Conception optimale pour l'ingénieur (Aerospace)

C5 by Prof. J. Morlier

2025

Institut Supérieur de l'Aéronautique et de l'Espace
SUPAERO

AU PROGRAMME
Python based

lundi 31 mars 2025			
		09h15 - 12h45	MORLIER Joseph
		14h00 - 16h15	MORLIER Joseph
mardi 01 avril 2025			
		09h15 - 12h45	MORLIER Joseph
		14h00 - 16h15	MORLIER Joseph
mercredi 02 avril 2025			
		09h15 - 12h45	MORLIER Joseph MURADÁS ODRIOZOLA Daniel
		14h00 - 16h15	MAS COLOMER JOAN MURADÁS ODRIOZOLA Daniel
jeudi 03 avril 2025			
		09h15 - 12h45	MAS COLOMER JOAN MURADÁS ODRIOZOLA Daniel

Intro: Sustainable Aviation (Materials) With Both Eyes Open

Design optimization 1: constrained optimization, MOO, Sensibility with examples

Project DO 1 2 3

Topology Optimization with examples

Material ecoselection, Ashby Diagram and more

Projet DO 1 2 3

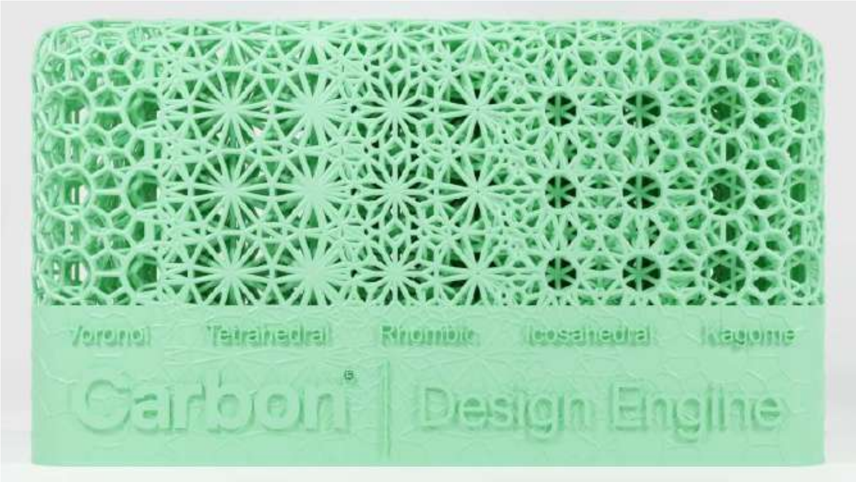
Wrap up and demo from students

Intro to MDAO
Static Aeroelastic problem is a MDAO problem

Airbus PROJECT by TEAM of 3 (marked*)

vendredi 04 avril 2025		**ORAL MARKED***	
		09h15 - 11h30	MORLIER Joseph MURADÁS ODRIOZOLA Daniel

Eco informed Material Selection?

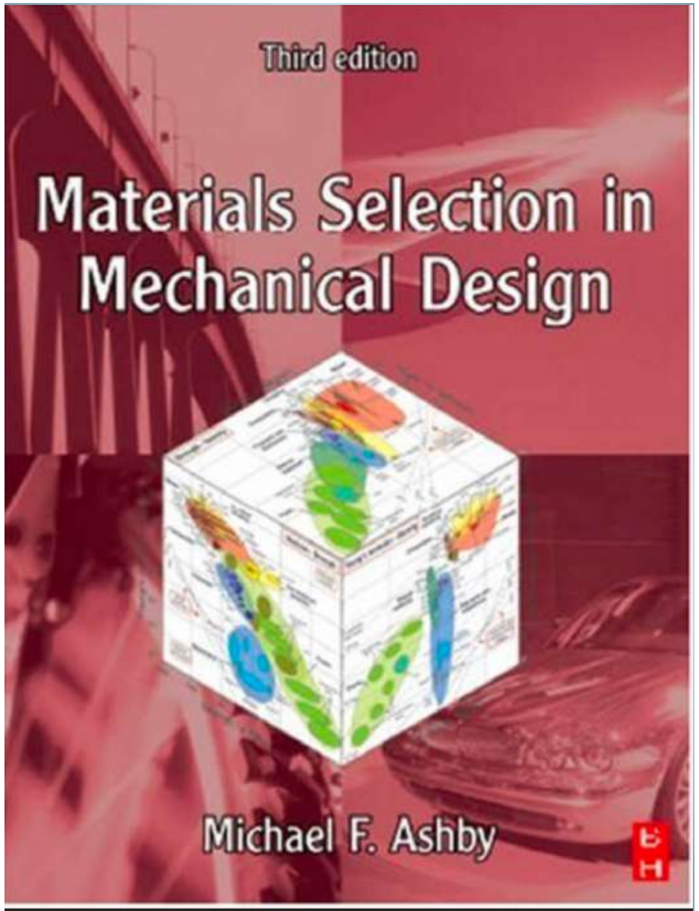


Au programme (14H-17H15)

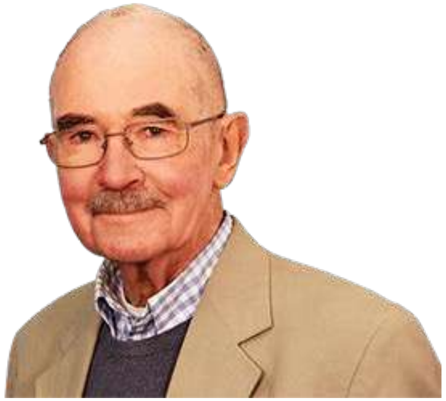
- 1/ Eco Material selection course
- 2/ Exercices using Ahsby diagram (pen&paper)

Learning outcomes

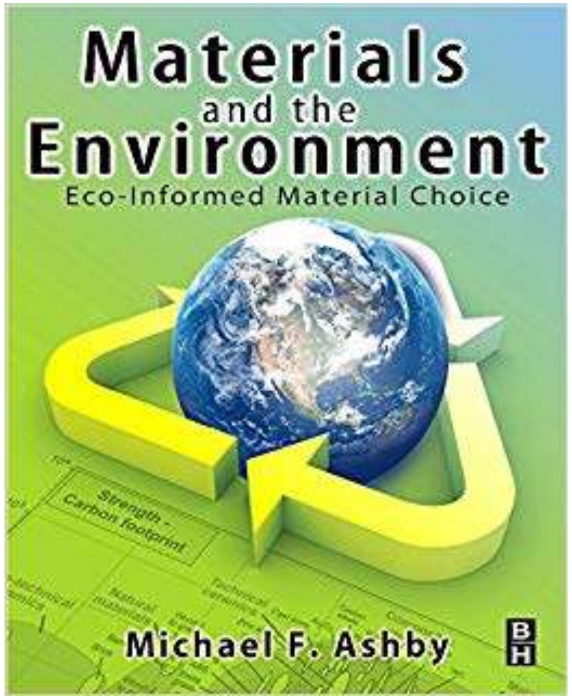
- 1/ Discovering Material profiles
- 2/ Use Ashby's diagram
- 3/ Select material (mono& multiobjective)
- 4/ Discovering Eco Properties

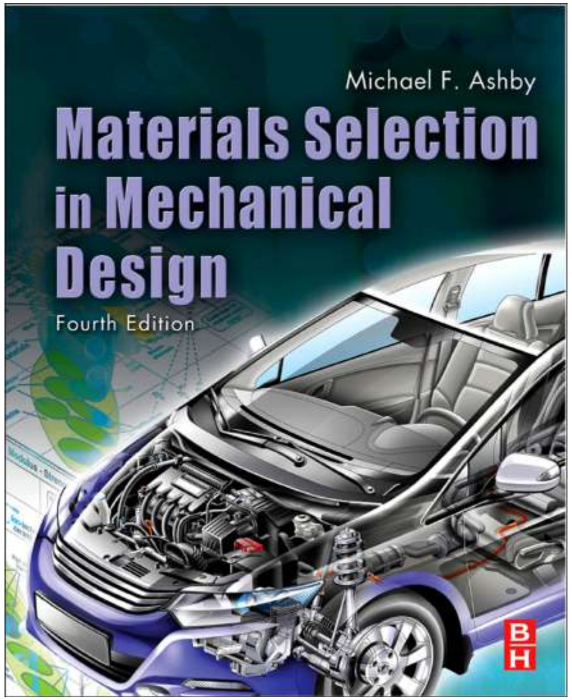


Prof. Ashby



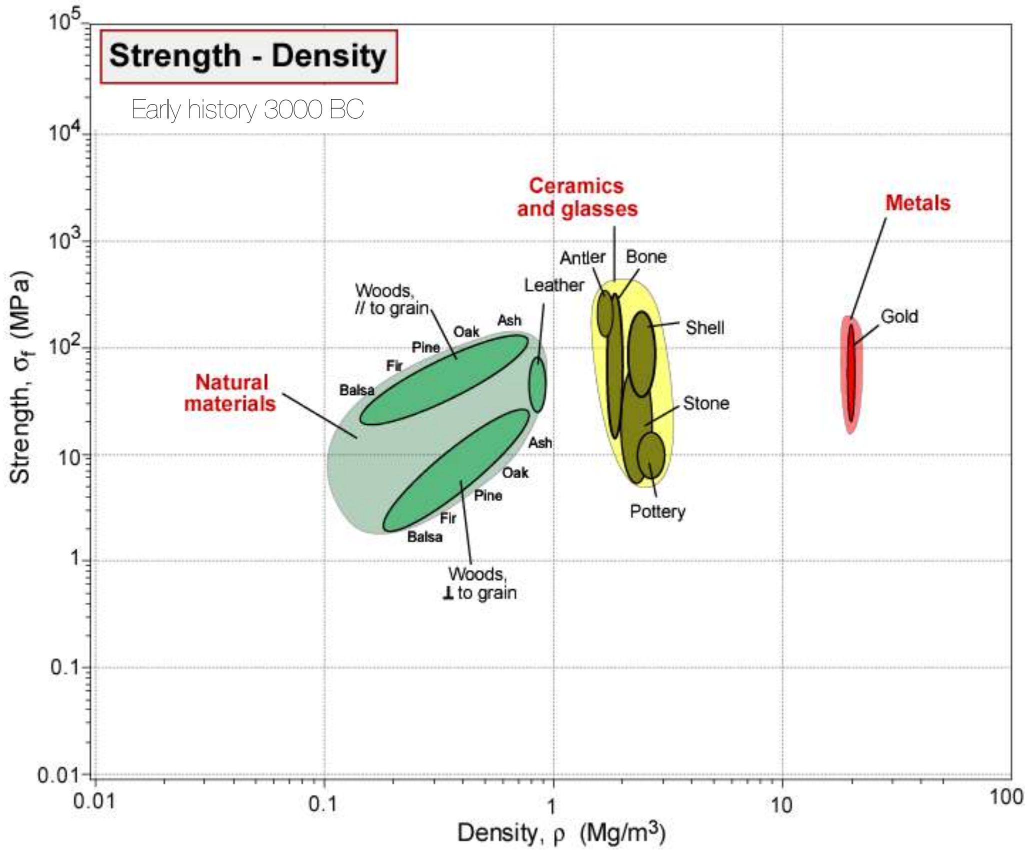
To Start


Materials
and the
Environment
Eco-Informed Material Choice
Michael F. Ashby


Michael F. Ashby
Materials Selection in Mechanical Design
Fourth Edition

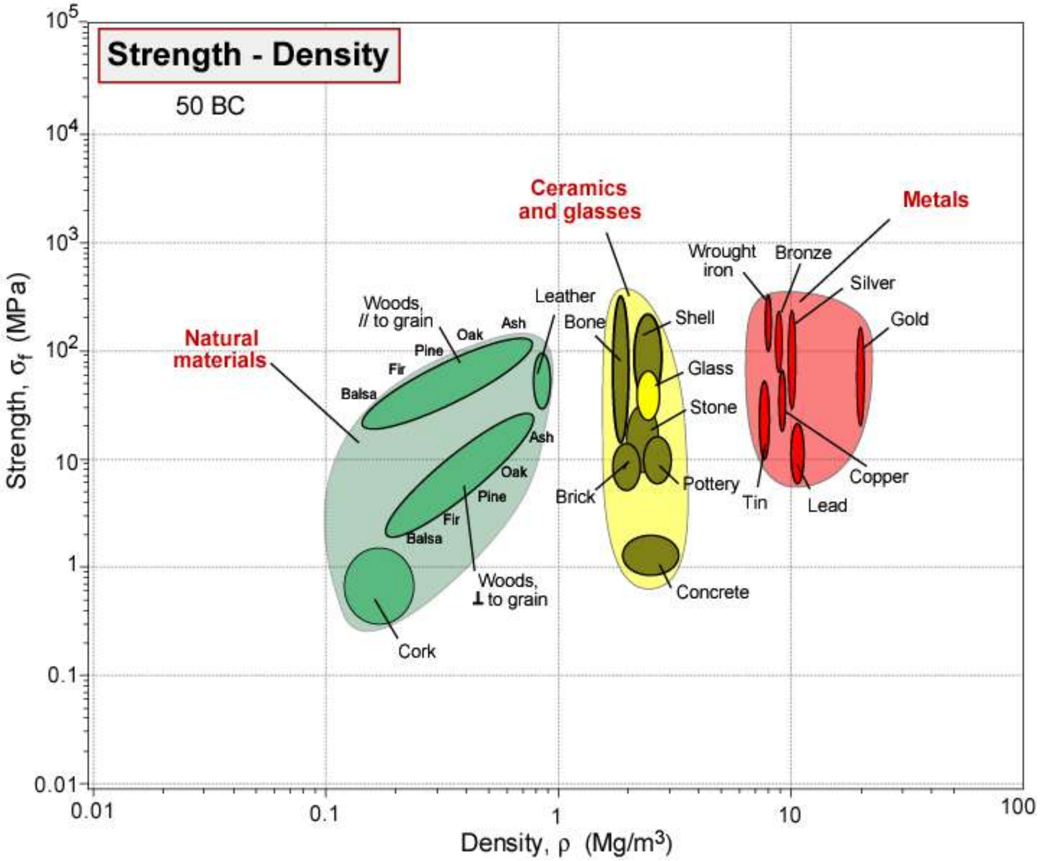
The evolution of structural materials

from Mike Ashby, 2018



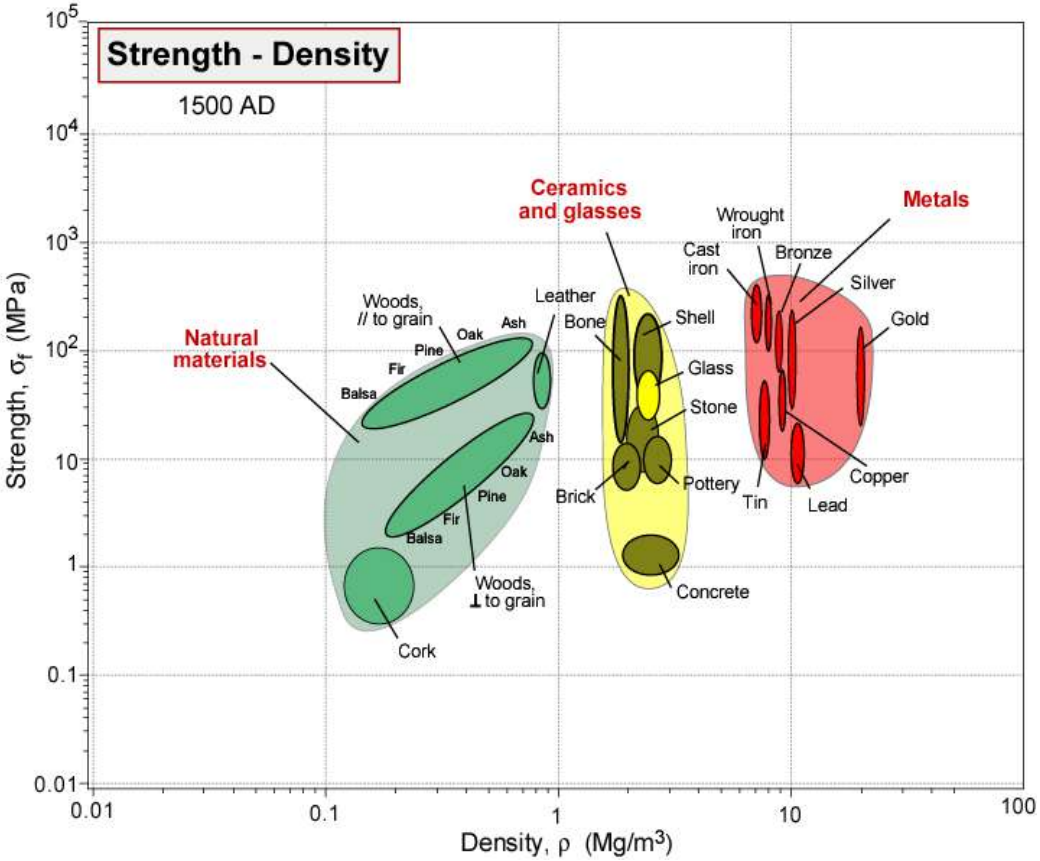
Egyptian Pyramids

50BC



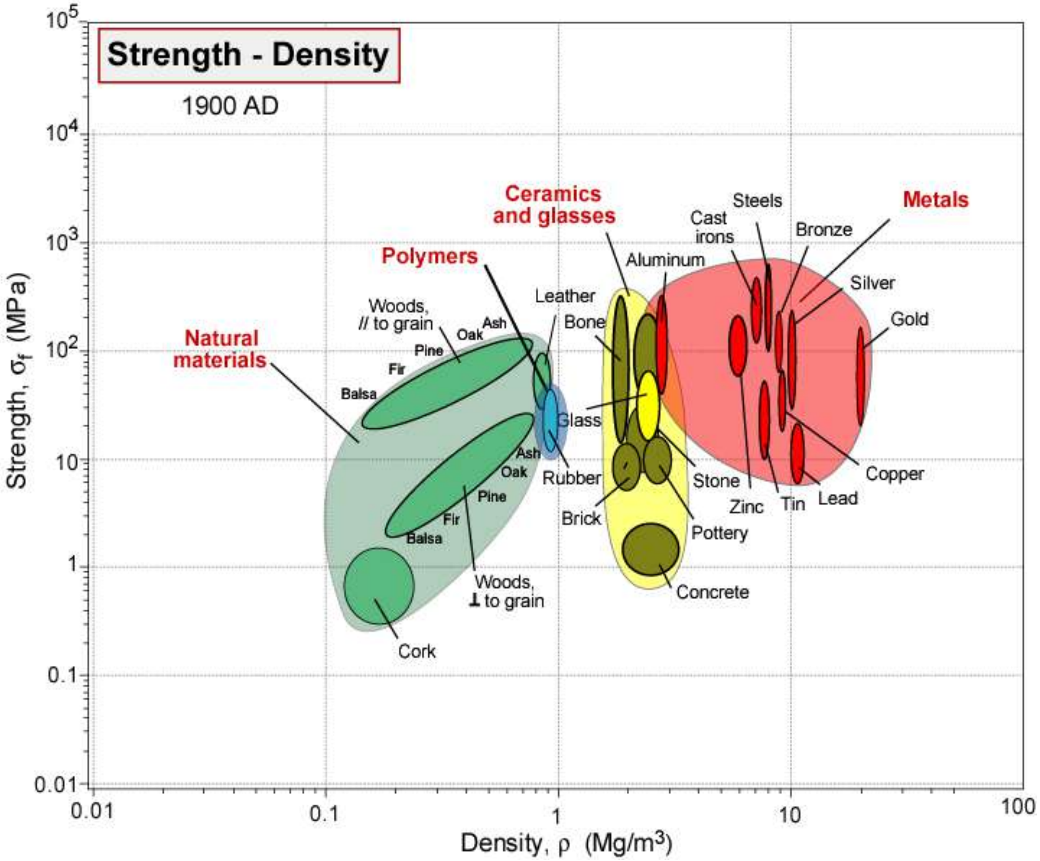
Roman Temples

1500 AD

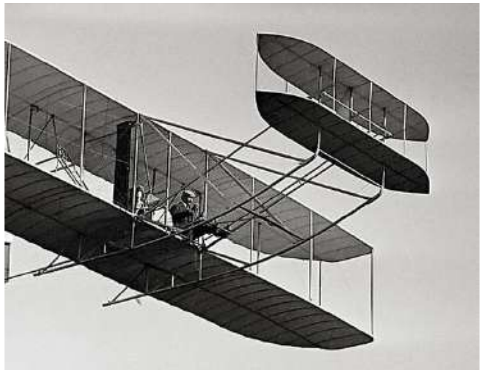


Medieval Castles

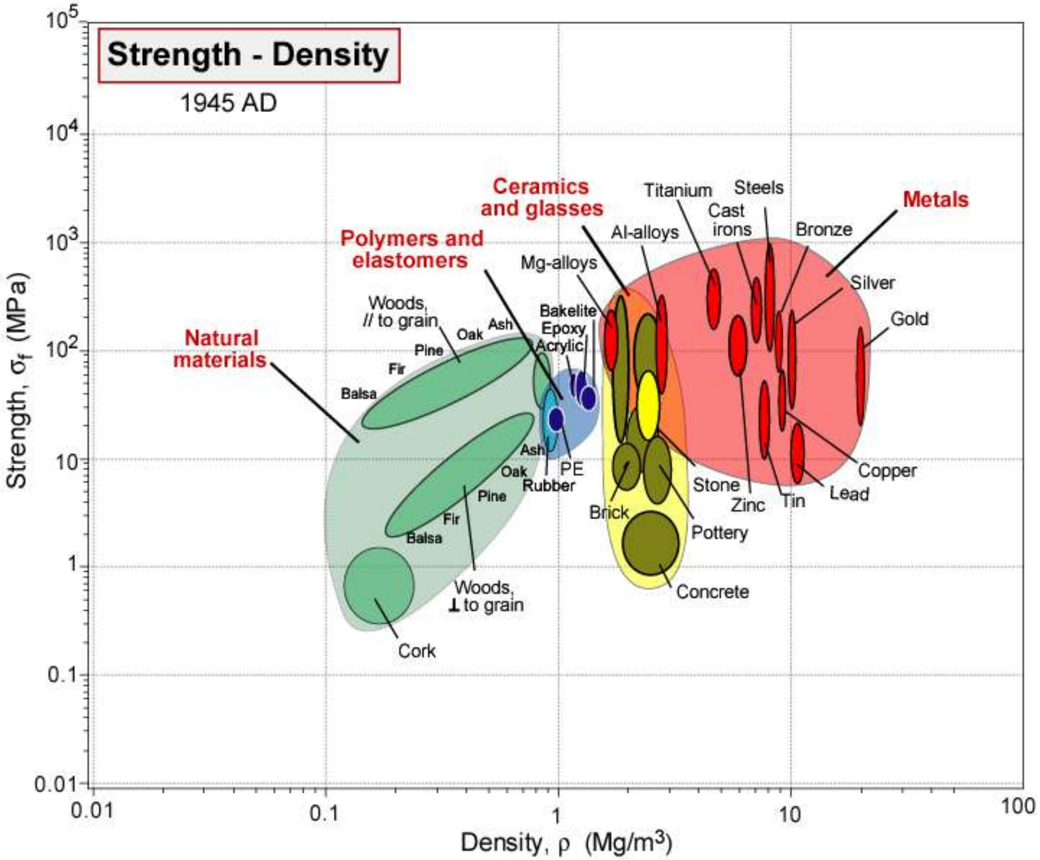
1900 AD



Art Nouveaux

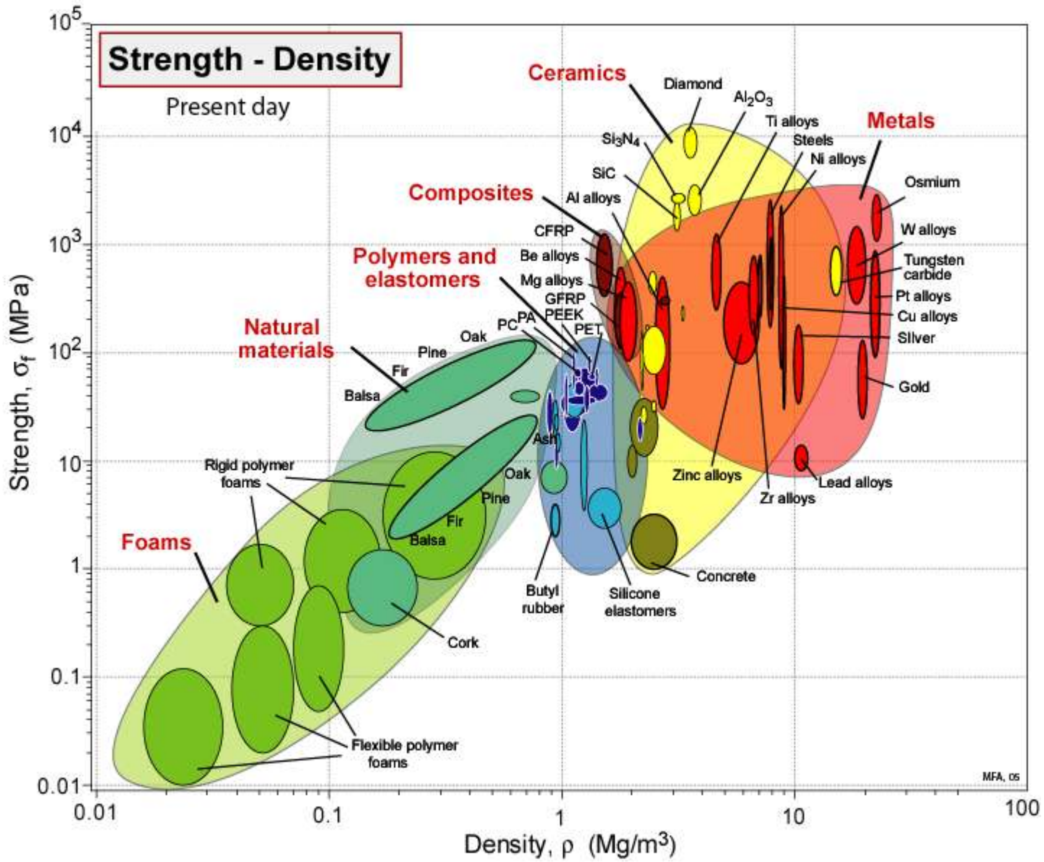


1945 AD



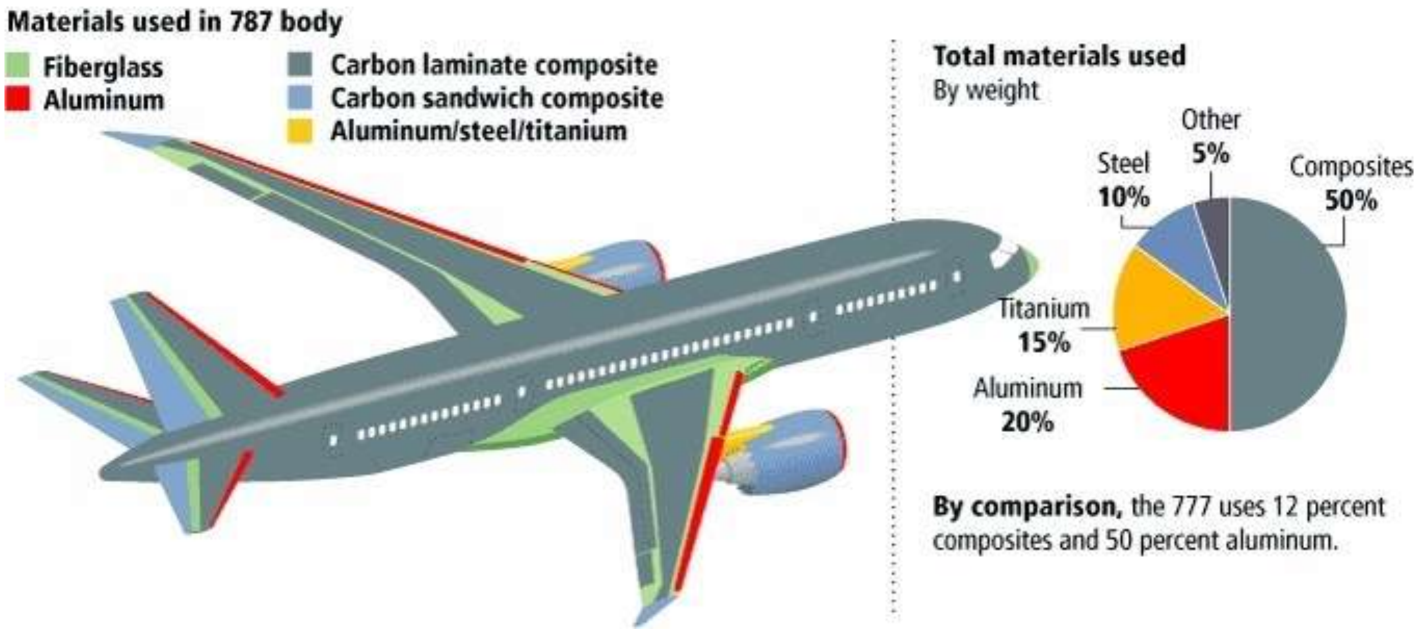
Skyscrapers

PRESENT DAY

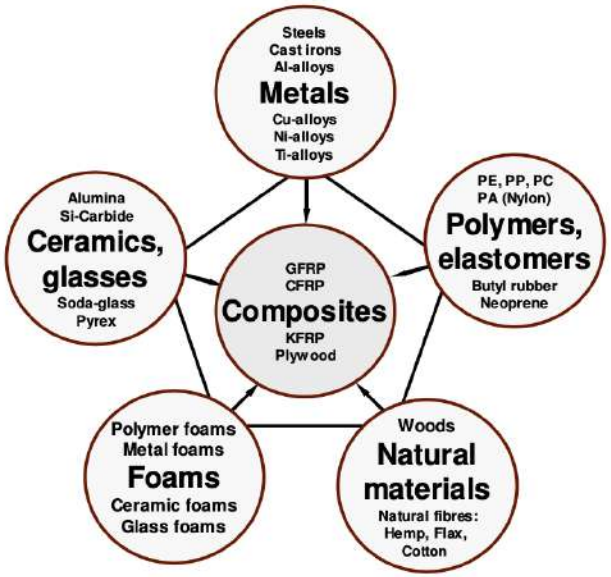


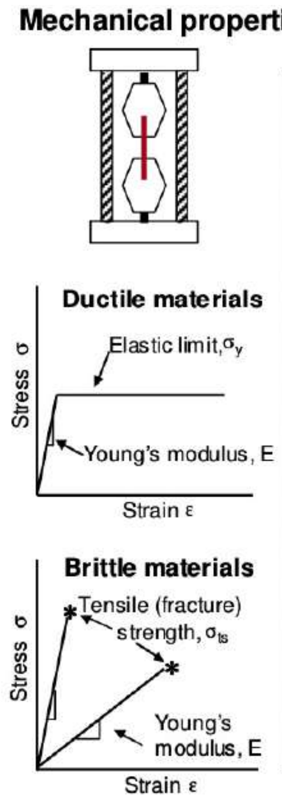
21st Century

Why ?


