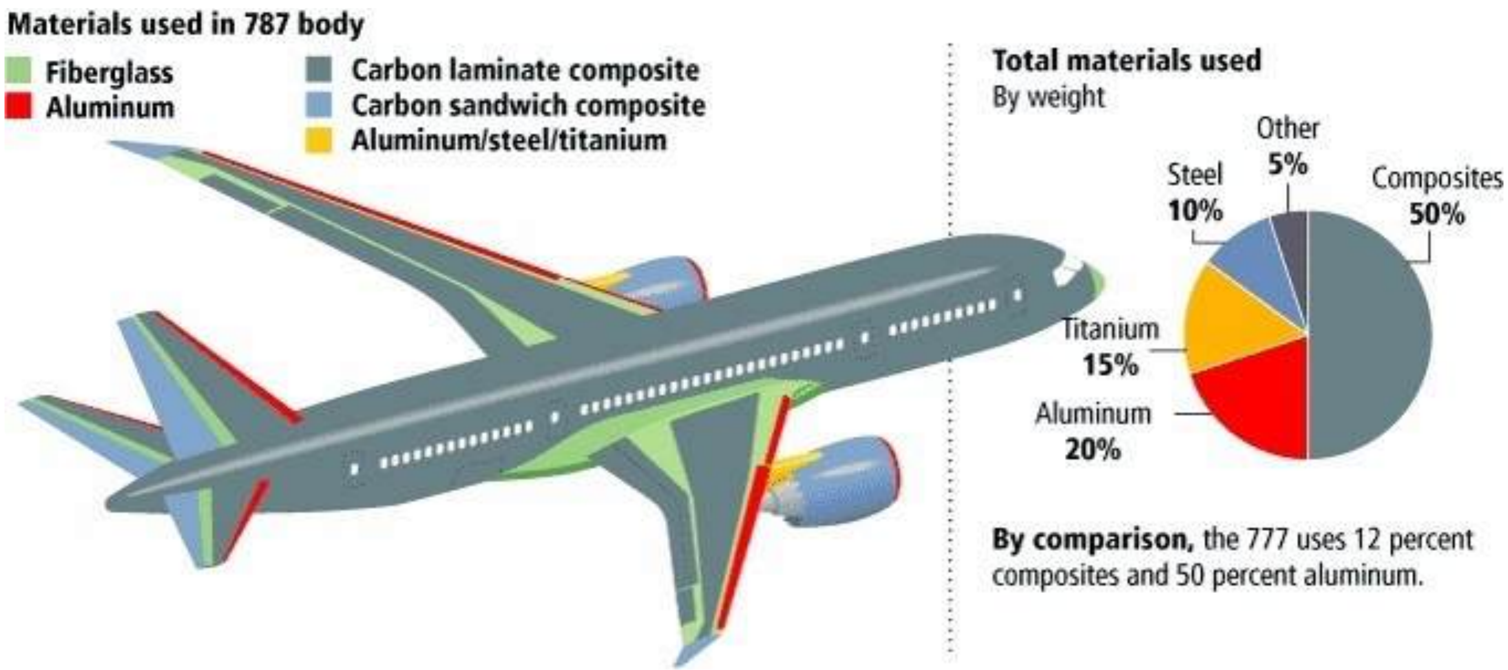
Skyscrapers

PRESENT DAY

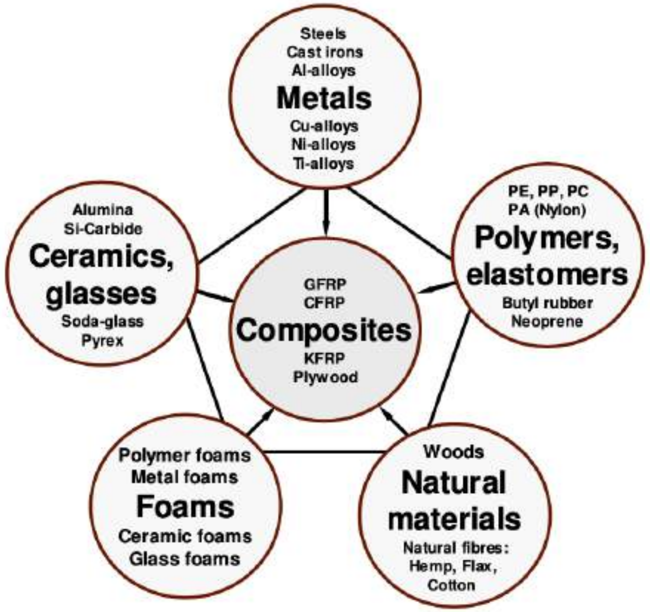


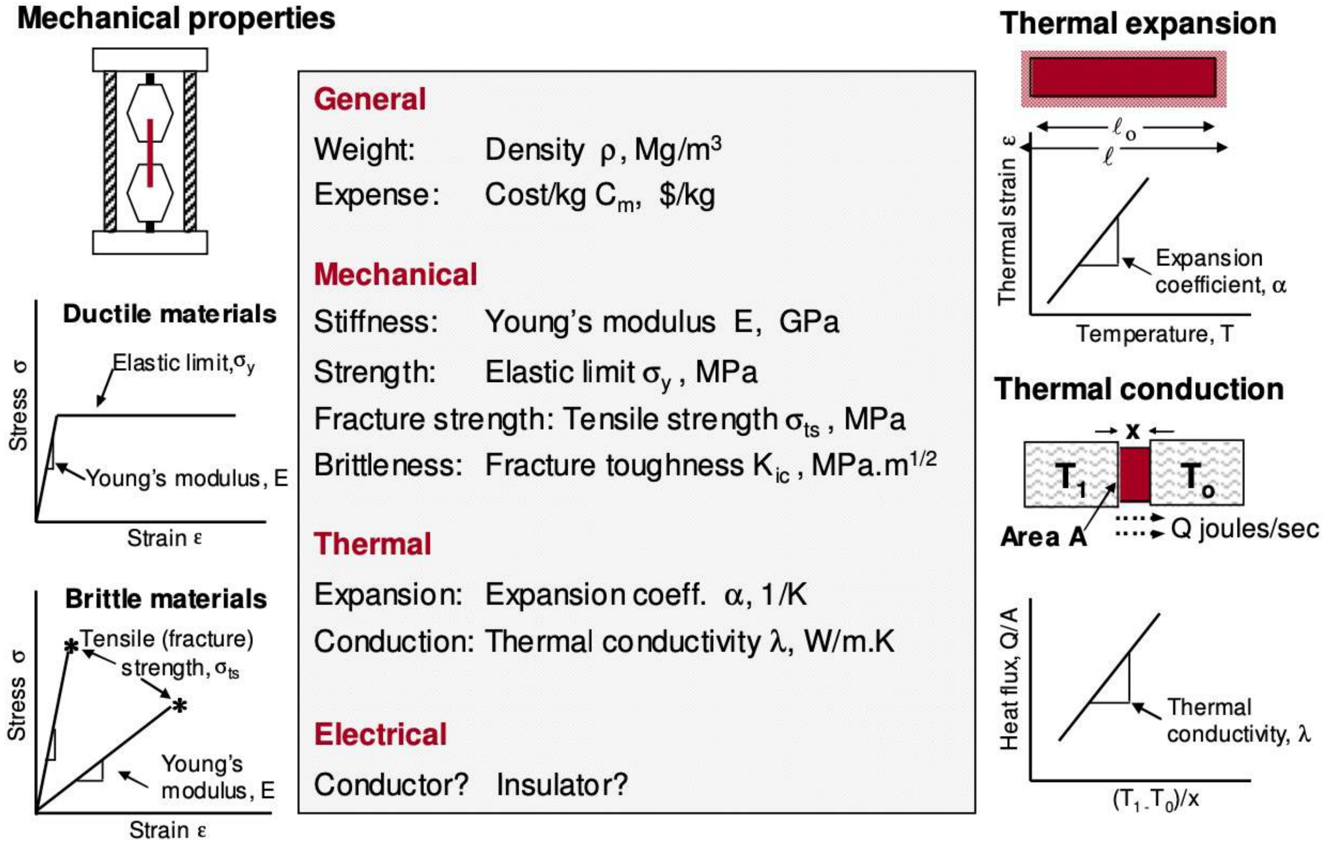
21st Century

Why ?


Materials used in 787 body
Fiberglass
Aluminum
Carbon laminate composite
Carbon sandwich composite
Aluminum/steel/titanium
Total materials used
By weight
Other 5%
Steel 10%
Composites 50%
Titanium 15%
Aluminum 20%
By comparison, the 777 uses 12 percent composites and 50 percent aluminum.

The world of materials





MATERIAL AS A NEW DESIGN VARIABLE

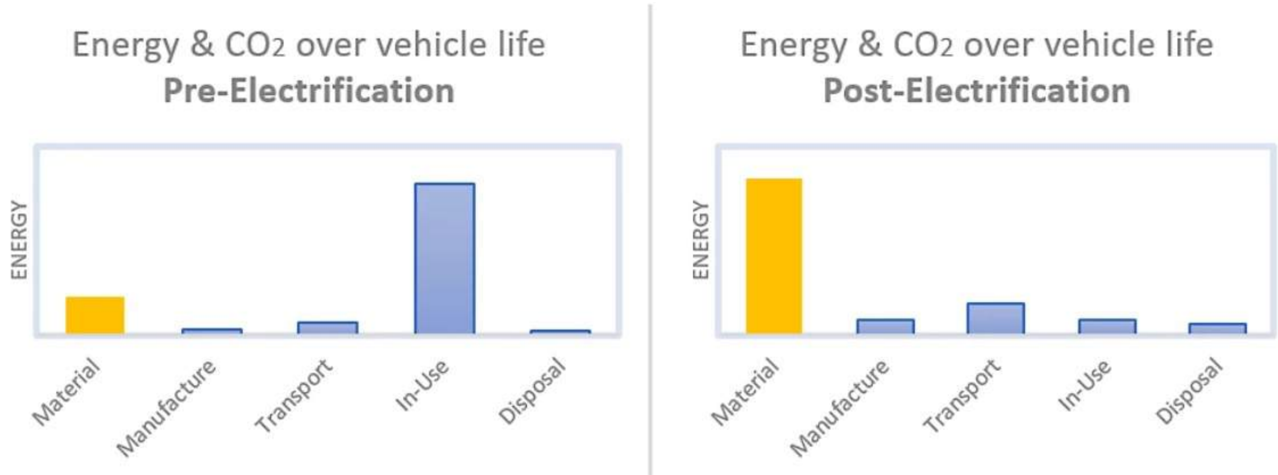
More DATA than {E, rho} in DMO

Le matériau composite dans la structure : objet d'étude et variable de conception

par

François-Xavier IRISARRI

Mémoire provisoire, en vue de l'obtention de l'Habilitation à Diriger des Recherches
Université Jean Monnet, Saint-Étienne



The overall importance on material selection will increase post-electrification

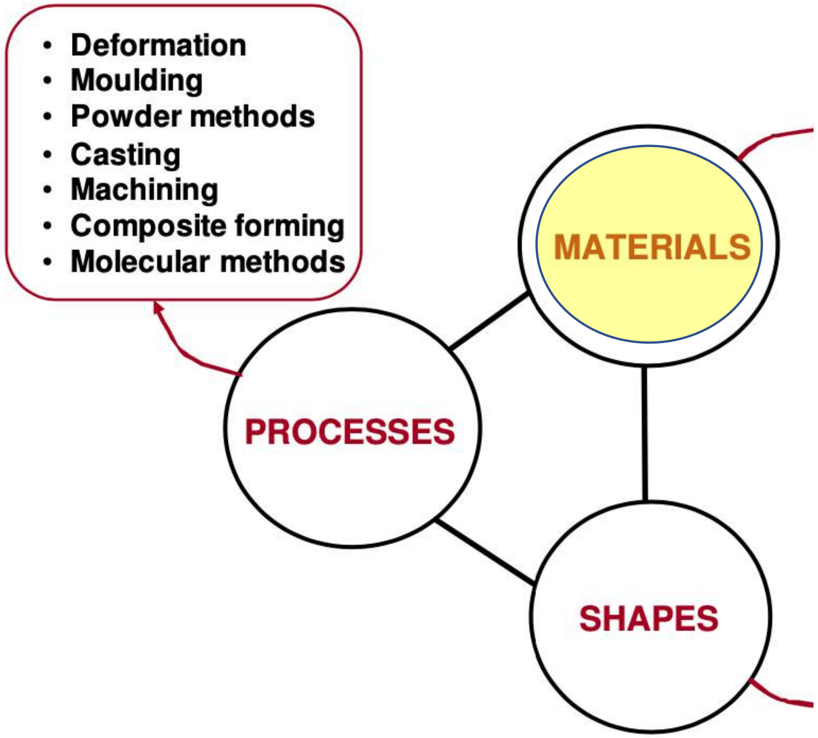
Au programme

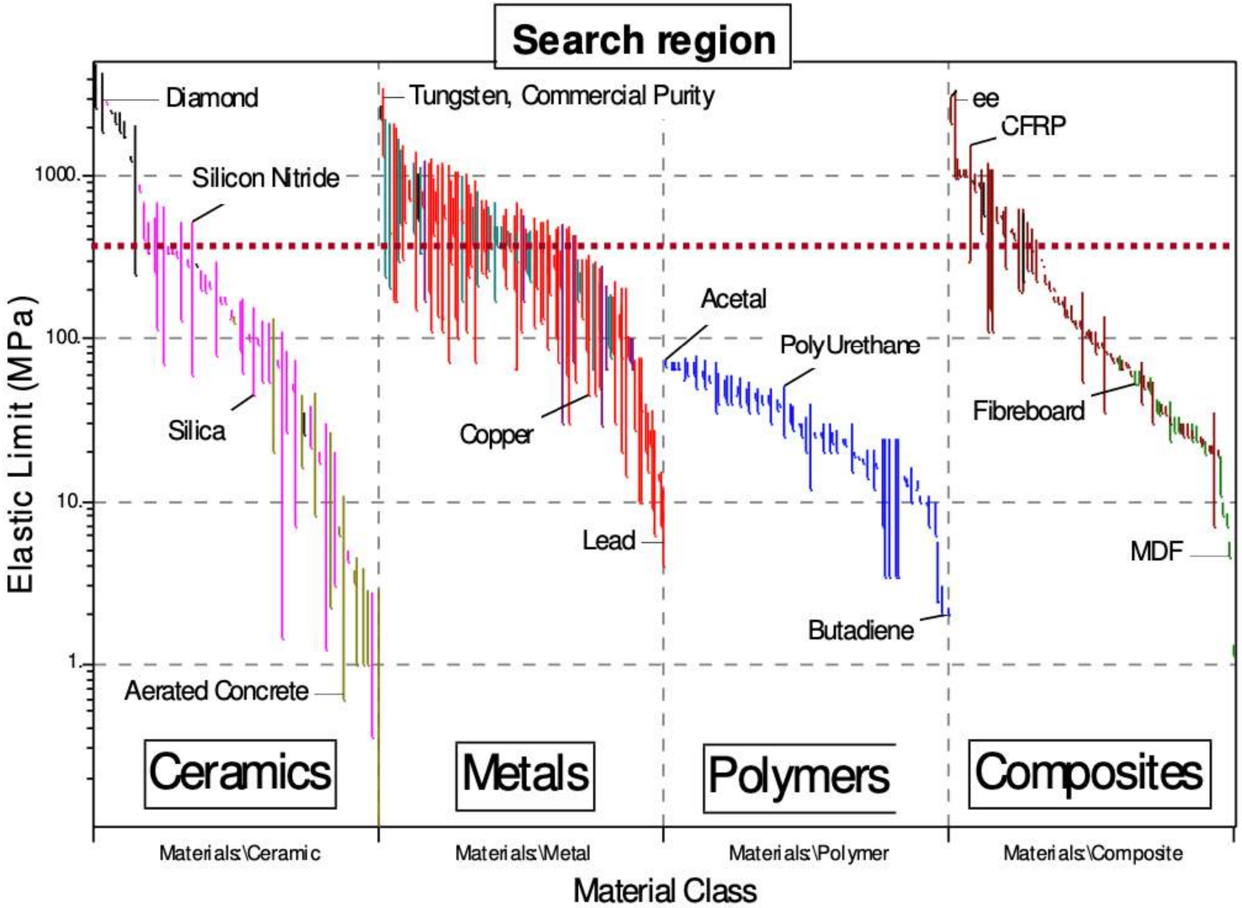
- Part 1 Overview of material's world
- Part 2 Lightest strong
- Part 3 Lightest stiff
- Part 4 GHG footprint
- Part 5 Optimization with multiple objectives (Mass-cost i.e. 2 objectives)

PART1

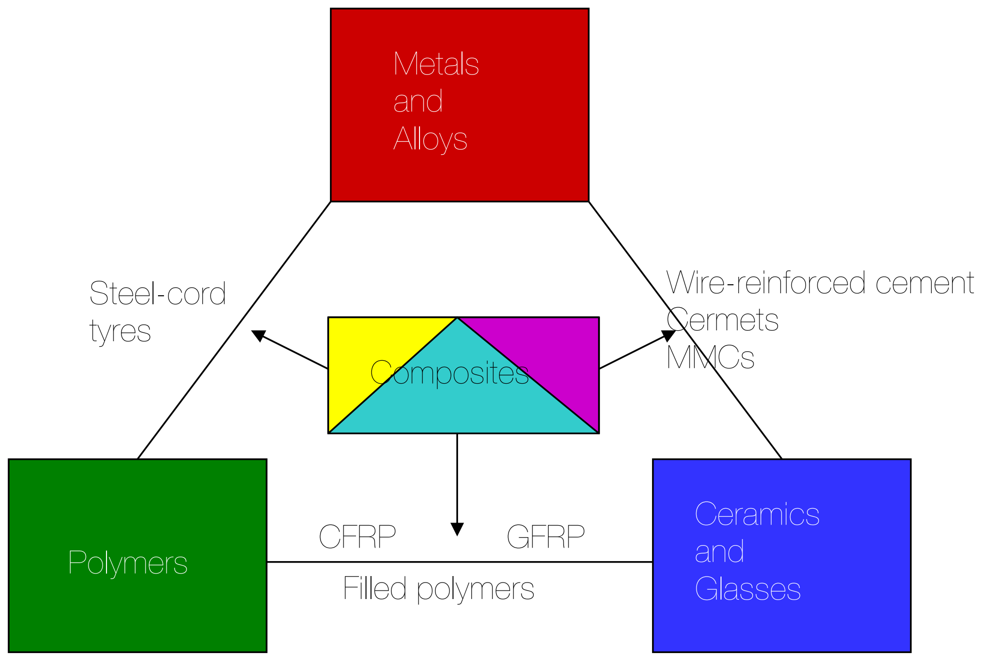
- Overview

Focus on Materials





Materials Selection



Materials Properties

STRUCTURAL MATERIALS

- Mechanical
  - tribology
  - fatigue
  - K_{IC}
  - σ_y
  - UTS
  - E
- Thermal
  - α
  - K
  - H
  - T_m
  - $T_{Transition}$

- Chemical
  - corrosion
  - oxidation

MATERIAL

- Environmental
  - recycling
  - energy consumption
  - waste

FUNCTIONAL MATERIALS

- Physical
  - optical
  - magnetic
  - electrical
- Other
  - feel
  - look

Database of Materials (quantitative/qualitative/*estimated)

- Table: Materials
  - Ceramic
  - Composite
  - Foam
  - Metal
    - Commercially Pure
    - Ferrous Alloys
    - Light Alloys
      - Aluminium
        - Cast
          - 1xxx Group
          - 2xxx Group
          - 3xxx Group
          - 4xxx Group
            - Cast aluminium alloy (A413.2)
            - Cast aluminium alloy (S413.0)
            - Cast aluminium alloy (S443.0)
          - 5xxx Group
          - 6xxx Group
          - 7xxx Group
          - 8xxx Group
        - Wrought
      - Beryllium
      - Magnesium
      - Titanium
    - Non-ferrous Alloys
    - Precious Metal Alloys
    - Refractory Alloys
  - Natural
  - Polymer
    - Elastomer
    - Thermoplastic
    - Thermoset

Cast aluminium alloy (A413.2)

General

Designation

Al alloy: A413.2 (cast)

Composition

Al-12Si

Atomic Volume (average)	0.01	- 0.011	m^3/kmol
Density	2.65	- 2.66	Mg/m^3
Energy Content	235	- 335	MJ/kg
Price	9.75297	- 11.5214	FRF/kg
Recycle Fraction	* 0.8	- 0.9	

Mechanical

Bulk Modulus	65	- 86	GPa
Compressive Strength	70	- 80	MPa
Ductility	0.08	- 0.13	
Elastic Limit	70	- 80	MPa
Endurance Limit	* 37	- 45	MPa
Fracture Toughness	* 25	- 28	MPa.m^1/2
Hardness	550	- 600	MPa
Loss Coefficient	* 1e-004	- 2e-003	
Modulus of Rupture	70	- 80	MPa
Poisson's Ratio	0.32	- 0.36	
Shape Factor	47		
Shear Modulus	26	- 28	GPa
Tensile Strength	170	- 200	MPa
Young's Modulus	71	- 71.5	GPa

Thermal

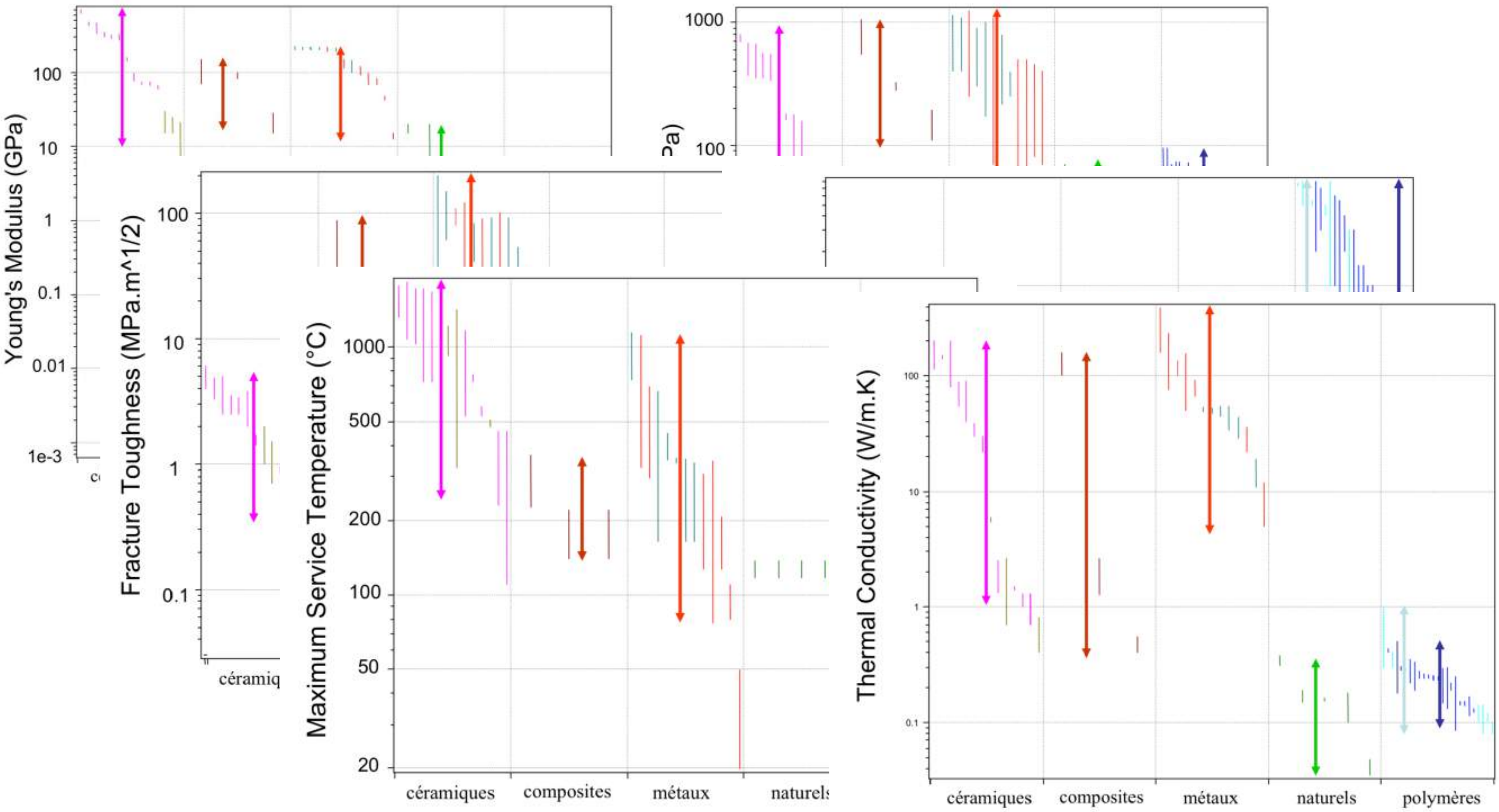
Glass Temperature	Not Applicable		K
Latent Heat of Fusion	384	- 393	kJ/kg
Maximum Service Temperature	400	- 450	K
Melting Point	838	- 848	K
Minimum Service Temperature	0	- 0	K
Specific Heat	* 910	- 960	J/kg.K
Thermal Conductivity	137	- 147	W/m.K
Thermal Expansion	23	- 23.1	10^-6/K

Electrical

Breakdown Potential	Not Applicable		MV/m
Dielectric Constant	Not Applicable		
Resistivity	4.7	- 4.88	10^-8 ohm.m
Power Factor	Not Applicable		

Environmental Resistance

Flammability	Good
Fresh Water	Very Good
Organic Solvents	Very Good
Oxidation at 500C	Very Poor
Sea Water	Good
Strong Acid	Very Good
Strong Alkalis	Poor
UV	Very Good


Young's Modulus (GPa)
100
10
1
0.1
0.01
1e-3
1000
100
Fracture Toughness (MPa.m^1/2)
100
10
1
0.1
céramiq
Maximum Service Temperature (°C)
1000
500
200
100
50
20
céramiques
composites
métaux
naturels
Thermal Conductivity (W/m.K)
100
10
1
0.1
céramiques
composites
métaux
naturels
polymères

Specific Properties

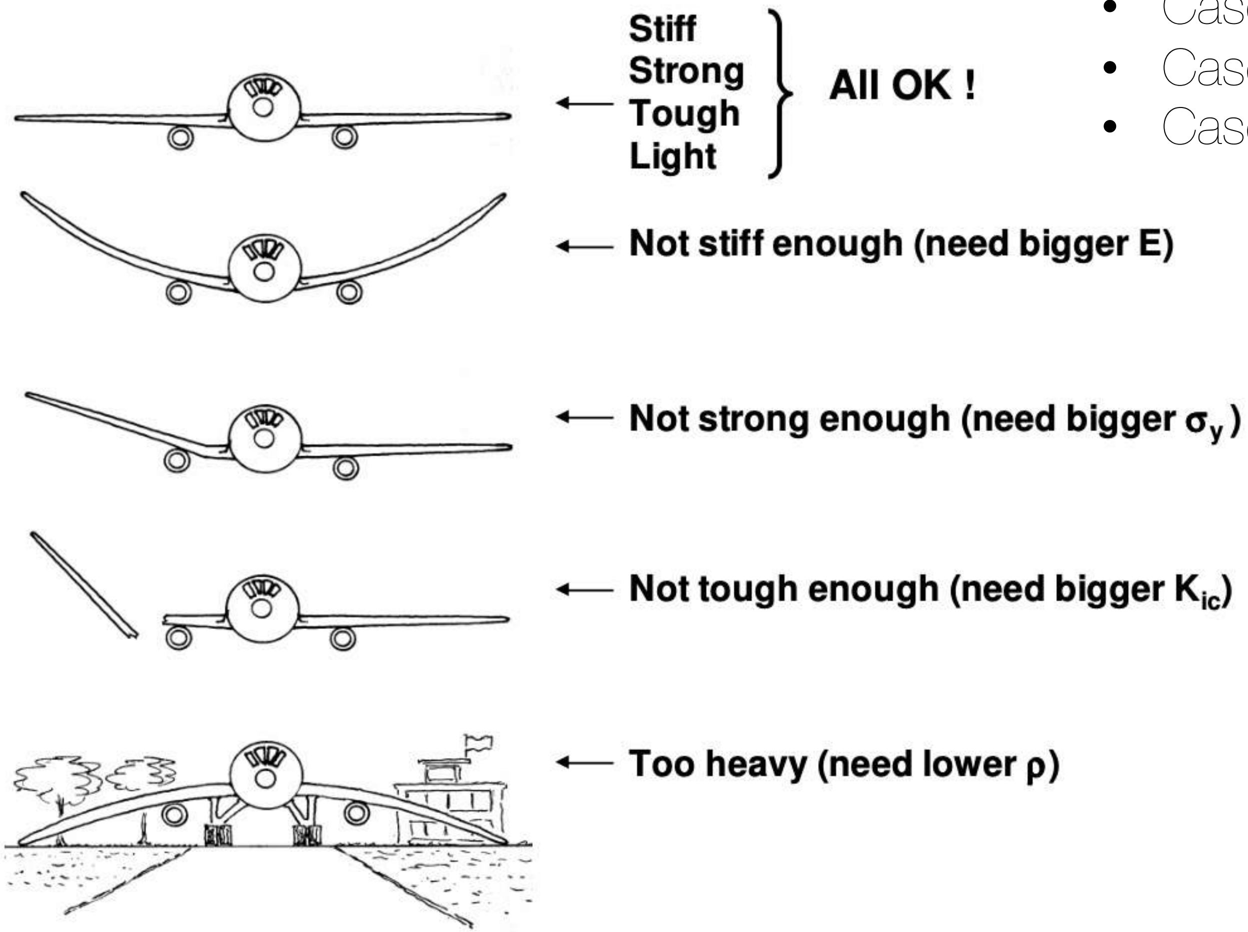
Stiffness vs Strenght

Material	E (GPa)	σ (MPa)	ρ (Mgm^{-3})	E/ρ_{mean} ($10^6\ m^2s^{-2}$)	σ/ρ_{mean} ($10^3\ m^2s^{-2}$)
Cobalt/WC cermets	400-530	400-900	11-12.5	34-45	34-77
Beryllium and alloys	200-289	34-276	1.8-2.1	103-148	17-141
Low-alloy steels	200-207	500-1980	7.8	26-27	64-253
CFRP	70-200	640-670	1.5-1.6	45-129	413-432
Aluminium alloys	69-79	100-627	2.6-2.9	25-45	36-228
Common woods, ∥ to grain	9-16	35-55	0.4-0.6	15-27	58-92
Lead and alloys	14	11-55	10.7-11.3	1.3	1.0-5.0
Polypropylene	0.9	19-36	0.88-0.91	1.0	21-40
Foamed polymers	0.001-0.1	0.2-10	0.01-0.6	0.003-0.03	0.66-33

E	E/ρ_{mean}	σ	σ/ρ_{mean}
Cobalt/WC cermets	Beryllium and alloys	Low-alloy steels	CFRP
Beryllium and alloys	CFRP	Cobalt/WC cermets	Low-alloy steels
Low-alloy steels	Cobalt/WC cermets	CFRP	Aluminium alloys
CFRP	Aluminium alloys	Aluminium alloys	Beryllium and alloys
Aluminium alloys	Low-alloy steels	Beryllium and alloys	Common woods, ∥ to grain
Common woods, ∥ to grain	Common woods, ∥ to grain	Common woods, ∥ to grain	Cobalt/WC cermets
Lead and alloys	Lead and alloys	Lead and alloys	Polypropylene
Polypropylene	Polypropylene	Polypropylene	Lead and alloys
Foamed polymers	Foamed polymers	Foamed polymers	Foamed polymers

Remember

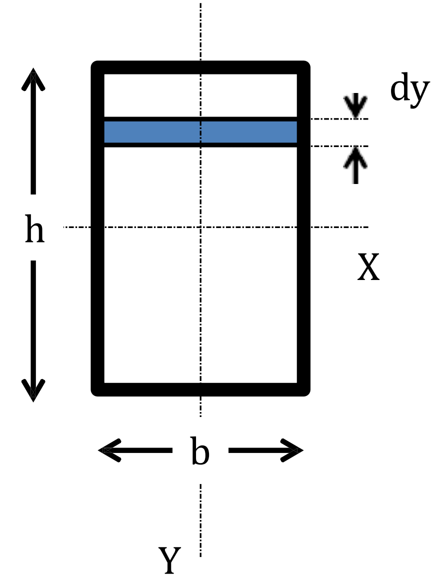
- Case Study 1: The Lightest STIFF Beam
- Case Study 2: The Lightest STIFF Tie-Rod
- Case Study 3: The Lightest STIFF Panel
- Case Study 6: The Lightest STRONG Tie-R
- Case Study 7: The Lightest STRONG Beam
- Case Study 8: The Lightest STRONG Panel

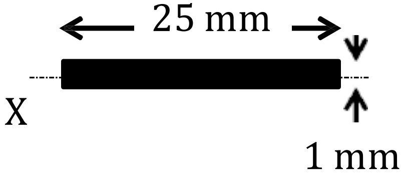


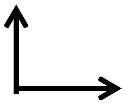
PART2

- LIGHTEST STRONG

Simplification







For a beam under flexion, the moment of inertia : $I_{XX} = \dfrac{bh^3}{12}$

Length (L): 300 mm

Thickness (h) = 1 mm

Width (b) = 25 mm

$$I_{XX} = \frac{25 \cdot 1^3}{12} = 2{,}1\ mm^4$$

$$I_{YY} = \frac{1 \cdot 25^3}{12} = 1300\ mm^4$$

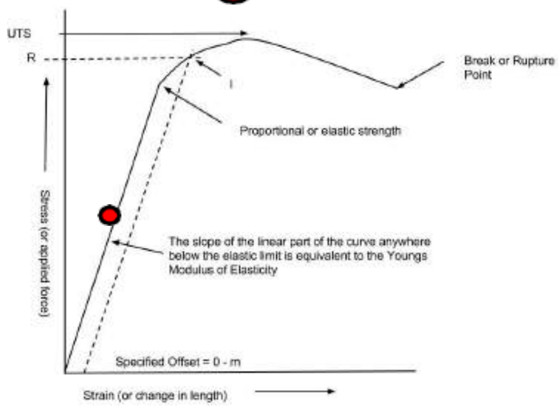
In the case of the mechanical properties, it is important to consider the forces applied, but it is the weakest point that determine the selection.

