Some Calculus background you should be familiar with, or review for Math 404

- Integration and differentiation of common functions
- Understanding partial derivatives
- Integration-by-parts
- Convergence of series, uniform convergence if terms depend on a variable
- Addition formulas (trig):
  $\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$
  $\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$
- Hyperbolic functions: $\sinh(x) = (e^x - e^{-x})/2$, $\cosh(x) = (e^x + e^{-x})/2$, $\tanh(x)$, etc.
- If $f = f(x, y, z)$ is a scalar function, and $F = (F_1, F_2, F_3)$ is a vector function, then $\nabla f = grad(f) = (f_x, f_y, f_z)$, where $f_x = \partial f / \partial x$, etc., and

  $divF = \nabla \bullet F = \frac{\partial F_1}{\partial x} + \frac{\partial F_2}{\partial y} + \frac{\partial F_3}{\partial z}$. Therefore, the Laplacian of u is

  $$\Delta u(x, y, z) = \nabla^2 u = divgrad(u) = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \quad .$$

  Also, $|\nabla u|^2 = |grad(u)|^2 = (u_x)^2 + (u_y)^2 + (u_z)^2$.

More background: we are going to be dealing with series of functions (Fourier series), so you should recall a few things about sequences and series.

Def'n: **convergence of a series** of (real) numbers: $\sum_{n=1}^{\infty} a_n = a_1 + a_2 + \cdots$ converges if the tail end can be made arbitrarily small; i.e. given any tolerance $\varepsilon > 0$, there is an $M > 1$ such that for $m > M$, $|\sum_{n=m}^{\infty} a_n| < \varepsilon$.

Def'n: **absolute convergence** of a series: $\sum_1^{\infty} a_n$ converges absolutely if $\sum_1^{\infty} |a_n|$ converges.

Remark: the **ratio test**: the series $\sum_{n=1}^{\infty} a_n$ converges absolutely if $\frac{|a_{n+1}|}{|a_n|} \le \rho < 1$ for some constant ρ , and for $n \ge N \ge 1$. (We don't care if the inequality is met for the first N terms.)

Def'n: **uniform convergence of sequence of functions**: assume the sequence $\{f_n(x)\}_{n=1,2,\ldots}$ of functions is defined on an interval *I* of the real numbers. Then $\{f_n\}$ converges uniformly on *I* to f(x) if for any tolerance $\varepsilon > 0$, there is an M such that for $m > M$, $|f_m(x) - f(x)| < \varepsilon$ for all x in *I*.

Def'n: **uniform convergence of a series of functions**: with f_n 's defined on interval I, $\sum_1^\infty f_n(x)$ converges uniformly on I to *f(x)* if the sequence of partial sums $\{s_N\}_{N=1,2,\ldots}$, $s_N = \sum_{n=1}^N f_n(x)$, converges uniformly to *f(x)* on *I*.