Materials used in 787 body
Fiberglass
Aluminum
Carbon laminate composite
Carbon sandwich composite
Aluminum/steel/titanium
Total materials used
By weight
Other
5%
Steel
10%
Composites
50%
Titanium
15%
Aluminum
20%
By comparison, the 777 uses 12 percent composites and 50 percent aluminum.

The world of materials





General

Weight: Density ρ, Mg/m^3

Expense: Cost/kg C_m, \$/kg

Mechanical

Stiffness: Young's modulus E, GPa

Strength: Elastic limit σ_y , MPa

Fracture strength: Tensile strength σ_{ts} , MPa

Brittleness: Fracture toughness K_{ic} , MPa.m$^{1/2}$

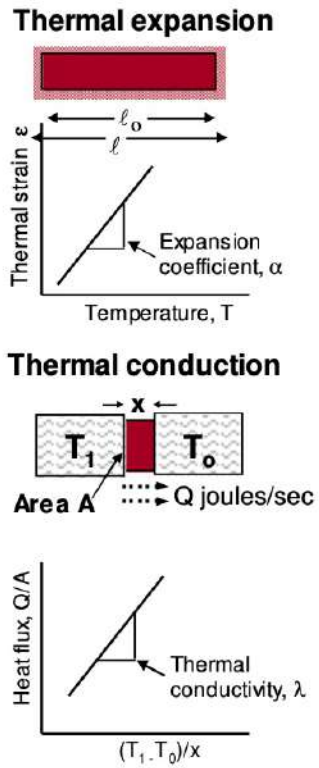
Thermal

Expansion: Expansion coeff. α, 1/K

Conduction: Thermal conductivity λ, W/m.K

Electrical

Conductor? Insulator?



MATERIAL AS A NEW DESIGN VARIABLE

More DATA than {E, rho} in DMO

ONERA

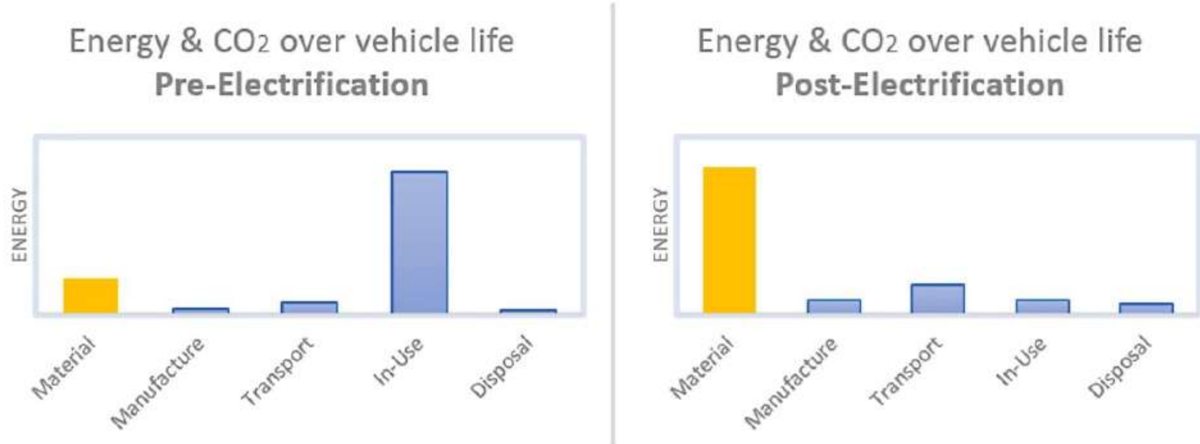
THE FRENCH AEROSPACE LAB

Le matériau composite dans la structure : objet d'étude et variable de conception

par

François-Xavier IRISARRI

Mémoire provisoire, en vue de l'obtention de l'Habilitation à Diriger des Recherches
Université Jean Monnet, Saint-Étienne



The overall importance on material selection will increase post-electrification

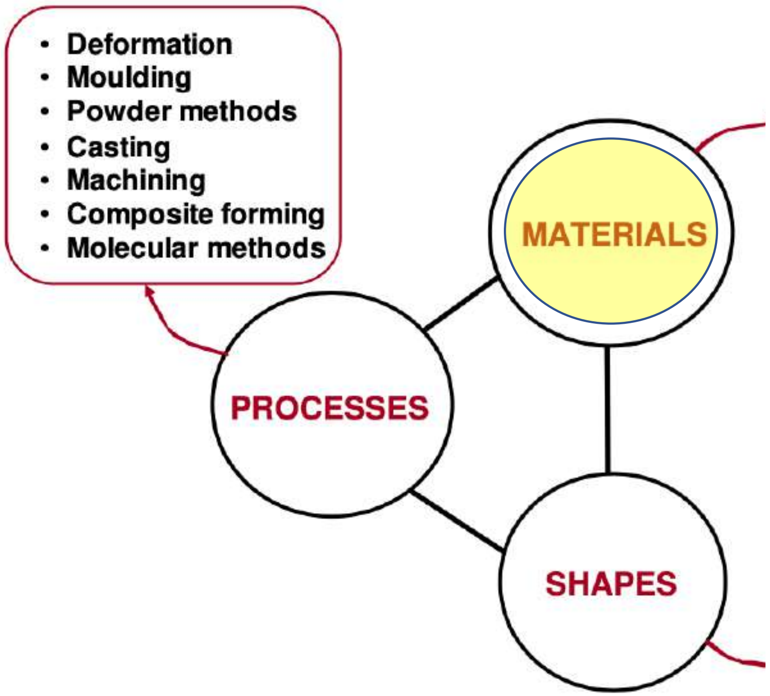
Au programme

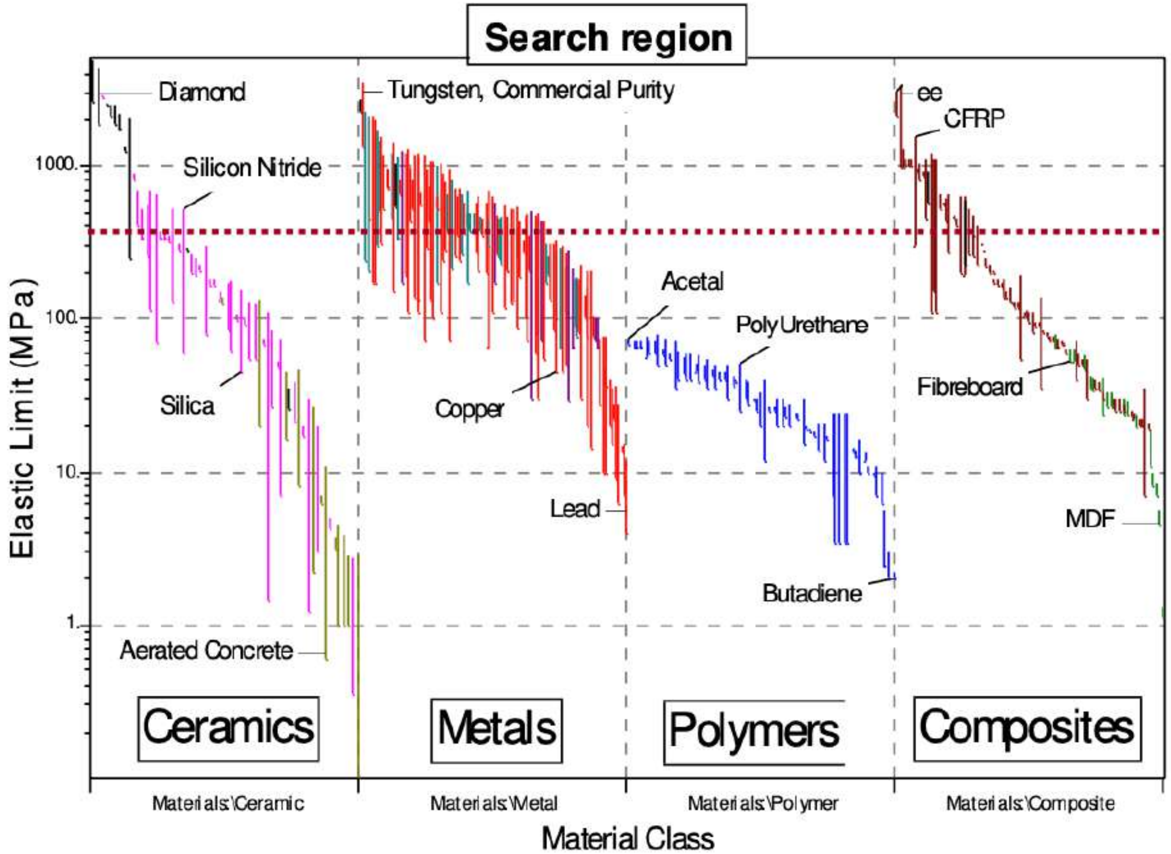
- Part 1 Overview of material's world
- Part 2 Lightest strong
- Part 3 Lightest stiff
- Part 4 GHG footprint
- Part 5 Optimization with multiple objectives (Mass-cost i.e. 2 objectives)

PART 1

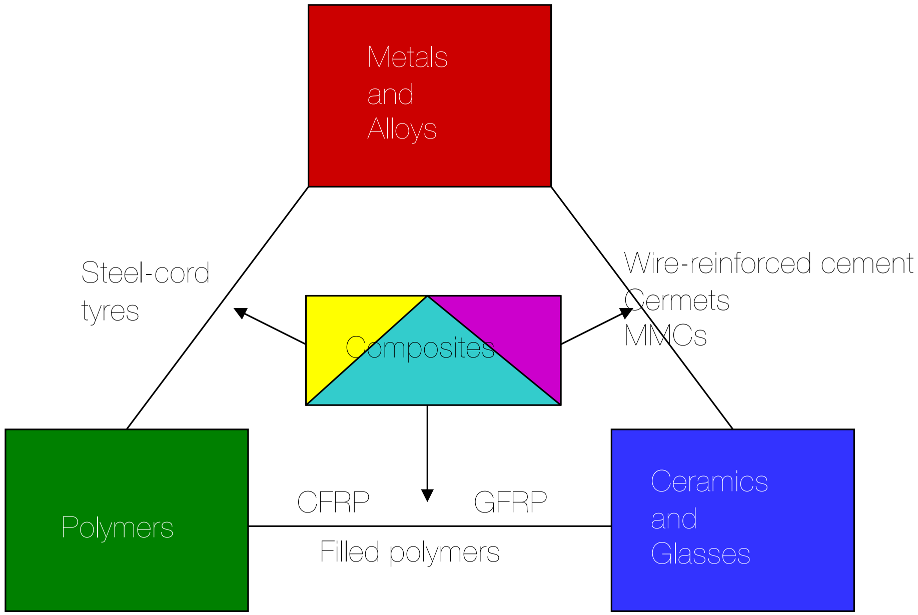
- Overview

Focus on Materials





Materials Selection



Materials Properties

STRUCTURAL MATERIALS

Mechanical
- tribology
- fatigue
- K_{IC}
- σ_y
- UTS
- E

Thermal
- α
- K
- H
- T_m
- $T_{Transition}$

Chemical
- corrosion
- oxidation

MATERIAL

Environmental
- recycling
- energy consumption
- waste

FUNCTIONAL MATERIALS

Physical
- optical
- magnetic
- electrical

Other
- feel
- look

Database of Materials
(quantitative/qualitative/*estimated)

- Table: Materials
  - Ceramic
  - Composite
  - Foam
  - Metal
    - Commercially Pure
    - Ferrous Alloys
    - Light Alloys
      - Aluminium
        - Cast
          - 1xx.x Group
          - 2xx.x Group
          - 3xx.x Group
          - 4xx.x Group
            - Cast aluminium alloy (A413.2)
            - Cast aluminium alloy (S413.0)
            - Cast aluminium alloy (S443.0)
          - 5xx.x Group
          - 6xx.x Group
          - 7xx.x Group
          - 8xx.x Group
        - Wrought
      - Beryllium
      - Magnesium
      - Titanium
    - Non-ferrous Alloys
    - Precious Metal Alloys
    - Refractory Alloys
  - Natural
  - Polymer
    - Elastomer
    - Thermoplastic
    - Thermoset

Cast aluminium alloy (A413.2)

General

Designation
Al alloy: A413.2 (cast)

Composition
Al-12Si

Atomic Volume (average)	0.01	- 0.011	m^3/kmol
Density	2.65	- 2.66	Mg/m^3
Energy Content	235	- 335	MJ/kg
Price	9.75297	- 11.5214	FRF/kg
Recycle Fraction	* 0.8	- 0.9	

Mechanical

Bulk Modulus	65	- 86	GPa
Compressive Strength	70	- 80	MPa
Ductility	0.08	- 0.13	
Elastic Limit	70	- 80	MPa
Endurance Limit	* 37	- 45	MPa
Fracture Toughness	* 25	- 28	MPa.m^1/2
Hardness	550	- 600	MPa
Loss Coefficient	* 1e-004	- 2e-003	
Modulus of Rupture	70	- 80	MPa
Poisson's Ratio	0.32	- 0.36	
Shape Factor	47		
Shear Modulus	26	- 28	GPa
Tensile Strength	170	- 200	MPa
Young's Modulus	71	- 71.5	GPa

Thermal

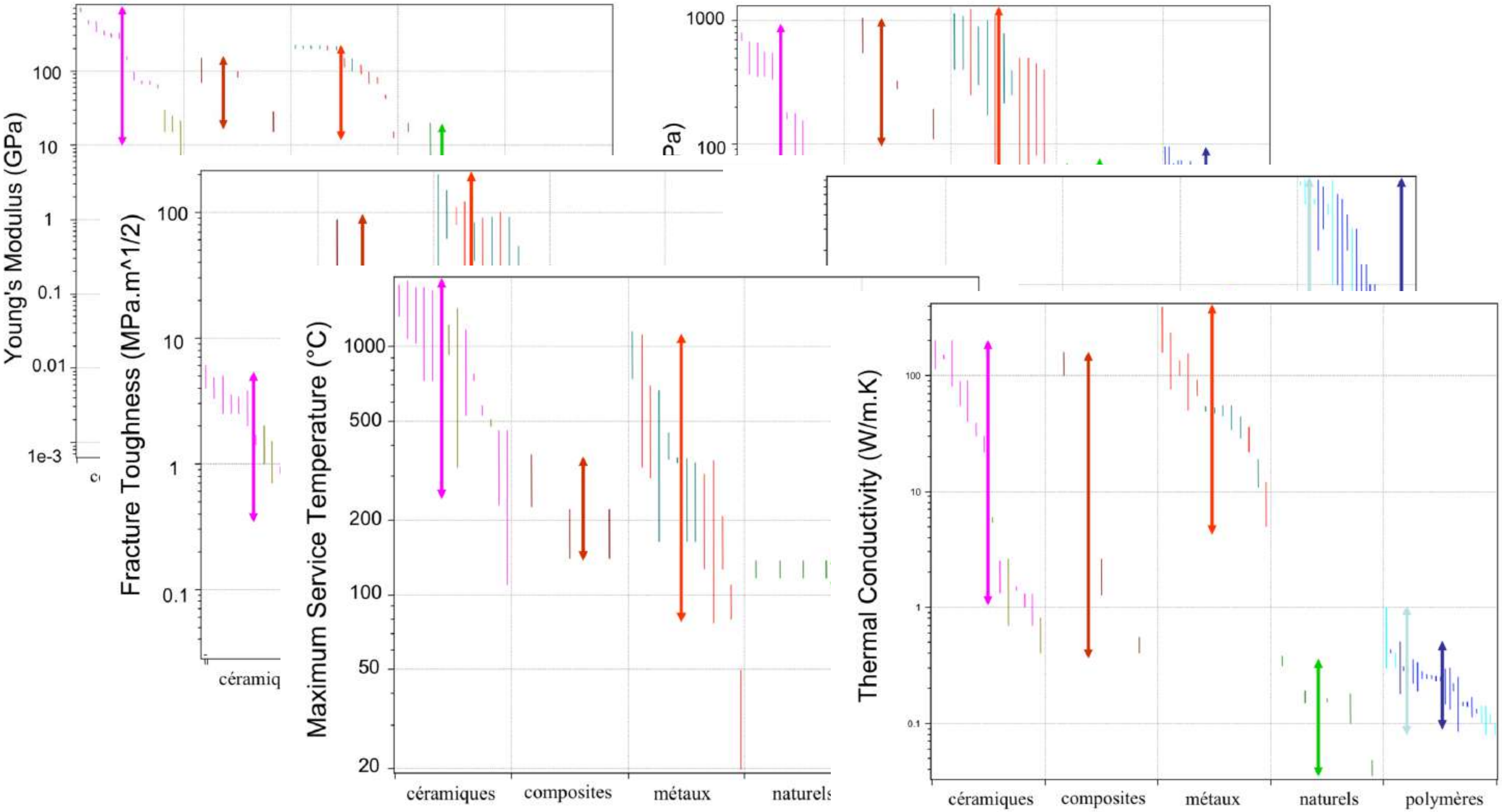
Glass Temperature	Not Applicable		K
Latent Heat of Fusion	384	- 393	kJ/kg
Maximum Service Temperature	400	- 450	K
Melting Point	838	- 848	K
Minimum Service Temperature	0	- 0	K
Specific Heat	* 910	- 960	J/kg.K
Thermal Conductivity	137	- 147	W/m.K
Thermal Expansion	23	- 23.1	10^-6/K

Electrical

Breakdown Potential	Not Applicable		MV/m
Dielectric Constant	Not Applicable		
Resistivity	4.7	- 4.88	10^-8 ohm.m
Power Factor	Not Applicable		

Environmental Resistance

Flammability	Good
Fresh Water	Very Good
Organic Solvents	Very Good
Oxidation at 500C	Very Poor
Sea Water	Good
Strong Acid	Very Good
Strong Alkalis	Poor
UV	Very Good


Young's Modulus (GPa)
Fracture Toughness (MPa.m^1/2)
Maximum Service Temperature (°C)
Thermal Conductivity (W/m.K)
1000
500
200
100
50
20
10
1
0.1
0.01
1e-3
céramiques
composites
métaux
naturels
polymères

Specific Properties

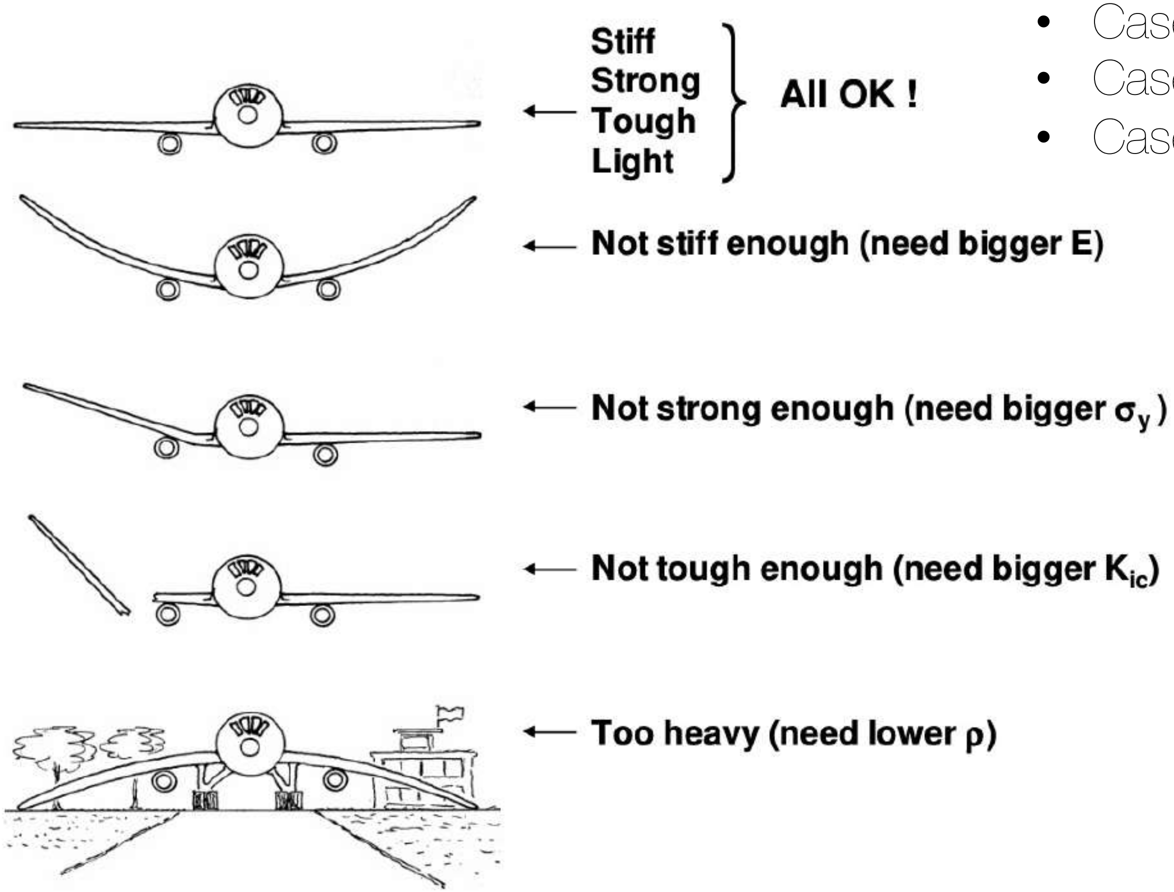
Stiffness vs Strenght

Material	E (GPa)	σ (MPa)	ρ (Mgm^{-3})	E/ρ_{mean} ($10^6\ m^2s^{-2}$)	σ/ρ_{mean} ($10^3\ m^2s^{-2}$)
Cobalt/WC cermets	400-530	400-900	11-12.5	34-45	34-77
Beryllium and alloys	200-289	34-276	1.8-2.1	103-148	17-141
Low-alloy steels	200-207	500-1980	7.8	26-27	64-253
CFRP	70-200	640-670	1.5-1.6	45-129	413-432
Aluminium alloys	69-79	100-627	2.6-2.9	25-45	36-228
Common woods, ‖ to grain	9-16	35-55	0.4-0.6	15-27	58-92
Lead and alloys	14	11-55	10.7-11.3	1.3	1.0-5.0
Polypropylene	0.9	19-36	0.88-0.91	1.0	21-40
Foamed polymers	0.001-0.1	0.2-10	0.01-0.6	0.003-0.03	0.66-33

E	E/ρ_{mean}	σ	σ/ρ_{mean}
Cobalt/WC cermets	Beryllium and alloys	Low-alloy steels	CFRP
Beryllium and alloys	CFRP	Cobalt/WC cermets	Low-alloy steels
Low-alloy steels	Cobalt/WC cermets	CFRP	Aluminium alloys
CFRP	Aluminium alloys	Aluminium alloys	Beryllium and alloys
Aluminium alloys	Low-alloy steels	Beryllium and alloys	Common woods, ‖ to grain
Common woods, ‖ to grain	Common woods, ‖ to grain	Common woods, ‖ to grain	Cobalt/WC cermets
Lead and alloys	Lead and alloys	Lead and alloys	Polypropylene
Polypropylene	Polypropylene	Polypropylene	Lead and alloys
Foamed polymers	Foamed polymers	Foamed polymers	Foamed polymers

Remember

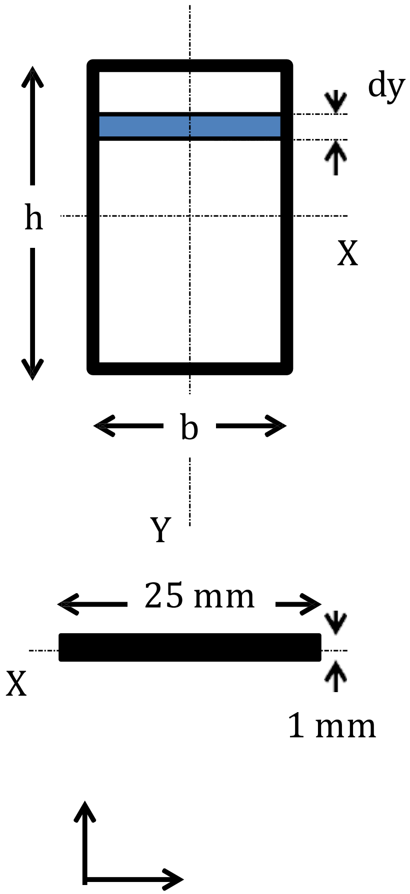
- Case Study 1: The Lightest STIFF Beam
- Case Study 2: The Lightest STIFF Tie-Rod
- Case Study 3: The Lightest STIFF Panel
- Case Study 6: The Lightest STRONG Tie-R
- Case Study 7: The Lightest STRONG Beam
- Case Study 8: The Lightest STRONG Panel



PART2

- LIGHTEST STRONG

Simplification



For a beam under flexion, the moment of inertia : $I_{XX} = \frac{bh^3}{12}$

Length (L): 300 mm

Thickness (h) = 1 mm

Width (b) = 25 mm

$$I_{XX} = \frac{25 \cdot 1^3}{12} = 2{,}1\ mm^4$$

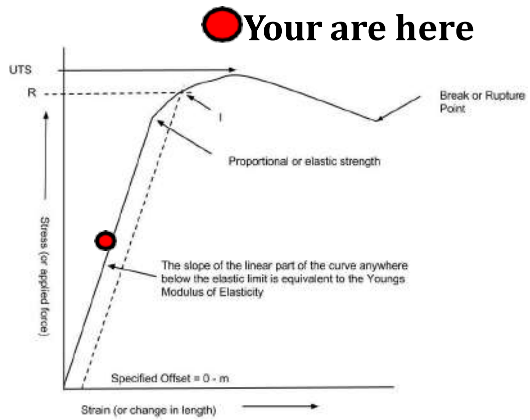
$$I_{YY} = \frac{1 \cdot 25^3}{12} = 1300\ mm^4$$

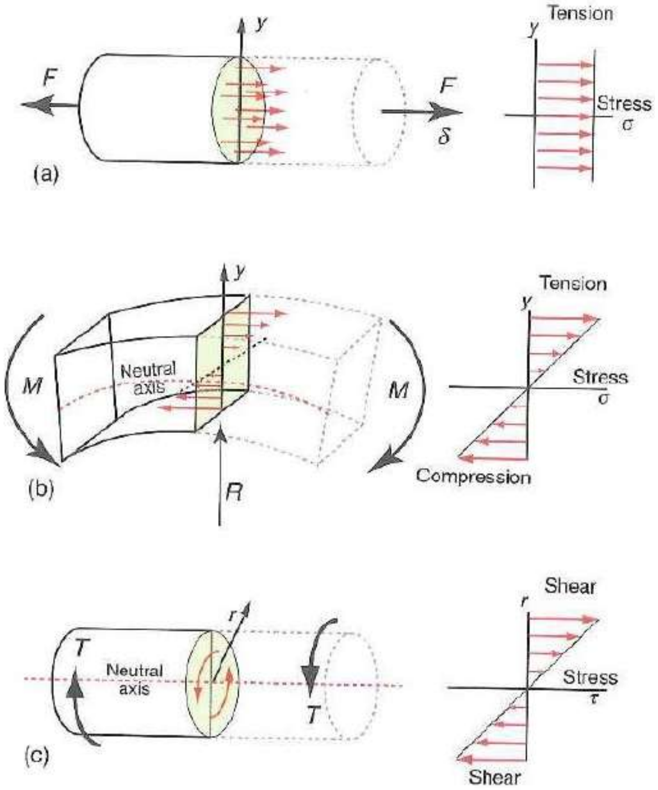
In the case of the mechanical properties, it is important to consider the forces applied, but it is the weakest point that determine the selection.

