Conception optimale pour l'ingénieur (Aerospace)

C2 by Prof. J. Morlier

2025

ISAE
Institut Supérieur de l'Aéronautique et de l'Espace
SUPAERO

AU PROGRAMME
Python based

lundi 31 mars 2025			
		09h15 - 12h45	MORLIER Joseph
		14h00 - 16h15	MORLIER Joseph
mardi 01 avril 2025			
		09h15 - 12h45	MORLIER Joseph
		14h00 - 16h15	MORLIER Joseph
mercredi 02 avril 2025			
		09h15 - 12h45	MORLIER Joseph MURADÁS ODRIOZOLA Daniel
		14h00 - 16h15	MAS COLOMER JOAN MURADÁS ODRIOZOLA Daniel
jeudi 03 avril 2025			
		09h15 - 12h45	MAS COLOMER JOAN MURADÁS ODRIOZOLA Daniel

Intro: Sustainable Aviation (Materials) With Both Eyes Open

Design optimization 1: constrained optimization, MOO, Sensibility with examples

Project DO 1 2 3

Topology Optimization with examples

Material ecoselection, Ashby Diagram and more

Projet DO 1 2 3

Wrap up and demo from students

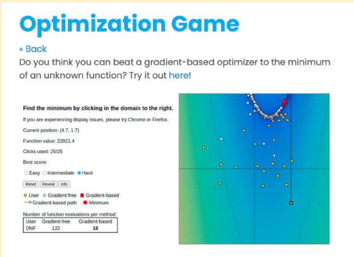
Intro to MDAO
Static Aeroelastic problem is a MDAO problem

Airbus PROJECT by TEAM of 3 (marked*)

vendredi 04 avril 2025		**ORAL MARKED***	
		09h15 - 11h30	MORLIER Joseph MURADÁS ODRIOZOLA Daniel

KEEP STUDENTS ACTIVE

First Exercice



https://mdolab.engin.umich.edu/assets/optimizationGame/

SM305 49

HOW?

Example of Optimizers

CHALLENGE #1 On Rosenbrock function minimization

https://colab.research.google.com/drive/1rGHIklpVM4Gqy9eq1Y10fIQW9umIjeEx

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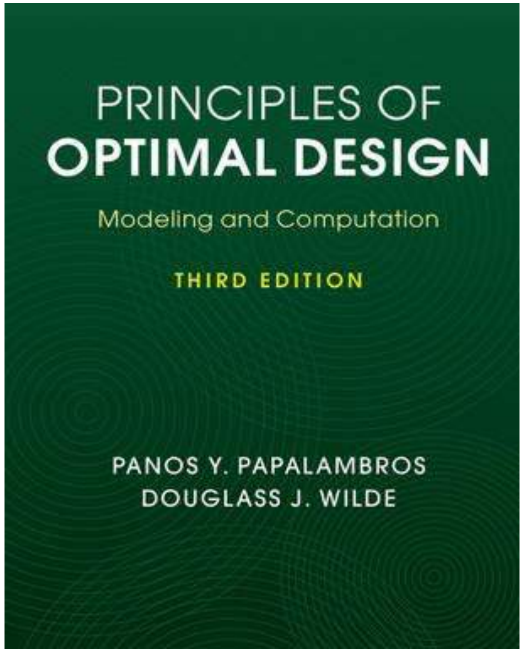
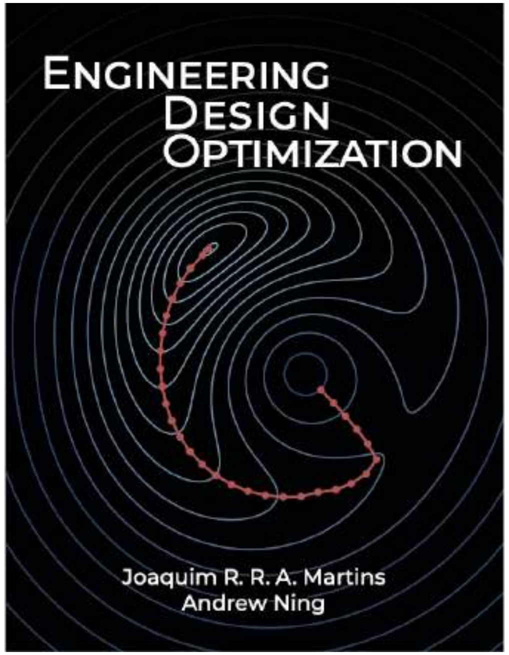
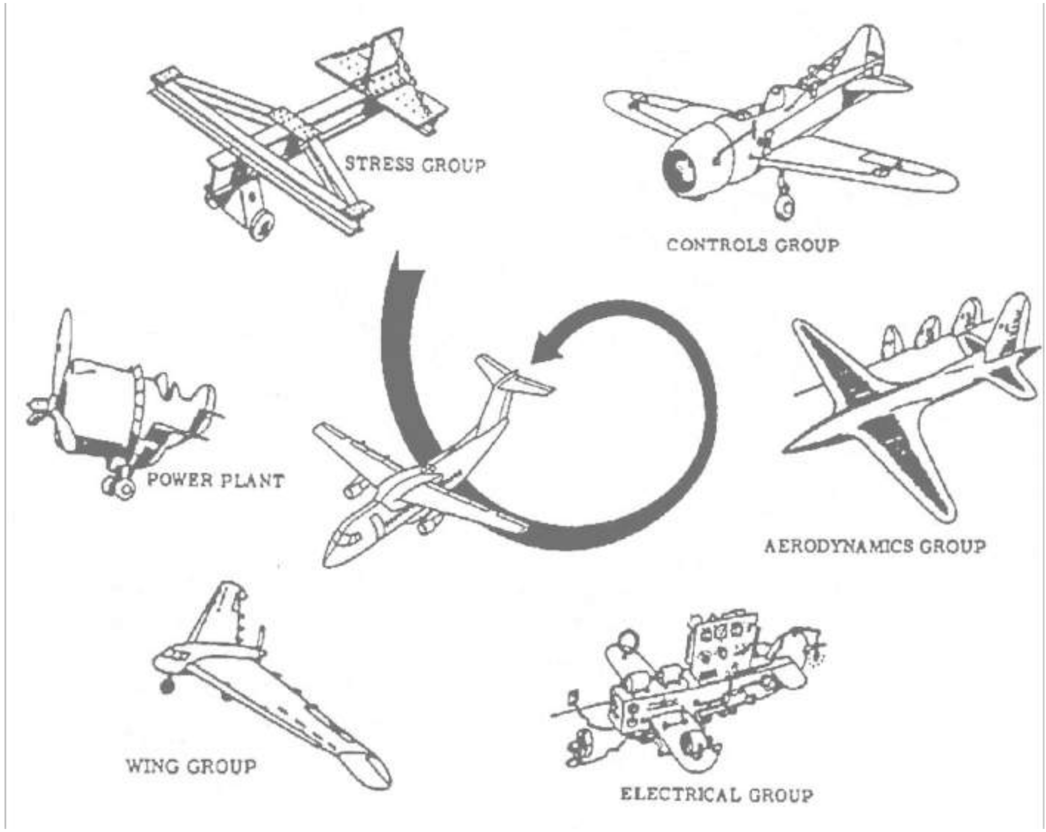
Part1 : Constrained Optimization

A recap of past courses (and more)?

Good Starting Point (x0)



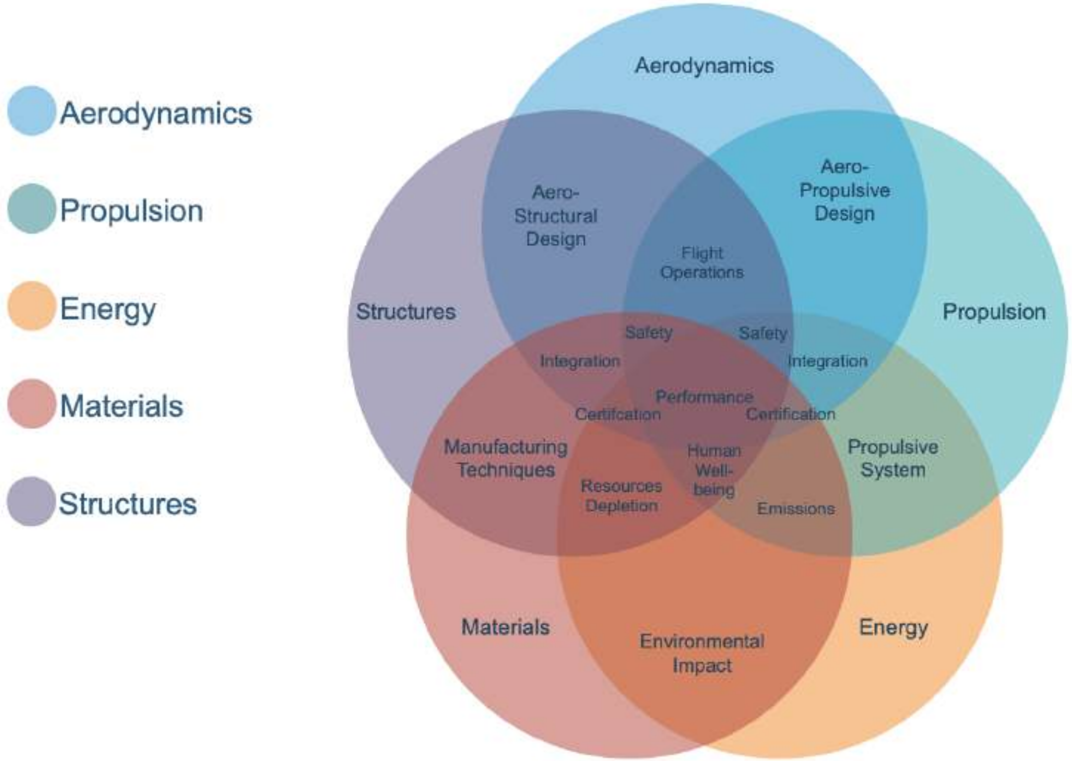




Multidisciplinary Design Optimization (MDO)

Venn Diagram

To illustrate the multidisciplinary nature of designing an aircraft and how the fundamentals disciplines are interlinked we can recur to a Venn diagram



F. **Afonso**, et al., "Strategies towards a more sustainable aviation: a systematic review", *Progress in Aerospace Sciences*, Vol. 137, 100878, 2023, https://doi.org/10.1016/j.paerosci.2022.100878

A Curated list of (mostly) opensource softwares

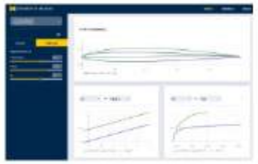
A good example

https://mdolab.engin.umich.edu/software

Software

The software packages listed below are all distributed under open source licenses. These are research codes, so they require a strong background in programming and some persistence to get them to work. Unfortunately we are not able to provide support except for collaborators and sponsors. However, we strive to provide as much documentation as we can and continually work towards improving the usability.

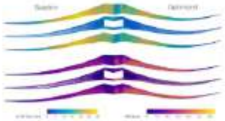
Webfoil: This is an online tool for airfoil analysis and optimization. It also includes a vast database of airfoils. [Webfoil site] [Paper]



interface to various optimization packages. pyOptSparse includes OptView, a visualization tool to explore the optimization history. [Code] [Documentation]

pyOptSparse

TACS: A general purpose structural finite-element code with adjoint derivatives that is developed by Prof. Graeme Kennedy. [Code] [Paper]



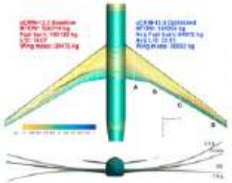
SMT: The surrogate modeling toolbox (SMT) is an open-source Python package consisting of libraries of surrogate modeling methods (e.g., radial basis functions, kriging), sampling methods, and benchmarking problems. SMT is designed to make it easy for developers to implement new surrogate models in a well-tested and well-document platform, and for users to have a library of surrogate modeling methods with which to use and compare methods. [Code] [Paper]

SMT

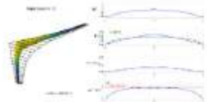
ADflow: (pronounced "A-D-flow") CFD solver that can handle structured multi-block and overset meshes. It includes an adjoint solver for computing derivatives and can be used in the MACH-Aero framework for aerodynamic shape optimization. [Code] [Documentation] [Paper]

DAFOAM: (pronounced "dahfoam") is a suite of adjoint solvers for OpenFOAM that enable the computation of derivatives for aerodynamic shape optimization. [Code] [Documentation] [Paper]

MACH-Aero: A framework for aerodynamic design optimization that couples a CFD solver (e.g. ADflow or OpenFOAM), geometry parametrization (e.g. pyGeo), mesh deformation (e.g., IDWarp), and optimizer interface (pyOptSparse). [Code] [Documentation and Tutorials]



OpenAeroStruct: A lightweight aerostructural optimization code that can optimize a wing design in minutes on a laptop. [Code] [Documentation]



OpenMDAO: A framework for coupling multiple numerical models and performing multidisciplinary analysis and optimization. OpenMDAO is developed by NASA and uses numerical techniques developed in the MDO Lab. [OpenMDAO in a nutshell] [OpenMDAO site] [Paper]

openMDAO

A Curated list of (mostly) opensource softwares

Tools for decarbonizing air transportation:

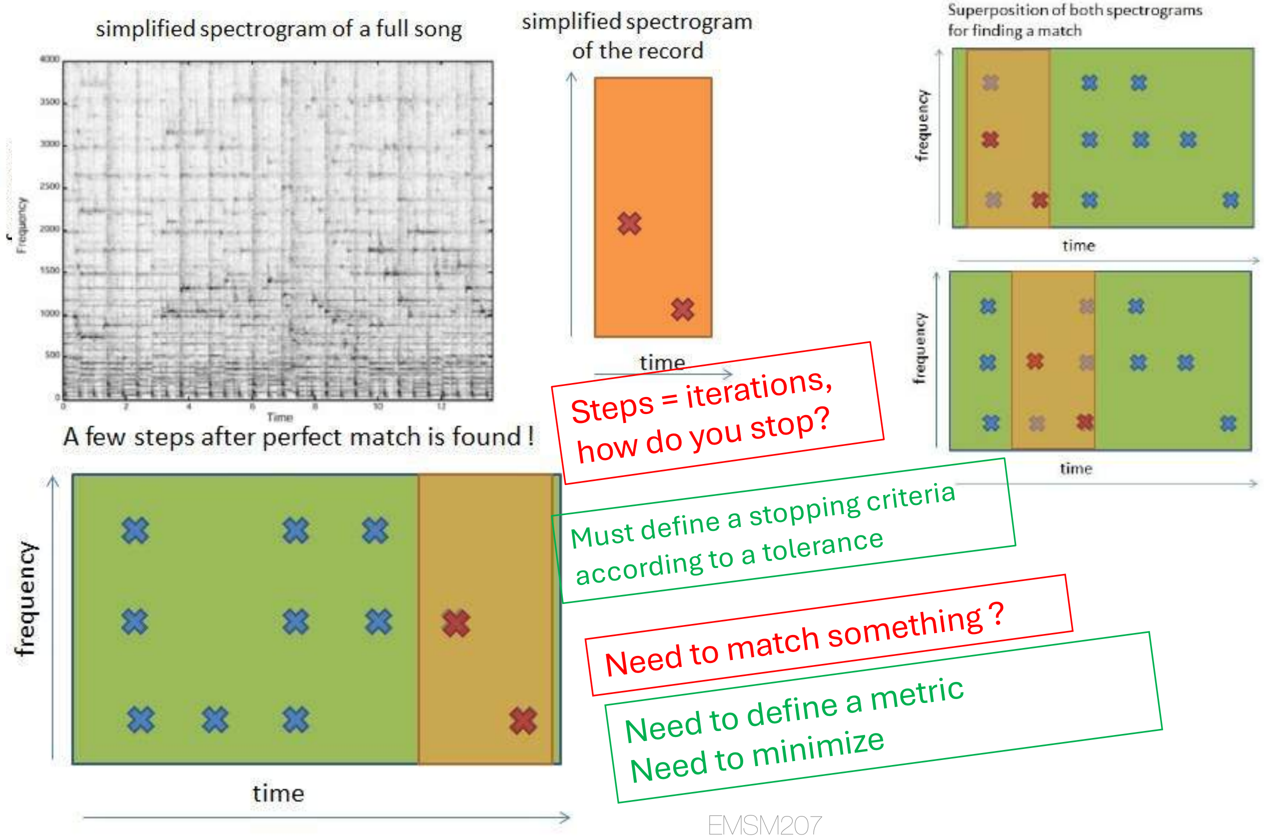
https://cascade.boeing.com/
https://aeromaps.eu
https://github.com/AeroMAPS/AeroMAPS
https://www.leadsresearchgroup.com/technology-dashboard
https://github.com/contrailcirrus/pycontrails?tab=readme-ov-file
https://github.com/sustainableaviation
https://github.com/Aircraft-Operations-Lab
https://psesh.github.io/aviation.html
https://github.com/leadsgroup
https://www.leadsresearchgroup.com/software
https://github.com/protontypes/open-sustainable-technology
https://www.witness4climate.org/optimizing-investments-in-energy-production-technology
https://junzis.com/open-source

Tools for aircraft design

https://github.com/peterdsharpe/AeroSandbox/tree/master
https://openmdao.github.io/Aviary/examples/OAS_subsystem.html
https://github.com/OpenMDAO/Aviary
https://github.com/ideas-um/FAST
https://github.com/MIT-LAE/TASOPT.jl
https://github.com/mdolab/OpenAeroStruct
https://github.com/mdolab/openconcept
https://github.com/fast-aircraft-design/FAST-OAD
https://web.mit.edu/drela/Public/web/
https://lsdo.eng.ucsd.edu/software
https://github.com/mid2SUPAERO/LCA4MDAO
https://github.com/ImperialCollegeLondon/sharpy
https://www.aircraftflightmechanics.com/NotesIntroduction.html
https://github.com/MIT-LAE
https://github.com/OpenVSP/OpenVSP
https://github.com/suavecode
https://github.com/camUrban/PteraSoftware
https://github.com/cfsengineering/CEASIOMpy
https://github.com/DLR-AE/PanelAero
https://github.com/DLR-AE
https://github.com/ImperialCollegeLondon/PinhoLab-WingBox
https://commonresearchmodel.larc.nasa.gov
https://github.com/facebookarchive/FBHALE
https://github.com/mid2SUPAERO/ecoHALE

Optimization is everywhere

http://coding-geek.com/how-shazam-works/



Steps = iterations, how do you stop?

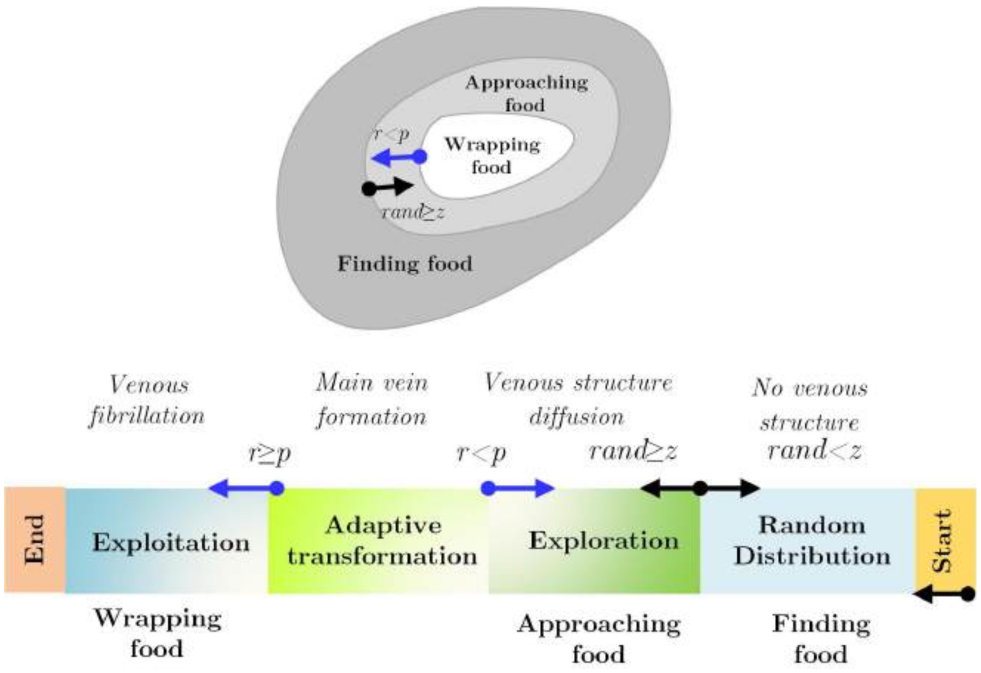
Must define a stopping criteria according to a tolerance

Need to match something ?

