Homework # 6

Due : Wednesday, April 5

1. For the damped vibrating string equation

$$\frac{\partial^2 u}{\partial t^2} + r\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} \qquad 0<x<1 \ , t>0 \ , r = \text{constant} > 0$$

$$u(0,t) = 0 = u(1,t)$$

show that the energy decreases.

2. For the motion of a string with density proportional to $(1+x)^{-2}$, i.e.

$$(1+x)^{-2}\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2} \qquad 0<x<l \ , t>0$$

$$u(0,t) = 0 = u(l,t) \qquad + \text{ initial conditions}$$

after separating variables, $u(x,t) = T(t)\,\varphi(x)$, what are the resulting eigenvalues and associated eigenfunctions?

Using the superposition principle, what would the series representation for $u(x,t)$ be?

3. Write each of the following equations in the Stürm-Liouville form and identify the coefficients $p(x), q(x), \sigma(x)$.

(a) $x^2\varphi'' + \lambda\varphi = 0$

(b) $\sin(x)\,\varphi'' + \cos(x)\,\varphi' + \lambda\sin(x)\,\varphi = 0$

(c) $(x\varphi')' + (\lambda - 1/x^2)\,\varphi = 0$

(d) $\varphi'' - x\varphi' + \lambda\varphi = 0$

(e) $\varphi'' - \varphi' + \lambda\varphi = 0$

(f) $(x\varphi)'' + \lambda x\varphi = 0$

4. Find the eigenvalue-eigenfunction pairs $\{\lambda_n, \varphi_n\}_{n\geq 1}$ to

$$\varphi'' + \frac{1}{x}\varphi' + \frac{\lambda}{x^2}\varphi = 0 \qquad 1<x<2 \ , \ \varphi(1) = 0 \ , \ \varphi(2) = 0$$

(hint: use change of variables $x = e^z$ to reduce equation to constant coeffic.)