Conception optimale pour l'ingénieur (Aerospace)

C4 by Prof. J. Morlier
2025

Institut Supérieur de l'Aéronautique et de l'Espace
SUPAERO

EMSM207

AU PROGRAMME

Python based

lundi 31 mars 2025			
		09h15 - 12h45	MORLIER Joseph
		14h00 - 16h15	MORLIER Joseph
mardi 01 avril 2025			
		09h15 - 12h45	MORLIER Joseph
		14h00 - 16h15	MORLIER Joseph
mercredi 02 avril 2025			
		09h15 - 12h45	MORLIER Joseph MURADÁS ODRIOZOLA Daniel
		14h00 - 16h15	MAS COLOMER JOAN MURADÁS ODRIOZOLA Daniel
jeudi 03 avril 2025			
		09h15 - 12h45	MAS COLOMER JOAN MURADÁS ODRIOZOLA Daniel

Intro: Sustainable Aviation (Materials) With Both Eyes Open
Design optimization 1: constrained optimization, MOO, Sensibility with examples
Project DO 1 2 3

Topology Optimization with examples
Material ecoselection, Ashby Diagram and more

Projet DO 1 2 3
Wrap up and demo from students

Intro to MDAO **Static Aeroelastic problem is a MDAO problem**
Airbus PROJECT by TEAM of 3 (marked*)

vendredi 04 avril 2025		**ORAL MARKED***	
		09h15 - 11h30	MORLIER Joseph MURADÁS ODRIOZOLA Daniel

Part1 : On topolgy optimization

Lego?

https://labs.sogeti.com/numerical-simulation-as-product-sculpture-topological-optimization/

History

- Homogenization of Microstructures was introduced by mathematics in the 1970s.
- First paper by Martin Bendsoe (Technical University of Denmark) and Noboru Kikuchi (University of Michigan) in 1988

A topology optimisation problem can be written in the general form of an optimization problem as

$$\min_{\rho} \; F = F(\mathbf{u}(\rho), \rho) = \int_{\Omega} f(\mathbf{u}(\rho), \rho) \mathrm{d}V$$

subject to

- $\rho \in \{0, 1\}$
- $G_0(\rho) = \int_{\Omega} \rho(\mathbf{u}) \mathrm{d}V - V_0 \leq 0$
- $G_j(\mathbf{u}(\rho), \rho) \leq 0$ with $j = 1, \ldots, m$

TopOpt

Je choisis un bloc de marbre et j'enlève tout ce
dont je n' ai pas besoin..
Auguste Rodin (1840-1917)

Define the design space (marble block, fixed mesh)

Apply loads & BCs

Start optimization with hyper parameters

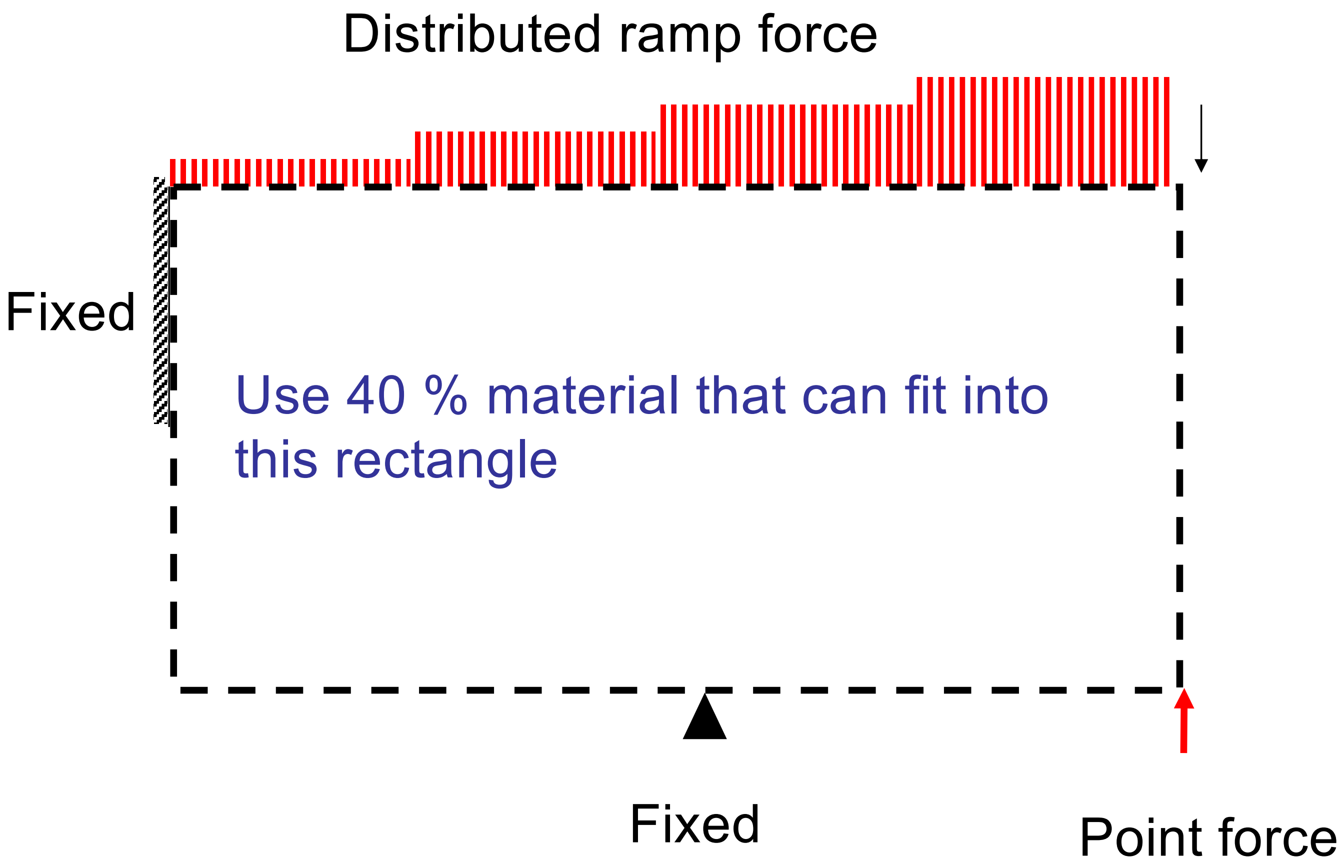
Interpreting the results

Optimal distribution of material (One can have an idea of the part to be reinforced, in addition to giving an excellent initial design …)

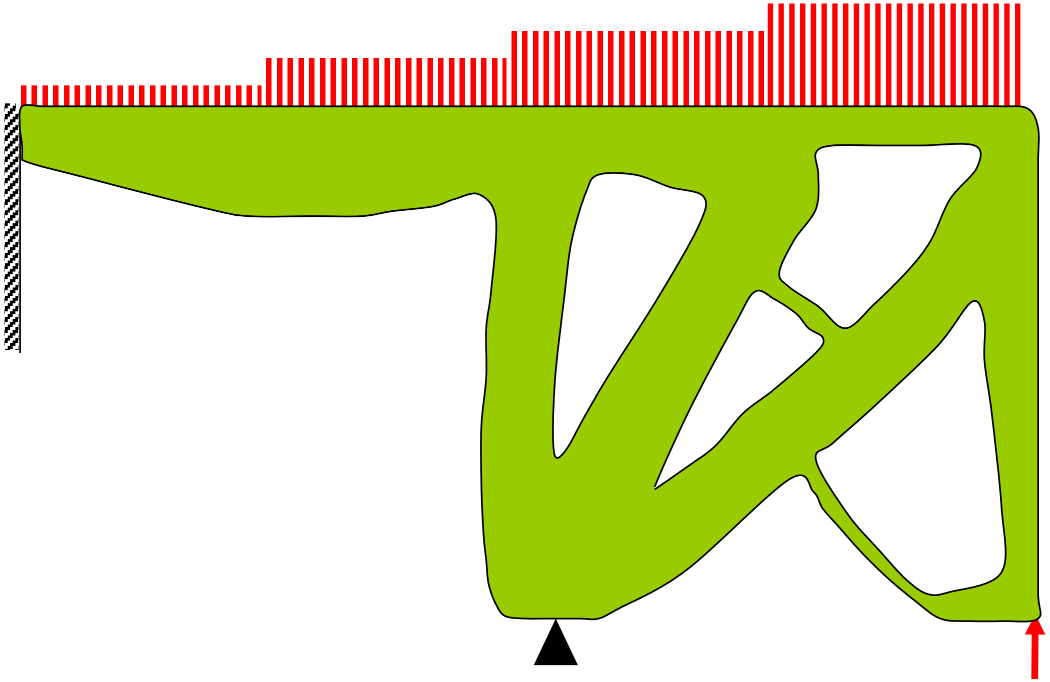
Why is it so powerful?

→There is a lot of possible redistribution of FORCES (internal)

Stiff structure for your specifications



Stiff structure for your specifications



Well-Known example

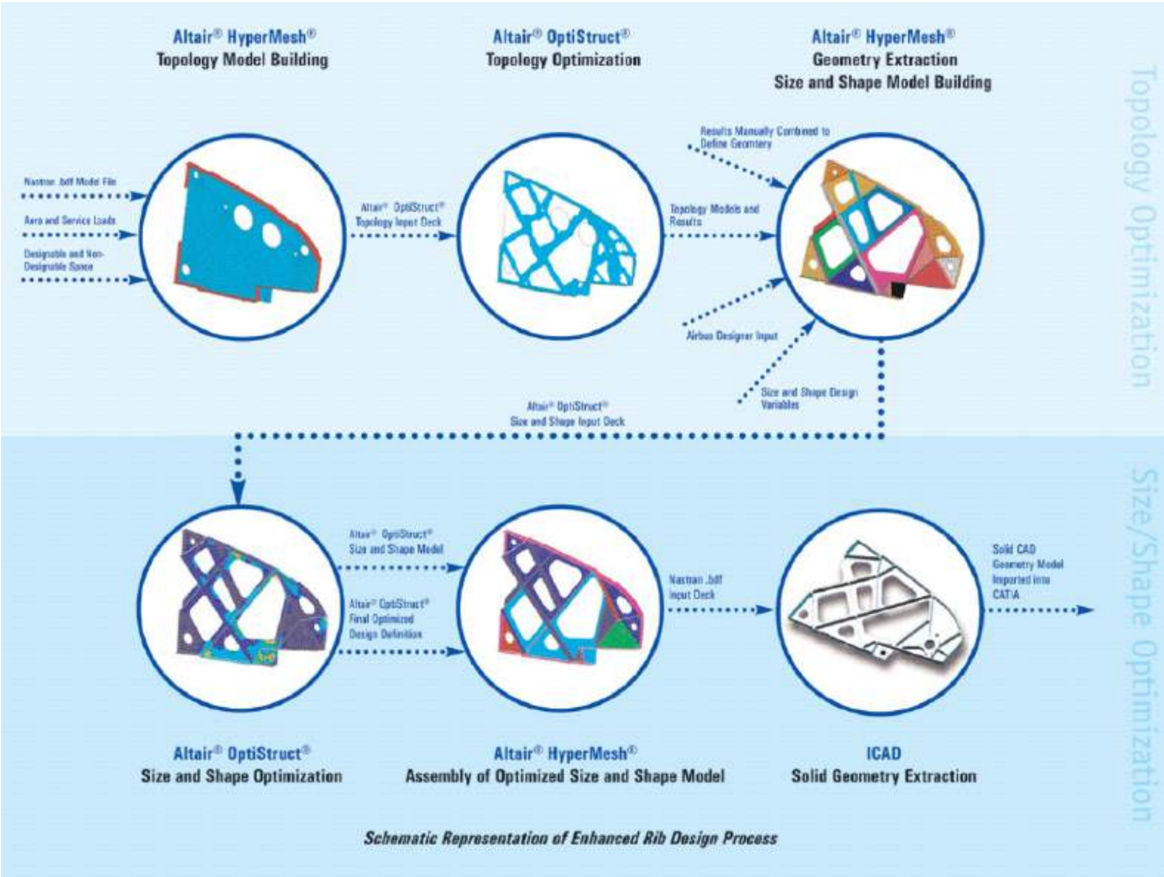
- Wing stiffening ribs of Airbus A380:

AIRBUS

- Objective: reduce weight
- Constraints: stress, buckling

Altair Engineering

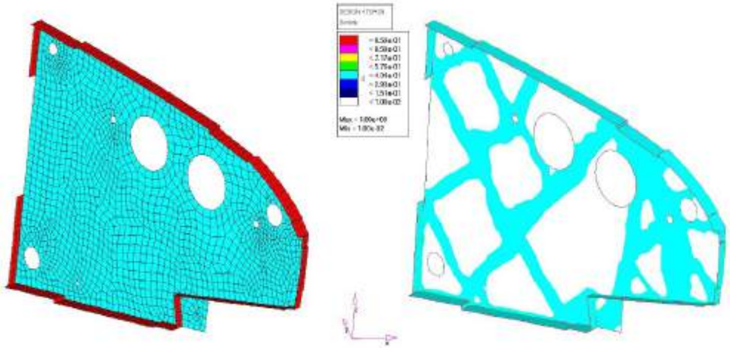
Topology and shape optimization



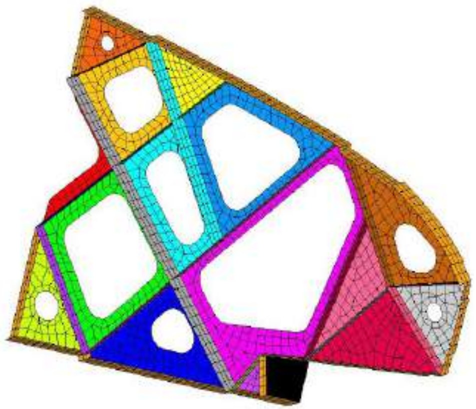
Altair Engineering

Airbus A380 example (cont.)

- Topology optimization:

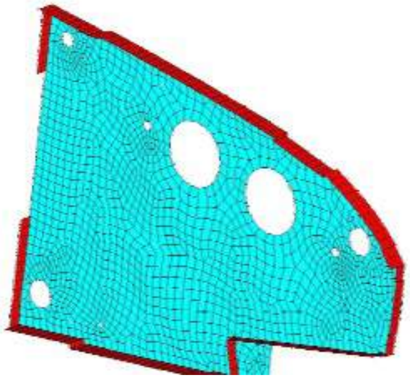


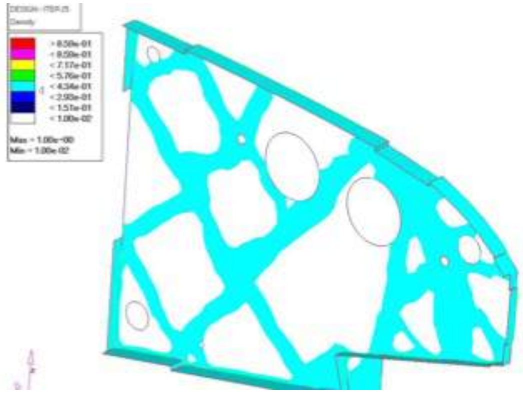
- Sizing / shape optimization:



Altair Engineering

Finally…





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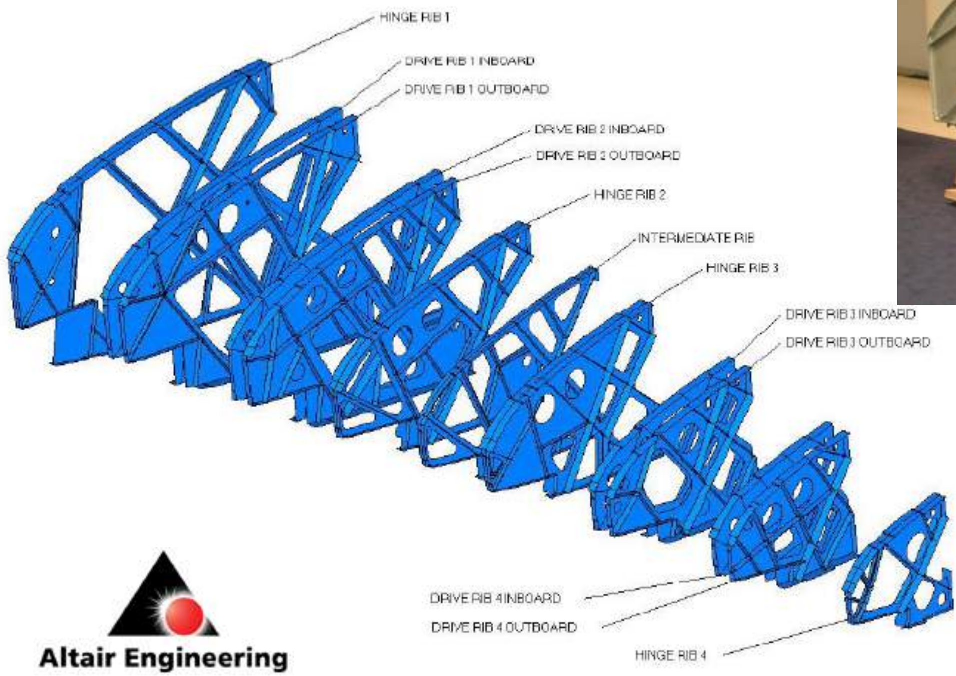
Airbus A380 example (cont.)