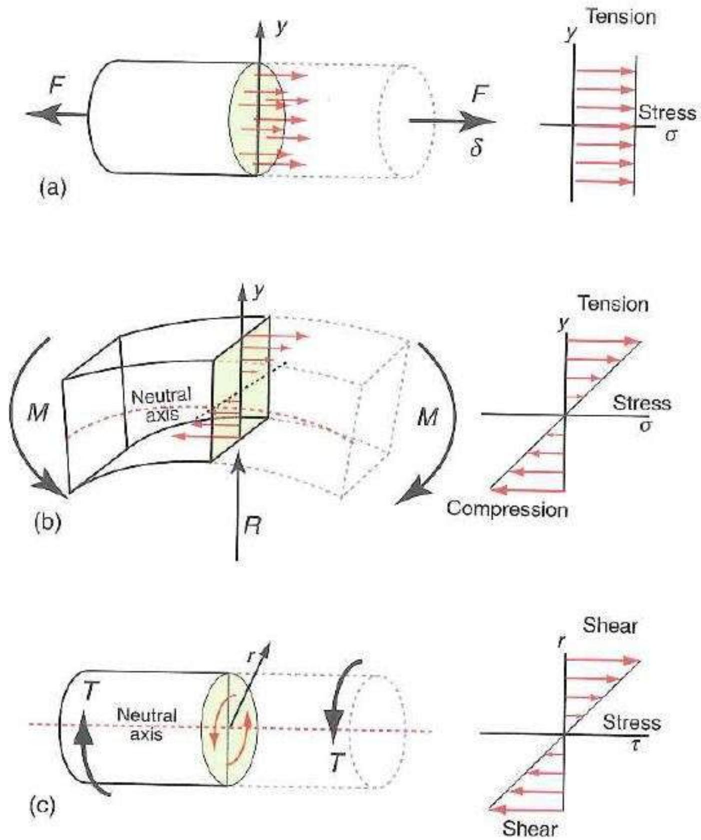
It is possible to change the geometry, but if you cannot
What can we do?

Stiffness design

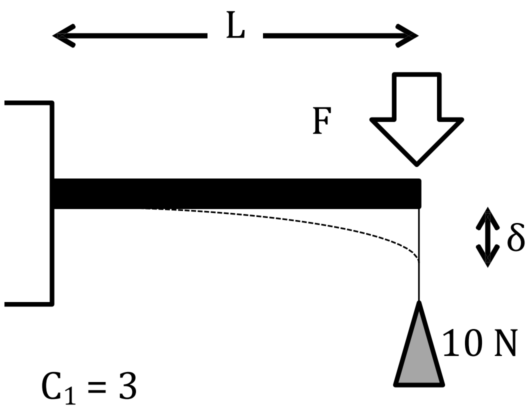
The Stiffness design is important to avoid excessive ELASTIC deflection

Your are here





Example 1



$C_1 = 3$

$$S = \frac{F}{\delta} = \frac{C_1 EI}{L^3}$$

$$\delta = \varepsilon \cdot L$$

EI = Flexural rigidity
I = Second Moment of inertia
E = Young's Modulus
δ = Deflexion

Length (L): 300 mm
Thickness (h)= 1 mm
Width (b)= 25 mm

$$I_{XX} = \frac{25 \cdot 1^3}{12} = 2{,}1\ mm^4$$

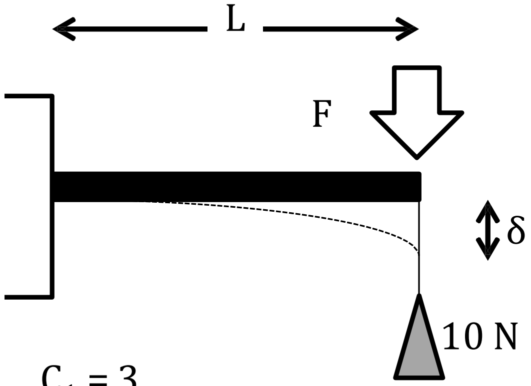
$$I_{YY} = \frac{1 \cdot 25^3}{12} = 1300\ mm^4$$

Problem :

δ??
IF we consider that the beam is made of Stainless Steel (E = 200 GPa)

Which are the consequences if I want to use Polystyrene (E = 2 GPa)?
IF I can change the thickness and hold the same deflection.

Example bis



$C_1 = 3$

$$S = \frac{F}{\delta} = \frac{C_1 EI}{L^3}$$

EI = Flexural rigidity
I = Second Moment of inertia
E = Young's Modulus

Stainless Steel (E = 200 GPa; ρ = 7800 kg/m^3)
Polystyrene (E = 2 GPa; ρ = 1040 kg/m^3)

$$I_{YY} = \frac{1 \cdot 25^3}{12} = 1300\ mm^4 \longrightarrow \delta = \frac{10 \cdot (0{,}25)^3}{3 \cdot (200 \cdot 10^9) \cdot (1300 \cdot 10^{-12})} = 0{,}02\ mm$$

$$I_{XX} = \frac{25 \cdot 1^3}{12} = 2{,}1\ mm^4 \longrightarrow \delta = \frac{FL^3}{C_1 EY_{XX}} = 124\ mm$$

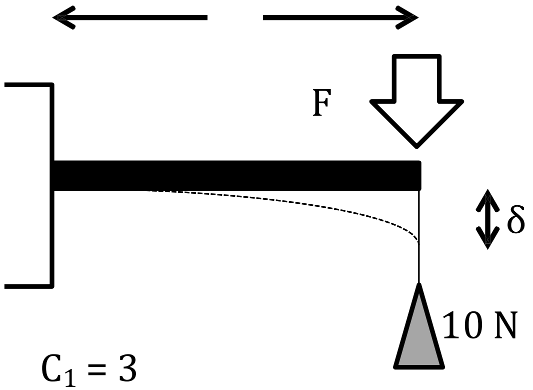
Steel

$$With\ \delta = 124\ mm \qquad I_{XX} = \frac{10 \cdot (0{,}25)^3}{3 \cdot (2 \cdot 10^9) \cdot (0{,}124)} = 210\ mm^4$$

$$h = \left(\frac{12 I_{XX}}{w}\right)^{1/3} = \left(\frac{12 \cdot 210}{25}\right)^{1/3} = 4{,}6\ mm \qquad When\ h(Steel) = 1\ mm$$

PS

Example 1 ter



$C_1 = 3$

$$S = \frac{F}{\delta} = \frac{C_1 EI}{L^3}$$

Length: 300 mm

Width = 25 mm

EI = Flexural rigidity

I = Second Moment of inertia

E = Young's Modulus

δ = Deflexion

Stainless Steel ($E = 200$ GPa; $\rho = 7800$ kg/m³)
Polystyrene ($E = 2$ GPa; $\rho = 1040$ kg/m³)

Thickness = 1 mm

Thickness = 4,6 mm

About the weight?

$$m_{SS} = 7800 \cdot 0{,}3 \cdot 0{,}025 \cdot 0{,}001 = 59\ gr$$

$$m_{PS} = 1040 \cdot 0{,}3 \cdot 0{,}025 \cdot 0{,}046 = 36\ gr$$

BIGGER Section
BUT LIGHTER

Depends on what you need and the conditions

Generic Materials Selection

p: Performance of component; f(F,G,M)

F: Functional requirement, e.g. withstanding a force

G: Geometry, e.g. diameter, length etc.

M: Materials properties, e.g. E, K_{IC}, ρ

Separable function if:

$$P = f_1(F) \cdot f_2(G) \cdot f_3(M)$$

TASK: Maximize $f_3(M)$ where M is the "performance index"

Procedure for Deriving "M"

(a) Identify the *attribute* to be maximized or minimized (weight, cost, energy, stiffness, strength, safety, environmental damage, etc.).

(b) Develop an equation for this attribute in terms of the functional requirements, the geometry, and the material properties (the *objective function*).

(c) Identify the *free* (unspecified) *variables.*

(d) Identify the *constraints*; rank them in order of importance.

(e) Develop *equations* for the constraints (no yield, no fracture, no buckling, maximum heat capacity, cost below target, etc.).

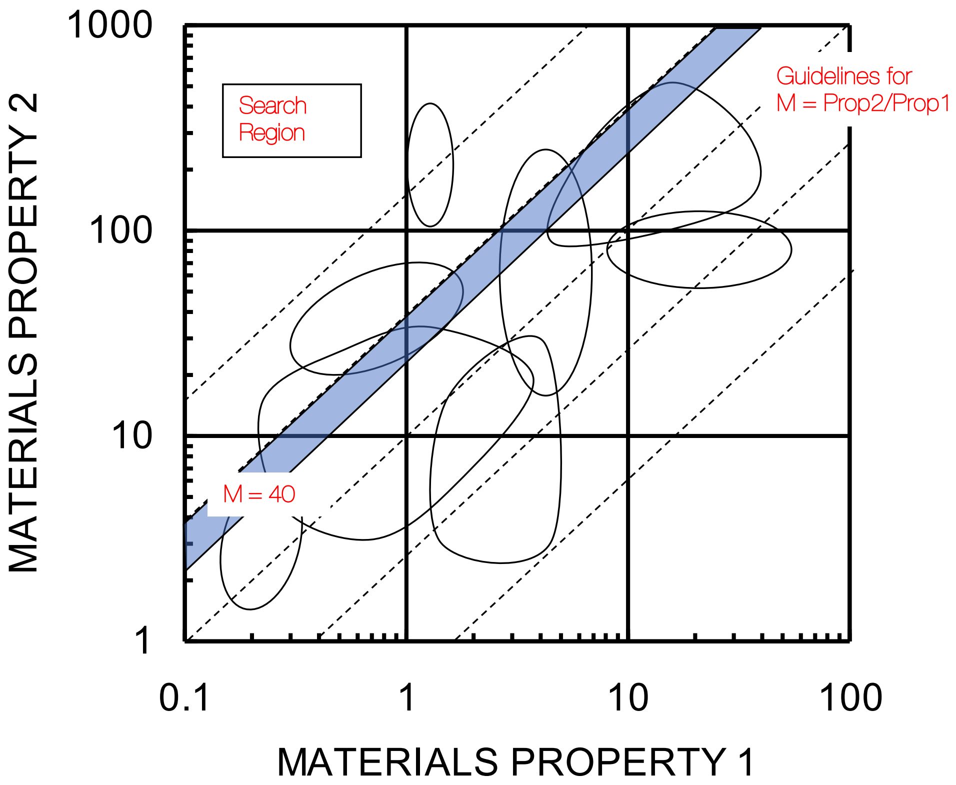
(f) S*ubstitute* for the free variables from the constraints into the objective function.

(g) G*roup the variables* into three groups: functional requirements, F, geometry, G, and materials properties, M.

(h) R*ead off* the performance index, expressed as a quantity, M, to be maximized.

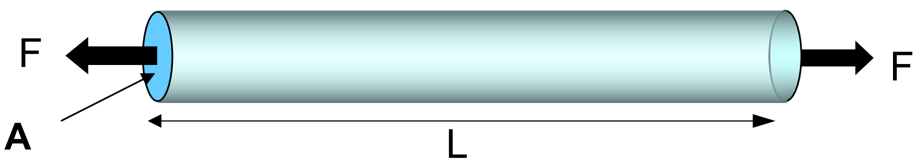
(i) N*ote* that a full solution is not necessary in order to identify the material property group.

The Materials Selection Map



Index for a strong, light tie-rod

Strong tie of length L and minimum mass



Function — *Tie-rod*

Constraints
- *Length L is specified*
- *Must not fail under load F*

Equation for constraint on A:

$$\boxed{F/A < \sigma_y}$$

Objective — *Minimize mass m:*

$$\boxed{m = A\,L\,\rho}$$

m = mass
A = area
L = length
ρ = density
σ_y = yield strength

Performance metric

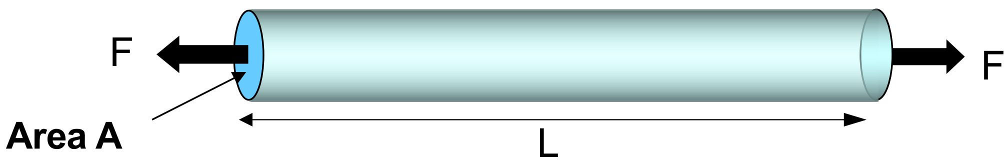
$$m = F\,L\left(\frac{\rho}{\sigma_y}\right)$$

Chose materials with largest $M = \left(\frac{\sigma_y}{\rho}\right)$

Example I:

Function — ***Tie-rod***

Strong tie of length L and minimum mass



Objective — Mass: $m = AL\rho \quad \Rightarrow A = \dfrac{m}{L\rho}$

Stress: $\sigma_f = \dfrac{F}{A} = \dfrac{FL\rho}{m}$

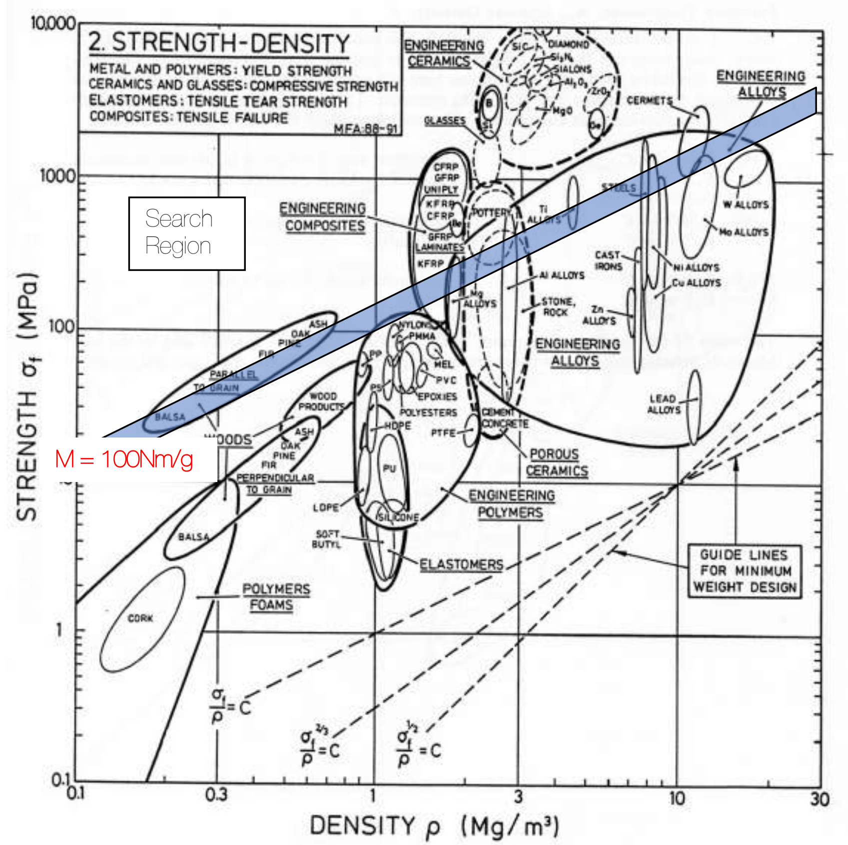
Constraints

- ***Length L is specified***
- ***Must not fail under load F***

$$\Rightarrow m = \underbrace{F}_{f_1(F)} \cdot \underbrace{L}_{f_2(G)} \cdot \underbrace{\frac{\rho}{\sigma_f}}_{f_3(M)}$$

So, to minimize mass m,

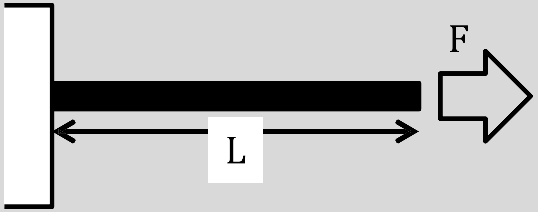
Chose materials with largest $\boxed{M = \dfrac{\sigma_f}{\rho}}$ — **Performance metric**



EMSM207

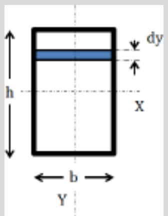
Lightest Tie-Rod (Traction conditions)

Case Study 6:
Find the Lightest STRONG <u>Tie-Rod</u>



Objective	• Minimize the mass
Constraints	• Support tensile load F without yielding • Length L
Free Variables	• Area (A) of the cross-section • Choice of the material

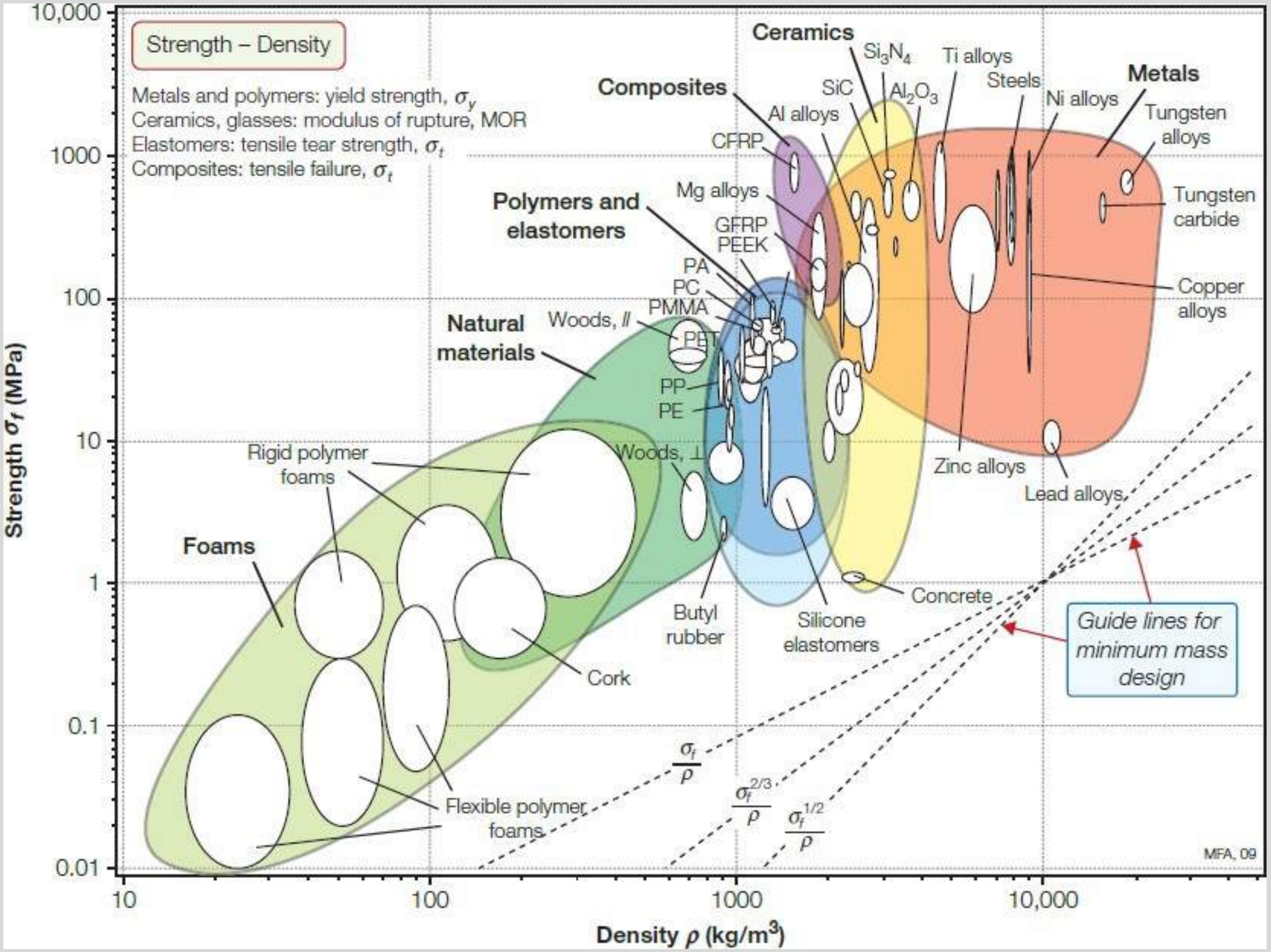
DATA
F =1000 N
Dimensions:
Length: 300 mm
Thickness = 1 mm
Width = 25 mm



In Traction,
the shape of the cross-section is not important

$$\begin{cases} \boxed{m = A \cdot L \cdot \rho} \longrightarrow A = \dfrac{m}{L \cdot \rho} \\ From\ material: \ \dfrac{F}{A} \leq \sigma_y \end{cases} \Rightarrow m \geq F \cdot L \cdot \frac{\rho}{\sigma_y}$$

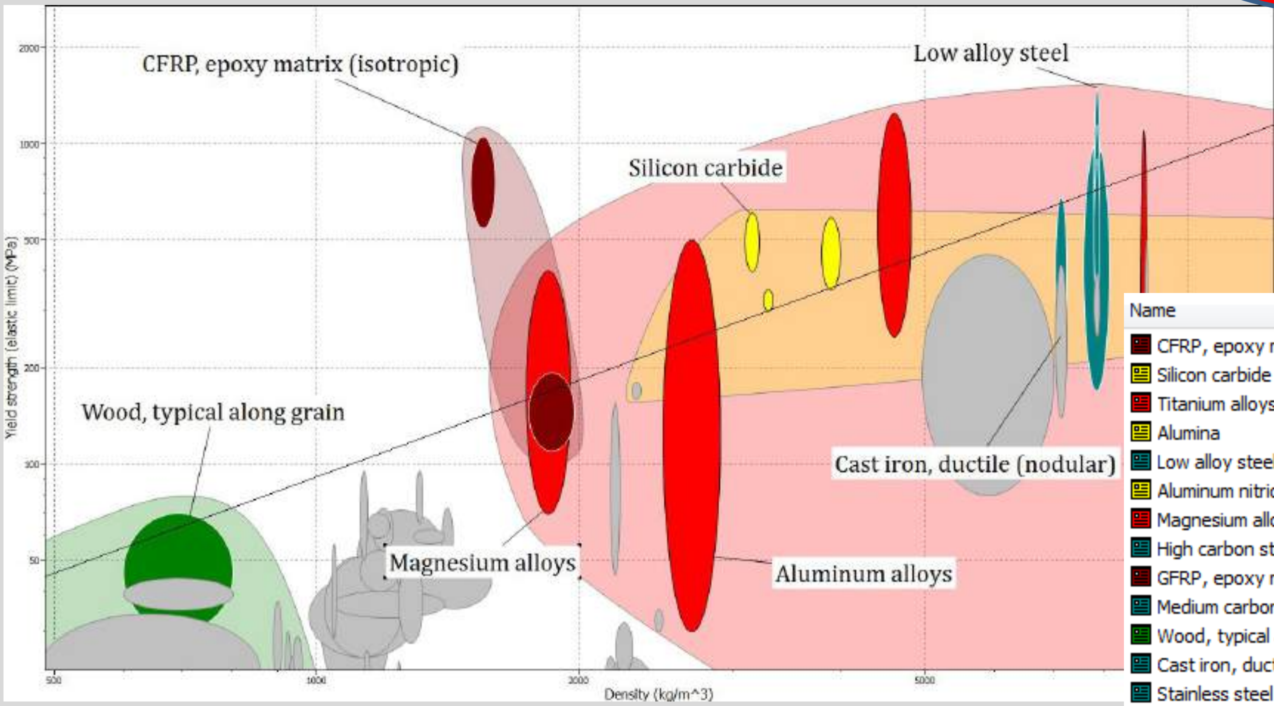
Ashby Diagrams



CES

Case Study 6:
Find the Lightest STRONG <u>Tie-Rod</u>

$m \downarrow \qquad \frac{\sigma_y}{\rho} \uparrow$



Name	Stage 1: Index
CFRP, epoxy matrix (isotropic)	0.491
Silicon carbide	0.157
Titanium alloys	0.121
Alumina	0.117
Low alloy steel	0.0987
Aluminum nitride	0.0983
Magnesium alloys	0.0908
High carbon steel	0.0866
GFRP, epoxy matrix (isotropic)	0.0783
Medium carbon steel	0.0667
Wood, typical along grain	0.0661
Cast iron, ductile (nodular)	0.0577
Stainless steel	0.0526
Aluminum alloys	0.0455
Nickel alloys	0.0312

Lightest Tie-Rod (Traction conditions)

Case Study 6:
Find the Lightest STRONG <u>Tie-Rod</u>

m ↓ $\frac{\sigma_y}{\rho}$ ↑

$$m \geq F \cdot L \cdot \frac{\rho}{\sigma_y}$$

It is possible to do as before, but let's calculate the maximum F on the precedent Tie-Rod

Material	Weight (kg)	Width and Thickness (mm)
Al Alloys	1,25	63

Stainless Steel (σ_y = 600 MPa; ρ = 7800 kg/m^3)
Wood (σ_y = 50 MPa; ρ = 700 kg/m^3)
Al Alloys (σ_y = 270 Mpa; ρ = 75 kg/m^3)

$$F \leq \frac{m}{L} \cdot \frac{\sigma_y}{\rho} = 416\, kN$$

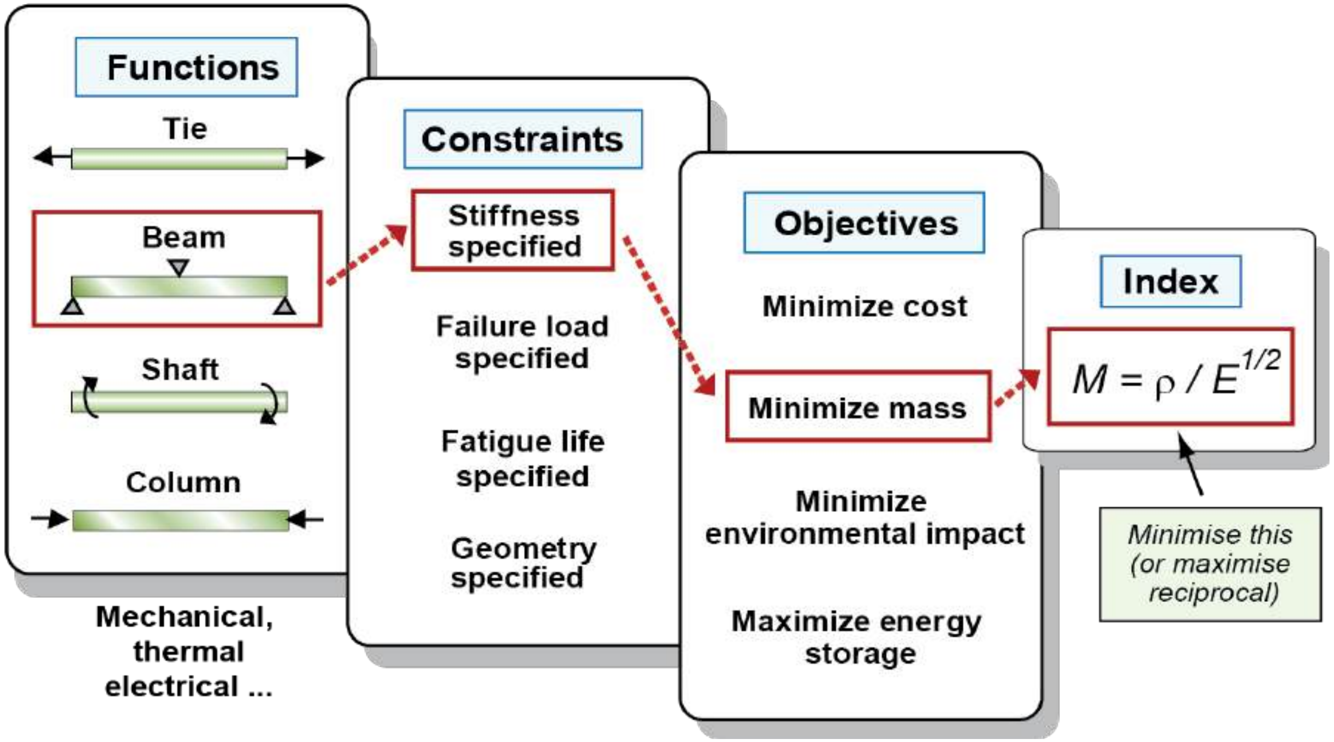
0 kN → **X kN**

Elastic Throughout **Plastic deformation/ Collapse**

PART3

- LIGHTEST STIFF

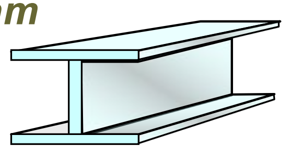
A performance index is a group of material properties that limits the performance of a design



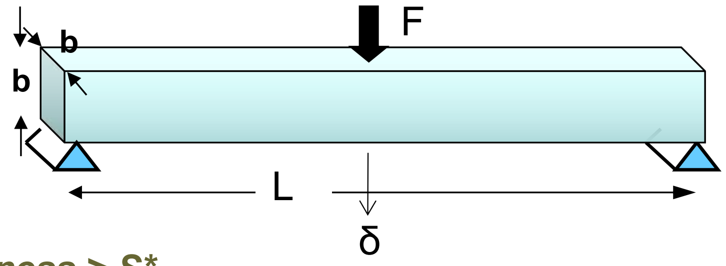
Index for a stiff, light beam

Function

Beam



Stiff beam of length L and minimum mass



Square section, area $A = b^2$

Constraints

- ***Length L is specified***
- ***Must have bending stiffness > S****

Equation for constraint on A:

$$S = \frac{F}{\delta} = \frac{CEI}{L^3} = \frac{CEA^2}{12L^3}$$

m = mass
A = area
L = length
ρ = density
S = stiffness (F/δ)
This beam: $\delta = FL^3/CEI$
C = constant (here, 48)
E = Young's modulus
I = second moment of area
($I = b^4/12 = A^2/12$)

