Homework 2

(10 points each)

1. Consider again $R/\rho << 1$ so we let $e^{-R/\rho} \simeq 1 - R/\rho$ only, and substitute into the R equation:

$$\frac{dR}{dt} = \mu\{N - R - S_0(1 - R/\rho)\} = \mu(1 - S_0/\rho)\left\{\frac{N - S_0}{1 - S_0/\rho} - R\right\}.$$ With

$R(0) = 0$, then $R(t) = \dfrac{N - S_0}{1 - S_0/\rho}\left(1 - e^{-\mu(1 - S_0/\rho)t}\right)$

Thus,
$$\frac{dR}{dt} = \mu(N - S_0)\, e^{\mu(S_0/\rho - 1)t}$$

So, for the epidemic case ($S_0 > \rho$), $\frac{dR}{dt} > 0$ for all $t > 0$ and, hence, there is no way to fit the data (figure) shown in class.

2. If you first look at the case $Y = 0$, then we obtain $Z = 0$ and are led to the (healthy population) equilibrium state $(X, Y, Z) = (N, 0, 0)$. If $Y \neq 0$, then $X = \frac{b+r}{\beta}$, and working through some elementary algebra, we obtain another equilibrium state $(X, Y, Z) = \left(\frac{b+r}{\beta}, \frac{\beta b N - b(b+r)}{\beta(b+r)}, \frac{\beta r N - r(b+r)}{\beta(b+r)}\right)$
But we can only have this ~~state~~ exist meaningfully, i.e. have positive components, is N is large enough. That is, we need $N > \frac{b+r}{\beta} \doteq N_c$. Otherwise we only have the stable healthy rest state. (Actually, its not strickly stable since the eigenvalues are $\lambda = 0, -b, \beta N - r - b$, the latter being negative $\Leftrightarrow N < N_c$; but we have a zero eigenvalue.)

3. $v(t) = at \rightarrow \frac{dx}{dt} = -atx$, $x(0) = 1 \rightarrow x(t) = e^{-at^2/2} \rightarrow$
$\frac{dy}{dt}(t) = at\, e^{-at^2/2}$, $y(0) = 0$, $\rightarrow y(t) = 1 - e^{-at^2/2}$.