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% FRE 6251 Numerical and Simulation Techniques in Finance

% Assignment #4

% Name: Surya L Gurung ID: 0449604

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Brief Reports of Assignment #4:

(1). This simulation has four matlab files, three functions and one script files.

Function “**geometricAvgOption**” gives a closed-form solution for the geometric average Asian options, call or put. Function “**AssetPaths**” gives a matrix of asset values at each time step of many paths and each column represents one path. Function “**AsianVarReduction**” estimates price of Arithmetic average Asian options, call or put, using Control Variates method in which geometric average Asian option price is used as control variable. Script file “**VarReduction**” calls all three functions to compute option prices based on Control Variate method and without using it for different sets of paths ,up to 1,000,000 paths, in the multiple of 10. It also calculates correlation between Arithmetic average option price and Geometric average option price, two variances of Arithmetic average option price with and without Control Variate method. Finally, it shows the speed gain depending on the correlation.

In “VarReduction”, I used example of an asset with following parameters and values:

Example asset:

So = 100;

K = 110;

r = 0.05;

sigma = 0.15;

T = 1;

cORp = 1; % this is for call option

tSteps = 20;

pNum = 100;

row = 1;

VarReduction calls function “AsianVarReduction(100, 110, 0.05, 0.15, 1, 1, 20, 100)” for different value of pNum which increase at multiple of 10 up to 1,000,000.

AsianVarReduction uses Control Variate method to estimate Arithmetic average Asian call option price using closed-form solution of Geometric average of Asian call option and option prices calculated by using Monte-Carlo simulation. So, if Yi is price of arithmetic average Asian option and Xi is price of geometric average Asian option calculated using Monte-Carlo simulation and EX is closed form solution for geometric average option then, control variate estimator Yb is given by

Yb = mean(Yi - b\*(Xi - EX)) and b = covarianc of X and Y/ variance of X;

Y = mean(Yi);

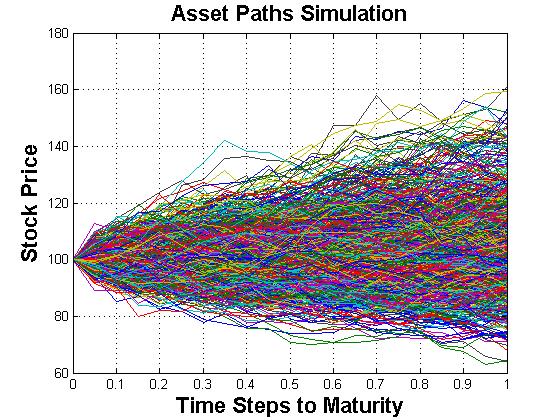
variance of Yb = (varY - 2\*b\*stdX\*stdY\*corrXY + b^2\*varX)/pNum;

variance of Y = varY / pNum;

speed = 1/ (1- correlation^2)

Please take a look at function for more detail

Results of running “VarReduction.M” included:



**The result of running the Matlab script file “varReduction.m” for upto 1,000,000 paths.**

>> VarReduction

Result Interpretation:

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Paths = Number of path

Yb = Estimation of Arithmetic Average Asian Option Price using variance reduction

Y = Estimation of Arithmetic Average Asian Option without variance reduction

CorrXY = Correlation coefficient between X and Y

varYb = variance of Yb

varY = variance of Y

Speed = gain depending on correlation

Paths Yb Y CorrXY varYb varY Speed

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100 1.096190 0.995531 0.998237 0.000198 0.056265 284

1000 1.067641 1.096778 0.999307 0.000013 0.009084 722

10000 1.072325 1.052651 0.999233 0.000001 0.000908 652

100000 1.072071 1.038073 0.999243 0.000000 0.000087 661

1000000 1.071510 1.039317 0.999257 0.000000 0.000009 673

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% VarReduction.M

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% Monte Carlo Simulation of Asian Option Variance reduction

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% Parameter Description:

% So = initial price of asset

% K = strike price

% r = risk-free rate

% sigma = standard deviation of asset

% T = time to maturity in year

% cORp = option type, 1 for call and -1 for put put option

% tSteps = number of time steps

% pNum = number of paths

%-----------

% Example 1:

%-----------

So = 100;

K = 110;

r = 0.05;

sigma = 0.15;