Objective

Minimize mass m:

$$m = A\,L\,\rho$$

Performance metric

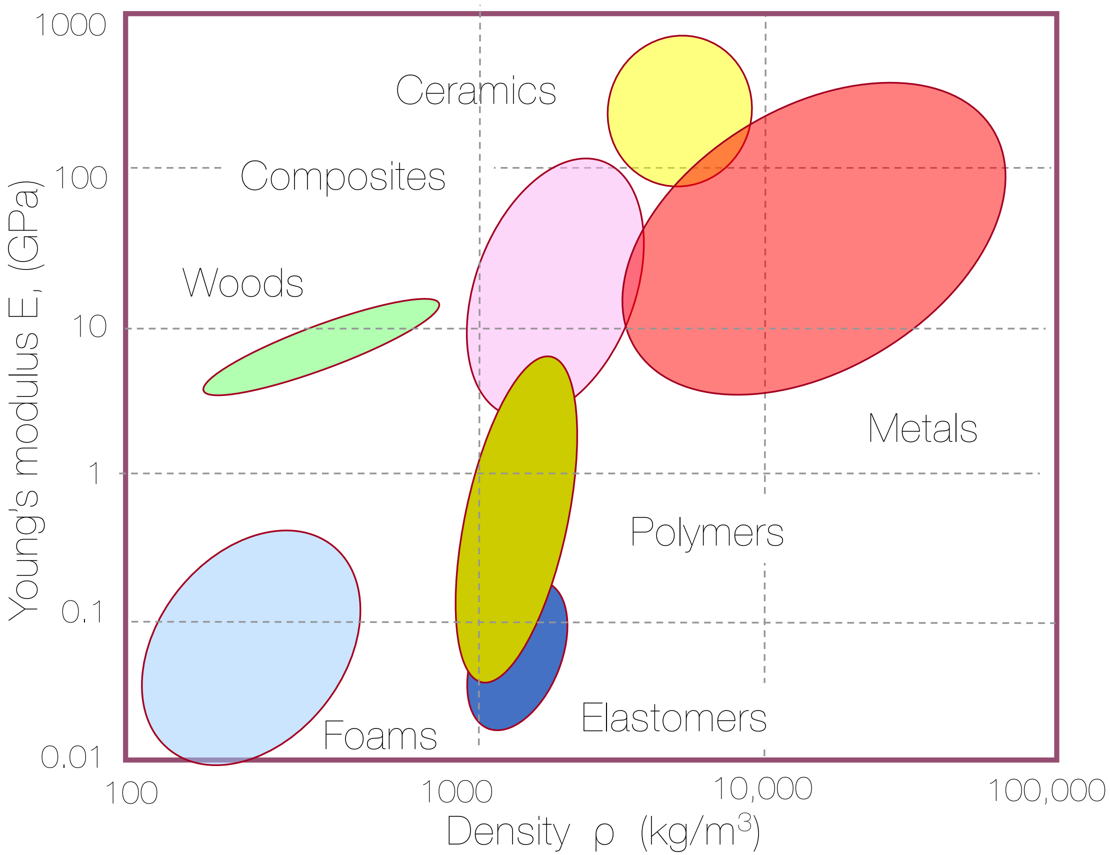
$$m = \left(\frac{12L^5 S^*}{C}\right)^{1/2} \left(\frac{\rho}{E^{1/2}}\right)$$

Chose materials with largest $M = \left(\frac{E^{1/2}}{\rho}\right)$

Back to Ashby diagrams

- We need to find the material Maximizing

$$M = \left(\frac{\mathbf{E^{1/2}}}{\boldsymbol{\rho}} \right)$$



Back to Ashby diagrams

- We need to find the material

Maximizing

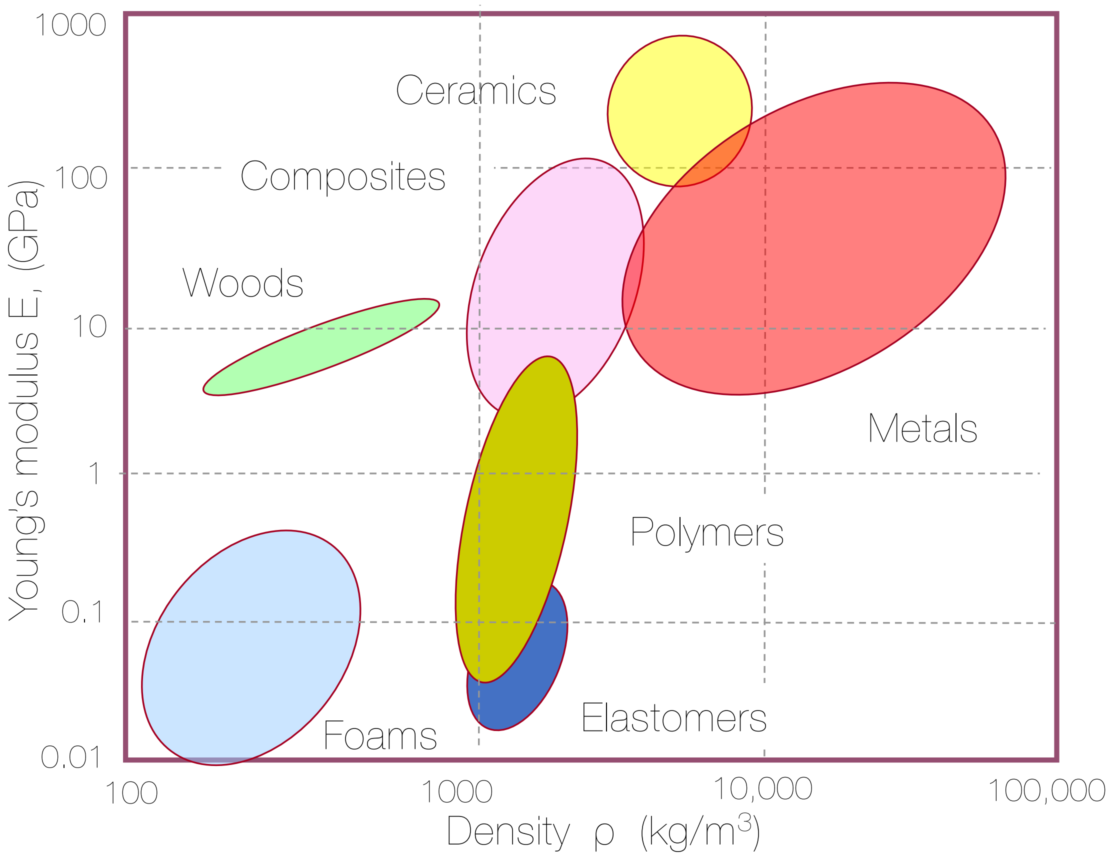
$$M = \left(\frac{E^{1/2}}{\rho} \right)$$

We reorganise this :

E = M^2 * ρ^2

Therefore log(E) = 2log(M) + 2log(ρ)

⇒ Advantage of the log-log diagram: Materials on a line of slope 2 have the same index!



Back to Ashby diagrams

- We need to find the material Maximizing

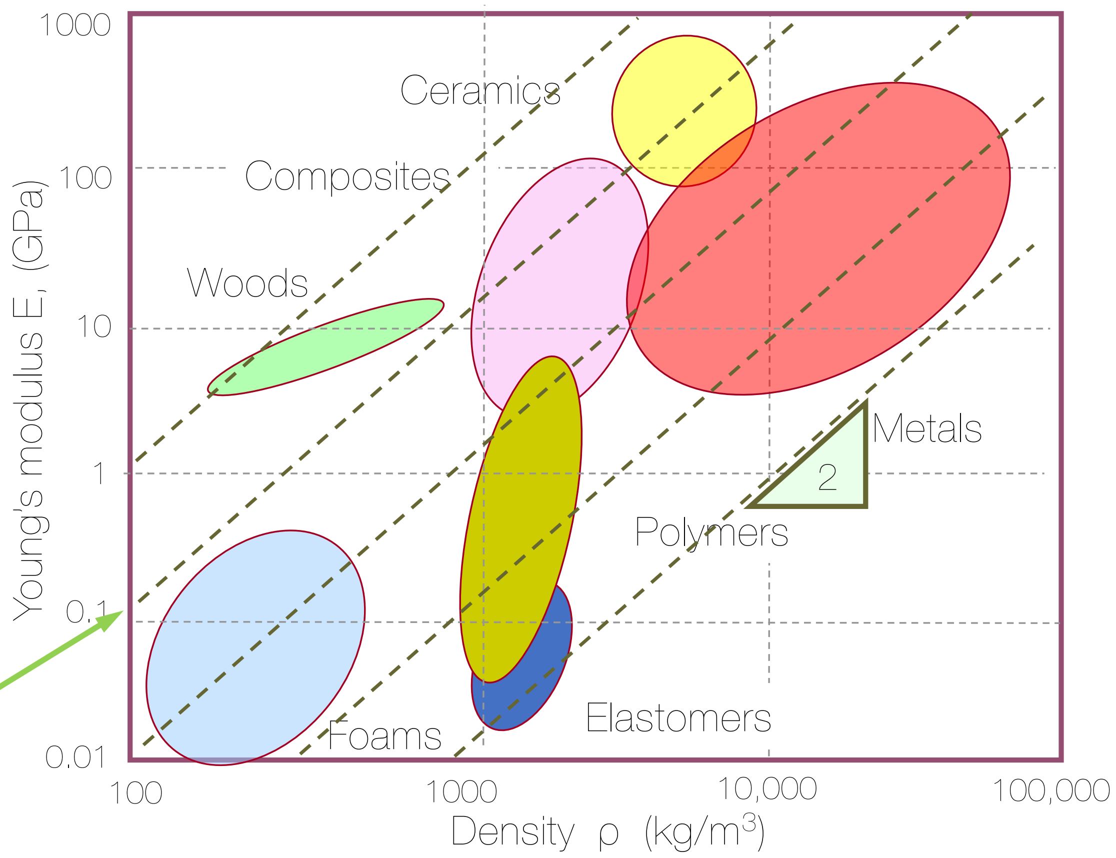
$$M = \left(\frac{\mathbf{E^{1/2}}}{\boldsymbol{\rho}}\right)$$

We reorganise this :
E = M^2 * ρ^2

Therefore log(E) = 2log(M) + 2log(ρ)

⇒ Advantage of the log-log diagram: Materials on a line of slope 2 have the same index!

Part	Index	Slope
Bar	$\mathbf{E/\rho}$	1
Beam	$\mathbf{E^{1/2}/\rho}$	2
Panel	$\mathbf{E^{1/3}/\rho}$	3



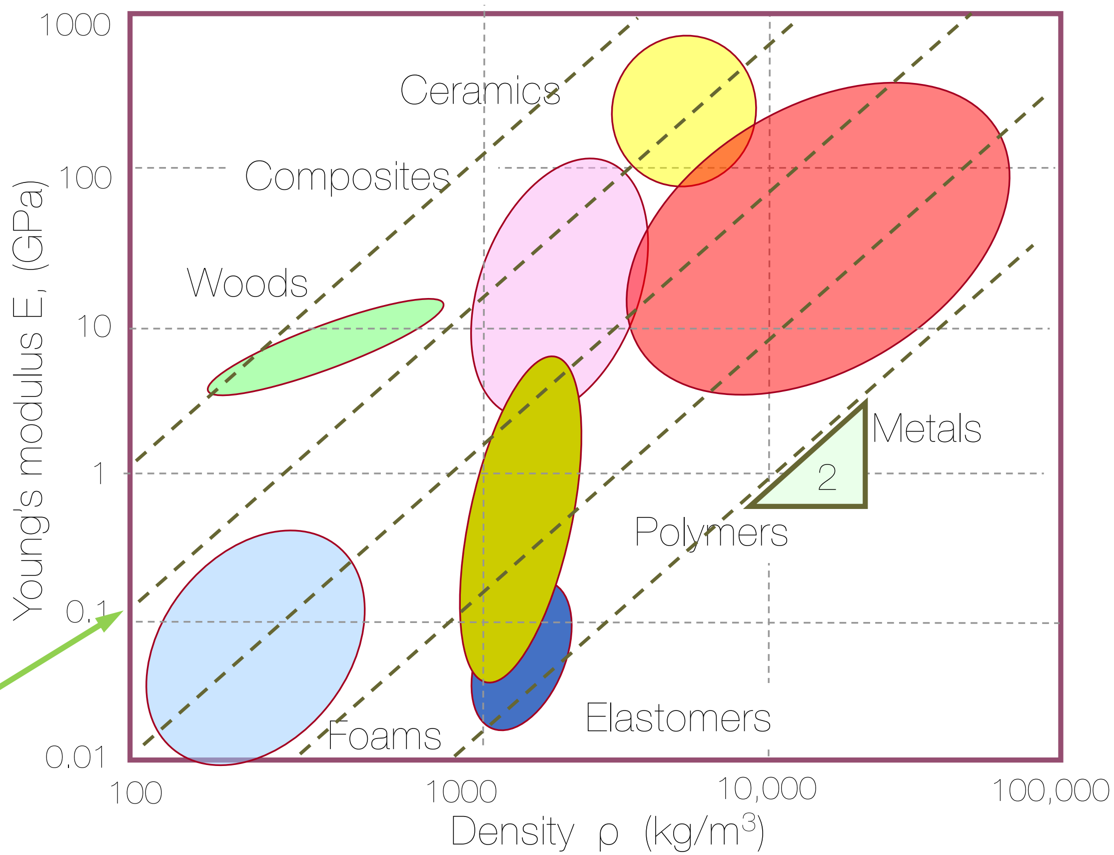
Back to Ashby diagrams

- Looking at the diagram, what type of material enables to minimize the mass of a stiff beam?

$$\log(E) = 2\log(M) + 2\log(\rho)$$

A: Woods
B : Elastomers
C : Ceramics
D : Metals

Part	Index	Slope
Bar	E/ρ	1
Beam	$E^{1/2}/\rho$	2
Panel	$E^{1/3}/\rho$	3



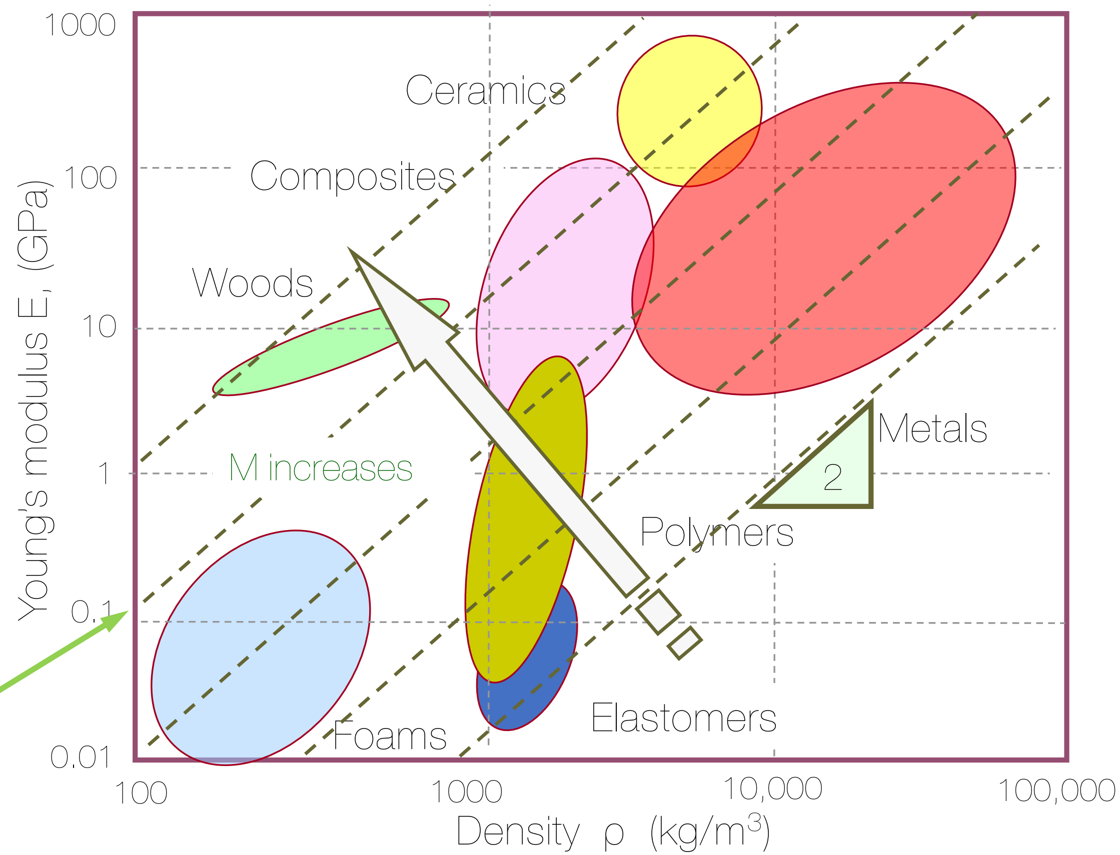
Back to Ashby diagrams

- Looking at the diagram, what type of material enables to minimize the mass of a stiff beam?

$$\log(E) = 2\log(M) + 2\log(\rho)$$

A : Woods
B : Elastomers
C : Ceramics
D : Metals

Part	Index	Slope
Bar	**E/ρ**	1
Beam	**$E^{1/2}/\rho$**	2
Panel	**$E^{1/3}/\rho$**	3



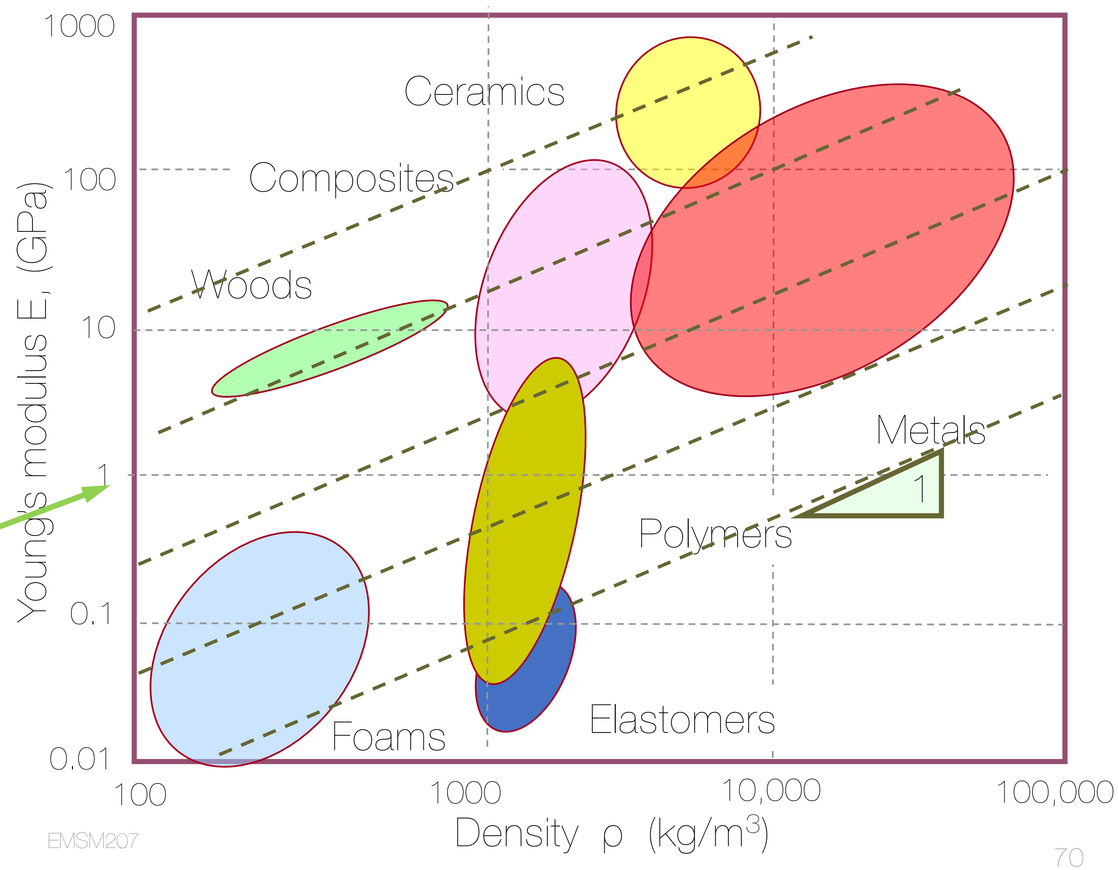
Back to Ashby diagrams

- Looking at the diagram, what type of material enables to minimize the mass of a stiff bar?

$\log(E) = 1\log(M) + 1\log(\rho)$

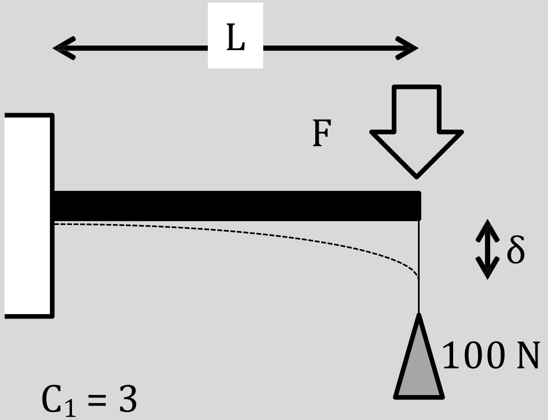
Part	Index	Slope
Bar	$\mathbf{E/\rho}$	1
Beam	$\mathbf{E^{1/2}/\rho}$	2
Panel	$\mathbf{E^{1/3}/\rho}$	3

=> ceramics



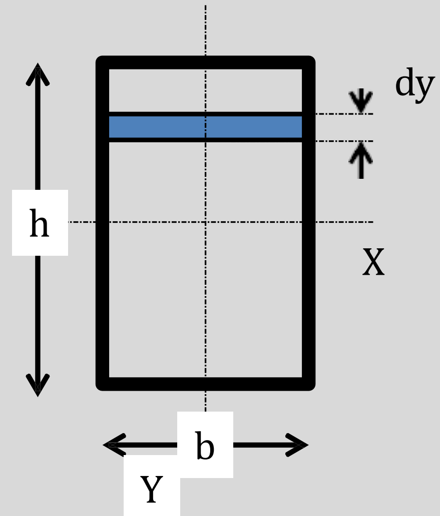
The Materials Selection approach

Case Study 1:
Find the Lightest STIFF <u>Beam</u>



$C_1 = 3$

<u>Beam: Square Section</u>
b=h



$$\left\{ \begin{aligned} \frac{F}{\delta} &= \frac{C_1 E I}{L^3} \geq S_{min} \\ A &= \frac{m}{L \cdot \rho} \end{aligned} \right.$$

Since A = b^2

$$I = \frac{bh^3}{12} = \frac{A^2}{12}$$

$$\left\{ \begin{aligned} A &= \frac{m}{L \cdot \rho} \\ m &\geq \left(\frac{12 \cdot S}{C_1 \cdot L}\right)^{1/2} \cdot L^3 \cdot \frac{\rho}{E^{1/2}} \end{aligned} \right.$$

The Area will be the Free Variable

$$m \downarrow \qquad \frac{E^{1/2}}{\rho} \uparrow$$

Just remember:

Constraints	• Stiffness specified • Length L • Square shape

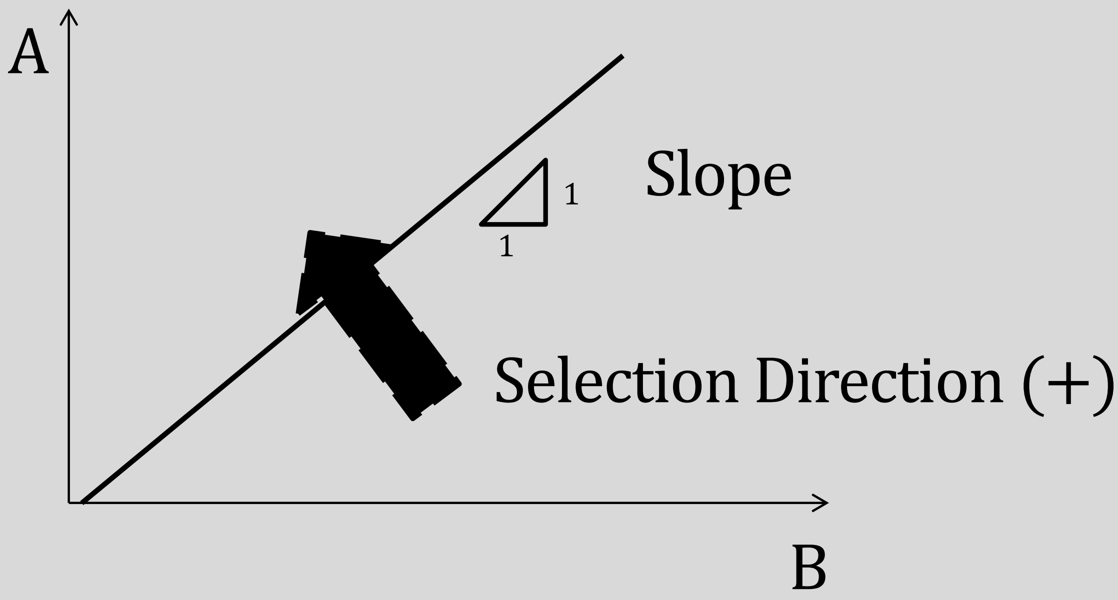
The Material Index (M)

Case Study 1:
Find the Lightest STIFF <u>Beam</u>

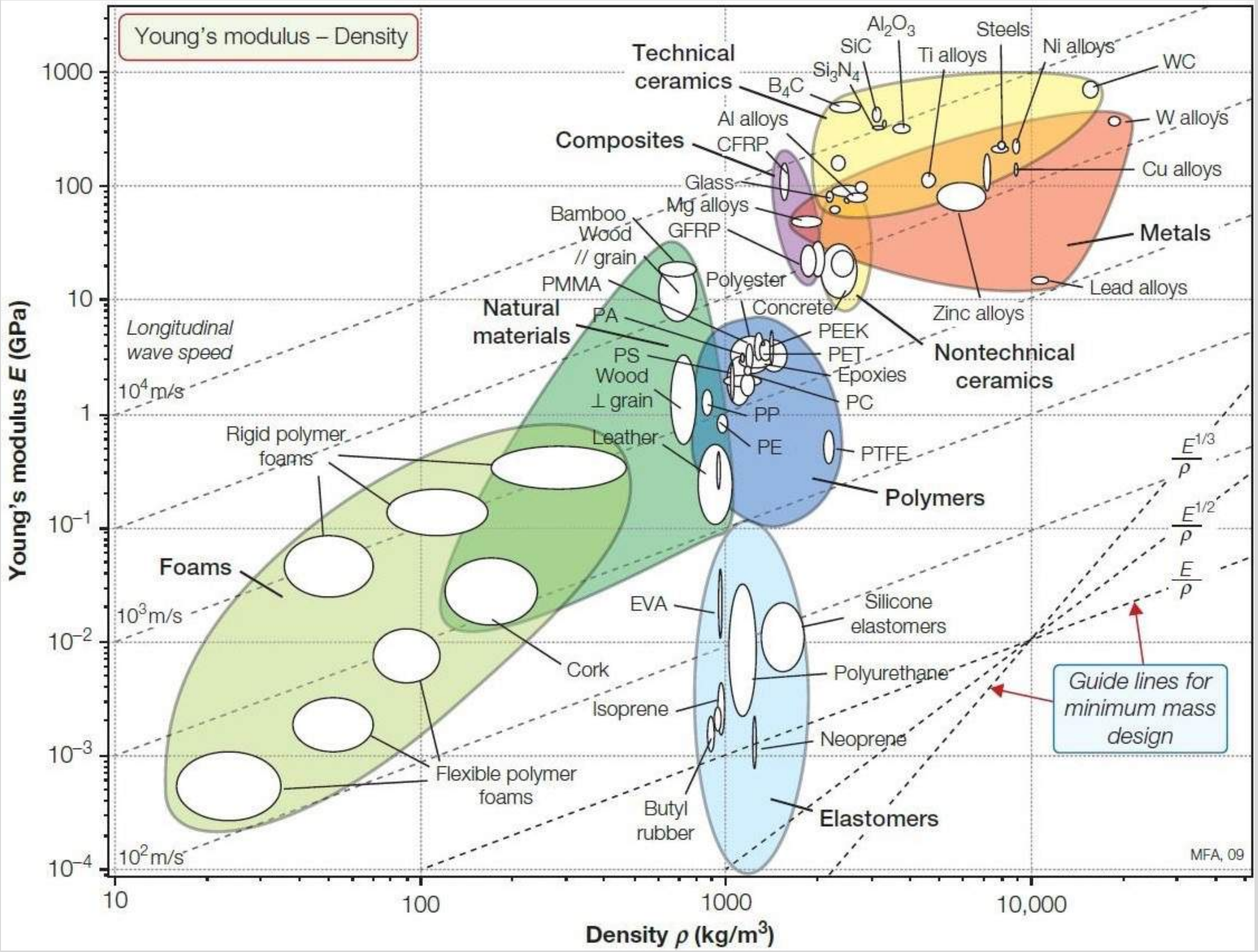
$$\boxed{M = \frac{A}{B}} \longrightarrow \boxed{\mathrm{Log(M)} = Log\ (A) - Log(B)}$$

$$\boxed{\mathrm{Log(A)} = Log(B) + Log\ (M)}$$

For instance $\frac{E^{1/2}}{\rho}$



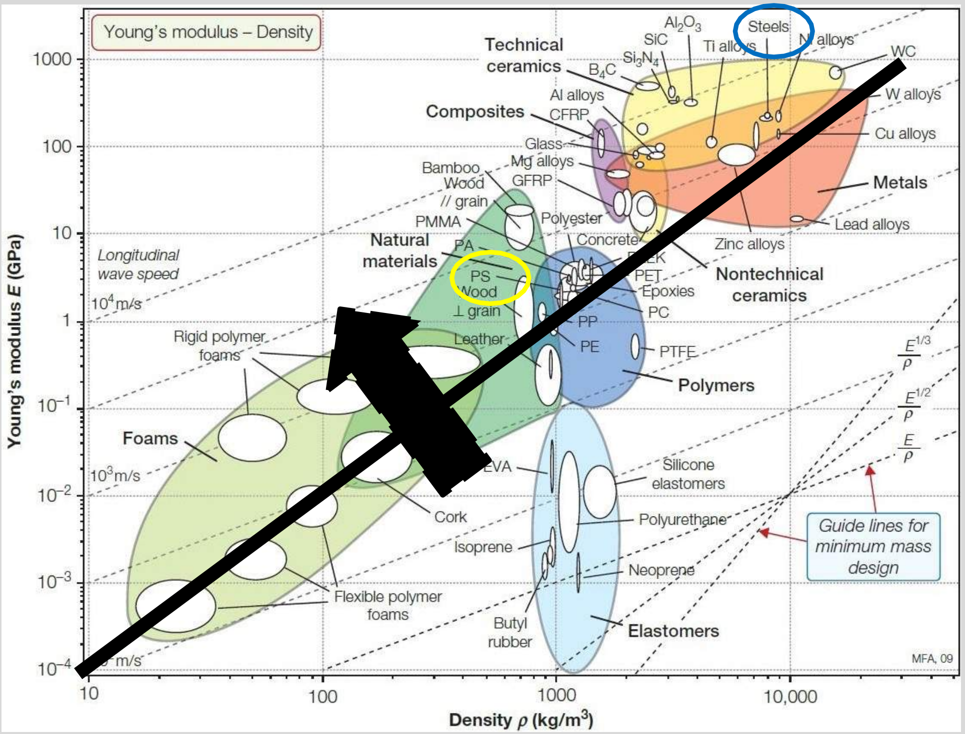
Ashby Diagrams



The Material Index (M)

Case Study 1:
Find the Lightest STIFF <u>Beam</u>

$m \downarrow \quad \frac{E^{1/2}}{\rho} \uparrow$



1 2 Slope

Stainless Steel
(E = 200 GPa; ρ = 7800 kg/m³)
Polystyrene
(E = 2 GPa; ρ = 1040 kg/m³)

Lightest Beam (Bending conditions)

Case Study 1:
Find the Lightest STIFF <u>Beam</u>

Length: 300 mm
~~Thickness = 1 mm~~
~~Width = 25 mm~~

Width and thickness ?

