Math 404 Introduction to Partial Differential Equations

Spring Semester, 2010
Tuesday-Thursday 10:00-11:15am room:
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Text (recommended, but not required): **Partial Differential Equations for Scientists and Engineers**, by Stanley J. Farlow, Dover, 1993.
I post my (hand-written) lecture notes on Blackboard, and most students rely mainly on those notes since they are pretty complete, and have a fair number of example either worked, or given as exercises.

Mathematical modeling has been a central methodology for physics and engineering for a long time, and the physical laws are written in terms of partial differential equations (pdes); examples come from fluid mechanics (e.g. Navier-Stokes equations), solid mechanics (e.g. the plate equation), electrodynamics (e.g. Maxwell's equations), heat transfer (diffusion equation), genetics (e.g. Fisher's equation), neurobiology (e.g. the Hodgkin-Huxley equations), financial engineering (Black-Scholes equation), etc. Partial differential equations have helped advance areas like probability theory, differential geometry, Lie algebra theory, to name a few mathematical topics (whose theories have fed back into a variety of application areas and suggested other questions to ask of partial differential equations).

Since the subject stretches into almost every application area, we can only cover a small piece of the subject. We will concentrate on second order linear equations where we can find explicit solutions. These have played a significant part in the development of models in physics and engineering. But I will also discuss some general qualitative behavior of solutions to expect, since the form of the solutions is more complicated than solutions for ordinary differential equations.

Comments on Course Content: Here is an outline of the topics to be covered: these lay a foundation for a vast subject.

1. PDEs from physical examples, order, superposition principle, homogeneous and non-homogeneous equations, boundary conditions, classification of 2^{nd} order equations. There will be a little discussion about conservation principles, constitutive laws, and quick derivations of basic equations.
2. Introduction to first-order linear and quasi-linear equations, characteristic equations.
3. PDEs on unbounded domains, and in particular, initial-value problems for the diffusion (heat) equation, and the wave (string) equation. Laplace and Fourier transforms with applications to heat and wave equation. Semi-infinite domains; equations with sources.
4. Orthogonal expansions and Fourier's method; convergence results on Fourier series; integration and differentiation of series.
5. PDEs on bounded domains, starting with the heat equation: some properties of solutions for different boundary conditions, steady state solutions; separation of variables method and representation of solutions in terms of Fourier series; eigenvalue problems; 2D and 3D problems
6. Introduction to Sturm-Liouville eigenvalue theory and its relation to boundary value problems, Rayleigh quotient and the minimization principle.
7. Wave equation: solution of various problems, Bessel functions and circularly symmetric vibrating membranes.
8. Laplace equation: solution on various domains. Solution of Poisson's equation.
9. Beam equation as an example of a higher order pde.
10. Well-posedness; compatibility conditions; maximum principle; integral energy method.
11. Green's function technique to solve pde boundary value problems.

Grading Policy: The course grade will be based on two mid-term exams (25% each), homework (20%), and a final exam (30%). Letter grades will be based on the weighted sum of scores and generally follow the

85-100% being A, 70-84% being B, 55-69% being a C, etc. In actuality I compose a final distribution and then look for significant gaps between successive scores before assigning letter grades. So the grade cut-offs and distribution varies a little from class to class.

Homework: Homework assignments will generally be given weekly and posted on the course **Blackboard** website Thursday evenings. I do not grade the homework. I will have a grader for homework, but if there is an issue with homework grading, point it out to me and I will check into it. Most of the time *homework will be due on Thursday, at the beginning of class*. I encourage you to work in groups and discuss homework problems among yourselves, or go over problems with me. However, any work you turn in for grading must represent your own work. Any questions about a homework assignment must be raised *immediately* after return of the homework. Remember, I am often available to help you if you are struggling with the homework assignment. Students do find the homework assignments time consuming, so make sure you budget enough time to study the material and do the assigned work.

Homework Policy: *Late homework submissions are **not** accepted. Unless otherwise specified I must receive the homework in Thursday's class.*

Exams: I probably won't put out a day-by-day schedule. I am estimating right now that exam 1 will come around *February 25* and exam 2 will come around *April 1*. **No** make-up exams will be given except in the case of a documented serious emergency, which will require *written documentation*. In such case, I need to be notified ***before*** the exam period except in the most exceptional cases (and I determine what is exceptional). All exams are closed-book, closed-notes, and calculators/ computers (or any other electronic device) will not be allowed to be used during exams. (You wouldn't want to take an open book exam from me anyway!) *Any question about the grading of an exam must be brought to my attention within one week after the exam is handed back to you*!

