Homework Assignment # 11

Due: Wednesday, May 10. (We may have only 1 more homework.)

1. Find the Fourier Cosine transform of e^{-ax} $(a>0)$

2. Use the Fourier Sine transform to solve the problem
$$\begin{cases} \nabla^2 u = 0 & \text{in } \{(x,y) \mid x>0, y>0\} \\ u(0,y)=1 & y>0 \\ u(x,0)=0 & x>0 \end{cases}, \quad u \text{ remains bounded}$$
(Remark: leave $u(x,y)$ as an integral since we haven't computed $\mathcal{F}_S^{-1}\left(\frac{1}{k}e^{-ky}\right)$.)

3. If $\mathcal{L}(f(t)) = F(s) = \int_0^\infty e^{-st} f(t)\,dt$, derive the operational properties
(a) $\mathcal{L}(-t f(t)) = \frac{dF}{ds}(s)$ (b) $\mathcal{L}(H(t-b) f(t-b)) = e^{-bs}F(s)$, $b>0$.

4. What is $F(s)$, given $f(t) = \begin{cases} 0 & \text{if } t<5 \text{ or } t>8 \\ t^2 & \text{if } 5<t<8 \end{cases}$

5. Use Laplace transform to find the bounded solution to
$$\begin{cases} \dfrac{\partial^2 u}{\partial t^2} = c^2 \dfrac{\partial^2 u}{\partial x^2} & |x|<\infty,\ t>0 \\ u(x,0) = \sin x, \ \dfrac{\partial u}{\partial t}(x,0) = 0 \end{cases}$$