Home work Assignment #1

1. text, p5, #2:

(a) is linear; (b) is not because of term uu_y; (c) is not because of term u_y^2; (d) is not because of the term 1 ($\mathcal{L}(u_1+u_2) \neq \mathcal{L}(u_1)+\mathcal{L}(u_2)$); (e) is linear.

#3

(a) $u_t - u_{xx} = 1$ is 2^{nd} order, linear inhomogeneous (nonhomogeneous).

(b) $u_t - u_{xx} + x\,u = 0$ is 2^{nd} order, linear homogeneous

(c) $u_t - u_{xxt} + uu_x = 0$ is 3^{rd} order, nonlinear

(d) $u_{tt} - u_{xx} + x^2 = 0$ is 2^{nd} order, linear, inhomogeneous

(e) $iu_t - u_{xx} + x^{-1}u = 0$ is 2^{nd} order, linear homogeneous

(f) $u_x(1+u_x^2)^{-1/2} + u_y(1+u_y^2)^{-1/2} = 0$ is 1^{st} order, nonlinear

(g) $u_x + e^y u_y = 0$ is 1^{st} order, linear homogeneous

(h) $u_t + u_{xxxx} + \sqrt{1+u} = 0$ is 4^{th} order, nonlinear

2. a) $\frac{dy}{dt} + 4t^{-1}y = t$ has integrating factor $t^4 = e^{4\ln t}$, so

$$t^4\frac{dy}{dt} + 4t^3 y = \frac{d}{dy}(t^4 y) = t^5 \to t^4 y = \frac{t^6}{6} + C \to y = \frac{t^2}{6} + Ct^{-4}$$

b) $3\frac{d^2y}{dt^2} - 5\frac{dy}{dt} - 2y = 0$. For a constant coefficient linear homogeneous 2^{nd} order equation,

let $y = e^{rt} \to e^{rt}\{3r^2 - 5r - 2\} = 0 \to 3r^2 - 5r - 2 = 0 \to r = 2, -\frac{1}{3}$

so a fundamental set of solutions is $y_1 = e^{2t}$, $y_2 = e^{-t/3}$; so the general solution is $y(t) = C_1 e^{2t} + C_2 e^{-t/3}$.

3. a) $\frac{d^2y}{dt^2} + 4\frac{dy}{dt} = 2$ $\quad y(0) = 0$, $\frac{dy}{dt}(0) = 4$

Since y term is missing, let $w = \frac{dy}{dt} \to \frac{dw}{dt} + 4w = 2$. The integrating factor is e^{4t}, so $\frac{d}{dt}(e^{4t}w) = 2e^{4t}$

$\to e^{4t}w = \frac{1}{2}e^{4t} + C$, or $w = \frac{dy}{dt} = \frac{1}{2} + Ce^{-4t} \to y = \frac{t}{2} - \frac{C}{4}e^{-4t} + C_2$,

or just write $y = \frac{t}{2} + C_2 - C_1 e^{-4t}$; then $y(0) = 0 = C_2 - C_1$

and $\frac{dy}{dt}(0) = \frac{1}{2} + 4C_1 = 4 \to y(t) = \frac{t}{2} + \frac{7}{8} - \frac{7}{8}e^{-4t}$

One can also 1st consider the homogeneous equation $\frac{d^2y}{dt^2} + 4\frac{dy}{dt} = 0$, let $y = e^{rt} \Rightarrow$ characteristic equation is $r(r+4) = 0 \Rightarrow$ a fundamental set of solutions is $y_1 = e^{-4t}, y_2 = 1$.
A particular solution is $y_p = \frac{t}{2}$, so write $y = \frac{t}{2} + C_1 e^{-4t} + C_2$ and substitute i.c.s to obtain $y = \frac{t}{2} + \frac{7}{8} - \frac{7}{8}e^{-4t}$.

3 b) $t\frac{d^2y}{dt^2} + \frac{dy}{dt} = -1$, $t > 1$, $y(1) = 0$, $\frac{dy}{dt}(1) = 0$

Note that $t\frac{d^2y}{dt^2} + \frac{dy}{dt} = \frac{d}{dt}\left(t\frac{dy}{dt}\right) = -1 \rightarrow t\frac{dy}{dt} = -t + C$

so $\frac{dy}{dt} = -1 + C/t \rightarrow y = -t + C\ln t + D$. The i.c.s give $C = 1, D = 1 \rightarrow y = 1 - t + \ln t$.

4. a) $y(t) \rightarrow 5$ as $t \rightarrow \infty$ so that is the steady state.

b) $te^{-t} \rightarrow 0$ as $t \rightarrow \infty$, so $y(t) \rightarrow 4$ as $t \rightarrow \infty$.