It is possible to change the geometry, but if you cannot What can we do?

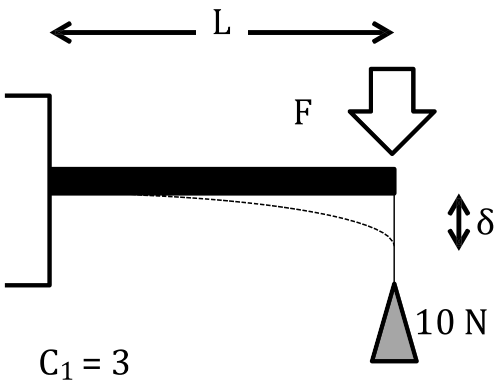
Stiffness design

The Stiffness design is important to avoid excessive ELASTIC deflection





Example 1



$C_1 = 3$

$$S = \frac{F}{\delta} = \frac{C_1 EI}{L^3}$$

$$\delta = \varepsilon \cdot L$$

EI = Flexural rigidity
I = Second Moment of inertia
E = Young's Modulus
δ = Deflexion

Length (L): 300 mm
Thickness (h)= 1 mm
Width (b)= 25 mm

$$I_{XX} = \frac{25 \cdot 1^3}{12} = 2{,}1\ mm^4$$

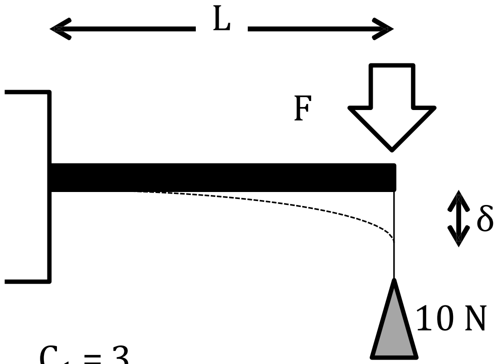
$$I_{YY} = \frac{1 \cdot 25^3}{12} = 1300\ mm^4$$

Problem :

δ??
IF we consider that the beam is made of Stainless Steel (E = 200 GPa)

Which are the consequences if I want to use Polystyrene (E = 2 GPa)?
IF I can change the thickness and hold the same deflection.

Example bis



$C_1 = 3$

$$S = \frac{F}{\delta} = \frac{C_1 EI}{L^3}$$

EI = Flexural rigidity
I = Second Moment of inertia
E = Young's Modulus

Stainless Steel (E = 200 GPa; ρ = 7800 kg/m^3)
Polystyrene (E = 2 GPa; ρ = 1040 kg/m^3)

Steel

$$I_{YY} = \frac{1 \cdot 25^3}{12} = 1300\ mm^4 \longrightarrow \delta = \frac{10 \cdot (0{,}25)^3}{3 \cdot (200 \cdot 10^9) \cdot (1300 \cdot 10^{-12})} = 0{,}02\ mm$$

$$I_{XX} = \frac{25 \cdot 1^3}{12} = 2{,}1\ mm^4 \longrightarrow \delta = \frac{FL^3}{C_1 E Y_{XX}} = 124\ mm$$

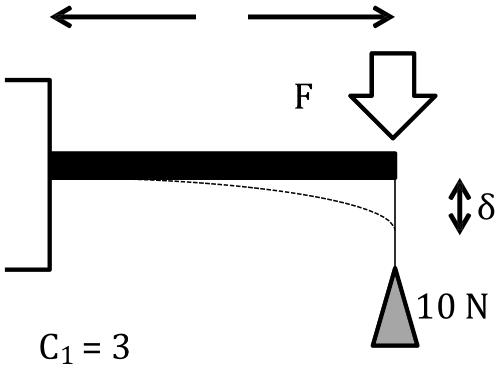
PS

$$With\ \delta = 124\ mm \qquad I_{XX} = \frac{10 \cdot (0{,}25)^3}{3 \cdot (2 \cdot 10^9) \cdot (0{,}124)} = 210\ mm^4$$

$$h = \left(\frac{12 I_{XX}}{w}\right)^{1/3} = \left(\frac{12 \cdot 210}{25}\right)^{1/3} = 4{,}6\ mm$$

$When\ h(Steel) = 1\ mm$

Example 1 ter



$C_1 = 3$

$$S = \frac{F}{\delta} = \frac{C_1 EI}{L^3}$$

Length: 300 mm

Width = 25 mm

EI = Flexural rigidity
I = Second Moment of inertia
E = Young's Modulus
δ = Deflexion

Stainless Steel (E = 200 GPa; ρ = 7800 kg/m^3)
Polystyrene (E = 2 GPa; ρ = 1040 kg/m^3)

Thickness = 1 mm
Thickness = 4,6 mm

About the weight?

$$m_{SS} = 7800 \cdot 0{,}3 \cdot 0{,}025 \cdot 0{,}001 = 59\ gr$$

$$m_{PS} = 1040 \cdot 0{,}3 \cdot 0{,}025 \cdot 0{,}046 = 36\ gr$$

BIGGER Section
BUT LIGHTER

Depends on what you need and the conditions

Generic Materials Selection

p: Performance of component; f(F,G,M)

F: Functional requirement, e.g. withstanding a force

G: Geometry, e.g. diameter, length etc.

M: Materials properties, e.g. E, K_{IC}, ρ

Separable function if:

$$P = f_1(F) \cdot f_2(G) \cdot f_3(M)$$

TASK: Maximize $f_3(M)$ where M is the "performance index"

Procedure for Deriving "M"

(a) Identify the *attribute* to be maximized or minimized (weight, cost, energy, stiffness, strength, safety, environmental damage, etc.).

(b) Develop an equation for this attribute in terms of the functional requirements, the geometry, and the material properties (the *objective function*).

(c) Identify the *free* (unspecified) *variables.*

(d) Identify the *constraints*; rank them in order of importance.

(e) Develop *equations* for the constraints (no yield, no fracture, no buckling, maximum heat capacity, cost below target, etc.).

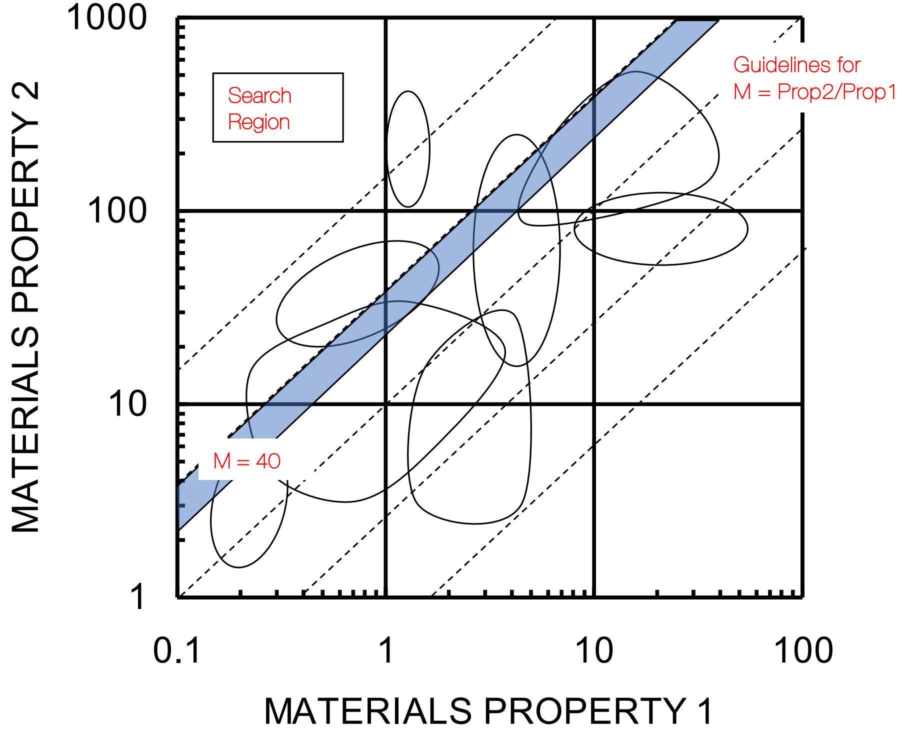
(f) S*ubstitute* for the free variables from the constraints into the objective function.

(g) G*roup the variables* into three groups: functional requirements, F, geometry, G, and materials properties, M.

(h) R*ead off* the performance index, expressed as a quantity, M, to be maximized.

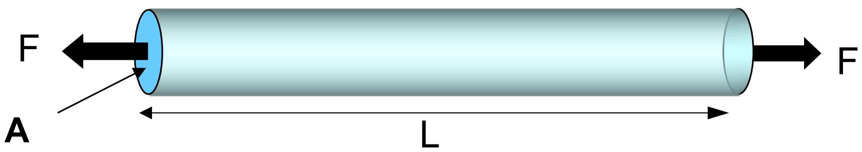
(i) N*ote* that a full solution is not necessary in order to identify the material property group.

The Materials Selection Map



Index for a strong, light tie-rod

Strong tie of length L and minimum mass



Function — *Tie-rod*

Constraints

- *Length L is specified*
- *Must not fail under load F*

Equation for constraint on A:

$$\boxed{F/A < \sigma_y}$$

Objective

Minimize mass m:

$$\boxed{m = A\,L\,\rho}$$

Performance metric

$$m = F\,L\left(\frac{\rho}{\sigma_y}\right)$$

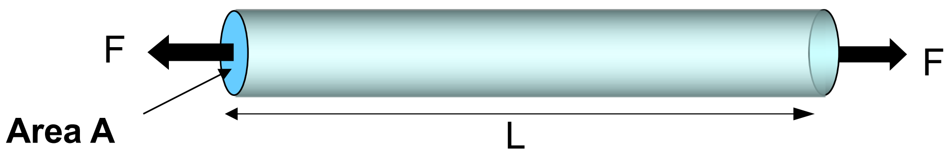
Chose materials with largest $M = \left(\frac{\sigma_y}{\rho}\right)$

m = mass
A = area
L = length
ρ = density
σ_y **=** yield strength

Example I:

Function — *Tie-rod*

Strong tie of length L and minimum mass



Objective — Mass: $m = AL\rho \quad \Rightarrow A = \dfrac{m}{L\rho}$

Stress: $\sigma_f = \dfrac{F}{A} = \dfrac{FL\rho}{m}$

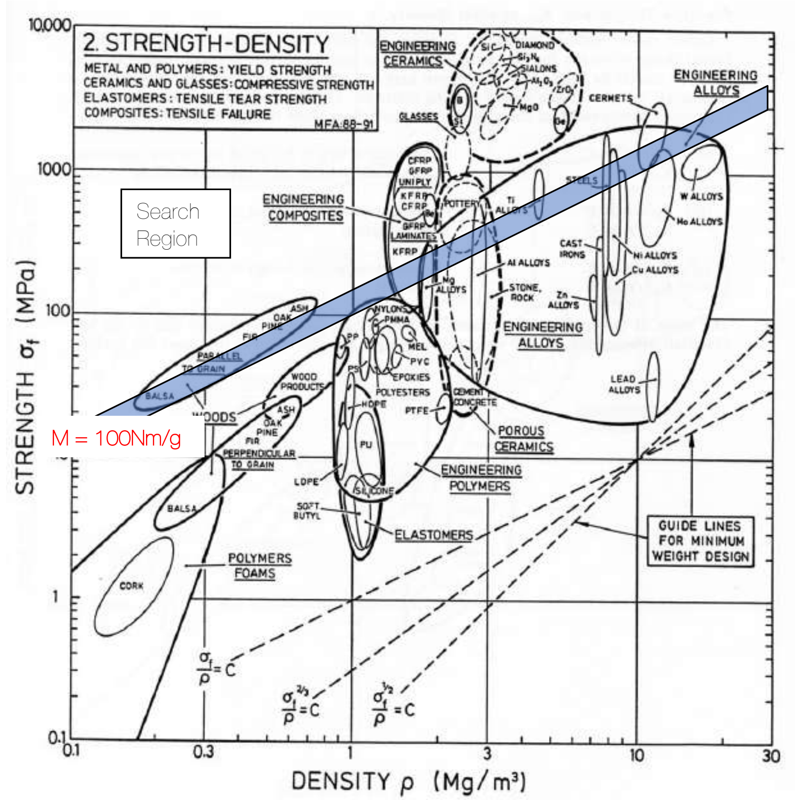
Constraints

- *Length L is specified*
- *Must not fail under load F*

$$\Rightarrow m = \underbrace{F}_{f_1(F)} \cdot \underbrace{L}_{f_2(G)} \cdot \underbrace{\frac{\rho}{\sigma_f}}_{f_3(M)}$$

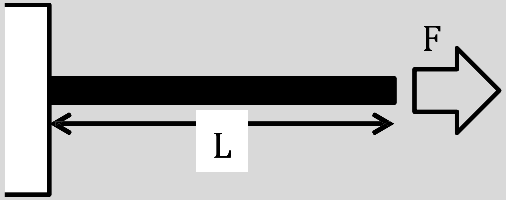
So, to minimize mass m,

Chose materials with largest $\boxed{M = \dfrac{\sigma_f}{\rho}}$ **Performance metric**

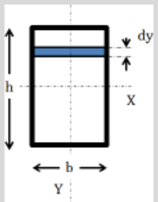


Lightest Tie-Rod (Traction conditions)

Case Study 6:
Find the Lightest STRONG <u>Tie-Rod</u>



DATA
F =1000 N
Dimensions:
Length: 300 mm
Thickness = 1 mm
Width = 25 mm

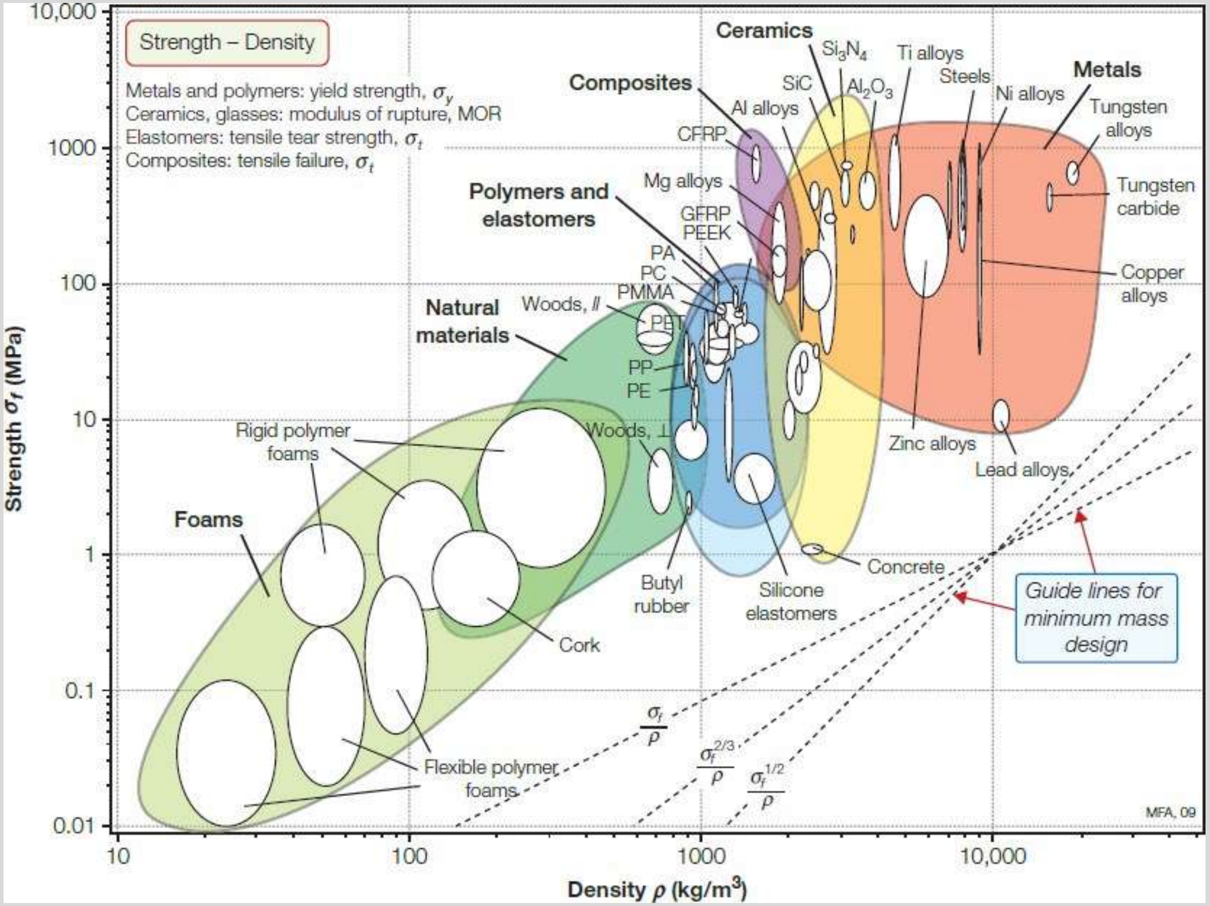


Objective	• Minimize the mass
Constraints	• Support tensile load F without yielding • Length L
Free Variables	• Area (A) of the cross-section • Choice of the material

In Traction,
the shape of the cross-section is not important

$$\begin{cases} \boxed{m = A \cdot L \cdot \rho} \longrightarrow A = \dfrac{m}{L \cdot \rho} \\ From\ material: \ \dfrac{F}{A} \leq \sigma_y \end{cases} \Rightarrow m \geq F \cdot L \cdot \frac{\rho}{\sigma_y}$$

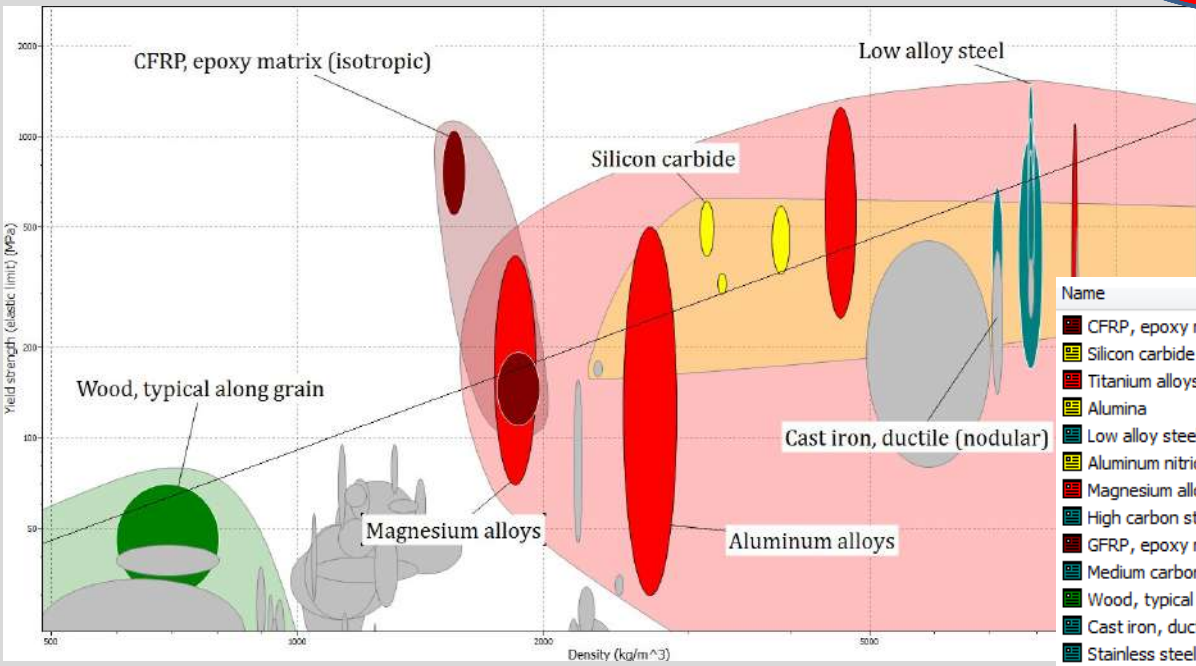
Ashby Diagrams



CES

Case Study 6:
Find the Lightest STRONG Tie-Rod

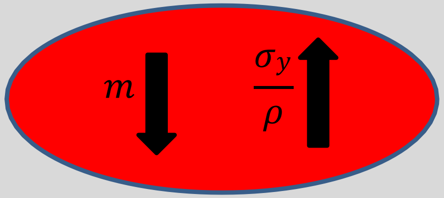
$m \downarrow \quad \frac{\sigma_y}{\rho} \uparrow$



Name	Stage 1: Index
CFRP, epoxy matrix (isotropic)	0.491
Silicon carbide	0.157
Titanium alloys	0.121
Alumina	0.117
Low alloy steel	0.0987
Aluminum nitride	0.0983
Magnesium alloys	0.0908
High carbon steel	0.0866
GFRP, epoxy matrix (isotropic)	0.0783
Medium carbon steel	0.0667
Wood, typical along grain	0.0661
Cast iron, ductile (nodular)	0.0577
Stainless steel	0.0526
Aluminum alloys	0.0455
Nickel alloys	0.0312

Lightest Tie-Rod (Traction conditions)

Case Study 6:
Find the Lightest STRONG <u>Tie-Rod</u>



$$m \geq F \cdot L \cdot \frac{\rho}{\sigma_y}$$

It is possible to do as before, but let's calculate the maximum F on the precedent Tie-Rod

Material	Weight (kg)	Width and Thickness (mm)
Al Alloys	1,25	63

Stainless Steel (σ_y = 600 MPa; ρ = 7800 kg/m^3)
Wood (σ_y = 50 MPa; ρ = 700 kg/m^3)
Al Alloys (σ_y = 270 Mpa; ρ = 75 kg/m^3)

$$F \leq \frac{m}{L} \cdot \frac{\sigma_y}{\rho} = 416\, kN$$

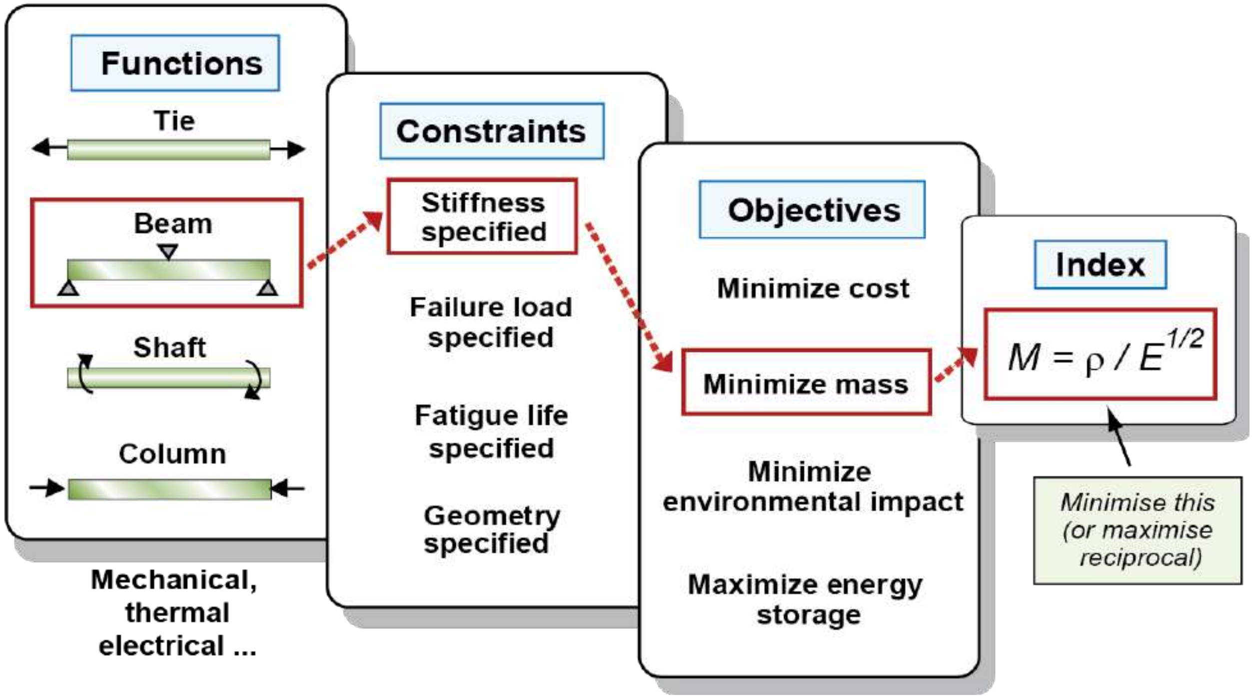
0 kN → **X kN**

Elastic Throughout — **Plastic deformation/ Collapse**

PART3

- LIGHTEST STIFF

A performance index is a group of material properties that limits the performance of a design

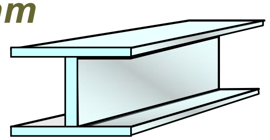


Index for a stiff, light beam

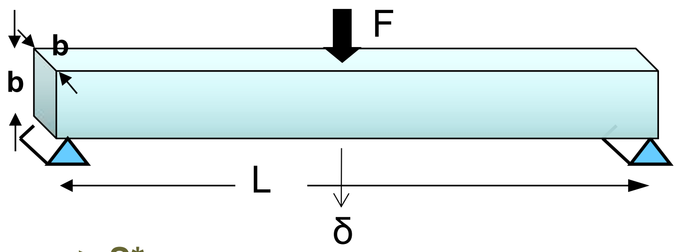
GRANTA

Function

Beam



Stiff beam of length L and minimum mass



Square section, area $A = b^2$

Constraints

- ***Length L is specified***
- ***Must have bending stiffness > S****

Equation for constraint on A:

$$\boxed{S = \frac{F}{\delta} = \frac{CEI}{L^3} = \frac{CEA^2}{12L^3}}$$

m = mass
A = area
L = length
ρ = density
S = stiffness (F/δ)
This beam: $\delta = FL^3/CEI$
C = constant (here, 48)
E = Young's modulus
I = second moment of area
($I = b^4/12 = A^2/12$)

