

Math 404/0101

Introduction to Partial Differential Equations

Spring Semester, 2009 [4036]
Tuesday-Thursday 2:30-3:45pm room: ACIV 145

Instructor: Jonathan Bell M/P 427

Office hours: right after class, or by appointment

Contact information: Email: jbell@math.umbc.edu
Phone: 410-455-3298

Text: *Partial Differential Equations, An Introduction*, 2nd ed., by Walter A. Strauss, John Wiley & Sons, 2008.

Mathematical modeling has been a central methodology for physics and engineering for a long time, and the physical laws are written in terms of partial differential equations (pdes); examples come from fluid mechanics (e.g. Navier-Stokes equations), solid mechanics (e.g. the plate equation), electrodynamics (e.g. Maxwell's equations), heat transfer (diffusion equation), genetics (e.g. Fisher's equation), neurobiology (e.g. the Hodgkin-Huxley equations), financial engineering (Black-Scholes equation), etc. Partial differential equations have helped advance areas like probability theory, differential geometry, Lie algebra theory, to name a few mathematical topics (whose theories have fed back into a variety of application areas).

Since the subject stretches into almost every application area, we can only cover a small piece of the subject. We will concentrate on second order linear equations where we can find explicit solutions. These have played a significant part in the development of models in physics and engineering. But I will also discuss some general qualitative behavior of solutions to expect, since the form of the solutions is more complicated than solutions for ordinary differential equations.

Comments on Course Content: Here is an outline of the topics to be covered: these lay a foundation for a vast subject. I expect to cover parts of Chapters 1-6, and maybe bits of other chapters as time permits.

1. PDEs from physical examples, order, superposition principle, homogeneous and non-homogeneous equations, boundary conditions, classification of 2nd order equations. There will be a little discussion about conservation principles, constitutive laws, and quick derivations of basic equations.
2. Introduction to first-order linear and quasi-linear equations, characteristic equations.
3. PDEs on unbounded domains, and in particular, initial-value problems for the diffusion (heat) equation, and the wave (string) equation. Laplace and Fourier transforms with applications to heat and wave equation. Semi-infinite domains. Equations with Sources.
4. Orthogonal expansions and Fourier's method; convergence results on Fourier series; integration and differentiation of series; Sturm-Liouville problems: regular problems, Rayleigh quotient and minimization principle.
5. PDEs on bounded domains, starting with the heat equation: some properties of solutions for different boundary conditions, steady state solutions; separation of variables method and representation of solutions in terms of Fourier series; eigenvalue problems; 2D and 3D problems
6. Wave equation: solution of various problems, Bessel functions and circularly symmetric vibrating membranes.
7. Laplace equation: solution on various domains. Solution of Poisson's equation
8. Comments about pdes and well-posedness; compatibility conditions; maximum principle; integral energy method
9. As time permits the introduction of the Green's function technique to solve ode and pde boundary value problems, Fredholm Alternative. Also, a brief introduction to numerical methods.

Grading Policy: The course grade will be based on two mid-term exams (25% each), homework (20%), and a final exam (30%). Letter grades will be based on the weighted sum of scores and generally follow the 85-100% being A, 70-84% being B, 55-69% being a C, etc. Grade cut-offs depend on the nature of the overall distribution, and this varies from class to class.

Homework: Homework assignments will generally be given weekly and posted on the course **Blackboard** website. I do not grade the homework. I will have a grader for homework, but if there is an issue with homework grading, point it out to me and I will check into it. Most of the time *homework will be due on Thursdays, at the beginning of class*. I encourage you to work in groups and discuss homework problems among yourselves. However, any work you turn in for grading must represent your own work. Any questions about a homework assignment must be raised *immediately* after return of the homework. Remember, I am often available to help you if you are struggling with the homework assignment.