F = 100 N
δ = 0,34 mm
$S_{min} = 296 \cdot 10^3$ N/m

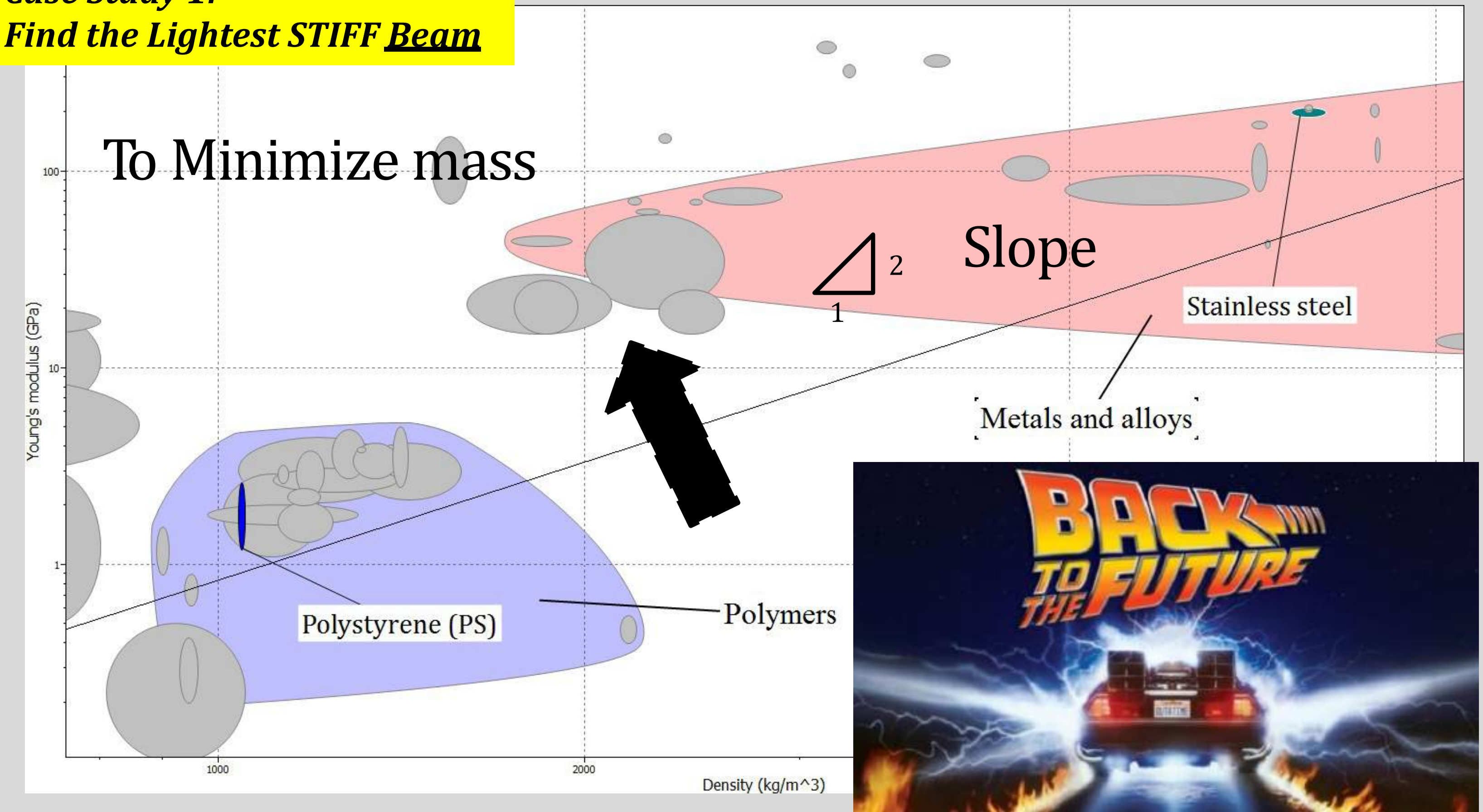
Stainless Steel (E = 200 GPa; ρ = 7800 kg/m³)
Polystyrene (E = 2 GPa; ρ = 1040 kg/m³)

$$\begin{cases} m \geq \left(\dfrac{12 \cdot S}{C_1 \cdot L}\right)^{1/2} \cdot L^3 \cdot \dfrac{\rho}{E^{1/2}} \\ A = \dfrac{m}{L \cdot \rho} \end{cases}$$

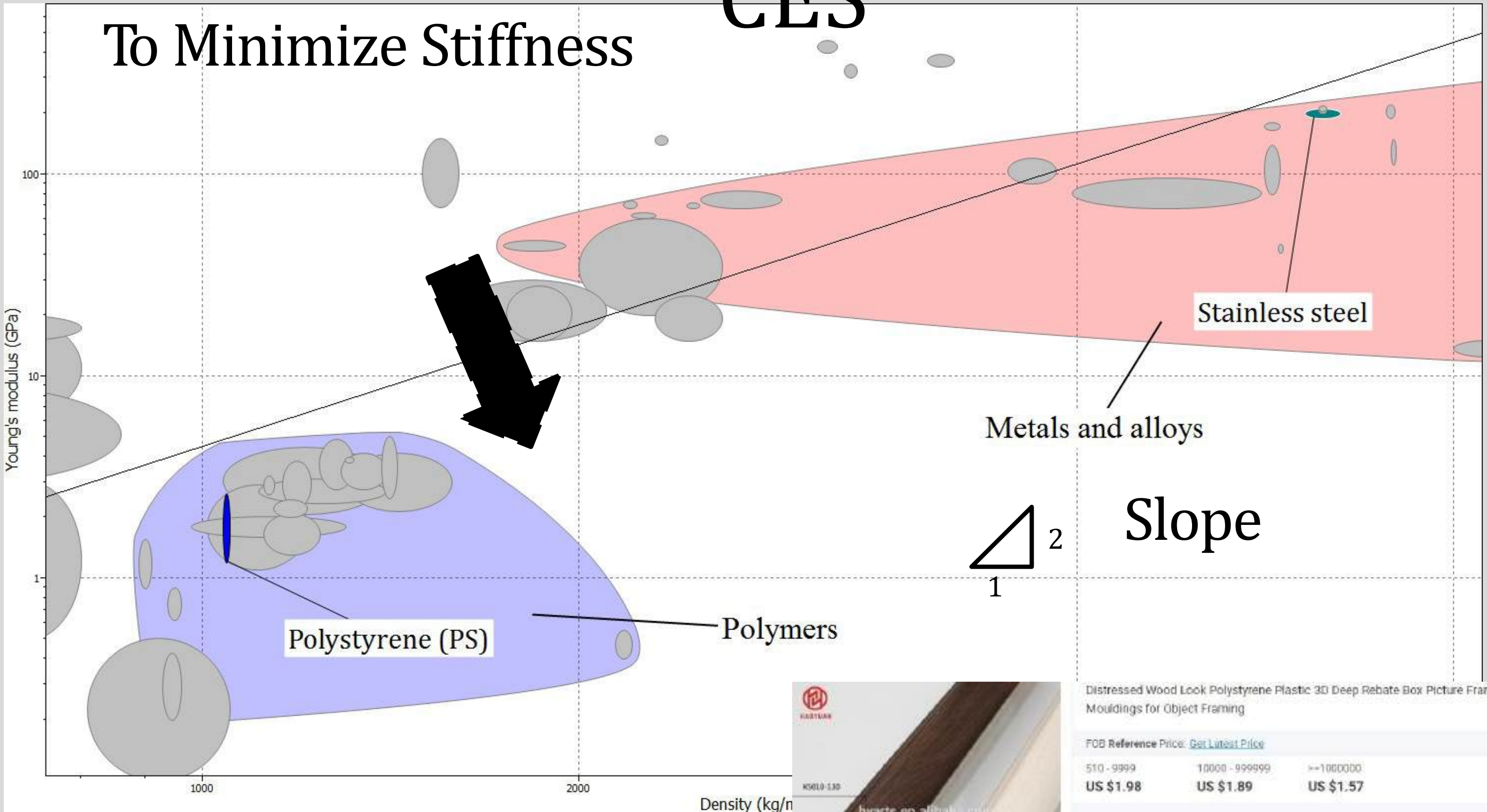
Material	**Weight (kg)**	**A (mm^2)**	**Width and Thickness (mm)**
Stainless Steel	0,935	400	20
Polystyrene	1,25	4000	63

CES

Case Study 1:
Find the Lightest STIFF <u>Beam</u>



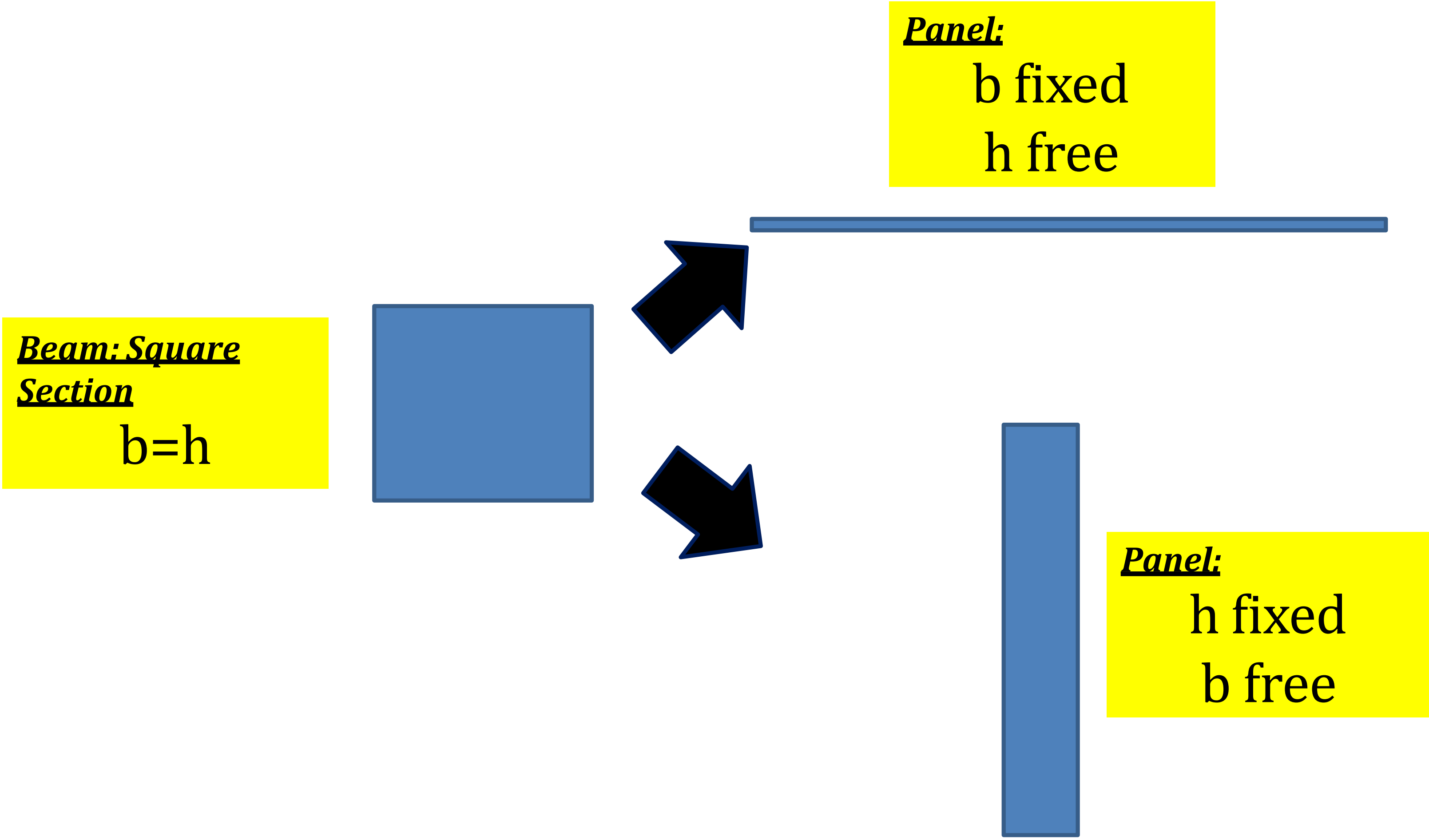
CES



Depends on what you need→

<u>It is not in flexion</u>

Change of the section



Summary (to minimize the mass)

Stiffness – Traction :

Name	Stage 1: Index
Silicon carbide	0.136
Aluminum nitride	0.0984
Alumina	0.094
CFRP, epoxy matrix (isotropic)	0.0657
Silicon	0.0634
Tungsten carbides	0.0425
Silica glass	0.0323
Soda-lime glass	0.0284
Borosilicate glass	0.0278
Aluminum alloys	0.0277
Bamboo	0.025
Wood, typical along grain	0.0158

Stiffness – Bending (Beam):

Name	Stage 1: Index
Silicon carbide	0.00657
CFRP, epoxy matrix (isotropic)	0.00651
Bamboo	0.00601
Aluminum nitride	0.00546
Silicon	0.00522
Alumina	0.00492
Wood, typical along grain	0.00478
Rigid Polymer Foam (LD)	0.00413
Silica glass	0.00384
Magnesium alloys	0.00362

Stiffness – Bending (Panel):

Name	Stage 1: Index
Rigid Polymer Foam (LD)	0.00697
Rigid Polymer Foam (MD)	0.00442
Bamboo	0.00373
Flexible Polymer Foam (VLD)	0.00335
Wood, typical along grain	0.00321
CFRP, epoxy matrix (isotropic)	0.00301
Paper and cardboard	0.00269
Rigid Polymer Foam (HD)	0.00239
Flexible Polymer Foam (LD)	0.00233
Flexible Polymer Foam (MD)	0.00212
Cork	0.00173

$\frac{E}{\rho}$	$\frac{E^{1/2}}{\rho}$	$\frac{E^{1/3}}{\rho}$
$\frac{\sigma_y}{\rho}$	$\frac{{\sigma_y}^{2/3}}{\rho}$	$\frac{{\sigma_y}^{1/2}}{\rho}$

Strength – Traction :

Name	Stage 1: Index
CFRP, epoxy matrix (isotropic)	0.491
Silicon carbide	0.157
Titanium alloys	0.121
Alumina	0.117
Low alloy steel	0.0987
Aluminum nitride	0.0983
Magnesium alloys	0.0908
High carbon steel	0.0866
GFRP, epoxy matrix (isotropic)	0.0783
Medium carbon steel	0.0667
Wood, typical along grain	0.0661
Cast iron, ductile (nodular)	0.0577
Stainless steel	0.0526
Aluminum alloys	0.0455
Nickel alloys	0.0312

Strength – Bending :

Name	Stage 1: Index
CFRP, epoxy matrix (isotropic)	0.0538
Silicon carbide	0.0198
Wood, typical along grain	0.0185
Magnesium alloys	0.0165
Rigid Polymer Foam (LD)	0.0159
Titanium alloys	0.0147
Rigid Polymer Foam (MD)	0.00986
Aluminum alloys	0.00916

Strength – Bending (Panel):

Name	Stage 1: Index
CFRP, epoxy matrix (isotropic)	0.0178
Rigid Polymer Foam (LD)	0.0168
Wood, typical along grain	0.00977
Rigid Polymer Foam (MD)	0.00959
Bamboo	0.00904
Flexible Polymer Foam (VLD)	0.00787
Paper and cardboard	0.0074
Magnesium alloys	0.00702
Rigid Polymer Foam (HD)	0.00623
Flexible Polymer Foam (LD)	0.0054

Chart stage: select performance index finder

Identify function (choice of 43)

Panel in bending

Identify free variables

Panel thickness

Identify variables and constraints

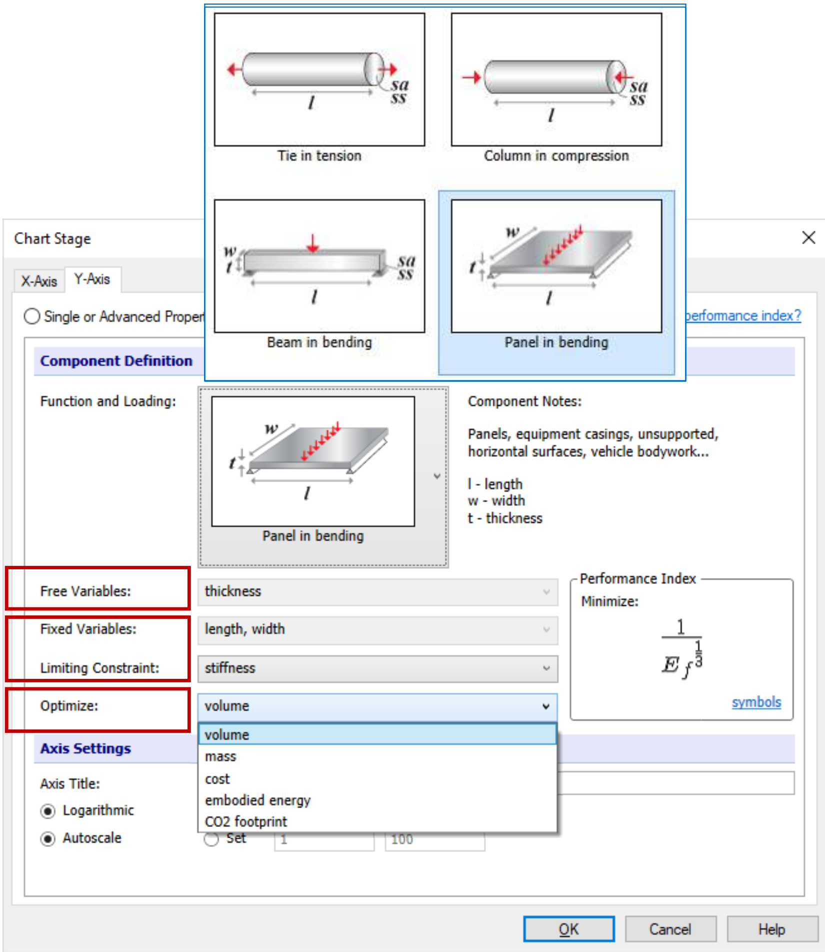
Panel length

Panel width

Stiffness limited design

Identify objective

Default is to minimize



Minimum weight design - indices

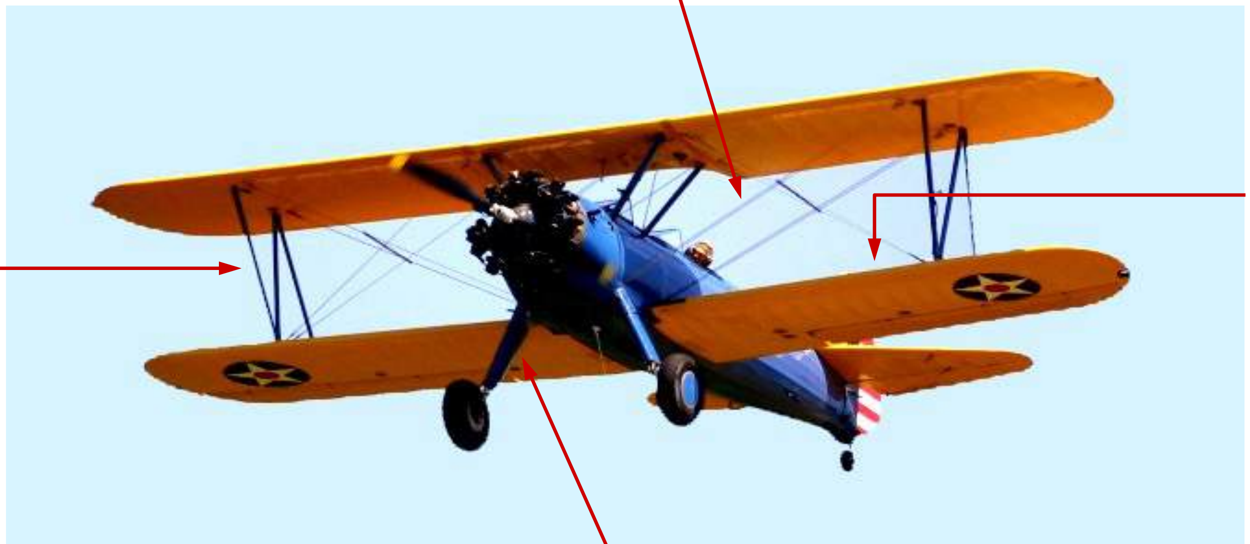
GRANTA

Tensile ties

$$\left(\frac{\sigma_y}{\rho}\right)$$

Compression strut

$$\left(\frac{E^{1/2}}{\rho}\right)$$



Main spar - beam

$$\left(\frac{E^{1/2}}{\rho}\right)$$

Undercarriage - bending and compression

$$\left(\frac{\sigma_y^{2/3}}{\rho}\right)$$

E = Young's modulus
ρ = Density
σ_y = Yield strength

The marked components of this plane perform different functions. The ties carry tension, the struts carry compression (they act as columns) and the spars carry bending moments – they are beams. They are chose to be as light as possible: thus the objective is to minimize mass.

The mass of a tensile tie of prescribed strength depends on two material properties – yield strength and density – in the combination σ_y/ρ; it is the material index for this component.

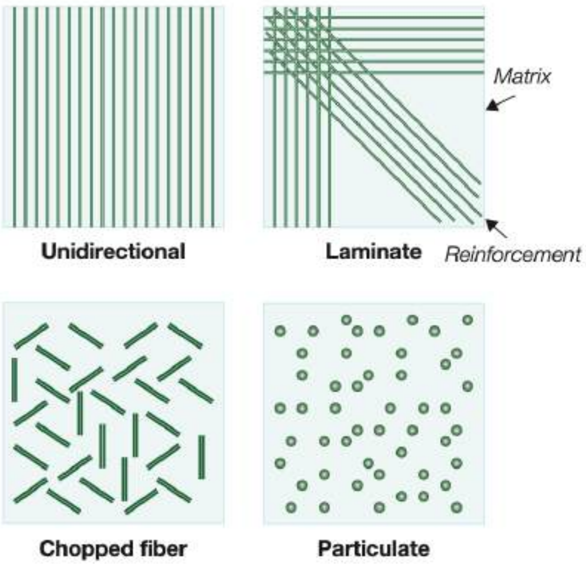
The mass of a strut that must carry a compressive load without buckling elastically is proportional to the material group $E^{1/2}/\rho$ so this becomes the material index.

The mass of a beam, loaded in bending, with restriction on elastic deflection is also proportional to $E^{1/2}/\rho$ so the index here is the same as that for the strut.

Thus the index depends on the mode of loading (tension, compression, bending) and on the requirement for stiffness or strength.

PART3bis

- On small exercices



Exercise E4.15 (a) Use the material property data in the table below to find the composite density $\tilde{\rho}$ and upper bound modulus $\tilde{E}_U$ for the following composites: (i) carbon-fibre/epoxy resin ($f = 0.5$), (ii) glass-fibre/polyester resin ($f = 0.5$).

Material	Density (Mg m^{-3})	Young's modulus (GPa)
Carbon fibre	1.90	390
Glass fibre	2.55	72
Epoxy resin	1.15	3
Polyester resin	1.15	3

(b) A magnesium company has developed an experimental metal-matrix composite (MMC), by casting magnesium containing 20% (by volume) of particulate SiC. Use the lower bound estimate for Young's modulus to estimate $\tilde{E}_L$ for this MMC, with the following data for matrix and particles: Mg alloys, $E = 44.5$ GPa, $\rho = 1.85$ Mgm^{-3}; SiC, $E = 380$ GPa, $\rho = 3.1$ Mgm^{-3}.

(c) Find the *specific stiffness* $\tilde{E}/\tilde{\rho}$ for the three composites in (a,b). Compare the composites with steels, for which $E/\rho \approx 28$ GPa Mg^{-1} m^3.

RoM

In materials science, a **general rule of mixtures** is a weighted mean used to predict various properties of a composite material .[1][2][3] It provides a theoretical upper- and lower-bound on properties such as the elastic modulus, ultimate tensile strength, thermal conductivity, and electrical conductivity.[3] In general there are two models, one for axial loading (Voigt model),[2][4] and one for transverse loading (Reuss model).[2][5]

In general, for some material property E (often the elastic modulus[1]), the rule of mixtures states that the overall property in the direction parallel to the fibers may be as high as

$$E_c = fE_f + (1-f)\,E_m$$

where

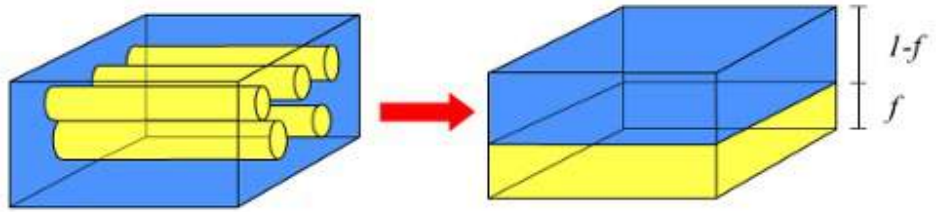
- $f = \dfrac{V_f}{V_f + V_m}$ is the volume fraction of the fibers
- E_f is the material property of the fibers
- E_m is the material property of the matrix

Similar derivations give the rules of mixtures for

- mass density:

$$\rho_c = \rho_f \cdot f + \rho_M \cdot (1-f)$$

where f is the atomic percent of fiber in the mixture.



Composite material showing aligned fibres within the matrix

The approximation used in the Rule of Mixtures

Answers

Exercise E4.15 (a) Use the material property data in the table below to find the composite density $\tilde{\rho}$ and upper bound modulus $\tilde{E}_U$ for the following composites: (i) carbon-fibre/epoxy resin ($f = 0.5$), (ii) glass-fibre/polyester resin ($f = 0.5$).

Material	Density (Mg m^{-3})	Young's modulus (GPa)
Carbon fibre	1.90	390
Glass fibre	2.55	72
Epoxy resin	1.15	3
Polyester resin	1.15	3

Answer.

(a) Density and Young's moduli of the fibre composites are both derived using the rule of mixtures:

Carbon-fibre epoxy: $\tilde{\rho} = (0.5 \times 1.90) + (0.5 \times 1.15) = 1.53$ Mgm^{-3}

$$\tilde{E}_U = (0.5 \times 390) + (0.5 \times 3) = 197 \text{ GPa}$$

Glass-fibre polyester: $\tilde{\rho} = (0.5 \times 2.55) + (0.5 \times 1.15) = 1.85$ Mgm^{-3}

$$\tilde{E}_U = (0.5 \times 72) + (0.5 \times 3) = 37.5 \text{ GPa}$$

Answers

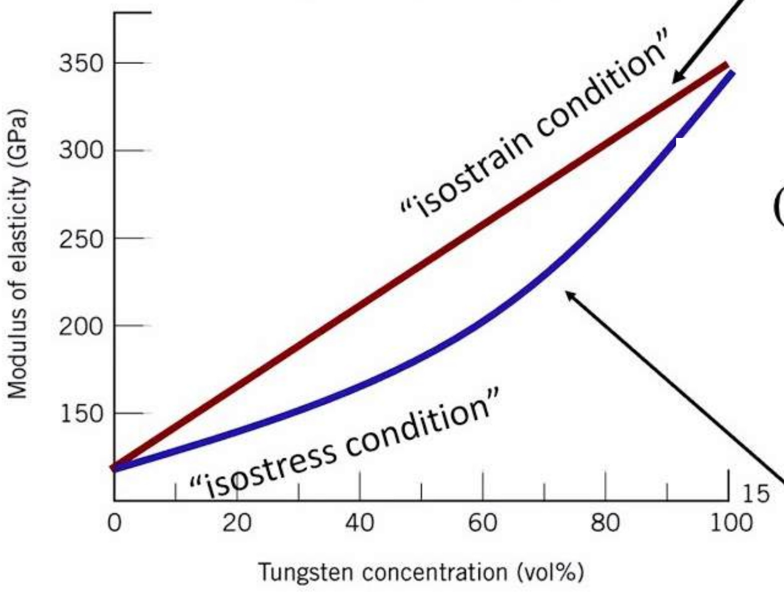
https://www.youtube.com/watch?v=ko73yHImLPU

(b) A magnesium company has developed an experimental metal-matrix composite (MMC), by casting magnesium containing 20% (by volume) of particulate SiC. Use the lower bound estimate for Young's modulus to estimate $\widetilde{E}_L$ for this MMC, with the following data for matrix and particles: Mg alloys, $E = 44.5$ GPa, $\rho = 1.85$ Mgm^{-3}; SiC, $E = 380$ GPa, $\rho = 3.1$ Mgm^{-3}.

Composites: particle-reinforced

- Elastic modulus of composites, E_c, (example, tungsten particles in Cu):

-- two "rule of mixture" guides to predict properties:



upper bound:

$$E_c = V_m E_m + V_p E_p$$

volume fraction-weighted average

lower bound:

$$\frac{1}{E_c} = V_m \frac{1}{E_m} + V_p \frac{1}{E_p}$$

Volume fraction-weighted inverse sum

EMSM207

(b) Magnesium – SiC: $\widetilde{\rho} = (0.2 \times 3.1) + (0.8 \times 1.85) = 2.1$ Mg m^{-3}

$$\widetilde{E}_L = 1/[(0.2/380) + (0.8/44.5)] = 54.0 \text{ GPa}$$

Answers

(c) Find the *specific stiffness* $\tilde{E}/\tilde{\rho}$ for the three composites in (a,b). Compare the composites with steels, for which $E/\rho \approx 28$ GPa Mg^{-1} m^3.

(c)

Composite	$\tilde{E}/\tilde{\rho}$ (GPa Mg^{-1}m^3)
Carbon-fibre/epoxy resin	129
Glass-fibre/polyester resin	20
Mg/SiC particulate	26
Steels	28

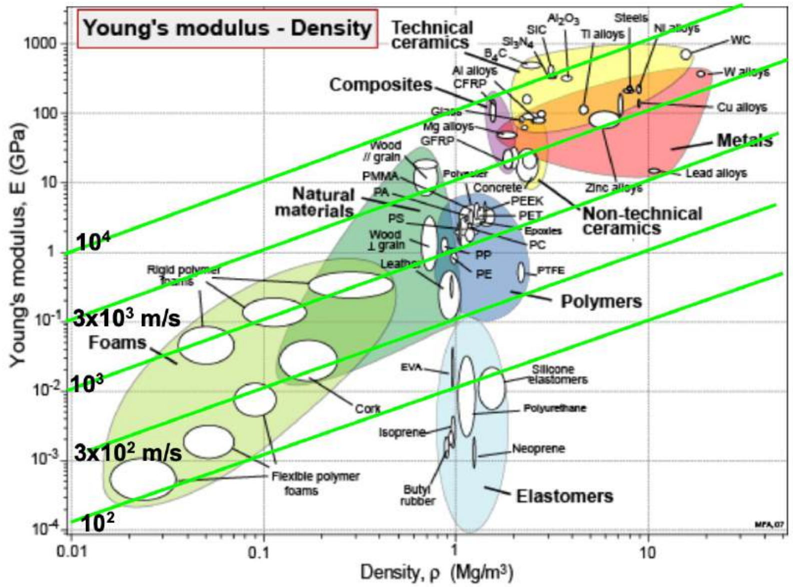
Carbon-fibre/epoxy resin has much higher specific stiffness than steels, while the MMC is comparable to steels, and the glass-fibre polyester has a lower value.

[*Footnote*: specific stiffness is a good indicator for tensile loading, but remember that other combinations of E and ρ are usually appropriate in *bending* – e.g. $E^{1/3}/\rho$. In this case it is readily shown that all of the composites perform better than steels. But there will be other important factors to consider in using one of the composites instead of steel: geometry and shape limits, cost, strength, durability, ability to manufacture and join etc.]

A simple map of E/rho

Exercise E4.18 The speed of longitudinal waves in a material is proportional to $\sqrt{E/\rho}$. Plot contours of this quantity onto a copy of a modulus-density chart allowing you to read off approximate values for any material on the chart. Which metals have the about the same sound velocity as steel? Does sound move faster in titanium or glass?

Answer. The figure shows the chart of Figure 4.6 onto which contours of $\sqrt{E/\rho}$ are plotted (remember to multiply E by 10^9 and ρ in the units of this chart by 10^3 in order to get the velocity in m/s). Tungsten, titanium, nickel, aluminum, magnesium and steel all have about the same value of $\sqrt{E/\rho}$, and thus similar sound velocities. The sound velocity in glass is a little higher than that in titanium.



Speed of longitudinal waves in a solid rod:

$$\boxed{v = \sqrt{\frac{E}{\rho}}} \qquad \boxed{\rho = m/V}$$

E - Young's modulus
ρ - density
m - mass
V - volume

Young's modulus (optional):

$$\boxed{E = \frac{\tau}{\varepsilon}}$$

Stress: $\tau = F_{\perp}/A$

Strain: $\varepsilon = \Delta L / L_0$

The Young's modulus is the stress divided by the strain.

PART4

- Environnemental impact

Eco-properties	Energy required per unit mass to produce material (embodied energy)	H_m	MJ/kg
	CO_2 footprint (CO_2 mass produced per unit mass of material produced)	CO_2	kg/kg

GHG footprint per mass

- Usually in databases :

Primary production CO2 footprint (Eco)
Processing CO2 footprint (Ecp)
Recycling CO2 footprint (Ecr)
Recycle fraction in current supply (Fr)

Ec= Fr*Ecr + (1-Fr)*Eco + Ecp
-> enables to take into account multiple stages of the life cycle.

