# Homework Assignment #3

Due:

1. $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ $\quad 0 < x < 1 \quad , t > 0$

   $u(0,t) = 0.5 \quad , \quad u(1,t) = 2$

   a) What is the steady state solution $u(x,t) = U(x)$ ?

   b) If we let $u(x,0) = U(x) + \sin 2\pi x$, what is the solution $u(x,t)$ to the diffusion problem?

2. $$\begin{cases} \frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} & 0 < x < 1 \ , \ t > 0 \\ u(x,0) = f(x) & 0 < x < 1 \\ \frac{\partial u}{\partial x}(0,t) = 0 = u(1,t) & t > 0 \end{cases}$$

   Solve for $u(x,t)$ and give the Fourier coefficients for $f(x)$

3. Here is a problem with an "oblique" boundary condition (motivated by a problem that arose in neurobiology):

   $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ $\quad 0 < x < 1 \ , \ t > 0$

   $u(0,t) = 0, \quad \frac{\partial u}{\partial t}(1,t) + a \frac{\partial u}{\partial x}(1,t) = 0 \quad t > 0 \qquad a \neq 0$ , a is a constant.

   a) What is the eigenvalue problem?

   b) given $a = 1$, calculate the first three eigenvalues

4. Do problem # 6 in textbook, p 87