Math 481 Homework #1
Due: Thursday, March 9
Reading Assignment: Murray, *Mathematical Biology*, sections 10.1-10.2

1. Our simplest epidemic model involved two populations, namely the infective population and the susceptible population, but because we have a fixed population size, N > 0 constant, we only have the single equation for the number of infectives, *I(t),* to investigate: $\frac{dI}{dt}(t) = \beta I(t)(N - I(t))$.
   If $I(0) = I_0 > 0$, solve this equation explicitly for $I(t)$ to obtain
   $$I(t) = \frac{I_0 \exp(N\beta t)}{1 - (I_0 / N)(1 - \exp(N\beta t))} \quad .$$
   Then show that $\frac{dI}{dt} = \frac{\beta(N - I_0)I_0 \exp(N\beta t)}{[1 - (I_0 / N)(1 - \exp(N\beta t))]^2}$.

2. A host-viral population model: let $x = x(t)$ be the population of human hosts, and $y = y(t)$ be the size of the viral (disease carrier) population. Consider the following assumptions: (i) there is a constant human birth rate a; (ii) the viral infection causes an increased mortality rate of humans due to disease and is given by a function g(y) > 0, for y > 0: (iii) the reproduction of viral particles depends on human presence and is assumed to be mutually proportional to the product of the two population sizes; (iv) in the absence of human hosts, viral particles “die”, or become nonviable, with constant rate c.
   a) Write down the model differential (rate) equations for *x(t), y(t).*
   b) Assume g is a smooth, monotone increasing function with g(0) = 0. Show that the first quadrant of the x, y plane is an invariant region for the model. Why is this property important to the model?
   c) What are the equilibrium points for this model system? If you linearize the system about each equilibrium point, what type of point is each equilibrium point, and what is their stability?
   d) (*) What is the main failing of this type model for infectious disease?