Homework #7.

Due: Wednesday, April 12

1. Use the Monotonicity theorem to obtain a lower and an upper bound on the lowest eigenvalue, λ_1, of the problem (Sturm-Liouville EVP)

$$\frac{d}{dx}\left((1+x^2)\frac{d\varphi}{dx}\right) + \lambda(1+x^2)\varphi = 0 \qquad 0<x<1$$

$$\varphi(0) = 0 = \varphi(1)$$

2. Work through the details of getting a series representation of the solution to the following (slightly revised) problem from class:

$$\begin{cases} u_t = u_{xx} + e^{-t}\sin(3x) & 0<x<\pi,\ t>0 \\ u(0,t) = A(t),\ u(\pi,t) = 0 \\ u(x,0) = 0 \end{cases}$$

$A(t)$ is a continuous, differentiable function with $A(0)=0$.

3. Solve the following forced, damped, wave equation:

$$u_{tt} = u_{xx} - \beta u_t + q_0\cos(\omega t) \qquad 0<x<1,\ t>0$$

$$u(0,t) = 0 = u(1,t)$$

$$u(x,0) = 0,\ u_t(x,0) = 0$$

ω, q_0, β are positive constants with $\beta < 2\pi$.