- Result: 500 kg weight savings!



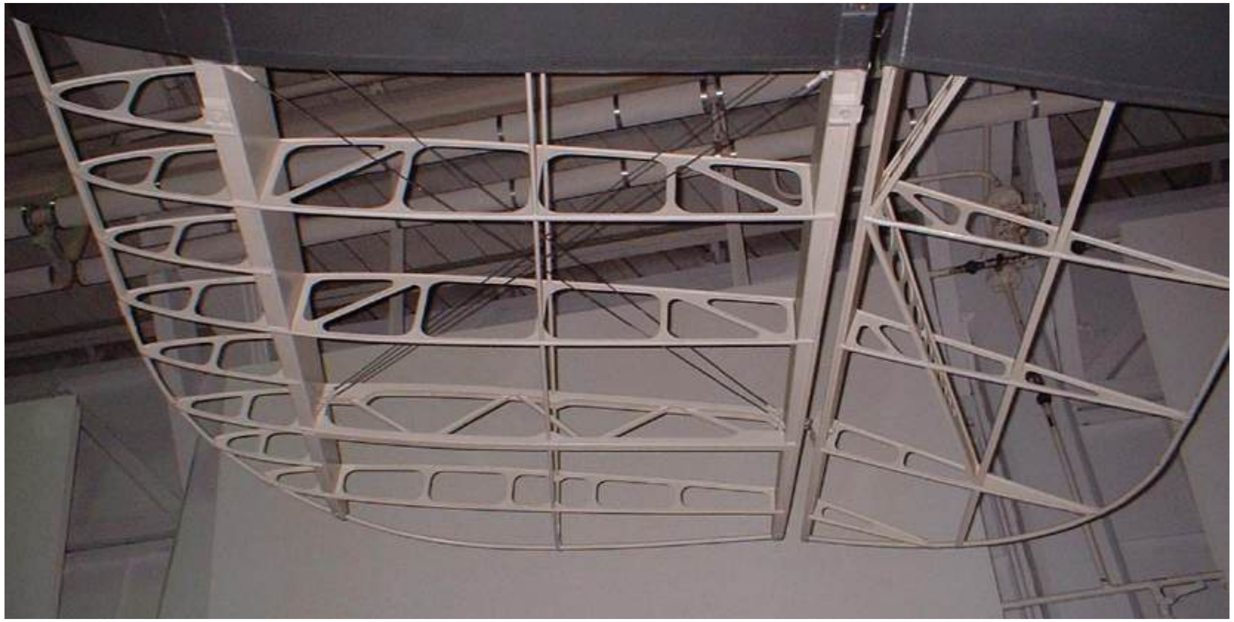
Altair Engineering

Wing rib designs

The perforated plates were replaced by reinforced lattice structures (think of the path of preferential intern foces)

Is this really a discovery?

Supermarine Southampton, 1925



System approach automates the process!

Industrial probems

TopOpt: Preliminary phases of a project

The idea is to find the best path of stiffening in a given volume of matter.

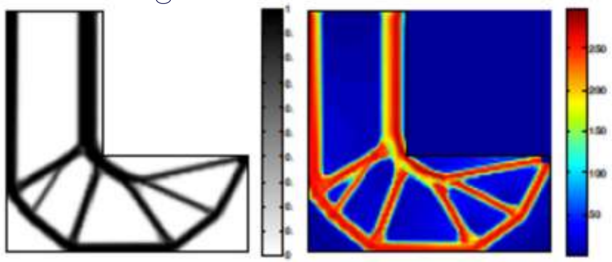
The mass is only found where it is needed, which is a good starting point for optimization of shape or dimensioning.

Adapted formulation:
To find the structure with the best static behavior.
The paths of internal forces identified are those which help to rigidify the structure as well as possible

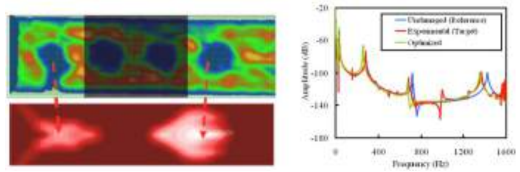
→The structure will deform less, and stress levels will be possibly limited. But not only…

→Lots of Actuall Researches

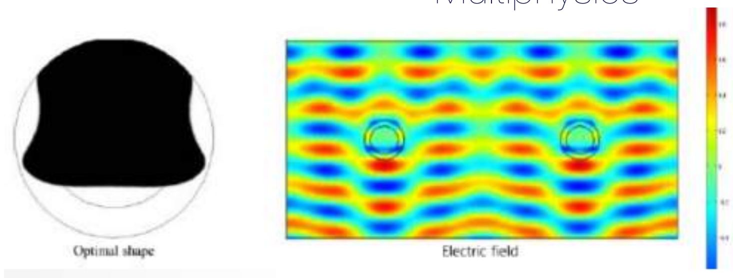
Fatigue



Global and clustered approaches for stress constrained topology optimization and deactivation of design variables. Erik Holmberg, Bo Torstenfelt, Anders Klarbring



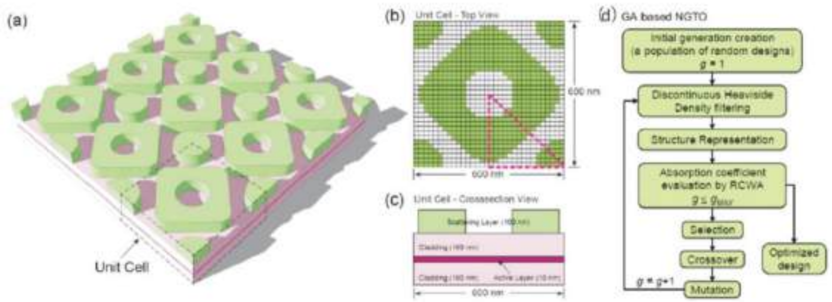
Multiphysics



Topology Optimization of silver nano-particles in thin film solar cells Soohwan Byun, Jeonghoon Yoo

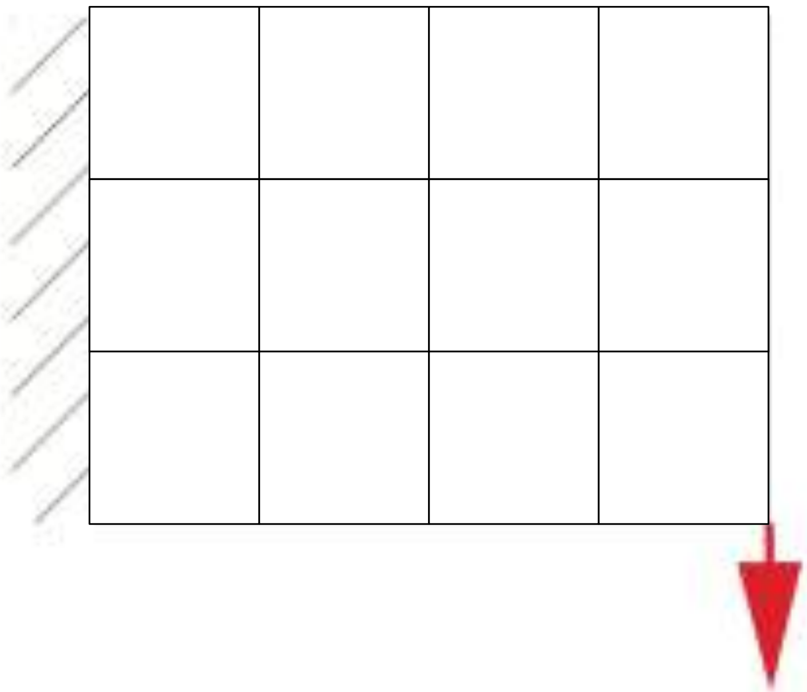
H. Niemann, J. Morlier, A. Shahdin and Y. Gourinat. Damage Localization using Experimental Modal Parameters and Topology Optimization. Mechanical Systems and Signal Processing. 24(3)636-652. 2010

Microstructures



Topology Optimization for Highly-efficient Light-trapping Structure in Solar Cell Shuangcheng Yu, Chen Wang, Cheng Sun, Wei Chen*

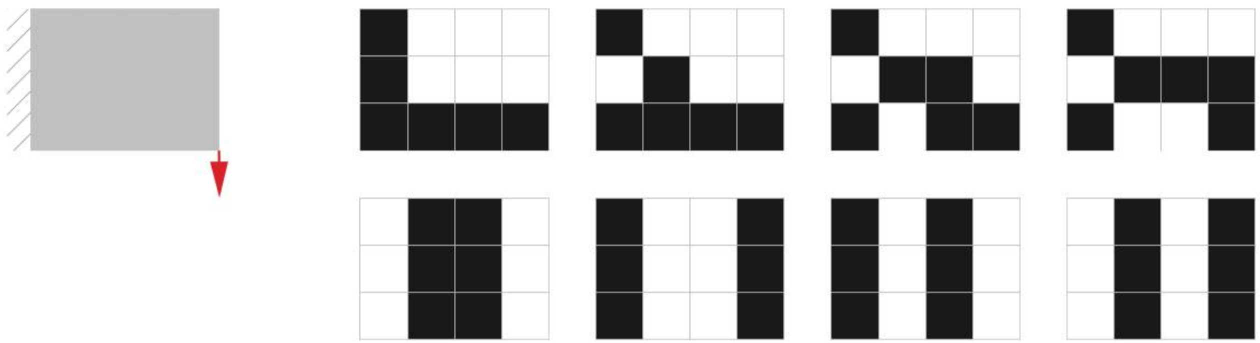
Quiz ! draw material (black) or void (white)



Goal: 50% gain in mass

Pixels

- Finding a solution by checking all the possible combinations IS impossible since the number of topologies nT increases exponentially with the number of finite elements n
- nT = 2^n,



The legal (top) and some illegal (bottom) topologies with 4 by 3 elements

Division into elements (pixels or voxels) and binary decision for each
or example 10,000 elements --> 210,000 possible configurations!

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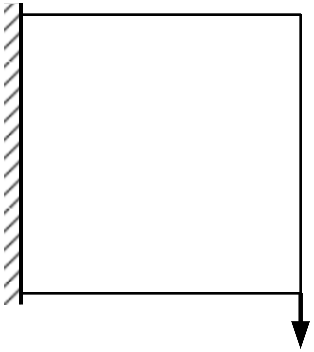
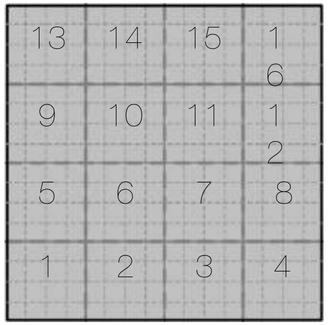
Maximum stiffness in the plane of a plate by selecting the best orientations of fibers



Loads and boundary conditions



Design model with 4∗ 4 patches

Table 4 Material properties

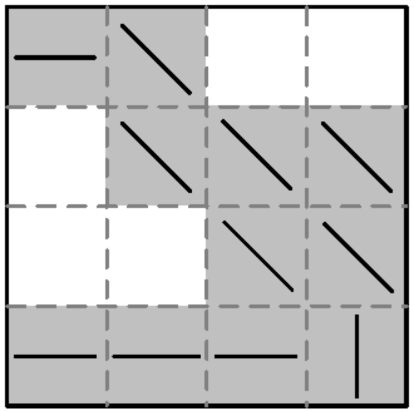
E_x	E_y	G_{xy}	v_{xy}
146.86GPa	10.62GPa	5.45GPa	0.33

Table 3 Orientations

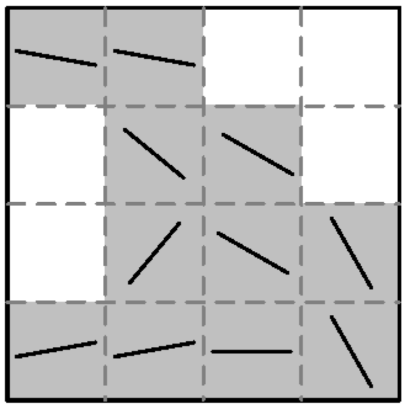
Number of material phases (m)	Number of design variables for each region (m_v)	Discrete orientation angle (°)
4	2	90/45/0/-45
9	4	80/60/40/20/0/-20/-40/-60/-80
12	4	90/75/60/45/30/15/0/-15/-30/-45/-60/-75

Discrete Material Optimization: exemple

Topological optimization: vacuum + composite laminate
Volume constraints: V <11/16



