Homework Assignment # 10

Due: Friday, May 5.

1. Find the harmonic function in the semi-infinite strip $\{(x,y) \mid 0<x<\pi,\ 0<y<\infty\}$ that satisfies the boundary conditions
$$u(0,y) = u(\pi,y), \quad y \geq 0$$
$$u(x,0) = h(x), \quad 0 \leq x \leq \pi$$
$$\lim_{y\to\infty} u(x,y) = 0 \quad \text{for all } x \in [0,\pi]$$

2. Check the validity of the maximum principle for the harmonic function
$$u(x,y) = (1-x^2-y^2)/(1-2x+x^2+y^2)$$ in the domain $\Omega = \{(x,y) \mid x^2+y^2 \leq 1\}$. Explain.

3. A spherical shell (in 3 space) with inner radius 1 and outer radius 2 has a steady state temperature distribution. Its inner boundary is held at 100°C. Its outer boundary has boundary condition $\partial u/\partial r = -\gamma < g$, γ being a constant.
   (a) Find the temperature
   (b) What are the hottest and coldest temperatures?
   (c) Can you choose γ so that the temperature on its outer boundary is 20°C?

4. Find the Fourier transform of (a) $e^{-|x|}$; (b) e^{-ax^2} $(a>0)$.

5. Use the Fourier transform to solve
$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} - tu, \quad |x|<\infty,\ t>0$$
$$u(x,0) = f(x) \qquad u \text{ is bounded} \qquad (f \in L^1(\mathbb{R})).$$