Objective

Minimize mass m:

$$\boxed{m = A\,L\,\rho}$$

Performance metric

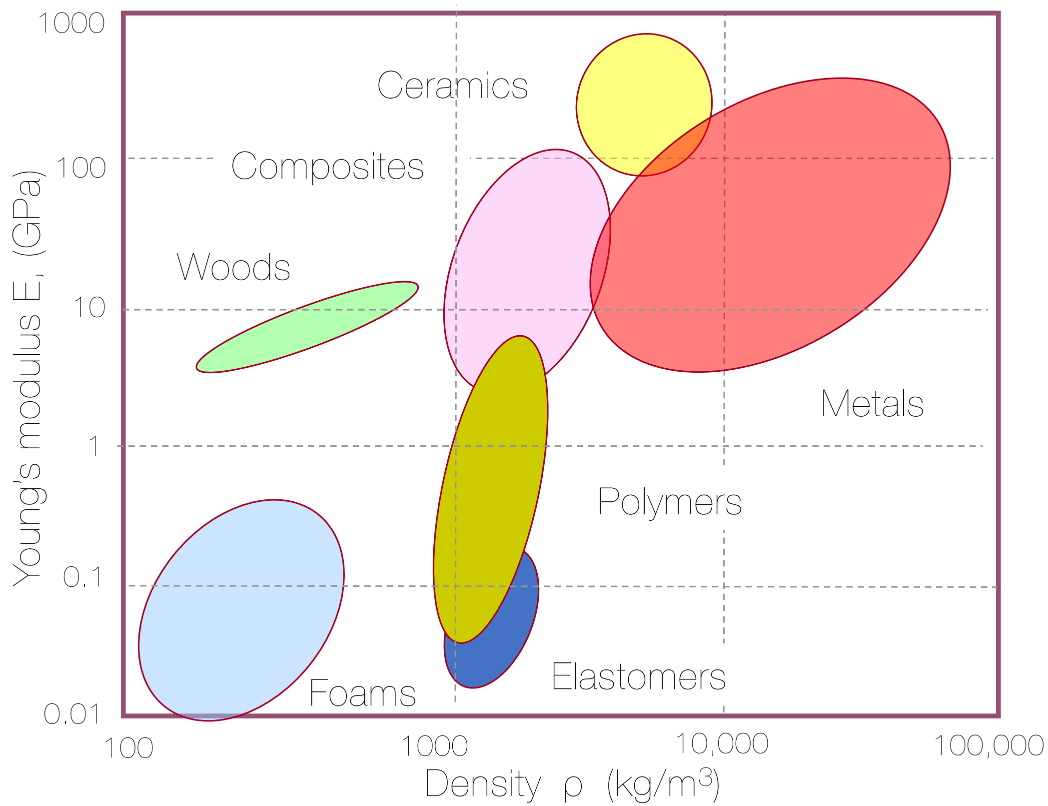
$$m = \left(\frac{12L^5 S^*}{C}\right)^{1/2} \left(\frac{\rho}{E^{1/2}}\right)$$

Chose materials with largest $M = \left(\dfrac{E^{1/2}}{\rho}\right)$

Back to Ashby diagrams

- We need to find the material Maximizing

$$M = \left(\frac{E^{1/2}}{\rho} \right)$$



Back to Ashby diagrams

- We need to find the material

Maximizing

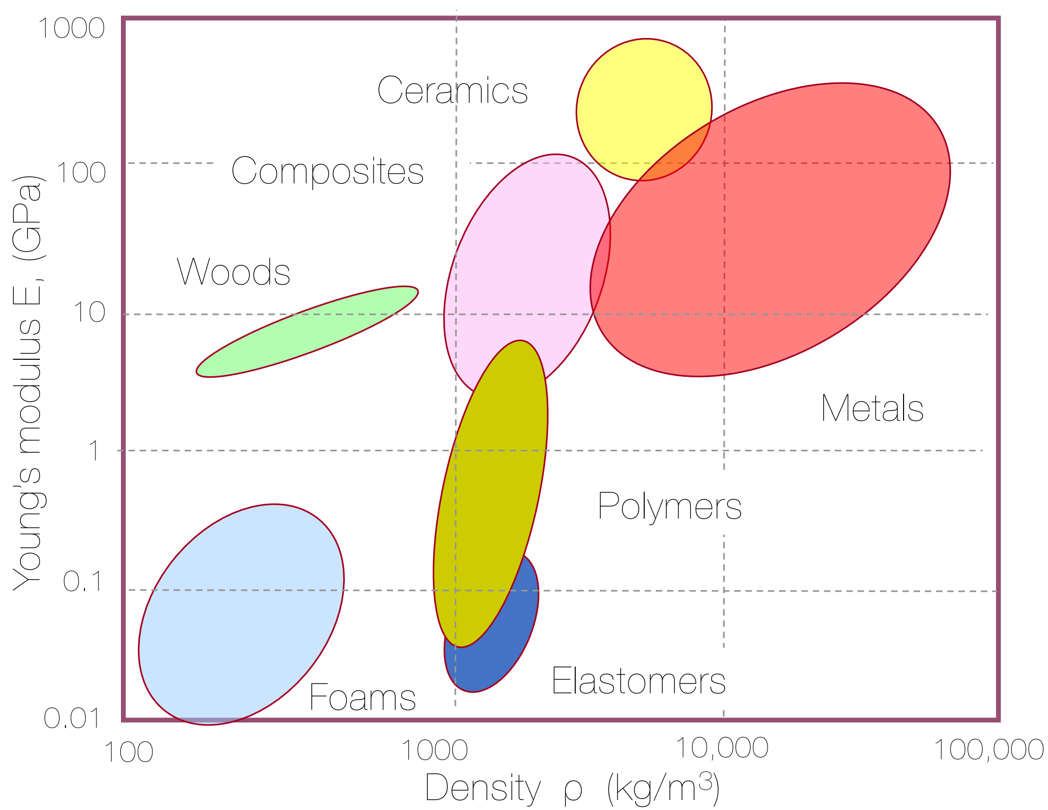
$$M = \left(\frac{\mathbf{E^{1/2}}}{\boldsymbol{\rho}} \right)$$

We reorganise this :

E = M^2 * ρ^2

Therefore log(E) = 2log(M) + 2log(ρ)

⇒ Advantage of the log-log diagram: Materials on a line of slope 2 have the same index!



Back to Ashby diagrams

- We need to find the material

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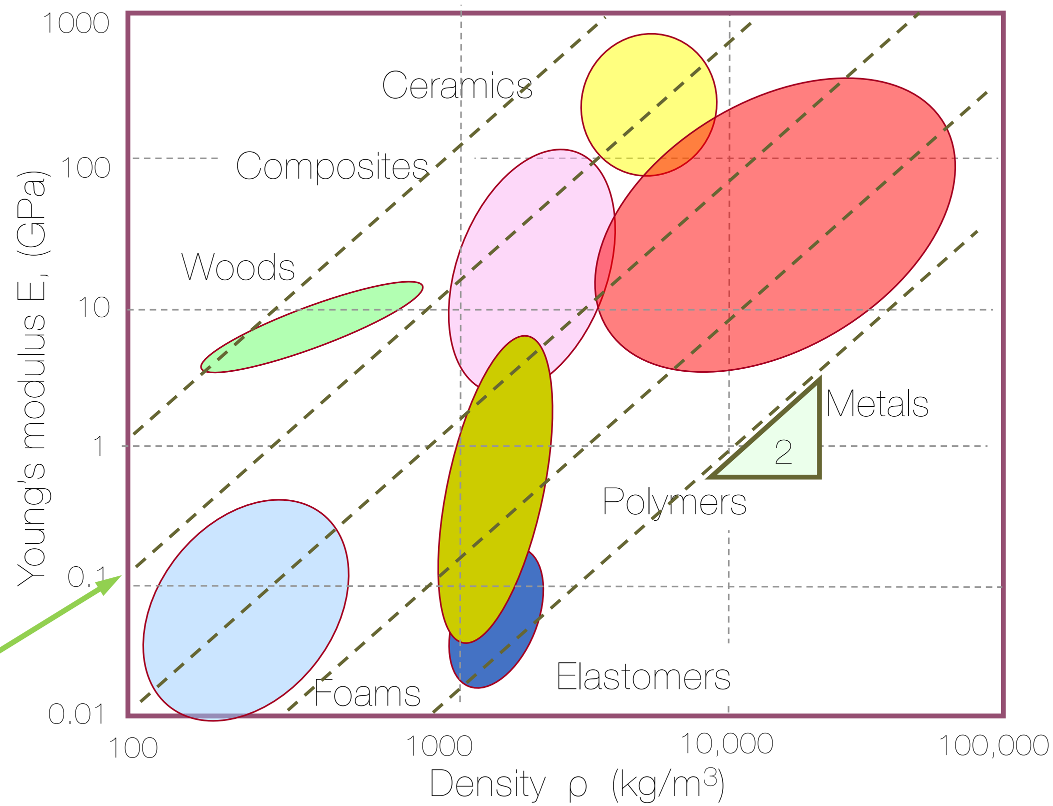
Therefore log(E) = 2log(M) + 2log(ρ)

⇒ Advantage of the log-log diagram: Materials on a line of slope 2 have the same index!

Part	Index	Slope
Bar	**E/ρ**	1
Beam	$\mathbf{E^{1/2}/\rho}$	2
Panel	$\mathbf{E^{1/3}/\rho}$	3



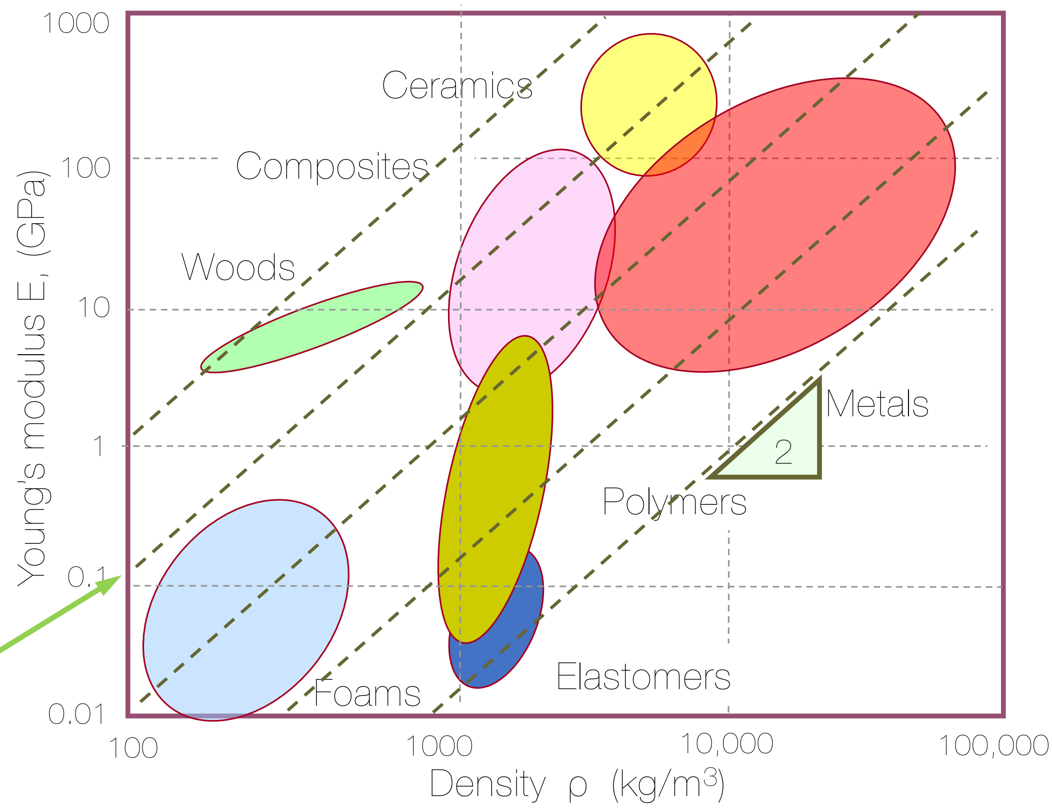
Back to Ashby diagrams

- Looking at the diagram, what type of material enables to minimize the mass of a stiff beam?

$$\log(E) = 2\log(M) + 2\log(\rho)$$

A: Woods
B : Elastomers
C : Ceramics
D : Metals

Part	Index	Slope
Bar	**E/ρ**	1
Beam	**$E^{1/2}/\rho$**	2
Panel	**$E^{1/3}/\rho$**	3



Back to Ashby diagrams

- Looking at the diagram, what type of material enables to minimize the mass of a stiff beam?

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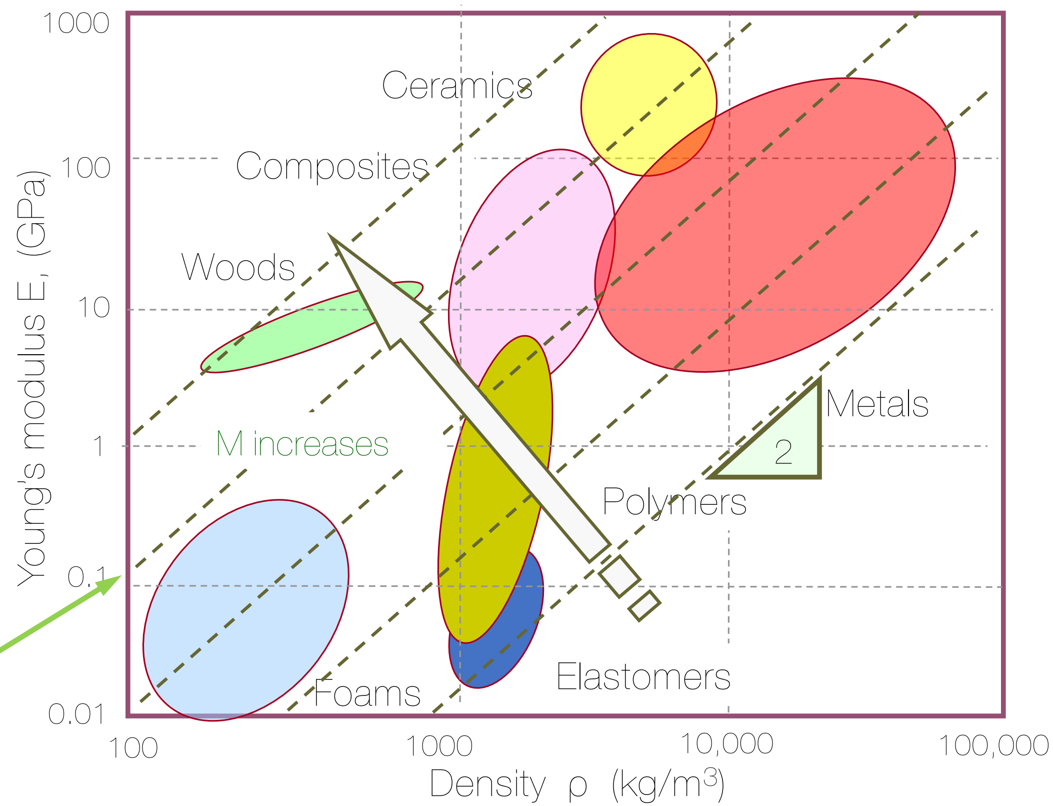
A : Woods
B : Elastomers
C : Ceramics
D : Metals

Part	Index	Slope
Bar	E/ρ	1
Beam	$E^{1/2}/\rho$	2
Panel	$E^{1/3}/\rho$	3



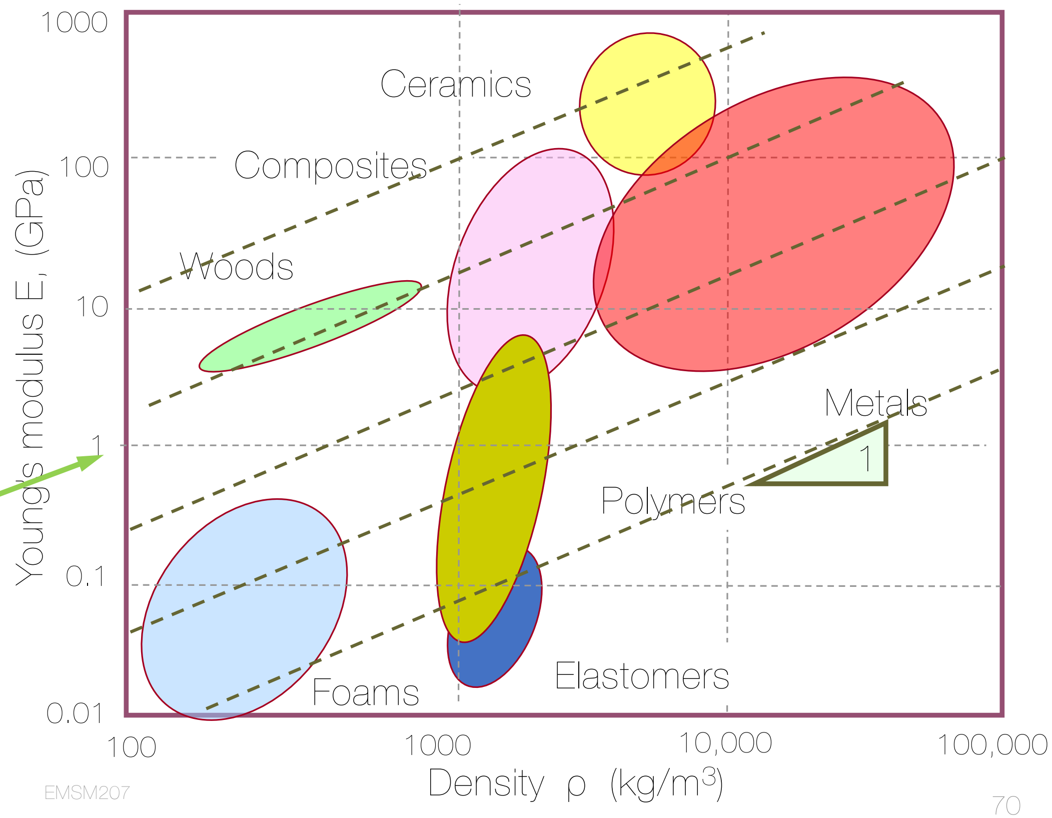
Back to Ashby diagrams

- Looking at the diagram, what type of material enables to minimize the mass of a stiff bar?

=> ceramics

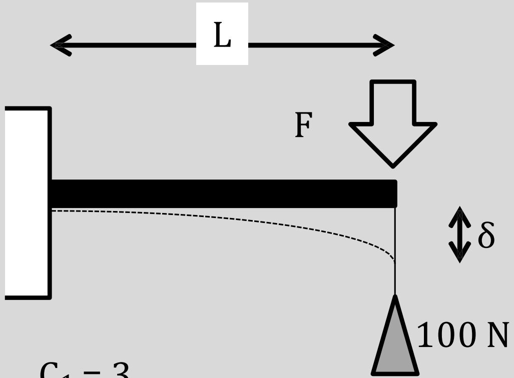
log(E) = 1log(M) + 1log(ρ)

Part	Index	Slope
Bar	**E/ρ**	1
Beam	**$E^{1/2}/\rho$**	2
Panel	**$E^{1/3}/\rho$**	3



The Materials Selection approach

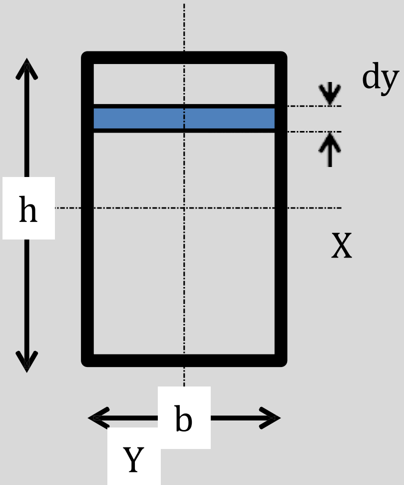
Case Study 1:
Find the Lightest STIFF Beam



$C_1 = 3$

Beam: Square Section

b=h



$$\begin{cases} \dfrac{F}{\delta} = \dfrac{C_1 E I}{L^3} \geq S_{min} \\ A = \dfrac{m}{L \cdot \rho} \end{cases}$$

Since $A = b^2$

$$I = \frac{bh^3}{12} = \frac{A^2}{12}$$

$$\begin{cases} A = \dfrac{m}{L \cdot \rho} \\ m \geq \left(\dfrac{12 \cdot S}{C_1 \cdot L}\right)^{1/2} \cdot L^3 \cdot \dfrac{\rho}{E^{1/2}} \end{cases}$$

The Area will be the Free Variable

$m \downarrow \quad \dfrac{E^{1/2}}{\rho} \uparrow$

Just remember:

Constraints	• Stiffness specified • Length L • Square shape

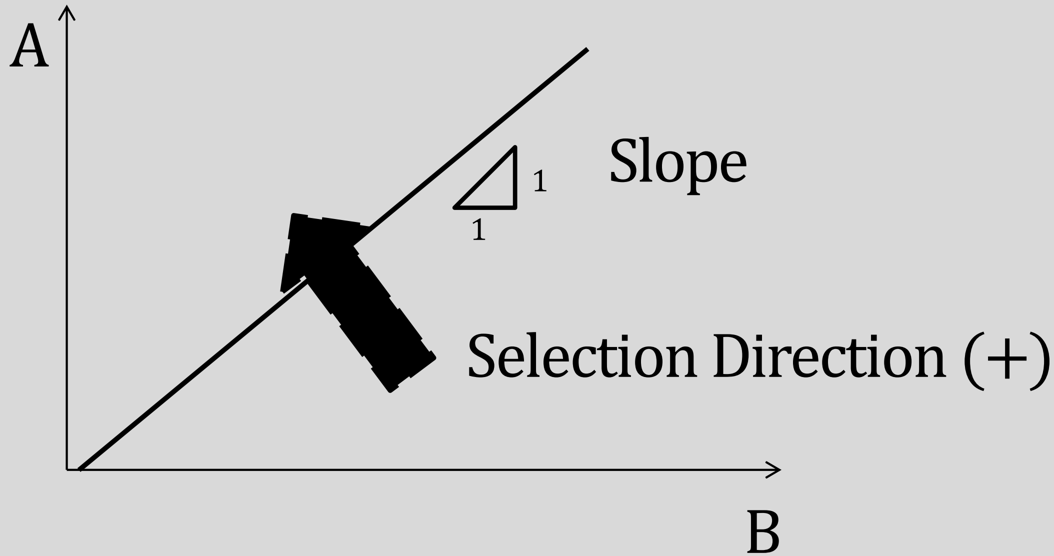
The Material Index (M)

Case Study 1:
Find the Lightest STIFF <u>Beam</u>

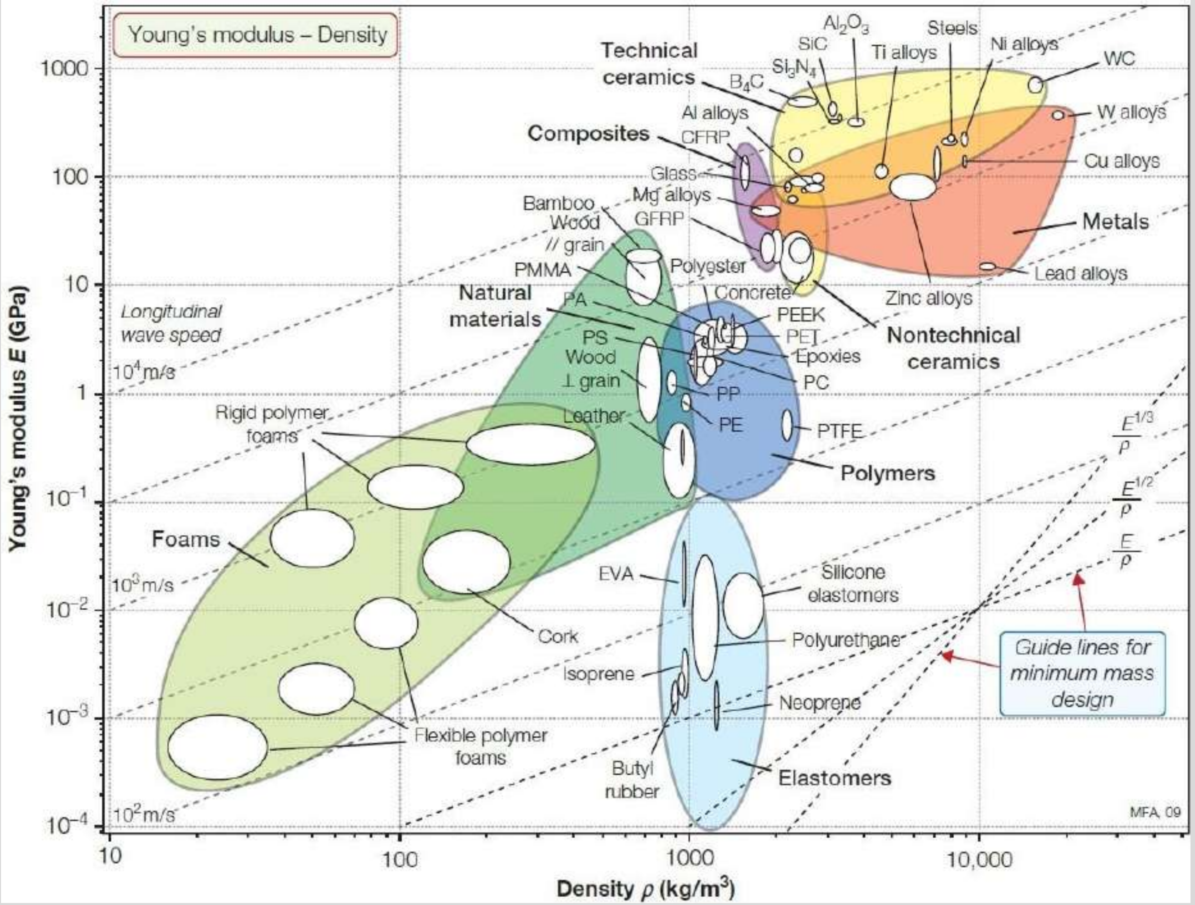
$$M = \frac{A}{B} \longrightarrow \boxed{\mathrm{Log}(M) = Log\,(A) - Log(B)}$$

$$\boxed{\mathrm{Log}(A) = Log(B) + Log\,(M)}$$

For instance $\frac{E^{1/2}}{\rho}$

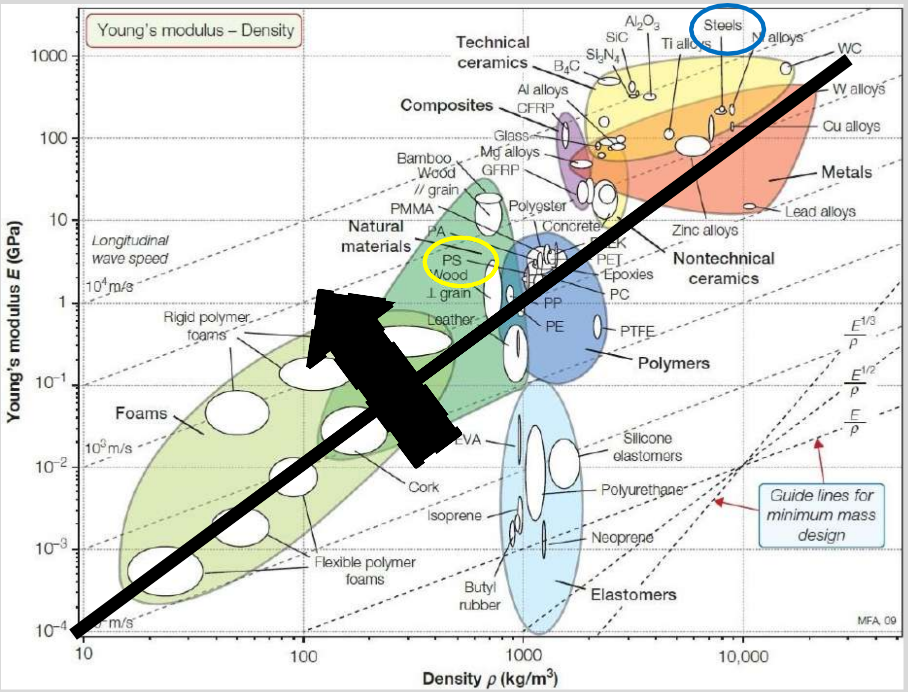


Ashby Diagrams



The Material Index (M)

Case Study 1:
Find the Lightest STIFF Beam



$m \downarrow \quad \frac{E^{1/2}}{\rho} \uparrow$

Slope 2 / 1

Stainless Steel
(E = 200 GPa; ρ = 7800 kg/m^3)
Polystyrene
(E = 2 GPa; ρ = 1040 kg/m^3)

Lightest Beam (Bending conditions)

Case Study 1:
Find the Lightest STIFF Beam

Length: 300 mm
~~Thickness = 1 mm~~
~~Width = 25 mm~~

Width and thickness ?

