Homework Assignment # 9

Due Wednesday, April 26

1. Solve Laplace's equation in the rectangle $\Omega = \{(x,y) \mid 0 < x < L,\ 0 < y < K\}$ with the boundary conditions $u_x(0,y) = u_x(L,y) = 0$ $(0 \le y \le K)$, $u(x,0) = 0$, $u(x,K) = 6\cos\left(\frac{8\pi x}{L}\right)$ $(0 \le x \le L)$

2. Solve Laplace's equation for a bounded solution in the exterior of the disk, i.e. in $\Omega = \{(r,\theta) \mid r > a,\ 0 \le \theta < 2\pi\}$, with boundary condition $u(a,\theta) = \ln(2) + 4\cos(3\theta)$.

3. Show that there is no solution of the problem

$$\nabla^2 u = f \quad \text{in } \Omega \subset \mathbb{R}^3$$
$$\frac{\partial u}{\partial \eta} = g \quad \text{on } \partial\Omega$$

unless

$$\int_{\Omega} f \, d\Omega = \int_{\partial\Omega} g \, dS$$

4. (a) What is the general solution to $\nabla^2 u = 0$ in $\mathbb{R}^3$ in the case u only depends on r?

   (b) What is the general solution to $\nabla^2 u = k^2 u$ in $\mathbb{R}^3$ ($k > 0$ constant) in the case u only depends on r? (hint: substitute $u = v/r$).