Home work #3

1. The authors are very honest about the complexity of factors that can effect such a study. There is the natural difficulty of establishing transmission rates, partly because of the tenuous reliance on body size. Other factors include age structure, spatial diversity, seasonality factors, predation issues, etc. I wasn't looking for a comprehensive list from you, but a real sense that you dug into the paper and understood some strengths and limitations of the study.

2. In the Asian flu case, children would be much more at risk and transmission rates are much higher than in the adult population. In the Hong Kong flu case the attack rates are roughly age independent. The authors calibrated their model closer to the Asian flu case.

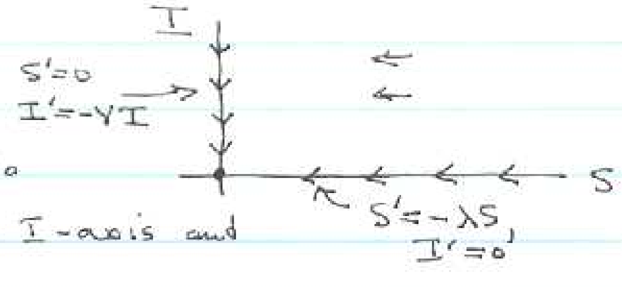
The authors also considered social networks for contact sufficient to transmit flu as a large set of connecting mixing groups. These groups include households and household clusters, preschool groups, schools, markets, shops, temples and churches, hospitals, etc.

3. starts on the next page.

3. The normal $g(I)$ we have used has been $g(I)=\beta I$, which as a "force of infection" in DeLeo's terminology, represented a density-dependent infection rate. Here, as the number of infections, I, increases, the contact rate reaches a maximum, then declines. This could be due to self-isolation or other mechanisms. γ is the normal recovery rate, and λ is a rate of emigration of susceptibles. Hence, the component of the population that is taken out of participation in the disease are those that emigrated (moving away ~~or~~ quaranteening themselves) and those that recovered from the disease and have immunity from the disease.

As a steady state $S' = 0 = -S\{g(I)+\lambda\} \to S=0$ only, since $g(I)+\lambda \geq \lambda > 0$. $I' = 0 = g(I)S - \gamma I\big|_{S=0} = -\gamma I \to I = 0 \to (S,I)=(0,0)$ is the only steady state.

Also, in the 1st quadrant
$$S' = -S\{g(I)+\lambda\} < 0$$
so $S(t)$ is always decreasing, so eventually trajectories must get close to the I-axis and be driven down to the origin.



4. Here β is the relative infection (transmission) rate of susceptibles to infectives, while βr is the relative transmission rate from the susceptible class to the incubating class. So r is a percentage of these two rates; $r = \beta r/\beta < 1$ since, as stated, it is harder to catch the disease from an incubating individual, than a fully infected one. b is the transmission rate from incubating to infectious class, and c is the recovery rate. Since $N = S+E+I+R$ and we are only interested in a local analysis about the healthy population case, consider the steady state $(S,E,I)=(N,0,0)$.

(We need not include R because initially $R=0$ and once we move away from the healthy state, knowing $I(t)$ gives us knowledge of $R(t)$ by integration.)

The Jacobian of the (S,E,I) system is

$$J = \begin{bmatrix} -\frac{\beta}{N}(I+rE) & -\frac{\beta r S}{N} & -\frac{\beta S}{N} \\ \frac{\beta}{N}(I+rE) & \frac{\beta r S}{N} - b & \frac{\beta S}{N} \\ 0 & b & -c \end{bmatrix}$$

so at $(N,0,0)$,

$$J = \begin{bmatrix} 0 & -\beta r & -\beta \\ 0 & \beta r - b & \beta \\ 0 & b & -c \end{bmatrix}$$

Since, with a column of zeros, $\det J = 0 \rightarrow \lambda = 0$ is an eigenvalue. This reduces the characteristic equation to a quadratic, namely

$$\lambda^2 + (c+b-\beta r)\lambda + a_2 = 0 \qquad \text{where}$$

$a_2 = -c\beta r + bc - \beta b$

For $r < 1$ small it is reasonable to consider $\beta r < c+b$ so we can only have instability if $a_2 < 0$; i.e.

$$bc < \beta b + c\beta r = \beta(b+cr) \rightarrow 1 < (\beta/bc)(b+cr).$$

That condition we will consider our threshold condition for onset of the disease since the health state becomes unstable. Hence it is consistent with our other model studies to define R_0 to be the right-hand side.