<u>Homework #4</u>

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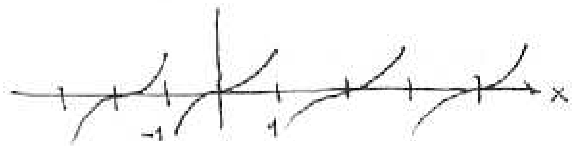
#2 (a) $\phi(x) = x^2 = \sum_{n=1}^{\infty} b_n \sin(n\pi x)$

$$\frac{b_n}{2} = \int_0^1 x^2 \sin(n\pi x)\,dx = \frac{(-1)^{n+1}}{n\pi} + \frac{2}{(n\pi)^3}\left[(-1)^n - 1\right]$$

$$\rightarrow b_n = \begin{cases} -\frac{2}{n\pi}, & n = \text{even} \\ \frac{2}{n\pi}\left[1 - \frac{4}{n^2\pi^2}\right], & n = \text{odd} \end{cases}$$



because $\cos(n\pi) = (-1)^n$

(b) $\phi(x) = x^2 = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos(n\pi x)$

$$\frac{a_n}{2} = \int_0^1 x^2 \cos(n\pi x)\,dx = \frac{2(-1)^n}{(n\pi)^2} \qquad \frac{a_0}{2} = \int_0^1 x^2\,dx = \frac{1}{3}$$

$n \geq 1$

$$\rightarrow x^2 = \frac{1}{3} + \frac{4}{\pi^2}\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}\cos(n\pi x)$$

#5. From example 3, $\phi(x) = x = \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{l}\right) = \frac{2l}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}\sin\left(\frac{n\pi x}{l}\right)$

$$\int_0^x \phi(y)\,dy = \int_0^x y\,dy = \frac{x^2}{2} = \frac{2l}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}\int_0^x \sin\left(\frac{n\pi y}{l}\right)dy$$

$$= \frac{2l^2}{\pi^2}\sum_1^{\infty}\frac{(-1)^n}{n^2}\left[\cos\left(\frac{n\pi x}{l}\right) - 1\right]$$

$$= \frac{2l^2}{\pi^2}\sum_1^{\infty}\frac{(-1)^{n+1}}{n^2} + \frac{2l^2}{\pi^2}\sum_1^{\infty}\frac{(-1)^n}{n^2}\cos\left(\frac{n\pi x}{l}\right)$$

Note – from problem #2, $\frac{2l^2}{\pi^2}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n^2} = l^2/6$ or

$$\sum_1^{\infty}\frac{(-1)^{n+1}}{n^2} = \frac{\pi^2}{12}$$

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#4. (a) $\phi(x) = \frac{a_0}{2} + \sum_1^{\infty} \left\{ a_n \cos\left(\frac{n\pi x}{l}\right) + b_n \sin\left(\frac{n\pi x}{l}\right) \right\}$ $\phi =$ odd function

so

$$a_n = \frac{1}{l} \int_{-l}^{l} \underbrace{\phi(x) \cos\left(\frac{n\pi x}{l}\right)}_{\text{odd}} dx = 0 \qquad n = 0, 1, 2, \ldots$$

$$b_n = \frac{1}{l} \int_{-l}^{l} \underbrace{\phi(x) \sin\left(\frac{n\pi x}{l}\right)}_{\text{even}} dx = \frac{2}{l} \int_0^l \phi(x) \sin\left(\frac{n\pi x}{l}\right) dx, \quad n = 1, 2, \ldots$$

$$\rightarrow \phi(x) = \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{l}\right) dx$$

(b) similar argument

#9. ϕ is π-periodic, $\phi(x) = \sum_{n=1}^{\infty} a_n \sin(nx)$ for all x.

$$\phi(x+\pi) = \sum_{n=1}^{\infty} a_n \sin[n(x+\pi)]$$

$$= \sum_1^{\infty} a_n \left\{ \sin(nx)\cos(n\pi) + \cos(n\pi)\sin(n\pi) \right\}$$

$$= \sum_1^{\infty} a_n (-1)^n \sin(nx)$$

$$\rightarrow 0 = \sum_{n=1}^{\infty} a_n [1 - (-1)^n] \sin(nx) = 2 \sum_{\substack{n=1 \\ n=\text{odd}}} a_n \sin(nx) \rightarrow a_n = 0 \quad n = 1, 3, 5, \ldots$$