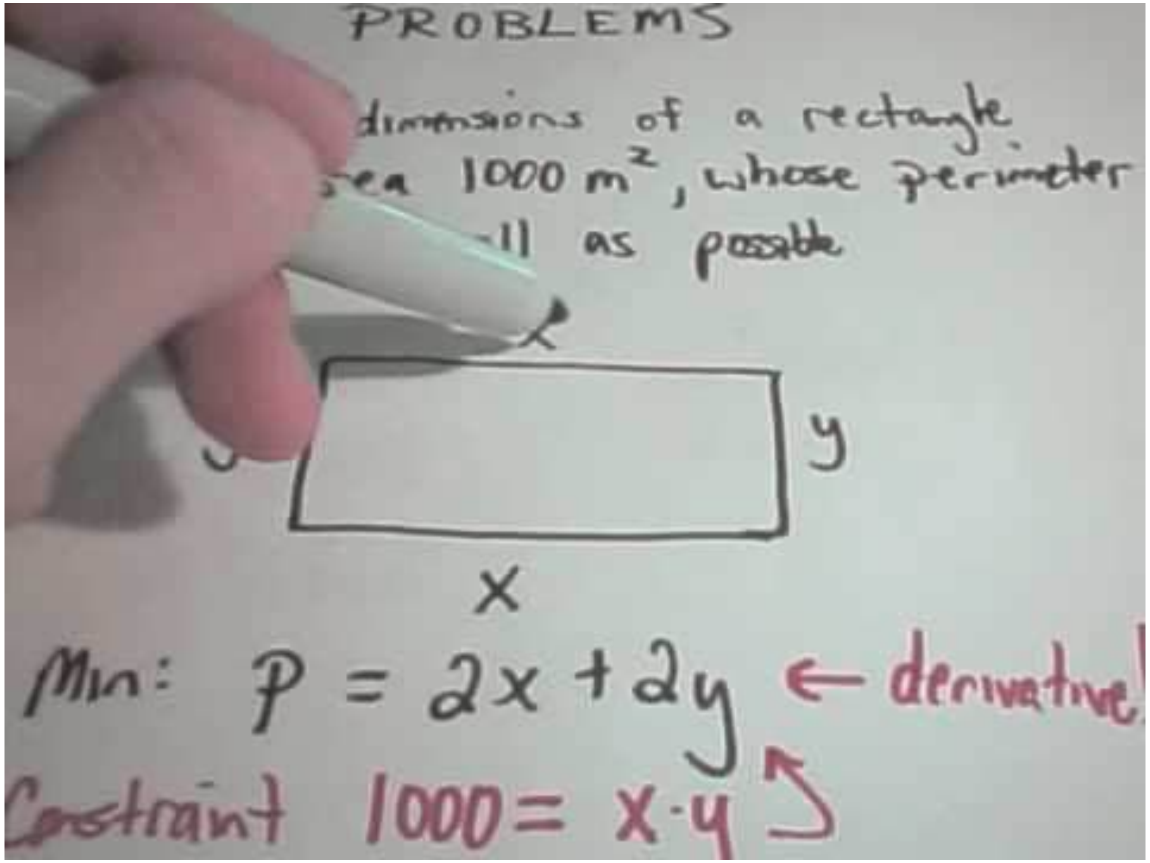
Need to define a metric
Need to minimize

Optimization is Everywhere

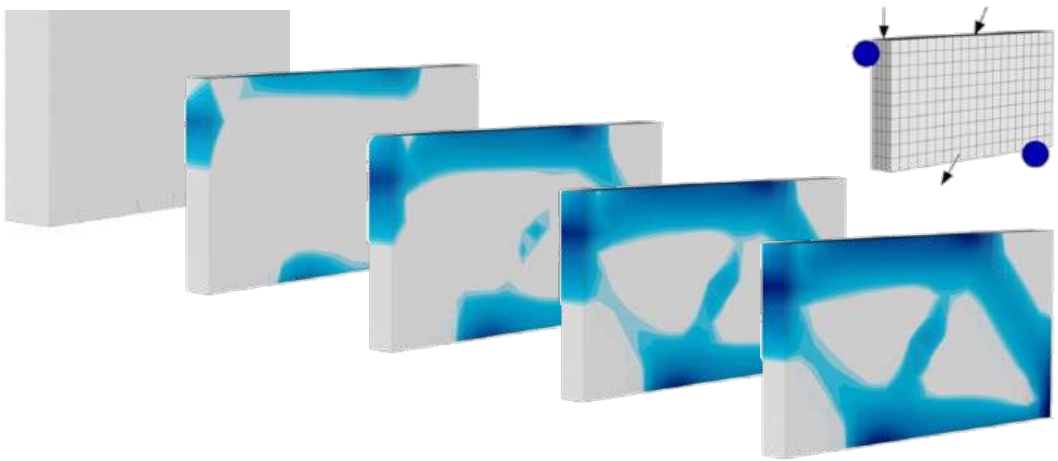
https://www.aliasgharheidari.com//SMA.html



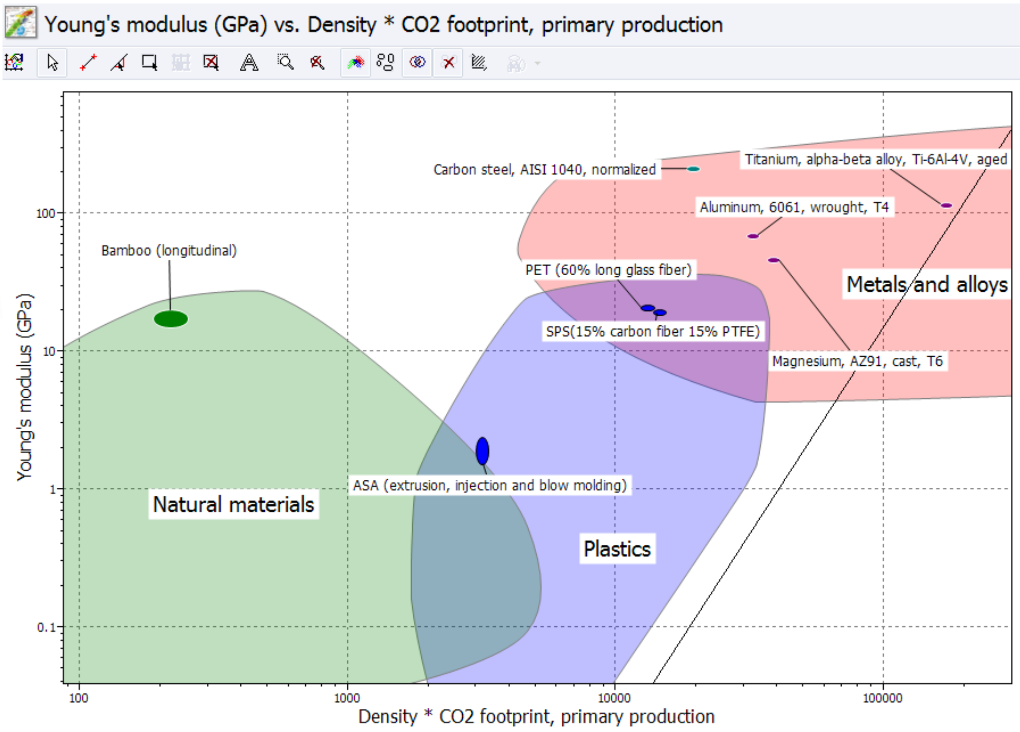
From engineering to maths (*next course)



design of bicycle frame by optimization



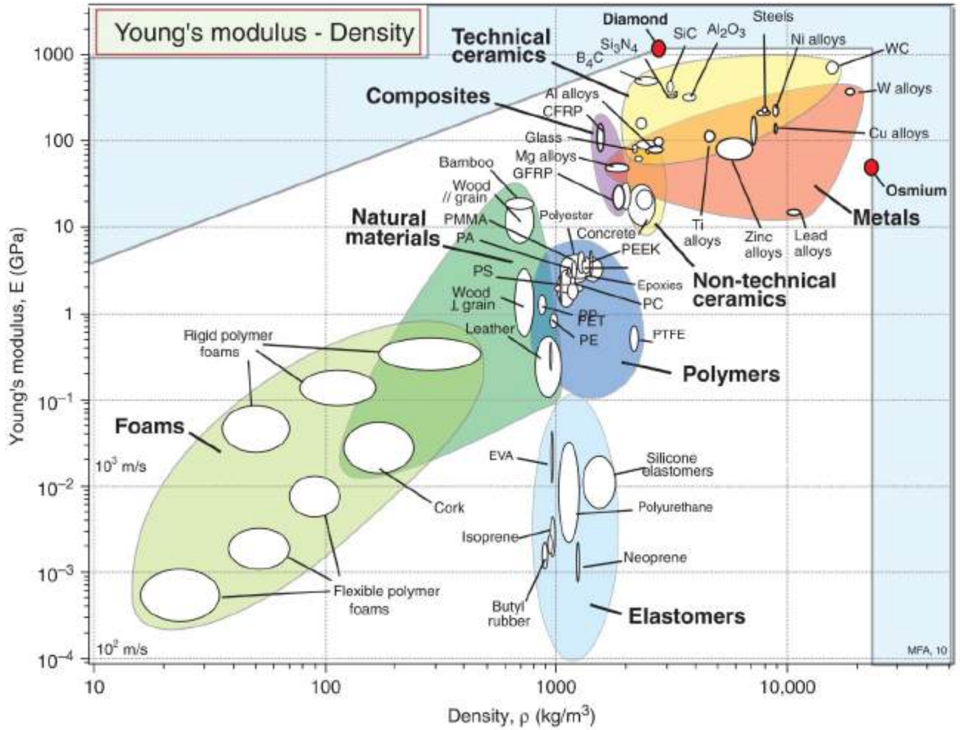
design of bicycle frame by EcoDesign



Optimization is Everywhere

To get started, you need to select the right material. Why use steel if you can get away with aluminum?

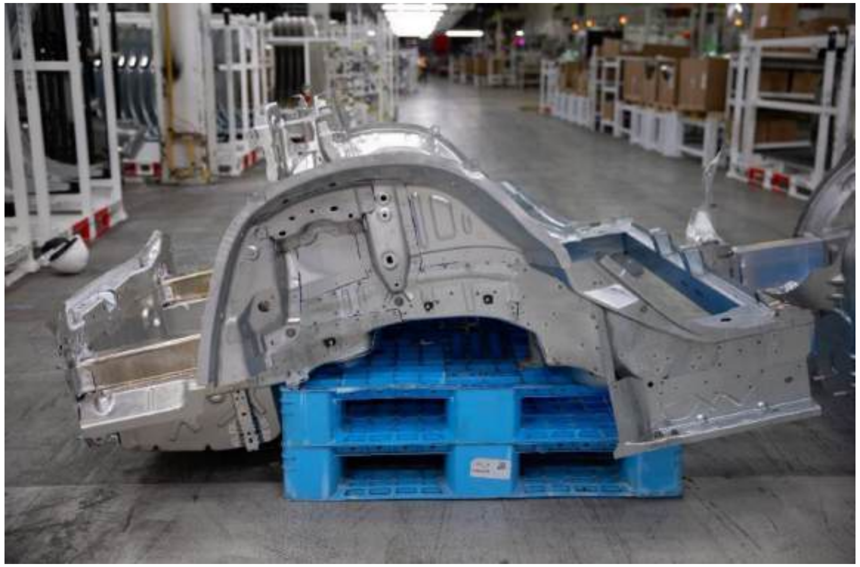
There's a weight reduction of approximately two thirds on the table, albeit with compromises: reduced stiffness, higher cost, more difficulty in welding. A great way of visualizing these tradeoffs and rankings of mechanical properties is with Ashby Charts, which essentially represent the menu of materials that an engineer can select.



[Ashby](#) Chart for evaluating stiffness-to-weight ratio of conventional materials. This essentially represents the menu of (conventional) materials that engineers have to choose from.

Optimization is Everywhere

https://www.3dprintingmedia.network/tesla-shows-massive-generatively-designed-3d-printed-part-in-model-y-underbody/

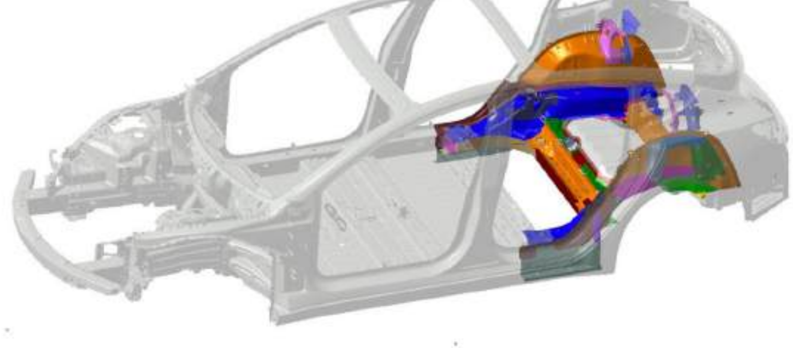


The current underbody part made of 70 different components

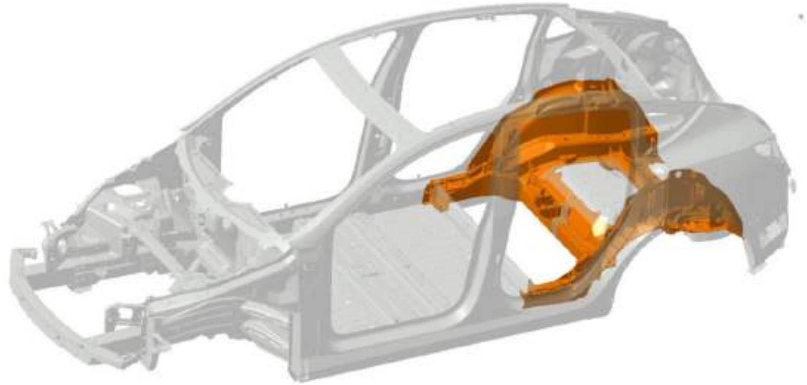
The generatively designed underbody, made of 2 and eventually 1 single piece.

Optimization is Everywhere

Think different!!!



Model 3 rear underbody
70 pieces of metal



Model Y rear underbody
2 pieces of metal (eventually a single piece)

the use of 3D printing for sand casts such as that offered by voxeljet and ExOne for to enable the reduction of subassemblies (form 70 to 1) in a custom cast can bring about a significant transition even before metal AM can be used to produce such large metal parts directly. Producing a complex cast that can reduce the number of parts to this degree needs digital casting technology

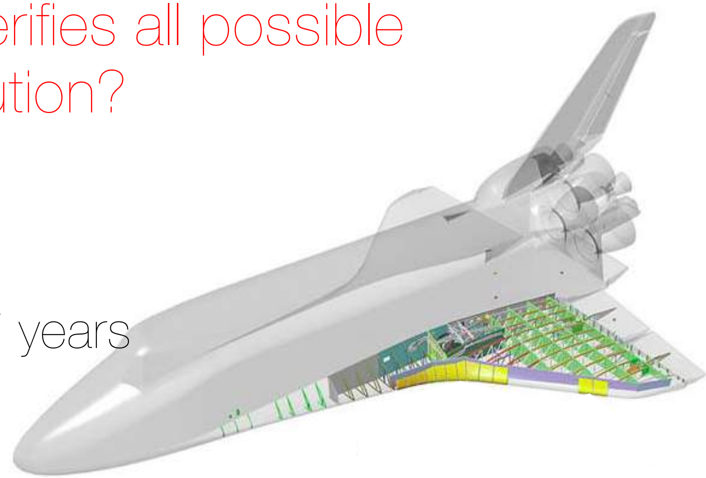
Let's take the example of a structure with 10 structural elements (10 design variables).

Each variable is associated with a section type (U, Z, I etc …: 10 sections available per variable).

A structural analysis "costs" only 1 second of calculation.

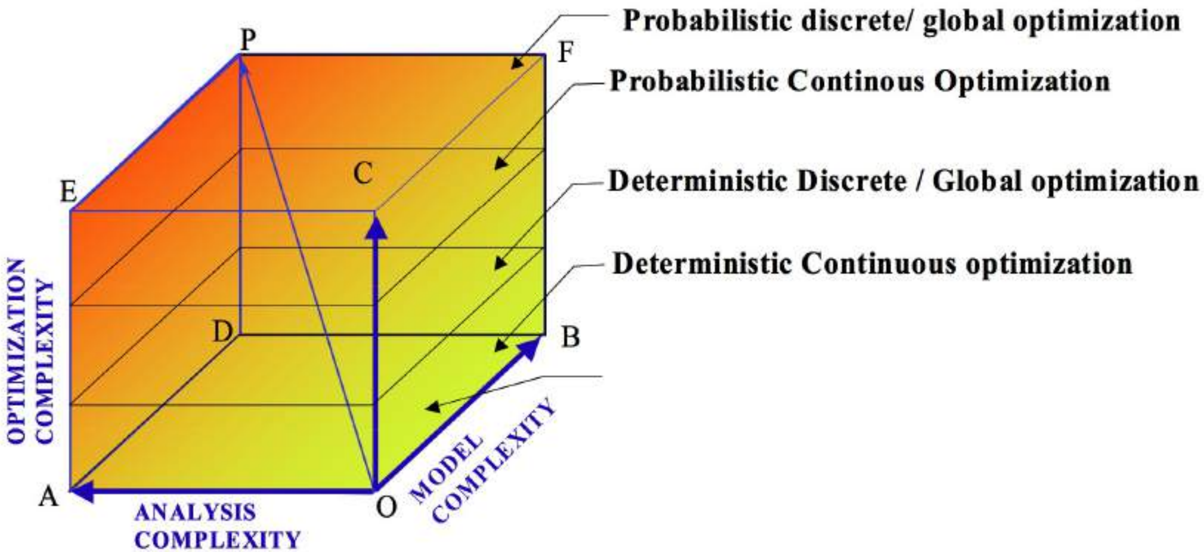
Question:

How long will the calculation last that verifies all possible combinations to ensure an optimal solution?

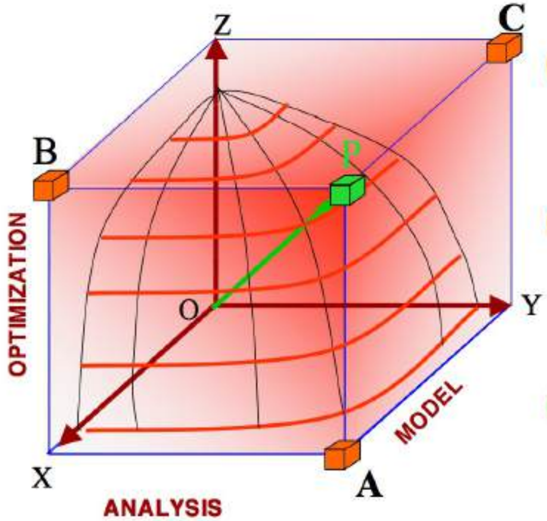


… 10^{10} seconds At least 317 years

Haftka's principle



Haftka’s Examples



(A) Model and Analysis Complexity

– e.g., Complex 3-D structure requiring non-linear analysis

(B) Model and Optimization Complexity

– e.g., Probabilistic design of a complex 3-D structure

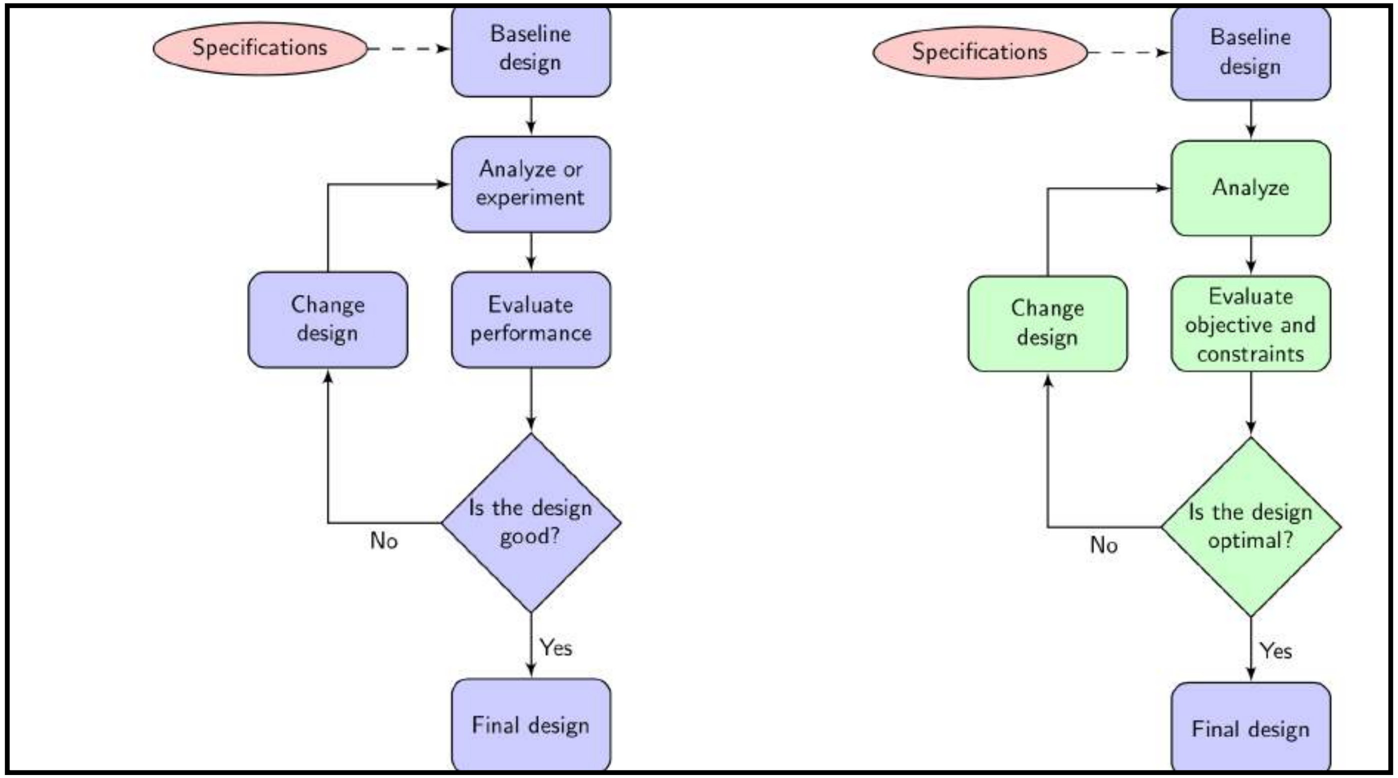
(C) Analysis and Optimization Complexity

– e.g., Non-deterministic optimization with non-linear analysis (or combined macro and micro mechanical failure models

(P) Model, Analysis and Optimization Complexity

e.g., Non-deterministic optimization of Complex 3-D structure requiring nonlinear analysis

Conventional vs Optimal Design



Definition ?

