Physics 111: Theoretical Mechanics
Computational Project

Most problems in physics (and certainly all of the interesting ones!) cannot be solved strictly by analytical means and need to be studied with the aid of numerical methods. In this project, you will derive and numerically solve the equations of motion for an analytically intractable system, and then write a report showing any physical insights you have gained into the system from your study. **The study and report will be done in groups of one or two.**

Physical System for Study

You will select a physical system that will be the subject of your numerical project. A number of suggested project ideas are listed in the Project Ideas document. You are also free to design and study your own system. If you pick your own project, please run it by me first.

If you wish to use one of the ideas from the Project Ideas section, please sign your name in the Google Sheet linked on Canvas. There is no cap on the number of groups doing each project, but you are not allowed to give or get help from anyone working on the *same* project.

Analysis Steps

Your analysis should proceed in a few iterated stages, all of which will be included in the final report:

1. **Derivation of Equations of Motion:** In order to numerically study a system, you first need to derive the Lagrangian and the equations of motion! Choose one or more generalized coordinates (one is needed for each degree of freedom), derive the Lagrangian in these coordinates, and derive and simplify the Euler-Lagrange equations as much as possible (feel free to use any conserved quantities).

2. **Simplifying the Equations of Motion:** We want to reduce the number of parameters in our equation to as few as possible, and to normalize them to "reasonable" values. There is a procedure for re-casting your differential equations purely in terms of dimensionless numbers and variables: this will make it easier to systematically solve and study the solution to your equations of motion (see the "Dimensionless Equations" note on Canvas).

3. **Solving the Equations of Motion:** Using a numerical program of your choice (or of your own making), solve the equations of motion for a *single* choice of the parameters and initial conditions. To solve the equation, you will also need to specify a time over which you integrate the equation. I am personally most familiar with Python and its open-source libraries and can provide the most support for it. But you are free to use whatever you like (Mathematica, MATLAB, C++ libraries, etc).

4. **Characterizing the Equations of Motion:** This is the meat of the project. Solve the equations of motion for various choices of parameters in the equation, initial conditions, and integration times. You should try to do this in a quasi-systematic way: first, see if you can identify a choice of parameters and/or initial conditions that you expect will make your system undergo "simple" motion (*e.g.,* simple oscillations). Then, slowly move away from these special points and identify more interesting/rich behaviours of the system.

Report

You will submit a report that describes your physical system and articulates your findings. Your report should use be written in an accessible manner such that a fellow Physics 111 student could read your report and understand the main features of your physical system. The report must be written in LaTeX. The report should contain:

- An introduction, featuring an overview of the physical system to be studied.
- The definition of the generalized coordinates for the system, and the derivation of the Lagrangian and Euler-Lagrange equations for the system. The reduction of the Euler-Lagrange equations to dimensionless form and a list of the dimensionless parameters of the model should be provided
- Sections that illustrate the different types of dynamical behaviour exhibited by the system. These sections should include both numerical results in the form of (labelled, captioned) plots indicating the evolution of the system for particular parameters/initial conditions, as well as a physical explanation of the observed effects where possible. Craft a physical argument and then show only the (well-labeled!) plots needed to support the physical arguments in the text.
- A conclusion summarizing your main results and indicating any open questions for future study.

Accompanying the report should be file(s) containing the code you used to numerically solve your equations of motion and make any plots. You don't have to include the code for all plots, but only representative examples that illustrate how you did your work.

On Canvas you will find a link to a LaTeXtemplate on Overleaf.com, which is MUCH easier than trying to install LaTeXand the packages you need on your own computer. Click on the link on Canvas (you may need to make a free overleaf acocunt), click on Menu on the top left, and then copy project. The figures for

Timeline: see Course Calendar for due dates

- Selection of project topic on Google Sheet
- Checkpoint 1: Derivation of Euler-Lagrange equations: Due as part of PS7;
- Checkpoint 2: Euler-Lagrange equations in dimensionless form and numerical solution to equations of motion for *one* point;

- Final report: 11:59 pm on Friday of exam week; automatic extension to 9 am the following Monday (this is a hard, final deadline).

Grading Breakdown

The report is worth 10% of your final grade. The grade will be determined as follows:

- On-time submission of checkpoints: 5%
- Derivation of equations and motion and overview of system: 15%
- Computational methods for solving equations of motion: 15%
- Identification of several phases of motion and their characterization: 40%
- Format, clarity, and accessibility of report: 25%

Tips for a Successful Project

1. One way to study the system with generalized coordinate $q(t)$ is to plot $q(t)$ as a function of t. However, we can often gain more information from a parametric plot of $x = q(t)$, $y = \dot{q}(t)$. This is called a **state space plot**; we will discuss them in class, and you can read about them in Taylor 12.7.

2. Try to vary the parameters, initial conditions, and/or integration time in a systematic way (for example, see what the effect is of holding all but one parameter fixed)

3. Classify the motions according to their qualitative characteristics. For example, try looking for parameter choices that give rise to simple, bounded motions like approximate harmonic oscillation, or unbounded, monotonic rotations (imagine a pendulum that just keeps swinging around by 2π angles in an arc). For more chaotic motions, try see if you can identify any discernible patterns in the motion, even if they are somewhat irregular (sort of like how we observed elliptical precession even in gravitational 3-body motion).

4. Try to be quantitative about identifying the *boundaries* between phases. For example, try to identify for what parameters your system executes bounded vs. unbounded motion (or whatever behaviours you see in your particular system!)

5. Vary the initial conditions to see how sensitive the resulting dynamics are to the initial conditions. When the system depends sensitively on the initial conditions, this is a sign of **chaotic motion**. If you observe chaotic motion, try changing the initial condition by only some small amount $q(t = 0) \to q(t = 0) + \Delta q$, and then try to quantify how different the trajectories are at the final time, t_{f}.

6. Try changing the time interval over which you are solving the equations of motion. It should be long enough to capture at least a few oscillations or characteristic motions of the system. Does integrating the system for longer time give rise to unexpected

motions? If you observe simple patterns amidst complicated, chaotic motion, how does the time interval of the simple pattern compare with the time interval over which sudden, chaotic jumps happen?