F = 100 N
δ = 0,34 mm
$S_{min} = 296 \cdot 10^3$ N/m

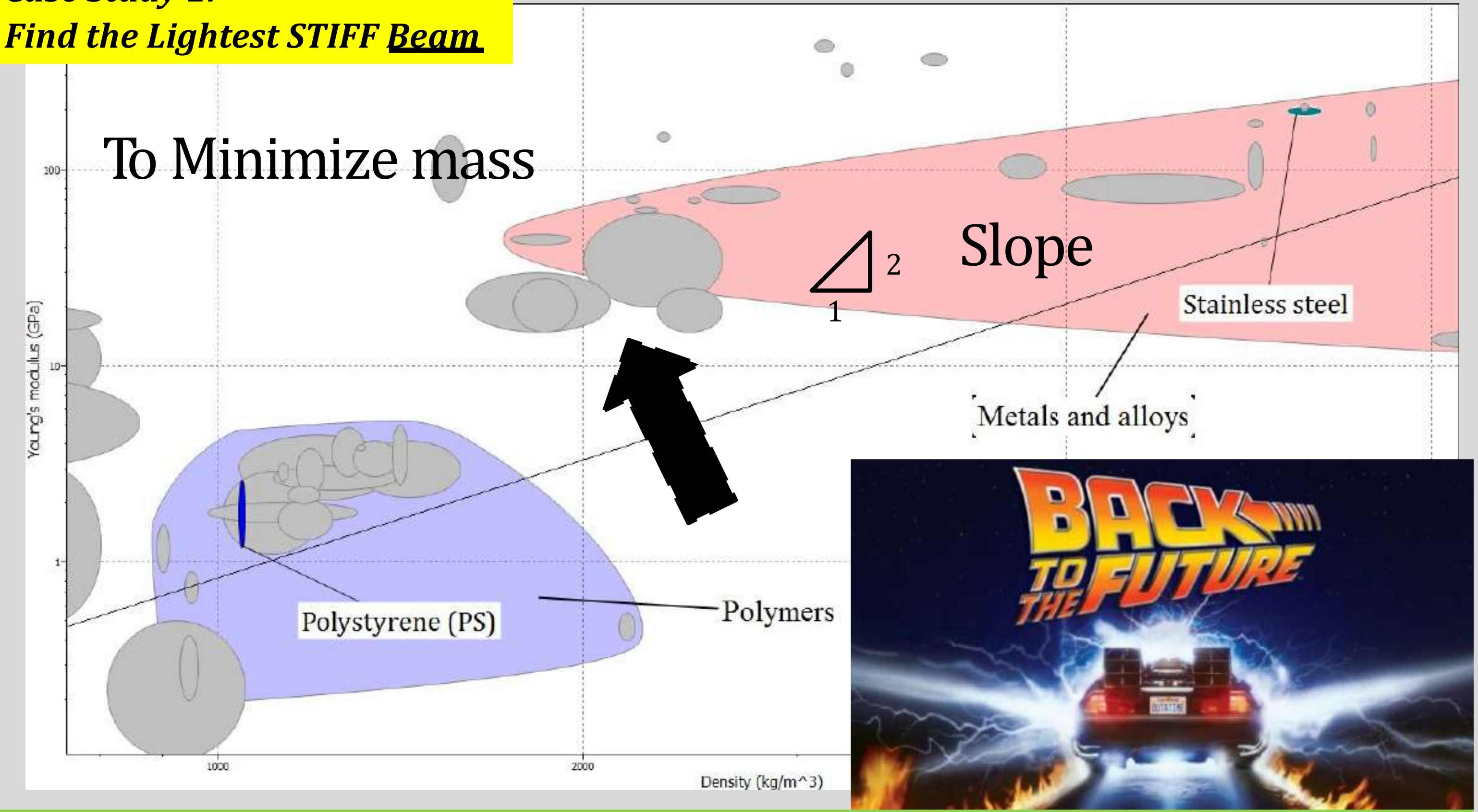
Stainless Steel (E = 200 GPa; ρ = 7800 kg/m^3)
Polystyrene (E = 2 GPa; ρ = 1040 kg/m^3)

$$\begin{cases} m \geq \left(\dfrac{12 \cdot S}{C_1 \cdot L}\right)^{1/2} \cdot L^3 \cdot \dfrac{\rho}{E^{1/2}} \\ A = \dfrac{m}{L \cdot \rho} \end{cases}$$

Material	**Weight (kg)**	**A (mm^2)**	**Width and Thickness (mm)**
Stainless Steel	0,935	400	20
Polystyrene	1,25	4000	63

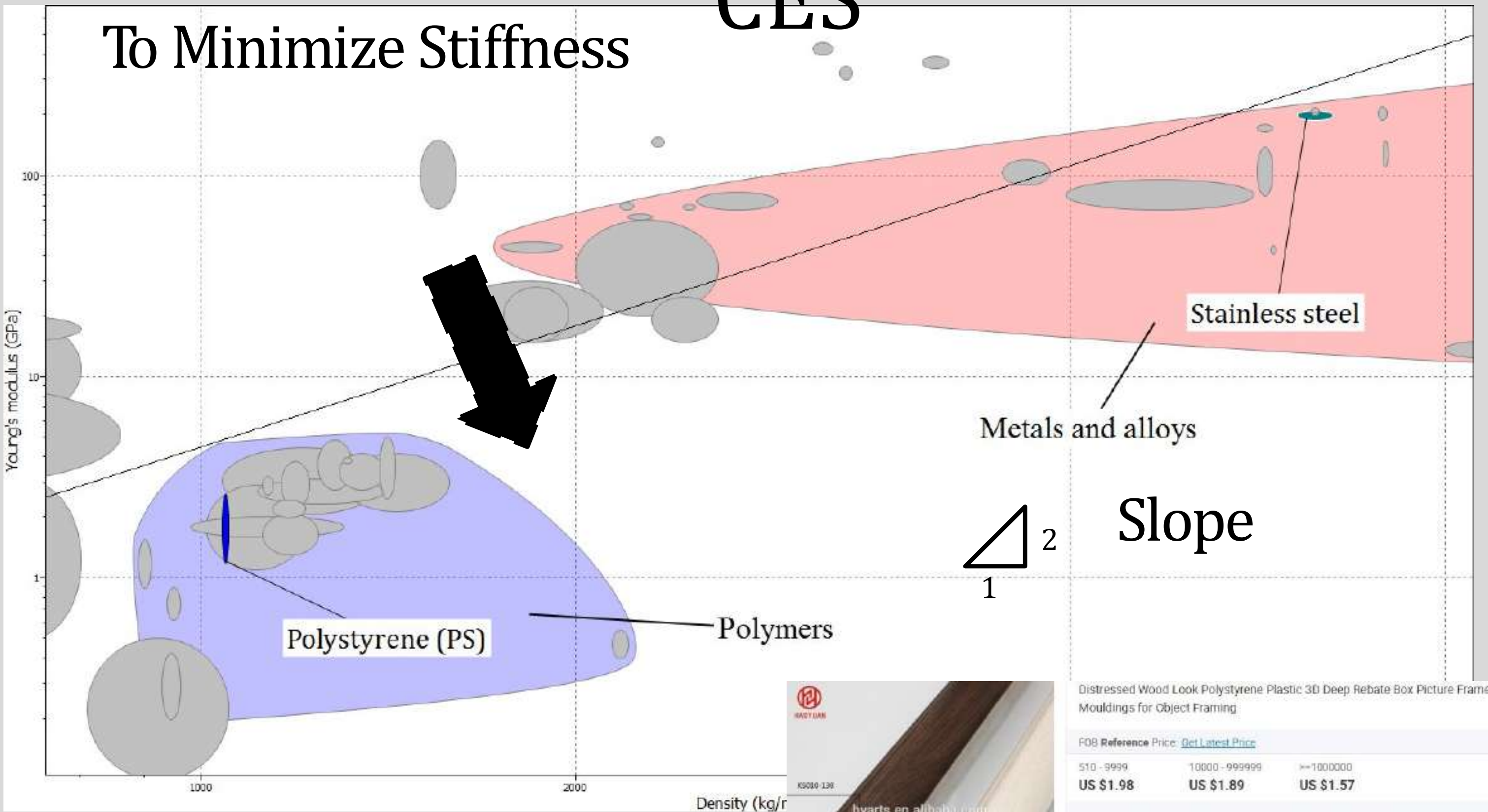
CES

Case Study 1:
Find the Lightest STIFF Beam



CES

To Minimize Stiffness



Slope

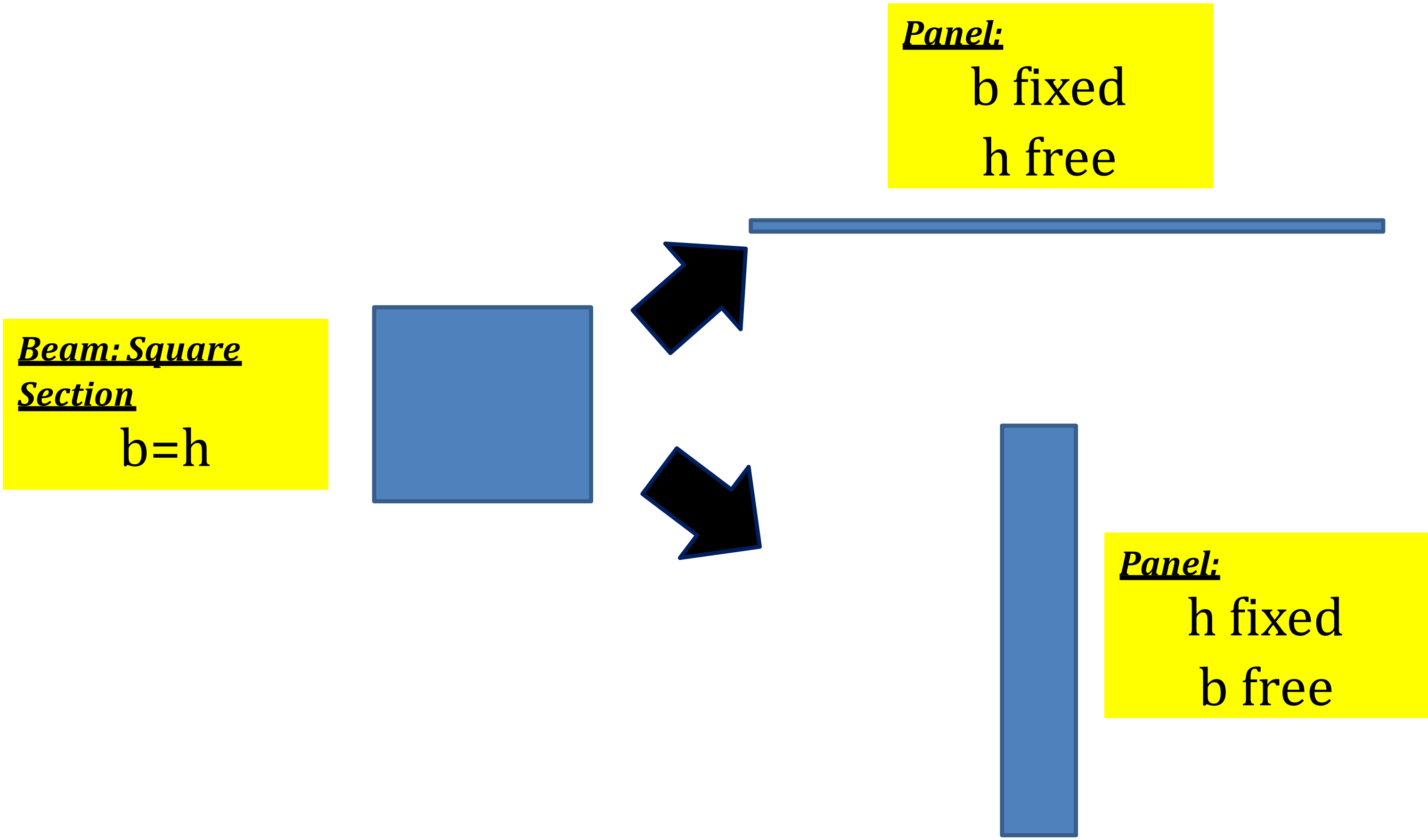
2

1

Depends on what you need→

<u>It is not in flexion</u>

Change of the section



Summary (to minimize the mass)

$\dfrac{E}{\rho}$	$\dfrac{E^{1/2}}{\rho}$	$\dfrac{E^{1/3}}{\rho}$
$\dfrac{\sigma_y}{\rho}$	$\dfrac{\sigma_y^{2/3}}{\rho}$	$\dfrac{\sigma_y^{1/2}}{\rho}$

Stiffness – Traction :

Name	Stage 1: Index
Silicon carbide	0.136
Aluminum nitride	0.0984
Alumina	0.094
CFRP, epoxy matrix (isotropic)	0.0657
Silicon	0.0634
Tungsten carbides	0.0425
Silica glass	0.0323
Soda-lime glass	0.0284
Borosilicate glass	0.0278
Aluminum alloys	0.0277
Bamboo	0.025
Wood, typical along grain	0.0158

Stiffness – Bending (Beam):

Name	Stage 1: Index
Silicon carbide	0.00657
CFRP, epoxy matrix (isotropic)	0.00651
Bamboo	0.00601
Aluminum nitride	0.00546
Silicon	0.00522
Alumina	0.00492
Wood, typical along grain	0.00478
Rigid Polymer Foam (LD)	0.00413
Silica glass	0.00384
Magnesium alloys	0.00362

Stiffness – Bending (Panel):

Name	Stage 1: Index
Rigid Polymer Foam (LD)	0.00697
Rigid Polymer Foam (MD)	0.00442
Bamboo	0.00373
Flexible Polymer Foam (VLD)	0.00335
Wood, typical along grain	0.00321
CFRP, epoxy matrix (isotropic)	0.00301
Paper and cardboard	0.00269
Rigid Polymer Foam (HD)	0.00239
Flexible Polymer Foam (LD)	0.00233
Flexible Polymer Foam (MD)	0.00212
Cork	0.00173

Strength – Traction :

Name	Stage 1: Index
CFRP, epoxy matrix (isotropic)	0.491
Silicon carbide	0.157
Titanium alloys	0.121
Alumina	0.117
Low alloy steel	0.0987
Aluminum nitride	0.0983
Magnesium alloys	0.0908
High carbon steel	0.0866
GFRP, epoxy matrix (isotropic)	0.0783
Medium carbon steel	0.0667
Wood, typical along grain	0.0661
Cast iron, ductile (nodular)	0.0577
Stainless steel	0.0526
Aluminum alloys	0.0455
Nickel alloys	0.0312

Strength – Bending :

Name	Stage 1: Index
CFRP, epoxy matrix (isotropic)	0.0538
Silicon carbide	0.0198
Wood, typical along grain	0.0185
Magnesium alloys	0.0165
Rigid Polymer Foam (LD)	0.0159
Titanium alloys	0.0147
Rigid Polymer Foam (MD)	0.00986
Aluminum alloys	0.00916

Strength – Bending (Panel):

Name	Stage 1: Index
CFRP, epoxy matrix (isotropic)	0.0178
Rigid Polymer Foam (LD)	0.0168
Wood, typical along grain	0.00977
Rigid Polymer Foam (MD)	0.00959
Bamboo	0.00904
Flexible Polymer Foam (VLD)	0.00787
Paper and cardboard	0.0074
Magnesium alloys	0.00702
Rigid Polymer Foam (HD)	0.00623
Flexible Polymer Foam (LD)	0.0054

Performance Index finder in the advanced databases

GRANTA

Chart stage: select performance index finder

Identify function (choice of 43)

Panel in bending

Identify free variables

Panel thickness

Identify variables and constraints

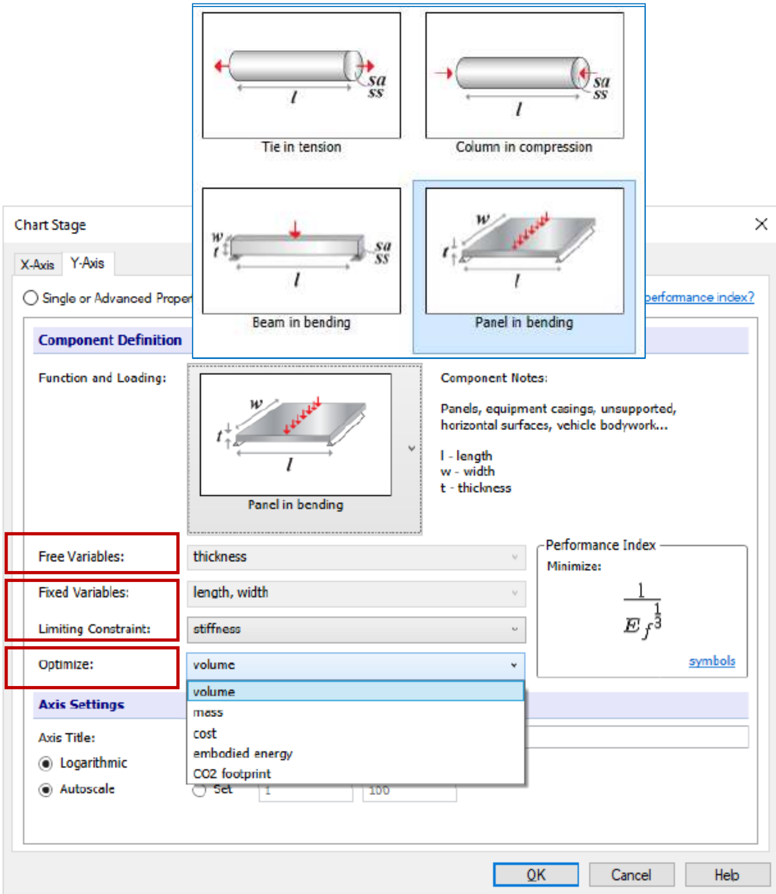
Panel length

Panel width

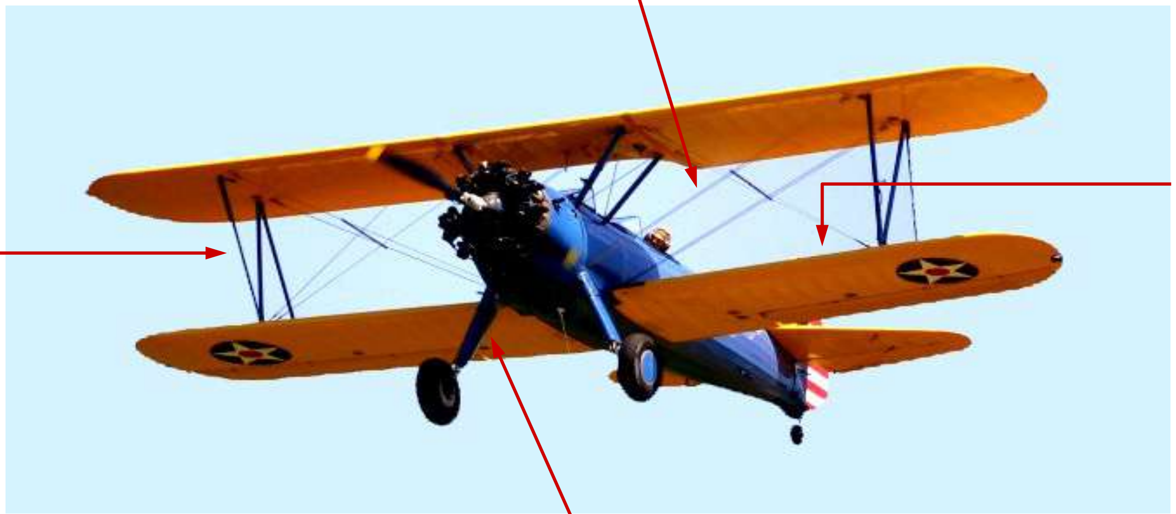
Stiffness limited design

Identify objective

Default is to minimize



Minimum weight design - indices



E = Young's modulus
ρ = Density
σ_y = Yield strength

The marked components of this plane perform different functions. The ties carry tension, the struts carry compression (they act as columns) and the spars carry bending moments – they are beams. They are chose to be as light as possible: thus the objective is to minimize mass.

The mass of a tensile tie of prescribed strength depends on two material properties – yield strength and density – in the combination σ_y/ρ; it is the material index for this component.

The mass of a strut that must carry a compressive load without buckling elastically is proportional to the material group $E^{1/2}/\rho$ so this becomes the material index.

The mass of a beam, loaded in bending, with restriction on elastic deflection is also proportional to $E^{1/2}/\rho$ so the index here is the same as that for the strut.

Thus the index depends on the mode of loading (tension, compression, bending) and on the requirement for stiffness or strength.

PART3bis

- When materials are loaded in bending, in torsion, or are used as slender columns, section shape becomes important

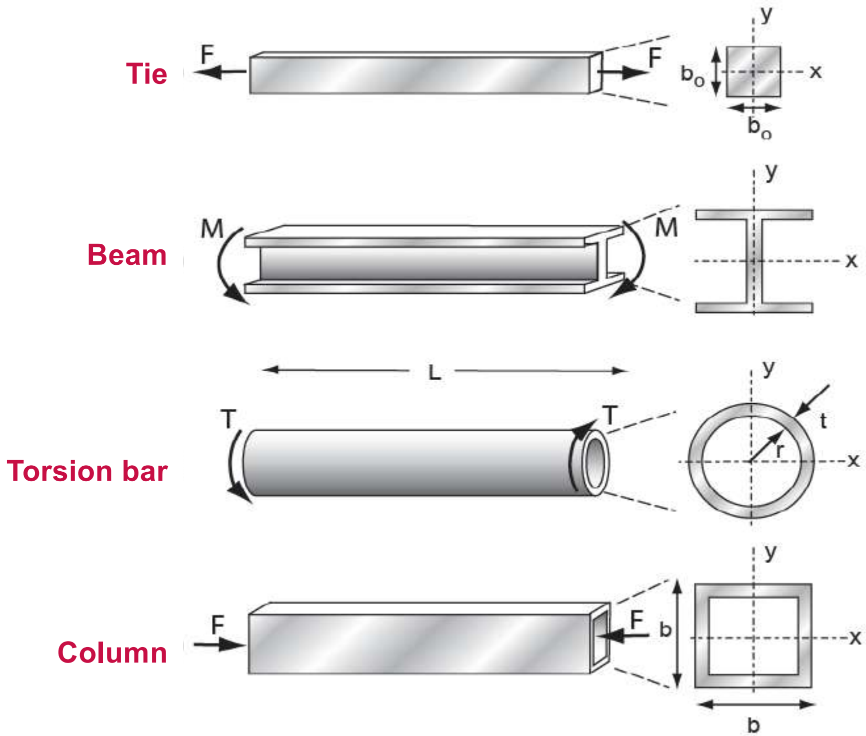
- "Shape" = cross section formed to a
  tubes
  I-sections
  tubes
  hollow box-section
  ~~sandwich panels~~
  ~~ribbed panels~~

- "Efficient" = use least material for given stiffness or strength

- Shapes to which a material can be formed are limited by the material itself

- Goals: co-selecting material and shape

Standard structural members



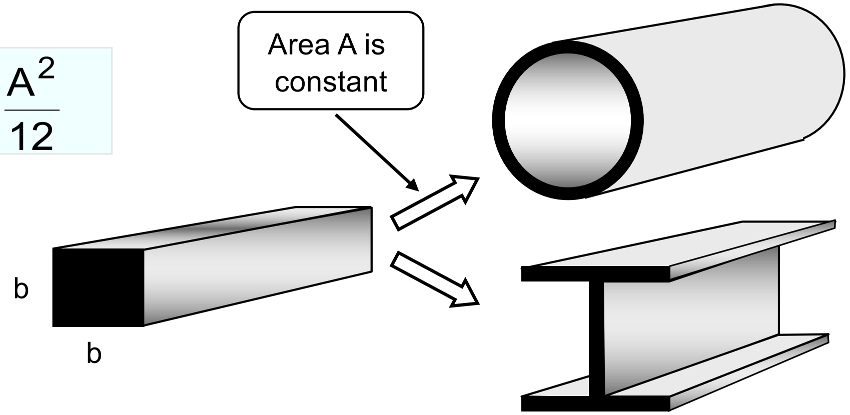
Tie	Area A matters, not shape
Beam	Area A and shape I_{XX}, I_{YY} matter
Torsion bar	Area A and shape J matter
Column	Area A and shape I_{min} matter

Certain materials can be made to certain shapes: what is the best combination?

- Take ratio of bending stiffness S of shaped section to that (S_o) of a neutral reference section of the same cross-section area
- Define a standard reference section: a solid square with area $A = b^2$
- Second moment of area is I; stiffness scales as EI.

$$I_o = \frac{b^4}{12} = \frac{A^2}{12}$$

Area $A = b^2$



$$I = \int y^2 b(y) dy$$

Area A and modulus E unchanged

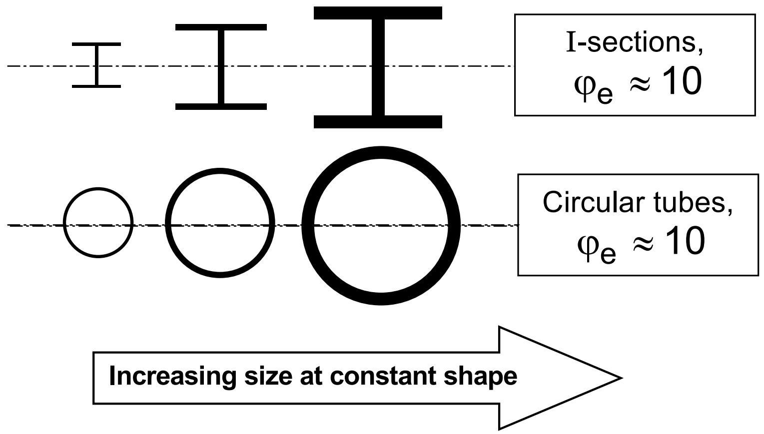
- Define **shape factor for elastic bending,** measuring efficiency, as

$$\varphi_e = \frac{S}{S_o} = \frac{EI}{EI_o} = 12\frac{I}{A^2}$$

Properties of the shape factor

GRANTA

- The shape factor is dimensionless – a pure number.
- It characterizes shape.

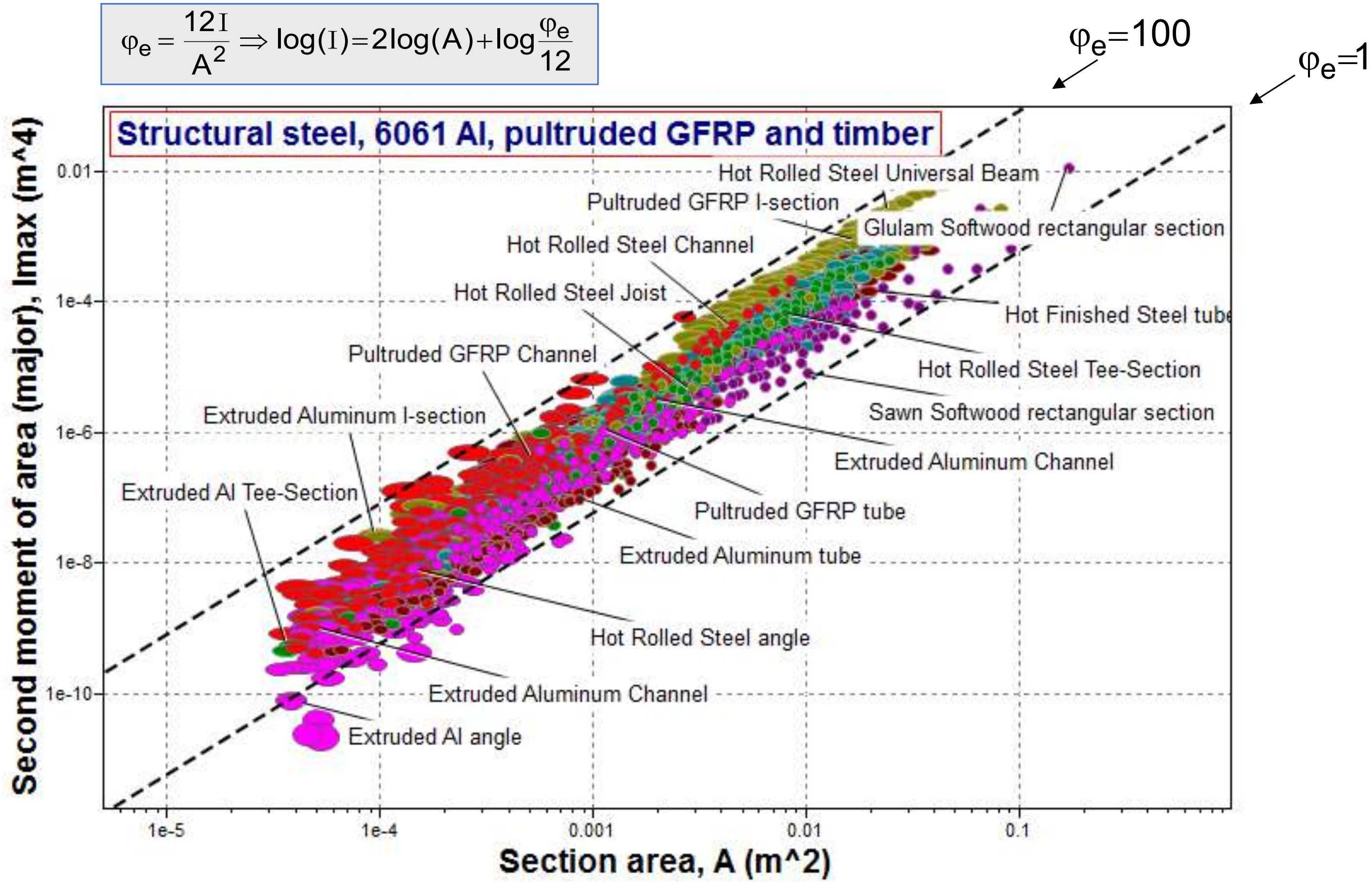


- The "shape efficiency" is the amount of material needed to carry the load. It is measured by the shape factor, φ.