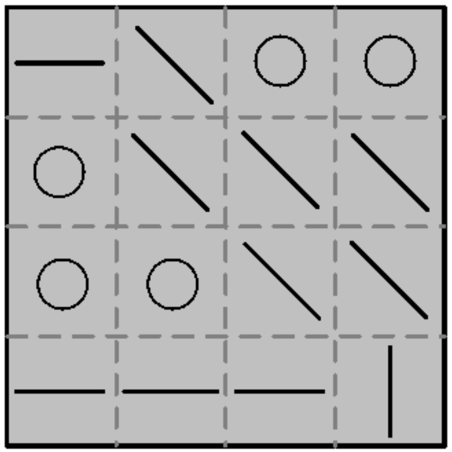
4 orientations
90/45/0/-45



18 orientations
90/80/70/60/50/40/30/20/10/0
/
-10/-20/-30/-40/-50/-60/-70/-
80

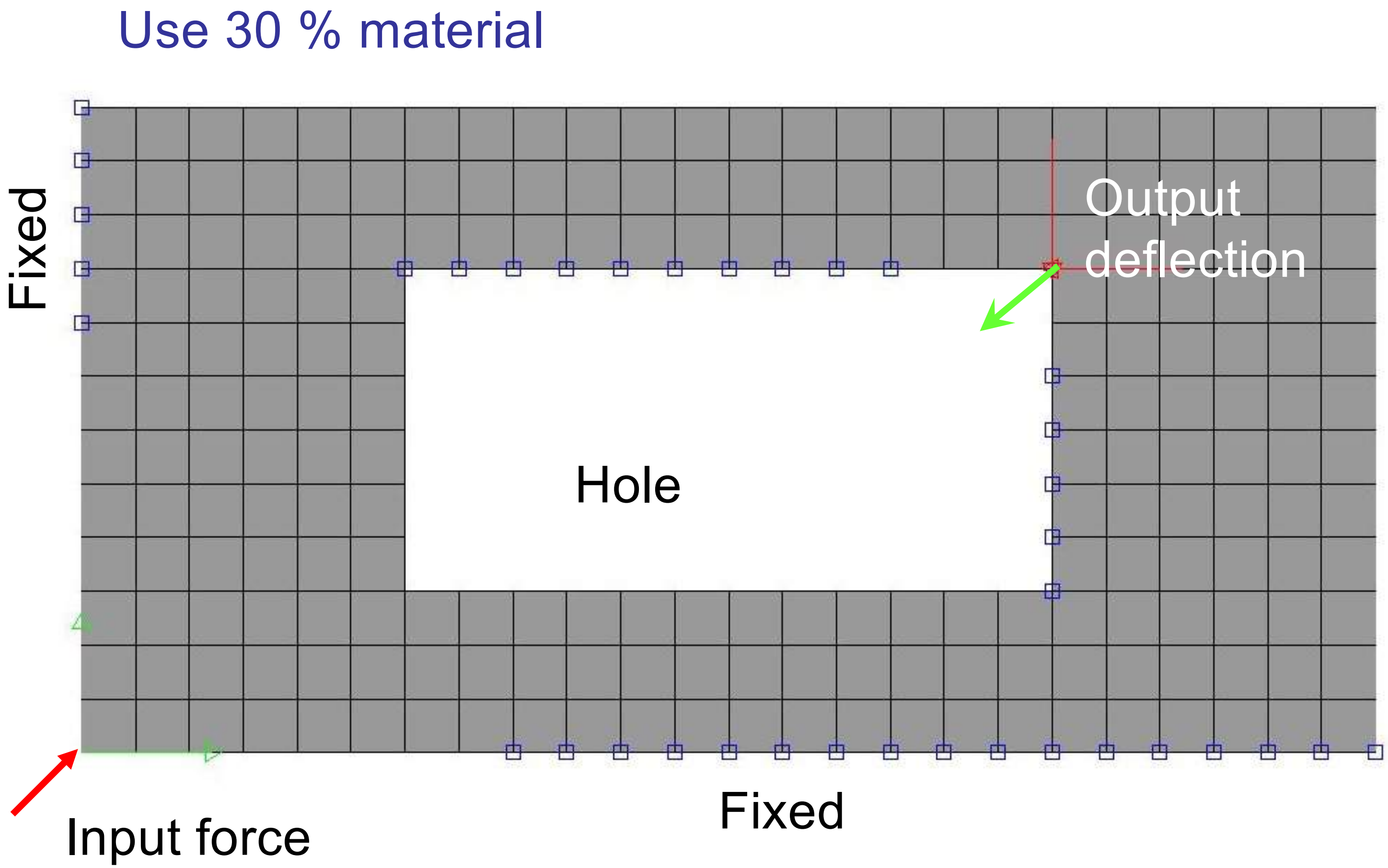
Discrete Material Optimization: exemple

Use of both glass fibers and foam
Limitation of the number of domains occupied by the fiber of glass

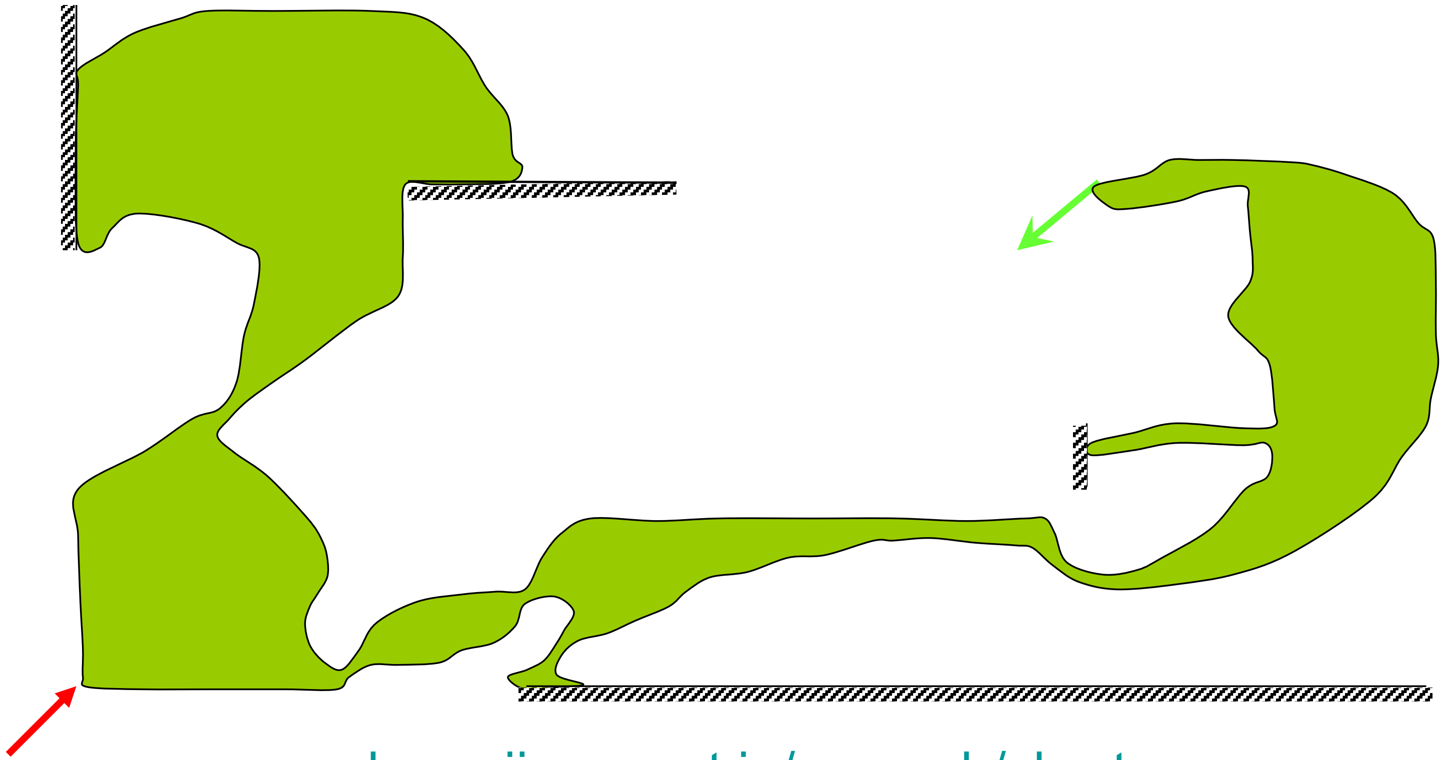


Optimization result of the square plate under vertical force with volume constraint
Glass-epoxy with 4 orientations (90/45/0/-45) and polymer-foam

Compliant mechanism to your specifications

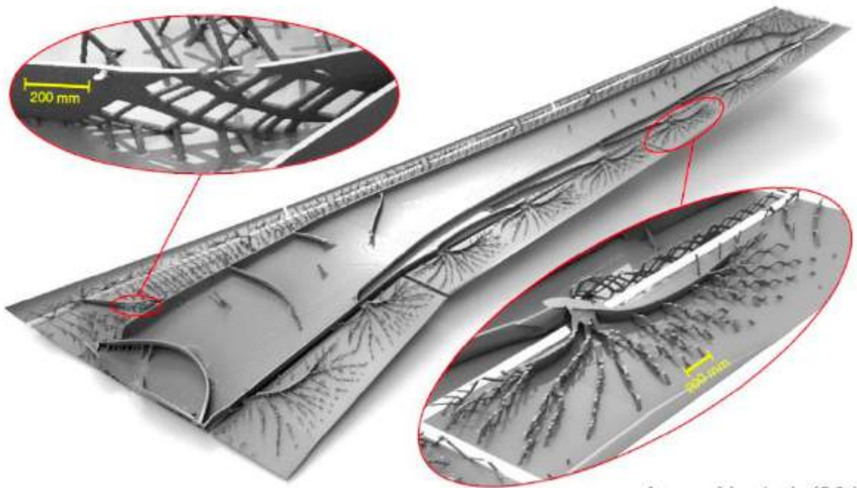


Compliant mechanism to your specifications

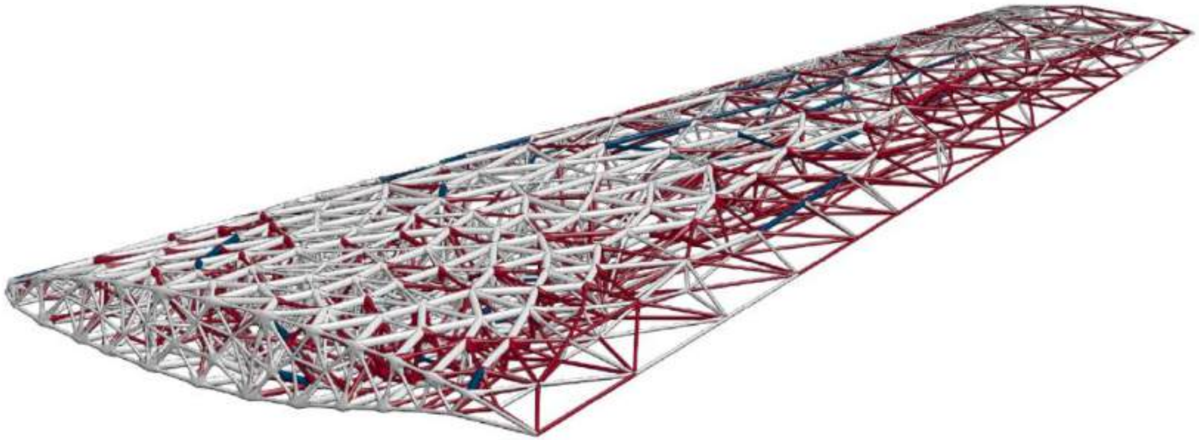


www.mecheng.iisc.ernet.in/~suresh/shortcourse

Concurrent method TOPOPT vs LAYOPT (continuous vs discrete)

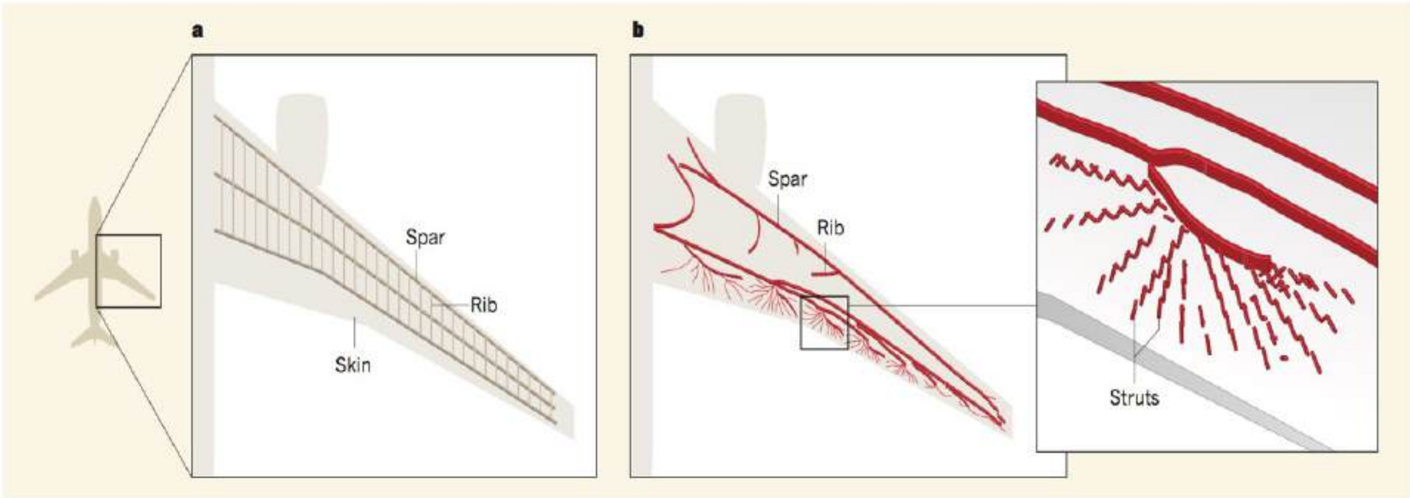


Aage, N. et al. (2017)



Opgenoord, M. M. and Willcox, K. E. (2018)

HPC



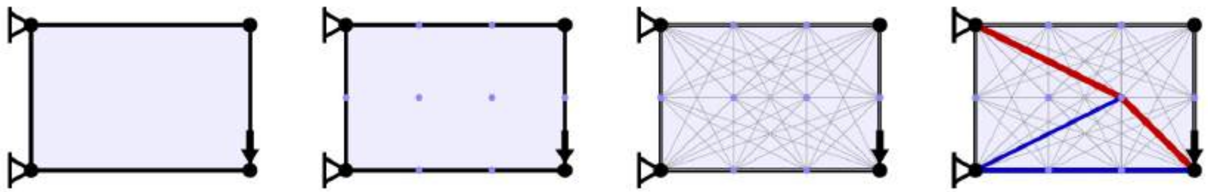
https://www.nature.com/articles/nature23911

More recently

https://www.layopt.com/truss/
https://www.youtube.com/watch?v=8OuU5K4iwSM

Layout Optimization

It was realized in the 1960s that minimum volume structures could be identified computationally. The steps in the numerical layout optimization procedure used by LayOpt are shown in the image below. First, the design domain (the region within which the structure is permitted to lie) is populated with nodes. Each pair of nodes is then connected with a potential structural member to create a "fully connected ground structure". Finally, a mathematical optimization problem is solved to identify the minium volume subset of members and their sizes.



Step 1: The problem is specified (domain, loads and supports).

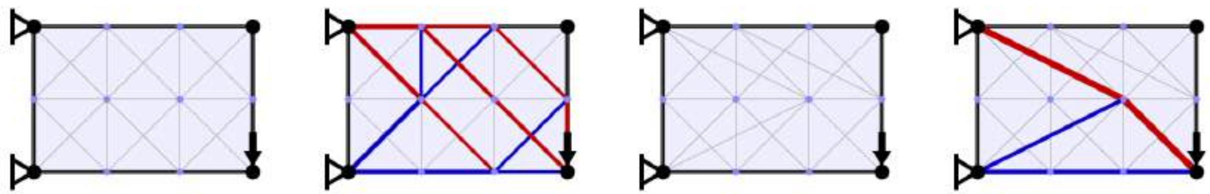
Step 2: Nodes are added to the design domain

Step 3: The nodes are connected by potential members.

Step 4: The optimal set of members is identified.

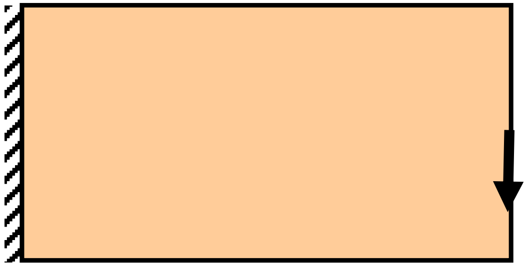
In LayOpt, nodes are located at the intersections of grid lines, and at locations where the grid lines cross the edges of the design domain. The coarseness of the grid can be adjusted using the slider. When a very fine nodal grid is employed a very close approximation of the true minimum volume is obtained. However, increasing the number of nodes also increases the computational resources required, and usually the complexity of the resulting structure.

To increase the computational efficiency LayOpt makes use of the adaptive member adding method proposed by Gilbert and Tyas (2003). This only requires a subset of the possible connections to be considered initially (here only adjacent nodes are initially connected). Once this problem is solved, each initially neglected potential member is checked to see if it is likely to reduce the calculated volume. If so, then is considered for addition to the problem in the next iteration. This process continues until no potential members can be found that have the potential to reduce the volume of the structure. At this point the volume of the structure will be identical to the solution of the corresponding problem which included all potential member connections from the outset. LayOpt shows the result of each member adding iteration as it is calculated, allowing you to see how the optimal design is being identified.