- “Making things better”
- “Generating more profit”
- “Determining the best”
- “Do more with less”

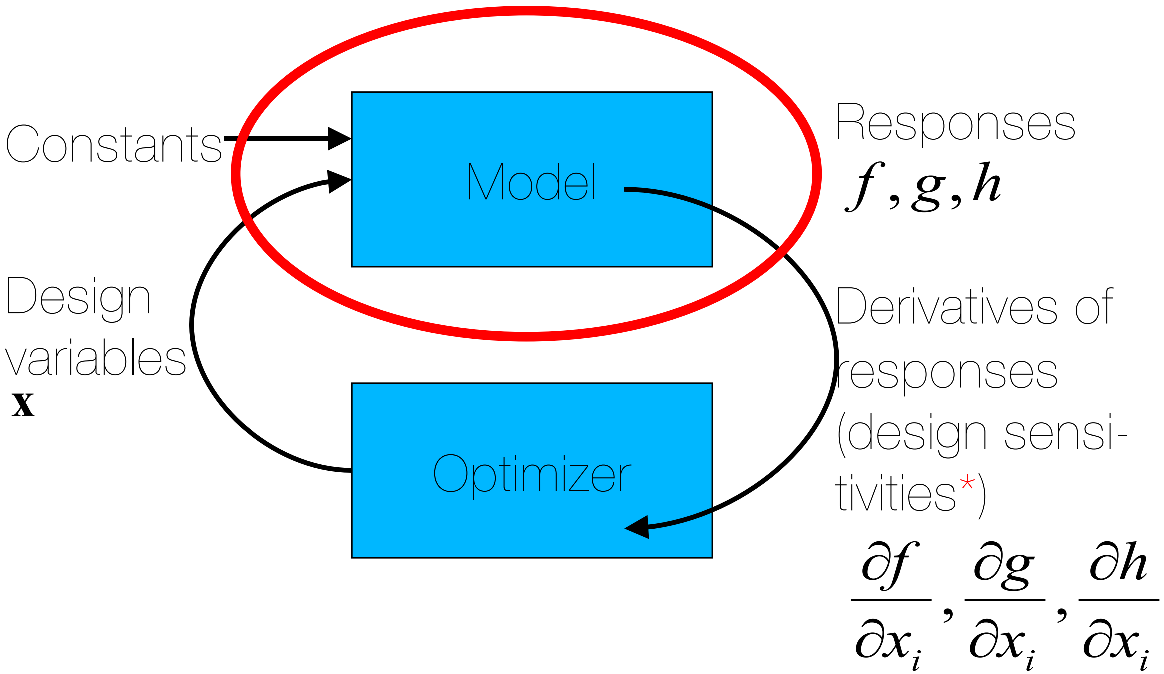
- “The determination of values for design variables which minimize (maximize) the objective, while satisfying all constraints” : Optimal design

Engineer's goals?

- The engineer must often answer a contradictory problem:
  - Airplane wing, F1: rigidity vs mass
  - Confort in turbulence vs speed of flutter
  - Pressurized tank: use (form) / buckling load

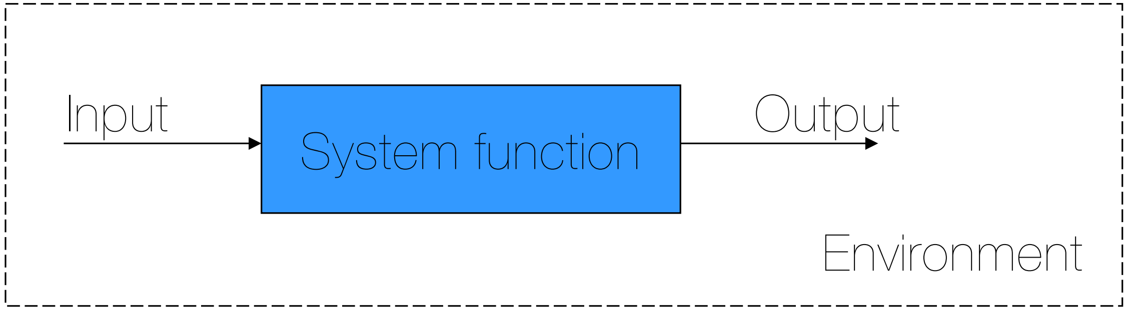
  Everything else is trivial!

How?



*see doc on LMS

System approach



- Wonder:
  - What are the inputs / outputs?
  - What is the system / environment?
  - Hierarchize the system

Optimization types

$$\min J(x) \quad x \in \mathbb{N}$$

Discrete:
Operational research, combinatory, theory of graph

$$\min J(x) \quad x \in D \subset \mathbb{R}^N$$

Finite dimension:
Optimality criteria, differentiability, convex analysis, constraints/unconstraints

$$\min J(u) \quad u \in H$$

Variational method:
Physics, mechanics, economy

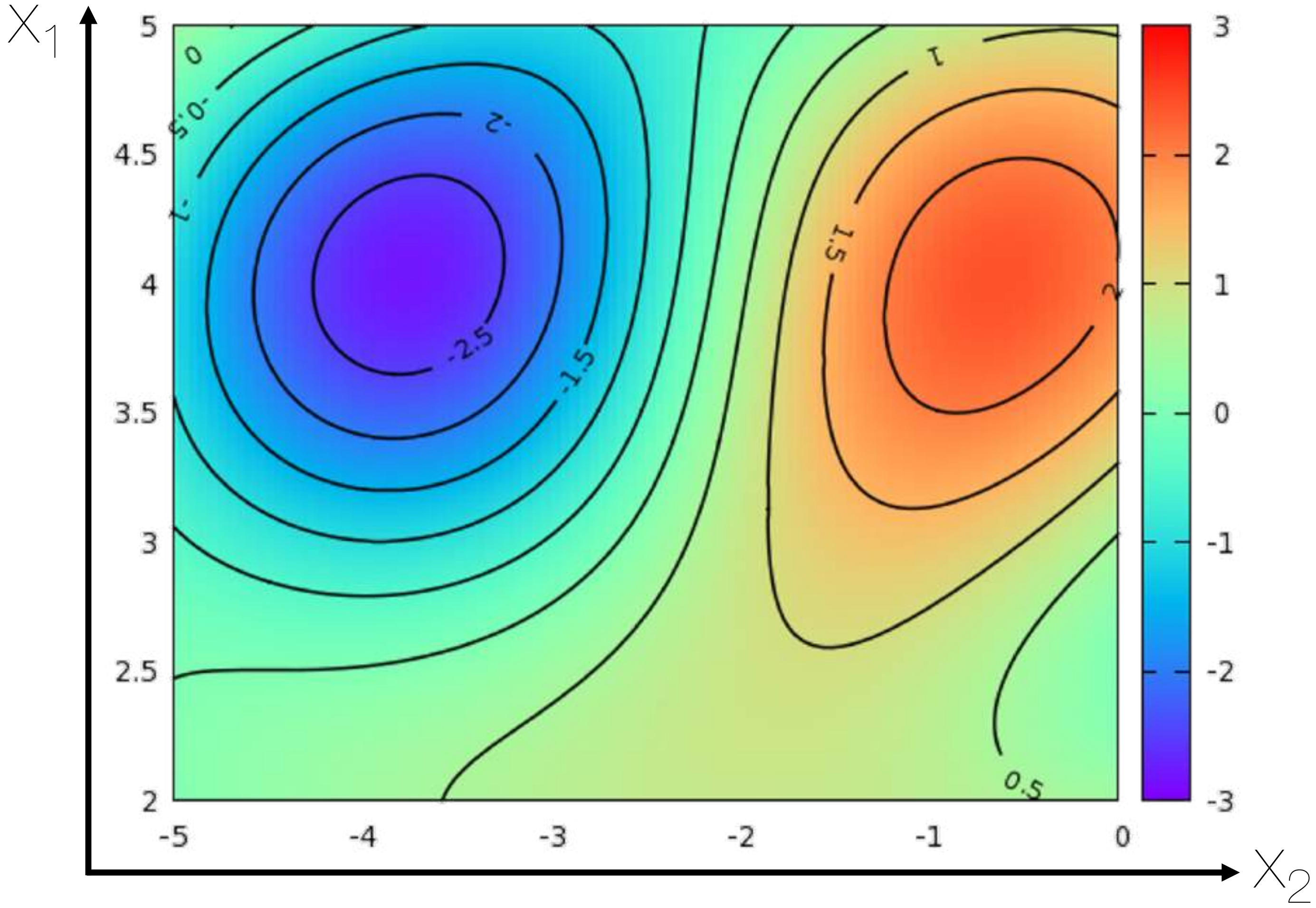
$$\min J(\Omega) \quad \Omega \subset \mathbb{R}^N$$

Shape:
Properties, optimality criteria

https://neos-guide.org/content/optimization-tree-alphabetical

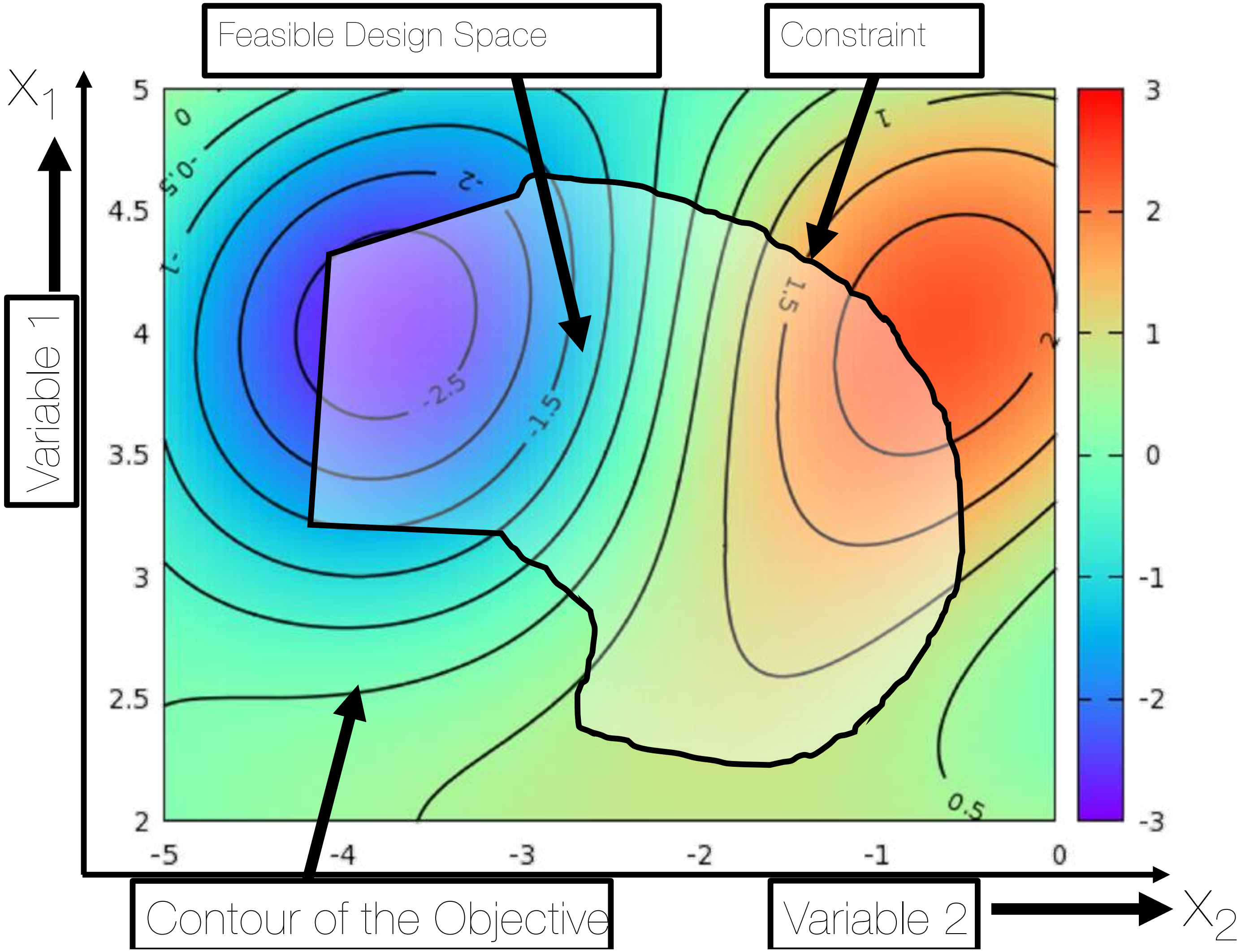
The BIG picture

Variables and the Objective



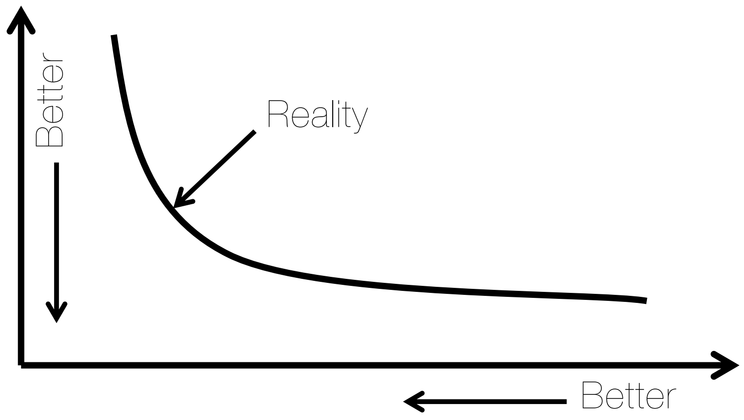
The BIG picture

Variables, Objective & Constraints

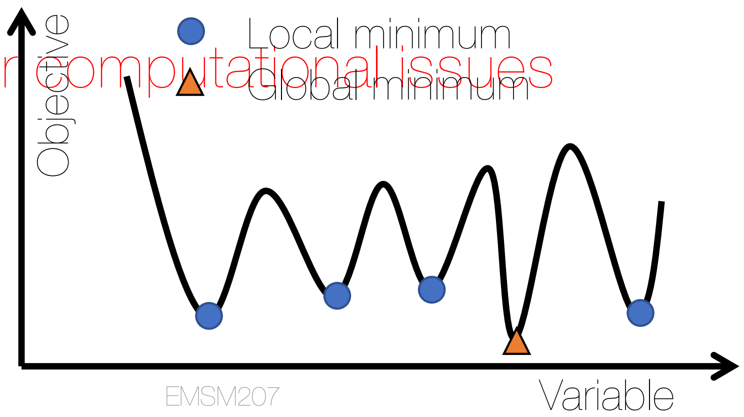


Common issues in the optimization of engineering systems

Multiple objectives

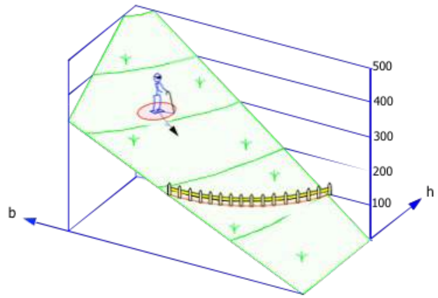


Multiple local solutions and other computational issues



Gradient based Optimization (Maximize/Minimize)

The optimum (the solution) will depend on the starting point !!! And this point of arrival may be a local minimum and not an overall minimum.

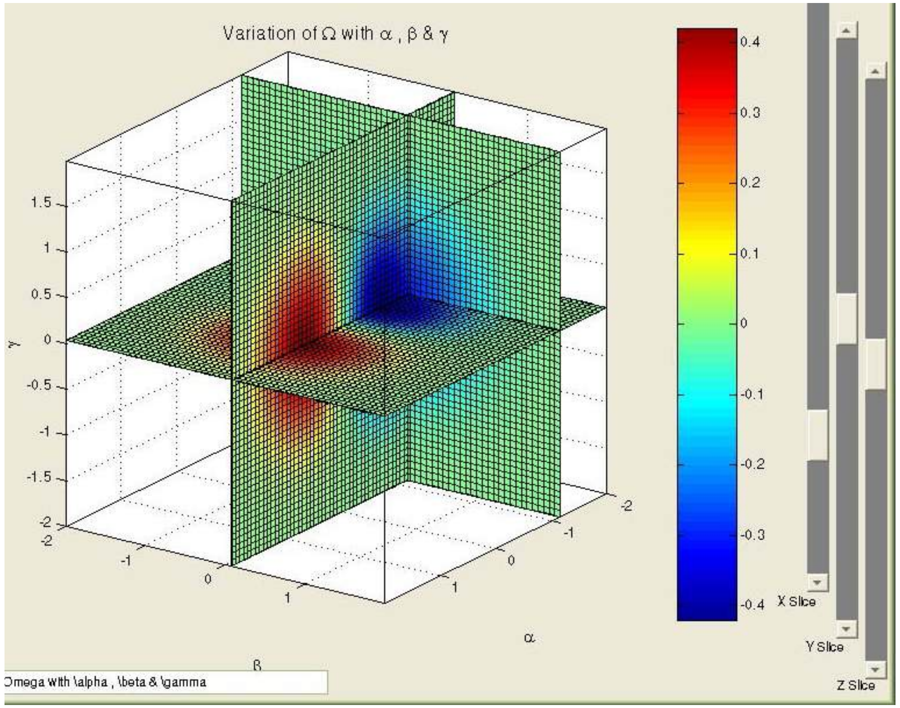


Transformation des programmes de minimisation : tout programme de minimisation peut aisément être transformé en programme de maximisation en remplaçant la fonction objectif f par son opposée $-f$:

$$\left|\begin{array}{ll} \min\limits_{\tilde{x}} & f(\tilde{x}) \\ \text{s.c.} & contraintes \end{array}\right. \Leftrightarrow \left|\begin{array}{ll} \max\limits_{\tilde{x}} & -f(\tilde{x}) \\ \text{s.c.} & contraintes \end{array}\right.$$

In N dimension?

Of course in N-D,
N Design variables
P constraints, it may be difficult to visualize…



Derivatives ?

Quasi-Newton method
Gradient descent
Gauss–Newton algorithm
Levenberg–Marquardt algorithm
Trust region
Nelder–Mead method
MATLAB, SOL200 etc…

In optimization

$$\min_{x \in \mathbf{X}} J(x)$$

Why computing derivative of criteria J ?

→Optimality criteria J'(x)=0

→To calculate an approximated solution:

→Steepest descent Xn+1=Xn-rho*J'(Xn)

→Newton Xn+1=Xn-$[D^2J(Xn)]^{-1}$J'(Xn)

Negative null form [MATLAB]

$$\begin{aligned} \underset{\mathbf{x}}{\text{Minimize}} \quad & f(\mathbf{x}) \\ \text{subject to} \quad & \mathbf{h}(\mathbf{x}) = 0 \\ & \mathbf{g}(\mathbf{x}) \leq 0 \\ & \mathbf{x} \in \mathcal{X} \subseteq \mathbb{R}^n \end{aligned}$$

$\mathbf{x}$ = (column) vector of design variables

$\mathbf{h} = [h_1, h_2, \ldots, h_{m_h}]^T$

$\mathbf{g} = [g_1, g_2, \ldots, g_{m_g}]^T$

Other formulations:

- positive null form ($\mathbf{g}(\mathbf{x}) \geq 0$) PYTHON
- negative unity form ($\mathbf{g}(\mathbf{x}) \leq 1$)
- positive unity form ($\mathbf{g}(\mathbf{x}) \geq 1$)

Criteria to optimize (performance)

- Some examples of criteria to minimize (or maximize)
  - Cost
  - Mass
  - Aerodynamic drag
  - Lift
  - Etc.

Design variables

The section of a beam in an assembly (lattice)
The number of ribs in a wingbox
The skin thickness of a wingbox
The orientation and sequencing of a composite
The coordinates defining a NACA profile
Etc …

In optimization of structures, we distinguish the variables of shape, geometry, and materials

Constraints

Mechanical

- Von Mises (Stresses)
- Buckling load
- Eigenfrequencies

$$g = \frac{\sigma_{\max}(\mathbf{x})}{\sigma_{allowed}} - 1 \leq 0$$

$$g = \sigma_{\max}(\mathbf{x}) - \sigma_{allowed} \leq 0$$

Scaled (Reserve Factor) vs. Unscaled

Optimization Steps

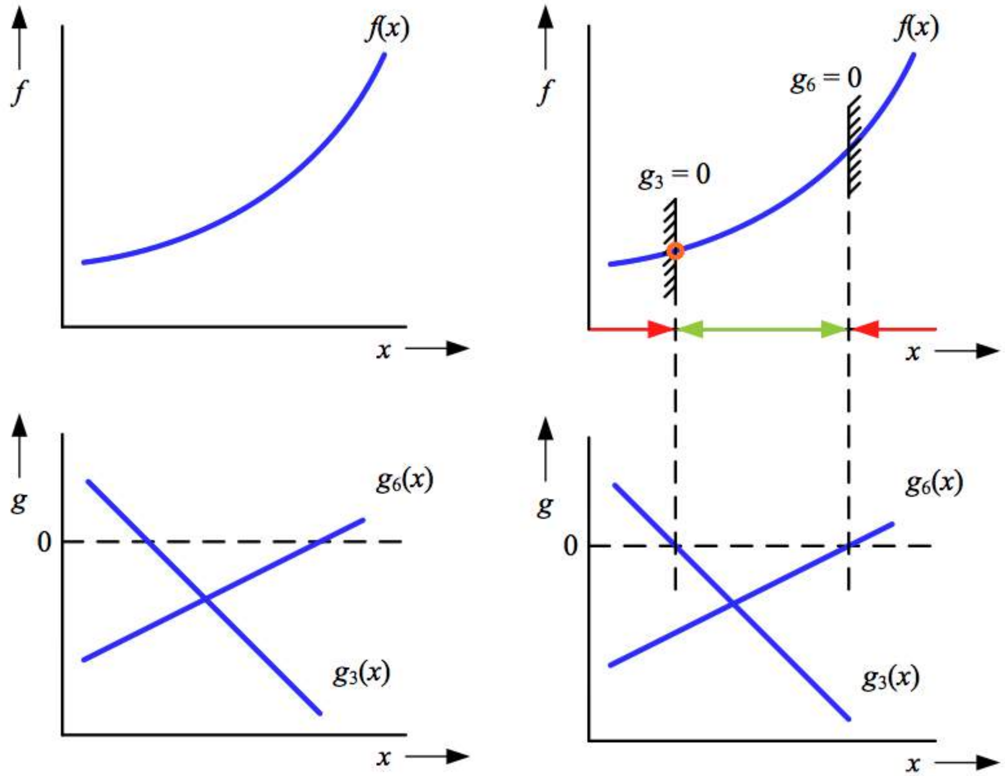
- 1 Select design variables
- 2 Select objective criterion in terms of design variables (to minimize or maximize)
- 3 Determine constraints in terms of design variables, which must be satisfied
- 4 Determine design variable values which minimize (maximize) the objective while satisfying all constraints

[Papalambros & Wilde 2000: Principles of optimal design]