- Each of these is roughly 10 times stiffer in bending than a solid square section of the same cross-sectional area

What values of φ_e exist in reality?

$$\varphi_e = \frac{12I}{A^2} \Rightarrow \log(I) = 2\log(A) + \log\frac{\varphi_e}{12}$$



- Theoretical limit $$\varphi_e \approx 2\sqrt{\frac{E}{\sigma_y}}$$

Modulus

Yield strength

E= Young's modulus

σy= Yield strength (for polymers and composites, we use an approximate failure strength) s)

Material	Young's Modulus $E(GPa)$	Yield Strength σ_y(MPa)	φ_e
Steels	~ 200	$\sim 250 - 1000$	$\sim 9 - 28$
Aluminum Alloys	~ 70	$\sim 100 - 500$	$\sim 6 - 14$
GFRP (Glass Fiber)	$\sim 20 - 40$	$\sim 50 - 300$	$\sim 5 - 18$
CFRP (Carbon Fiber)	$\sim 70 - 150$	$\sim 500 - 1000$	$\sim 5 - 11$
Unreinforced Polymers	$\sim 1 - 5$	$\sim 10 - 100$	$\sim 2 - 6$
Woods	$\sim 5 - 20$	$\sim 20 - 100$	$\sim 3 - 9$

Limits for Shape Factors φ_e and φ_f

- There is an upper limit to shape factor for each material

Material	Max φ_e
Steels	65
Aluminum alloys	44
GFRP and CFRP	39
Unreinforced polymers	12
Woods	8
Elastomers	<6
Other materials	...can calculate

- Theoretical limit $$\varphi_e \approx 2\sqrt{\frac{E}{\sigma_y}}$$

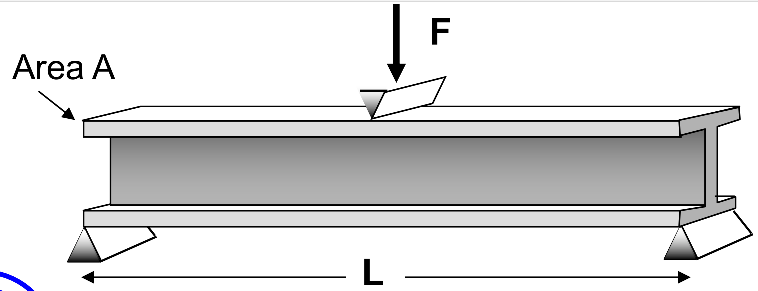
Modulus

Yield strength

- Refined Limit set by: (a) manufacturing constraints
  (b) local buckling

GRANTA

Function	*Beam* (shaped section).
Constraint	*Bending stiffness* = S:



$$S = \frac{CEI}{L^3}$$

I is the second moment of area:

$$\varphi_e = 12\frac{I}{A^2} \qquad A = \left(\frac{12I}{\varphi_e}\right)^{1/2}$$

Objective *Minimise mass*, m, where:

$$m = AL\rho$$

m = mass
A = area
L = length
ρ = density
b = edge length
S = stiffness
I = second moment of area
E = Youngs Modulus

$$m = \left(\frac{12\,S\,L^5}{C}\right)^{1/2}\left(\frac{\rho}{(\varphi_e E)^{1/2}}\right)$$

Chose materials with smallest $\left(\frac{\rho}{(\varphi_e E)^{1/2}}\right)$

- Materials for stiff, *shaped* beams of minimum weight
- Fixed shape (φ_e fixed): choose materials with low $\dfrac{\rho}{E^{1/2}}$
- Shape φ_e a variable: choose materials with low $\dfrac{\rho}{(\varphi_e E)^{1/2}}$

Material	ρ, Mg/m³	E, GPa	$\varphi_{e,max}$	$\rho / E^{1/2}$	$\rho / (\varphi_{e,max} E)^{1/2}$
1020 Steel	7.85	205	65	0.55	0.068
6061 T4 Al	2.70	70	44	0.32	0.049
GFRP	1.75	28	39	0.35	0.053
Wood (oak)	0.9	13	8	0.25	0.088

- Commentary: Fixed shape (up to φ_e = 8): wood is best

  Maximum shape ($\varphi_e = \varphi_{e,max}$): Al-alloy is best

  Steel recovers some performance through high $\varphi_{e,max}$

- If two materials have the *same* shape, the standard indices for bending (eg $\rho / E^{1/2}$) guide the choice.

- If materials can be made -- or are available -- in different shapes, then indices which include the shape (eg $\rho / (\varphi E)^{1/2}$) guide the choice.

PART4

- Environnemental impact

Eco-properties	Energy required per unit mass to produce material (embodied energy)	H_m	MJ/kg
	CO_2 footprint (CO_2 mass produced per unit mass of material produced)	CO_2	kg/kg

GHG footprint per mass

- Usually in databases :

Primary production CO2 footprint (Eco)
Processing CO2 footprint (Ecp)
Recycling CO2 footprint (Ecr)
Recycle fraction in current supply (Fr)

Ec= Fr*Ecr + (1-Fr)*Eco + Ecp
-> enables to take into account multiple stages of the life cycle.

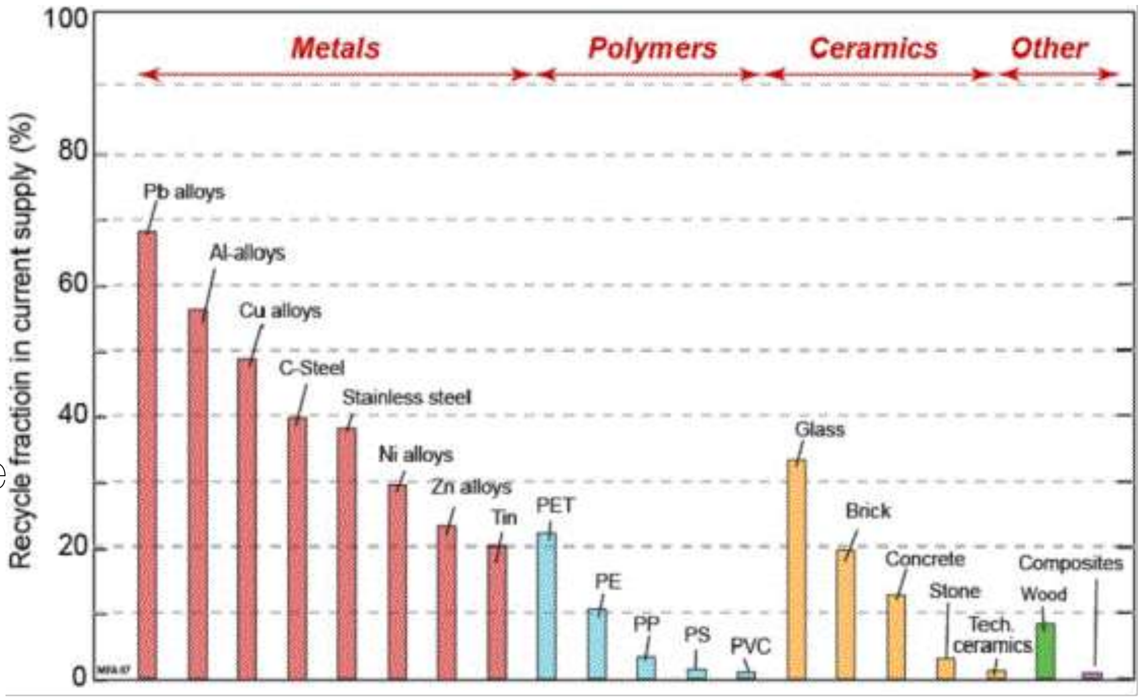


Figure 1. The fractional contribution of recycled material to current consumption. For metals, the contribution is large; for polymers, small (2005/06 data).

GHG footprint per mass

- Example for Aluminum in CES Edupack

Primary production energy, CO2 and water

Embodied energy, primary production	ⓘ	* 45	-	49,6	MJ/kg
CO2 footprint, primary production	ⓘ	* 3,46	-	3,81	kg/kg
Water usage	ⓘ	* 135	-	150	l/kg

Recycling and end of life

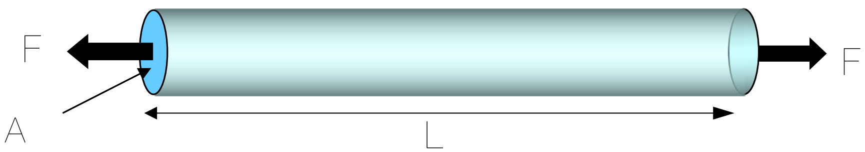
Recycle	ⓘ	✓			
Embodied energy, recycling	ⓘ	* 10,8	-	12	MJ/kg
CO2 footprint, recycling	ⓘ	* 0,85	-	0,94	kg/kg
Recycle fraction in current supply	ⓘ	52,3	-	57,8	%
Downcycle	ⓘ	✓			
Combust for energy recovery	ⓘ	✗			
Landfill	ⓘ	✓			
Biodegrade	ⓘ	✗			

Processing energy, CO2 footprint & water

Casting energy	ⓘ	* 10,9	-	12,1	MJ/kg
Casting CO2	ⓘ	* 0,818	-	0,904	kg/kg
Casting water	ⓘ	* 20,6	-	31	l/kg
Roll forming, forging energy	ⓘ	* 15,6	-	17,2	MJ/kg
Roll forming, forging CO2	ⓘ	* 1,17	-	1,29	kg/kg
Roll forming, forging water	ⓘ	* 8,2	-	12,3	l/kg
Extrusion, foil rolling energy	ⓘ	* 30,8	-	34,1	MJ/kg
Extrusion, foil rolling CO2	ⓘ	* 2,31	-	2,56	kg/kg
Extrusion, foil rolling water	ⓘ	* 14,7	-	22,1	l/kg
Wire drawing energy	ⓘ	* 115	-	127	MJ/kg
Wire drawing CO2	ⓘ	* 8,61	-	9,52	kg/kg
Wire drawing water	ⓘ	* 43,3	-	64,9	l/kg
Metal powder forming energy	ⓘ	* 37,5	-	41,4	MJ/kg
Metal powder forming CO2	ⓘ	* 3	-	3,32	kg/kg
Metal powder forming water	ⓘ	* 40,9	-	61,4	l/kg
Vaporization energy	ⓘ	* 1,09e4	-	1,2e4	MJ/kg
Vaporization CO2	ⓘ	* 815	-	901	kg/kg
Vaporization water	ⓘ	* 4,53e3	-	6,8e3	l/kg
Coarse machining energy (per unit wt removed)	ⓘ	* 2,77	-	3,06	MJ/kg
Coarse machining CO2 (per unit wt removed)	ⓘ	* 0,207	-	0,229	kg/kg
Fine machining energy (per unit wt removed)	ⓘ	* 23,4	-	25,8	MJ/kg
Fine machining CO2 (per unit wt removed)	ⓘ	* 1,75	-	1,94	kg/kg

Example of carbon performance index

- We want to minimize the GHG **GreenHouse Gas** footprint of a bar of length L under a load F, while staying in the elastic domain. The section and the material are free.



- P : $1/CO2 = 1/(Ec*\boldsymbol{\rho}*L*A)$ (1)
- F : $F/A = \sigma < \boldsymbol{\sigma_y}$

(stay in the elastic domain)

We go to the limit=> $A= F/\boldsymbol{\sigma_y}$ (2)

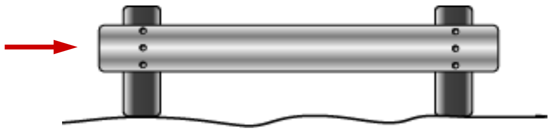
- (1)+(2) : $P = \boldsymbol{\sigma_y} /(Ec*\boldsymbol{\rho}*L*F)$
- $P = \underbrace{1/F}_{f_1(F)} * \underbrace{1/L}_{f_2(G)} * \underbrace{\boldsymbol{\sigma_y} /(Ec*\boldsymbol{\rho})}_{f_3(M)}$

m = mass
A = section
L = length
ρ = density
σ_y= yield strength
Ec= GHG footprint per mass (kg_{eqCO2}/kg)

=> We will choose the material maximizing the index $M=\boldsymbol{\sigma_y} /(Ec*\boldsymbol{\rho})$

Materials for crash barriers

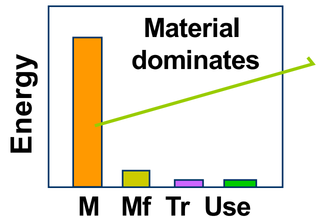
Static barrier



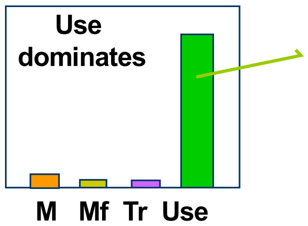
Mobile barrier

Function: *Absorb impact, transmit load to energy-absorbing units or supports*

Dominant phase of life:



Minimize embodied energy



Minimize mass

Criterion: *Bending strength per unit embodied energy* | *Bending strength per unit mass*

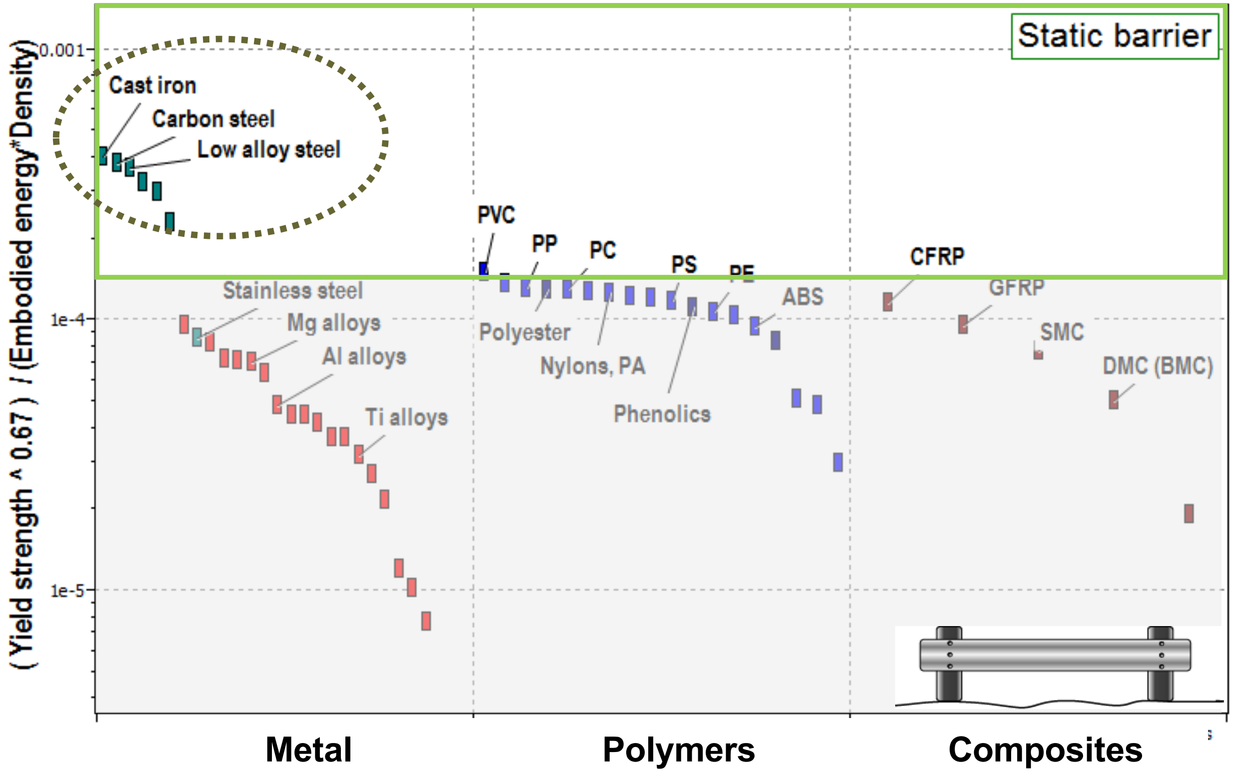
Index:

$$\frac{\sigma_y^{2/3}}{H_m \, \rho}$$

$$\frac{\sigma_y^{2/3}}{\rho}$$

Static barrier: the index as bar chart

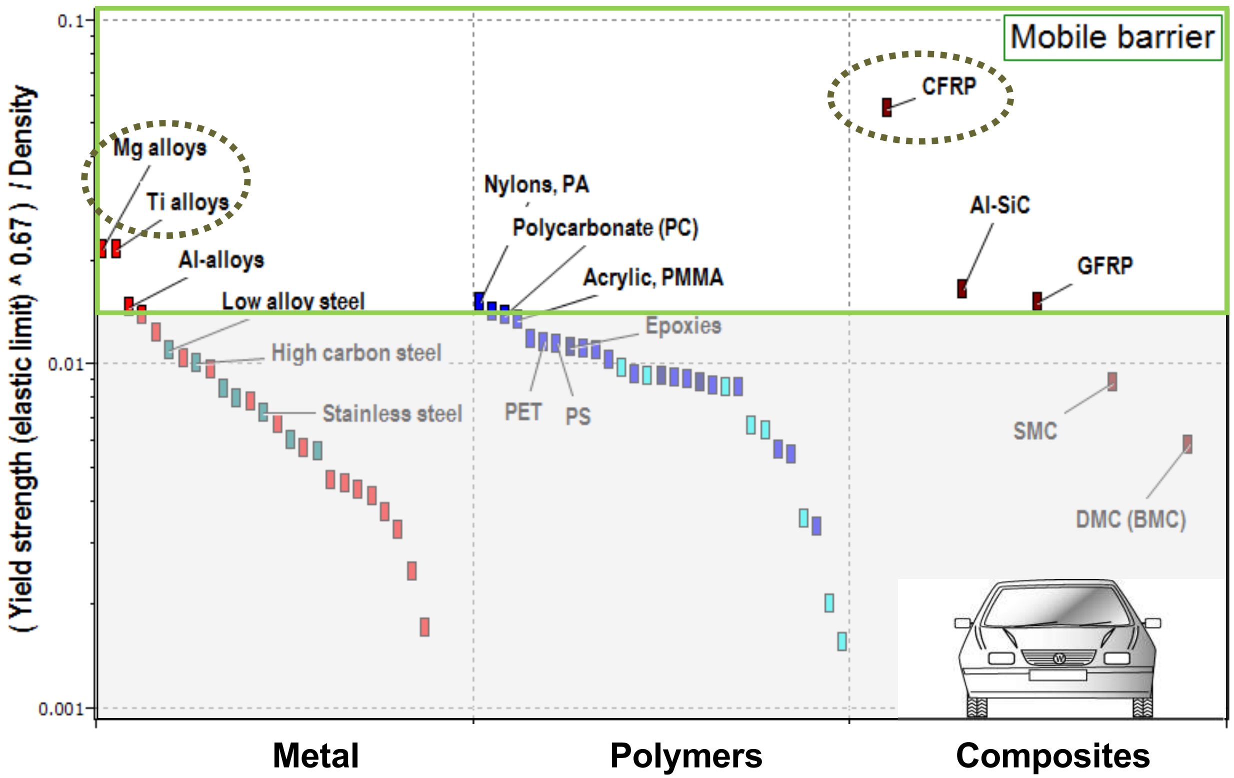
$$\frac{\sigma_y^{2/3}}{H_m\,\rho}$$



Selected materials: *Cast irons, steels*

Mobile barrier: the index as bar chart

$$\frac{\sigma_y^{2/3}}{\rho}$$



Selected materials: *CFRP, Mg alloys, Ti alloys, Al alloys*

So what?

- Eco-informed material choice is part of the eco-design process
- An Eco Audit identifies the most damaging phase of life and identifies strategies for overcoming it
- Systematic strategies, using material indices, optimize material choice to minimize life energy
- CES EduPack allows the strategy to be implemented and documents the steps taken to minimize eco-impact.

PART5

- Multiobjective optimization

Optimization with multiple objectives

- Ashby index are well adapted to cases where there is only one objective function in which appear multiple quantities, however in ecodesign we generally have multiple objectives…
- A first step is to make the difference between objectives (to be minimized/maximized) and constraints (to filter).

Typical constraints

The material must be:

- Electrical conductor
- Transparent…

And be enough:

- Stiff
- Strong

And be able to be:

- Cast
- Welded…

=> Easy to take into account

Typical objectives

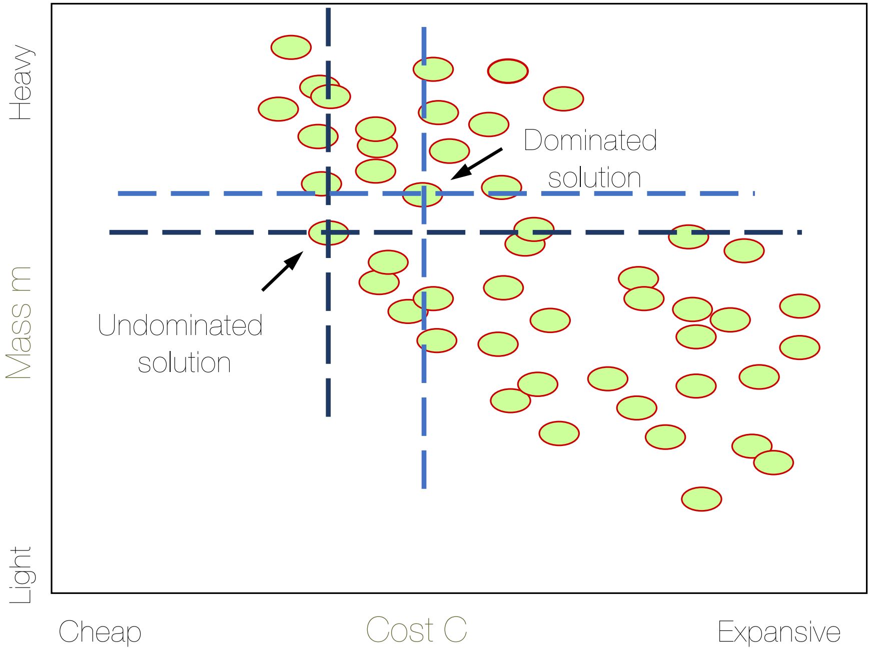
Minimize

- Mass m (satellite components)
- Volume (smartphones)
- Energy consumption (refrigerator)
- GHG footprint (everything)
- Grey energy (materials)
- Cost C (everything)

=> Easy if only one; complex otherwise

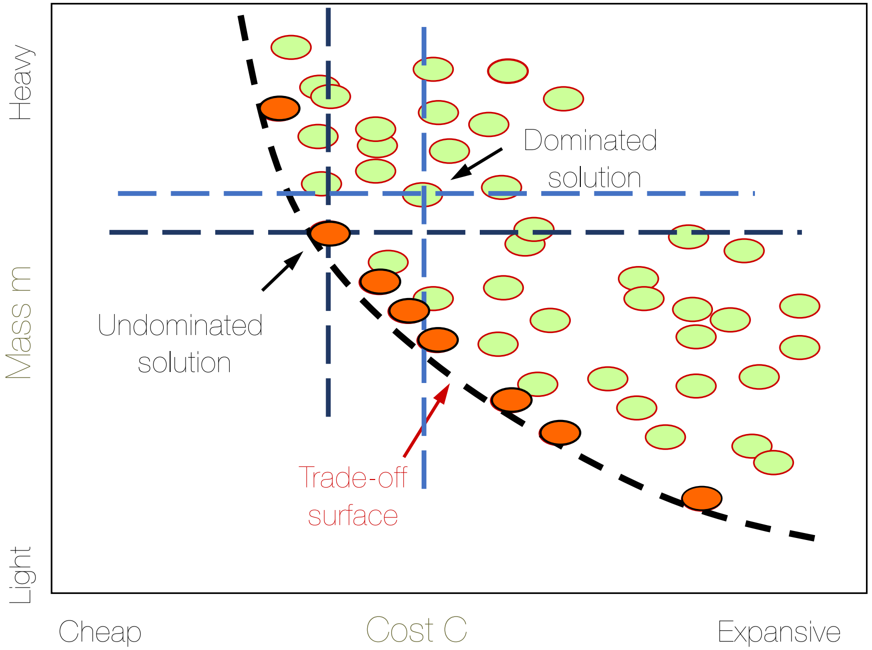
Mass-cost example

- Solution : respects the constraints (e.g. transparent, water resistant,…)
- Diagram with both indexes to be minimized
- Get rid of dominated solutions



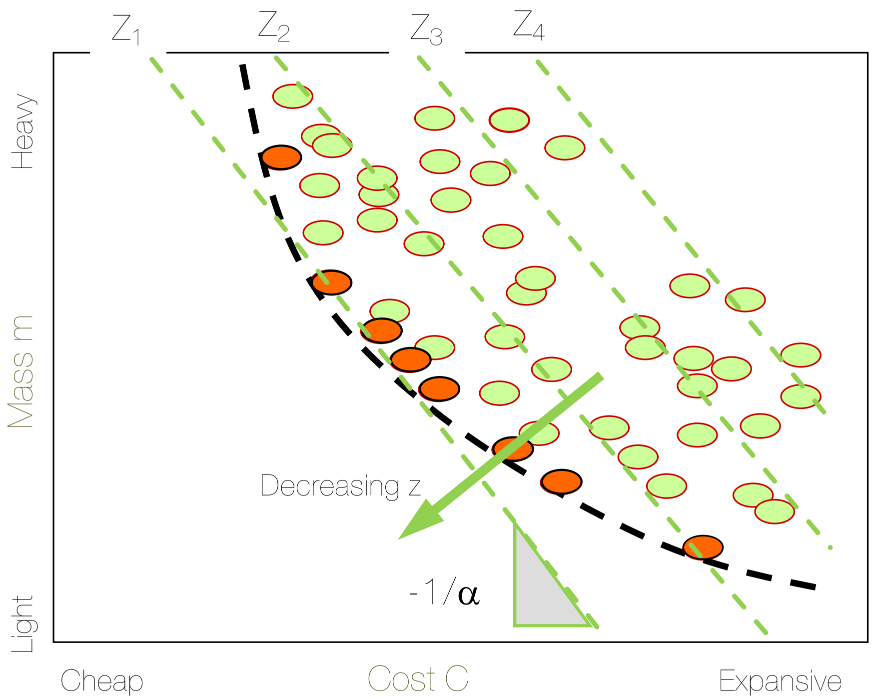
Mass-cost example

- Solution : respects the constraints (e.g. transparent, water resistant,…)
- Diagram with both indexes to be minimized
- Get rid of dominated solutions
- We get the Pareto front



Penalty function

- Give a relative importance to the objectives to choose a solution.
- Z = C + α * m : new objective
- With linear axis, lines of equation
  m = 1/α * Z - 1/α * C
  have the same objective value.



Penalty function

- Give a relative importance to the objectives to choose a solution.
- Z = C + α * m : new objective
- With linear axis, lines of equation
  m = 1/α * Z - 1/α * C

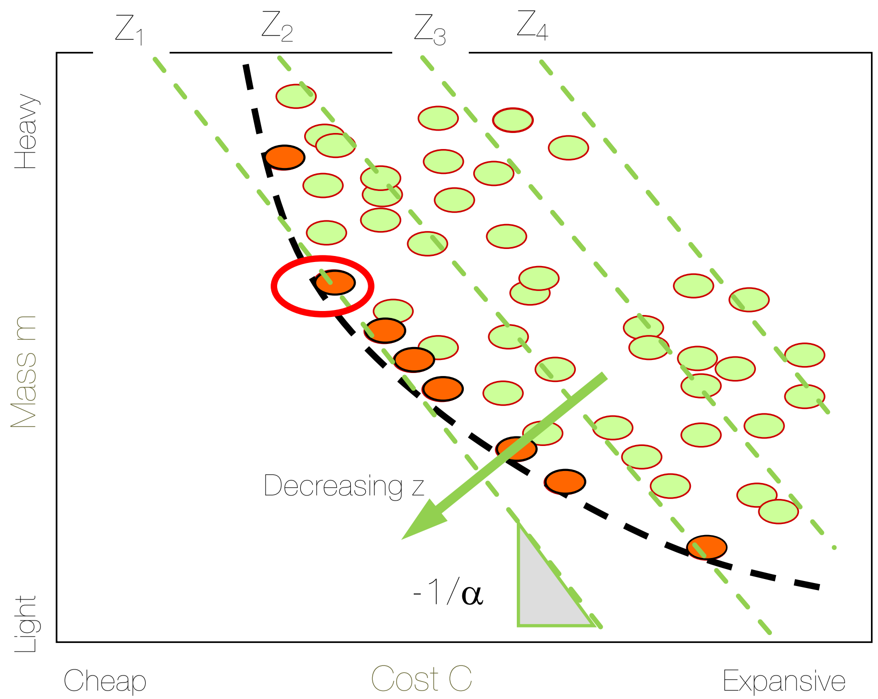
have the same objective value.