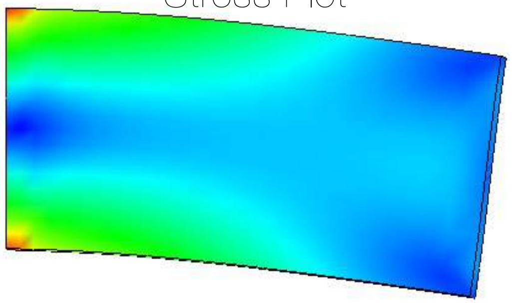


Intuition …

*See Prof Suresh’s work:
https://dl.acm.org/citation.cfm?id=1861606

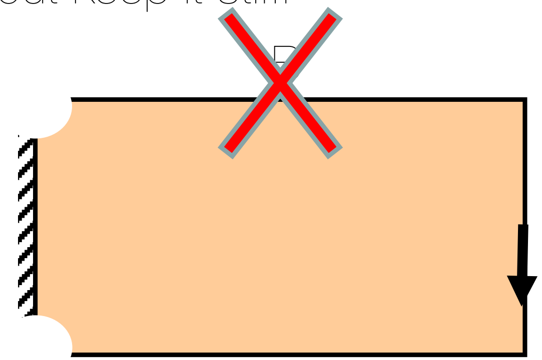


Stress Plot

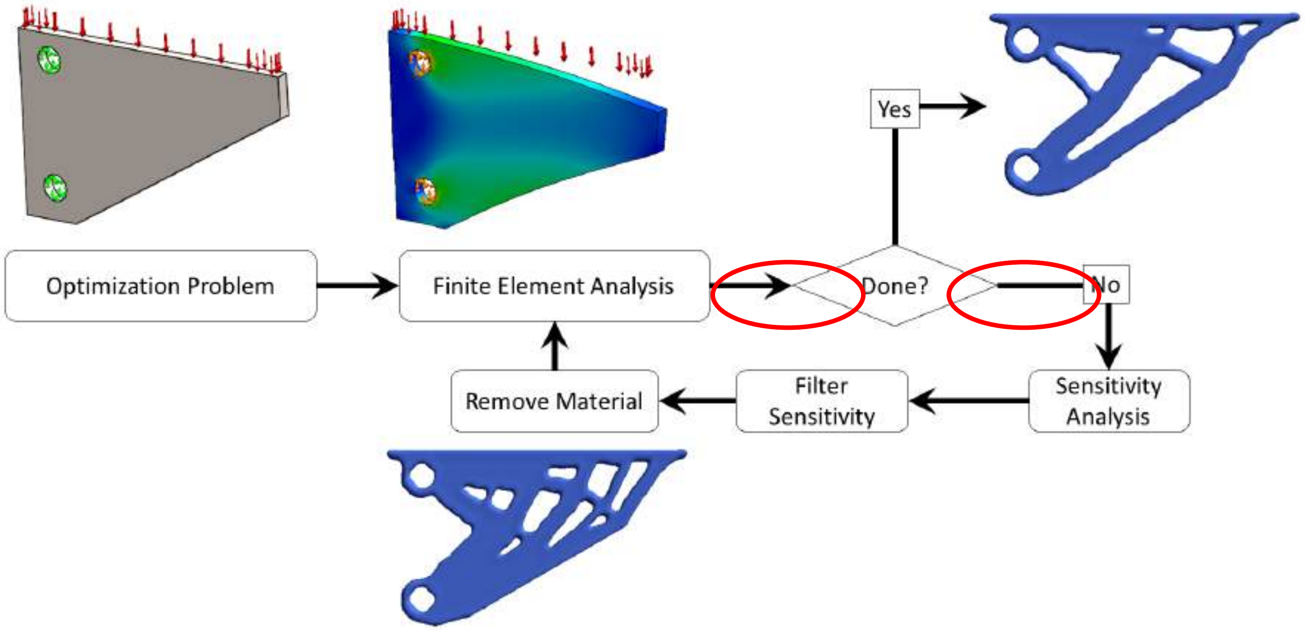


Remove material, but keep it ~~stiff!~~ Strong*!

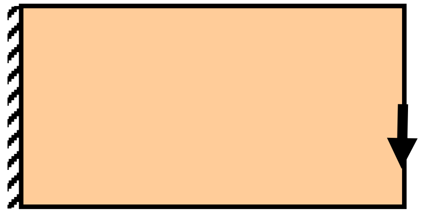
A



TopOpt relies on FEA
Online computation: http://www.cloudtopopt.com



TopOpt



$Ku = f$

1. Objective?
2. Constraints?
3. Method?

Minimize Compliance

Volume Constraint

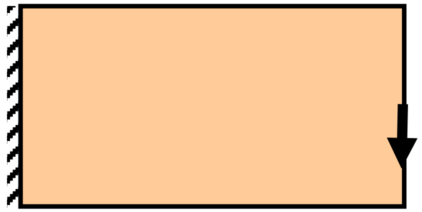
$\boxed{\begin{array}{l}\text{Minimize } J \\ \text{Vol.Frac} \leq 0.5\end{array}}$

Compliance $J = f^T u$

Compliance = 1/Stiffness

Method: Gradient based:
Need sensitivities…

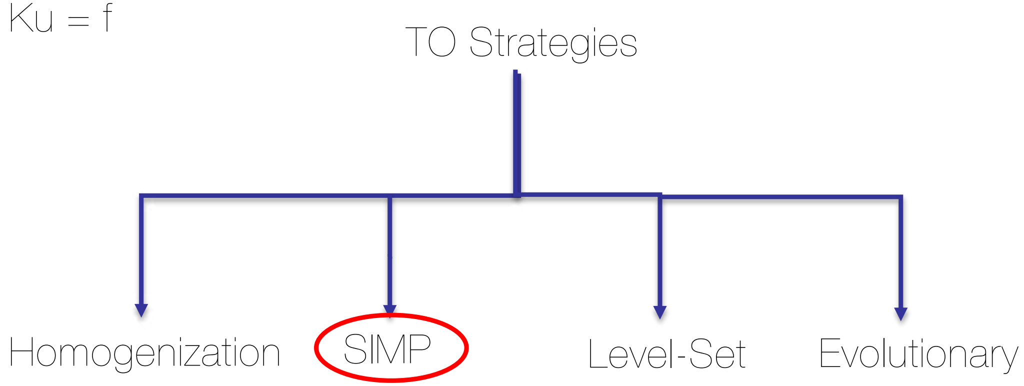
Current TO Strategies



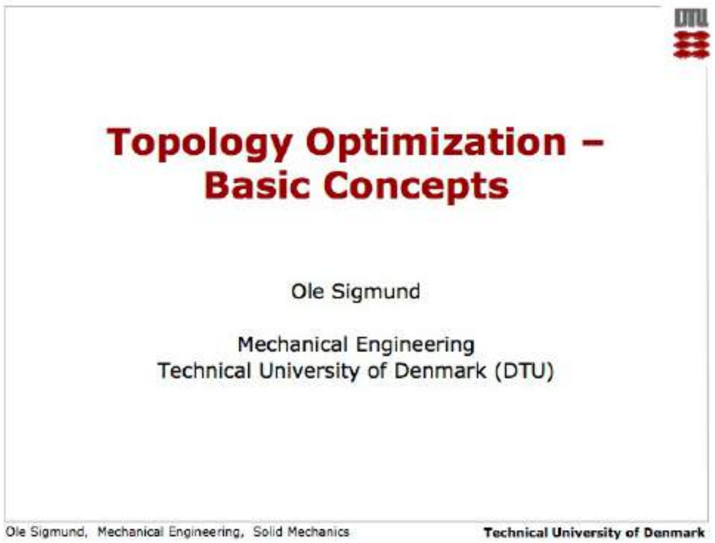
Minimize J

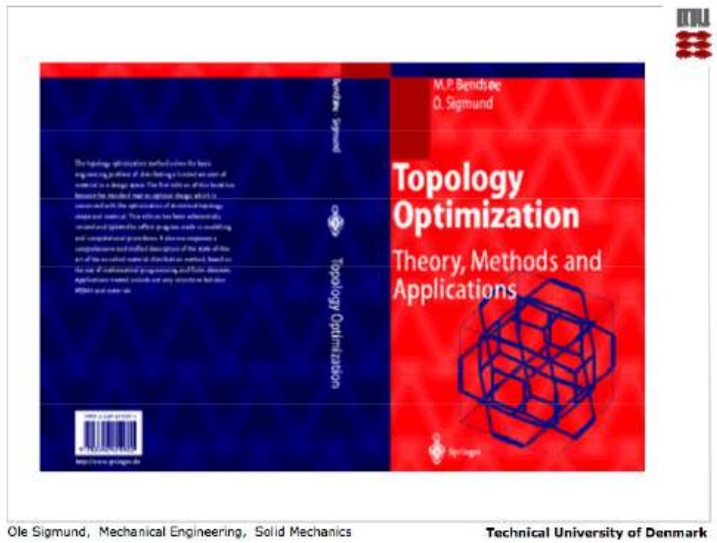
Vol.Frac ≤ 0.5

Ku = f



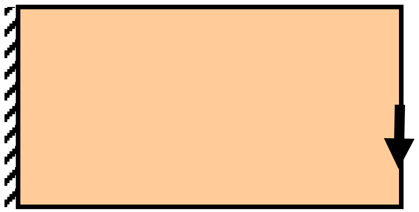
One pioneer, SIMP (Solid Isotropic Material with Penalization)

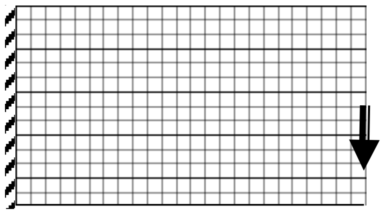




SIMP

SIMP: Solid Isotropic Material with Penalization





$\underset{}{Min}$ Compliance

$v = 0.5v_0$

$0 < \rho_e \leq 1$: 'PseudoDensity'

Where do we add holes?

$$\underset{\rho_e}{Min} \ \text{Compliance}$$

$$\sum \rho_e v_e = 0.5 v_0$$

Pixels?

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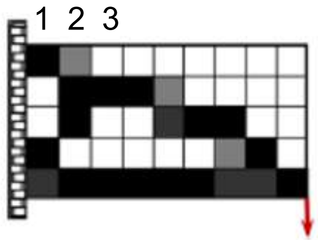
Intuitive Problem? Quadratic Form



$x_1 = 1$
$x_2 = 0.5$
$x_3 = 0$
$\dots$

- Objective function; Strain energy

$$\min c(\mathbf{x}) = \mathbf{U}^T\mathbf{F} = \mathbf{U}^T\mathbf{K}\mathbf{U} \qquad \text{with} \quad x_e = \frac{\rho_e}{\rho_0} \tag{4}$$

with $$\mathbf{K} = \mathbf{K}_0 \sum_{e=1}^{N} x_e^p$$ one can write:

$$\min c(\mathbf{x}) = \sum_{e=1}^{N} (x_e)^p \mathbf{u}_e^T \mathbf{k}_0 \mathbf{u}_e \tag{5}$$

Scalar

- Contraints: mass target

$$\frac{V(\mathbf{x})}{V_0} = f = const \Leftrightarrow \sum_{e=1}^{N} V_e x_e - V_0 f = 0 = h(\mathbf{x})$$

Scalar

$$0 < \rho_{\min} \le \rho_e \le 1$$

$$\min c(\mathbf{x}) = \sum_{e=1}^{N} (x_e)^p \mathbf{u}_e^T \mathbf{k}_0 \mathbf{u}_e$$

Quadratic Form

$x \in \mathbb{R}^{m \times 1}, \ \mathbb{A} \in \mathbb{R}^{m \times m}$

$$\boxed{\text{Quadratic form : } \quad x^T \mathbb{A} x}$$

$x^T \mathbb{A} x$ is a scalar value.

$(1 \times m) \times (m \times m) \times (m \times 1) \rightarrow 1 \times 1$

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K is linked through E and xe

Rozvany, G.I.N. , Zhou, M., and Gollub, M. (1989). Continuum Type Optimality Criteria Methods for Large Finite Element Systems with a Displacement Connstraint, Part 1. *Structural Optimization* 1:47-72.

$$\mathbf{K} = \mathbf{K}_0 \sum_{e=1}^{N} x_e^p \qquad x_e = \frac{\rho_e}{\rho_0}$$

What is p,
(simP)?????????

- But HOW ??

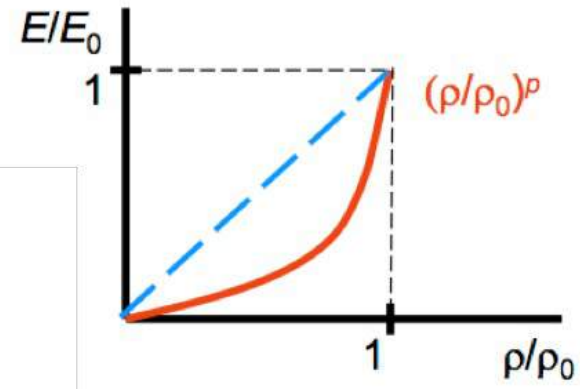
Avoid intermediate densities !

Solid Isotropic Material with Penalization (SIMP)

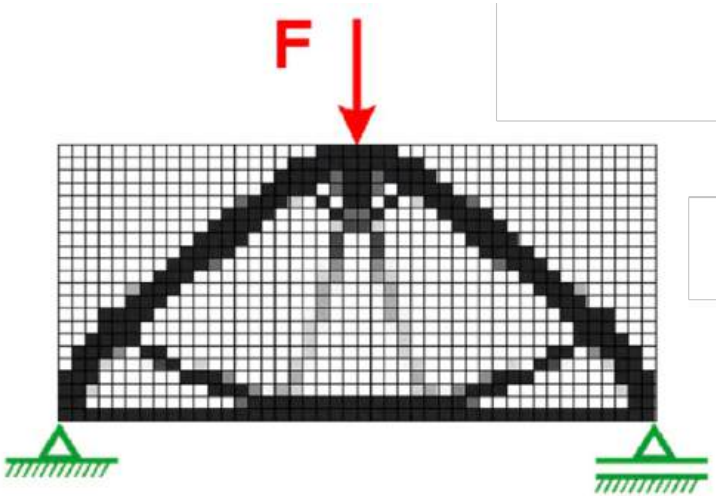
$$E(x) = E_{min} + (E_0 - E_{min})x^p$$

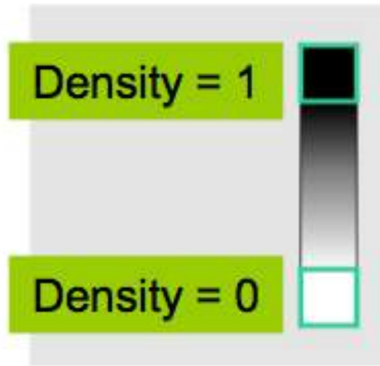
p is the penalty parameter to push densities to black (1) and white (0).