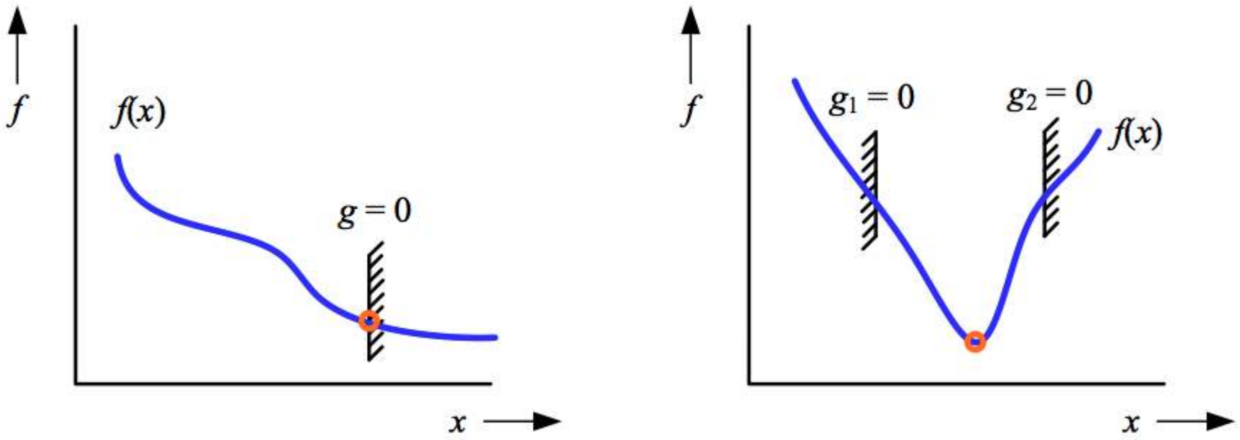
Classification

- Problems:
  - Constrained vs. unconstrained
  - Single level vs. multilevel
  - Single objective vs. multi-objective
  - Deterministic vs. stochastic
- Responses:
  - Linear vs. nonlinear
  - Convex vs. nonconvex (later!)
  - Smooth vs. nonsmooth
- Variables:
  - Continuous vs. discrete (integer)

Visualization: 1D example



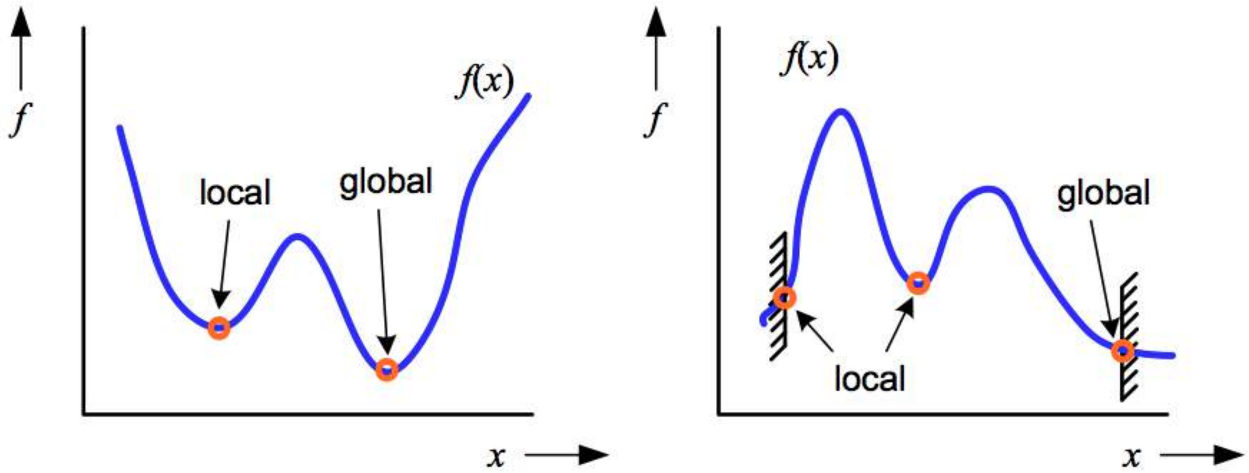
Constrained VS unconstrained optimum



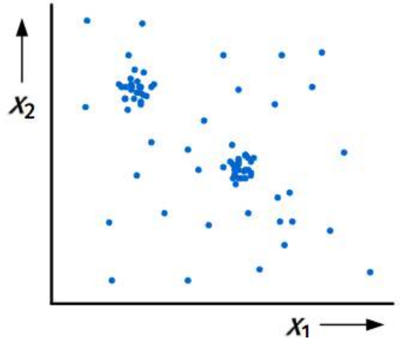
- constrained optimum
- bounded optimum

- unconstrained optimum
- interior optimum

Multimodality (multiple local minima)



Genetic Algorithms GA



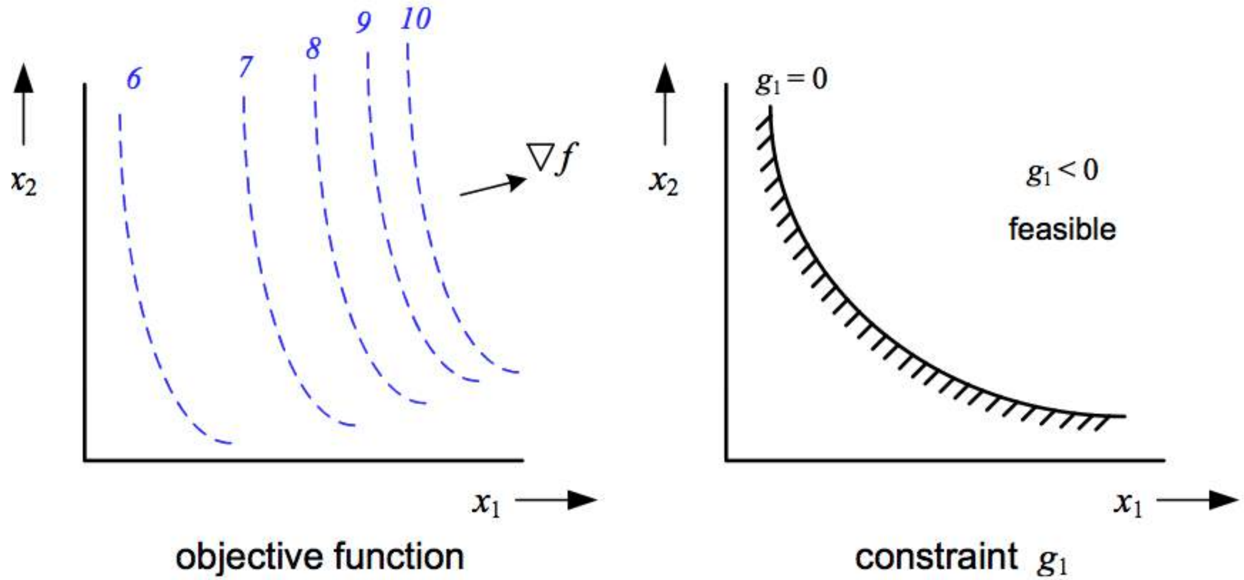
- The first "metaheuristic": 1975, John Holland publishes "Adaptation in natural and artificial systems »
- Widely used, easy to program, easy to use and robust.

1 / Problems of continuous GLOBAL optimization

2 / Problems of combinatorial (discrete) GLOBAL optimization:

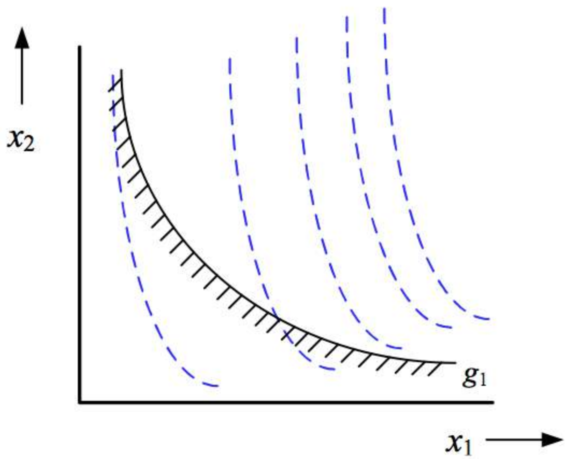
→ For these problems, either discrete values can only be used for the variables, or we must change elements of the problem to "change the values of the variables"

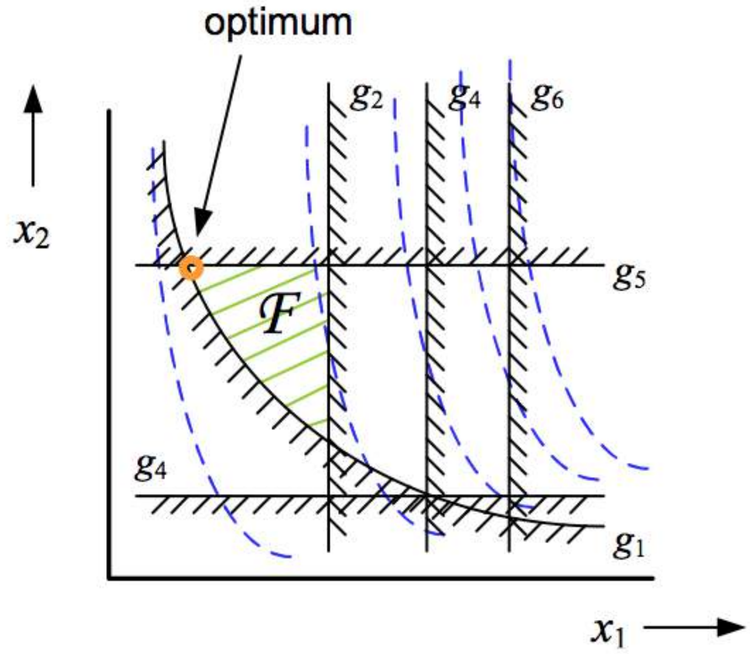
2D example



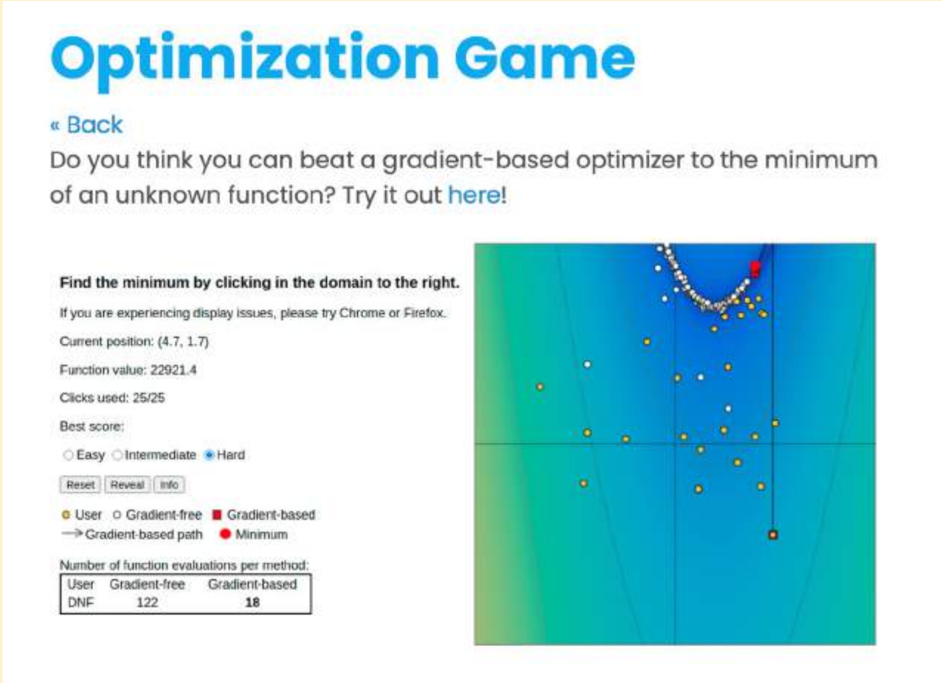
6
7
8
9
10
∇f
x_2
x_1
objective function
$g_1 = 0$
$g_1 < 0$
feasible
x_2
x_1
constraint g_1

2D example





First Exercice



https://mdolab.engin.umich.edu/assets/optimizationGame/

Stopping criteria

Condition on optimality

$\|\nabla L(\mathbf{x}_{k+1})\| < \varepsilon$ (Matlab: TolFun)

$g_j(\mathbf{x}_{k+1}) < \varepsilon$ and $|h_j(\mathbf{x}_{k+1})| < \varepsilon$ (Matlab: TolCon)

or a condition on change in $\mathbf{x}$

$\|\mathbf{x}_{k+1} - \mathbf{x}_k\| < \varepsilon$ (Matlab: TolX)

or a condition on the number of iterations

$k \leq k_{\max}$ (Matlab: MaxIter)

or a combination of these, with $\varepsilon > 0$

Type	Domaine	Function
Scalar Minimization	$\min_a f(a)$ such that $a_1 < a < a_2$	`fminbnd`
Unconstrained Minimization	$\min_x f(x)$	`fminunc` `fminsearch`
Linear Programming	$\min_x f^T(x)$ such that $A.x \leq b, Aeq.x = beq, lb \leq x \leq ub$	`linprog`
Quadratic Programming	$\min_x \frac{1}{2}x^T Hx + f^T x$ such that $A.x \leq b, Aeq.x = beq, lb \leq x \leq ub$	`quadprog`
Constrained Minimization	$\min_x f(x)$ such that $c(x) \leq 0, ceq(x) = 0$ $A.x \leq b, Aeq.x = beq, lb \leq x \leq ub$	`fmincon`
Goal Attainment	$\min_{x,\gamma} \gamma$ such that $F(x) - \omega\gamma \leq goal$ $c(x) \leq 0, ceq(x) = 0$ $A.x \leq b, Aeq.x = beq, lb \leq x \leq ub$	`fgoalattain`
MiniMax	$min_x max_{\{F_i\}}\{F_i(x)\}$ such that $c(x) \leq 0, ceq(x) = 0$ $A.x \leq b, Aeq.x = beq, lb \leq x \leq ub$	`fminimax`
Semi-Infinite Minimization	$min_x f(x)$ such that $K(x,\omega) \leq 0 \forall \omega$ $c(x) \leq 0, ceq(x) = 0$ $A.x \leq b, Aeq.x = beq, lb \leq x \leq ub$	`fseminf`

Linear constraints

$$\min \quad f(x) = \left(x_1^2 + x_2^2 - 1\right)^2$$

$$\text{s.t.} \quad -1 \le x_1 \le 1, -1 \le x_2 \le 1, \quad x_1 + x_2 \ge 1, \quad x_1 x_2 \ge 1/2, \; x_2 \ge x_1^2, \; x_1 \ge x_2^2$$

$$A = [-1, -1]; \; b = -1;$$

$$lb = [-1; -1]; \; ub = [1; 1];$$

$$c(x) = \begin{bmatrix} \frac{1}{2} - x_1 x_2 \\ x_1^2 - x_2 \\ x_2^2 - x_1 \end{bmatrix}; \; ceq(x) = [];$$

```
% myobj.m
function  f=myobj(x)
f = (x(1)^2+x(2)^2-1)^2;

% mycon.m
function [c, ceq]=mycon(x)
c=[1/2-x(1)*x(2);
  x(1)^2-x(2);
  x(2)^2-x(1)];  % nonlinear inequalities c(x) <= 0
ceq=[ ];  % nonlinear equalities ceq(x) = 0;

% main file for fmincon
[x,fval] = fmincon(@myobj,xo,A,b,[],[],lb,ub,
                   @mycon,options);
```

x1+x2 > 1

-x1 -x2<-1

xT=[x1,x2]

A=[-1 ,-1]

b=-1

Ax=b —> 1*2*2*1=1*1

- The syntax for fmincon

[x,fval,exitflag]=fmincon(objfun,x0,A,b,Aeq,beq,lb,ub, nonlcon,options);

  - x: optimal solution; fval: optimal value; exitflag: exit condition
  - objfun: objective function (usually written in a separate M file)
  - x0: starting point (can be infeasible)
  - A: matrix for linear inequalities; b: RHS vector for linear inequalities
  - Aeq: matrix for linear equalities; beq: RHS vector for linear equalities
  - lb: lower bounds; ub: upper bounds
  - Nonlcon: [c,ceq]=constraintfunction(x)

$$\min \quad f(x) = \left(x_1^2 + x_2^2 - a\right)^2$$

$$\text{s.t.} \quad -1 \le x_1 \le 1, -1 \le x_2 \le 1, \quad x_1 + x_2 \ge 1, \quad x_1 x_2 \ge 1/2, \; x_2 \ge x_1^2, \; x_1 \ge x_2^2$$

```
% myobj.m
function  f=myobj(x, a)
f = (x(1)^2+x(2)^2-a)^2;
% main file for fmincon
a = 1;
[x,fval] = fmincon(@(x) myobj(x,a),xo,A,b,[],[],lb,ub,
                   @mycon,options);
```

Demo Matlab

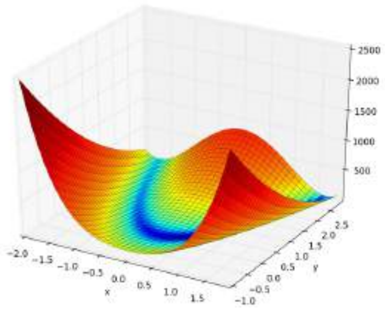
- Example1 (Local optimisation)
- Example2 (Global optimisation)
- Example3 (With/Without gradients)

- https://fr.mathworks.com/help/gads/solving-a-mixed-integer-engineering-design-problem-using-the-genetic-algorithm.html

For example, consider Rosenbrock's function

$$f(x) = 100\left(x_2 - x_1^2\right)^2 + (1 - x_1)^2,$$

which is described and plotted in Solve a Constrained Nonlinear Problem. The gradient of *f*(*x*) is



$$\nabla f(x) = \begin{bmatrix} -400\left(x_2 - x_1^2\right)x_1 - 2(1 - x_1) \\ 200\left(x_2 - x_1^2\right) \end{bmatrix},$$

and the Hessian *H*(*x*) is

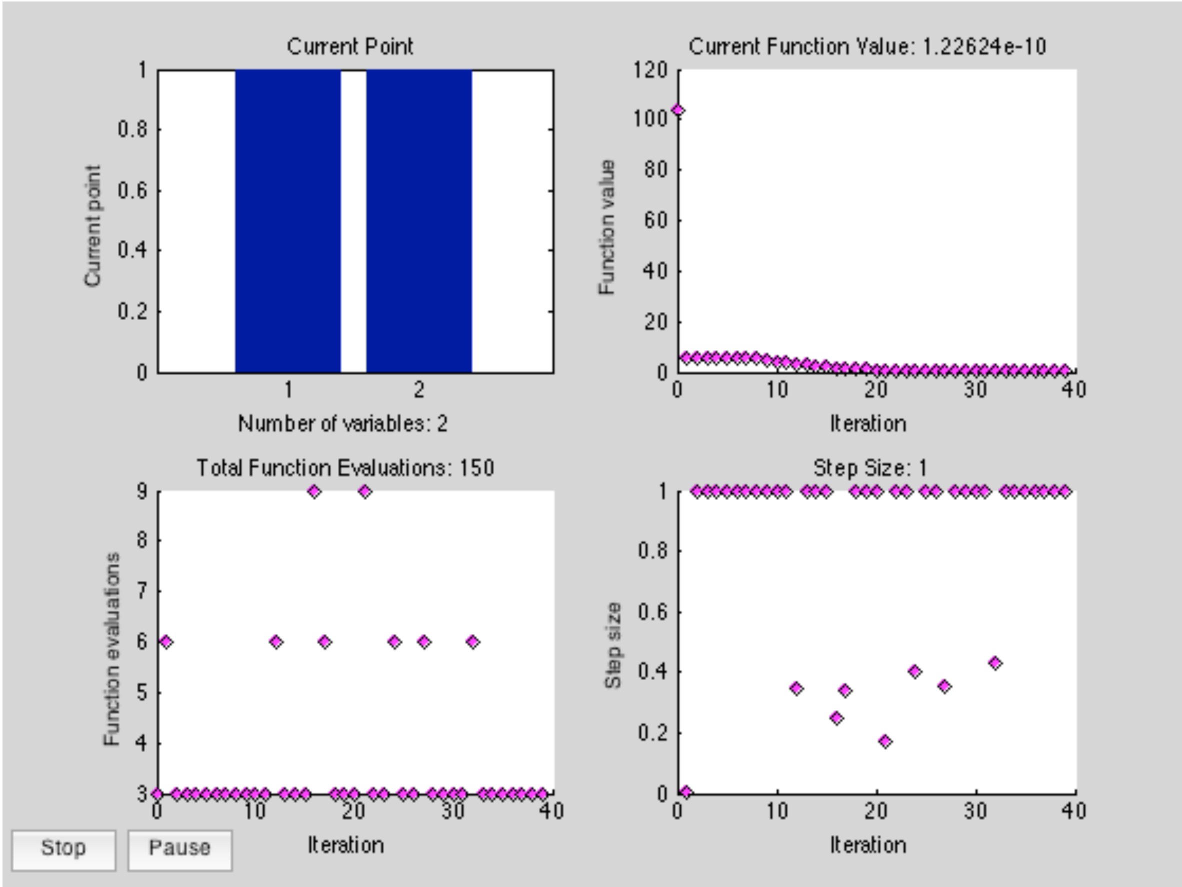
$$H(x) = \begin{bmatrix} 1200x_1^2 - 400x_2 + 2 & -400x_1 \\ -400x_1 & 200 \end{bmatrix}.$$

`rosenthree` is an unconditionalized function that returns the Rosenbrock function with its gradient and Hessian:

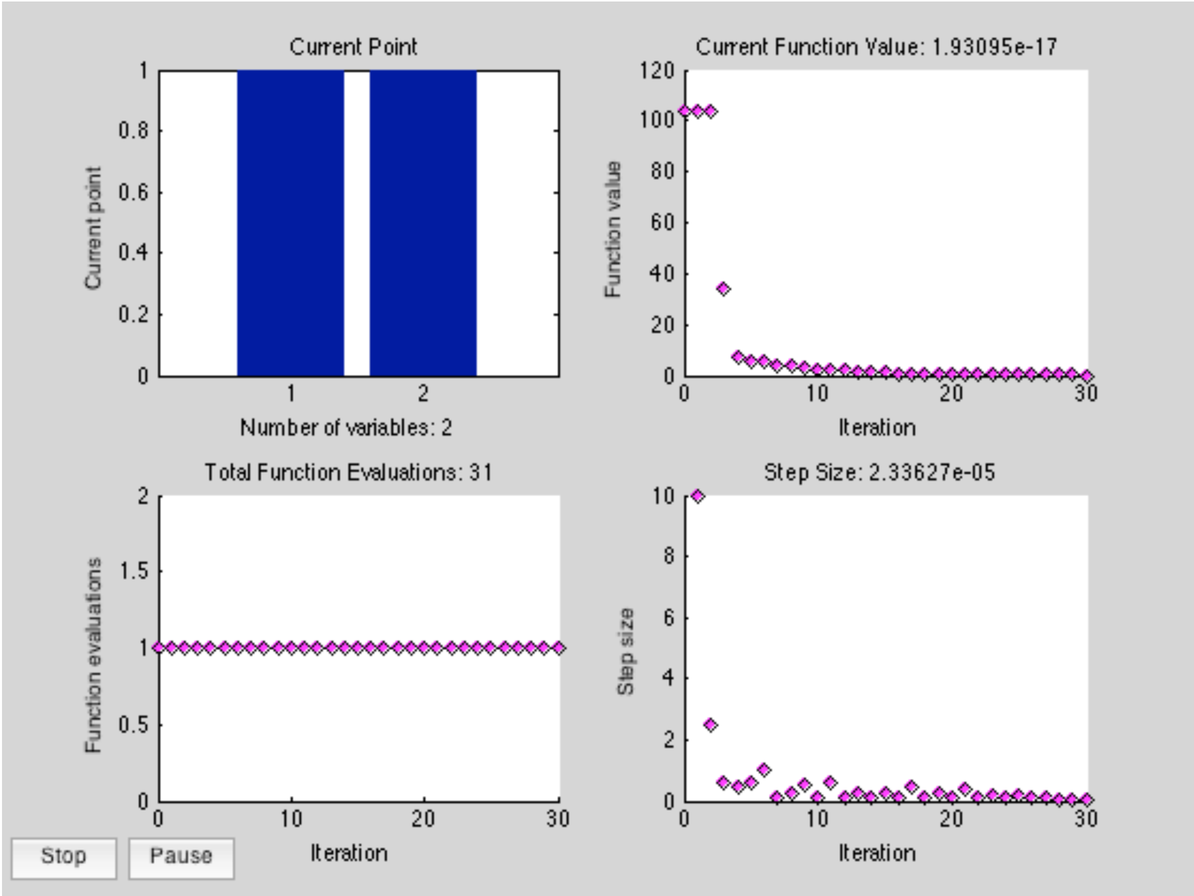
```
function [f g H] = rosenthree(x)
% Calculate objective f, gradient g, Hessian H
f = 100*(x(2) - x(1)^2)^2 + (1-x(1))^2;
g = [-400*(x(2)-x(1)^2)*x(1)-2*(1-x(1));
        200*(x(2)-x(1)^2)];
H = [1200*x(1)^2-400*x(2)+2, -400*x(1);
            -400*x(1), 200];
```

```
syms x y ;
    f=100*(y - x^2)^2 + (1-x)^2
    fx=diff(f,x)
    fy=diff(f,y)
%analytical gradients
f =(x - 1)^2 + 100*(- x^2 + y)^2
fx =2*x - 400*x*(- x^2 + y) – 2
fy =- 200*x^2 + 200*y
g=[fx;fy]
```

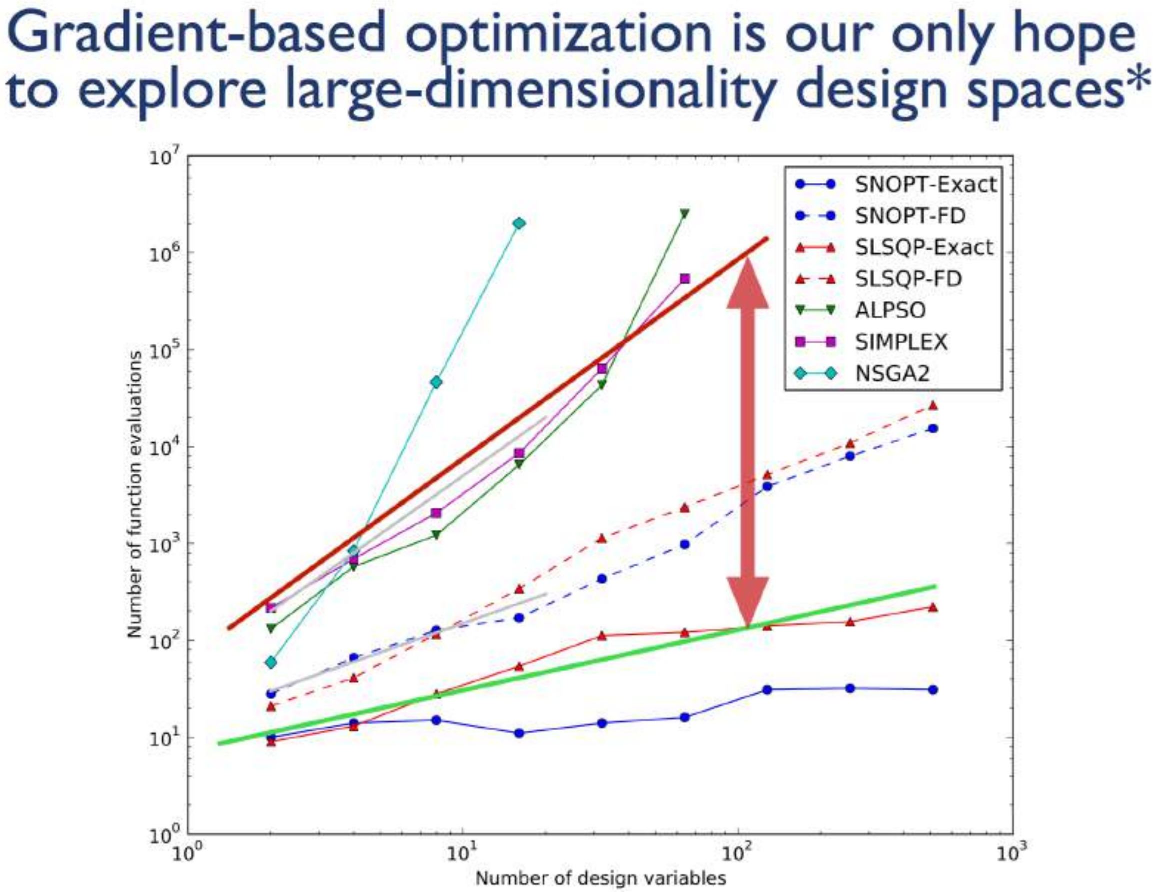
Results (w/o gradient)



Results (w/ gradient)



J. Martins, MDO course



Second exercice

1 Optimization problems in practice

Rewrite the following practical optimization problems using the following canonical form

$$\min_{x \in \Omega} f(x), \quad \Omega = \{x \in \mathbb{R}^n | h(x) = 0, g(x) \leq 0\}$$

where $h : \mathbb{R}^n \to \mathbb{R}^m$ represents m equality constraints ($h(x) = [h_i(x)]_{i=1}^m$) and $g : \mathbb{R}^n \to \mathbb{R}^p$ represents p inequality constraints ($g(x) = [g_j(x)]_{j=1}^p$).

1. Find two positive numbers whose sum is 300 and whose product is a maximum.
2. Find two positive numbers whose product is 750 and for which the sum of one and 10 times the other is a minimum.

Warning
Matlab Negative Null Form
Python Positive Null Form

Reformulation

Basic idea: convert to an unconstrained optimization problem

- Penalty function methods
- Append a penalty for violating constraints (exterior penalty methods)
- Append a penalty as you approach infeasibility (interior point methods)
- Method of Augmented Lagrange multipliers

Augmented Lagrange Method

- Adaptation of penalty method for equality constraints

$$p_{\text{Lagrange}}(\mathbf{x}) = \frac{1}{2}\rho \sum_i (h_i(\mathbf{x}))^2 - \sum_i \lambda_i h_i(\mathbf{x})$$

$$\boldsymbol{\lambda}^{(k+1)} = \boldsymbol{\lambda}^{(k)} - \rho \mathbf{h}(\mathbf{x})$$

- λ converges towards the Lagrange multiplier

Example of Optimizers

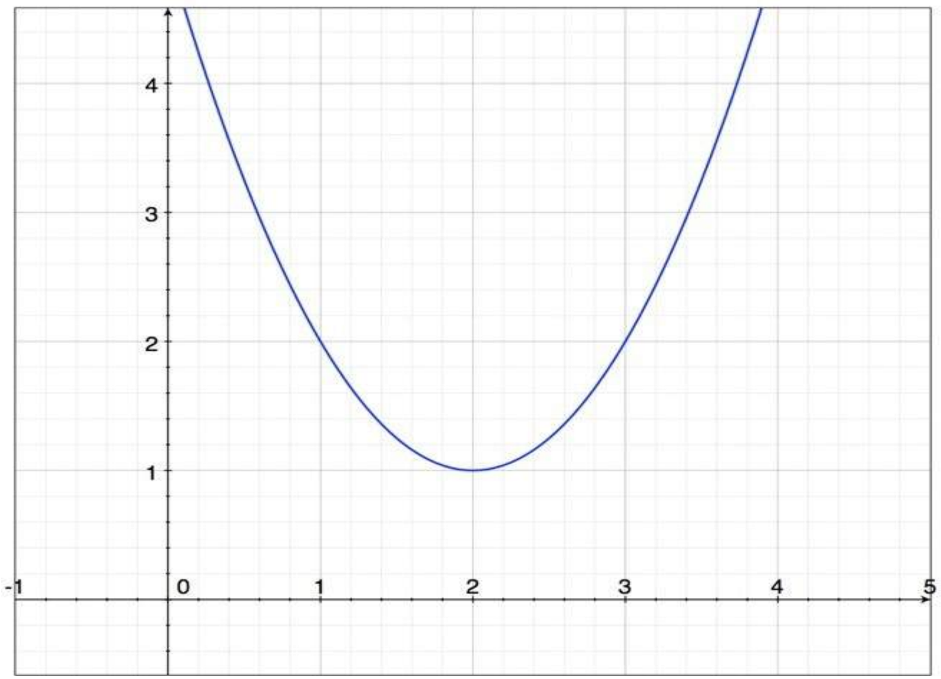
CHALLENGE #1 On Rosenbrock function minimization

https://colab.research.google.com/drive/1rGHIklpVM4Gqy9eq1Y10fIQW9umIjeEx

Example of using autograd

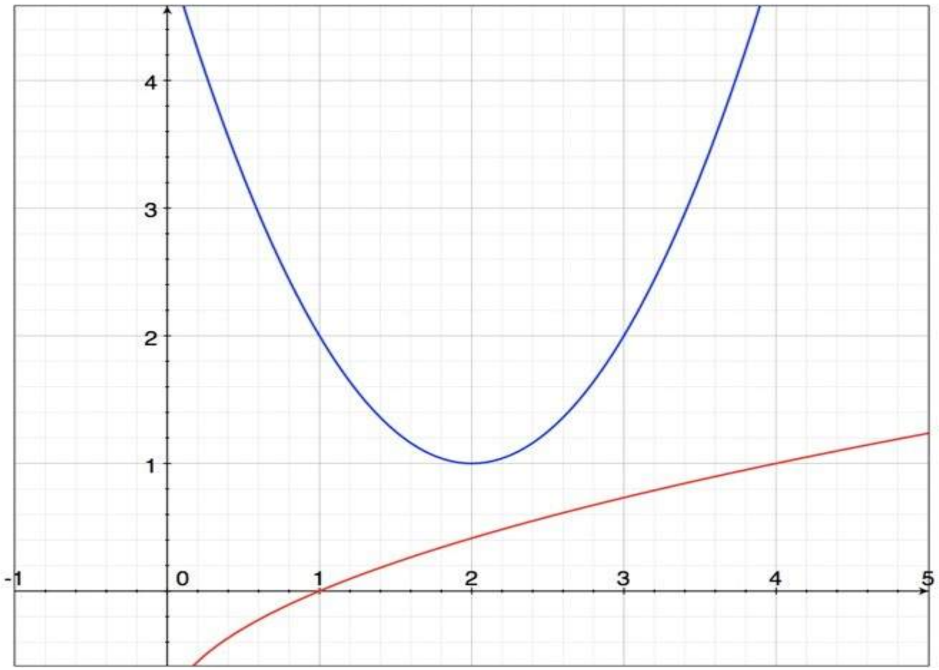
- https://colab.research.google.com/drive/1oDwKKU-WUnogMAxTg0Fx_NBuk5-8LD1x

Example



Objective: $f(x) = (x-2)^2 + 1$

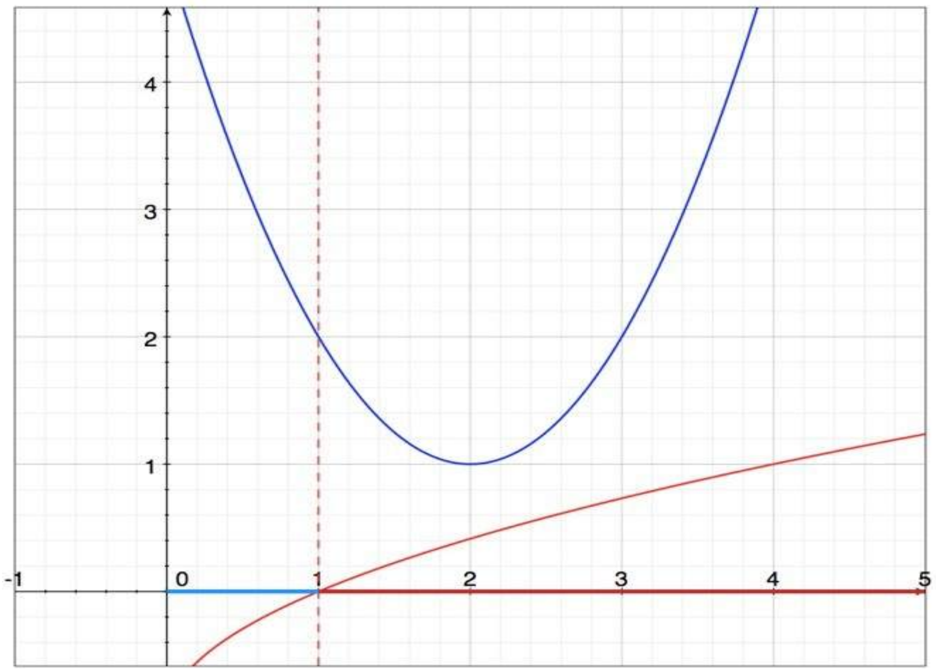
Example



Objective: $f(x) = (x-2)^2 + 1$

Constraint: $c(x) \leq \sqrt{x} - 1$

Example



Objective: $f(x) = (x-2)^2 + 1$

Constraint: $c(x) \leq \sqrt{x} - 1$

Bound: $x \geq 0$

Feasible Region: $0 \leq x \leq 1$

Penalty function methods

$$\pi(x, \rho) = \overbrace{f(x)}^{\text{Objective}} + \underbrace{\rho}_{\text{Penalty parameter (non-negative)}}\overbrace{\phi(x)}^{\text{Penalty function}}$$

1. Initialize penalty parameter
2. Initialize solution guess
3. Minimize penalized objective starting from guess
4. Update guess with the computed optimum
5. Go to 3., repeat

How to find the. Penalty function?

With the constraint $x - 5 \leq 0$, we need a penalty that is:

- 0 when $x - 5 \leq 0$ (the constraint is satisfied)
- positive when $x - 5$ is > 0 (the constraint is violated)

This can be done using the operation

$$P(x) = \max(0, x - 5)$$

which returns the maximum of the two values, either 0 or whatever $(x - 5)$ is.

We can make the penalty more severe by using

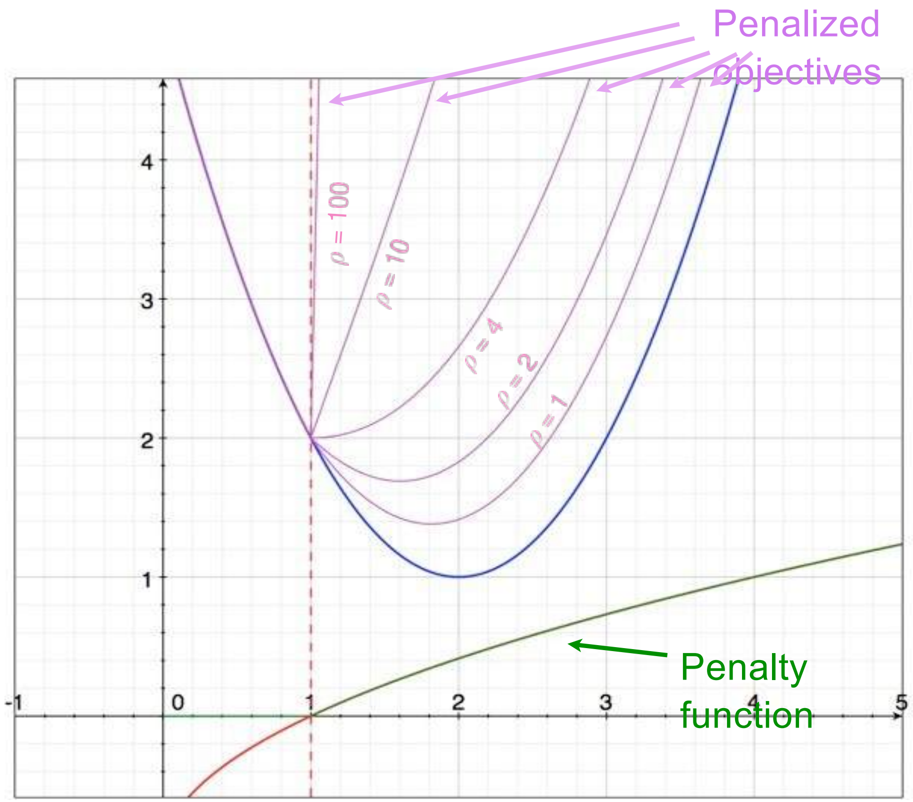
$$\boxed{P(x) = \max(0, x - 5)^2.}$$

This is known as a quadratic loss function.

Linear Exterior Penalty Function

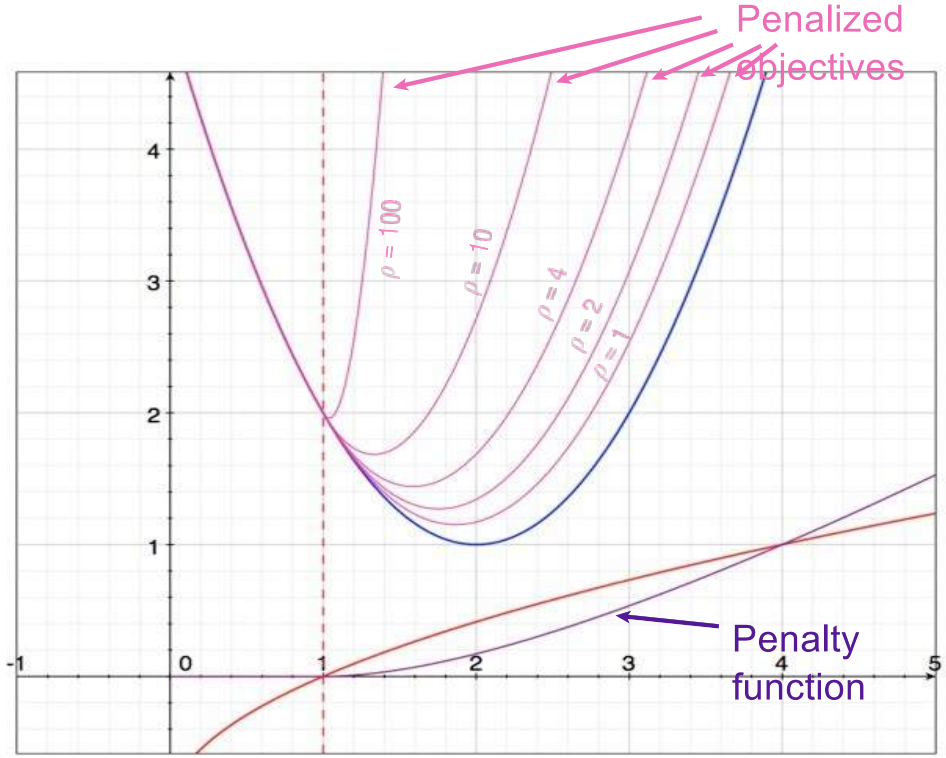
$$\phi_i(x) = \max(0, c_i(x) - u_i)$$

ui=RHS (negative null form)



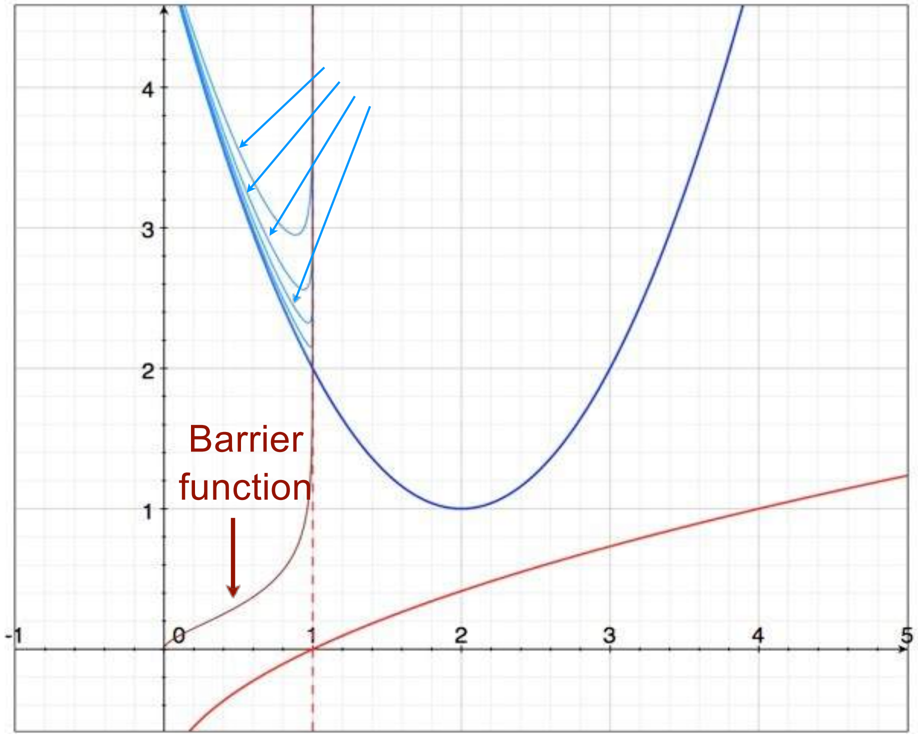
Quadratic Exterior Penalty Function

$$\phi_i(x) = \Big[\underbrace{\max(0, c_i(x) - u_i)}_{\text{Constraint violation}}\Big]^2$$