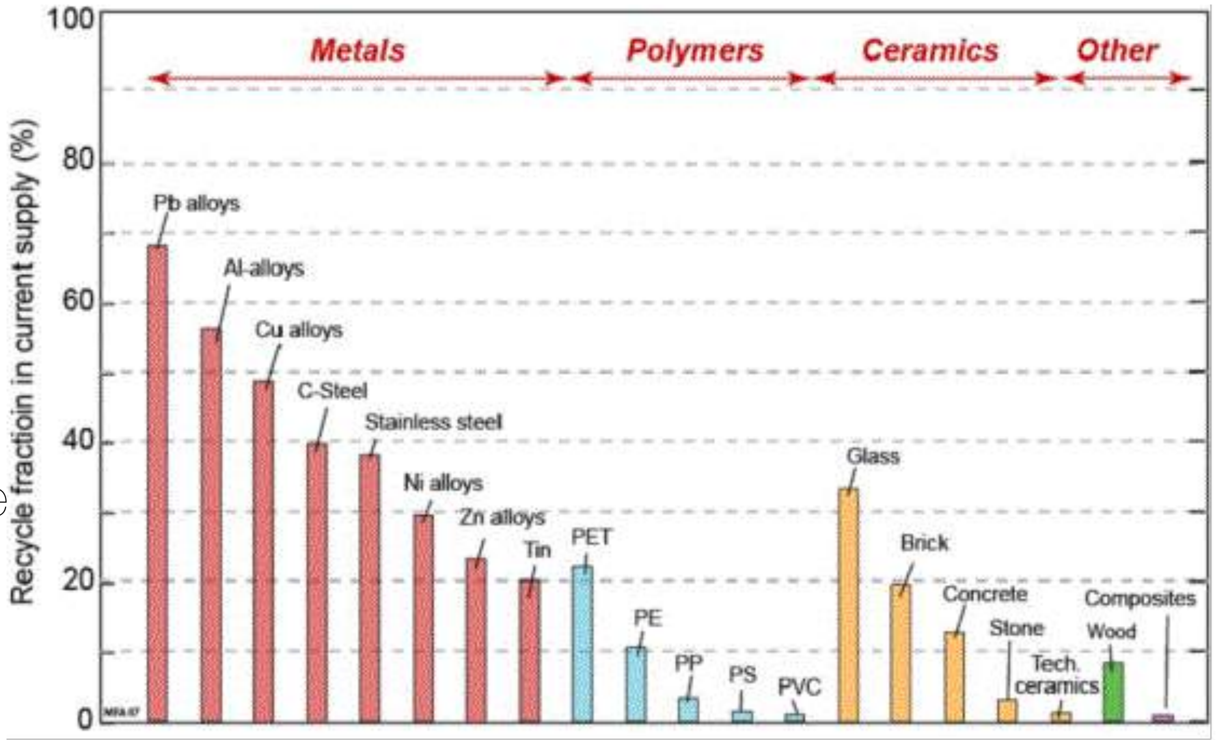


Figure 1. The fractional contribution of recycled material to current consumption. For metals, the contribution is large; for polymers, small (2005/06 data).

GHG footprint per mass

- Example for Aluminum in CES Edupack

Primary production energy, CO2 and water

Embodied energy, primary production	ⓘ	* 45	-	49,6	MJ/kg
CO2 footprint, primary production	ⓘ	* 3,46	-	3,81	kg/kg
Water usage	ⓘ	* 135	-	150	l/kg

Recycling and end of life

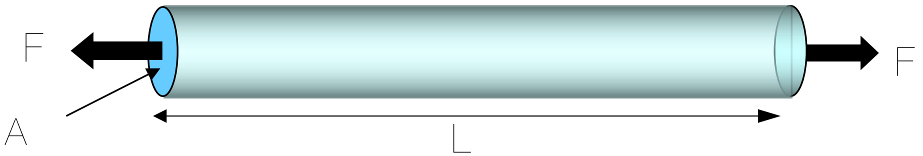
Recycle	ⓘ	✓			
Embodied energy, recycling	ⓘ	* 10,8	-	12	MJ/kg
CO2 footprint, recycling	ⓘ	* 0,85	-	0,94	kg/kg
Recycle fraction in current supply	ⓘ	52,3	-	57,8	%
Downcycle	ⓘ	✓			
Combust for energy recovery	ⓘ	✗			
Landfill	ⓘ	✓			
Biodegrade	ⓘ	✗			

Processing energy, CO2 footprint & water

Casting energy	ⓘ	* 10,9	-	12,1	MJ/kg
Casting CO2	ⓘ	* 0,818	-	0,904	kg/kg
Casting water	ⓘ	* 20,6	-	31	l/kg
Roll forming, forging energy	ⓘ	* 15,6	-	17,2	MJ/kg
Roll forming, forging CO2	ⓘ	* 1,17	-	1,29	kg/kg
Roll forming, forging water	ⓘ	* 8,2	-	12,3	l/kg
Extrusion, foil rolling energy	ⓘ	* 30,8	-	34,1	MJ/kg
Extrusion, foil rolling CO2	ⓘ	* 2,31	-	2,56	kg/kg
Extrusion, foil rolling water	ⓘ	* 14,7	-	22,1	l/kg
Wire drawing energy	ⓘ	* 115	-	127	MJ/kg
Wire drawing CO2	ⓘ	* 8,61	-	9,52	kg/kg
Wire drawing water	ⓘ	* 43,3	-	64,9	l/kg
Metal powder forming energy	ⓘ	* 37,5	-	41,4	MJ/kg
Metal powder forming CO2	ⓘ	* 3	-	3,32	kg/kg
Metal powder forming water	ⓘ	* 40,9	-	61,4	l/kg
Vaporization energy	ⓘ	* 1,09e4	-	1,2e4	MJ/kg
Vaporization CO2	ⓘ	* 815	-	901	kg/kg
Vaporization water	ⓘ	* 4,53e3	-	6,8e3	l/kg
Coarse machining energy (per unit wt removed)	ⓘ	* 2,77	-	3,06	MJ/kg
Coarse machining CO2 (per unit wt removed)	ⓘ	* 0,207	-	0,229	kg/kg
Fine machining energy (per unit wt removed)	ⓘ	* 23,4	-	25,8	MJ/kg
Fine machining CO2 (per unit wt removed)	ⓘ	* 1,75	-	1,94	kg/kg

Example of carbon performance index

- We want to minimize the GHG **GreenHouse Gas** footprint of a bar of length L under a load F, while staying in the elastic domain. The section and the material are free.



- P : $1/CO2 = 1/(Ec*\boldsymbol{\rho}*L*A)$ (1)
- F : $F/A = \sigma < \boldsymbol{\sigma_y}$

(stay in the elastic domain)

We go to the limit=> $A= F/\boldsymbol{\sigma_y}$ (2)

- (1)+(2) : $P = \boldsymbol{\sigma_y} /(Ec*\boldsymbol{\rho}*L*F)$
- $P = 1/F * 1/L * \boldsymbol{\sigma_y} /(Ec*\boldsymbol{\rho})$

  $f_1(F)$ $f_2(G)$ $f_3(M)$

m = mass
A = section
L = length
ρ = density
σ_y= yield strength
Ec= GHG footprint per mass (kg_{eqCO2}/kg)

=> We will choose the material maximizing the index $M=\boldsymbol{\sigma_y} /(Ec*\boldsymbol{\rho})$

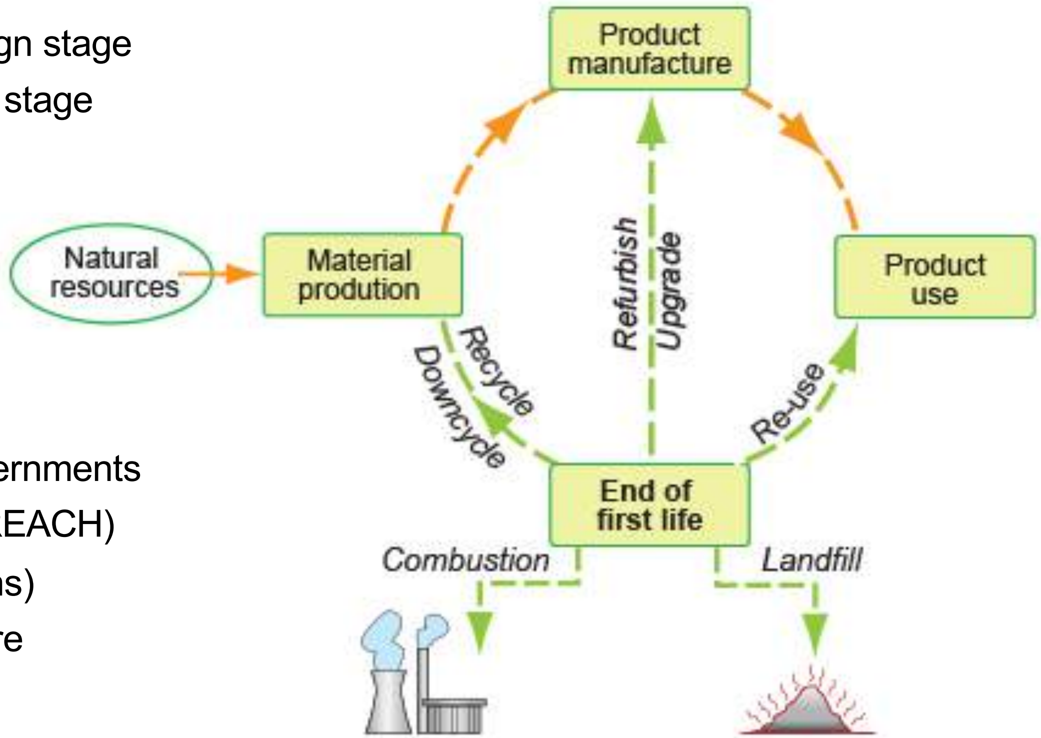
Eco-informed design

- 80% of eco-impact tied in at design stage
- Build-in eco criteria at the design stage

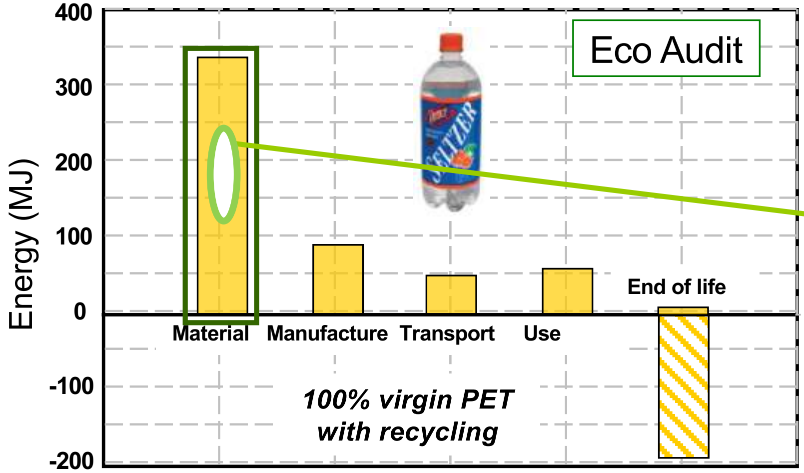
The drivers for eco-design

- Focus on carbon footprint by governments
- Legislation (Carbon taxes, EuP, REACH)
- Incentives (Subsidies, concessions)
- Urge for “responsible” manufacture
- Doing more with less = $$$

The materials life-cycle



Eco-selection for a fizzy drink bottle



Material dominates

Minimize embodied energy

Translation

Design brief

Improve green credentials of bottle

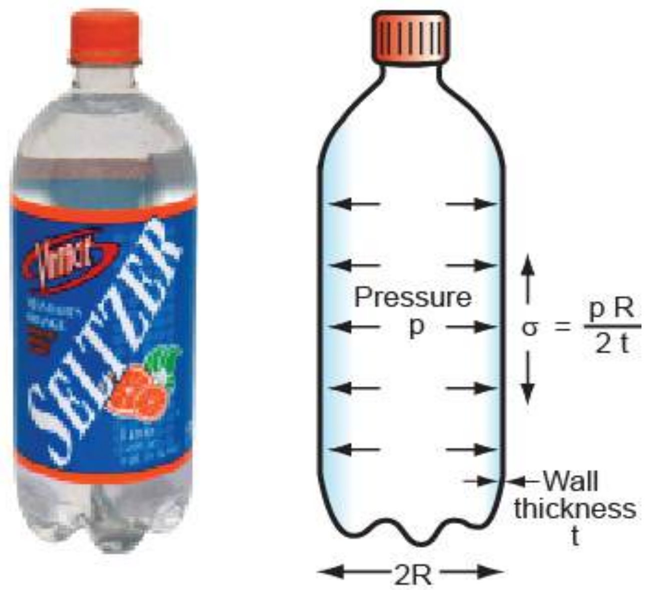
⇨

Constraints

- **Able to be molded**
- **Transparent / translucent**
- **Able to contain pressure**

Objectives

- **Minimize embodied energy of bottle**
- **Minimize material cost of bottle**



R= Bottle radius
t = Thickness of bottle wall
p = Internal pressure
σ_y = Yield strength of material
ρ = Density of material
H_m = Embodied energy of material/kg
E = Embodied energy/m^2 of wall
C_m = Material cost per kg

Cylindrical pressure vessel

- Circumferential stress $\sigma = \frac{pR}{t} < \sigma_y$
- Embodied energy per unit area of wall

$$E = t H_m \rho = pR \frac{H_m \rho}{\sigma_y}$$

Embodied energy / kg of material

- Find material with lowest energy, seek largest

$$\boxed{\frac{\sigma_y}{H_m \rho}}$$

- Find material with lowest cost, seek largest

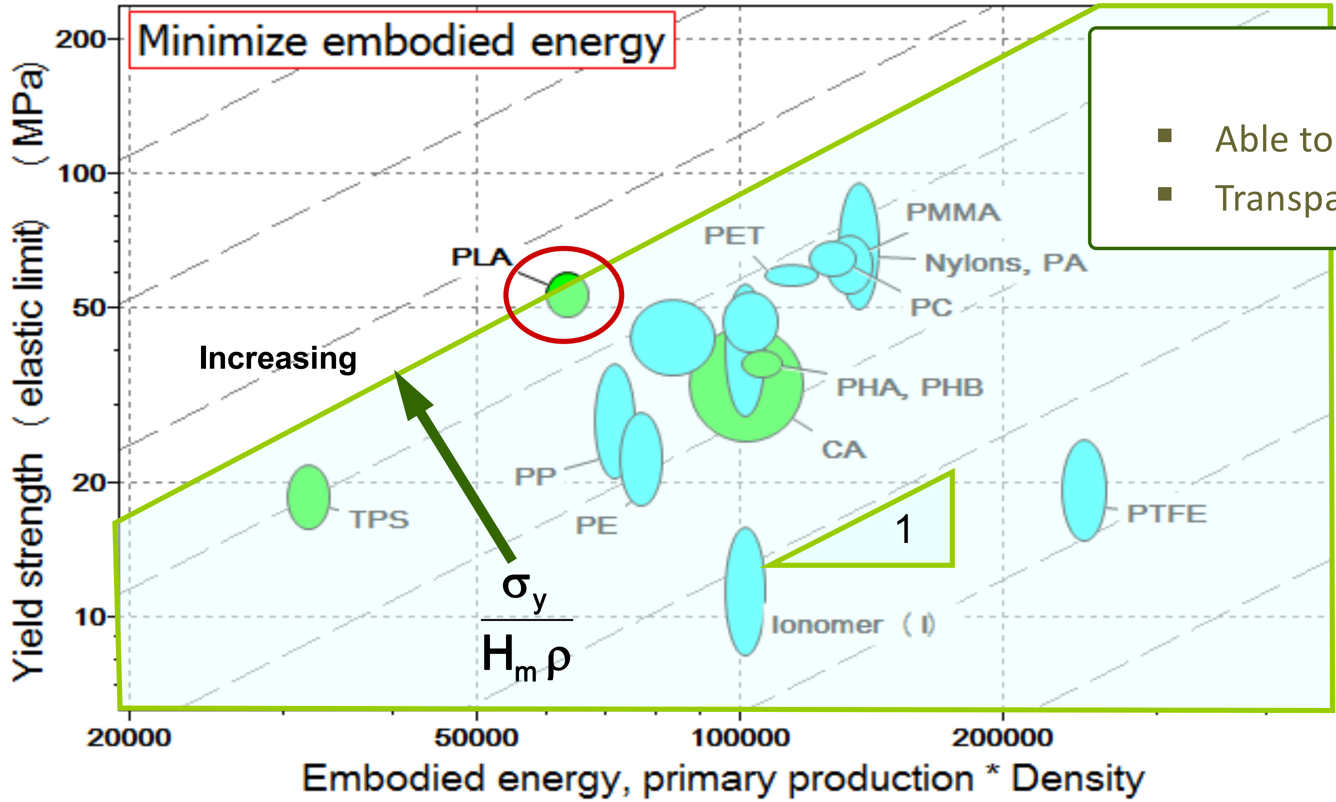
$$\boxed{\frac{\sigma_y}{C_m \rho}}$$

Price / kg of material

Selection to minimize embodied energy

GRANTA

First apply constraints, then use index to optimize choice



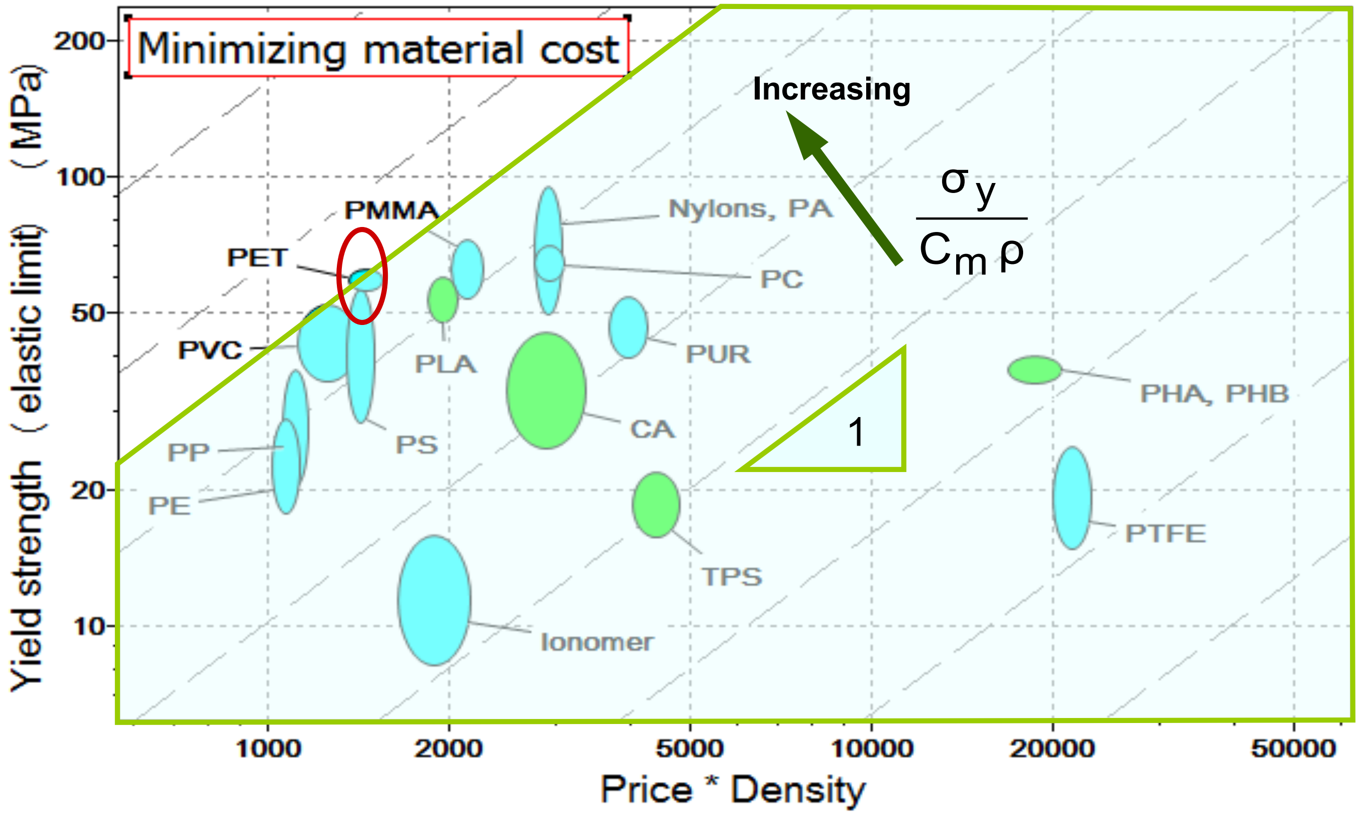
Constraints

- Able to be molded
- Transparent / translucent

Bio-polymers are colored green

PLA meets the constraints at lowest embodied energy

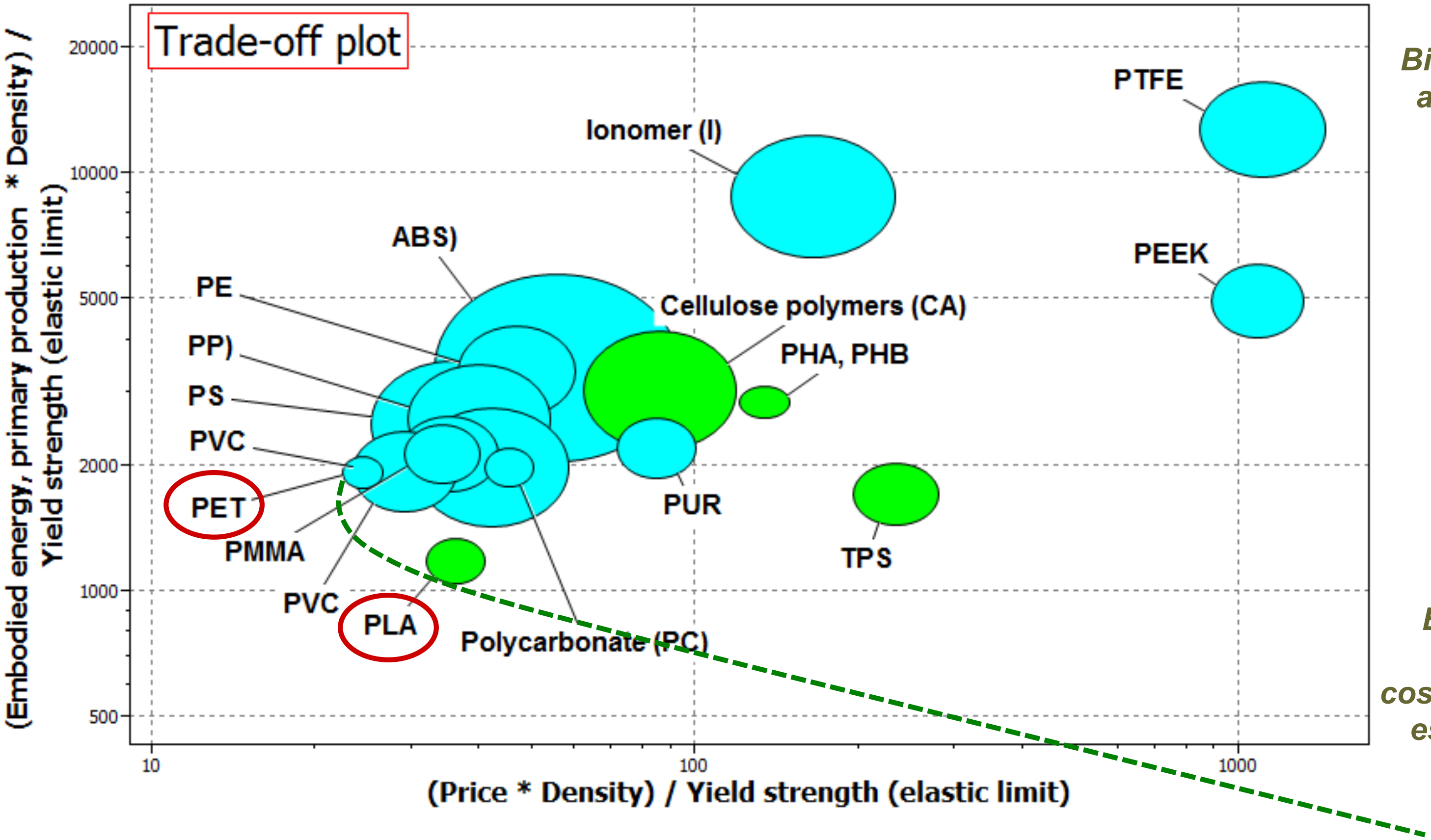
Can’t ignore cost



Bio-polymers are colored green

PET meets the constraints at lowest cost

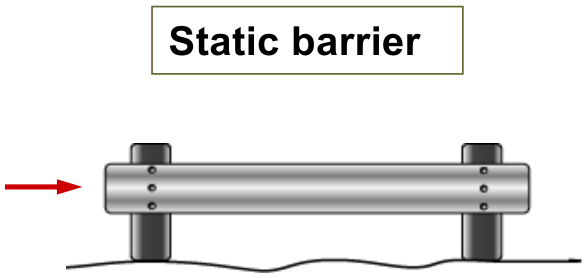
Minimizing both embodied energy and cost

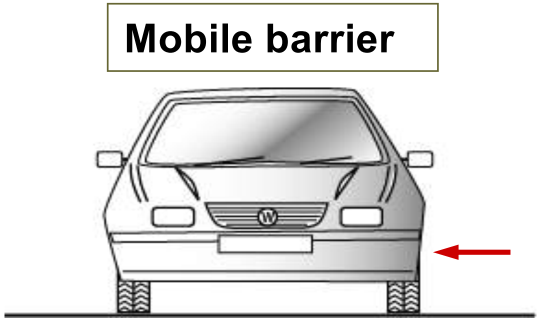


Bio-polymers are colored green

Exchange constant cost/energy not established

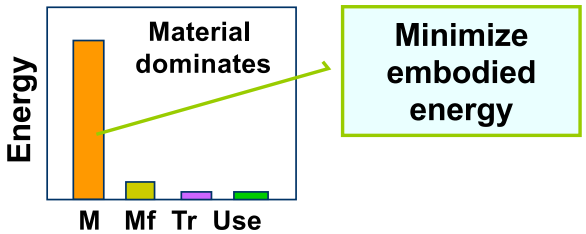
Materials for crash barriers

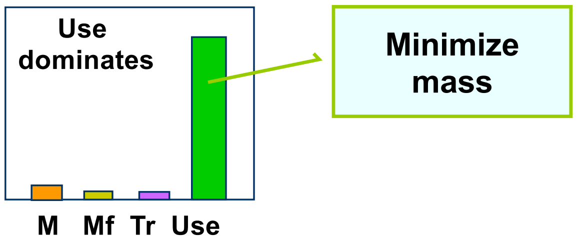




Function: *Absorb impact, transmit load to energy-absorbing units or supports*

Dominant phase of life:





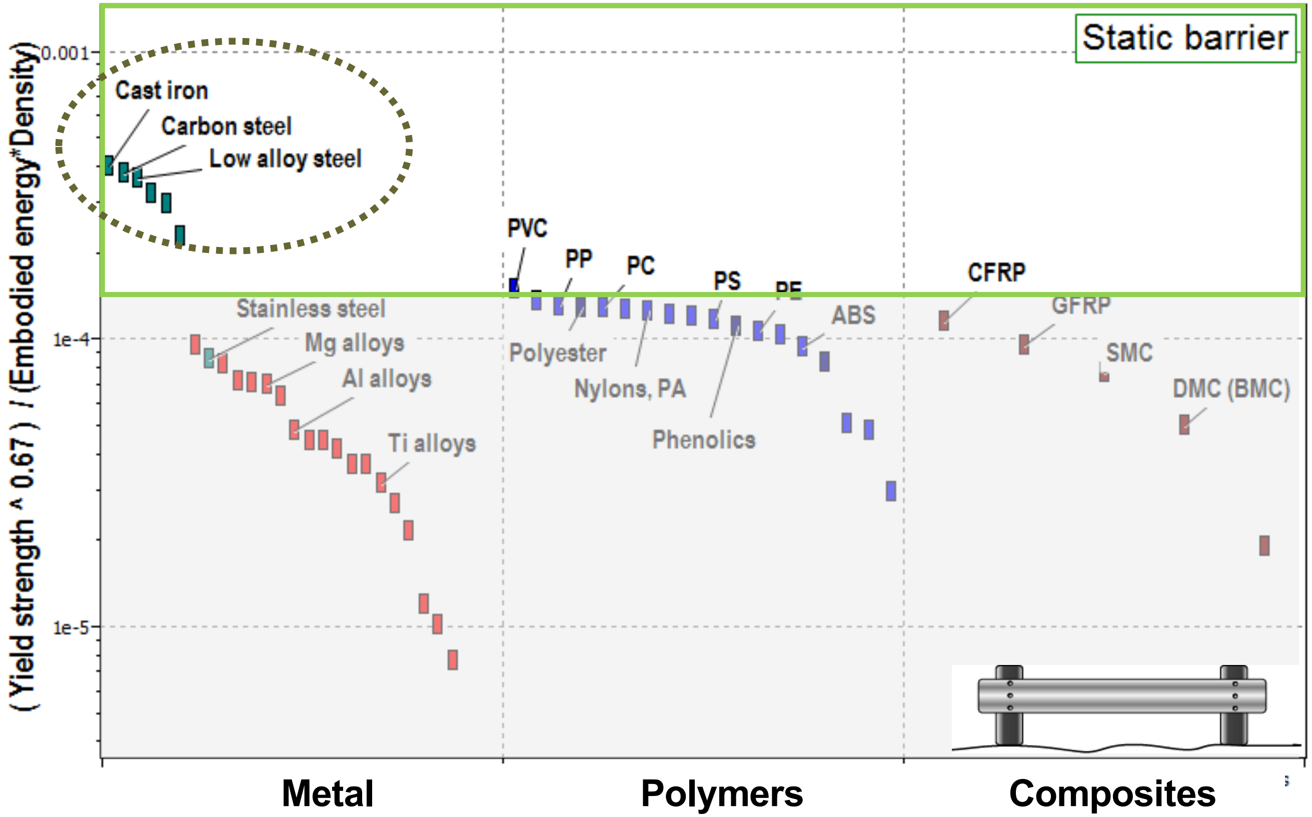
Criterion: *Bending strength per unit embodied energy* | *Bending strength per unit mass*

Index:

$$\frac{\sigma_y^{2/3}}{H_m\,\rho} \qquad \frac{\sigma_y^{2/3}}{\rho}$$

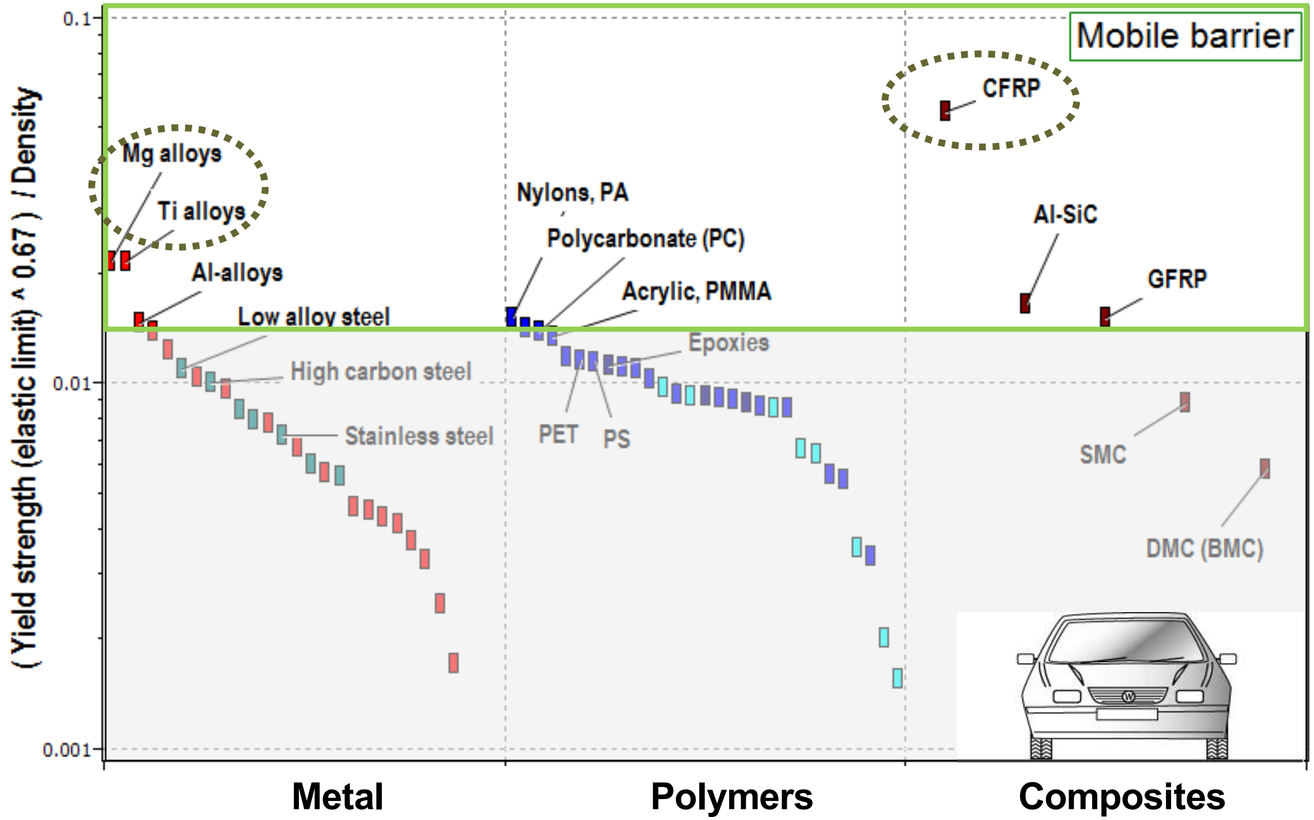
Static barrier: the index as bar chart

$$\frac{\sigma_y^{2/3}}{H_m \rho}$$



Selected materials: *Cast irons, steels*

$$\frac{\sigma_y^{2/3}}{\rho}$$



Selected materials: *CFRP, Mg alloys, Ti alloys, Al alloys*

So what?

