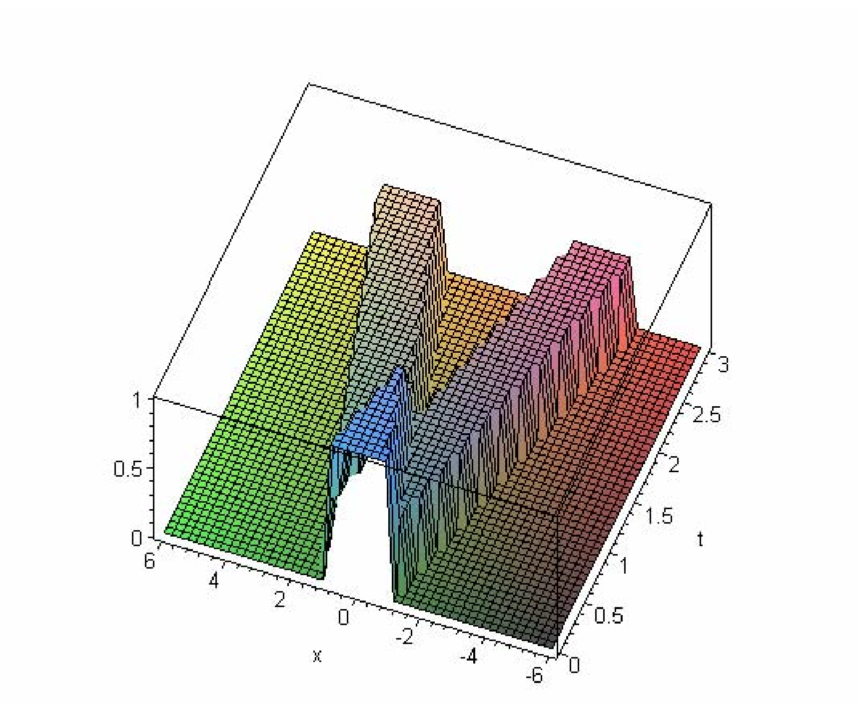
A comparison of what the wave equation does to an initial condition versus what the heat equation does to the same initial condition.

**Problem 1**:

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2} \qquad -\infty < x < \infty, \qquad t > 0$$

$$u(x,0) = f(x), \quad \frac{\partial u}{\partial t}(x,0) = 0, \quad -\infty < x < \infty$$

Here we take *f(x) = H(1 - |x|);* i.e. *f(x) = 1* for $-\infty < x < \infty$ , and 0 otherwise. Then



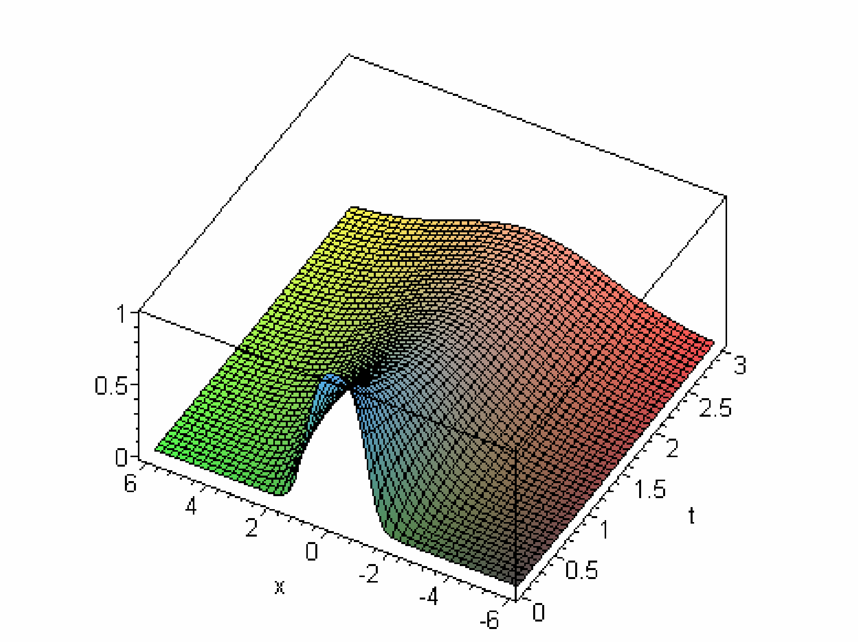

In this example we have "singularities" where the function $f$ jumps from a value of 0 to 1 (or vice versa). Notice that these singularities are transported along the characteristics of the equation, and that the wave form remains invariant as time increases. Now consider

**Problem 2**:

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} \qquad -\infty < x < \infty, \quad t > 0$$

$$u(x,0) = f(x), \quad -\infty < x < \infty$$

We now compute the solution with the same *f(x)* used above:




Notice that the solution becomes instantly smooth, even for $t << 1$. Even though I start with a piecewise constant initial condition $f(x)$, $u(x, t)$ for $t > 0$ is infinitely differentiable! The wave smears out and $u(x,t)$ forgets the initial condition as $t \to \infty$.