Interior-Point Methods

$$\pi(x,\mu) = f(x) - \underset{\substack{\uparrow \\ \text{Barrier parameter}}}{\mu} \overbrace{\log(u_i - c_i(x))}^{\text{Barrier function}}$$



Also called barrier methods, interior point methods ensure that each step is feasible

This allows premature termination to return a nearly optimal, feasible point

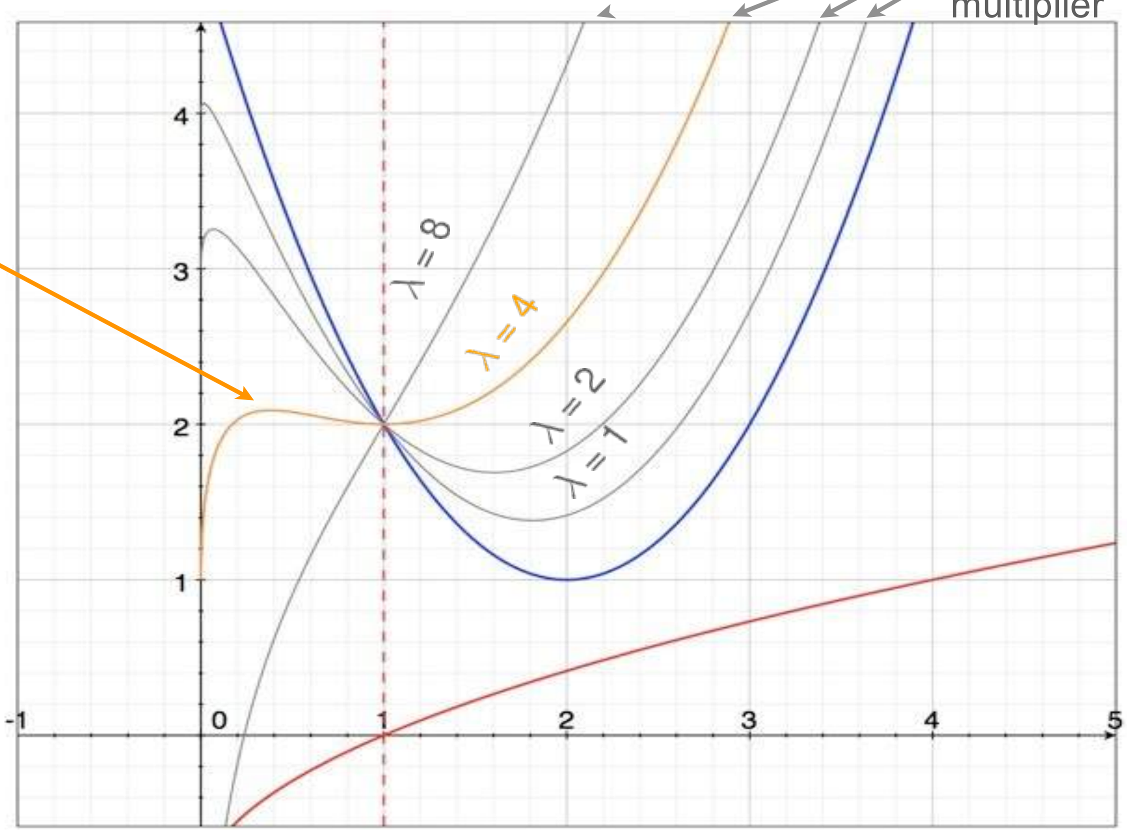
Barrier functions are implemented similar to penalties but must meet the following conditions

1. Continuous
2. Non-negative
3. Approach infinity as x approaches boundary

Lagrange Multipliers

Lagrangians for various values of the Lagrange multiplier

There is an optimal λ for which we obtain the constrained solution in x by minimizing the Lagrangian for that λ



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Summary

- Constraints are requirements on the design points that a solution must satisfy
- Some constraints can be transformed or substituted into the problem to result in an unconstrained optimization problem
- Analytical methods using Lagrange multipliers yield the generalized Lagrangian and the necessary conditions for optimality under constraints
- A constrained optimization problem has a dual problem formulation that is easier to solve and whose solution is a lower bound of the solution to the original problem

Summary

- Penalty methods penalize infeasible solutions and often provide gradient information to the optimizer to guide infeasible points toward feasibility
- Interior point methods maintain feasibility but use barrier functions to avoid leaving the feasible set

Supplementary materials & lecture notes

CHALLENGE #3 Reformulate the problem

$$\min_{x} f(x) = (x_1 - 3)^2 + (x_2 - 3)^2$$

subject to

$$h(x) = x_1 + x_2 - 4 = 0$$

to an unconstrained optimization problem using penalization.

$$P(x, R) = f(x) + \Omega(R, g(x), h(x))$$

$$\Omega = Rh_j^2(x)$$

find $x1^*, x2^*$ for $R = [10{,}100{,}1000{,}10000, 10\hat{}5]$

Supplementary materials

CHALLENGE #3 Reformulate the problem

https://colab.research.google.com/drive/1rGHIklpVM4Gqy9eq1
Y10fIQW9umIjeEx

Let's start minimizing

- Read and finish the notebook called 00-intro.ipynb and 02-constrained-optimization.ipynb

Part2: MultiObjective Optimization

Trade offs?

MultiObjective Optimization (MOO)

Definition:

Multi-objective optimization or Pareto optimization (also known as multi-objective programming, vector optimization, multicriteria optimization, or multiattribute optimization) is an optimization problem that involves

more than 1 objective function to be optimized simultaneously.

→ optimal decisions need to be taken in the presence of trade-offs between two or more conflicting objectives

https://ojs.aaai.org/aimagazine/index.php/aimagazine/article/download/2198/2030/

MultiObjective Optimization (MOO)

Main message: Multiobjective optimization is somewhat of a misnomer - you actually have to have predefined weightings for each of the objectives you care about, or implement them as constraints

https://openmdao.github.io/PracticalMDO/Notebooks/Optimization/multiobjective.html

Trade off

When doing multiobjective optimization,

I strongly recommend performing multiple single-objective optimizations instead of using a multiobjective optimizer.

- Let's view aerostructural wing design for an example. In a simple trade-off, if you extend the wingspan you get more lift and aerodynamic performance, but your structural masses and costs become greater to withstand the larger load.

- This trade-off means that you cannot maximize the aerodynamic performance and minimize the structural weight - there must be some sort of a balancing act. In reality we care about the total performance of the airplane, more than any subdiscipline, so our objective function usually captures effects from multiple disciplines at once.

Multi-Objective Optimization

$$\min f_i(x), \qquad i = 1, \dots, d$$

Subject to $\quad g(x) \geq, \quad h(x) = 0$

$$F(x) = [f_1(x), \dots . f_d(d)]$$

We know how to do this: $\boxed{\min f\,(x)}$

Solution: $f(x) = \sum_i w_i f_i(x)$

Multi-Objective Optimization

$$\min f_i(x), \qquad i = 1, \dots, d$$

Subject to $\quad g(x) \geq, \quad h(x) = 0$

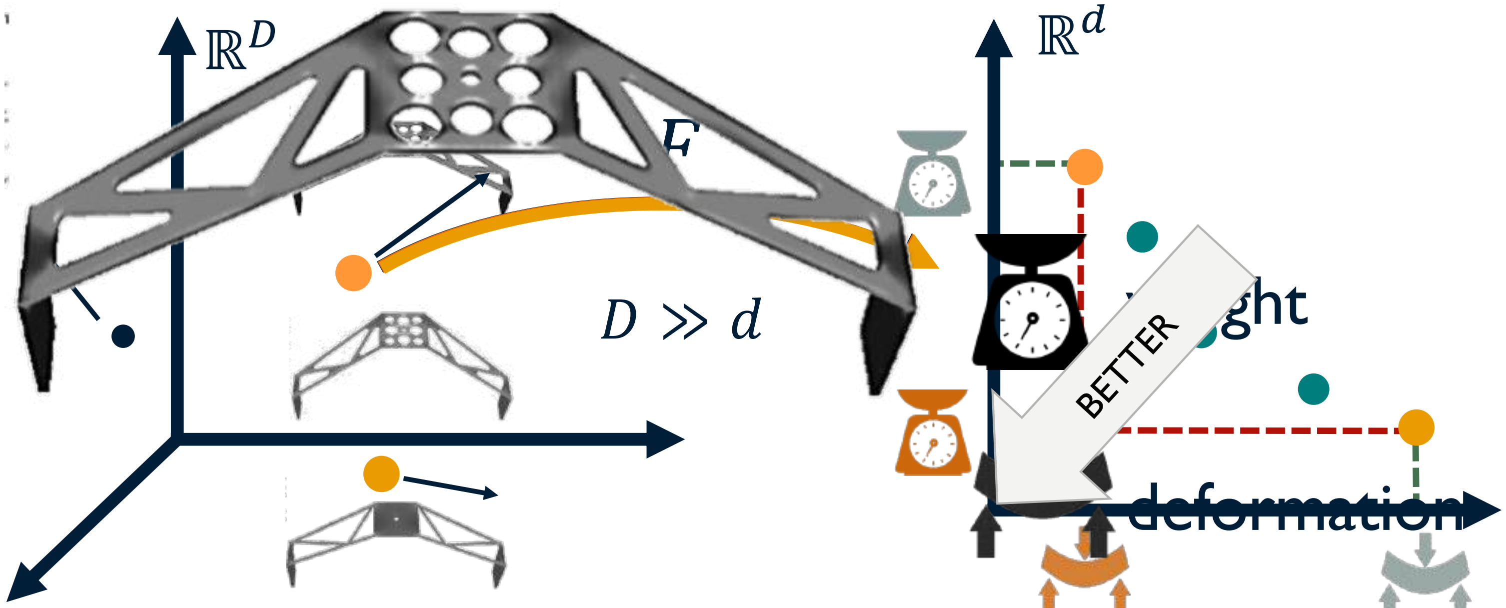
$$F(x) = [f_1(x), \dots . f_d(d)]$$

We know how to do this: $\boxed{\min f\,(x)}$

Solution: $f(x) = \sum_i w_i f_i(x)$

How do you pick the weights?

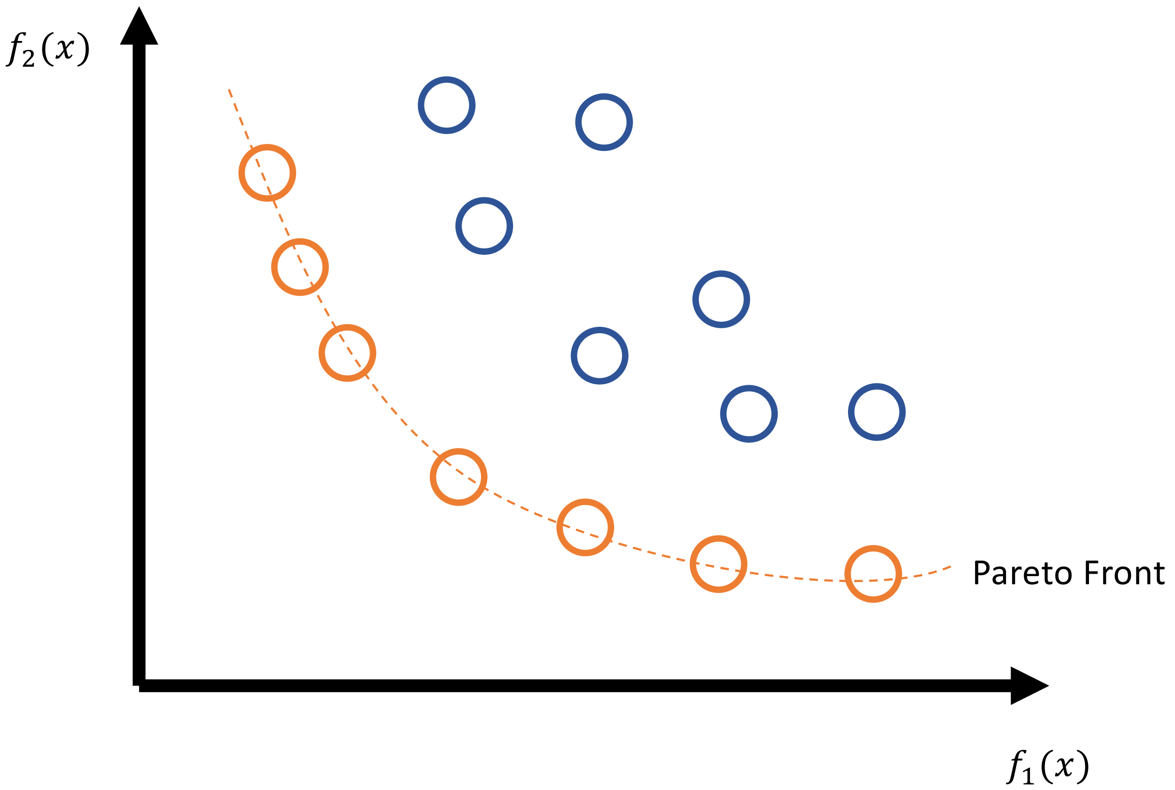
When Objectives are Conflicting



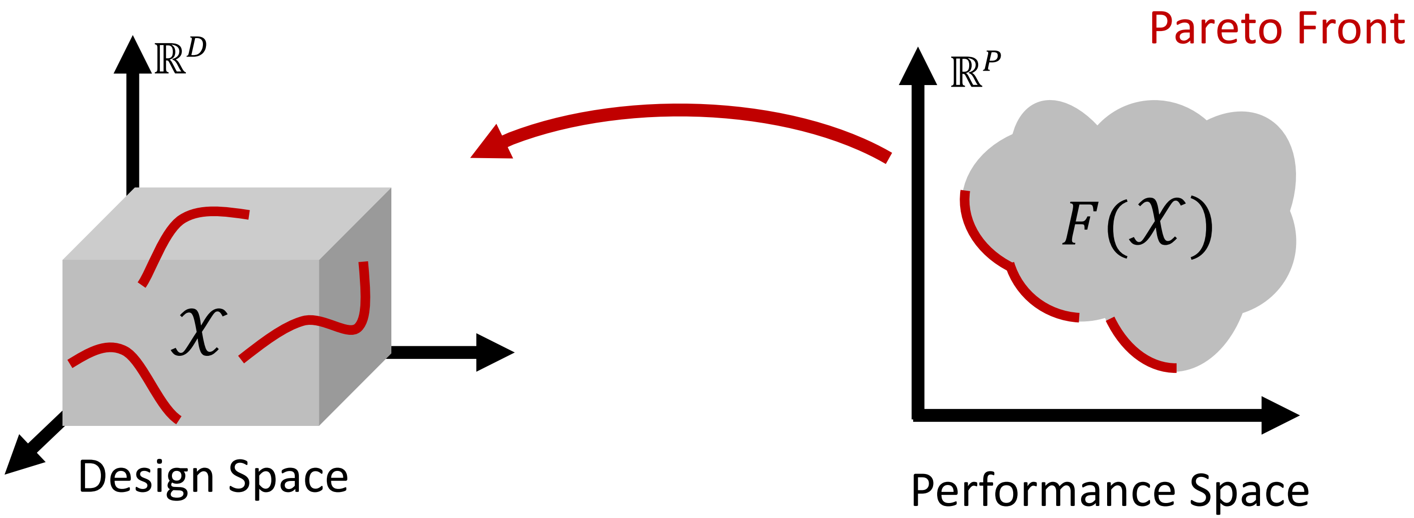
Design Space

Performance Space

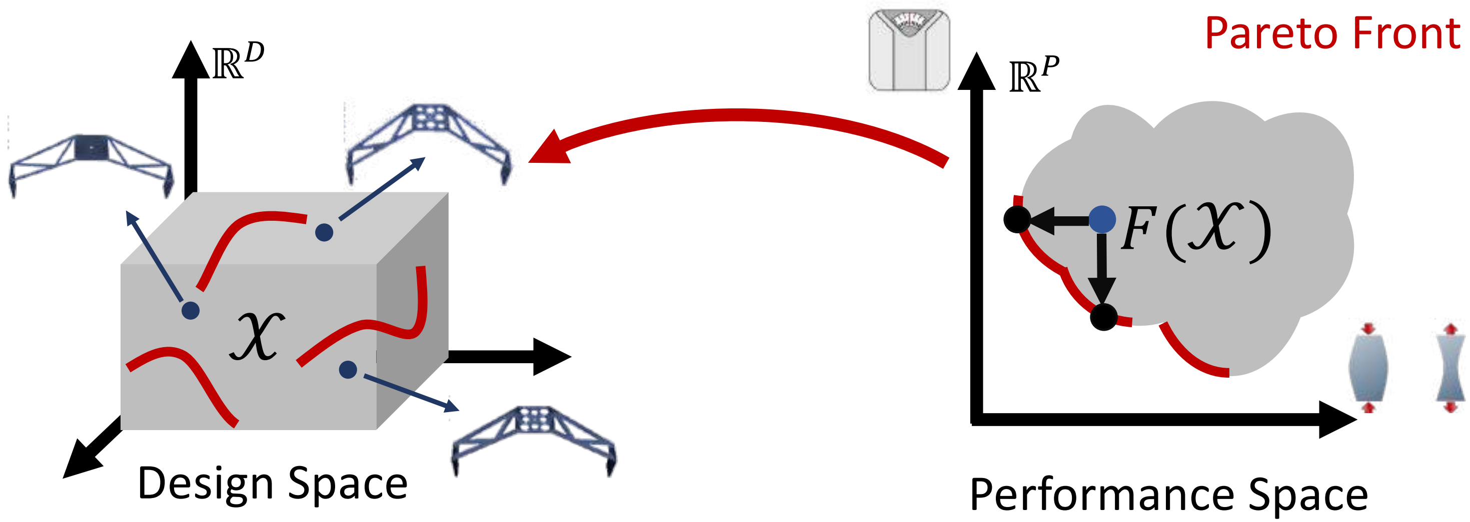
For Minimization



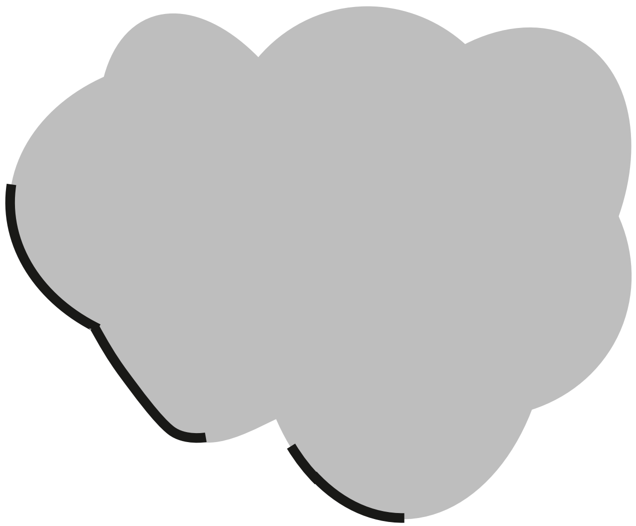
Space of Optimal Solutions



Space of Optimal Solutions



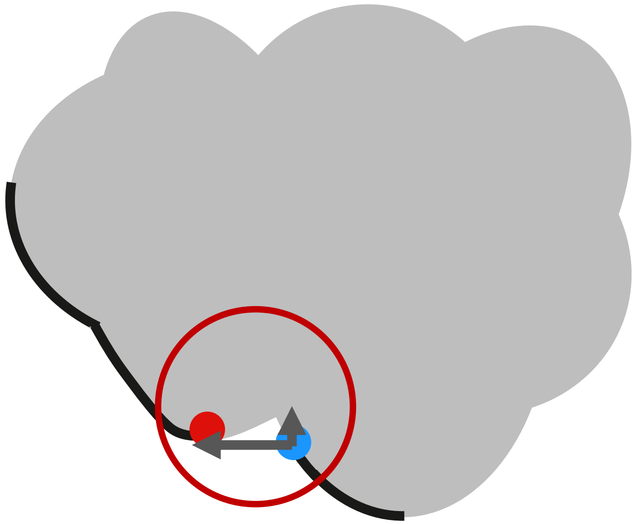
The Geometry of the Front



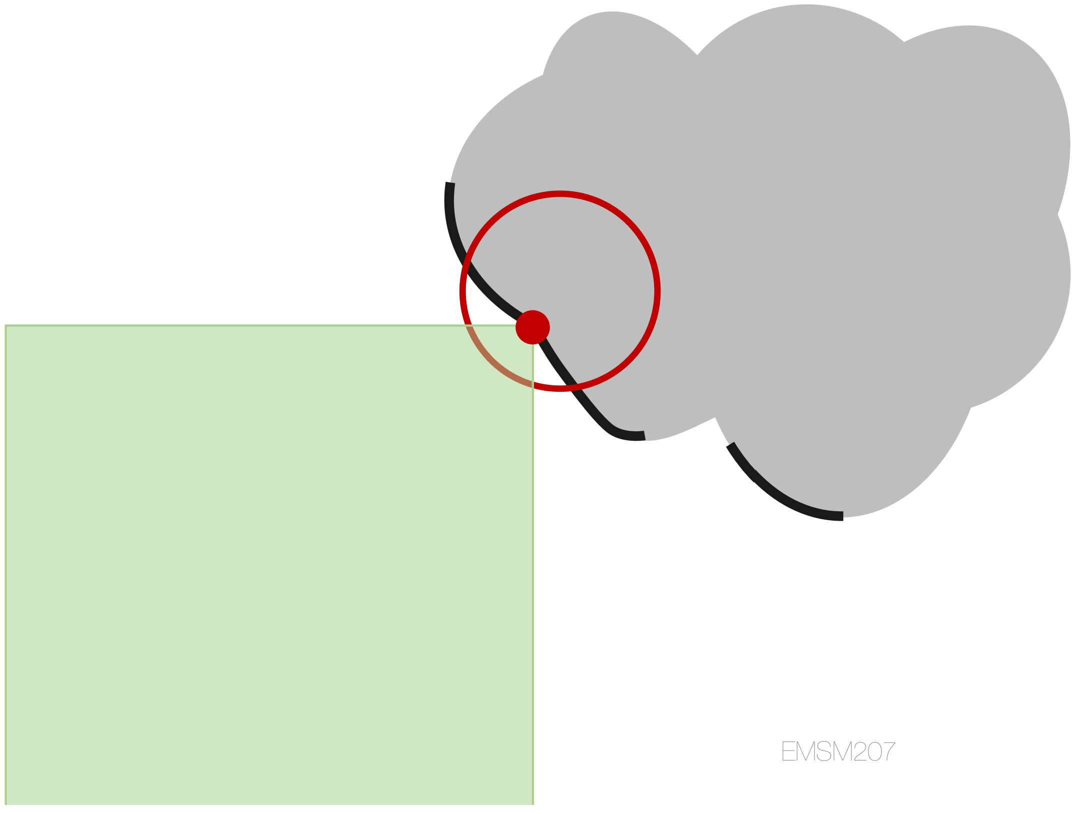
Not a straight line!