Learning Goals: By the end of the course you should

- Understand key definitions and concepts of the course. Examples include, but are not limited to, linearity, classification, fundamental solution, eigenfunctions, orthogonal series, domain of dependence, characteristic curves, separation of variables, some transformation methods
- Be able to identify the type of equation and the method appropriate for solving it. This includes use of transform methods on (spatially) unbounded domain problems, and series solution method on bounded domain problems.
- Be able to communicate orally and in writing the mathematical and physical ideas of the subject, using correct notation and terminology.

Further Comments: Many students consider this course difficult because there are many steps in solving problems, and many details to recall from calculus and ordinary differential equations (odes), and even algebra. I can not give all details of calculus or algebra in class, *so it is your responsibility to get out your calculus and ode book and review material pertinent to this class*. In fact, here are a couple "meta-statements" to keep in mind about the course:
>> Generally our methodological goal is to reduce the pde to solving odes, either by transforming the problem, separating variables, or using special properties of the problem or solutions sought after.
>> Once we have learned a mathematical method, we try to transform similar problems into problems we have already solved.

There are not as many concepts introduced in the course as it appears, but the course requires constant review of material. A general rule is to spend three to four hours studying outside the classroom for every hour in class. I encourage study in groups, but it is required that you write up homework individually. We can never assign enough homework because we have limited grading help. But I can give supplementary problems that you can work on, and though not collected, I can help you with, generally on an individual basis. That is, I encourage you seeing me whenever you are stuck on a problem or a concept. Don't spend three hours staring at any single problem. Sometimes something I can say in individual consultation can clear up a point in a few minutes.

Here are some keys to getting a lot out of the course:

Study before you work on the exercises: A poor strategy for conducting yourself in this course is to get the homework assignment and back-track through the examples in the notes and book to match with the examples closest to that of the exercises. You will learn less than you think you have, and it will show on the exams. I will post my (unpolished) lecture Notes on **Blackboard** and generally follow them, though I might modify or introduce new examples as I feel like it. Read ahead of time the material to be covered next, and attack the exercises buried within the Notes. Thus, when I discuss a topic in class, it will be the second time you have encountered it. If you take notes, re-read them within a day after class, and augment them with re-reading my Notes and the appropriate sections with in the text. Remember that very little learning takes place in class. Most learning takes place by your working to digest your notes and supplementary material, and discussing the material with other people. If we come into contact with a problem area or technique that is specifically relevant to something you are studying in another class, see if you can mentally strengthen that connection.

The course is more than methodology: There are definitions that must be learned in this course. Not many, but it isn't a matter of having an idea about a concept, but knowing the definition of a concept **exactly**. I will also state some results as theorems, lemmas, or propositions. In this course these three words will be equivalent. They are not there to extend theory and having a proof that needs memorization, but rather to summarize a development and make it easier to remember conditions needed for a conclusion. But these summary statements are important for you to digest.

As you are going along, frequently reflect on where you have been and decide what is important and what is crucial to your understanding. When I took the course I tried to compose my own exam ahead of taking the instructor's exam, and this became a successful mental organizing principle for myself.

We have discussed content with engineering, so the curriculum is pretty full and moves fast. Learning of material must take place outside the class room. (This is true of all courses, of course.) I can present informative examples in class, and build the key structures of the subject, but you need to practice by doing as many problems as you can. I plan to make my lecture notes available, and they will have extra problems imbedded within them that I do not discuss in class.

Academic Conduct and Policy: Academic integrity is an important value at UMBC and I support this. The following is the official UMBC statement on academic conduct.

By enrolling in a course, each student assumes the responsibilities of an active participant in UMBC's scholarly community in which everyone's academic work and behavior are held to the highest standards of honesty. Cheating, fabrication, plagiarism, and helping others to commit these acts are all forms of academic dishonesty, and they are wrong. Academic misconduct could result in disciplinary action that may include, but is not limited to suspension or dismissal.

To read the full Student Academic Conduct Policy, consult the UMBC Student Handbook, or go online to http://www.umbc.edu/integrity/.