10-bar Truss optimization with SciPY

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1 Problem Setup

One of the most famous Aerostructures is here under study and depicted in the next figure. A space truss consists of 10 bars. The initial model bar and node layout. This cantilever truss has 10 independent variables.

The material of the truss elements is Aluminium 1050-H14 whose characteristics are: Young modulus 69GPa , and density of 2700Kg/m^3.

Point load is F = 10kN, as shown in Fig.1.

The axial stress admissible is $\pm 170 MPa$ for all bars, and minimum area of all members is limited to 100 mm^2. .

The initial cross section area for all calculations is $300 mm^2$

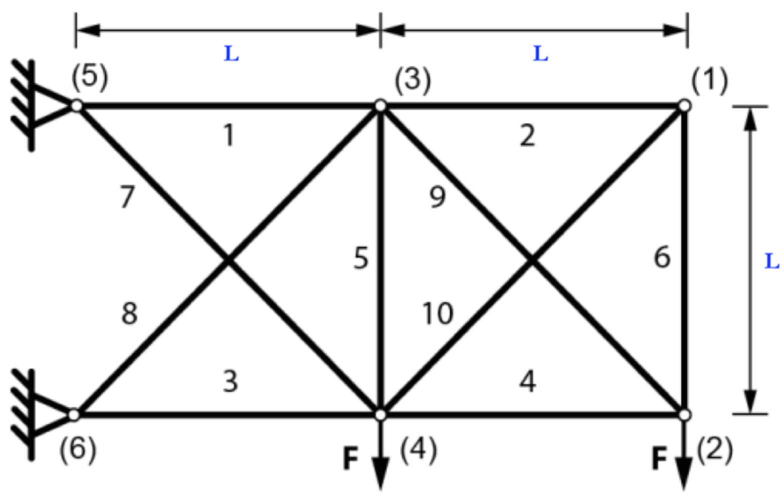

Figure 1: 10-bar truss

2 Theoretical background

Although not needed to solve the problem, an overview of the equations is provided. A truss element is the simplest type of structural finite element and only has an axial degree of freedom. The theory and derivation for truss elements are simple, but for our purposes, we skip to the result. Given a two-dimensional element oriented arbitrarily in space (Fig. D.12), we can relate the displacements at the nodes to the forces at the nodes through a stiffness relationship.

In matrix form, the equation for a given element is $K_e d = q$. In D Test Problems detail, the equation is

$$\frac{EA}{L}\begin{bmatrix} c^2 & cs & -c^2 & -cs \\ cs & s^2 & -cs & -s^2 \\ -c^2 & -cs & c^2 & cs \\ -cs & -s^2 & cs & s^2 \end{bmatrix}\begin{bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \end{bmatrix} = \begin{bmatrix} X_1 \\ Y_1 \\ X_2 \\ Y_2 \end{bmatrix}$$

where the displacement vector is $d = [u_1, v_1, u_2, v_2]$. The meanings of the variables in the equation are described in Table D.1.

Table D. 1 The variables used in the stiffness equation.

Variable	Description
X_i	Force in the x-direction at node i
Y_i	Force in the y-direction at node i
E	Modulus of elasticity of truss element material
A	Area of truss element cross section
L	Length of truss element
c	$\cos\phi$
s	$\sin\phi$
u_i	Displacement in the x-direction at node i
v_i	Displacement in the y-direction at node i

The stress in the truss element can be computed from the equation $\sigma = S_e d$, where σ is a scalar, d is the same vector as before, and the element S_e matrix (really a row vector because stress is one-dimensional for truss elements) is

$$S_e = \frac{E}{L}\begin{bmatrix} -c & -s & c & s \end{bmatrix}.$$

The global structure (an assembly of multiple finite elements) has the same equations, $Kd = q$ and $\sigma = Sd$, but now d contains displacements for all of the nodes in the structure, $d = [x_1, x_2, \ldots, x_n]$. If we have n nodes and m elements, then q and d are $2n$-vectors, K is a ($2n \times 2n$) matrix, S is an $(m \times 2n)$ matrix, and σ is an m-vector. The elemental stiffness and stress matrices are first computed and then assembled into the global matrices. This is straightforward because the displacements and forces of the individual elements add linearly.

After we assemble the global matrices, we must remove any degrees of freedom that are structurally rigid (already known to have zero displacement). Otherwise, the problem is ill-defined, and the stiffness matrix will be ill-conditioned.

Given the geometry, materials, and external loading, we can populate the stiffness matrix and force vector. We can then solve for the unknown displacements from

$$Kd = q$$

With the solved displacements, we can compute the stress in each element using

$$\sigma = Sd$$

2.1 Mass minimization

$$\begin{cases} \min W(A,n,l) = \sum_{i=1}^{i=n} \rho_i A_i l_i \text{ with } A = (A_1, \ldots, A_n) \\ \text{subjected to } \begin{cases} A_{\min} \leq A_i \leq A_{\max} \text{ for } i = 1, \ldots, n \\ \sigma_{\min} \leq \sigma_i \leq \sigma_{\max} \\ u_{\min} \leq u_j \leq u_{\max} \text{ for } i = 1, \ldots, n \end{cases} \\ \text{for } j = 1, \ldots, k \end{cases} . .$$

3 Possible extensions

- Use analytical gradients and or autograd.
- Add a constraint on maximum displacement of one critical point.
- Since the Euler critical buckling load equation (3) considers axial compression force, cross sectional characteristics, and bar length, buckling needs to be checked for all bars for each iteration. The proposed Euler buckling constraint defined by Euler's critical load is given in the following expressions:

$$|F_{Ai}^{comp}| \leq F_{Ki} \text{ for } i \quad 1, \ldots, n$$
$$F_{Ki} = \frac{\pi^2 \cdot E_i}{l_i^2}$$

- In order to allow for shape optimization coorddinates of nodes 1 and 3 are variables in examples which optimize this aspect of the truss. Node 5, as it is a support is not set as a variable. Topology is limited to the removal of 6 elements at most.
- Add a nice visualization of the stress

4 Simply with sciPy

Please check the notebook !!