⇒ α is important

How much is α for a low-cost bike?

A : 2€/kg C : 20€/kg

B : 5€/kg D : 100€/kg



Penalty function

How much is a for a low-cost bike?

A : 2€/kg

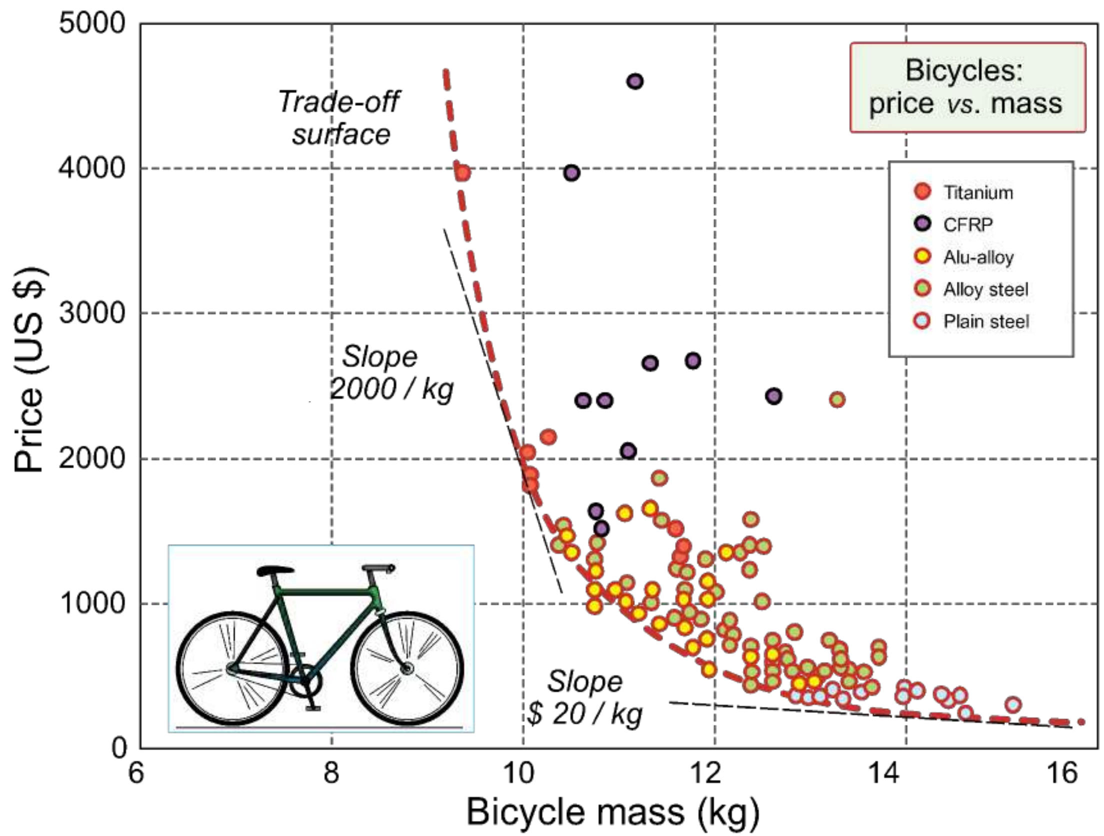
C : 20€/kg

B : 5€/kg

D : 100€/kg

=> Titanium / aluminum / steel?

: steel



Penalty function in transports

- For transports,   mass -> fuel consumption -> cost
- Costs over life time = acquisition cost (proportional to C) + fuel costs (proportional to m)

$\Rightarrow$ $Z = C + \alpha * m$

How much is α for a car?

A : 4€/kg   C : 50€/kg

B : 15€/kg   D : 200€/kg

Penalty function in transports

- For transports, mass -> fuel consumption -> cost
- Costs over life time = acquisition cost (proportional to C) + fuel costs (proportional to m)

$\Rightarrow Z = C + a * m$

How much is a for a car?

A : 4€/kg C : 50€/kg
B : 15€/kg D : 200€/kg

How much is a for space applications?

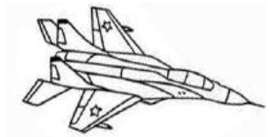
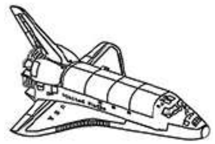
A : 100€/kg C : 2000€/kg
B : 500€/kg D : 8000€/kg

Penalty function in transports

- For transports,   mass -> fuel consumption -> cost
- Costs over life time = acquisition cost (proportional to C) + fuel costs (proportional to m)

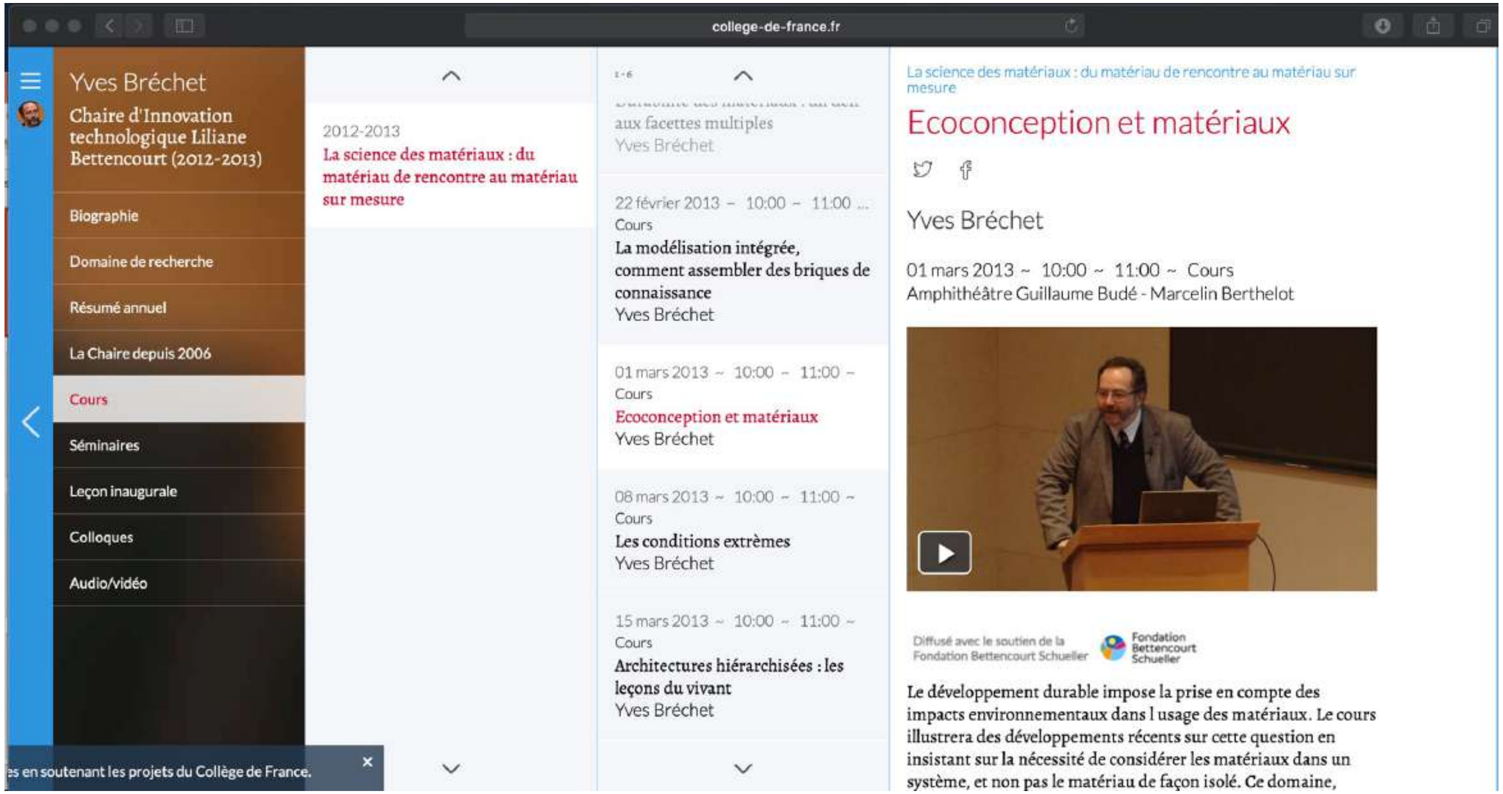
$\Rightarrow Z = C + \alpha * m$



	Steel	Steel / Alu	Alu / (composite)	Alu / Ti / composites	Composites
α ($/kg)	3 – 6	6 – 20	100 – 600	600 – 2,000 (?)	5,000 – 10,000

EN CONCLUSION



Eco Audit tool for rapid LCA

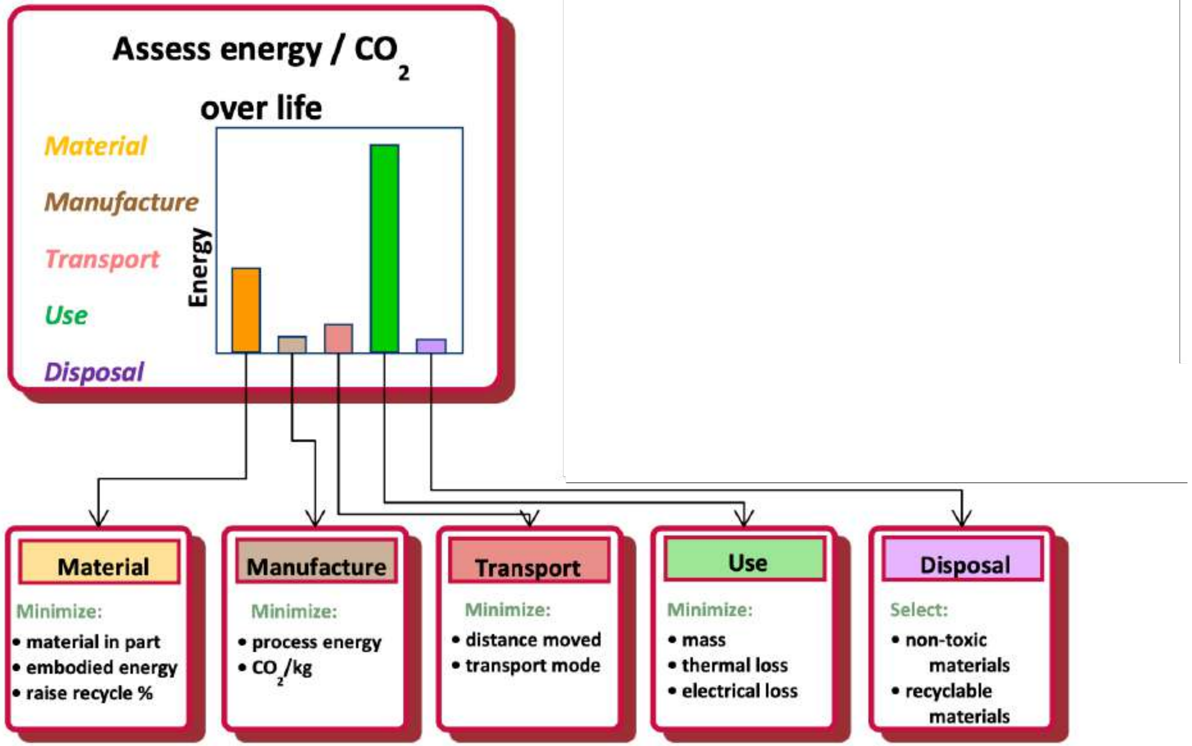
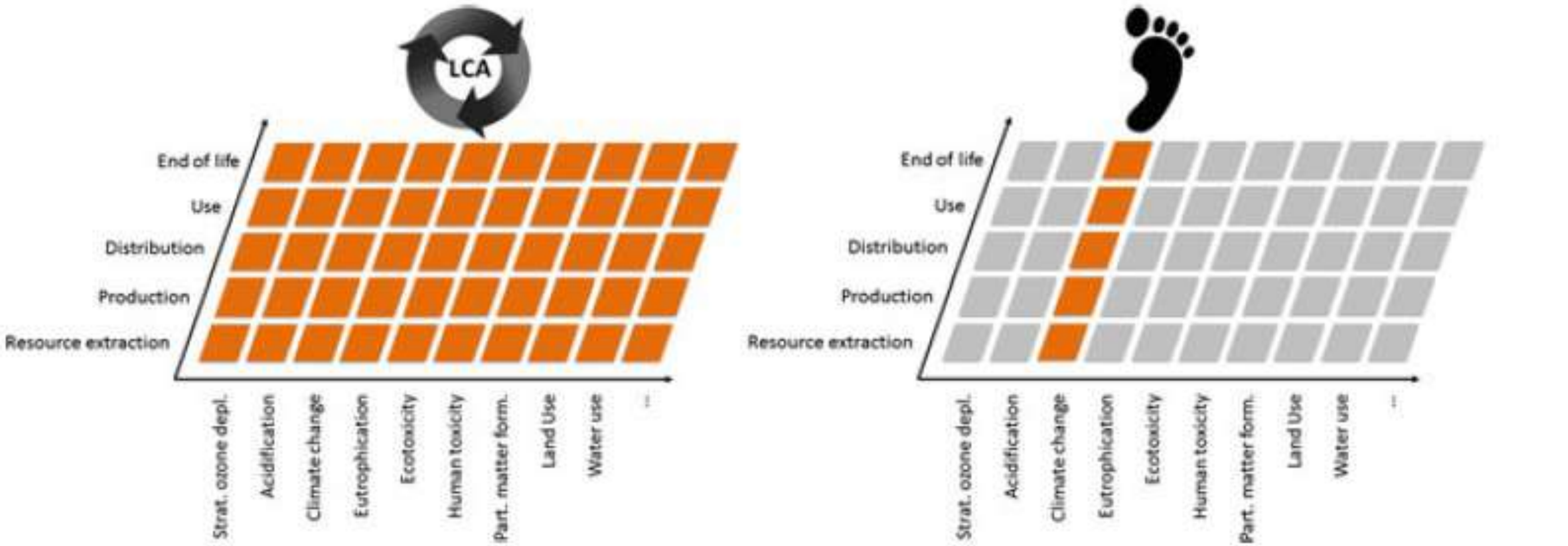


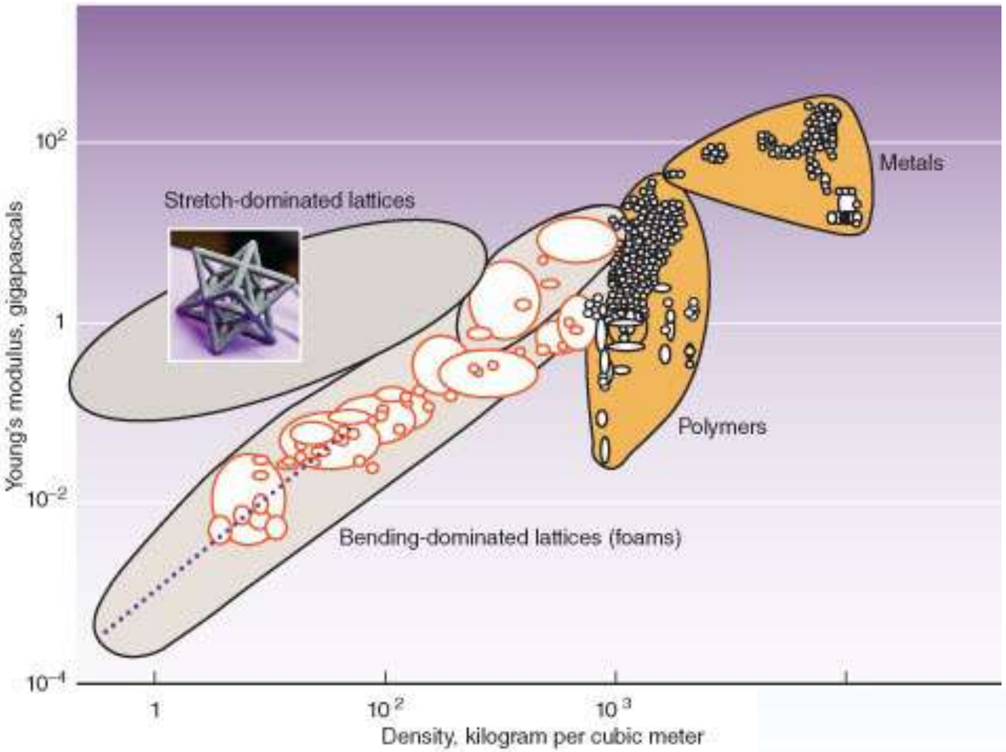
Figure 4: Environmental impact can be assessed for each life-stage of a product (Tip 3). Materials and process selection play an important role in determining environmental impacts and can be used in many eco design strategies (Tip 4.)

Global View with LCA



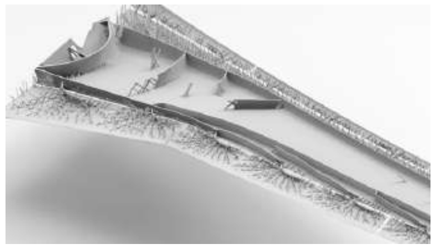
Research

AND TOMORROW?



(a) (b)

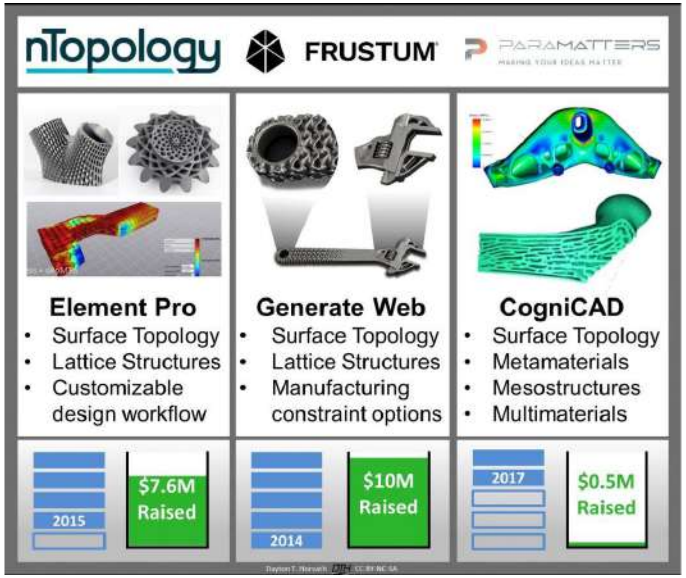
Chris Spadaccini (llnl,USA) "By controlling the architecture of a microstructure, we can create materials with previously unobtainable properties in the bulk form."



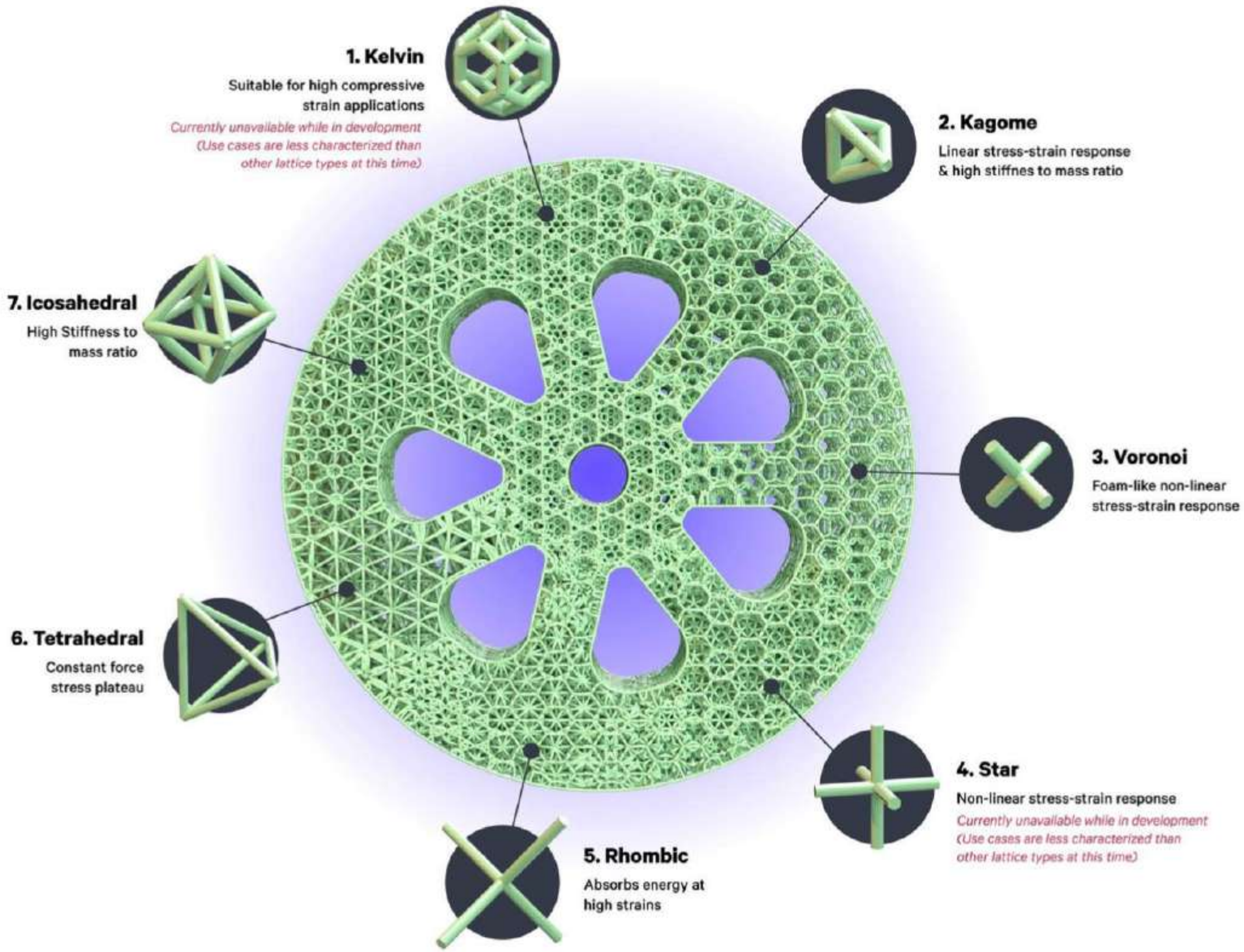
Software and algorithms for hierarchical design

Conventional CAD programs do not work well
New players are emerging

Examples:
- ntopology (see case studies): https://ntopology.com/
- additiveflow: https://www.additiveflow.com/
- Hyperganic
- ParaMatters: https://paramatters.com/
- Fusion 360 (Autodesk)




1: Carbon Design Engine™


1. Kelvin
Suitable for high compressive strain applications
Currently unavailable while in development (Use cases are less characterized than other lattice types at this time)
2. Kagome
Linear stress-strain response & high stiffnes to mass ratio
3. Voronoi
Foam-like non-linear stress-strain response
4. Star
Non-linear stress-strain response
Currently unavailable while in development (Use cases are less characterized than other lattice types at this time)
5. Rhombic
Absorbs energy at high strains
6. Tetrahedral
Constant force stress plateau
7. Icosahedral
High Stiffness to mass ratio

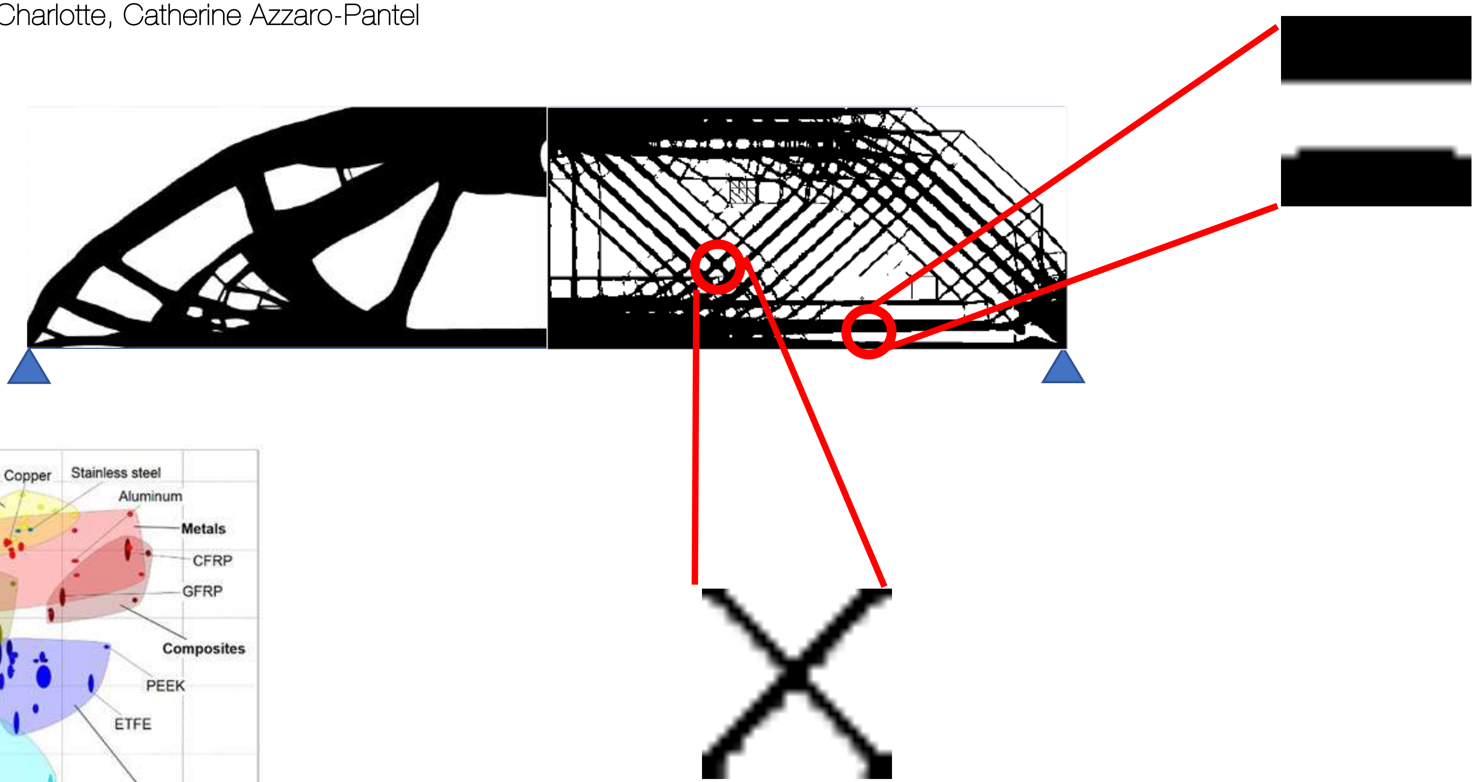
How to ECOdesign tomorrow's structures?