E_{min} is a small value that avoid stiffness matrix singularity

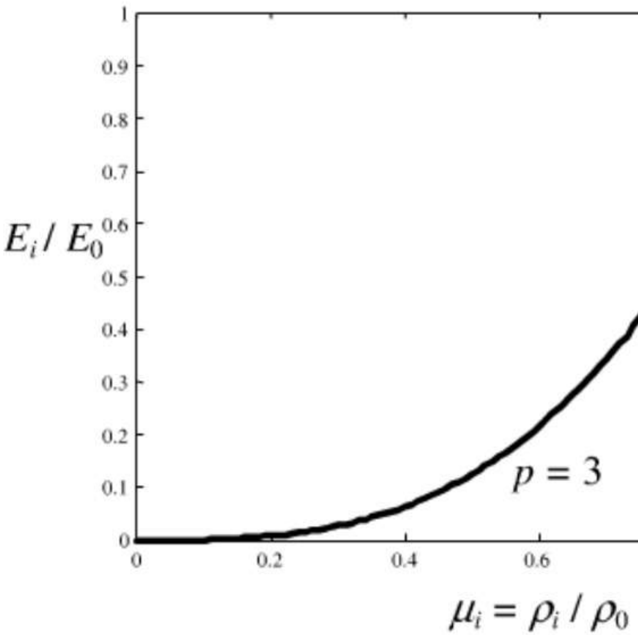


Penalization for altering stiffness localy

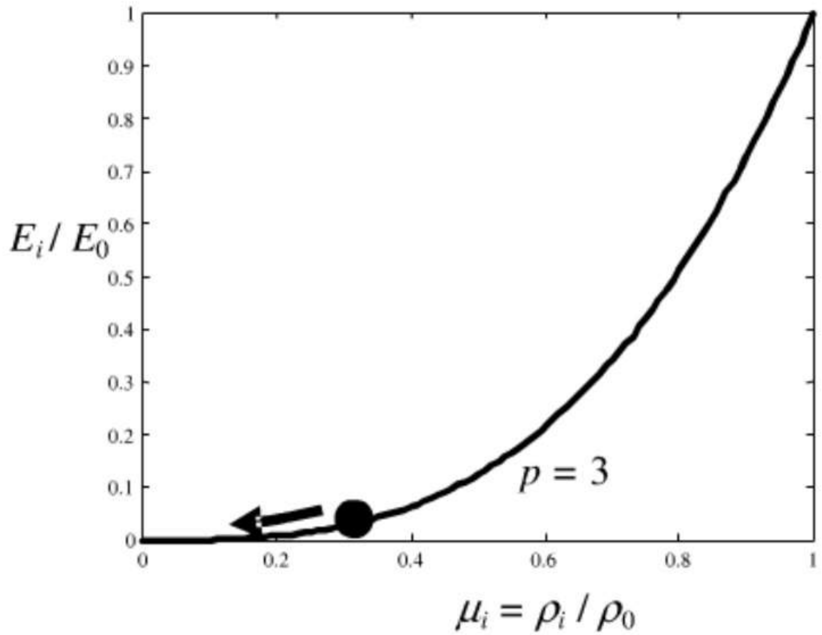




Let' s take p=3



E_i / E_0
$\mu_i = \rho_i / \rho_0$
$p = 3$
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1



E_i / E_0
$\mu_i = \rho_i / \rho_0$
$p = 3$
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

Penalty parameter in the SIMP method: Proof

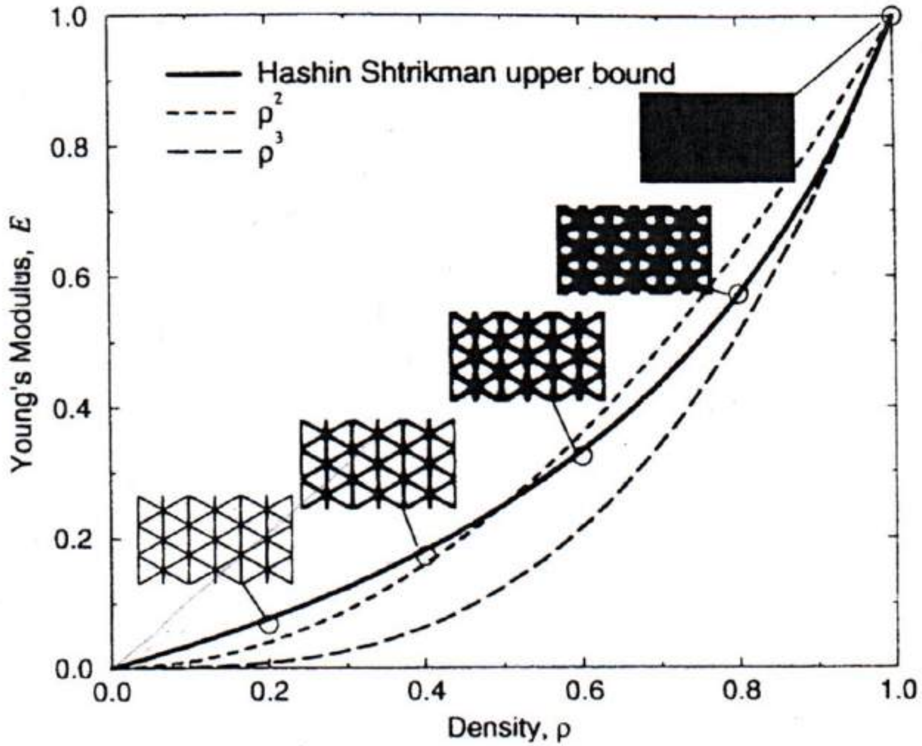
Hashin-Shtrikman bounds

$$0 \leq E \leq \frac{\rho E^0}{3-2\rho}$$

Therefore,

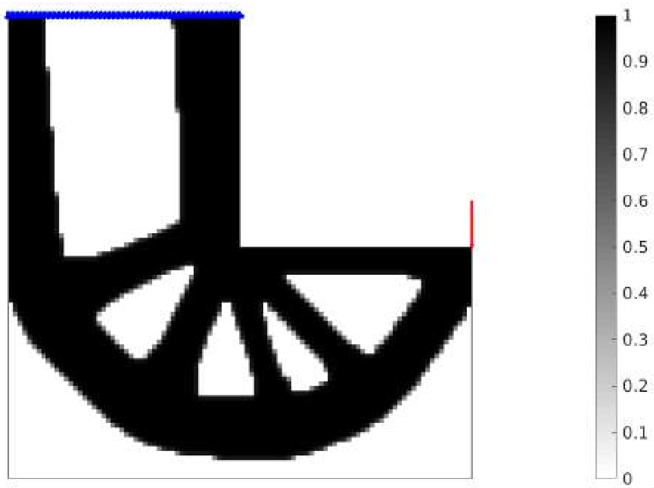
$$\rho^p E^0 \leq \frac{\rho E^0}{3-2\rho}$$

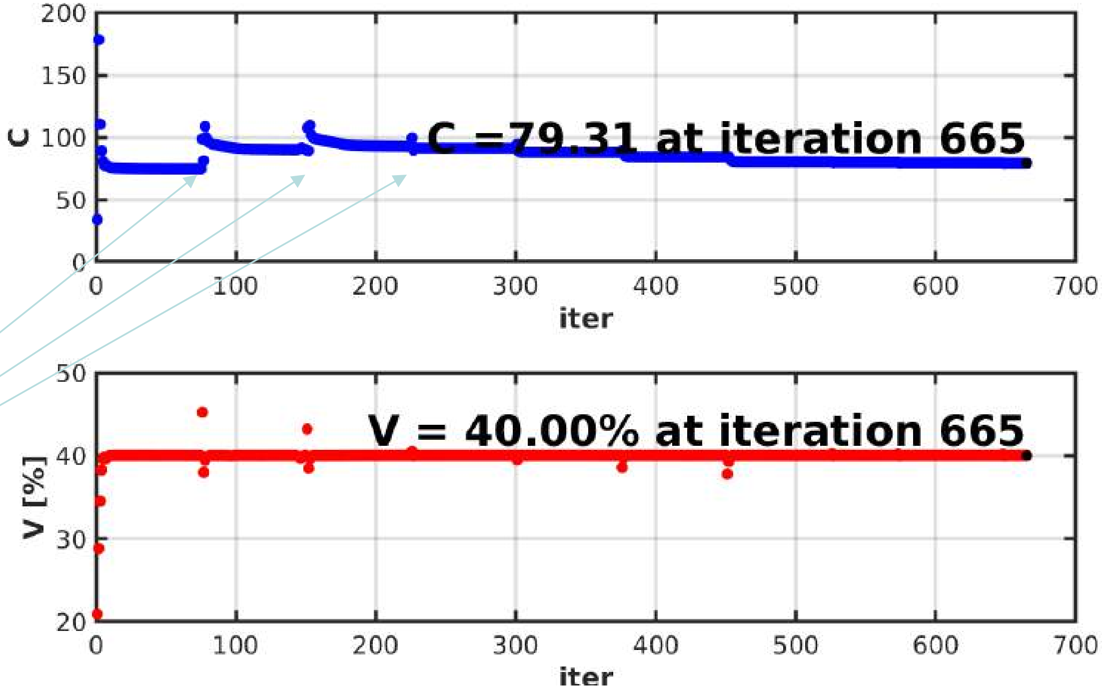
$$\Rightarrow p \geq 3$$



Bendsøe, M.P. and Sigmund, O., "Material Interpolation Schemes in Topology Optimization," *Archives in Applied Mechanics*, Vol. 69, (9-10), 1999, pp. 635-654.

Penalty parameter in the SIMP method: Continuation Methods





Increase p
Reduce the filter radius
Increase beta

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Nice idea !

1. Transform discrete variables continuously (TO USE gradient-based algorithms)
2. Find an objective function with "cheap" derivatives (we will see this later)

Others formulations

$$\min_{\boldsymbol{\mu}} \max_{l=1,\ldots,nc} C_l = \mathbf{F}_l^T \mathbf{q}_l$$
$$\sum_i \mu_i V_i \leq \overline{V}$$
$$0 < \underline{\mu}_i \leq \mu_i \leq 1$$

- If several load cases nc

→ we can minimize the maximal compliance

→ with ql obtained by solving Klql=Fl

$$\min_{\boldsymbol{\mu}} \sum_i \mu_i V_i$$
$$q_j \leq \overline{q}_j \qquad j = 1,\ldots,m$$
$$0 < \underline{\mu}_i \leq \mu_i \leq 1$$

- Prescribed displacement

→ we can minimize the volume (mass)

→ wrt amplitude at node j inferior to a certain displacement

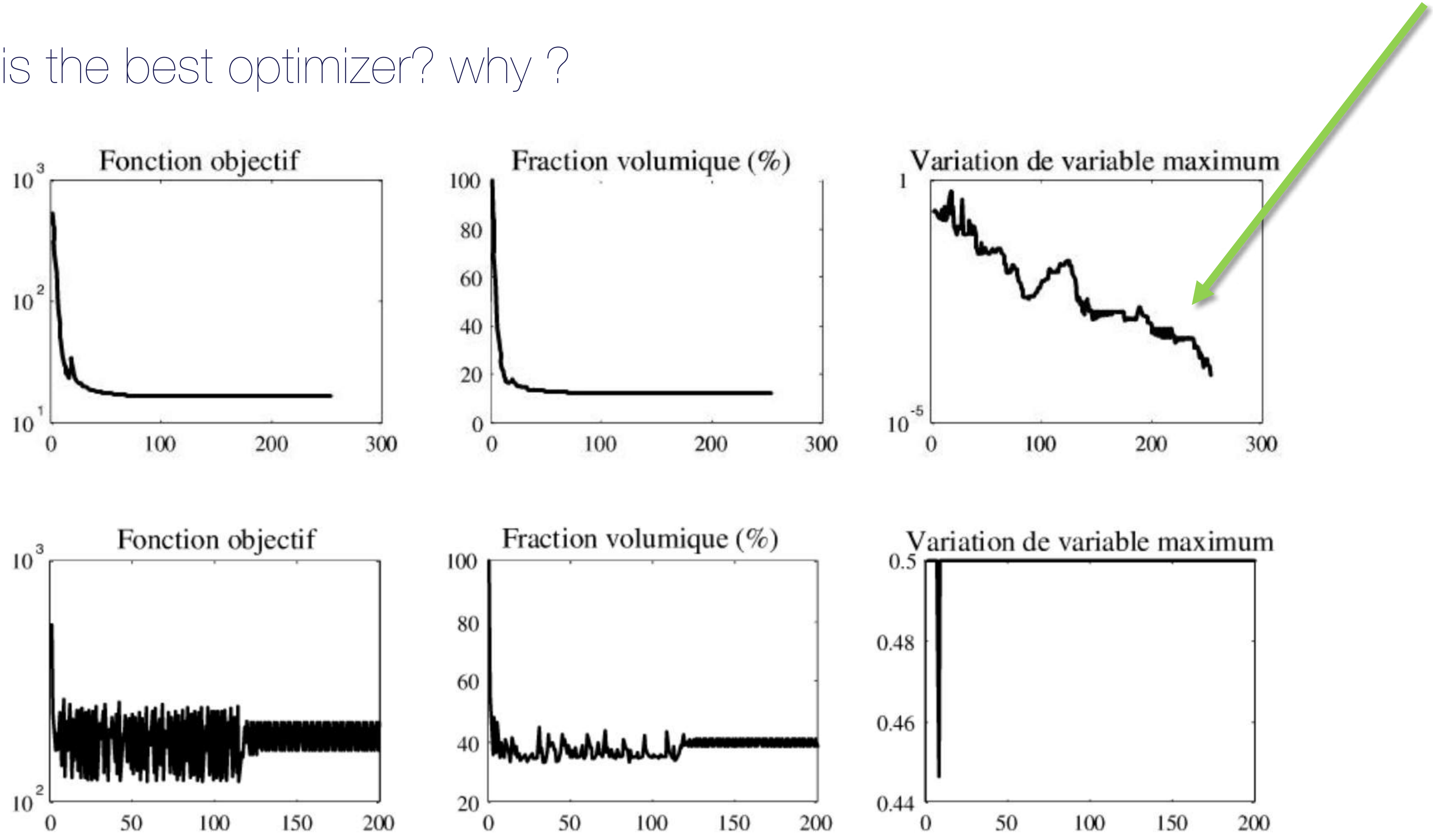
Others formulations

$$\max_{\boldsymbol{\mu}} \min_{k=1,nf} \omega_k$$
$$\sum_i \mu_i V_i \leq \overline{V}$$
$$0 < \underline{\mu}_i \leq \mu_i \leq 1$$

- Eigensolver to obtain the stiffest structure at a certain volfrac

→ wrt a vibration ccriteria

Which is the best optimizer? why ?



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Can you comment this ? Compliance are re

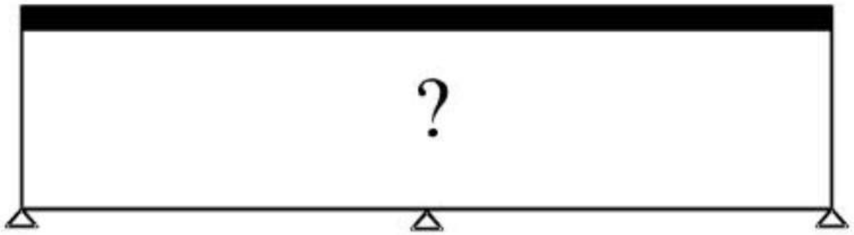


Figure 13.18 Problème du pont.

