Homework # 5

Due : Wednesday, March 15

1. text, page 36, # 1. (Use d'Alembert's formula)
2. text, page 37, # 8. (Again use d'Alembert's formula; keep in mind the initial data is specified for u, not v.)

3. Consider the (dimensional) slightly damped vibrating string problem

$$\begin{cases} \rho_0 \dfrac{\partial^2 u}{\partial t^2} = \tau_0 \dfrac{\partial^2 u}{\partial x^2} - \beta \dfrac{\partial u}{\partial t} & 0 < x < \ell, \ t > 0 \\ u(0,t) = 0 = u(\ell,t) \\ u(x,0) = 0, \quad \dfrac{\partial u}{\partial t}(x,0) = g(x) \end{cases}$$

Here g is piecewise smooth, ρ_0, τ_0, β are positive constants and represent, respectively, the string density, horizontal tension, and damping. By slightly damped, I mean you can assume $\beta^2 < 4\pi^2 \rho_0 \tau_0 / \ell^2$.

Compute the solution.

Remark: I the Supplemental Class Notes section I put the two figures up that I showed today in class (Wednesday, March 8). They are titled oneD wave_eqn_example2.pdf and oneD wave_eqn_example3.pdf .

I also spent time after dinner thinking about another illustrative figure and I came up with wave_vs_heat.pdf In that file I start with the initial data $f(x) = \begin{cases} 1 & -1 < x < 1 \\ 0 & \text{otherwise} \end{cases}$ and you can see how both the wave equation and the heat equation handle it.