- Eco-informed material choice is part of the eco-design process
- An Eco Audit identifies the most damaging phase of life and identifies strategies for overcoming it
- Systematic strategies, using material indices, optimize material choice to minimize life energy
- CES EduPack allows the strategy to be implemented and documents the steps taken to minimize eco-impact.

PART5

- Multiobjective optimization

Optimization with multiple objectives

- Ashby index are well adapted to cases where there is only one objective function in which appear multiple quantities, however in ecodesign we generally have multiple objectives…
- A first step is to make the difference between objectives (to be minimized/maximized) and constraints (to filter).

Typical constraints

The material must be:

- Electrical conductor
- Transparent…

And be enough:

- Stiff
- Strong

And be able to be:

- Cast
- Welded…

=> Easy to take into account

Typical objectives

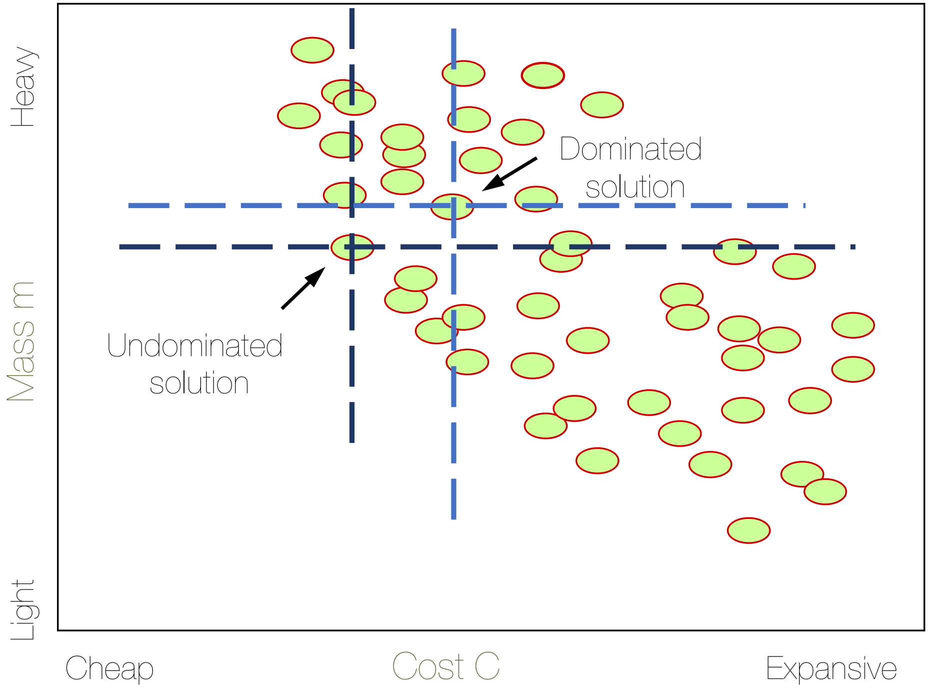
Minimize

- Mass m (satellite components)
- Volume (smartphones)
- Energy consumption (refrigerator)
- GHG footprint (everything)
- Grey energy (materials)
- Cost C (everything)

=> Easy if only one; complex otherwise

Mass-cost example

- Solution : respects the constraints (e.g. transparent, water resistant,…)
- Diagram with both indexes to be minimized
- Get rid of dominated solutions



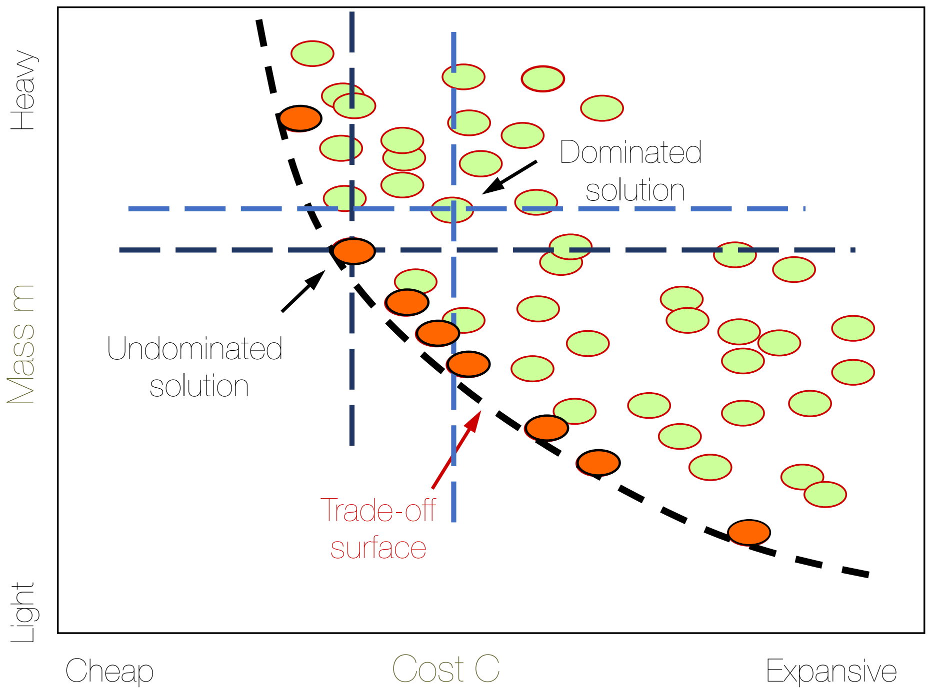
Mass-cost example

- Solution : respects the constraints
(e.g. transparent, water resistant,…)

- Diagram with both indexes to be minimized

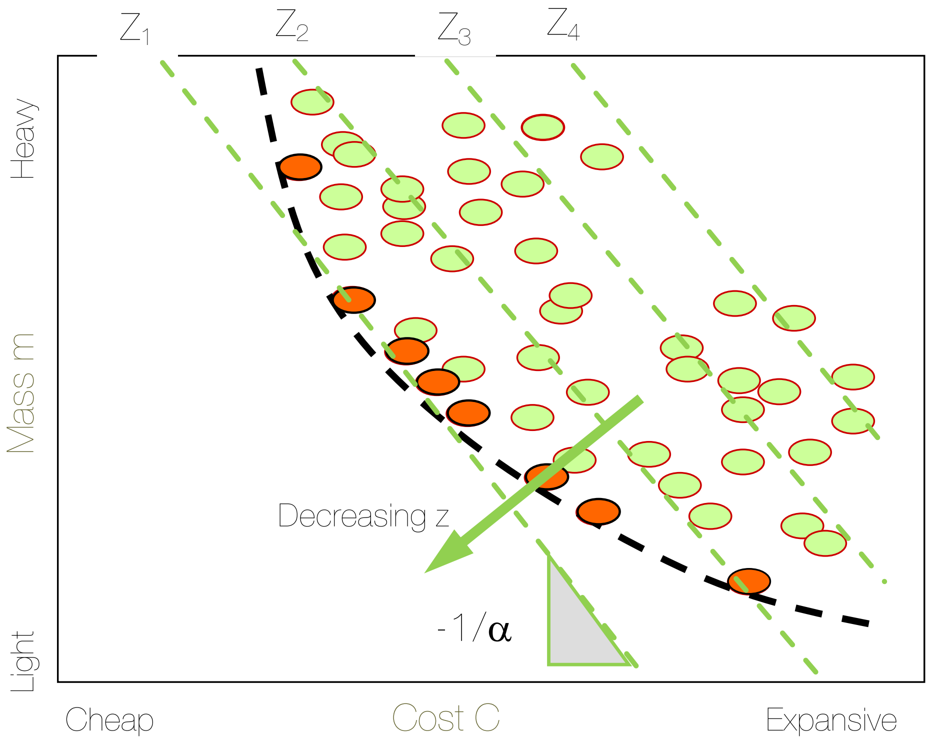
- Get rid of dominated solutions

- We get the Pareto front



Penalty function

- Give a relative importance to the objectives to choose a solution.
- $Z = C + \alpha * m$: new objective
- With linear axis, lines of equation
  $m = 1/\alpha * Z - 1/\alpha * C$
  have the same objective value.



Penalty function

- Give a relative importance to the objectives to choose a solution.
- $Z = C + \alpha * m$: new objective
- With linear axis, lines of equation
  $m = 1/\alpha * Z - 1/\alpha * C$

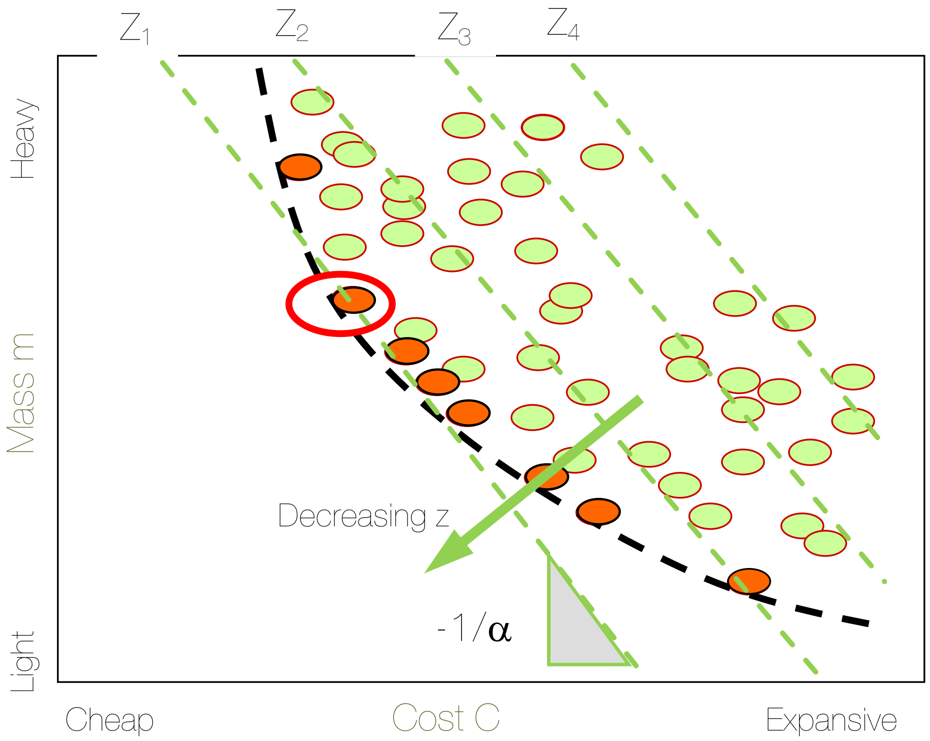
have the same objective value.

$\Rightarrow \alpha$ is important

How much is α for a low-cost bike?

A : 2€/kg C : 20€/kg

B : 5€/kg D : 100€/kg

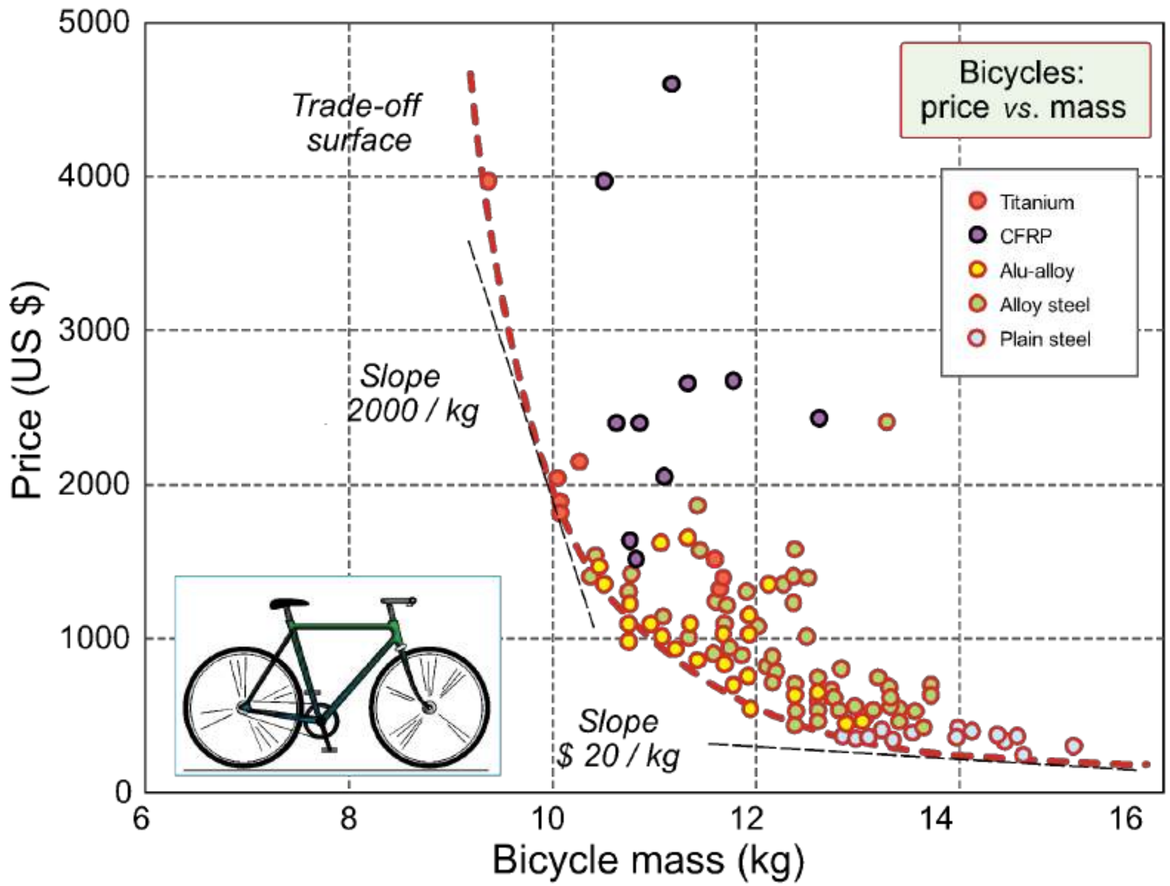


Penalty function

How much is a for a low-cost bike?
A : 2€/kg C : 20€/kg
B : 5€/kg D : 100€/kg

=> Titanium / aluminum / steel?

: steel



Penalty function in transports

- For transports,   mass -> fuel consumption -> cost
- Costs over life time = acquisition cost (proportional to C) + fuel costs (proportional to m)

$\Rightarrow Z = C + \alpha * m$

How much is α for a car?

A : 4€/kg   C : 50€/kg

B : 15€/kg   D : 200€/kg

Penalty function in transports

- For transports, mass -> fuel consumption -> cost
- Costs over life time = acquisition cost (proportional to C) + fuel costs (proportional to m)

$\Rightarrow Z = C + a * m$

How much is a for a car?

A : 4€/kg
B : 15€/kg
C : 50€/kg
D : 200€/kg

How much is a for space applications?

A : 100€/kg
B : 500€/kg
C : 2000€/kg
D : 8000€/kg

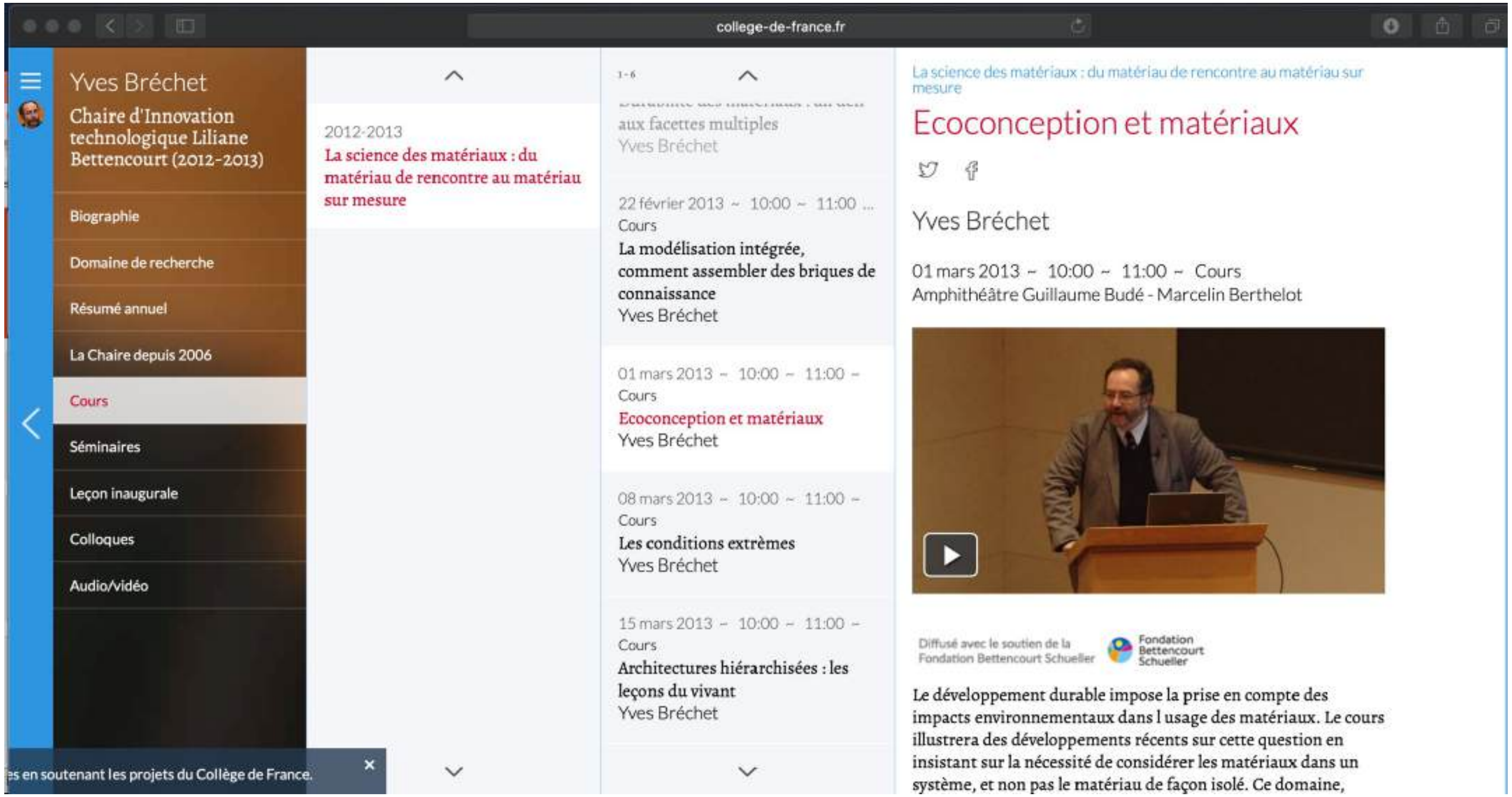
Penalty function in transports

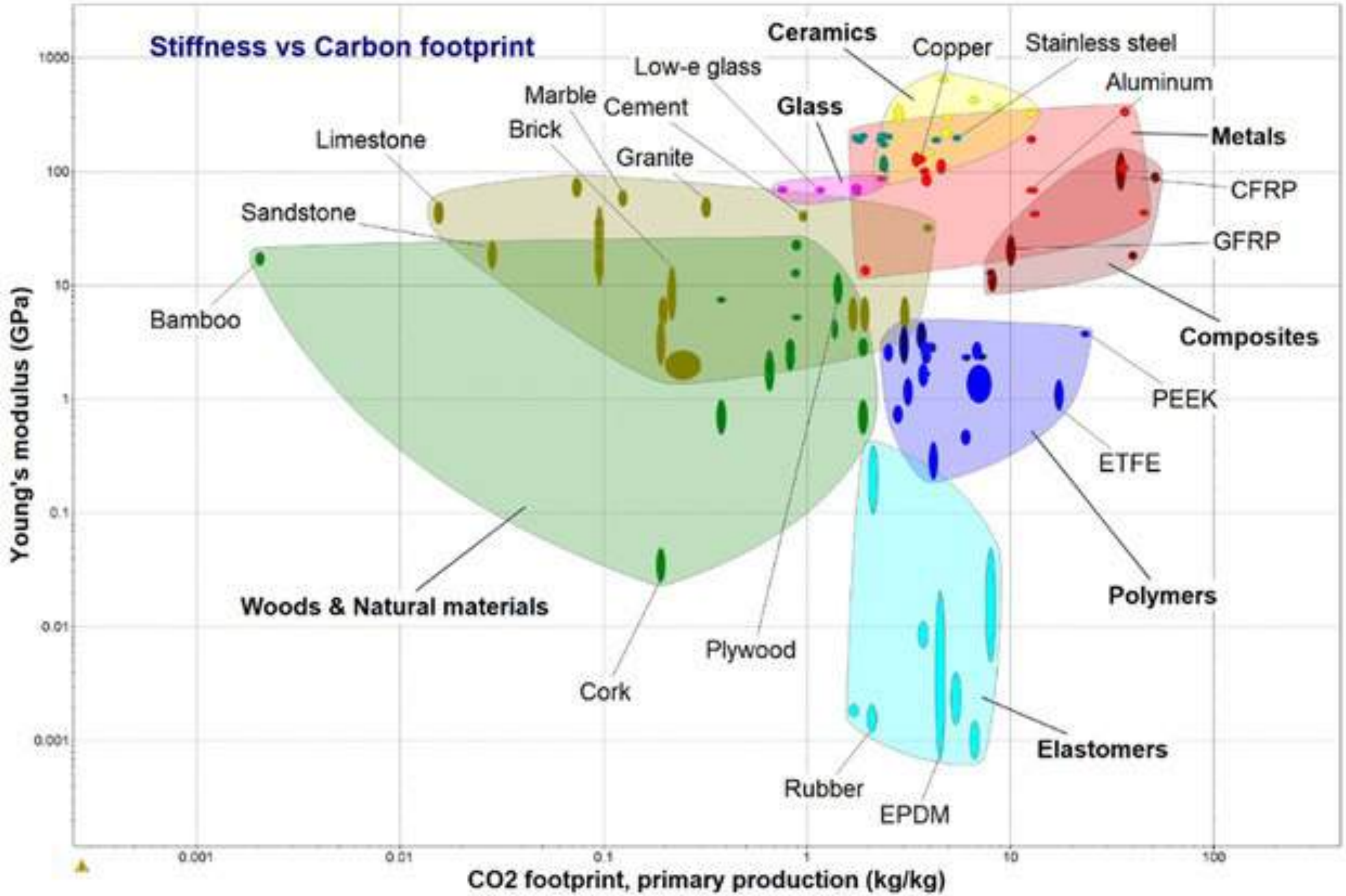
- For transports,   mass -> fuel consumption -> cost
- Costs over life time = acquisition cost (proportional to C) + fuel costs (proportional to m)

$\Rightarrow Z = C + \alpha * m$

	Steel	Steel / Alu	Alu / (composite)	Alu / Ti / composites	Composites
α ($/kg)	3 – 6	6 – 20	100 – 600	600 – 2,000 (?)	5,000 – 10,000

EN CONCLUSION




Stiffness vs Carbon footprint
Young's modulus (GPa)
CO2 footprint, primary production (kg/kg)
1000
100
10
1
0.1
0.01
0.001
0.001
0.01
0.1
1
10
100
Ceramics
Copper
Stainless steel
Aluminum
Low-e glass
Marble
Cement
Glass
Metals
Limestone
Brick
Granite
CFRP
Sandstone
GFRP
Bamboo
Composites
PEEK
ETFE
Polymers
Woods & Natural materials
Plywood
Cork
Elastomers
Rubber
EPDM

Eco Audit tool for rapid LCA

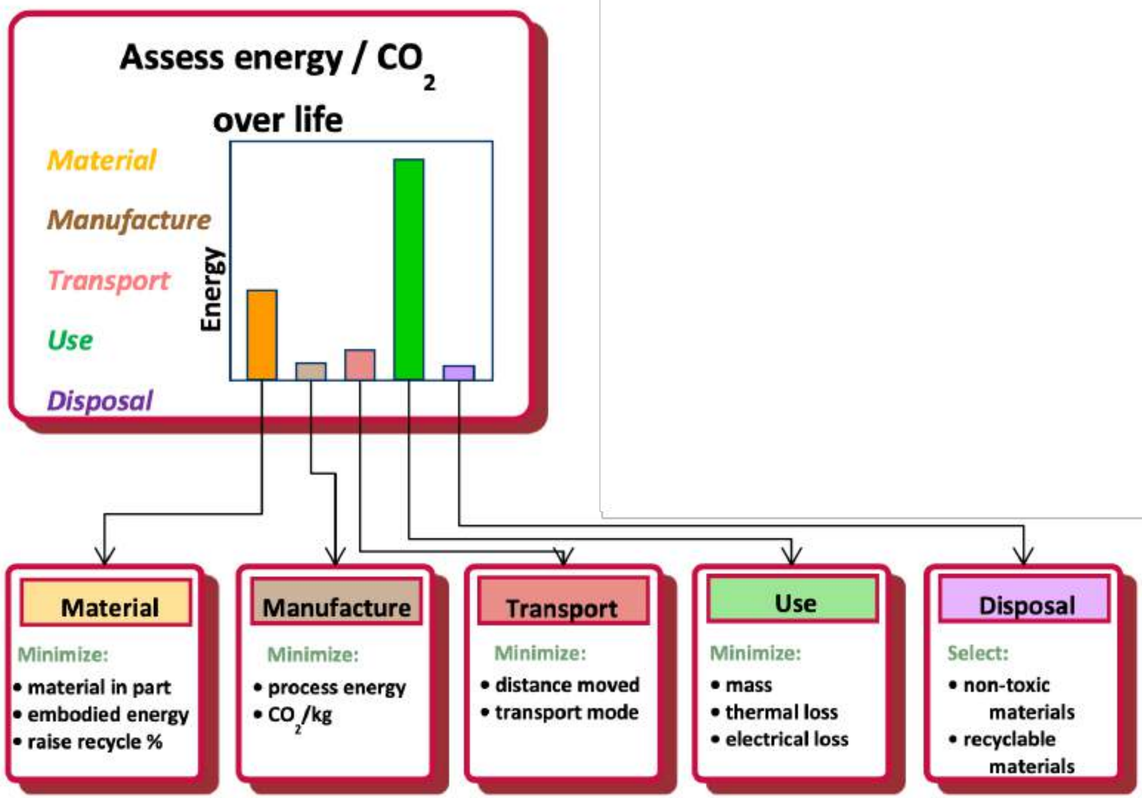
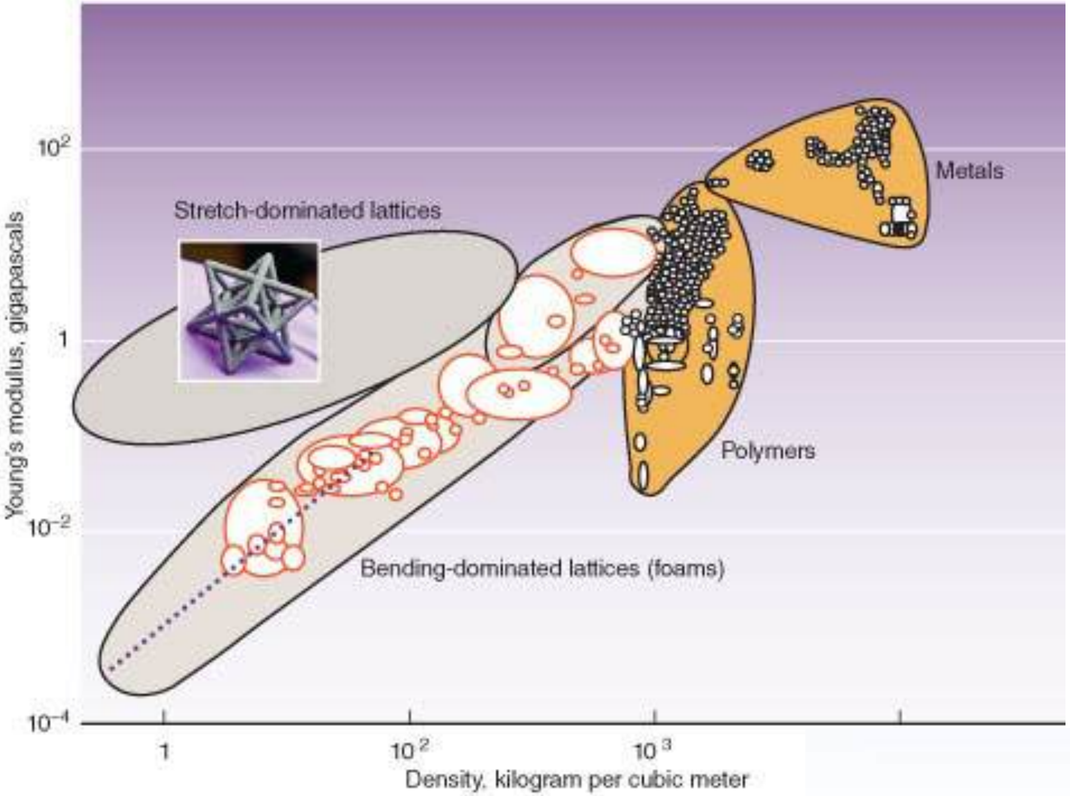


Figure 4: Environmental impact can be assessed for each life-stage of a product (Tip 3). Materials and process selection play an important role in determining environmental impacts and can be used in many eco design strategies (Tip 4.)

Research

AND TOMORROW?



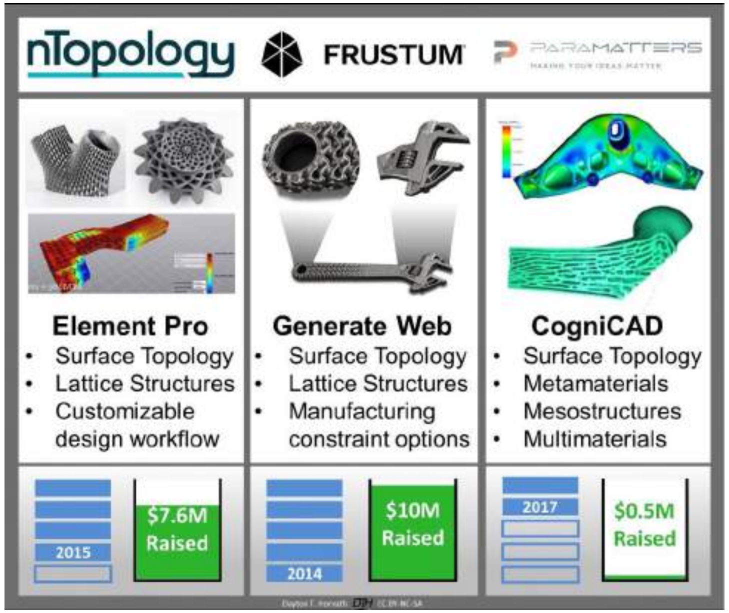
(a) (b)

Chris Spadaccini (llnl,USA) "By controlling the architecture of a microstructure, we can create materials with previously unobtainable properties in the bulk form."

