(Special*) Homework Assignment #12

Due: by 4:00 pm, Tuesday, May 16.

* Special means here that I will treat the score of this assignment as extra credit unto the homework grade. Recall my policy — I will throw out your lowest two homework scores, add in the score from this assignment, and scale the distribution so that homework counts for 35% of your grade.

1. In class I derived the solution to the quarter plane heat equation problem

$$\begin{cases} u_t = D u_{xx} & x>0,\ t>0 \\ u(x,0)=0 & x>0 \\ u(0,t) = g(t) & t>0 \end{cases}$$

via the Laplace transform method:

$$u(x,t) = \frac{x}{2\sqrt{\pi D}} \int_0^t g(t-\tau) \frac{e^{-x^2/4D\tau}}{\tau^{3/2}}\, d\tau .$$

Work out what ~~this~~ solution is for the case where $g(t)=1$ for $0 \le t < T$, and $g(t)=0$ for $t \ge T$, T a fixed constant.

2. Suppose $u=u(r,t)$ is the bounded solution to

$$\begin{cases} u_t = D\left(u_{rr} + \frac{1}{r} u_r \right) & 0<a<r<\infty,\ t>0 \\ u(r,0) = 0 & a<r \\ u(a,t) = 1 & t>0 \end{cases}$$

Using the Laplace transform, derive the solution representation in terms of modified Bessel functions. As a further application of getting small t behavior of $u(r,t)$ by considering large s behavior of $U(r,s)$, use the asymptotic approximation of the modified Bessel function given in class (large argument) to determine the behavior of u for small values of t.