Solution: $f(x) = \sum_i w_i f_i(x)$ ☹

The Front Can Have Gaps



The Front Can Have Non-Convex Regions



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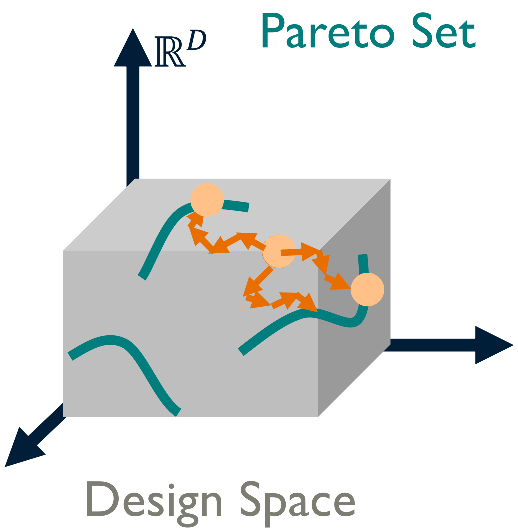
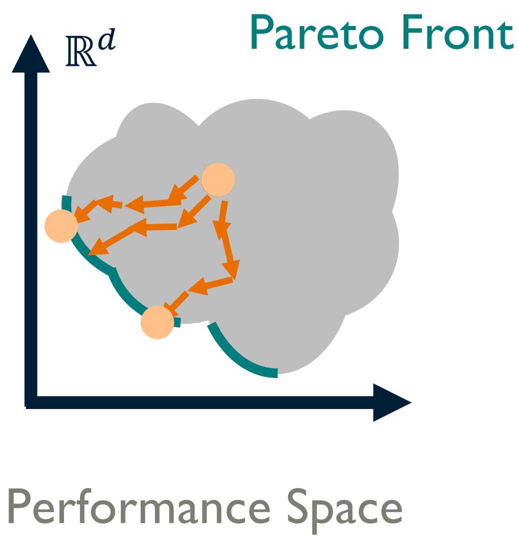
Pareto Front Discovery

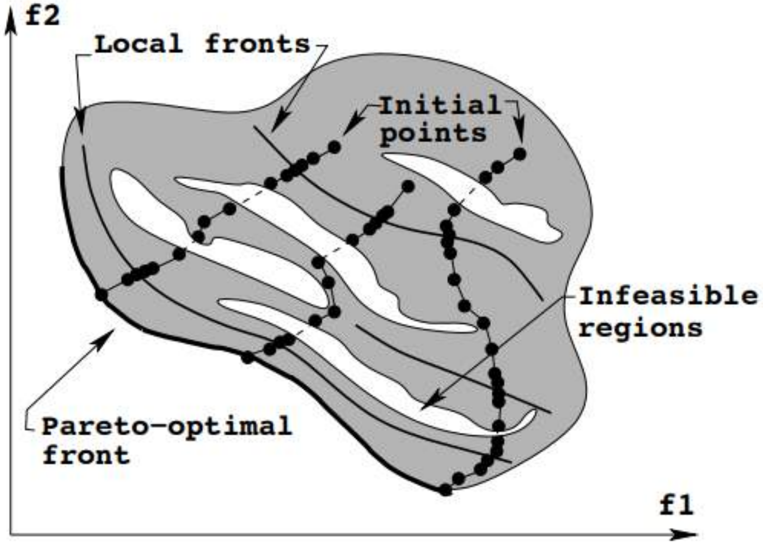
Main Challenge:

- Converge to optimal solutions
- Diverse set that describes the full front





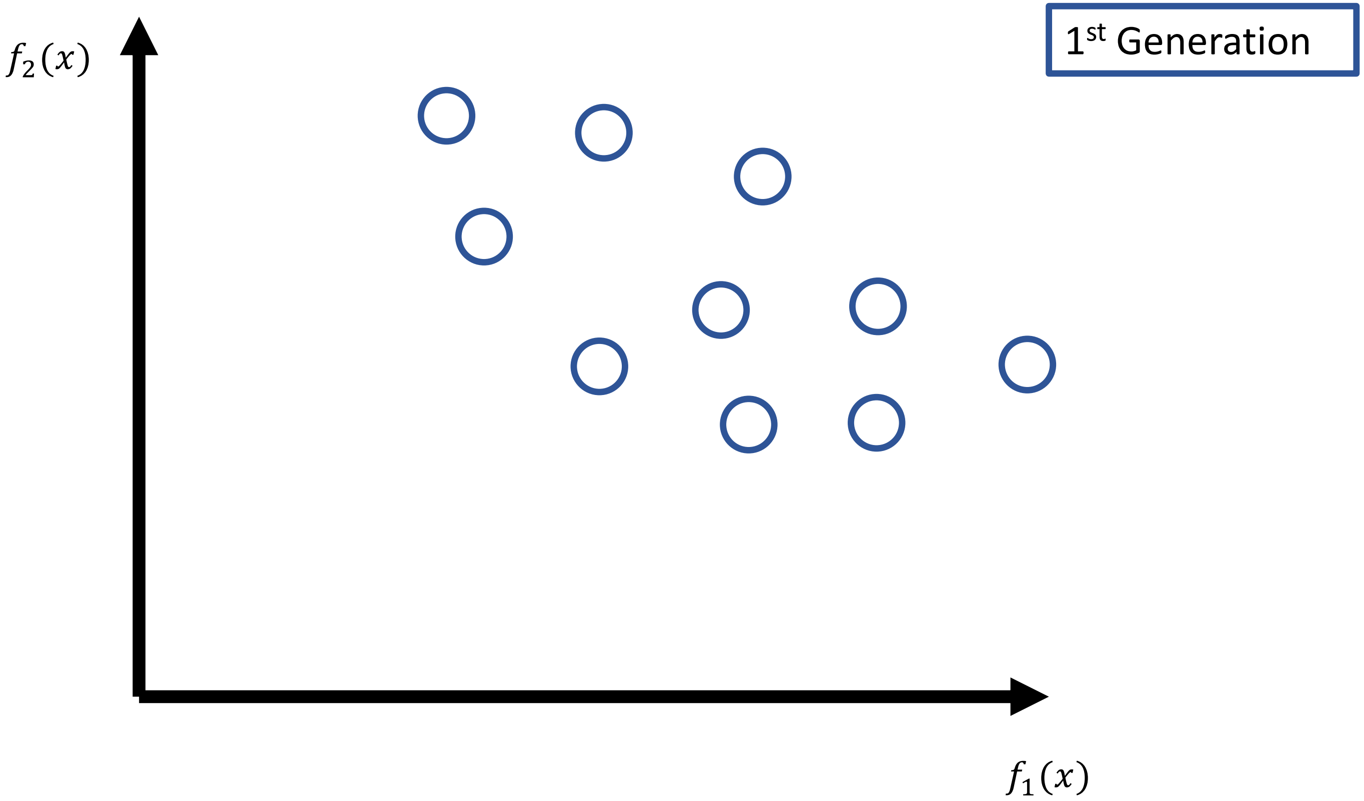
Problem: Each Single Objective Optimization is not SIMPLE!



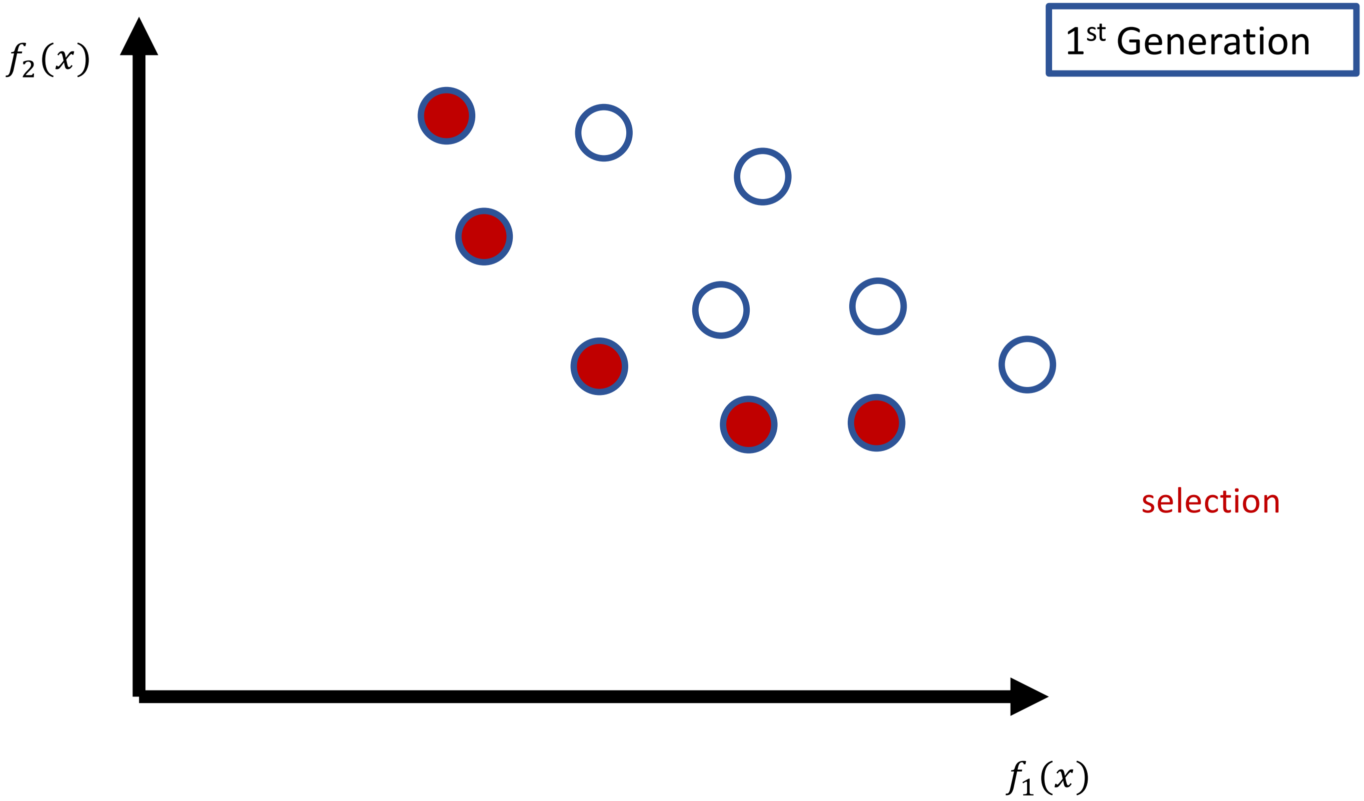
Move many points in parallel towards the front at the same time?