Software and algorithms for hierarchical design

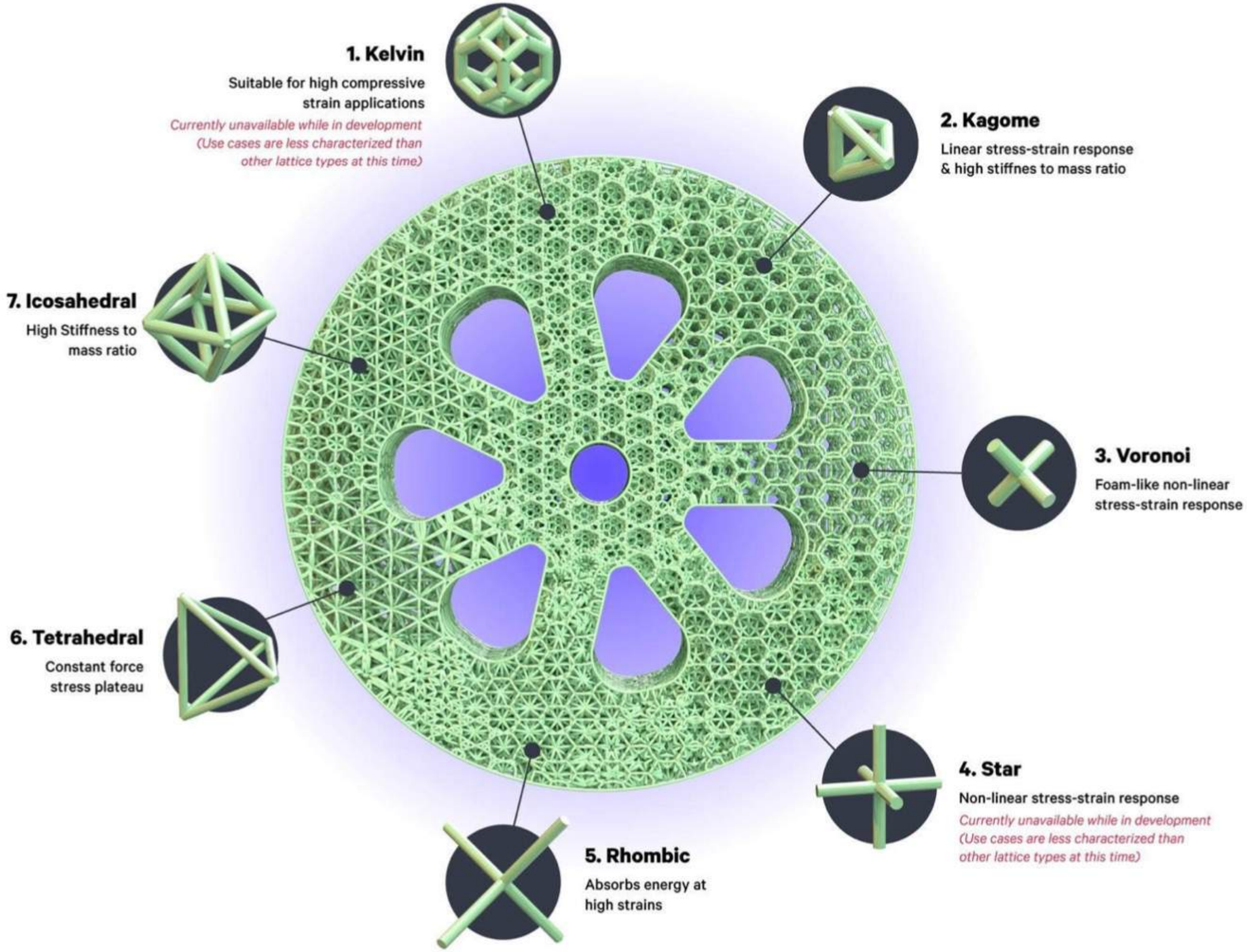
Conventional CAD programs do not work well
New players are emerging

Examples:
- ntopology (see case studies): https://ntopology.com/
- additiveflow: https://www.additiveflow.com/
- Hyperganic
- ParaMatters: https://paramatters.com/
- Fusion 360 (Autodesk)



MIT PROFESSIONAL EDUCATION


1: Carbon Design Engine™


1. Kelvin
Suitable for high compressive strain applications
Currently unavailable while in development (Use cases are less characterized than other lattice types at this time)
2. Kagome
Linear stress-strain response & high stiffnes to mass ratio
3. Voronoi
Foam-like non-linear stress-strain response
4. Star
Non-linear stress-strain response
Currently unavailable while in development (Use cases are less characterized than other lattice types at this time)
5. Rhombic
Absorbs energy at high strains
6. Tetrahedral
Constant force stress plateau
7. Icosahedral
High Stiffness to mass ratio

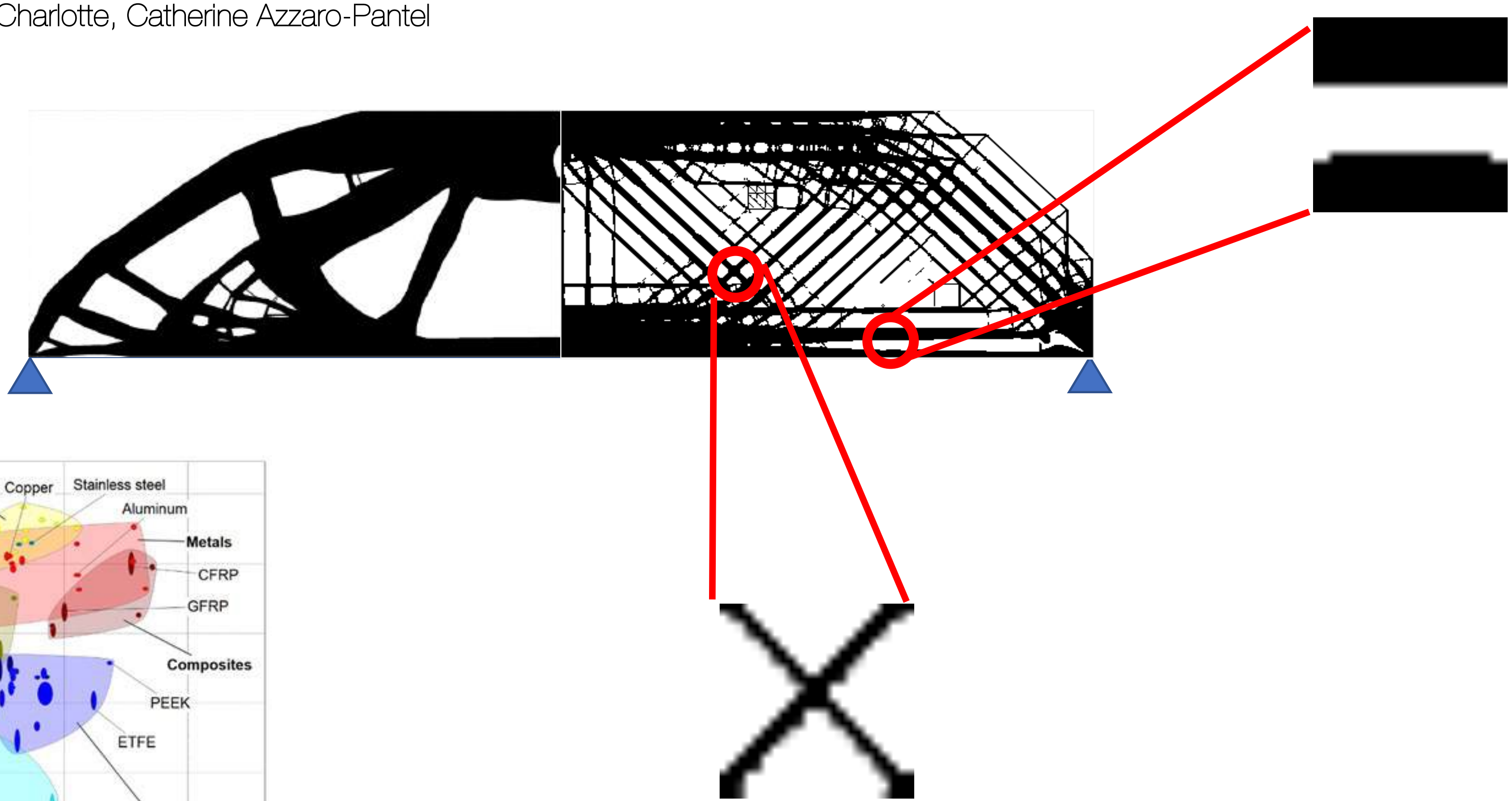
How to ECOdesign tomorrow's structures?

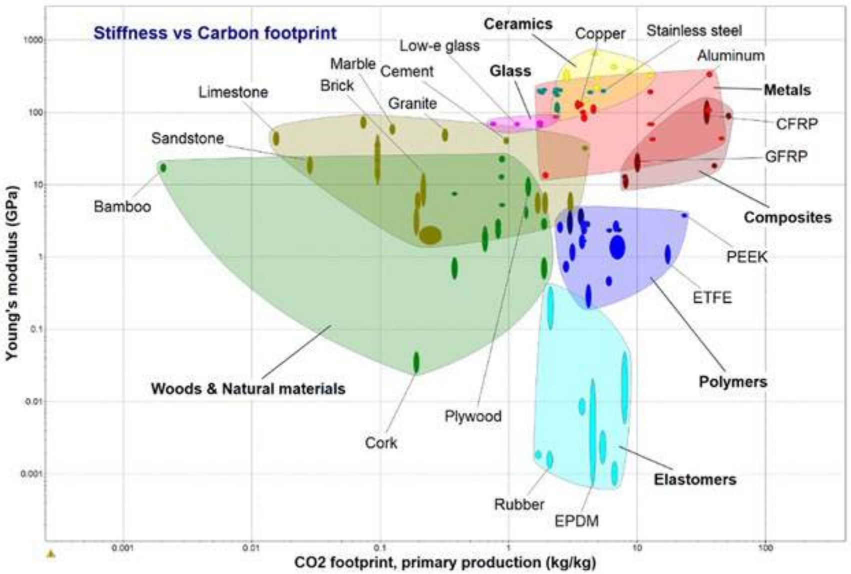
Prof. Joseph Morlier, Edouard Duriez, Miguel Charlotte, Catherine Azzaro-Pantel

#Mass vs CO2 footprint Minimization

#SIMP vs EMTO

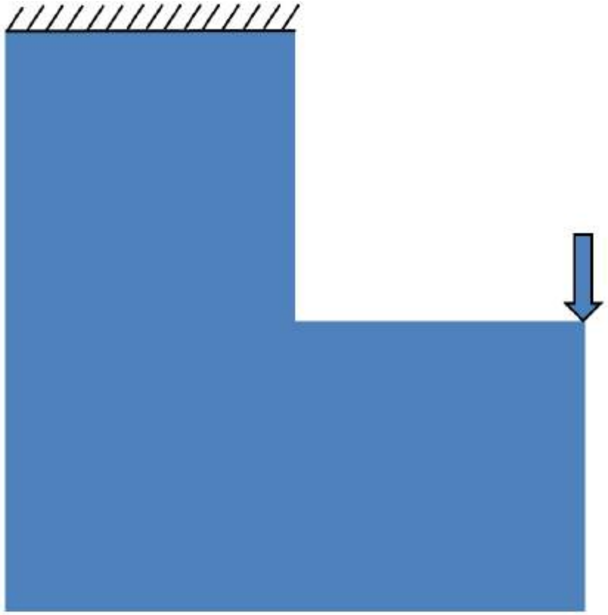
#3D printing process selection

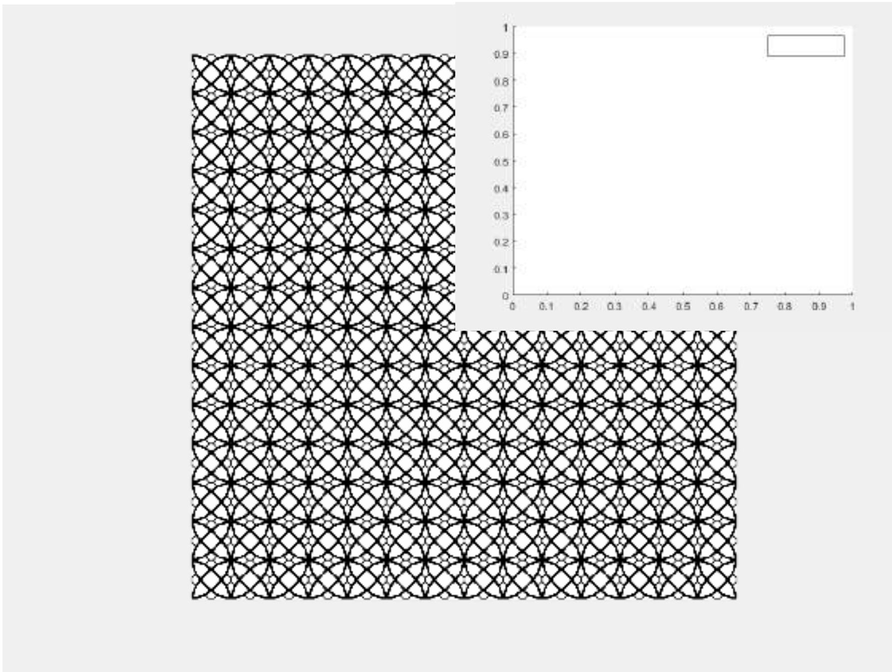




EMTO on L-shape (cellular /architectured materials)

Duriez, E., Morlier, J., Charlotte, M., & Azzaro-Pantel, C. (2021). A well connected, locally-oriented and efficient multi-scale topology optimization (EMTO) strategy. Structural and Multidisciplinary Optimization, 1-24.





https://github.com/mid2SUPAERO/EMTO

How to ECOdesign tomorrow's structures?

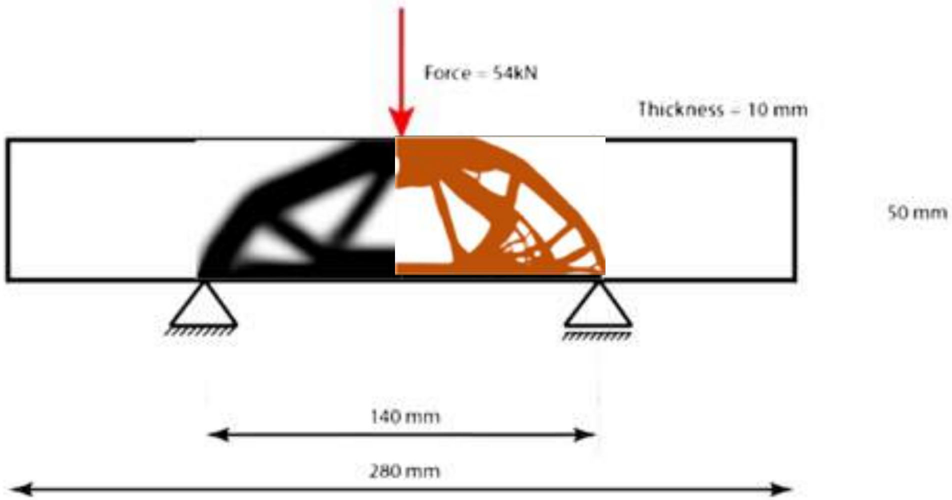
Prof. Joseph Morlier, Edouard Duriez, Miguel Charlotte, Catherine Azzaro-Pantel

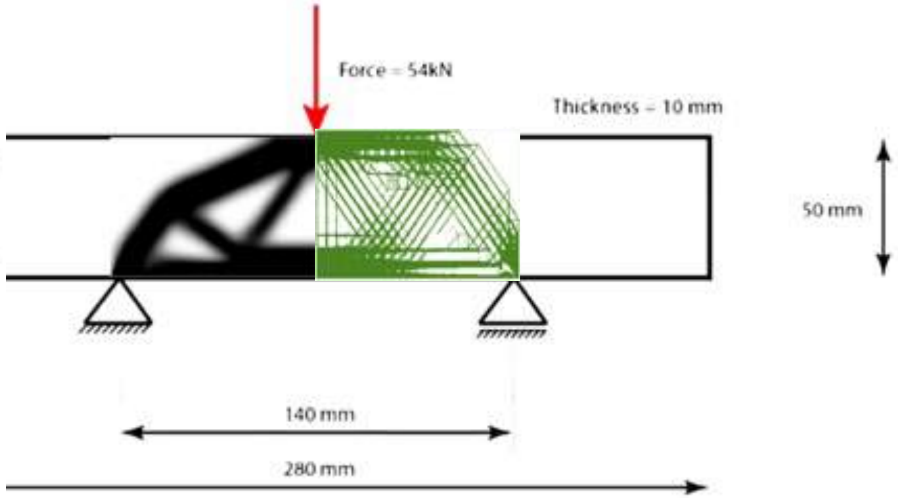
#Our very First Results

#SIMP vs ECOMA

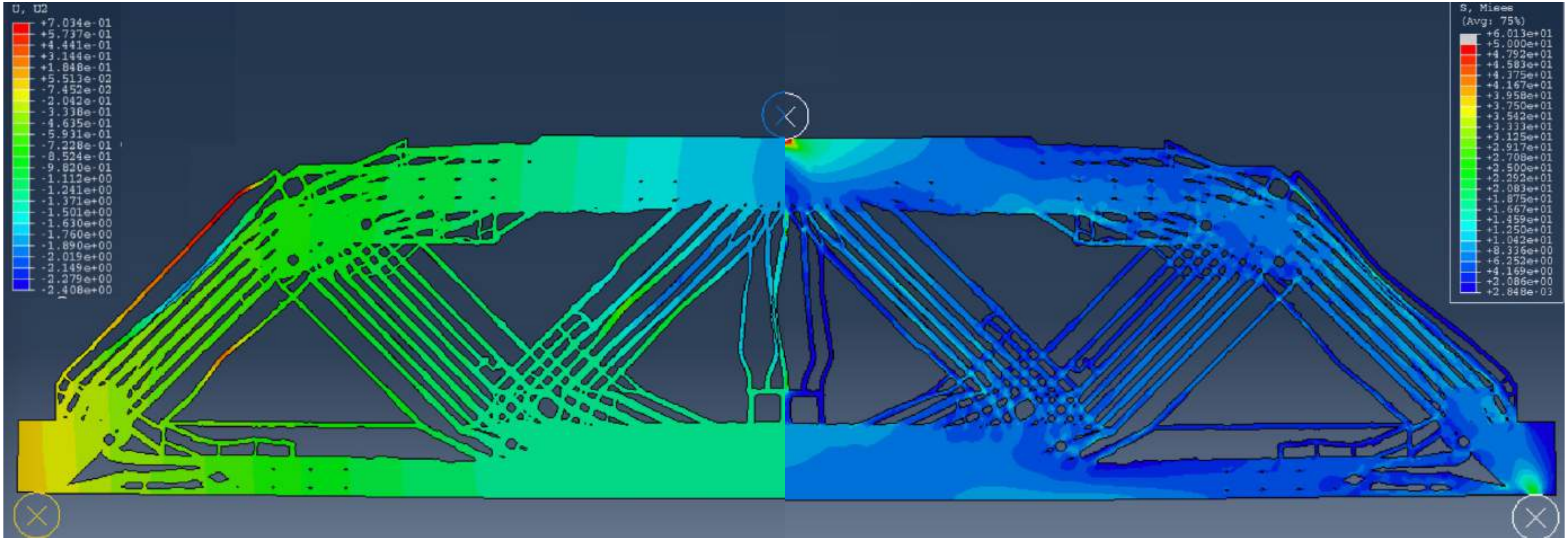
Print it , test it

(a)

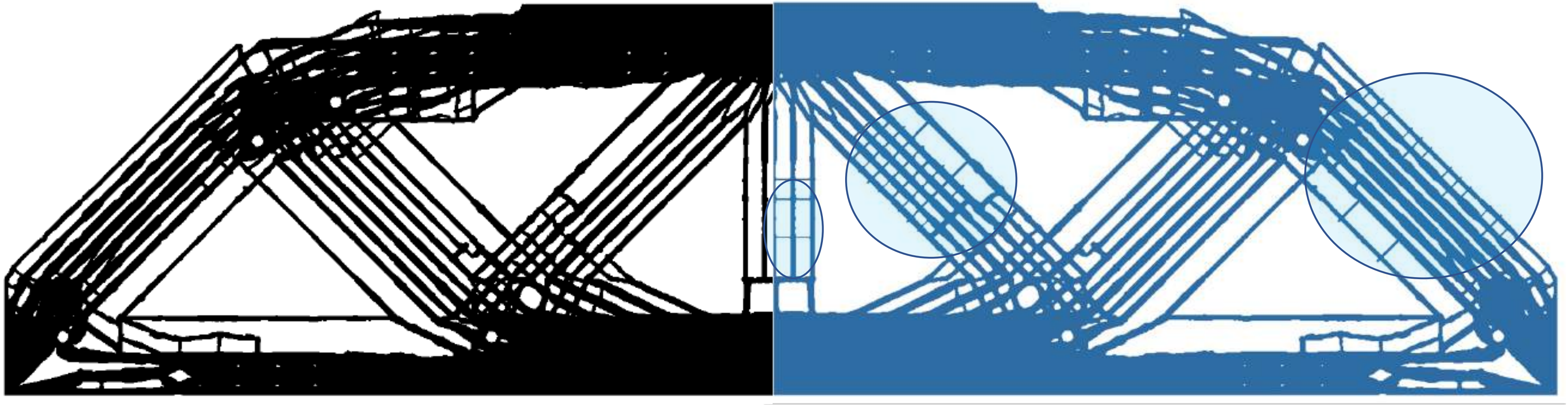




Disp or Stress ?



Do you see a difference (Left2Right)?



Engineer’s way

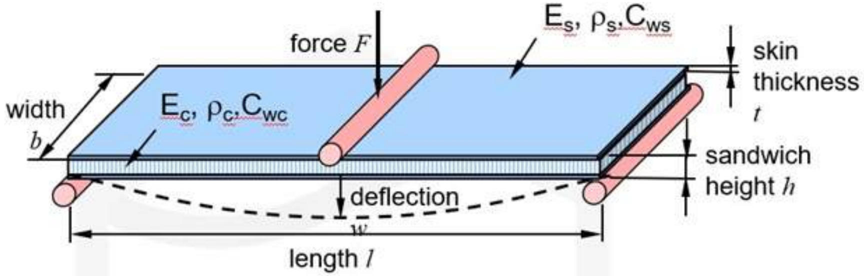
- https://renewable-carbon.eu/news/sustainable-lightweight-thermoplastic-honeycomb-panel-with-flax-fibre-composites-to-replace-glass-and-carbon-fibre-materials/
- In 2020 Flaxco approached EconCore suggesting combination of thermoplastic honeycomb technology with Flaxco’s flax fibre technology. The polypropylene honeycomb material produced by EconCore offers a high-performance-to-weight ratio along with efficient energy absorption.
- his is achieved through the materials synonymous honeycomb shaped hollow cells which support sandwich sheets to provide levels of high rigidity with minimal weight. Pioneering the use of flax with honeycomb technology, the partnership sees Flaxco’s flax fabric composite skins combined with EconCore polypropylene core to produce sandwich panels.
- In order to generate high level of stiffness, sandwich cores require surface layers, like the neatly woven layers that Flaxco have the means to create. Once created the panels can be thermoformed thanks to their thermoplastic properties. This is typically done by means of short cycle compression moulding.
- In order to achieve a product with well-balanced characteristics, pre-pregs of 0.5-1mm thickness were used. By optimising flax technology with the polypropylene honeycomb core Flaxco is able to achieve much higher levels of rigidity. The table below describes the bending performance of a sandwich panel with flax composite skins.

Bending Performance

- Flax fibres are a 100% natural fibre that produce lower CO_2 levels than competitor materials including glass and carbon fibres, an increasingly important element within the automotive industry. Furthermore, research and testing are already underway for the material to find its place within the sports industry.

- Sandwich example

<u>Example:</u> Simple supported 3pb
width b = 50 mm
length l = 150 mm
force F = 1000 N
allowable deflection w = 5 mm



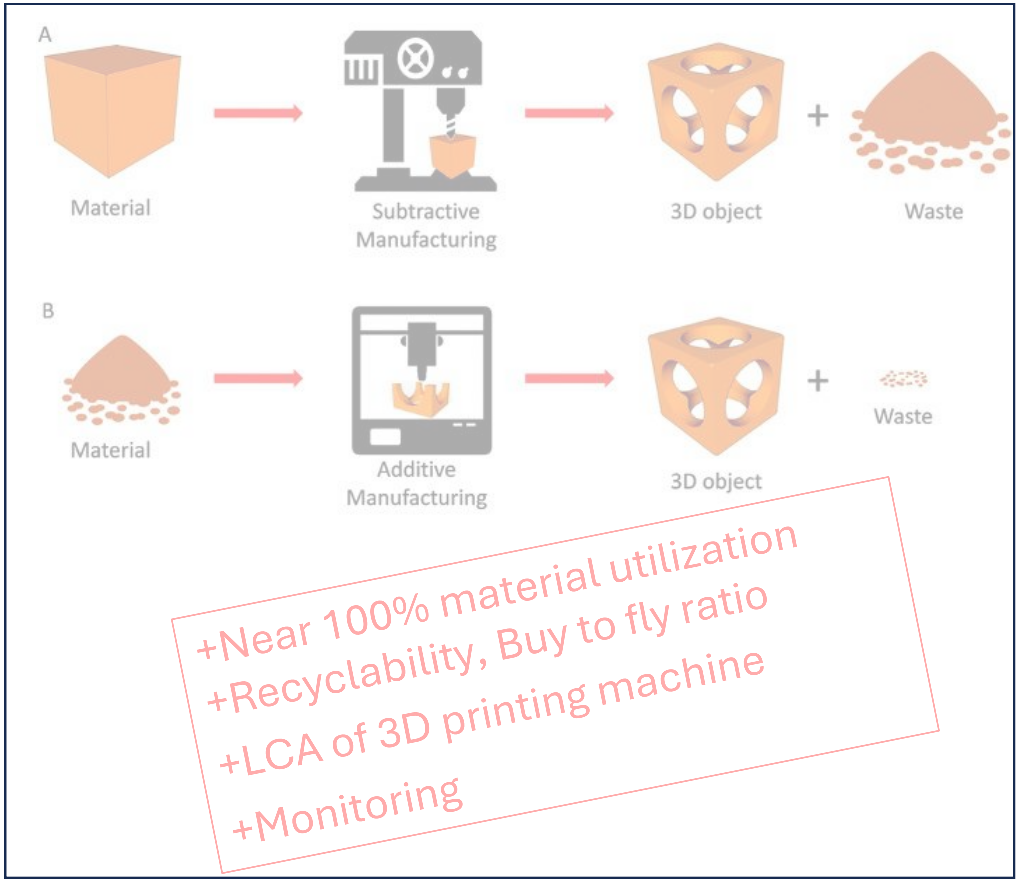
- Material comparison (Skin: FLAXCO , Core: 15 times lower density)

Panel Material (t [mm], h [mm])	t/h	E_H [GPa]	ρ_H [kg/dm³]	Bending stiffness per width EI/b [Nmm]	Panel weight W_a [kg/m²]	$ME_W = E_H^{1/3}/\rho_H$ [GPa$^{1/3}$/(kg/dm³)]
$h_1 = 2\ t_1$ = 6,082	0.5	15	1.2	281250	7.3	2.06
$h = 1.2\ h_1$ = 7,298 $t = 0.3\ t_1$ = 0.912	0.125	4.95	0.22	281250	1.94	7.74
$h = 2.4\ h_1$ = 14,60 $t = 0.06\ t_1$ = 0.182	0.0125	1.1	0.11	281250	1.57	9.55

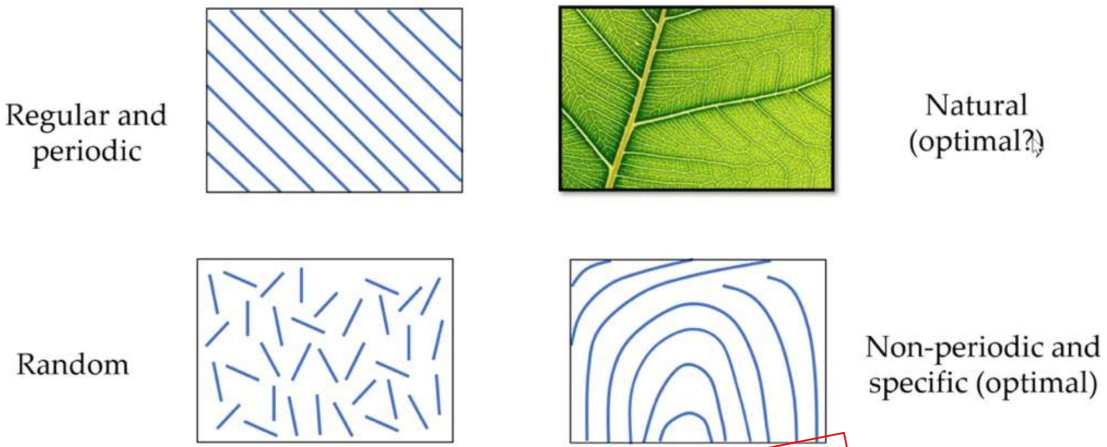
4,65 x less weight

Process is AM, but WHY?

https://dfam.substack.com/p/dfam-education-in-2022



+Near 100% material utilization
+Recyclability, Buy to fly ratio
+LCA of 3D printing machine
+Monitoring

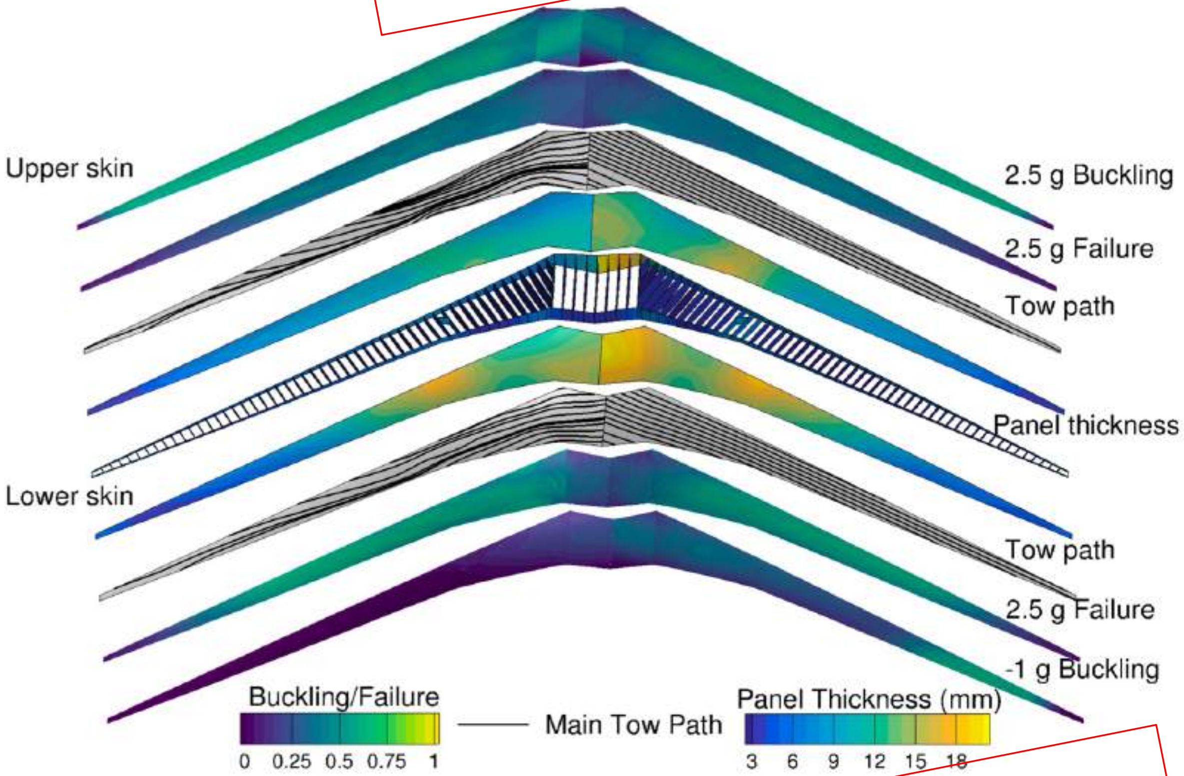


+ Automatic Fiber Placement + eco-fiber/resin selection
+Monitoring

Composites Fiber Placement as DVs

https://www.compositesworld.com/articles/tow-steering-part-2-the-next-generation

X* is a UHAR Wing ;)

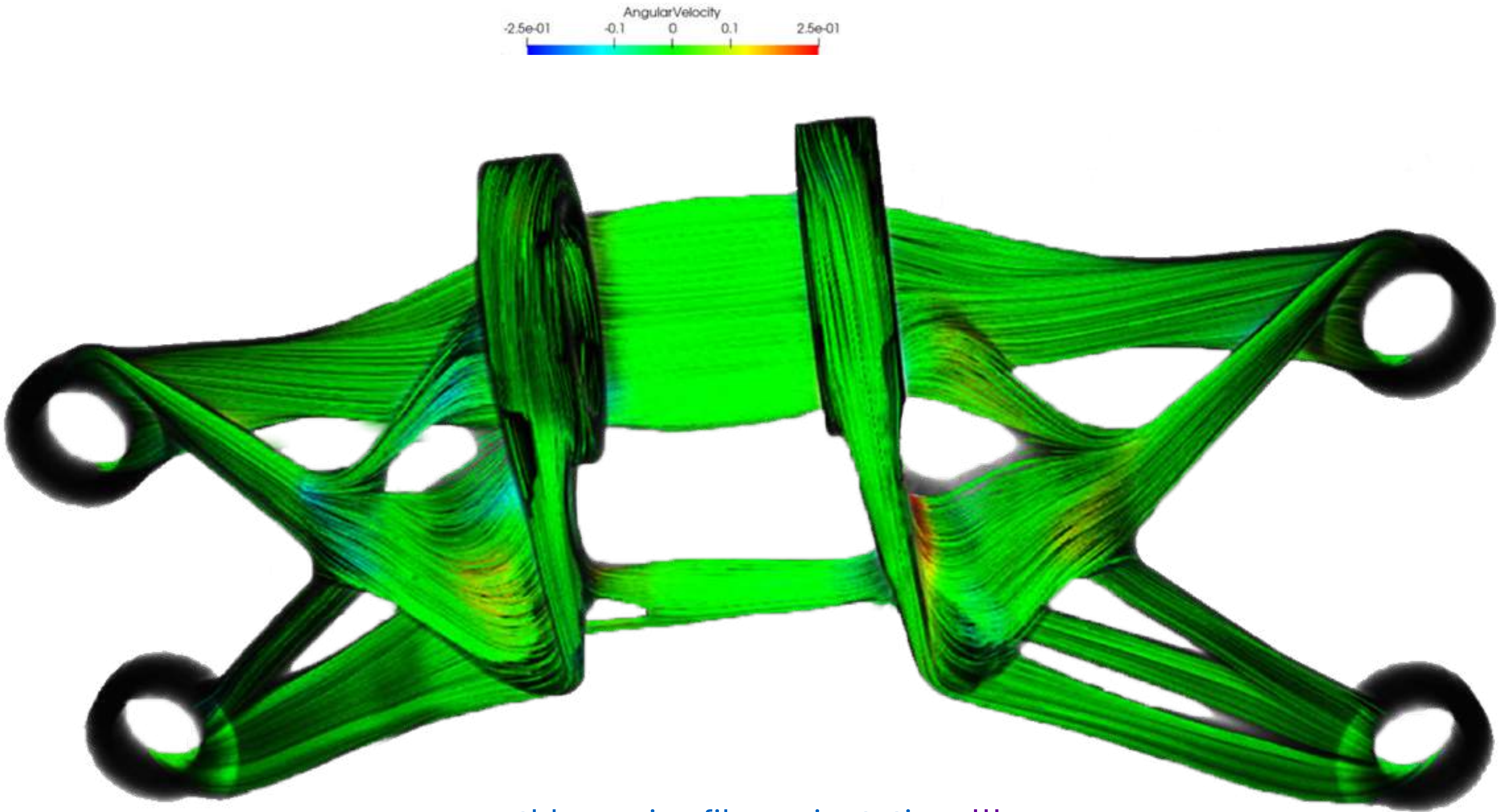


Brooks, T. R., Martins, J. R., & Kennedy, G. J. (2019). High-fidelity aerostructural optimization of tow-steered composite wings. Journal of Fluids and Structures, 88, 122-147.