Homework Policy: *Late homework submissions are **not** accepted. If we get in enough assignments then two homework assignments with lowest grades will be disregarded.*

Exams: Because of using a new textbook I probably won't put out a day-by-day schedule. I am estimating right now that exam 1 will come around *February 19* and exam 2 will come around *April 2*. **No** make-up exams will be given except in the case of a documented serious emergency, which will require *written documentation*. In such case, I need to be notified *before* the exam period except in the most exceptional cases (and I determine what is exceptional). All exams are closed-book, closed-notes, and calculators/computers (or any other electronic device) will not be allowed to be used during exams. (You wouldn't want to take an open book exam from me anyway!) Any question about the grading of an exam must be brought to my attention within one week after the exam is handed back to you!

Learning Goals: By the end of the course you should

- Understand key definitions and concepts of the course. Examples include, but are not limited to, linearity, classification, fundamental solution, eigenfunctions, orthogonal series, domain of dependence, characteristic curves, separation of variables, some transformation methods
- Be able to identify the type of equation and the method appropriate for solving it. This includes use of transform methods on (spatially) unbounded domain problems, and series solution method on bounded domain problems.
- Be able to communicate orally and in writing the mathematical and physical ideas of the subject, using correct notation and terminology.

Further Comments: Many students consider this course difficult because there are many steps in solving problems, and many details to recall from calculus and ordinary differential equations (odes), and even algebra. I can not give all details of calculus or algebra in class, *so it is your responsibility to get out your calculus and ode book and review material pertinent to this class*. In fact, here are a couple "meta-statements" to keep in mind about the course:

>> Generally our methodological goal is to reduce the pde to solving odes, either by transforming the problem, separating variables, or using special properties of the problem or solutions sought after.

>> Once we have learned a mathematical method, we try to transform similar problems into problems we have already solved.

There are not as many concepts introduced in the course as it appears, but the course requires constant review of material. A general rule is to spend multiple hours studying outside the classroom for every hour in class. I encourage study in groups, but it is required that you write up homework individually. We can never assign enough homework because we have limited grading help. But I can give supplementary problems that you can work on, and though not collected, I can help you with, generally on an individual basis. That is, I encourage you seeing me whenever you are stuck on a problem or a concept. Don't spend three hours staring at a problem. Sometimes something I can say in individual consultation can clear up a point in a few minutes.

Here are some keys to getting a lot out of the course:

>> Go over notes immediately and fill in gaps. If there is confusion, don't wait until exam time; see me about it.

>> We have discussed content with engineering, so the curriculum is pretty full and moves fast. Learning of material must take place outside the class room. (This is true of all courses, of course.) I can present informative examples in class, and build the key structures of the subject, but you need to practice by doing

as many problems as you can. I plan to make my lecture notes available, and they will have extra problems imbedded within them that I do not discuss in class.

Blackboard: Blackboard (Bb) is a course management system that allows posting and communicating among registered participants in a course. To log in, I suggest for you to go to **myUMBC** and then use the Blackboard link on the left side of the screen. Then look for this course under “My Courses”. I do not intend to use its full capability. I will use the “Course Documents” area of Bb for almost everything, including course syllabus, homework assignments, homework problem answers, lecture notes, statistical results of exams, and supplementary material when called for.

Because I will post problems on Bb and give announcements, you should check the site often! The grader for the course should be using the grade book on Bb so you can check to see he/she has recorded scores accurately. Finally, unless I say otherwise, the best way to reach me is to email me at the address above.

Academic Conduct and Policy: Academic integrity is an important value at UMBC and I support this. The following is the official UMBC statement on academic conduct.

By enrolling in a course, each student assumes the responsibilities of an active participant in UMBC’s scholarly community in which everyone’s academic work and behavior are held to the highest standards of honesty. Cheating, fabrication, plagiarism, and helping others to commit these acts are all forms of academic dishonesty, and they are wrong. Academic misconduct could result in disciplinary action that may include, but is not limited to suspension or dismissal.

To read the full Student Academic Conduct Policy, consult the UMBC Student Handbook, or go online to <http://www.umbc.edu/integrity/>.

Some Dates:

Class Begins	January 29
Last day to add a class	February 6
Last day to drop without “W”	Feb. 6 (grad students), Feb. 23 (undergrad students)
Spring Break	March 16-23
Last day to drop	April 13
Last day of class	May 12
Final exam period	May 14-20
Our final exam is scheduled for	May 14 (1:00-3:00pm)
Commencement	May 22