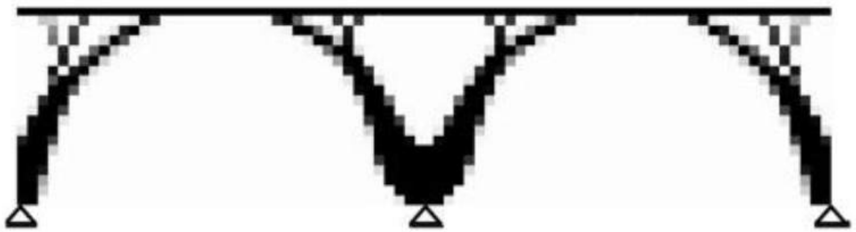
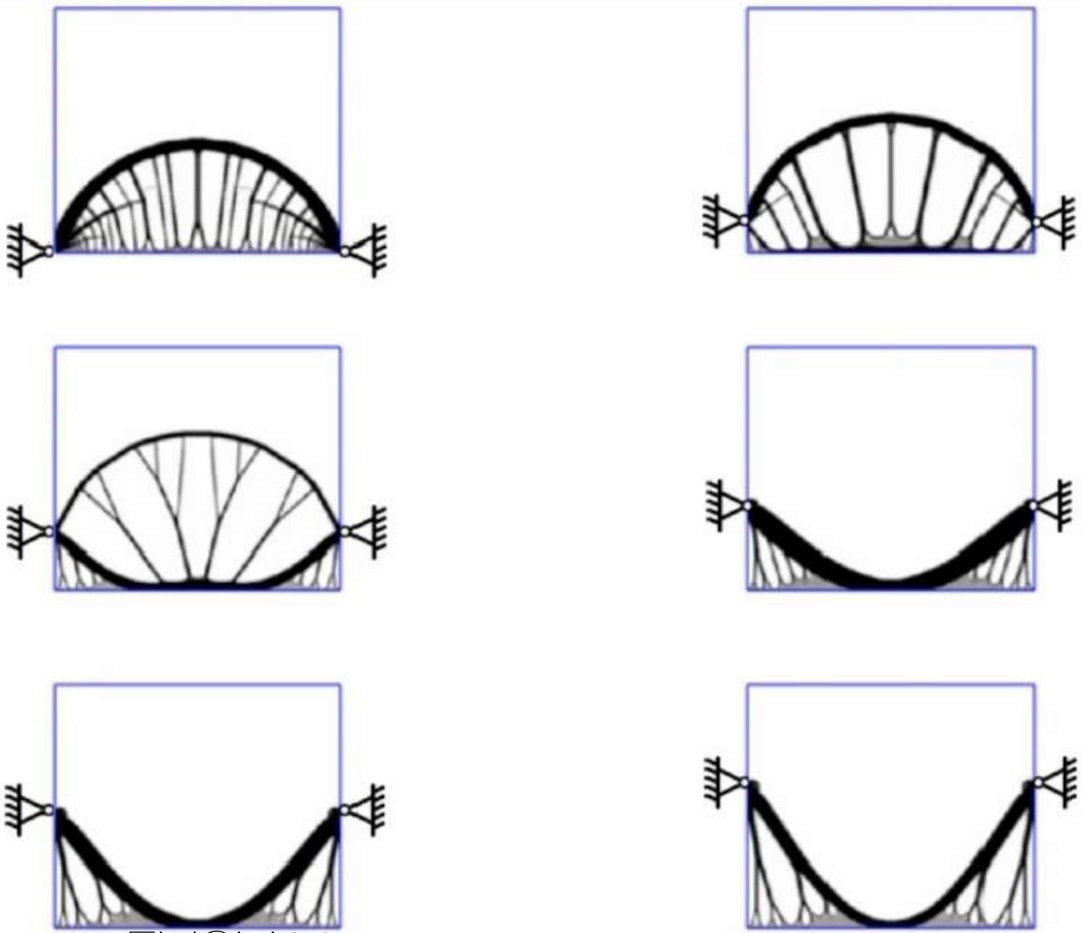


Figure 13.19 Solution du problème du pont avec MMA.

Figure 13.20 Solution du problème du pont avec GCMMA.

Figure 13.21 Solution du problème du pont avec GCM.

Small changes in BCs …



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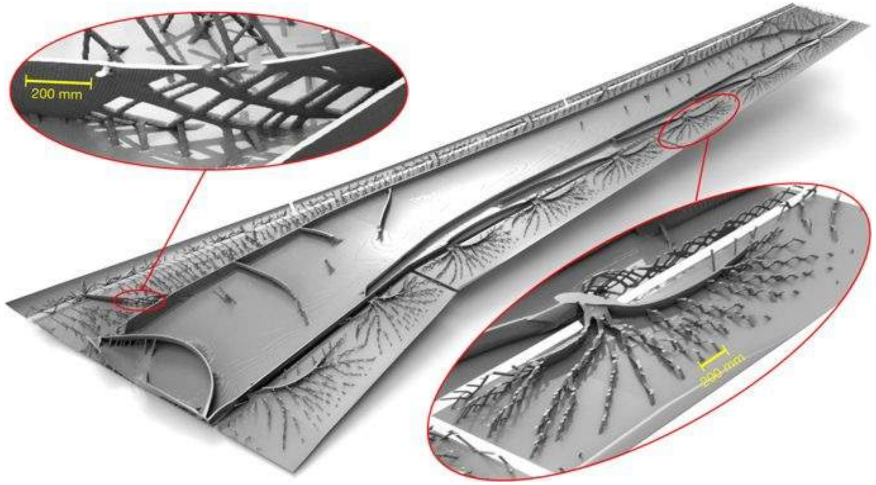
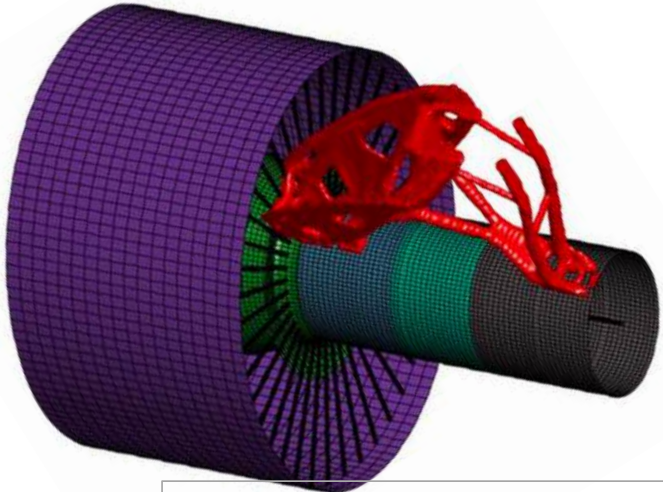
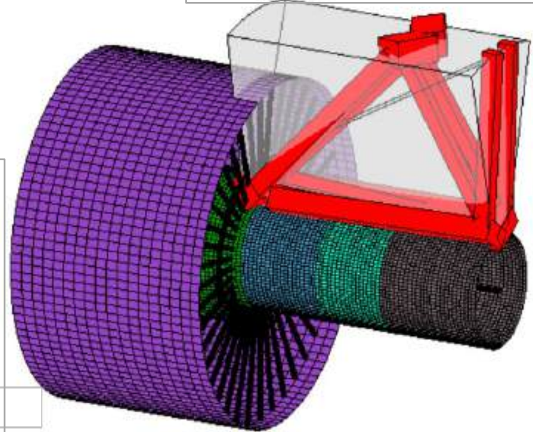
Interested in research ?

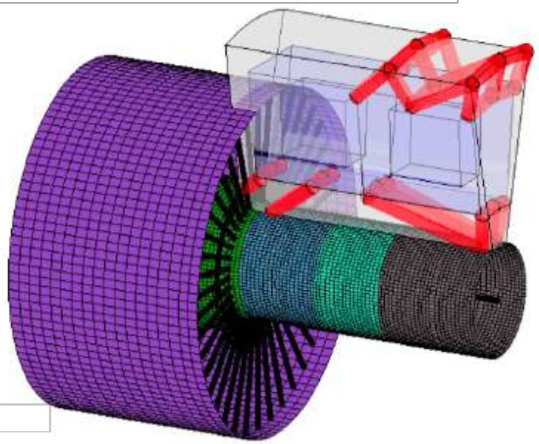


Niels Aage, Erik Andreassen, Boyan S Lazarov, and Ole Sigmund. Giga-voxel computational morphogenesis for structural design. Nature, 550(7674):84, 2017.



Coniglio, S., Gogu, C., Amargier, R., & Morlier, J. (2019). Engine pylon topology optimization framework based on performance and stress criteria. AIAA Journal, 57(12), 5514-5526.





Coniglio, S., Gogu, C., Amargier, R., & Morlier, J. (2019, May). Application of geometric feature based topology optimization to engine pylon architecture design including engine performance criteria. In 13th Wolrd Congress on Structural and Multidisciplinary Optimization.

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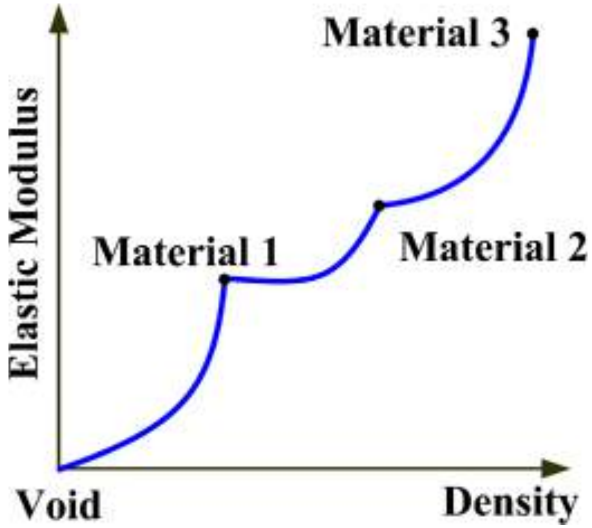
MULTIMATERIAL

- Solid Isotropic Material with Penalization (SIMP)
- $E_e(\rho_e) = A_E * \rho_e^p + B_E$,

$\rho_e \in [\rho_i , \rho_{i+1}]$, $A_E = \frac{E_i - E_{i+1}}{\rho_i^p - \rho_{i+1}^p}$, $B_E = E_i - A_E * \rho_i^p$



Zuo, W., & Saitou, K. (2016). Multi-material topology optimization using ordered SIMP interpolation. *Structural and Multidisciplinary Optimization, 55*(2), 477-491. doi:10.1007/s00158-016-1513-3

Part2: TOPOPT in practice

Lego?

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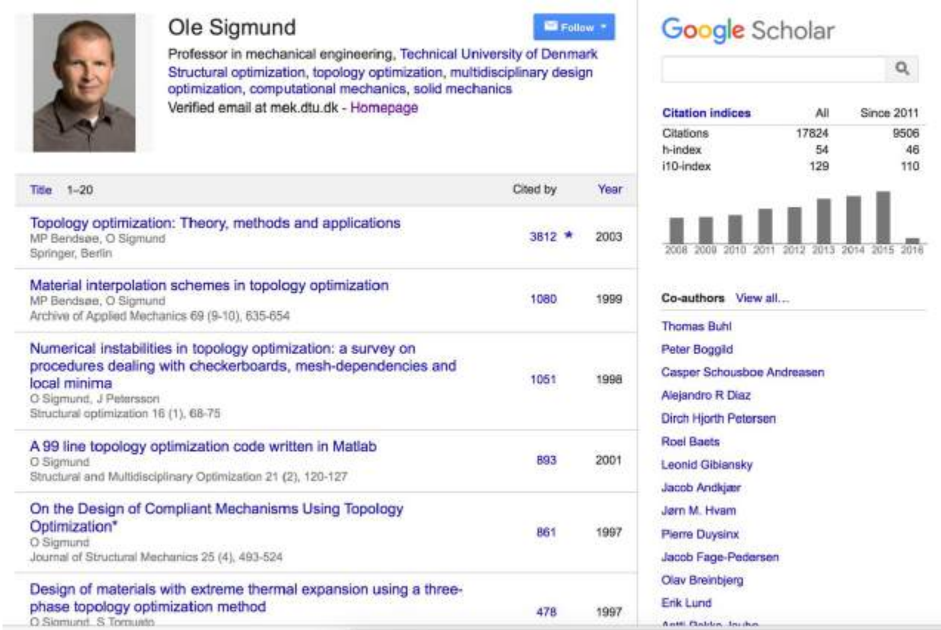
BUT …IN PRACTICE?

Educational article:

O. Sigmund , A 99 line topology optimization code written in Matlab Struct Multidisc Optim 21, 120–127 Springer-Verlag 2001

Heuristic formulation (intuitiive method of optimisation, but with no convergency proofs) to update xe by bi-section algorithm



History (1988, Bendsoe)

A topology optimization problem based on the power-law approach, where the objective is to minimize compliance can be written as

$$\left.\begin{aligned} \min_{\mathbf{x}}: &\quad c(\mathbf{x}) = \mathbf{U}^T\mathbf{K}\mathbf{U} = \sum_{e=1}^{N}(x_e)^p\, \mathbf{u}_e^T\, \mathbf{k}_0\, \mathbf{u}_e \\ \text{subject to}: &\quad \frac{V(\mathbf{x})}{V_0} = f \\ : &\quad \mathbf{K}\mathbf{U} = \mathbf{F} \\ : &\quad \mathbf{0} < \mathbf{x}_{\min} \le \mathbf{x} \le \mathbf{1} \end{aligned}\right\}, \tag{1}$$

where $\mathbf{U}$ and $\mathbf{F}$ are the global displacement and force vectors, respectively, $\mathbf{K}$ is the global stiffness matrix, $\mathbf{u}_e$ and $\mathbf{k}_e$ are the element displacement vector and stiffness matrix, respectively, $\mathbf{x}$ is the vector of design variables, $\mathbf{x}_{\min}$ is a vector of minimum relative densities (non-zero to avoid singularity), N ($=\texttt{nelx} \times \texttt{nely}$) is the number of elements used to discretize the design domain, p is the penalization power (typically $p = 3$), $V(\mathbf{x})$ and V_0 is the material volume and design domain volume, respectively and f (`volfrac`) is the prescribed volume fraction.

State-of-Art – SIMP[1,2]

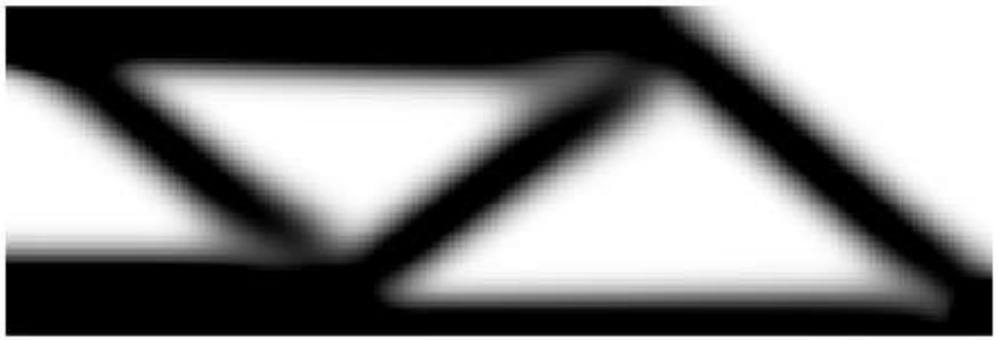
Predict an optimal distribution of the material in a design space for given boundary conditions and design constraints

Density-based topopt

$$E_e = E_{min} + x_e^p (E_0 - E_{min})$$

$$c(x) = U^T K U = \sum_{e=1}^{N} E_e(x_e)^p u_e^T k_0 u_e$$

MBB problem result by top88[3]



c = 222.3

1 - Bendsøe MP (1989) Optimal shape design as a material distribution problem.
2 - Zhou M, Rozvany G (1991) The COC algorithm, part II: topological, geometrical and generalized shape optimization.
3 - Andreassen E, Clausen A, Schevenels M, Lazarov B, Sigmund O (2010) Efficient topology optimization in MATLAB using 88 lines of code

Compliance minimization **self adjoint**

- Compliance is the opposite of stiffness

$$C = \mathbf{f}^T \mathbf{u} = \mathbf{u}^T K \mathbf{u}$$

- Inexpensive derivatives (use chain rule)

$$\frac{dC}{dx} = 2\mathbf{u}^T K \frac{d\mathbf{u}}{dx} + \mathbf{u}^T \frac{dK}{dx}\mathbf{u}$$

But since $K\mathbf{u} = \mathbf{f}$ if $\mathbf{f}$ does not depend on x

$$K\frac{d\mathbf{u}}{dx} = -\frac{dK}{dx}\mathbf{u}$$

$$\frac{dC}{dx} = -\mathbf{u}^T \frac{dK}{dx}\mathbf{u}$$

Knowing displacements you also know gradients

Density design variables

Need a DEMO ?

- Recall $$\frac{dC}{dx} = -\mathbf{u}^T \frac{dK}{dx}\mathbf{u}$$
- For density variables

$$\frac{dC}{d\rho^e} \propto -\mathbf{u}^T \rho^{p-1} K^e \mathbf{u}$$

- Want to increase density of elements with high strain energy and vice versa
- To minimize compliance for given weight can use an optimality criterion method.

And for other responses?

$$O = f(x, U)$$

$$\frac{\partial O}{\partial x} = \frac{\partial f}{\partial U}^T \frac{\partial U}{\partial x}$$

$$KU = F$$

$$\frac{\partial K}{\partial x} U + K \frac{\partial U}{\partial x} = 0$$

$$\frac{\partial O}{\partial x} = \frac{\partial f}{\partial U}^T \frac{\partial U}{\partial x} = -\frac{\partial f}{\partial U}^T K^{-1} \frac{\partial K}{\partial x} U = -\frac{\partial f}{\partial U}^T \delta$$

$K\lambda = \frac{\partial f}{\partial U}$ Adjoint Method

$K\delta = \frac{\partial K}{\partial x} U$ Direct Method

Either one solution per response
Either one solution per design variables
That's why Compliance!