T = 1;

cORp = 1; % this is for call option

tSteps = 20;

pNum = 100;

row = 1;

disp('Result Interpretation:')

disp('-----------------------')

disp('Paths = Number of path')

disp('Yb = Estimation of Arithmetic Average Asian Option Price using variance reduction')

disp('Y = Estimation of Arithmetic Average Asian Option without variance reduction')

disp('CorrXY = Correlation coefficient between X and Y')

disp('varYb = variance of Yb')

disp('varY = variance of Y')

disp('Speed = gain depending on correlation')

disp(' ')

disp(sprintf('%s', ' Paths Yb Y CorrXY varYb varY Speed' ))

disp(sprintf('%s', ' ---------- -------- -------- ----------- --------- -------- ------' ))

while pNum < 100000

[Yb, Y, corrXY, varYb, varY, speed] = AsianVarReduction(So, K, r, sigma, T, cORp, tSteps, pNum);

if row == 1

disp(sprintf(' %d %0.6f %0.6f %0.6f %0.6f %0.6f %d', pNum, Yb, Y, corrXY, varYb, varY, speed))

elseif row == 2

disp(sprintf(' %d %0.6f %0.6f %0.6f %0.6f %0.6f %d', pNum, Yb, Y, corrXY, varYb, varY, speed))

elseif row == 3

disp(sprintf(' %d %0.6f %0.6f %0.6f %0.6f %0.6f %d', pNum, Yb, Y, corrXY, varYb, varY, speed))

elseif row == 4

disp(sprintf(' %d %0.6f %0.6f %0.6f %0.6f %0.6f %d', pNum, Yb, Y, corrXY, varYb, varY, speed))

elseif row == 5

disp(sprintf(' %d %0.6f %0.6f %0.6f %0.6f %0.6f %d', pNum, Yb, Y, corrXY, varYb, varY, speed))

elseif row == 6

disp(sprintf(' %d %0.6f %0.6f %0.6f %0.6f %0.6f %d', pNum, Yb, Y, corrXY, varYb, varY, speed))

end

pNum = pNum \* 10;

row = row + 1;

end

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function [Yb, Y,corrXY,varYb, varY ] = AsianVarReduction(So, K, r, sigma, T, cORp, tSteps, pNum)

% Parameter Description:

% So = initial price of asset

% K = strike price

% r = risk-free rate

% sigma = standard deviation of asset

% T = time to maturity in year

% cORp = option type, 1 for call and -1 for put put option

% tSteps = number of time steps

% pNum = number of paths

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%closed form solution of geometric average asian option

EX = geometricAvgOption(So, K, r, sigma, T, cORp);

%matrix of asset paths, each column represents one path

stockPaths = AssetPaths(So, r, sigma, T, tSteps, pNum);

%discounted payoff of arithmetic average asian option

Yi = exp(-r \* T) \* max(cORp \*(mean(stockPaths)- K), 0);

%discounted payoff of geometric average asian option

Xi = exp(-r \* T) \* max(cORp \*(geomean(stockPaths)- K), 0);

%Y is process for arithmetic average

%X is process for geometric average

covXY = cov(Xi, Yi); %covariance matrix of X and Y

covXY = covXY(1,2);

varX = var(Xi); % variance of X

varY = var(Yi); % varicance of Y

stdX = sqrt(varX);

stdY = sqrt(varY);

corrXY = covXY/(stdX \* stdY); % correlation between X and Y

b = covXY/varX;

Yb = mean(Yi - b\*(Xi - EX));

Y = mean(Yi);

varYb = (varY - 2\*b\*stdX\*stdY\*corrXY + b^2\*varX)/pNum;

varY = varY / pNum;

if (pNum < 10000)

plot(0:T/tSteps:T,stockPaths,'Linewidth',1);

xlabel('Time Steps to Maturity','FontWeight','bold','Fontsize',16);

ylabel('Stock Price','FontWeight','bold','Fontsize',16);

title('Asset Paths Simulation','FontWeight','bold','Fontsize',16);

grid on

set(gcf,'Color','w');

end

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% Closed-form solution for Geometric Average Asian Option

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function [price] = geometricAvgOption(So, K, r, sigma, T, cORp)

% Parameter Description:

% So = initial price of asset

% K = strike price

% r = risk-free rate

% sigma = standard deviation of asset

% T = time to maturity in year

% cORp = option type, 1 for call and -1 for put put option

b = 0.5 \* (r- (sigma^2/6));

sigmaA = sigma / sqrt(3);

d1 = (log(So/K) + (b + 0.5 \* sigmaA^2)\* T)/(sigmaA \* sqrt(T));

d2 = d1 - sigmaA \* sqrt(T);

price = cORp\*(So\*exp((b - r)\*T)\*normcdf(cORp\*d1) - K\*exp(-r\*T)\*normcdf(cORp\*d2));

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% Matrix of asset prices, one column representing one path.

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function paths = AssetPaths(So,mu,sigma, T, tSteps,pNum)

% Parameters:

% So = initial asset price

% mu = expected return

% sigma = volatility

% T = maturity time in years

% tSteps = number of time steps

% pNum = number of paths

%

dT = T/tSteps; % size of one time step

drift = mu - (0.5 \* sigma^2); % drift of process

z = randn(tSteps, pNum); % matrix of normally distributed random values

% matrix of all paths

paths = So\*[ones(1,pNum); cumprod(exp(drift\*dT + sigma \* sqrt(dT)\* z),1)];