

Homework # 4

1. This is basically a straight forward differential equation problem. First we have $T^* ' = -sT^* \rightarrow T^*(t) = T_o^* e^{-st}$.

Then

$$V' = NsT^* - cV = NsT_o^* e^{-st} - cV$$

$$e^{ct} (V' + cV) = (e^{ct} V)' = NsT_o^* e^{(c-s)t} \quad c \neq s$$

$$\rightarrow e^{ct} V(t) - V_o = \frac{NsT_o^*}{c-s} (e^{(c-s)t} - 1)$$

or

$$V(t) = V_o e^{-ct} + \frac{NsT_o^*}{c-s} (e^{-st} - e^{-ct})$$

Assuming a quasi-steady state condition for the patient before treatment, $T_o^* = kV_o T_o / s$ and $NkT_o = c$
so

$$NsT_o^* = kNV_o T_o = cV_o \rightarrow$$

$$V(t) = V_o \left\{ e^{-ct} + \frac{c}{c-s} (e^{-st} - e^{-ct}) \right\}$$

$$= \frac{V_o}{c-s} \{ (c-s)e^{-ct} + ce^{-st} - ce^{-ct} \}$$

$$= \frac{V_o}{c-s} \{ ce^{-st} - se^{-ct} \}$$