Source: Deb et al 2002

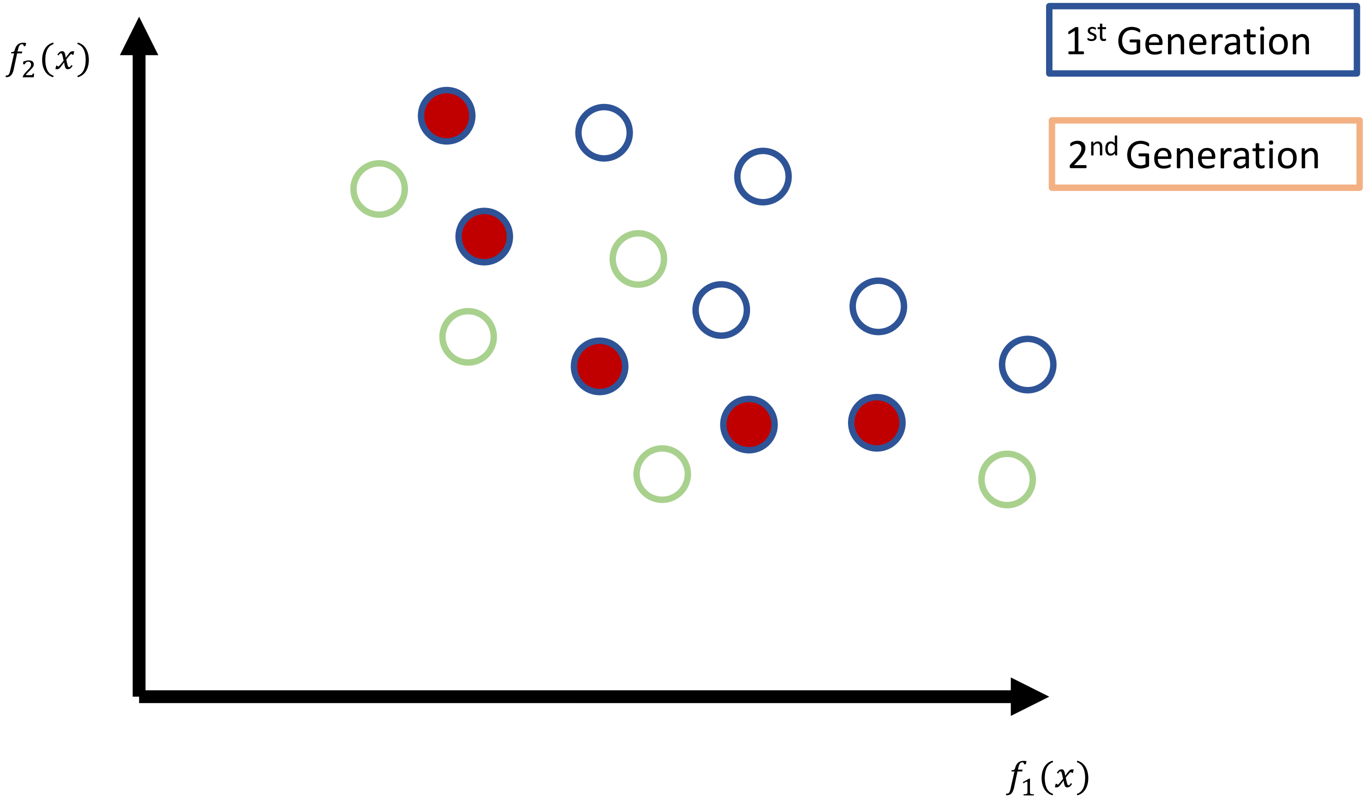
Evolutionary Algorithms



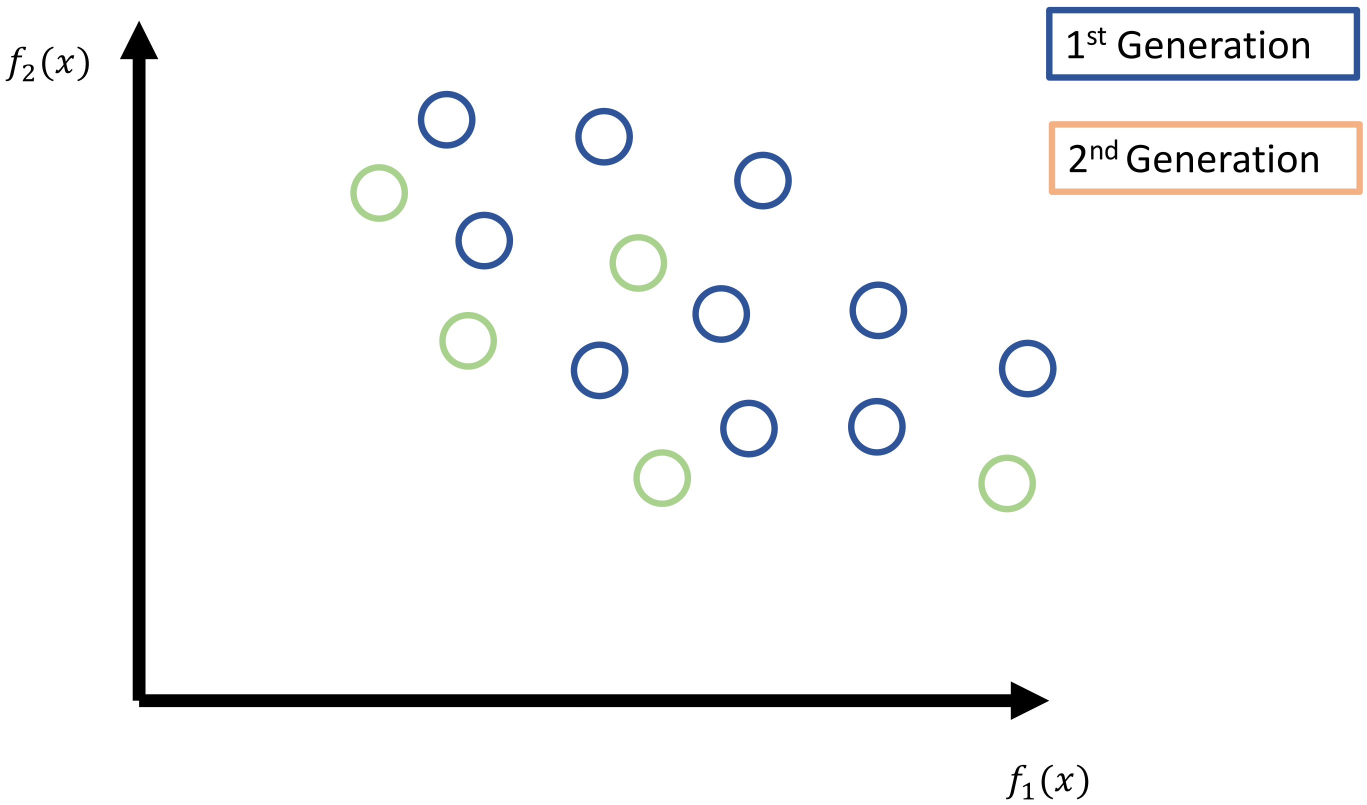
Evolutionary Algorithms



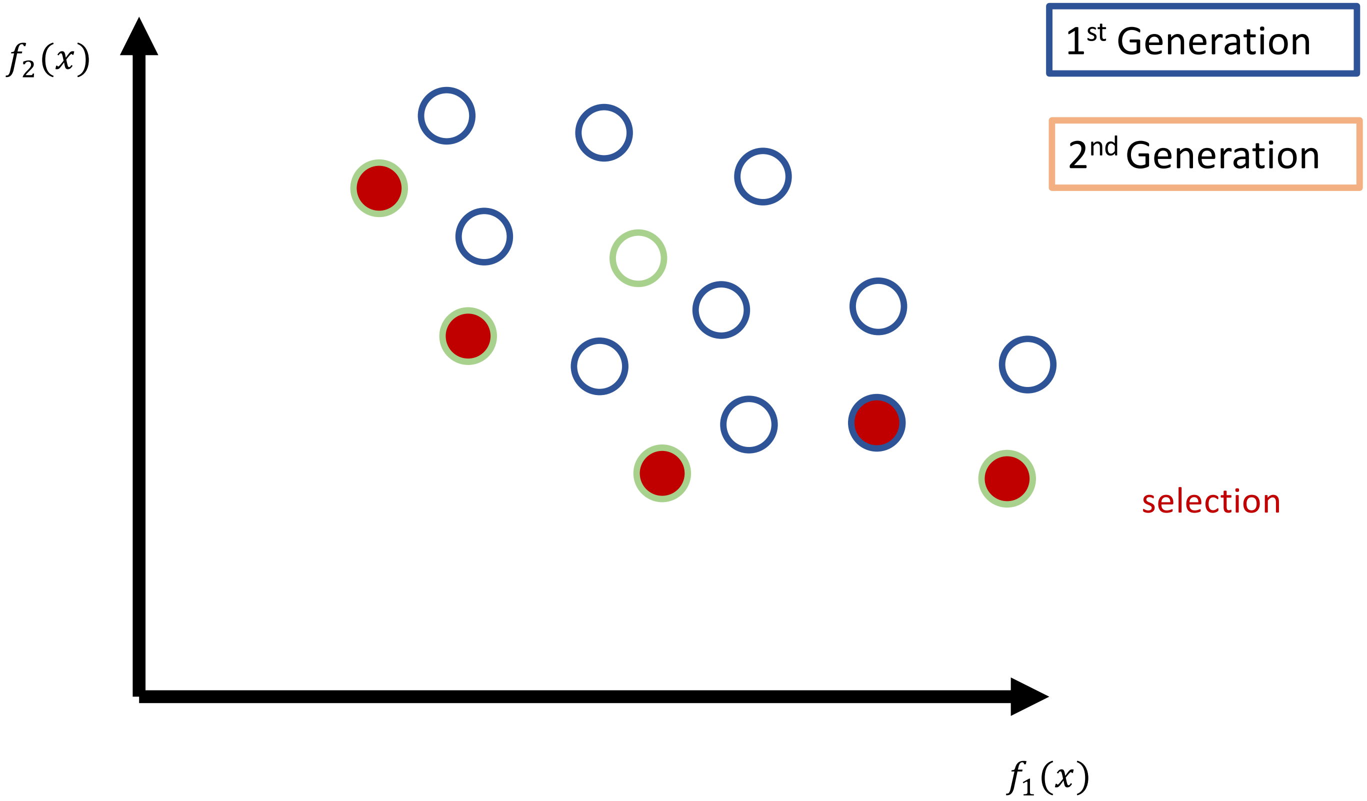
Evolutionary Algorithms



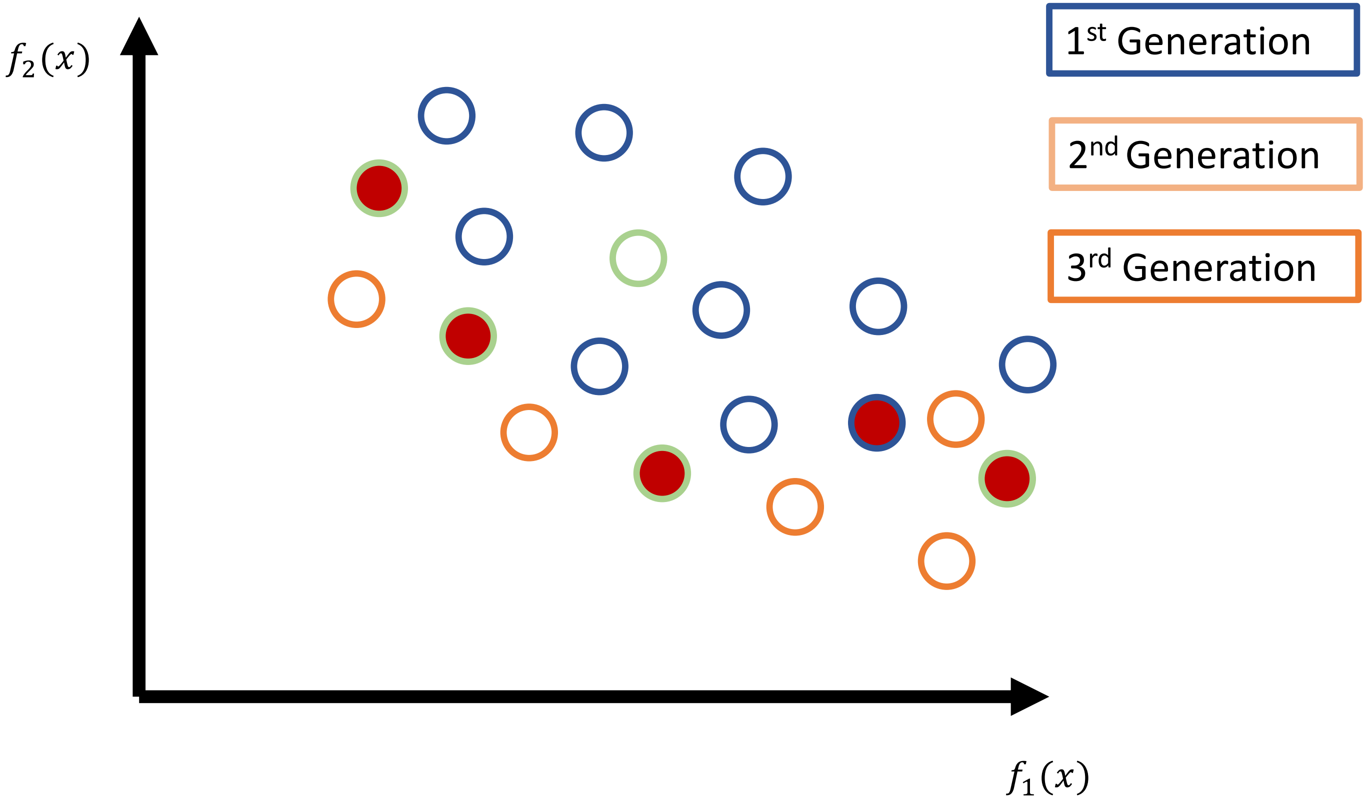
Evolutionary Algorithms



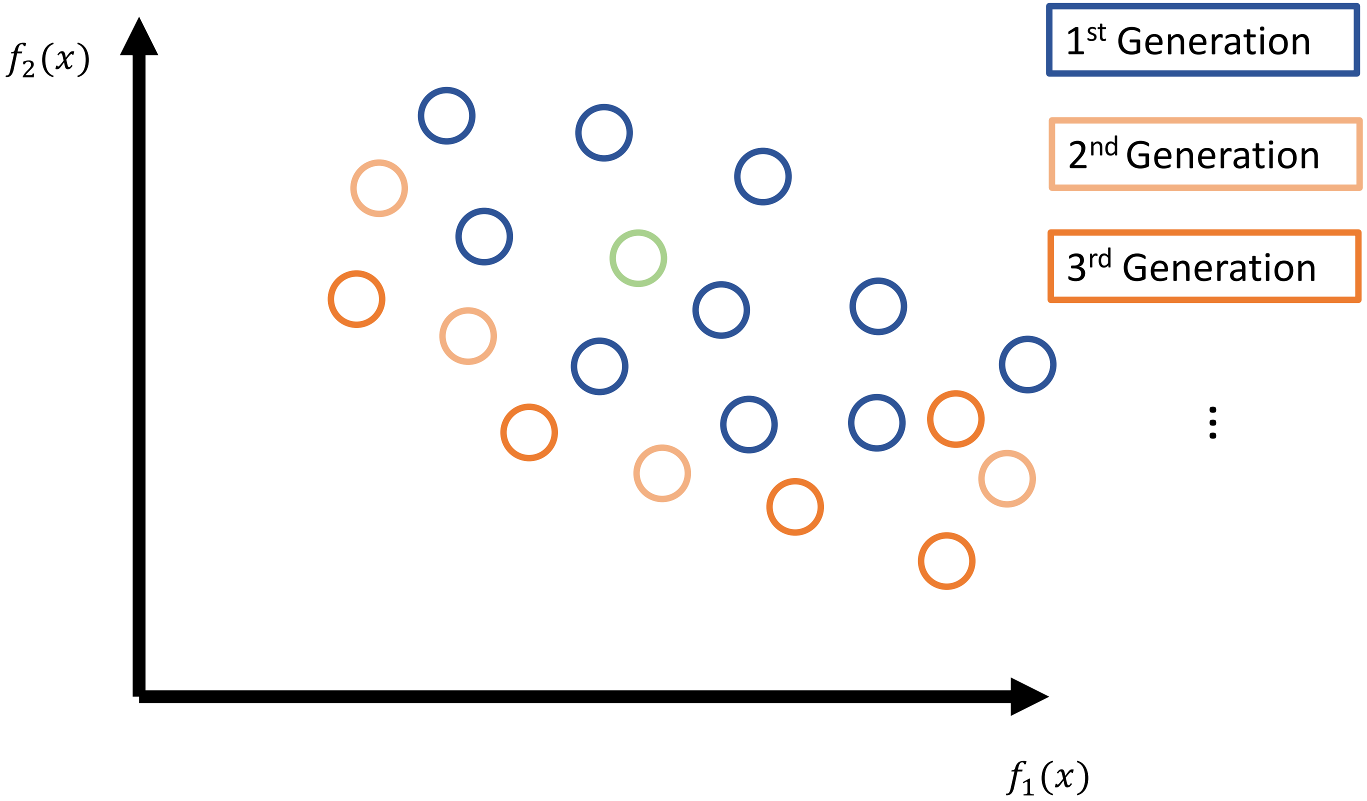
Evolutionary Algorithms



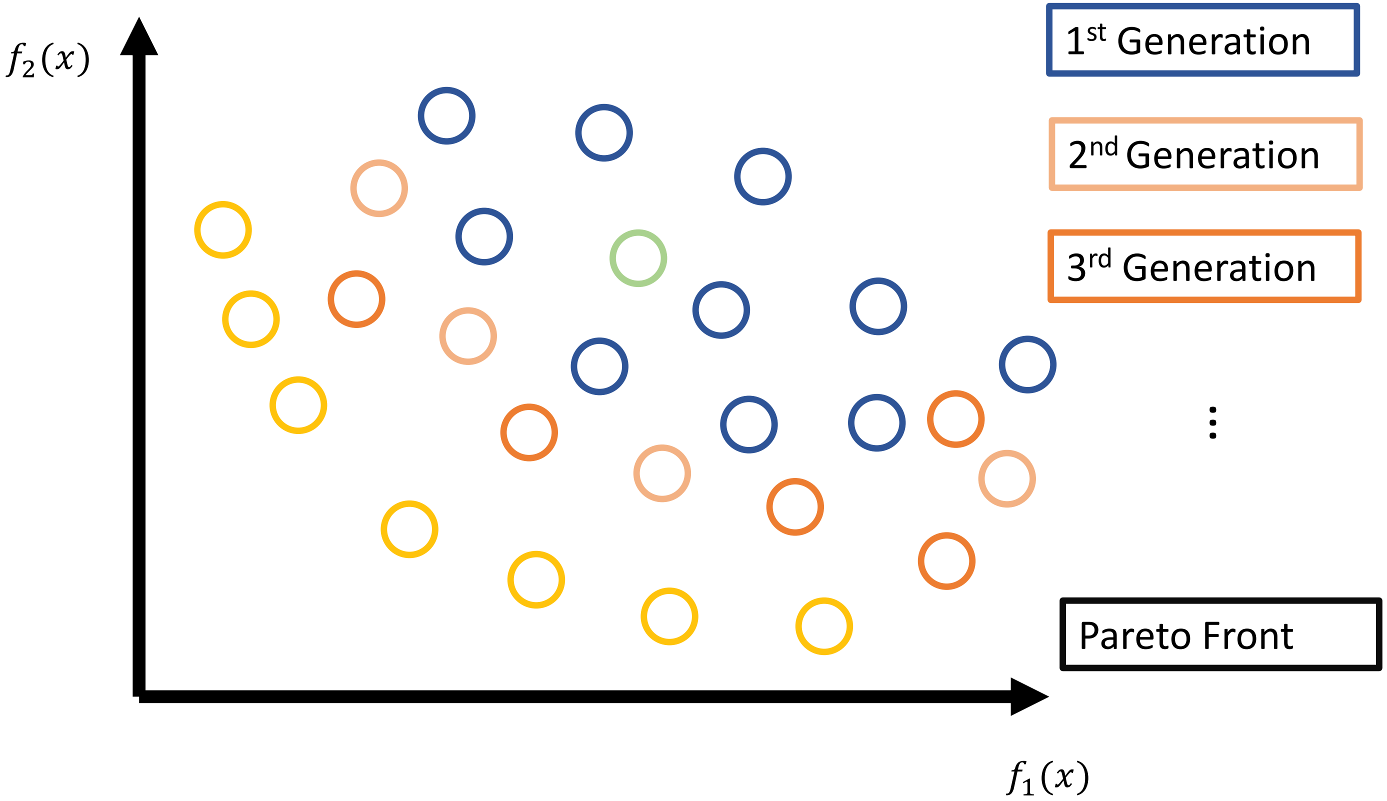
Evolutionary Algorithms



Evolutionary Algorithms



Evolutionary Algorithms



Elitist Non-dominated Sorting GA or NSGA-II

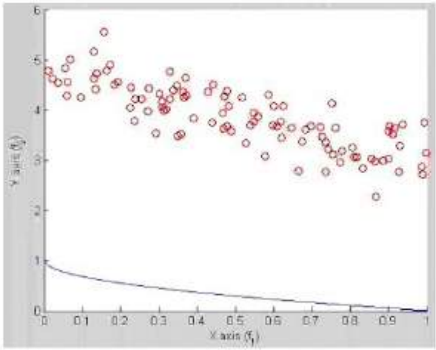


Figure 10: Initial population.

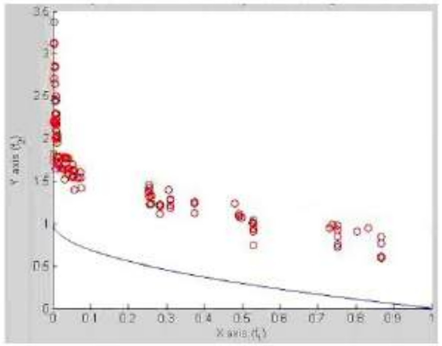


Figure 11: Population at generation 10.

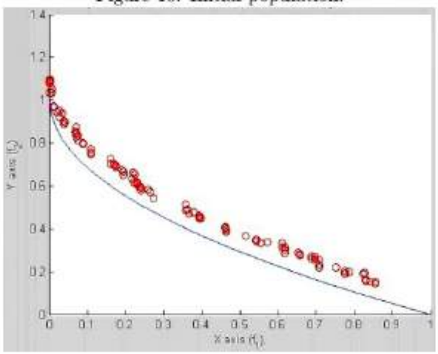


Figure 12: Population at generation 30.

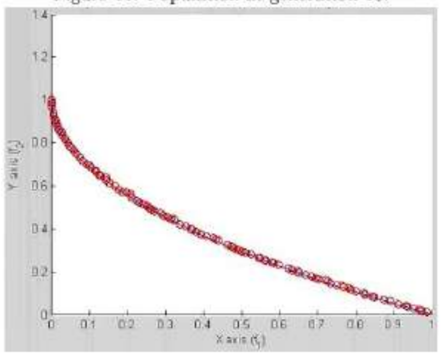


Figure 13: Population at generation 100.

Source: Deb et al 2002

To finish Complementary materials online

CHALLENGE #4 Could you use PYMOO?

https://pymoo.org/getting_started/preface.html

https://colab.research.google.com/drive/1EjHgXfbP21WvgWmGPGE7CRh2w42ejCJz

In Practice

Weighted sum method

The weighted sum method simply combines multiple objective functions by adding them together with some weights on each function.

An example is shown here,

$$f_{\mathrm{obj}}(x) = \alpha g(x) + \beta h(x)$$

where $g(x)$ and $h(x)$ are two objectives we're trying to optimize simultaneously

and α and β are weighting variables for each of the objective functions.

Weighted sum method

If you cared much more about the value of $g(x)$, you would set α to be larger than β.

However, it's not that simple as $g(x)$ and $h(x)$ might have radically different magnitudes.

If $g(x)$ is on the order 5,000 while $h(x)$ is on the order 0.1 ,

this would cause the optimizer to prioritize changing the design to make $g(x)$ smaller.

Weighted sum method

An example of this is if we're trying to optimize the structural weight (large magnitude) and the coefficient of drag C_D (small value) for an aircraft:

- $f_{obj}(x) = f_{\text{structural weight}}\,(x)[kg] + f_{C_D}$

If you care about both objectives approximately equally, you should scale both objective functions by an appropriate amount to make them approximately the same magnitude. The general form looks like this:

- $f_{obj}(x) = \alpha \frac{g(x)}{g_0} + (1-\alpha)\frac{h(x)}{h_0}$

where g_0 and h_0 are those magnitude scalars.

In the previously mentioned example, this might look like:

- $f_{obj}(x) = \alpha \frac{f_{\text{structural weight}}\,(x)^{[kg]}}{2700[\ kg]} + (1-\alpha)\frac{f_{C_D}}{0.025}$

Epsilon constraint method

- The epsilon-constrained (ϵ-constrained) method is another way to perform multiobjective optimization where one objective function is minimized by the optimizer while the other objective functions are constrained to specific values. This ensures that a given design is optimal is one objective for a given value in the other objective functions.
- The ϵ-constrained method is the preferred way to create Pareto fronts because it produces a more robust curve and avoids rare situations where the Pareto front is non-convex that may be troublesome if using the weighted-sum method.

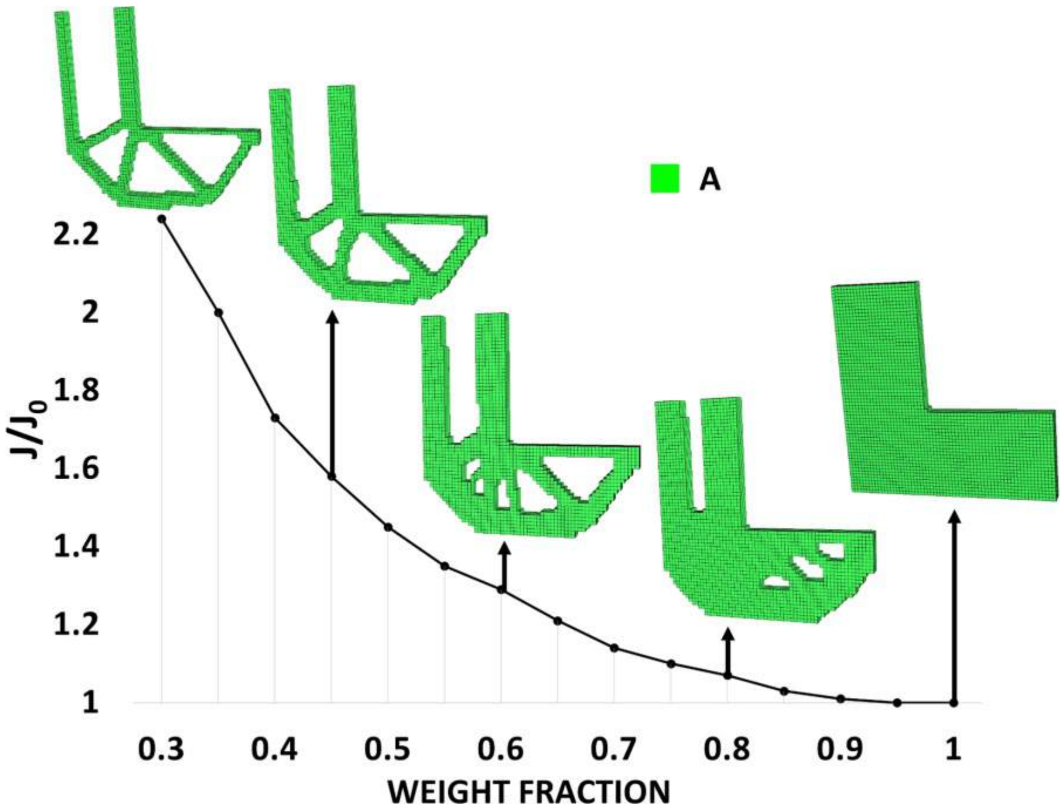
```
prob.model.add_subsystem('paraboloid_1',
om.ExecComp('g = (x-3)**2'), promotes=['*'])
prob.model.add_subsystem('paraboloid_2',
om.ExecComp('h = (x+1)**2 + 3*x'),
promotes=['*'])
prob.model.add_subsystem('objective',
om.ExecComp('f = alpha * g + beta * h'),
promotes=['*'])
```

Pareto Compliance vs volfrac

A topology optimization problem based on the power-law approach, where the objective is to minimize compliance can be written as

$$\left.\begin{aligned} \min_{\mathbf{x}}: &\quad c(\mathbf{x}) = \mathbf{U}^T\mathbf{K}\mathbf{U} = \sum_{e=1}^{N} (x_e)^p\, \mathbf{u}_e^T\, \mathbf{k}_0\, \mathbf{u}_e \\ \text{subject to}: &\quad \frac{V(\mathbf{x})}{V_0} = f \\ : &\quad \mathbf{K}\mathbf{U} = \mathbf{F} \\ : &\quad \mathbf{0} < \mathbf{x}_{\min} \leq \mathbf{x} \leq \mathbf{1} \end{aligned}\right\}, \tag{1}$$

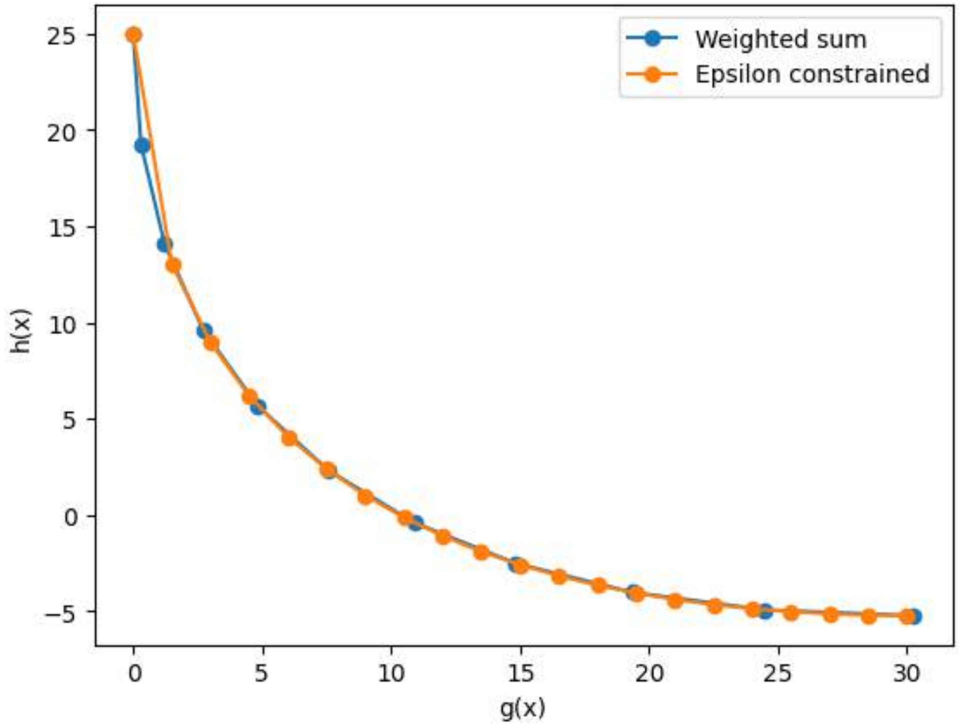
where $\mathbf{U}$ and $\mathbf{F}$ are the global displacement and force vectors, respectively, $\mathbf{K}$ is the global stiffness matrix, $\mathbf{u}_e$ and $\mathbf{k}_e$ are the element displacement vector and stiffness matrix, respectively, $\mathbf{x}$ is the vector of design variables, $\mathbf{x}_{\min}$ is a vector of minimum relative densities (non-zero to avoid singularity), N $(= \texttt{nelx} \times \texttt{nely})$ is the number of elements used to discretize the design domain, p is the penalization power (typically $p = 3$), $V(\mathbf{x})$ and V_0 is the material volume and design domain volume, respectively and f (`volfrac`) is the prescribed volume fraction.



Epsilon constraint method

This method also allows you to easily control the spacing for a Pareto front. Given a spread of $g(x)$ values for constraints, you can directly set the spacing produced by a series of optimizations.

For a bi-objective optimization, a good way to know what bounds to use for the constraint values are to run two optimizations first: one unconstrained using the first objective and the other unconstrained with the second objective only. From those two optimizations, the resulting values of the second and first objectives, respectively, could be your constraint limits.



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