Homework #2

1. In the SIR model discussed in class, the 'removed' model equation was written as $\frac{dR}{dt} = \mu(N - R - S_0 e^{-R/\rho})$. We then made an approximation by replacing $e^{-R/\rho}$ with 3 terms in its Taylor series, allowing the equation to be solved in closed form. Show that in retaining 2 terms of the Taylor series, the resulting solution provides bad agreement to the Bombay epidemic data of 1905

2. Consider the following model dynamics of a directly transmitted viral micro parasite:

$$\frac{dX}{dt} = bN - \beta XY - bX, \quad \frac{dY}{dt} = \beta XY - (b+r)Y, \quad \frac{dZ}{dt} = rY - bZ.$$

Here X, Y, Z are the number of susceptibles, infectives, and immune populations, respectively, and b, β, r are positive constants. Here the population is kept constant by births and deaths (with a contribution from each class) balancing. Show there is a threshold population size, $N_c \doteq \frac{b+r}{\beta}$, such that if $N < N_c$ the parasite cannot maintain itself in the population and both the infectives and immune class eventually die out.

3. Consider a population of haemophiliacs who were given infected blood and so were all infected with HIV at $t=0$. Let

$y(t)$ = fraction of the population who have AIDS at time t,

$x(t)$ = fraction of the population who are HIV-positive but do not yet have AIDS.

If $v(t)$ is the rate of conversion from infection to AIDS, a simple model of the dynamics is

$$\frac{dx}{dt} = -v(t)x, \quad \frac{dy}{dt} = v(t)x, \quad x(0) = 1, \ y(0) = 0$$

(In an article by Peterman, et al. Epidemiology Reviews 7: 7-21 (1985), they present data on 194 cases of blood transfusion-associated AIDS. With $v(t) = at$ the solution of the model system with $a = 0.237\ yr^{-1}$ applied to their data gives the rate of increase, dy/dt, in AIDS patients which compares very well with the data.)

Use Peterman's $v(t)$ form, solve the system for $x(t)$ and $y(t)$ and sketch the graphs of $x(t)$, $y(t)$ and $\frac{dy(t)}{dt}$.