FRE 6251: Numerical and Simulation Techniques in Finance

Assignment #6

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**Brief Reports of Assignment #6: Using Importance Sampling for Portfolio Credit Risk**

(1). This project has three matlab files,two functions (**ExpoTilting.m andMCSvarReduction**) and one script files (**ETImportantSampling.m**).

Function **“ExpoTilting**” use Important Sampling Algorithm for Credit Losses to gives estimator for

P( L > x | Z) . In this function, I am considering very simple case, where there is one common factor Z all Xi  are in the form

Xi = ρ Z + for i = 1, 2, …… m

and all original default probability pi = p = 0.002, number of obligors m = 1000, all credit exposure Ci = 1 and Z is a scalar. Now we have,

where  =  = 2.88 for all Xi. Since ρ and Z are also same for all Xi,  is also same for all Xi.

So, = = m

and

Now the conditional default probability is given by

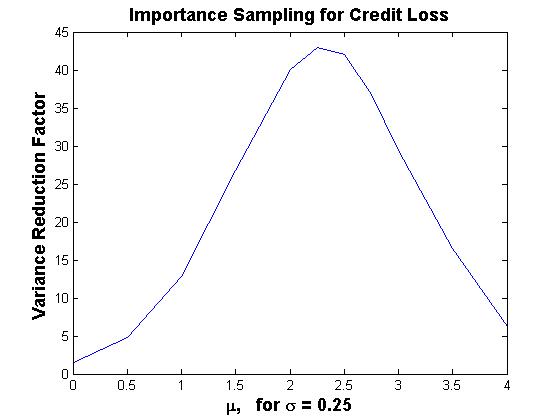
Given Z, default indicators all Yi takes 1 with probability and these are generated using binary distribution random number generator “binornd()” .

Finally, the estimator

exp(-µTZ + ½ µTµ - L + is calculated.

The function “MCSvarReduction” is used for calculation of variance without using “µ” to calculate Variance Reduction Factor. Only, script file “ETImportanceSampling” is executed at command window and this file calls both “ExpoTiltiling” and “MCSvarReduction”. Following are the outputs of running the script file. Because of running time I got the graph for only σ = 0.25 but other outputs are for all σ given in the book.

Please take a look the Code for more detail. I am able to get estimator for σ = 0.05, 0.1, 0.25. 0.5, and 0.8



**The result of running the Matlab script file “ETImportanceSampling.m”**

>> ETImportantSampling

rho x P(L > x)

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0.05 6 0.005789

0.10 6 0.010434

0.25 10 0.009311

0.50 26 0.008749

0.80 43 0.010307

>>

% The Script file “ETImportanceSampling.m”

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% Parameter Description:

% m = number of obligor in a portfolio

% Ci = credit exposure

% p = original default portfolio

% rho = correlation coefficient

% x = default threshold

%-----------

% Example 1:

%-----------

m = 1000;

Ci = 1;

p = 0.002;

rho = 0.05;

x = 6;

mu = [0, 0.5, 1, 1.5, 2, 2.25, 2.5, 2.75, 3, 3.5, 4];

disp(sprintf('%s', ' rho x P(L > x)' ))

disp(sprintf('%s', ' ------ ------ ---------' ))

for i= 1:length(mu)

[E\_mu(i), v1(i)] = ExpoTilting(m, Ci, p, rho, mu(i), x);

end

disp(sprintf(' %0.2f %d %0.6f', rho, x, mean(E\_mu)))

rho = 0.1;

x = 6;

for i= 1:length(mu)

[E\_mu(i), v1(i)] = ExpoTilting(m, Ci, p, rho, mu(i), x);

end

disp(sprintf(' %0.2f %d %0.6f', rho, x, mean(E\_mu)))

rho = 0.25;

x = 10;

for i= 1:length(mu)

[E\_mu(i), v1(i)] = ExpoTilting(m, Ci, p, rho, mu(i), x);

end

disp(sprintf(' %0.2f %d %0.6f', rho, x, mean(E\_mu)))

v3 = MCSvarReduction(m, Ci, p, rho, x);

vReductionFactors = v3./v1;

plot(mu, vReductionFactors,'Linewidth',1)

xlabel('\mu, for \sigma = 0.25','FontWeight','bold','Fontsize',14);

ylabel('Variance Reduction Factor','FontWeight','bold','Fontsize',14);

title('Importance Sampling for Credit Loss','FontWeight','bold','Fontsize',14);

rho = 0.5;

x = 26;

for i= 1:length(mu)

[E\_mu(i), v1(i)] = ExpoTilting(m, Ci, p, rho, mu(i), x);

end

disp(sprintf(' %0.2f %d %0.6f', rho, x, mean(E\_mu)))

rho = 0.8;

x = 43;

for i= 1:length(mu)

[E\_mu(i), v1(i)] = ExpoTilting(m, Ci, p, rho, mu(i), x);

end

disp(sprintf(' %0.2f %d %0.6f', rho, x, mean(E\_mu)))

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function [E\_mu, v1] = ExpoTilting(m, Ci, p, rho,mu, x)

format long g

xi = norminv(1-p, 0, 1);

for i = 1:100000

epsilon = randn(1, m);

Z = normrnd(mu, 1);

%Xi = rho \* Z + sqrt(1 - rho^2) \* epsilon;

pTilda = 1 - normcdf((xi - rho \* Z)/sqrt(1 - rho^2));

theta = log(x \* (1 - pTilda)/(pTilda \* (m - x)));

theta = max(theta, 0);

pTheta = (pTilda \* exp(theta\*Ci))/(pTilda \* exp(theta\*Ci) + 1 - pTilda);

L = binornd(m, pTheta);

cumulant = m \* log(pTilda\*exp(theta\*Ci) + 1 - pTilda);

indicator = (L > x);

estimatorWithMu(i) = exp(-mu\*Z + 0.5 \* mu^2 - theta \* L + cumulant)\* indicator;

end

E\_mu = mean(estimatorWithMu);

v1 = var(estimatorWithMu);

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function [v] = MCSvarReduction(m, Ci, p, rho, x)

format long g

xi = norminv(1-p, 0, 1);

for i = 1:100000

epsilon = randn(1, m);

Z = randn(1);

pTilda = 1 - normcdf((xi - rho \* Z)/sqrt(1 - rho^2));

theta = log(x \* (1 - pTilda)/(pTilda \* (m - x)));

theta = max(theta, 0);

pTheta = (pTilda \* exp(theta\*Ci))/(pTilda \* exp(theta\*Ci) + 1 - pTilda);

L = binornd(m, pTheta);

cumulant = m \* log(pTilda\*exp(theta\*Ci) + 1 - pTilda);

indicator = (L > x);

estimatorWithoutMu(i) = exp(- theta \* L + cumulant)\* indicator;

end

v = var(estimatorWithoutMu);