Matlab Code

```
x(1:nely,1:nelx) = volfrac;                                   % INITIALIZE

loop = 0; change = 1.;
while change > 0.01                                           % START ITERATION While Xk+1>>Xk
        loop = loop + 1;
        xold = x;
        [U]=FE(nelx,nely,x,penal);     % FE-ANALYSIS
        [KE] = lk;
        c = 0.;

for ely = 1:nely
  for elx = 1:nelx
    n1 = (nely+1)*(elx-1)+ely;
    n2 = (nely+1)* elx +ely;
    Ue = U([2*n1-1;2*n1; 2*n2-1;2*n2; 2*n2+1;2*n2+2; 2*n1+1;2*n1+2],1);
    c = c + x(ely,elx)^penal*Ue'*KE*Ue;                     % OBJECTIVE FUNCTION
    dc(ely,elx) = -penal*x(ely,elx)^(penal-1)*Ue'*KE*Ue; % SENSITIVITY ANALYSIS
end
  end
```

Sensitivity

$$\frac{\partial c}{\partial x_e} = -p(x_e)^{p-1}\, \mathbf{u}_e^T\, \mathbf{k}_0\, \mathbf{u}_e$$

Update rule

- OPTIMALITY CRITERIA METHOD

$$x_e^{new} = \begin{cases} \max(x_{\min}, x_e - m) & \text{if} \quad x_e B_e^\eta \le \max(x_{\min}, x_e - m), \\ x_e B_e^\eta & \text{if} \quad \max(x_{\min}, x_e - m) < x_e B_e^\eta < \min(1, x_e + m) \\ \min(1, x_e + m) & \text{if} \quad \min(1, x_e + m) \le x_e B_e^\eta, \end{cases}$$

$$B_e = \frac{-\dfrac{\partial c}{\partial x_e}}{\lambda \dfrac{\partial V}{\partial x_e}}$$

- Optimality criterion
  - If "$-\frac{\partial c}{\partial \rho_i}$" is large, increase ρ_i
  - Otherwise, decrease ρ_i
  - How to determine large or small?
  - Bisection search for a threshold

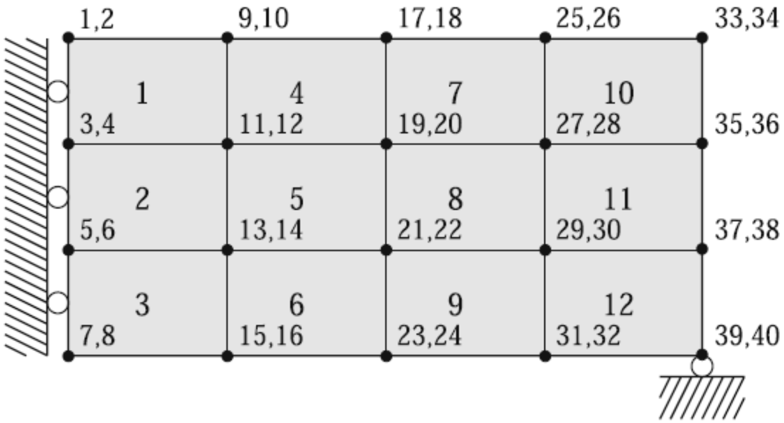
Element Stiffness Matrix

$$\underset{8\times 8}{\mathbf{K_e}} = \int_{-b}^{b}\int_{-a}^{a} \underset{8\times 3}{\mathbf{B}^T} \quad \underset{3\times 3}{\mathbf{E}} \quad \underset{3\times 8}{\mathbf{B}} \; t\,dx\,dy$$

```
function [KE]=lk
%Element Stiffness Matrix

E = 1.;
nu =1/3.;
k=[ 1/2-nu/6 1/8+nu/8 -1/4+nu/12 -1/8-nu/8
-1/4-nu/12 -1/8+3*nu/8 … nu/6 1/8-3*nu/8];
KE = E/(1-nu^2)* …
[ k(1) k(2) k(3) k(4) k(5) k(6) k(7) k(8)
k(2) k(1) k(8) k(7) k(6) k(5) k(4) k(3) k(3) k(8) k(1) k(6)
k(7) k(4) k(5) k(2) k(4) k(7) k(6) k(1) k(8) k(3) k(2) k(5)
k(5) k(6) k(7) k(8) k(1) k(2) k(3) k(4) k(6) k(5) k(4) k(3)
k(2) k(1) k(8) k(7) k(7) k(4) k(5) k(2) k(3) k(8) k(1) k(6)

k(8) k(3) k(2) k(5) k(4) k(7) k(6) k(1)];
```



FEM Analysis (2D mesh is invariant wrt to homotheties)

```
function [U]=FE(nelx,nely,x,penal)
[KE] = lk;
K = sparse(2*(nelx+1)*(nely+1), 2*(nelx+1)*(nely+1));
F = sparse(2*(nely+1)*(nelx+1),1); U = zeros(2*(nely+1)*(nelx+1),1);
for elx = 1:nelx
 for ely = 1:nely
  n1 = (nely+1)*(elx-1)+ely;
  n2 = (nely+1)* elx +ely;
  edof = [2*n1-1; 2*n1; 2*n2-1; 2*n2; 2*n2+1; 2*n2+2; 2*n1+1; 2*n1+2];
  K(edof,edof) = K(edof,edof) + x(ely,elx)^penal*KE;
end
 end
F(2*(nelx+1)*(nely+1),1)=-1;
fixeddofs=union([1,2],[2*nely+1:2*(nely+1)]);
alldofs = [1:2*(nely+1)*(nelx+1)];
freedofs = setdiff(alldofs,fixeddofs);
% SOLVING
U(freedofs,:) = K(freedofs,freedofs) \ F(freedofs,:);
U(fixeddofs,:)= 0;
```

TO HAVE REAL DISPLACEMENT

1) Choose consistent units N, mm, MPa for example *(Remember Nastran Course)*

2) Put the real Young's modulus E=210e3 MPa for example;

3) Multiply the unit load by true amplitude F for example 54*1e3 N ;

4) Multiply the elementary stiffness matrix by the thickness (mm)

5) 2D mesh is invariant wrt to homotheties ; Need to check that nelx and nely are related to the true value for example 140 and 50 mm

6) Apply the BCs

The compliance unit is mJ.

OPTIMALITY CRITERIA

```
function [xnew]=OC(nelx,nely,x,volfrac,dc)
l1 = 0; l2 = 100000; move = 0.2;
while (l2-l1 > 1e-4)
 lmid = 0.5*(l2+l1);
 xnew = max(0.001,max(x-move,min(1.,min(x+move,x.*sqrt(-dc./lmid)))));
 if sum(sum(xnew)) - volfrac*nelx*nely > 0;
   l1 = lmid;
 else
   l2 = lmid;
 end
end
```

lmid: 50.0000	l1: 0.0000	l2: 50.0000
lmid: 25.0000	l1: 0.0000	l2: 25.0000
lmid: 12.5000	l1: 0.0000	l2: 12.5000
lmid: 6.2500	l1: 6.2500	l2: 12.5000
lmid: 9.3750	l1: 6.2500	l2: 9.3750
lmid: 7.8125	l1: 6.2500	l2: 7.8125
lmid: 7.0313	l1: 7.0313	l2: 7.8125
lmid: 7.4219	l1: 7.0313	l2: 7.4219
lmid: 7.2266	l1: 7.2266	l2: 7.4219
lmid: 7.3242	l1: 7.3242	l2: 7.4219
lmid: 7.3730	l1: 7.3242	l2: 7.3730
lmid: 7.3486	l1: 7.3242	l2: 7.3486
lmid: 7.3364	l1: 7.3364	l2: 7.3486
lmid: 7.3425	l1: 7.3425	l2: 7.3486
lmid: 7.3456	l1: 7.3425	l2: 7.3456
lmid: 7.3441	l1: 7.3441	l2: 7.3456
lmid: 7.3448	l1: 7.3448	l2: 7.3456
lmid: 7.3452	l1: 7.3448	l2: 7.3452
lmid: 7.3450	l1: 7.3450	l2: 7.3452
lmid: 7.3451	l1: 7.3450	l2: 7.3451

Can also use:

- fmincon
- MMA…

The MMA approach, which was initially proposed by Svanberg (see Mini Project) is based on the first-order Taylor series expansion of the objective and constraint functions.

With this method, an explicit convex subproblem is generated to approximate the implicit nonlinear problem.

Matlab code command

top(nelx, nely, volfrac, penal, rmin)

- nelx and nely: number of elements in the horizontal and vertical directions,
- volfrac: volume fraction,
- penal: penalization power,
- rmin: filter size (divided by element size).

Example 1

Numerical instability

- top(40, 20, 0.5, 3, 1.0)

effect→ Checkerboard Pattern

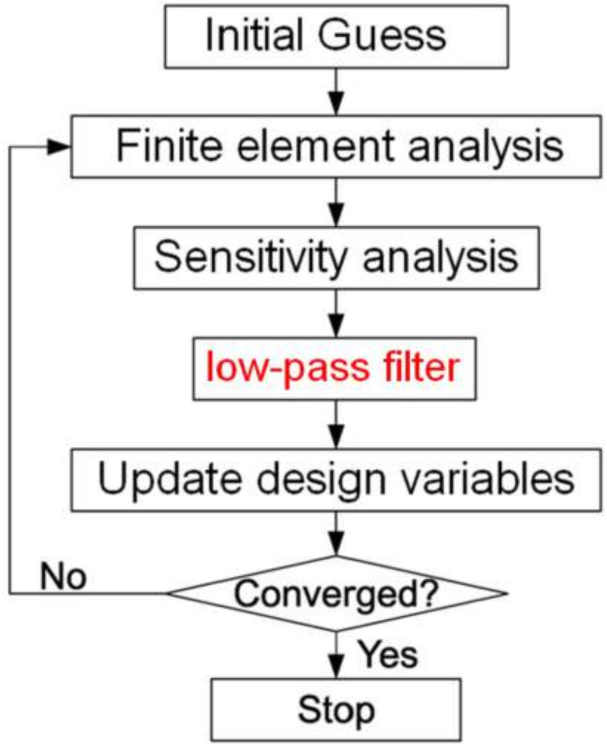
Example 1 -- Checkerboard Pattern Problem

→ Solution: LOW PASS Filter

$$\widehat{\frac{\partial c}{\partial x_e}} = \frac{1}{x_e \sum_{f=1}^{N} \hat{H}_f} \sum_{f=1}^{N} \hat{H}_f \, x_f \, \frac{\partial c}{\partial x_f} .$$

$$\hat{H}_f = r_{\min} - \operatorname{dist}(e, f) ,$$

$$\{f \in N \mid \operatorname{dist}(e, f) \le r_{\min}\}, \quad e = 1, \ldots, N$$

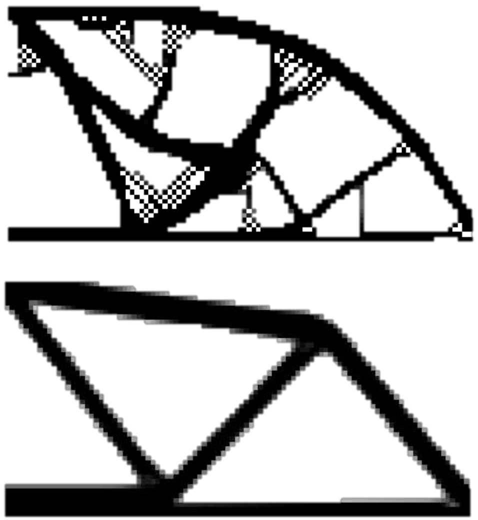


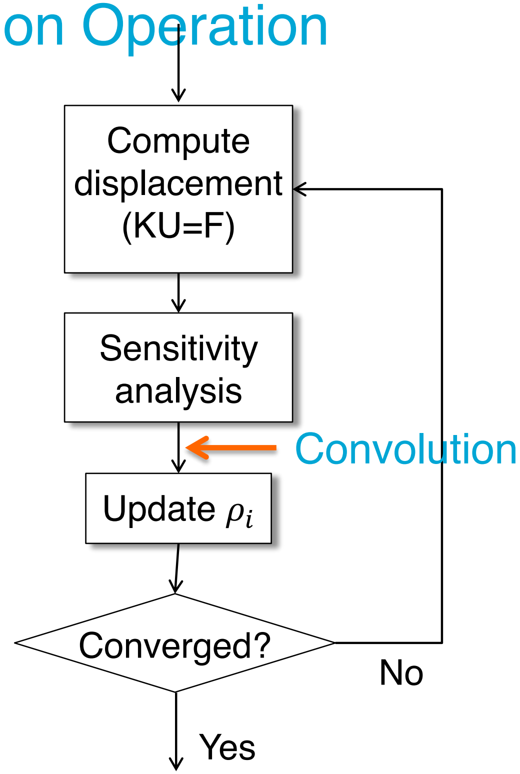
[dc] = check(nelx, nely, rmin, x, dc);

% FILTERING OF SENSITIVITIES

Filtering

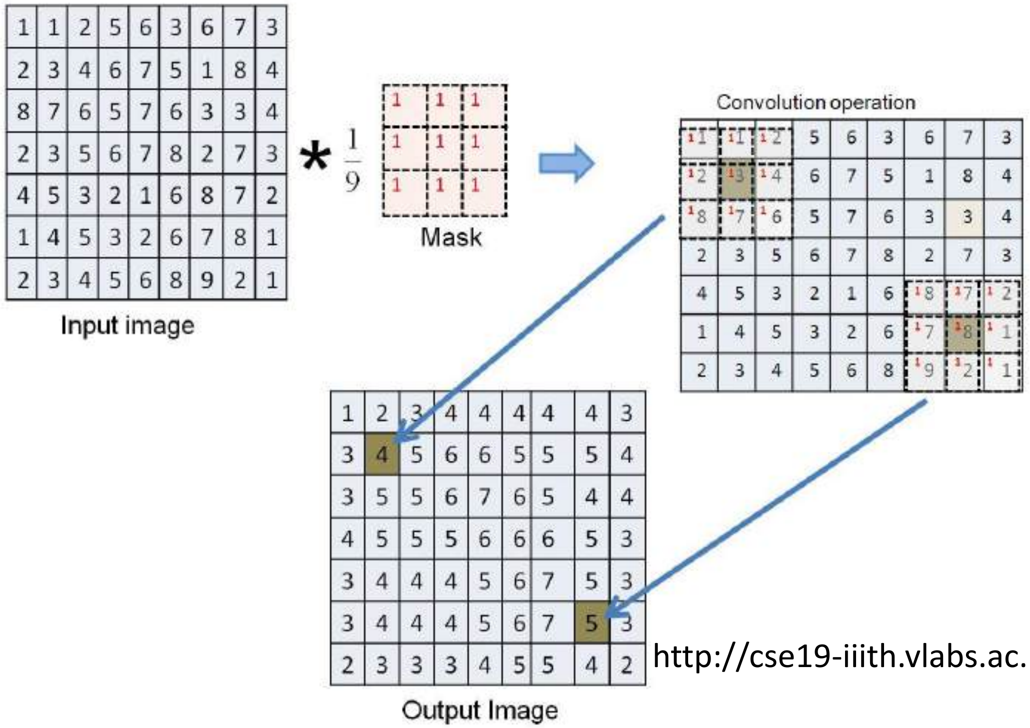
Sensitivity Filtering by a Convolution Operation