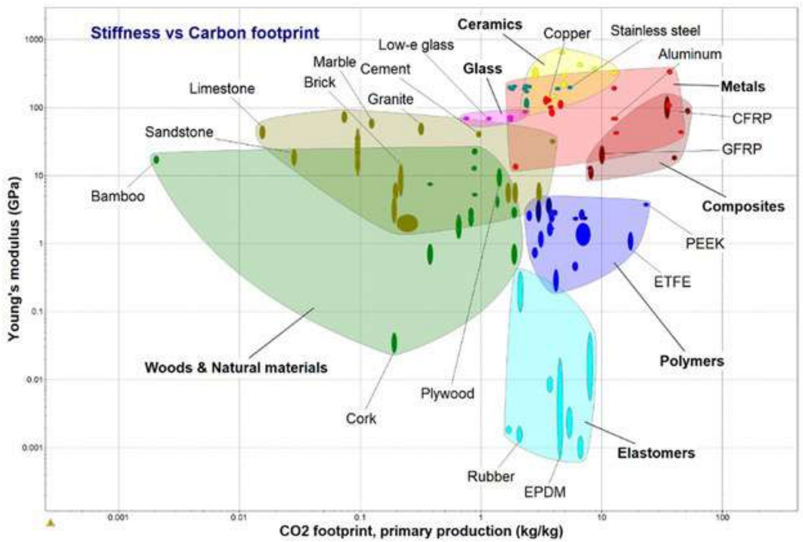
Prof. Joseph Morlier, Edouard Duriez, Miguel Charlotte, Catherine Azzaro-Pantel

#Mass vs CO2 footprint Minimization

#SIMP vs EMTO

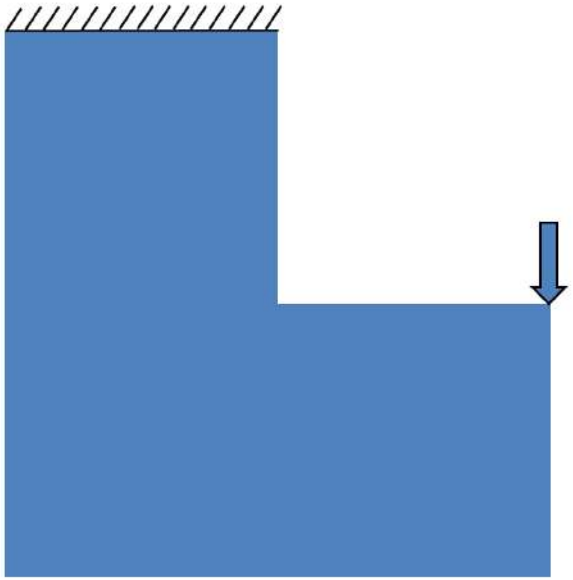
#3D printing process selection

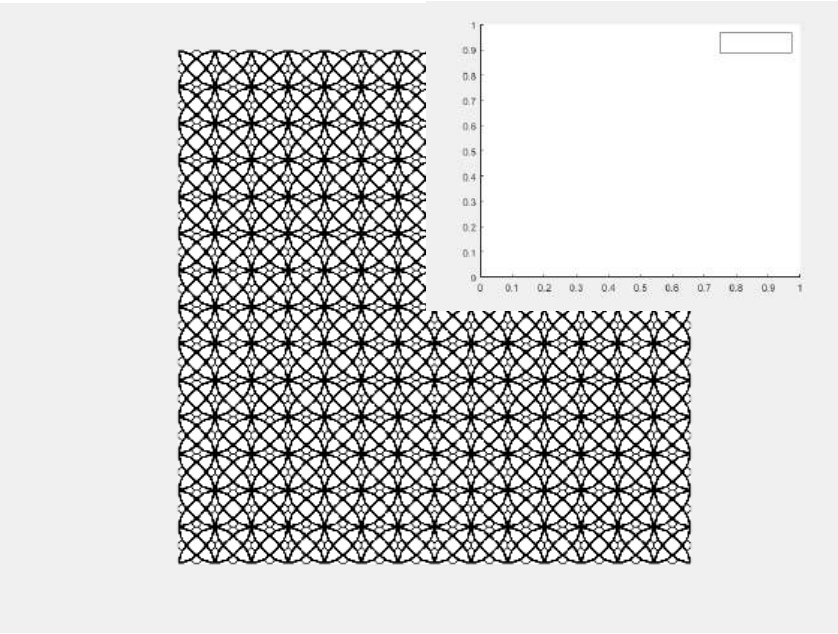




EMTO on L-shape (cellular /architectured materials)

Duriez, E., Morlier, J., Charlotte, M., & Azzaro-Pantel, C. (2021). A well connected, locally-oriented and efficient multi-scale topology optimization (EMTO) strategy. Structural and Multidisciplinary Optimization, 1-24.





https://github.com/mid2SUPAERO/EMTO

How to ECOdesign tomorrow's structures?

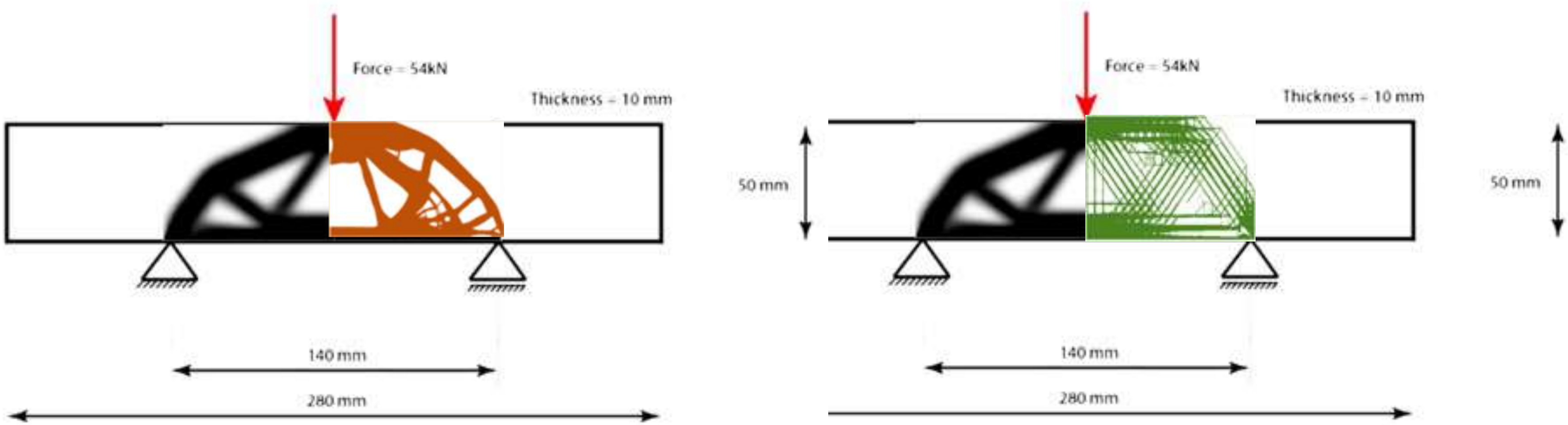
Prof. Joseph Morlier, Edouard Duriez, Miguel Charlotte, Catherine Azzaro-Pantel

#Our very First Results

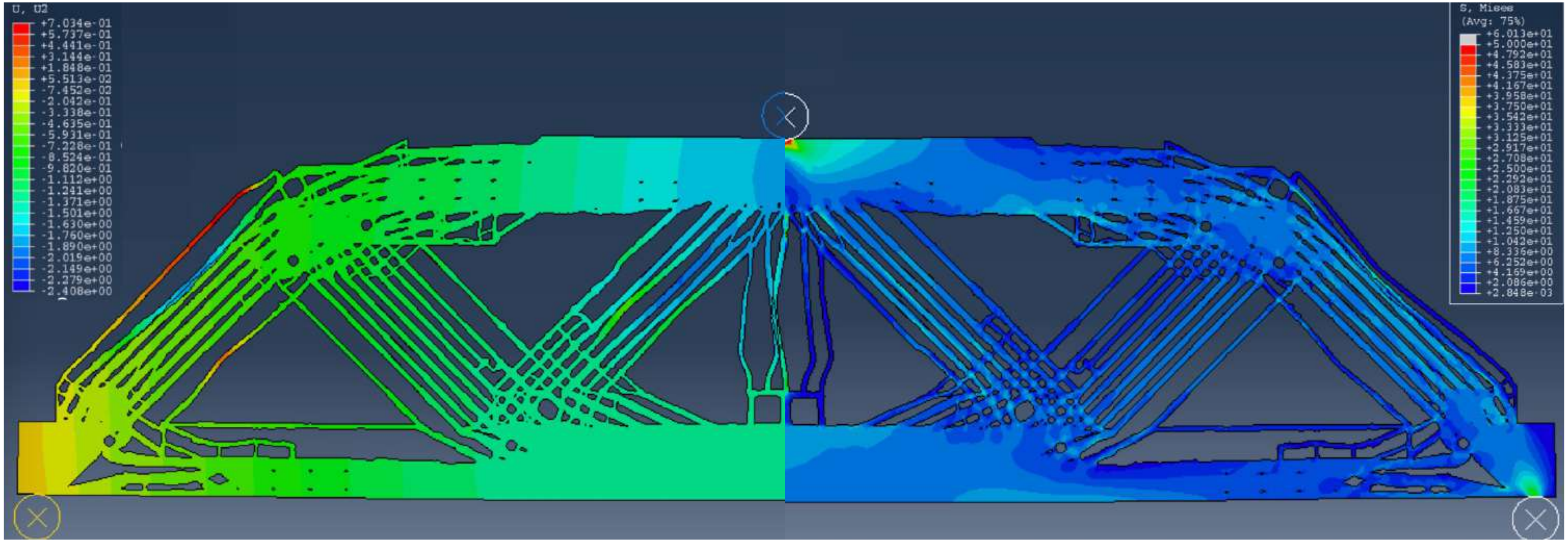
#SIMP vs ECOMA

Print it , test it

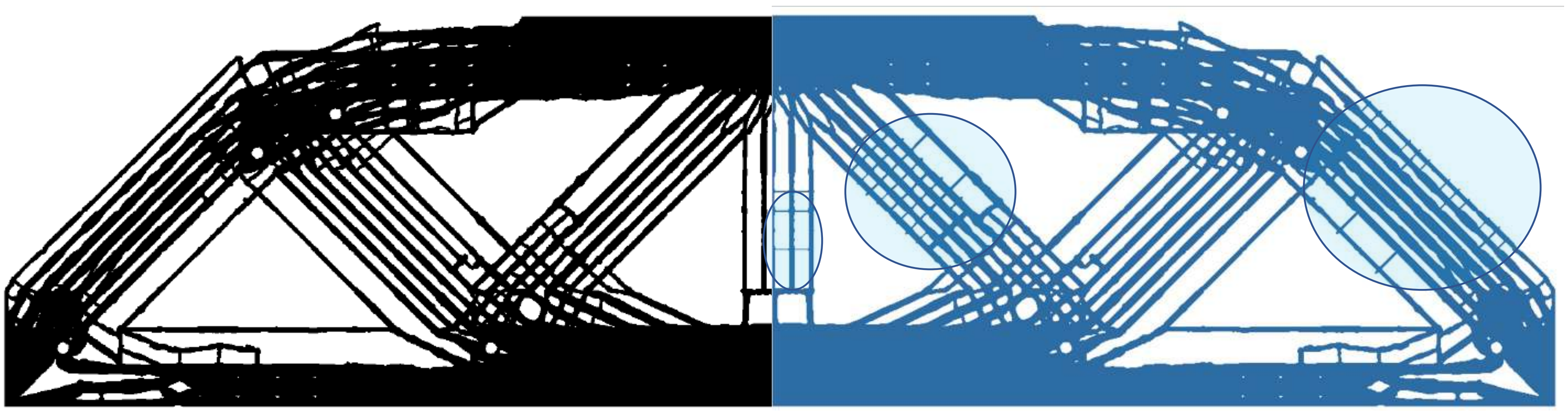
a)



Disp or Stress ?

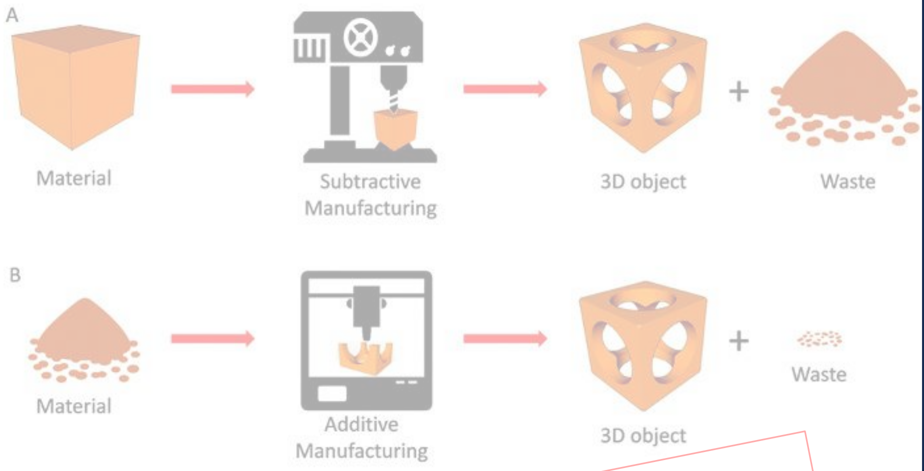


Do you see a difference (Left2Right)?

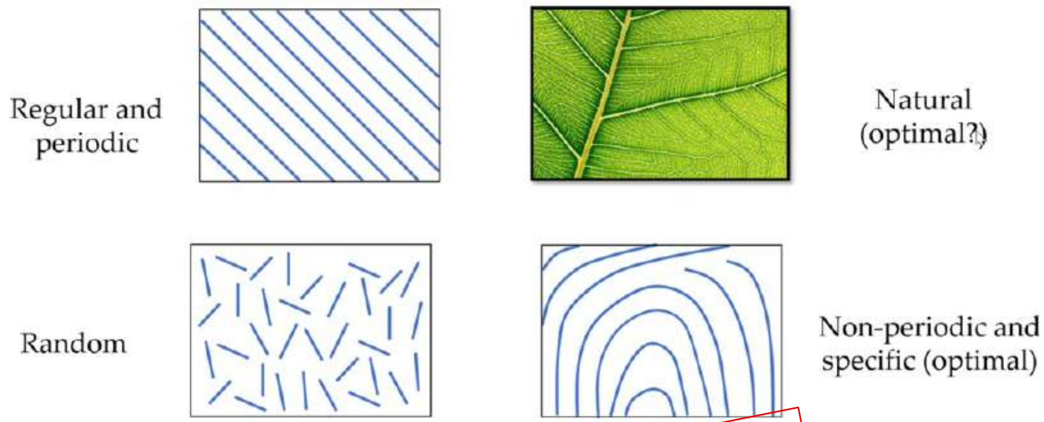


Process is AM, but WHY?

https://dfam.substack.com/p/dfam-education-in-2022



+Near 100% material utilization
+Recyclability, Buy to fly ratio
+LCA of 3D printing machine
+Monitoring

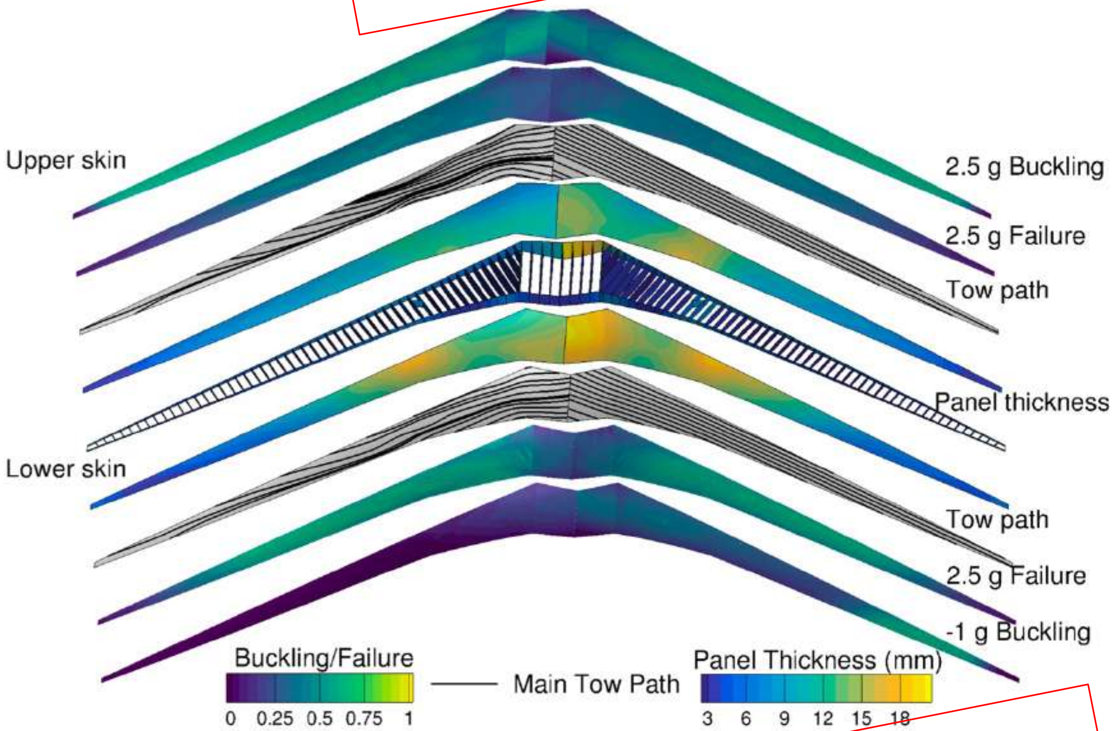


+ Automatic Fiber Placement + eco-fiber/resin selection
+Monitoring

Composites Fiber Placement as DVs

https://www.compositesworld.com/articles/tow-steering-part-2-the-next-generation

X* is a UHAR Wing ;)

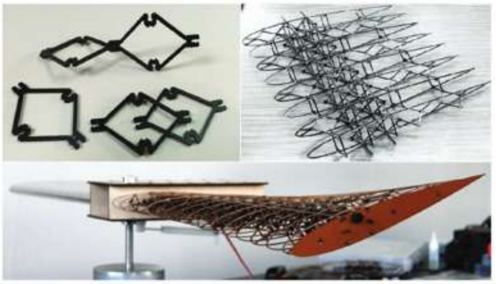


Brooks, T. R., Martins, J. R., & Kennedy, G. J. (2019). High-fidelity aerostructural optimization of tow-steered composite wings. Journal of Fluids and Structures, 88, 122-147.

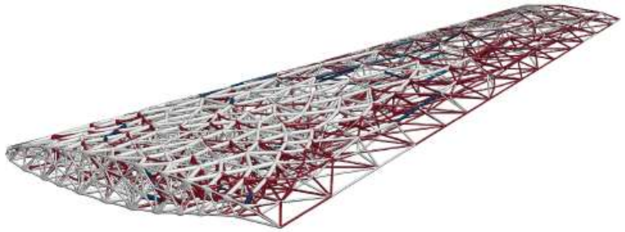
Brooks, T. R., Martins, J. R., & Kennedy, G. J. (2020). Aerostructural tradeoffs for tow-steered composite wings. Journal of Aircraft, 57(5), 787-799.

Lower wing mass,
Less fuel burn

Full wingbox concept



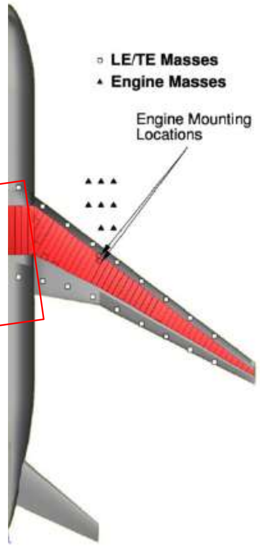
Jenett et al. (2017)



Opgenoord, M. M. and Willcox, K. E. (2018)

Cramer, N. B. et al. (2019)

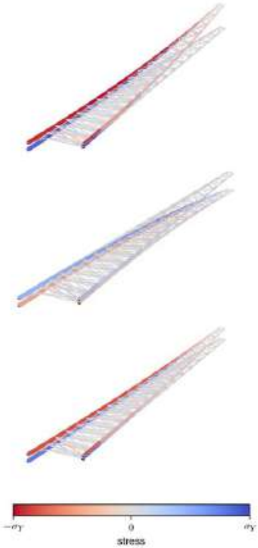
Thanks to Enrico Stragiotti and ONERA's



Wingspan, m : 58.76
MTOW, t : 297,55

3 load cases:

- +2.5 g manouver
- -1 g manouver
- Cruise with gust (+1.3 g)



Material	Aluminum alloy
E	69 GPa
σ_c	-270 MPa
σ_t	270 MPa
ρ	2.7 g/cm^3

Optimized mass = 21.342 t

-27.01% compared to 29.238 t (Fakhimi et al., 2021)

Table 6.3: Numerical results of the optimization of the CRM-315 model with three different maximum displacement constraints ($Z_{t,\ell} = 1\,\mathrm{m}$, $Z_{t,\ell} = 2\,\mathrm{m}$, $Z_{t,\ell} = 3\,\mathrm{m}$) and no maximum displacement constraints.

$Z_{t,\ell}$ [m]	1	2	3	–
V [m^3]	26.70	13.78	9.39	7.90
V [%]	4.421 %	2.283 %	1.556 %	1.309 %
Mass [t]	72.086	37.218	25.363	21.342
a_{max} [m^2]	0.615	0.293	0.197	0.198
C_{LC_1} [MJ]	1.05	1.96	2.79	3.23
C_{LC_2} [MJ]	0.37	0.71	1.04	1.28
C_{LC_3} [MJ]	0.26	0.47	0.67	0.76

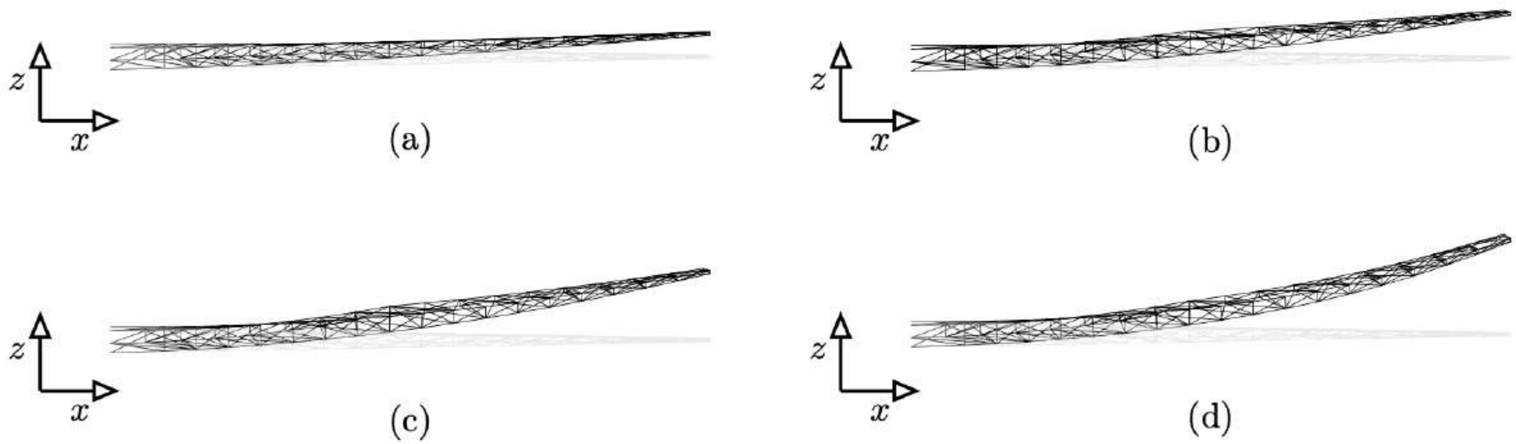


Figure 6.4: Undeformed (gray) and deformed (black) shapes of the optimized CRM-315 structures with a half wing span of 29.4 m for different values of maximum Z displacement $Z_{t,\ell}$ of the wing tip constraints for the LC_1 load case. (a) $Z_{t,\ell} = 1\,\mathrm{m}$; (b) $Z_{t,\ell} = 2\,\mathrm{m}$; (c) $Z_{t,\ell} = 3\,\mathrm{m}$; (d) no maximum displacement constraints.

Full wingbox concept

- Proof of concept of greener aerostructures with lattice wingbox
- Material as Design variable open new solutions:
- Who is the best?

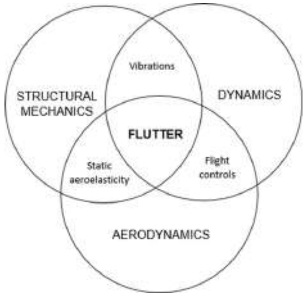
Material	Aluminium	Titanium	Steel	Pultruted CFRP
E	69 GPa	120 GPa	210 GPa	150 GPa
σ_c, σ_t	±270 MPa	±880 MPa	±355 MPa	+1200, −880 MPa
ρ	$2.7\,\mathrm{g\,cm^{-3}}$	$4.5\,\mathrm{g\,cm^{-3}}$	$7.8\,\mathrm{g\,cm^{-3}}$	$1.6\,\mathrm{g\,cm^{-3}}$

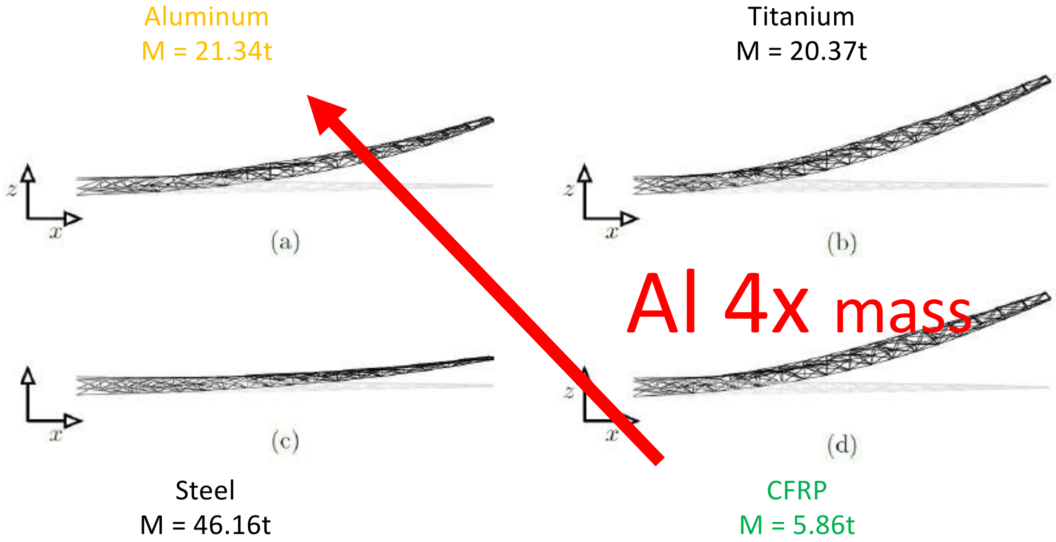
$$
\begin{aligned}
\min_{\substack{\boldsymbol{a},\boldsymbol{q}^0,\dots,\boldsymbol{q}^{N_p},\\ \boldsymbol{U}^0,\dots,\boldsymbol{U}^{N_p}}} \quad & V = \boldsymbol{\ell}^T \boldsymbol{a} \\
\text{s.t.} \quad & \boldsymbol{B}\boldsymbol{q}^p = \boldsymbol{f}^p && \forall p \in [0,\dots,N_p] \\
& \boldsymbol{q}^p = \frac{\boldsymbol{a}E}{\boldsymbol{\ell}} \boldsymbol{b}^T \boldsymbol{U}^p && \forall p \in [0,\dots,N_p] \\
& \boldsymbol{q}^p \geq -\frac{s\boldsymbol{a}^2}{\boldsymbol{\ell}^{*2}} && \forall p \in [0,\dots,N_p] \\
& -\sigma_c \boldsymbol{a} \leq \boldsymbol{q}^p \leq \sigma_t \boldsymbol{a} && \forall p \in [0,\dots,N_p] \\
& 0 \leq \boldsymbol{a} \leq \frac{4\pi\boldsymbol{\ell}^2}{\lambda_{\max}}.
\end{aligned}
$$

Stragiotti et al, (2024)

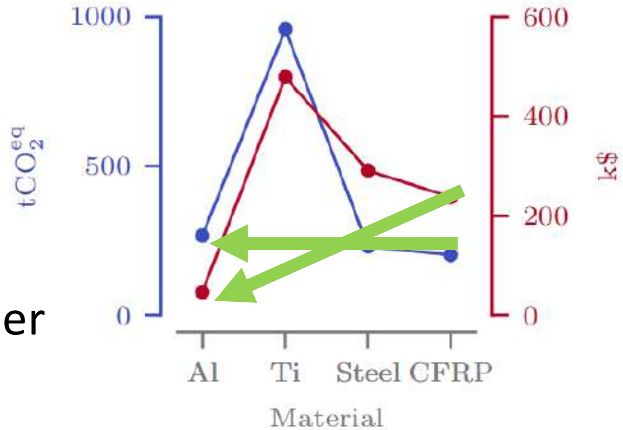
Full wingbox concept

- Lattice (truss) that respects all constraints
- Flutter ? Gust?





Material	Aluminium	Titanium	Steel	Pultruted CFRP
V [m^3]	7.90	4.53	5.88	3.67
V [%]	1.309 %	0.749 %	0.974 %	0.607 %
Mass [t]	21.342	20.372	46.168	5.868
a_{max} [m^2]	0.198	0.088	0.153	0.086
C_{LC_1} [MJ]	3.23	4.88	1.33	4.39
C_{LC_2} [MJ]	1.28	1.94	0.53	1.73
C_{LC_3} [MJ]	0.76	1.15	0.31	1.03
Z_t [m]	4.10	5.97	1.70	5.31
Cost [tCO_2^{eq}]	266.7	957.5	230.8	202.4
Cost [k$]	46.9	478.7	290.8	237.6



But finally
Al has approx same CO2pp footprint but cheaper