Homework Assignment # 8

Due : Wednesday , April 19

1. Find the smallest eigenvalue of the Sturm Liouville problem

$$\nabla^2 \varphi + \lambda \varphi = 0 \qquad \text{in } \Omega = \{(r,\theta) \mid 0 \le r < 1,\ 0 \le \theta \le \tfrac{\pi}{2}\}$$

$$\varphi(1,\theta) = 0 \qquad 0 \le \theta \le 2\pi$$

$$\varphi(r,0) = 0 = \varphi(r, \tfrac{\pi}{2}) \qquad 0 < r < 1$$

2. (a) Solve the following heat equation problem for a series solution for the temperature $u(x,y,t)$:

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \qquad \text{in } \Omega = \{(x,y) \mid 0 < x < \pi,\ 0 < y < \pi\}$$

$$u_x(0,y,t) = u_x(\pi,y,t) = 0 \qquad 0 \le y \le \pi \ , \ t > 0$$

$$u(x,0,t) = u(x,\pi,t) = 0 \qquad 0 \le x \le \pi \ , \ t > 0$$

$$u(x,y,0) = f(x,y) \qquad (x,y) \in \Omega$$

(b) What is the solution given in part (a) when $f(x,y) = \cos(2x)\sin(y)$?

3. For the damped wave equation in two space variables,

$$\frac{\partial^2 u}{\partial t^2} = c^2 \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) - \beta \frac{\partial u}{\partial t}$$

if we write $u(x,y,t) = f(x)g(y)h(t)$, what ordinary differential equations must f, g, h solve for u to be a solution to the equation ?

4. Solve the following wave equation problem (vibrating square problem):

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \qquad \text{in } \Omega = \{(x,y) \mid 0 < y < \pi,\ 0 < x < \pi\}$$

$$u(0,y,t) = 0 = u(\pi,y,t) \ , \quad t > 0 \ , \quad 0 \le y \le \pi$$

$$u_y(x,0,t) = 0 = u_y(x,\pi,t) \ , \quad t > 0 \ , \quad 0 \le x \le \pi$$

$$u(x,y,0) = 0 \ , \quad \frac{\partial u}{\partial t}(x,y,0) = g(x,y)$$