Filtering

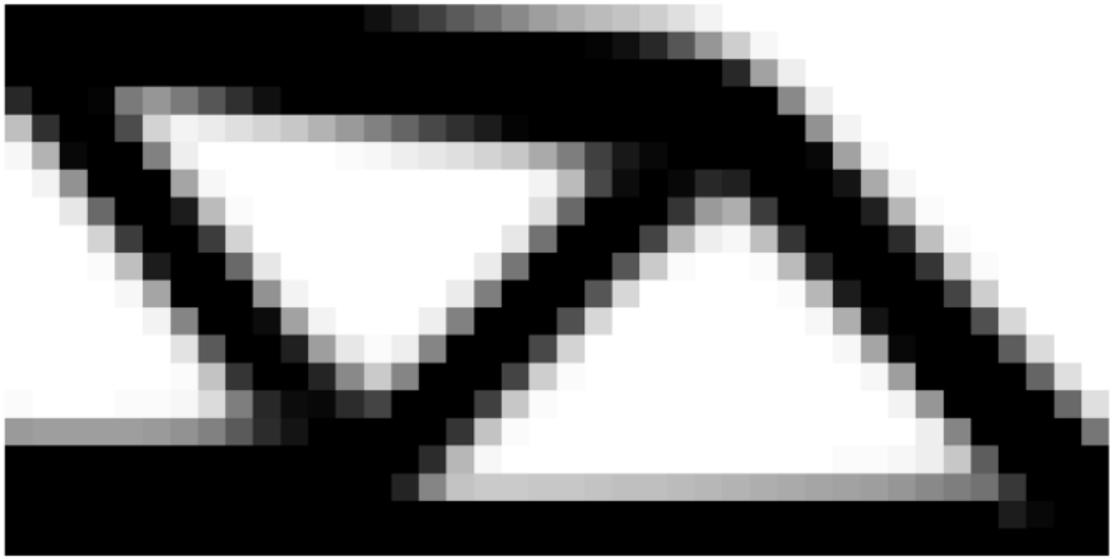
Convolution Operation



http://cse19-iiith.vlabs.ac.in/theory.php?exp=neigh

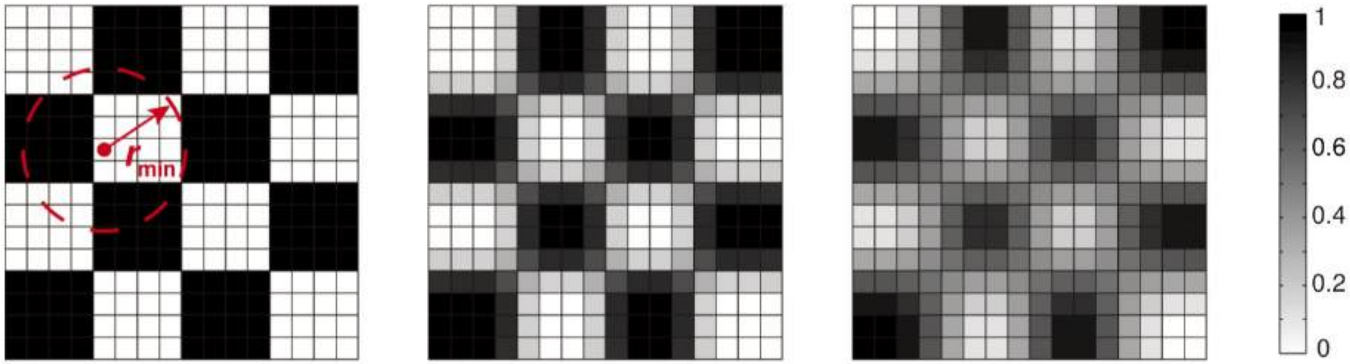
Example 2

- top(40, 20, 0.5, 3, 1.5)



- top(40, 20, 0.5, 3, 3) effect?

Filter



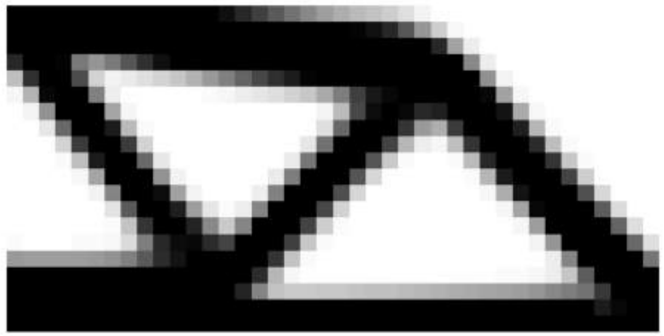
A checkerboard field and filtered fields ($r_{\min} = 1.5l_e$ and $3l_e$)

Example 3

→ Size of the filter makes it possible to obtain a more physical representation (Unblur?)

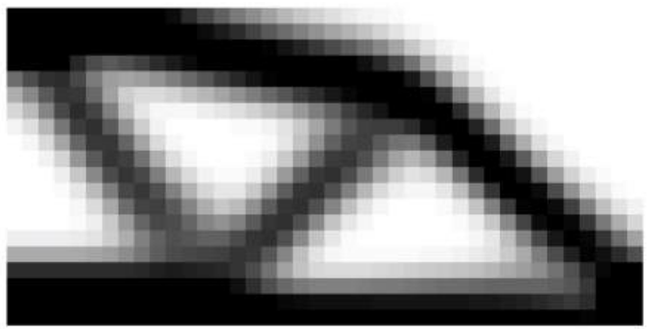
- rmin=1.5

Obj=82.7562;



- rmin=3

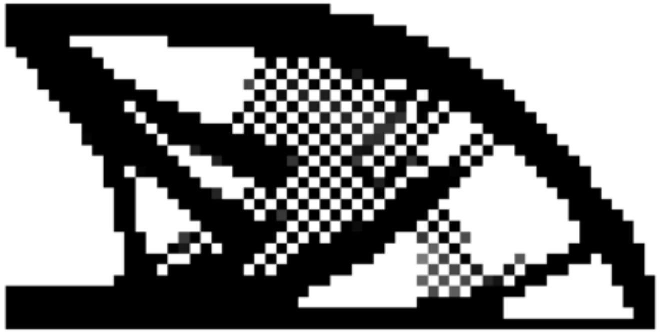
Obj=99.1929;



Example 3: Change mesh !

- top(60, 30, 0.5, 3, 1.0)

Obj: 83.0834



- top(40, 20, 0.5, 3, 1.0)

Obj: 80.4086;

Example 4: mixing

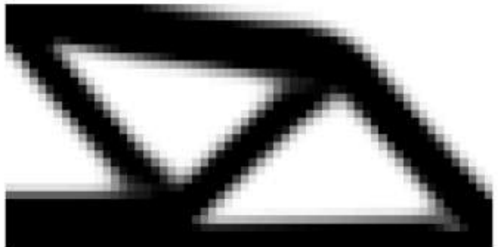
- top(60, 30, 0.5, 3, 1.5)

Obj: 81.3491

→ Filter size

- top(60, 30, 0.5, 3, 2.25)

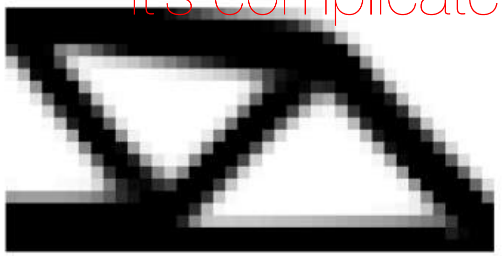
Obj: 83.5963



→ +mesh dependency, combined eddect It's complicated

- top(40, 20, 0.5, 3, 1.5)

Obj=82.7562;



Common questions ?

- Grayness level ?

```
%% Greyness Level
   gl = 4/nele*sum(xPhys(:).*(1-xPhys(:)));
```

- How to obtain B&W for the final design ?

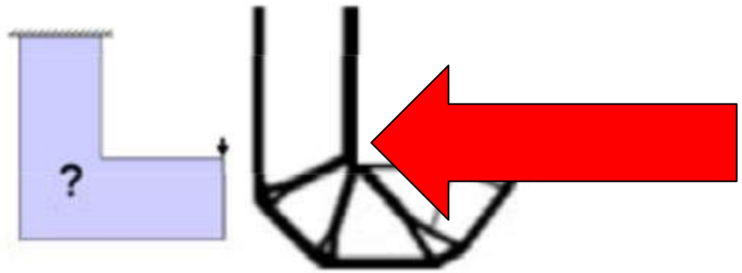
```
function xPhys=automatic_threshold(xPhys)
% %code to inputÂ xPhys and obtain the B&W design at respected volfrac

xPhys01=xPhys;
volfrac=mean(mean(xPhys))
lowlim=0;
highlim=1;
difVF=1;
while difVF>0.0001 && highlim-lowlim > 0.000001
midlim=(lowlim+highlim)/2;
xPhys01(xPhys>=midlim)=1;
xPhys01(xPhys<midlim)=0;
volfrac01=mean(mean(xPhys01));
difVF=abs(volfrac01-volfrac);
if volfrac01<volfrac
highlim=midlim;
else
lowlim=midlim;
end
end
xPhys=xPhys01;
```

First Exercice

Read and finish the notebook called top88_Student.ipynb

At this time the structure is rigid… but feasible?



Check the
stress?

EMSM207

Stress-based (your project)

$$
\begin{aligned}
&\textit{Find} \quad \mathbf{X} = [x_1, x_2, \cdots, x_N]^T \\
&\textit{minimize} \quad f(\mathbf{X}) = W(\mathbf{X}) \\
&\textit{subject} \quad \mathbf{K}(\mathbf{X})\mathbf{U}(\mathbf{X}) = \mathbf{F} \\
&\qquad \boldsymbol{\sigma}(\mathbf{X}) - [\boldsymbol{\sigma}] \le \mathbf{0} \\
&\qquad \frac{V(\mathbf{X})}{V_0} \le f \\
&\qquad 0 < x_{\min} \le x_e \le x_{\max} \le 1
\end{aligned}
$$

Constraints Aggregation (Reduce the Number of Constraints)

$\boxed{\hat{\rho}_i \le \alpha, \forall i}$ ⟺ $\boxed{\max_{i=1,\dots,n} |\hat{\rho}_i| \le \alpha}$ ⟺ $\boxed{\lim_{p\to\infty} \|\rho\|_p = \left(\sum_i (\hat{\rho}_i)^p\right)^{\frac{1}{p}} \le \alpha}$

Too many constraints!

A single constraint
But non-differentiable

A single constraint
and differentiable
Approximated with p =16

Stress Based TO

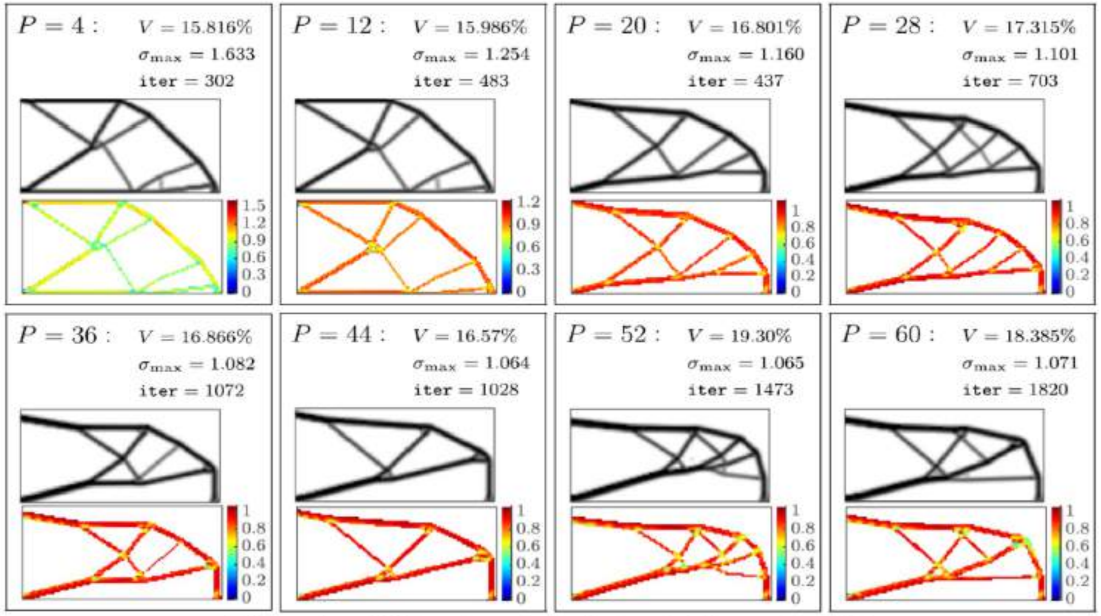
Microscopic stress tensor

$$\boxed{\boldsymbol{\sigma}_e = \mathbf{C}_e(E_0)\,\langle \boldsymbol{\epsilon}_e \rangle}$$

$$\overline{g}_j = \rho_j \left(\frac{\sigma_j}{\sigma_{\text{lim}}} - 1 \right) \leq 0.$$

Aggregation

$$\boxed{G_{\text{KS}}^{L} = \Psi_{\text{KS}}^{L} = \frac{1}{P} \ln \left(\frac{1}{N} \sum_{i=1}^{N} e^{P\overline{g}_i} \right) \leq 0}$$

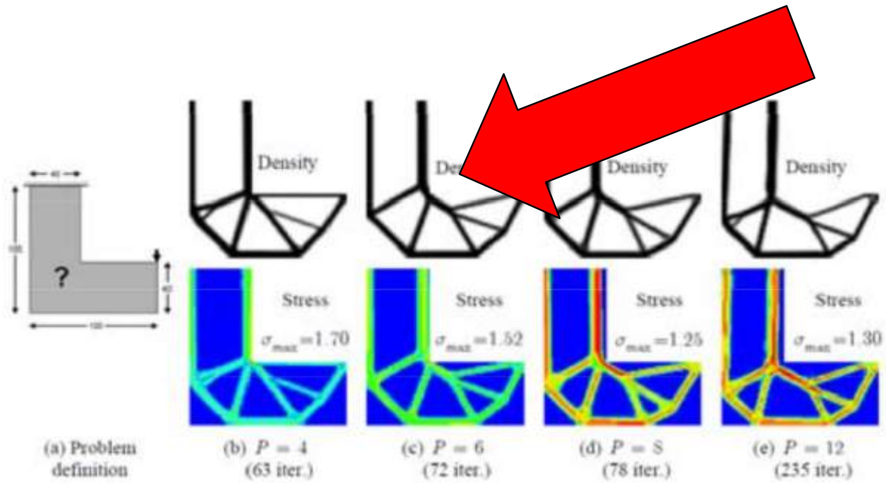


(a) Lower bound KS-function: optimized designs for different values of P.

Kreisselmeier, G., and R. Steinhauser. "Systematic control design by optimizing a vector performance index." *Computer aided design of control systems*. Pergamon, 1980. 113-117.

Verbart, Alexander, Matthijs Langelaar, and Fred Van Keulen. "A unified aggregation and relaxation approach for stress-constrained topology optimization." *Structural and Multidisciplinary Optimization* 55.2 (2017): 663-679.

Results



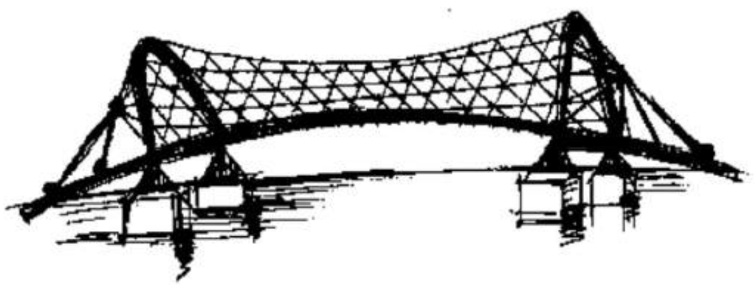
Le *et al.*, *SMO* (2010)

Is it better?

Conclusion

"The art of structure is where to put the holes"

Robert Le Ricolais
French-American engineer and philosopher
(1894-1977)



“What is the optimal form for a tall building? We don’t know...”— William Baker, Skidmore Owings and Merrill, Lecture at the Harvard Graduate School

“The significant design decision is not how big is the beam to be, but rather to have a beam.”— Sir Alan Harris, Co-founder, Harris & Sutherland