MATH 404 Introduction to Partial Differential Equations in Spring, 2006

Jonathan Bell, Professor and chair

I will be the instructor for MATH 404, scheduled for Monday-Wednesday, 5:30-6:45.

Why study Partial Differential Equations (PDEs)?

- Mathematical modeling has been a central technology for physics and engineering for a long time, and the physical "laws" are written in terms of PDEs; examples include fluid dynamics (Navier-Stokes equations), solid mechanics (biharmonic or plate equation), electrophysics/electrical engineering (Maxwell's equations), thermodynamics (diffusion equations), etc.
- PDEs have helped areas to advance, like probability theory, differential geometry, Lie algebras, etc.
- PDE modeling has played an important role in newer disciplines like financial engineering, image restoration and animation, neuroscience, spatial epidemiology, etc.

Topics likely to be discussed in Math 404, as time permits:

1. Introductory remarks: PDE examples, order, superposition principle, homogeneous and non-homogeneous equations, boundary conditions, classification of 2^{nd} order equations
2. Diffusion equation: remarks on derivation, some properties of solutions for different boundary conditions, steady state solutions; separation of variables method and representation of solutions in terms of Fourier series; eigenvalue problems
3. Laplace's equation: solution on various domains
4. Comments about PDEs and well-posedness; compatibility conditions; counterexamples
5. Fourier series: convergence results, integration and differentiation of series, rate of convergence
6. Wave equation: remarks on derivation, solution of various problems, Bessel functions and circularly symmetric vibrating membranes
7. Sturm-Liouville problems: regular problems, Rayleigh quotient and minimization principle, application to PDEs
8. Strategies for diffusion and wave equations with source terms and/or nonhomogeneous boundary conditions
9. Diffusion and wave equation problems in 2 and 3 space dimensions
10. Transforms: Fourier and Laplace transforms, quarter plane example problems
11. Green's function technique to solve boundary value problems
12. First order problems, method of characteristics, linear and nonlinear examples

Expected Grading Policy

The course grade will likely be based on one mid-term exam (25% each), homework (35%), and a comprehensive final exam (40%). Letter grades will be based on the weighted sum of scores and generally follow the 85-100% being A, 70-84% being B, 55-69% being a C, etc. That said, overall distribution of accumulated scores and consistency of homework performance may affect the final letter grade.