Brooks, T. R., Martins, J. R., & Kennedy, G. J. (2020). Aerostructural tradeoffs for tow-steered composite wings. Journal of Aircraft, 57(5), 787-799.

Lower wing mass,
Less fuel burn

GE Bracket by Schmidt et al., Struct. Multidiscip. Optim. (2020)



smoothly varying fiber orientations!!!

Collaboration with TU Delft

In-plane fibre orientations

The finite element analysis step calls the Ansys solver via the PyMAPDL interface

- Optimisation problem formulation

$$\min_{\boldsymbol{\rho},\boldsymbol{\theta}} c(\boldsymbol{\rho},\boldsymbol{\theta}) = \sum_e \rho_e^p \, \boldsymbol{u_e^T} \, \boldsymbol{k_0}(\theta_e) \, \boldsymbol{u_e^T}$$

$$\text{s.t.} \begin{cases} \frac{V(\boldsymbol{\rho})}{V_0} \leq f \\ \boldsymbol{KU} = \boldsymbol{F} \\ 0 < \rho_{min} \leq \boldsymbol{\rho} \leq 1 \\ -\pi \leq \boldsymbol{\theta} \leq \pi \end{cases}$$

solved with initial random point

- Filters

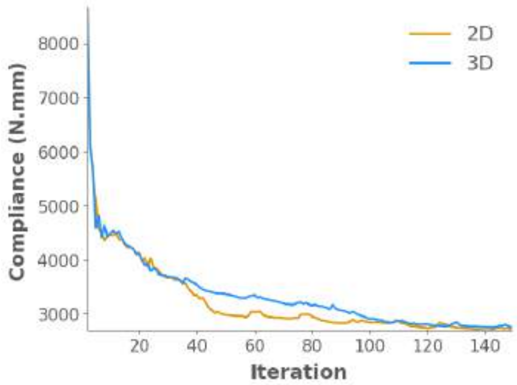
$$\rho_e \widetilde{\frac{\partial c}{\partial \rho_e}} = \frac{1}{\sum_i H_{ei}^{\rho}} \sum_i H_{ei}^{\rho} \rho_i \frac{\partial c}{\partial \rho_i}$$

$$H_{ei}^{\rho} = \max(0, r_\rho - \Delta(e,i))$$

$$\tilde{\theta}_e = \frac{1}{\sum_i H_{ei}^{\theta} \rho_i} \sum_i H_{ei}^{\theta} \rho_i \theta_i$$

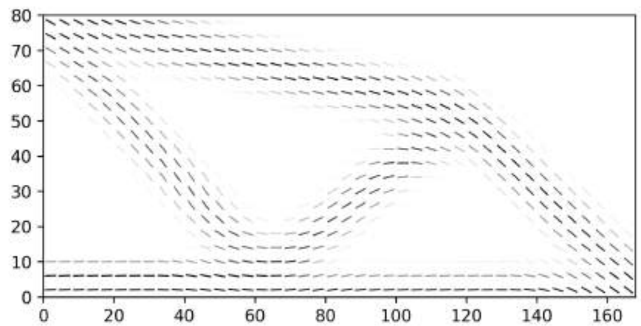
$$H_{ei}^{\theta} = \max(0, r_\theta - \Delta(e,i))$$

Problem 1 - MBB beam

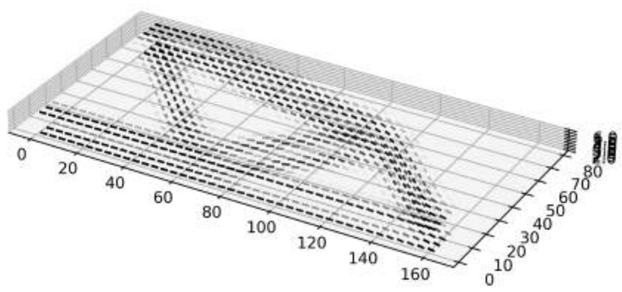


www.github.com/mid2SUPAERO/SOMP_Ansys/tree/csma

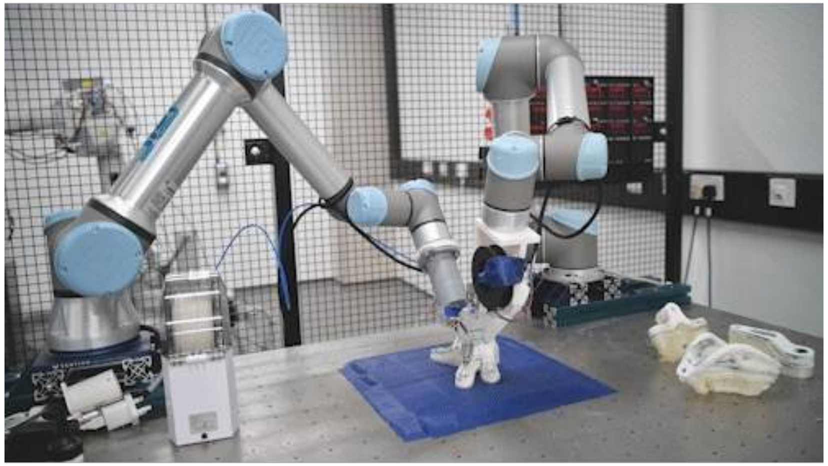
- 2D - Comp. = 2691 N.mm



- 3D - Comp. = 2733 N.mm



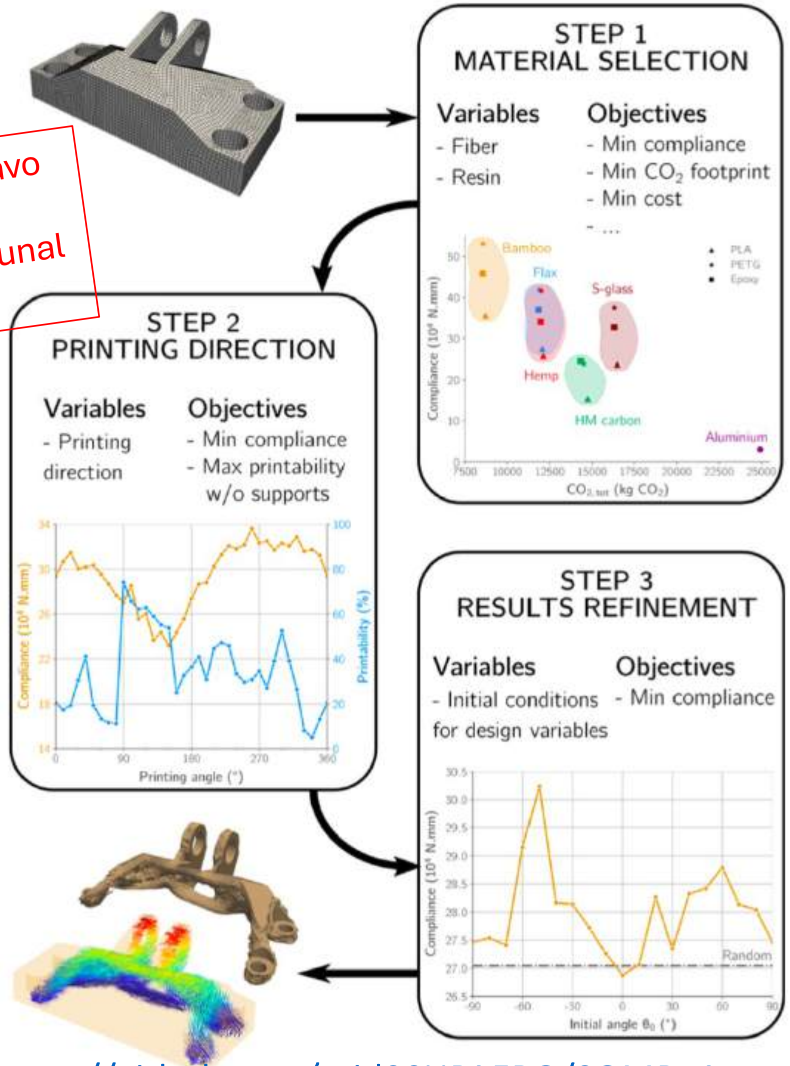
Inspired by spatial printing*



*Fang, G., Zhang, T., Huang, Y., Zhang, Z., Masania, K., & Wang, C. C. (2024). Exceptional mechanical performance by spatial printing with continuous fiber: Curved slicing, toolpath generation and physical verification. Additive Manufacturing, 104048.

https://www.youtube.com/watch?v=7Jxyu9uRMLo

Thanks to Gustavo Asai, Frederic Lachaud and Kunal Masania



https://github.com/mid2SUPAERO/SOMP_Ansys

Add Ecomaterial selection, printability

opensource 2D framework (but need an ANSYS LICENCE for 3D)

$$\min_{\boldsymbol{\rho},\boldsymbol{\theta},\boldsymbol{\alpha}} C(\boldsymbol{\rho},\boldsymbol{\theta},\boldsymbol{\alpha}) = \left(\sum_{i\in LC} c_i(\boldsymbol{\rho},\boldsymbol{\theta},\boldsymbol{\alpha})^n\right)^{\frac{1}{n}}$$

$$= \left(\sum_{i\in LC}\left(\sum_{e} \rho_e^p \boldsymbol{u}_{e,i}^{\boldsymbol{T}} \boldsymbol{k_0}(\theta_e,\alpha_e)\boldsymbol{u}_{e,i}\right)^n\right)^{\frac{1}{n}}$$

$$\text{s.t.} \begin{cases} \frac{V(\boldsymbol{\rho})}{V_0} \leq f \\ \boldsymbol{KU} = \boldsymbol{F} \\ 0 < \rho_{min} \leq \boldsymbol{\rho} \leq 1 \\ -\frac{\pi}{2} \leq \boldsymbol{\theta} \leq \frac{\pi}{2} \\ -\frac{\pi}{2} \leq \boldsymbol{\alpha} \leq \frac{\pi}{2} \end{cases}$$

$$\frac{\partial C}{\partial \cdot} = \sum_{i\in LC} c_i^{n-1} C^{1-n} \frac{\partial c_i}{\partial \cdot}$$

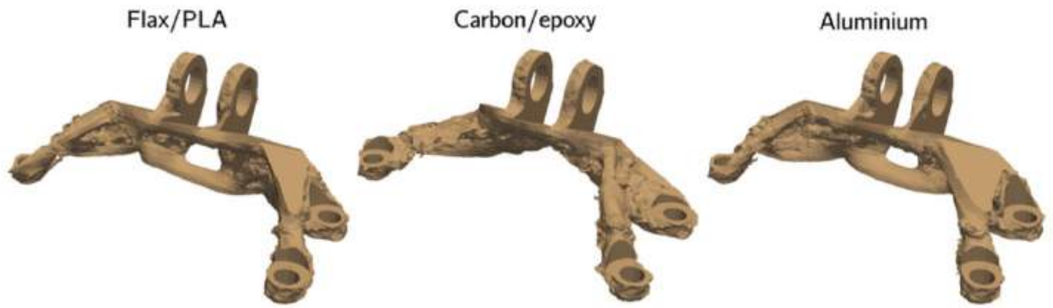


Figure 9: Isosurfaces of density 0.55 for different materials.

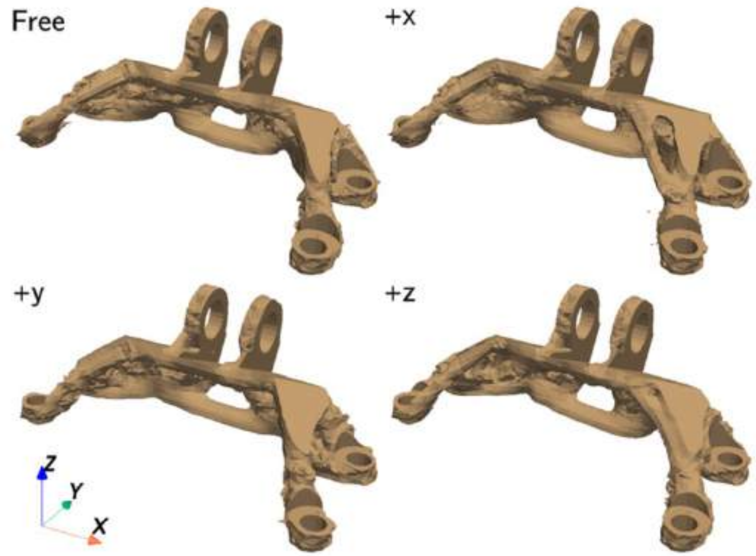
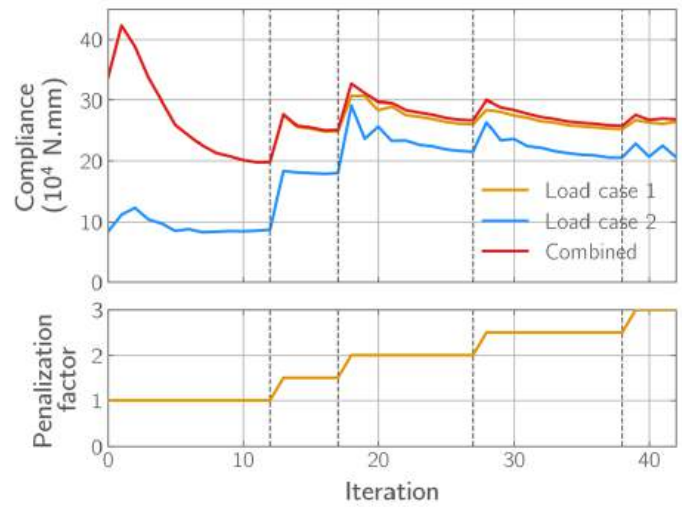
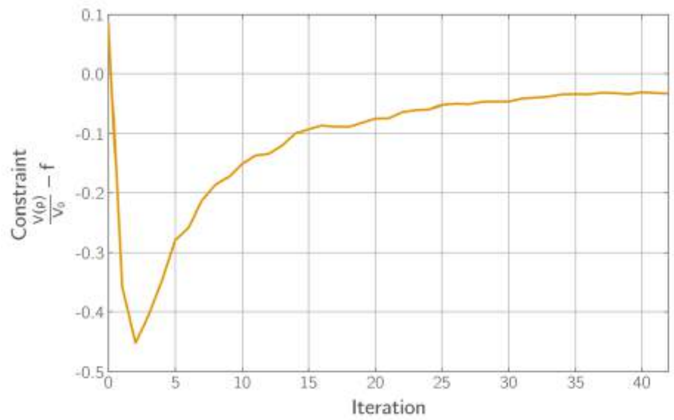


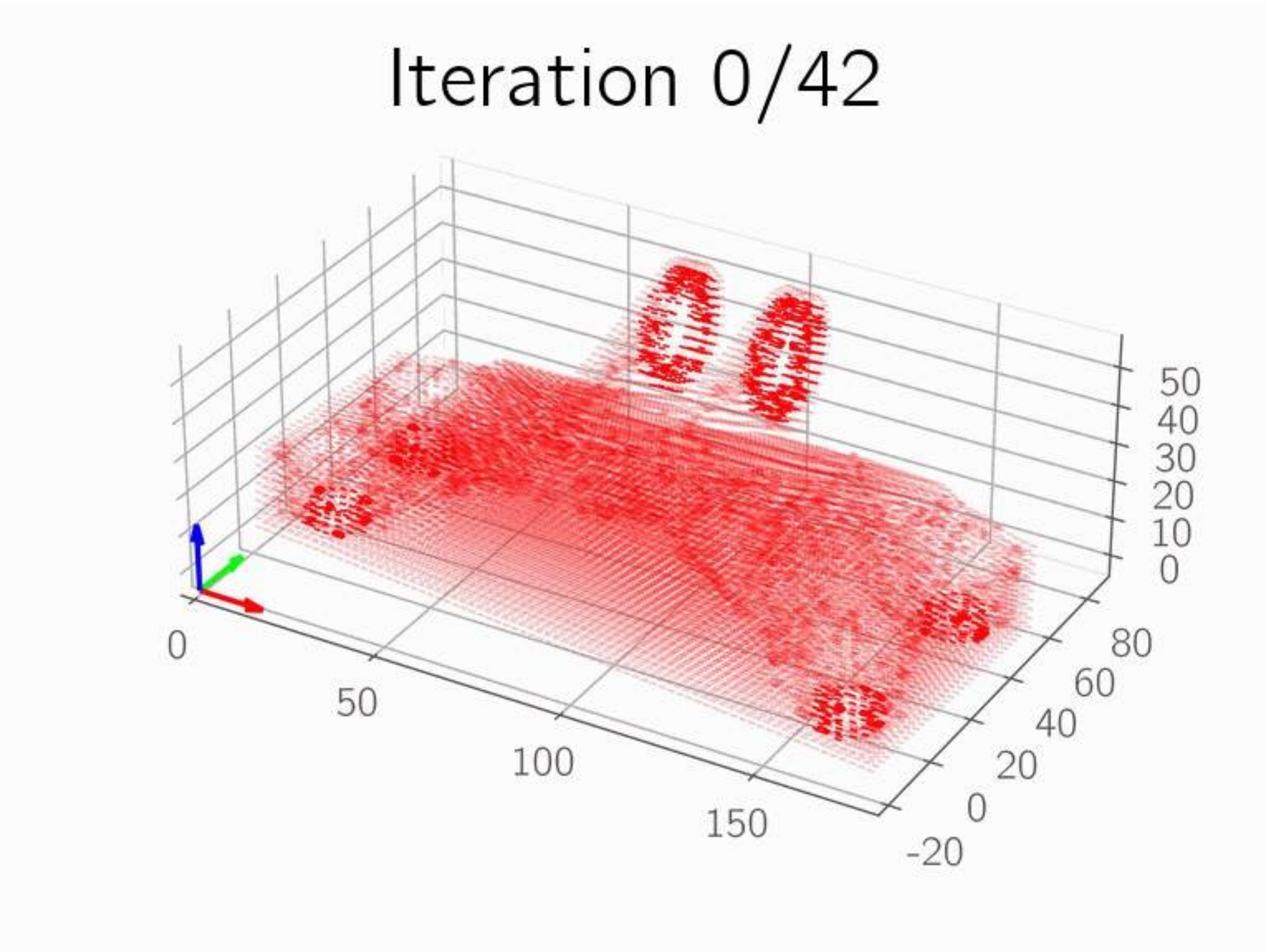
Figure 10: Isosurfaces of the optimal designs for each printing direction.

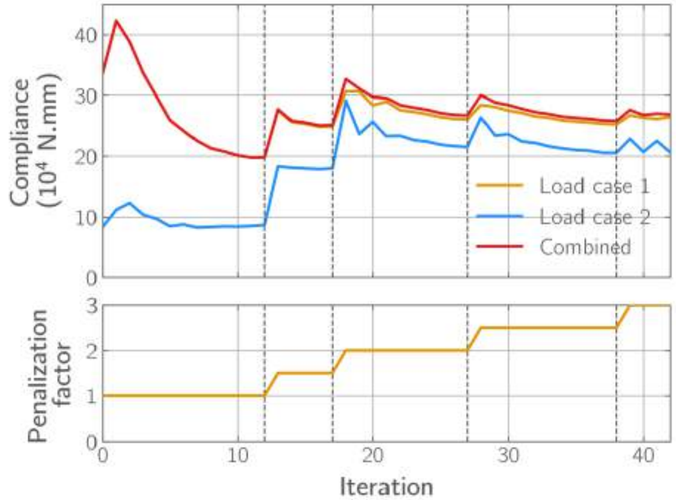
https://github.com/mid2SUPAERO/SOMP_Ansys

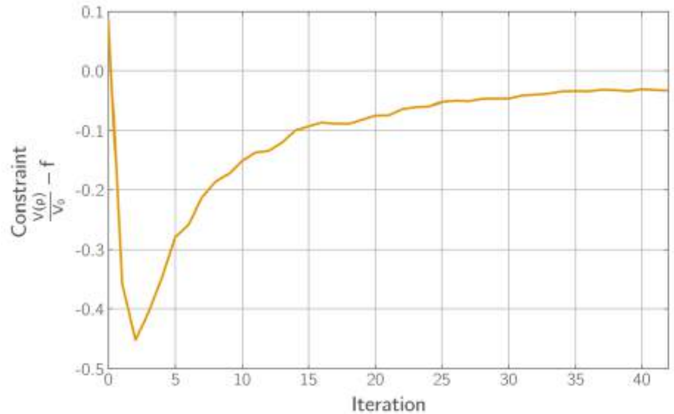
Iteration 1/42






Iteration 0/42
0
50
100
150
-20
0
20
40
60
80
0
10
20
30
40
50


Compliance
(10^4 N.mm)
Load case 1
Load case 2
Combined
Penalization
factor
Iteration


Constraint
$\frac{V(\rho)}{V_0} - f$
Iteration

Add Ecomaterial selection, printability

Results

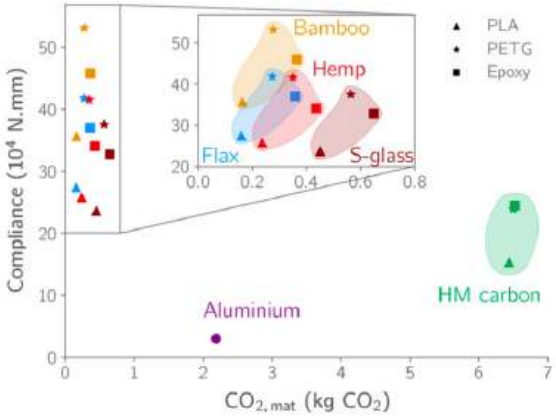


Figure 7: Compliance versus material production footprint of the optimal designs, grouped by fiber.

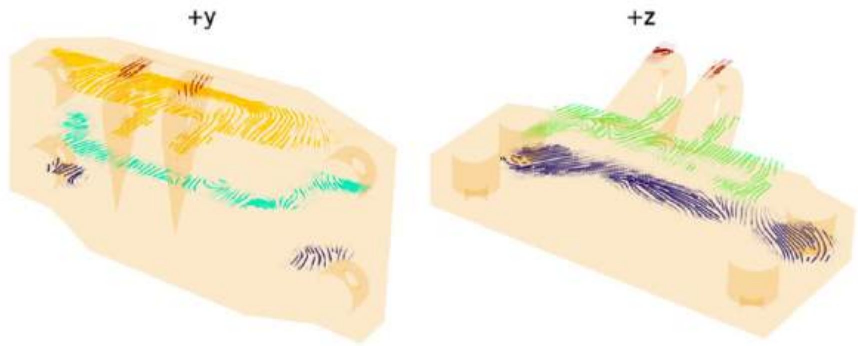


Figure 11: Examples of fiber distribution on slices of the optimal designs.

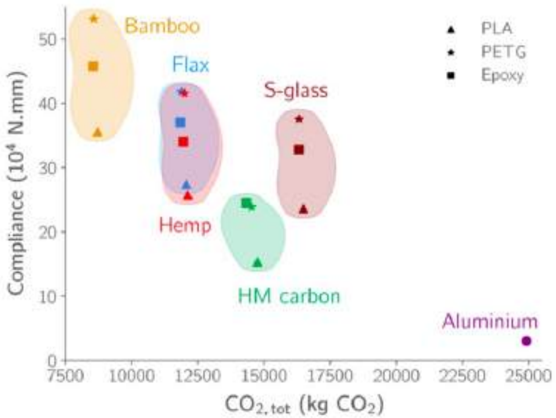


Figure 8: Compliance versus total footprint of the optimal designs, grouped by fiber.

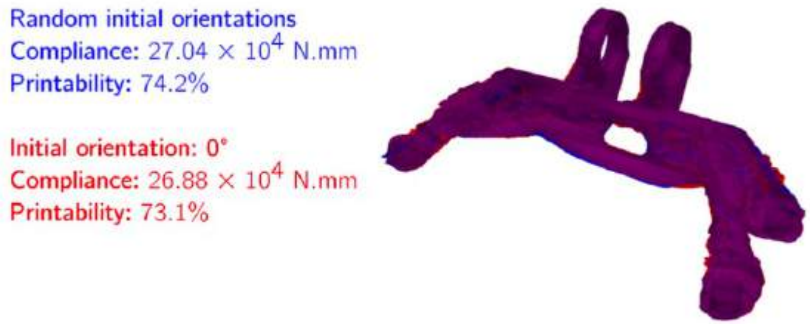
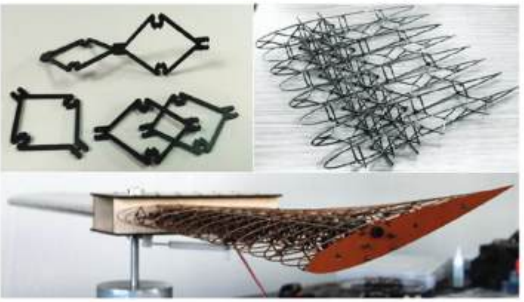


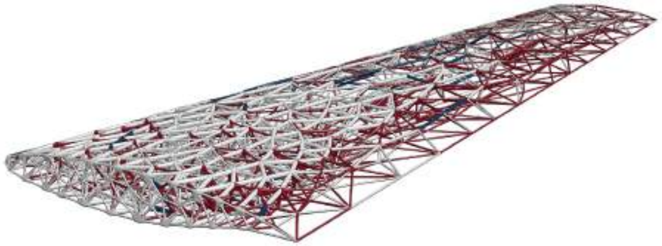
Figure 16: Isosurfaces of the optimal designs for random and 0° initial orientations.

https://github.com/mid2SUPAERO/SOMP_Ansys

Full wingbox concept

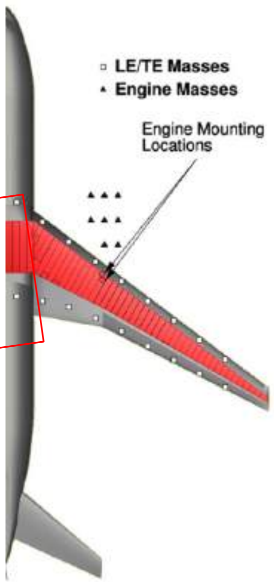


Jenett et al. (2017)



Opgenoord, M. M. and Willcox, K. E. (2018)

Cramer, N. B. et al. (2019)



Thanks to Enrico Stragiotti and ONERA's

Wingspan, m : 58.76
MTOW, t : 297,55

3 load cases:

- +2.5 g manouver
- -1 g manouver
- Cruise with gust (+1.3 g)

Material	Aluminum alloy
E	69 GPa
σ_c	-270 MPa
σ_t	270 MPa
ρ	2.7 g/cm^3

Optimized mass = 21.342 t

-27.01% compared to 29.238 t (Fakhimi et al., 2021)

Full wingbox concept

- Proof of concept of greener aerostructures with lattice wingbox
- Material as Design variable open new solutions:
- Who is the best?

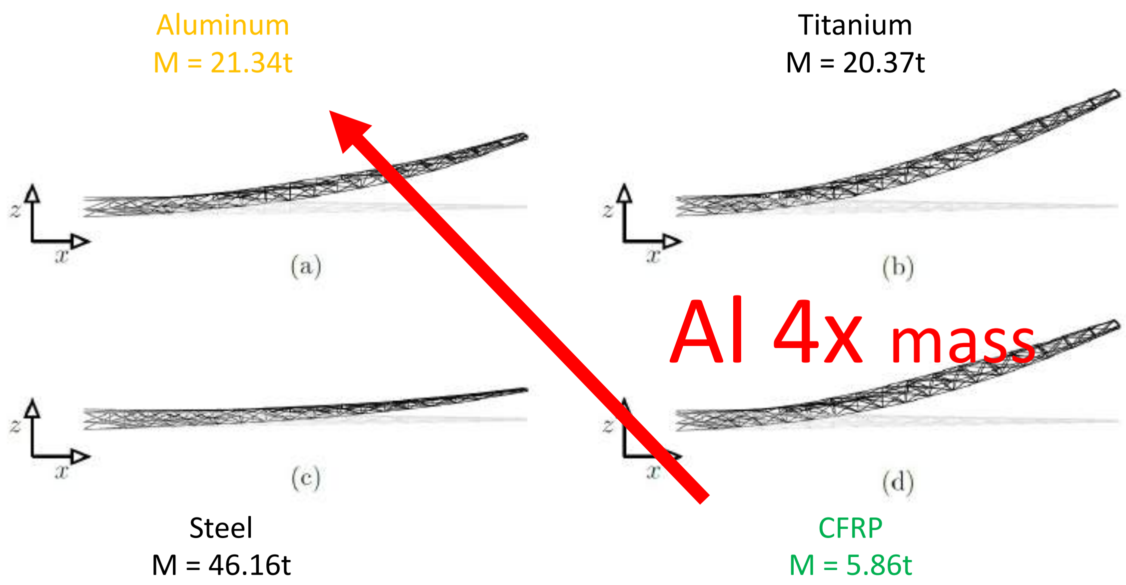
$$
\begin{aligned}
\min_{\substack{\boldsymbol{a},\boldsymbol{q}^0,\dots,\boldsymbol{q}^{N_p},\\ \boldsymbol{U}^0,\dots,\boldsymbol{U}^{N_p}}} \quad & V = \boldsymbol{\ell}^T \boldsymbol{a} \\
\text{s.t.} \quad & \boldsymbol{B}\boldsymbol{q}^p = \boldsymbol{f}^p && \forall p \in [0,\dots,N_p] \\
& \boldsymbol{q}^p = \frac{\boldsymbol{a}E}{\boldsymbol{\ell}} \boldsymbol{b}^T \boldsymbol{U}^p && \forall p \in [0,\dots,N_p] \\
& \boldsymbol{q}^p \geq -\frac{s\boldsymbol{a}^2}{\boldsymbol{\ell}^{*2}} && \forall p \in [0,\dots,N_p] \\
& -\sigma_c \boldsymbol{a} \leq \boldsymbol{q}^p \leq \sigma_t \boldsymbol{a} && \forall p \in [0,\dots,N_p] \\
& 0 \leq \boldsymbol{a} \leq \frac{4\pi\boldsymbol{\ell}^2}{\lambda_{\max}}.
\end{aligned}
$$

Stragiotti et al, (2024)

Material	Aluminium	Titanium	Steel	Pultruted CFRP
E	69 GPa	120 GPa	210 GPa	150 GPa
σ_c, σ_t	±270 MPa	±880 MPa	±355 MPa	+1200,−880 MPa
ρ	2.7 g cm^{-3}	4.5 g cm^{-3}	7.8 g cm^{-3}	1.6 g cm^{-3}

Full wingbox concept

- Proof of concept of greener aerostructures with lattice wingbox
- Material as Design variable open new solutions:
- Who is the best?



But finally
Al has approx same CO2pp footprint but cheaper