A brief history of Fourier series

The one-dimensional wave equation $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$ was introduced by d'Alembert in 1752, where he considered it a model of a vibrating string. He also gave a form of the solution as $u(x,t) = \frac{1}{2}\{f(x+ct) + f(x-ct)\}$, where $f(x)$ is the initial shape of the string at $t=0$. About 10 years later Daniel Bernoulli showed that a formal solution of the vibrating string problem was also $u(x,t) = \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi c t}{l}\right) \sin\left(\frac{n\pi x}{l}\right)$, where now the length of the string was finite and of length l. He also asserted, without proof, that this was the most general solution to the problem possible. (He was correct, for the boundary conditions used.) Neither d'Alembert nor Euler believed Bernoulli, and said such a series could not possibly converge to a function $f(x)$, such as $f(x) = x(l-x)$ at $t=0$.

Fourier (1822) proved for the first time that such series did converge in a large number of specific cases, while developing his analytic theory of heat. Others (Poisson, Cauch, Dirichlet, Bonnet) went on to develop more general proofs (some being incorrect); the first general proof of convergence was given by Dirichlet (1829). Later Riemann and Cantor developed the theory of trigonometric series generally, and Hurwitz and Fejér investigated properties of Fourier series when the series does not necessarily converge. All the mathematicians were among the best of their era.

ref: Whittaker and Watson's *A Course in Modern Analysis*, 1927.