Structural change point testing with application to stock returns

Jonathan Bown
Supervised by: Lajos Horváth (Committee Chair), Tom Alberts, Ron Reeder

Department of Mathematics
University of Utah

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Outline

Main Results

Boundary function for sequential change point detection procedures that generalizes.

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Sequential procedure that uses detector function compared with boundary function to detect change points.

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Sequential procedure that uses detector function compared with boundary function to detect change points.

Fixed sample test statistic that detects changes in the mean.

Predecessors

Structural breaks in time series, Aue, A. and Horvath, L. (2012)
Introduce open problem:

$$\tau_n = \inf\{k \geq 1 : |\Gamma_n(k)| \geq g_n(k)\}.$$

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Sequential change-point detection in GARCH(p, q) models, Berkes et al. (2004)

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The total observations we collect after the training sample is T.

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We have the following hypotheses

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Model for the means under H_A

$$\mu_i = \begin{cases} \mu, & M+1 \leq i \leq M+k^*, \\ \mu + \Delta, & i \geq M+k^*+1, \end{cases}$$

where $\Delta \neq 0$ and unknown.

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First choose rolling window $h << T$, then calculate

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Z_k is evaluated for crossing boundary function $g_\alpha(h,k)$. The stopping time is defined as

$$\tau_M = \min\left\{k < T - h : Z_k > g_\alpha(h,k),\ T - h\right\}.$$

Note $k \leq T - h$.

Properties of Sequential Procedure

Under H_0 we want

$$\lim_{M\to\infty} \mathrm{P}(\tau_M < T - h) = \alpha,$$

i.e. the probability of false alarm is α. We want to have under H_A that

$$\lim_{M\to\infty} \mathrm{P}(\tau_M < T - h) = 1.$$

Main Test Assumptions

Assumption

There exist partial sums of the stationary random variables $\{e_i, i \geq 1\}$ *and Wiener processes* $\{W_1(u), u \geq 0\}$ *and* $\{W_2(u), u \geq 0\}$ *such that*

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$$\sum_{i=1}^{M} e_i = \sigma W_1(M) + o(M^{\epsilon}) \;\; a.s. \;\; \text{for some} \;\; 0 < \epsilon < 1/2 \;\; \text{as} \;\; M \to \infty,$$

$$\sum_{i=M+1}^{k+M} e_i = \sigma W_2(k) + o(k^{\epsilon}) \;\; a.s. \;\; \text{for some} \;\; 0 < \epsilon < 1/2, \;\; \text{as } k \to \infty,$$

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and

$$\{W_1(s), 0 \leq s \leq M\} \text{ and } \{W_2(t), 0 \leq t < \infty\} \ \text{ are independent.}$$

Approximations

We make use of the Komlós-Major-Tusnády approximation (Csörgő and Révész (1981)) there is a Wiener process $\{W_1(u), u \geq 0\}$ such that

$$\sum_{t=1}^{M} \varepsilon_t - (\mathrm{Var}(\varepsilon_0))^{1/2} W_1(M) = o(M^{1/\nu}) \quad \text{a.s., as } M \to \infty.$$

for some $\nu > 2$. This requires that $E|\varepsilon_t|^\nu < \infty$.

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Theorem

If $W(t)$ is a Wiener process and $\Gamma(y) = W(y+1) - W(y)$ then

$$\limsup_{y \to \infty} \frac{|\Gamma(y)|}{\sqrt{2\log(y)}} = 1 \quad a.s.$$

Limit of Sequential Procedure

Theorem

If main assumptions hold, fix an $h > 0$ s.t. $h/M \to 0$ and $h/T \to 0$ as $M, T \to \infty$, and $g_\alpha(h, k)$ is defined accordingly, then

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$$\lim_{M \to \infty} \mathrm{P}\left(\tau_M < T - h\right) = \mathrm{P}\left(\sup_{0<u<\infty} \frac{|W(u+1) - W(u)|}{(u+1)^\beta} \leq c_{1-\alpha}\right),$$

where $\{W(u), u \geq 0\}$ is a Wiener process.

Test Statistic

$$\Theta_{M,T} = \max_{1 \leq k \leq T} \left| \overline{X}_M - \frac{1}{h} \sum_{i=k}^{k+h} X_i \right| \Big/ g_\alpha(h,k).$$

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Theorem

Let $X_i, 1 \leq i \leq M$, be i.i.d. random variables, also fix an $h > 0$ s.t. $h/M \to 0$ and $h/T \to 0$ as $M, T \to \infty$, and $g_\alpha(h,k)$ is defined accordingly, then

$$\Theta_{M,T} \xrightarrow{\mathcal{D}} \sup_{0<u<\infty} \frac{|W(u+1) - W(u)|}{(u+1)^\beta}$$

for $\beta > 1/2$ and $W(t)$ is a Wiener process, or standard Brownian motion.

Equivalence of Tests

Lemma

We assume that assumptions of Theorem 2.2 are satisfied. Then,

$$\mathrm{P}(\tau_M < T - h) \xrightarrow{\mathcal{D}} \mathrm{P}\left(\sup_{0<u<\infty} \frac{|W(u+1) - W(u)|}{(u+1)^{\beta}} \geq c_{1-\alpha} \right) = \alpha,$$

where $c_{1-\alpha}$ is the critical value chosen for test of size α.

Boundary Function Definition

Definition

Let $X_i, \;\; 0 \leq i \leq M$ be stationary random variables with mean 0 and unknown long-run variance σ^2. If main assumptions on partial sums hold, fix an $h = h(T) > 0$ s.t. $\; h/M \to 0$ and $h/T \to 0$ as $M, T \to \infty$ then the required boundary function is given by

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$$g(h,k) = \frac{\hat{\sigma}(h+k)^{\beta}}{h^{\beta+1/2}}$$

for $\beta > 1/2$ and $k \in \mathbb{Z}^+$.

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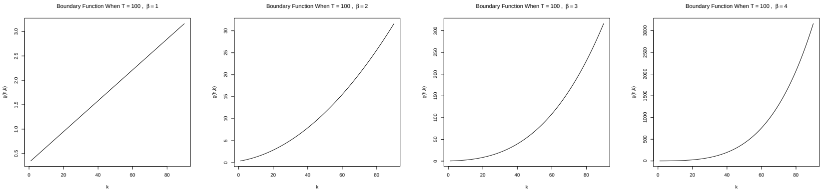
For i.i.d. random variables, $\hat{\sigma} = \sqrt{S^2}$.
For dependent r.v.s, $\hat{\sigma}$ is estimator of long-run variance.

Boundary Function Implementation

Our final boundary function is implemented as

$$g(h,k) = g(T^{1/2},k) = \frac{\hat{\sigma}(T^{1/2}+k)^{\beta}}{T^{\beta/2+1/4}}.$$

Found more consistent statistical results by using $h = \lfloor T^{1/2} \rfloor$.



Simulated Quantities

To obtain critical values for testing, we simulated

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Since

$$\sup_{u\leq t<\infty} \frac{|W(t+1) - W(t)|}{(t+1)^{\beta}} \to 0 \text{ a.s.},$$

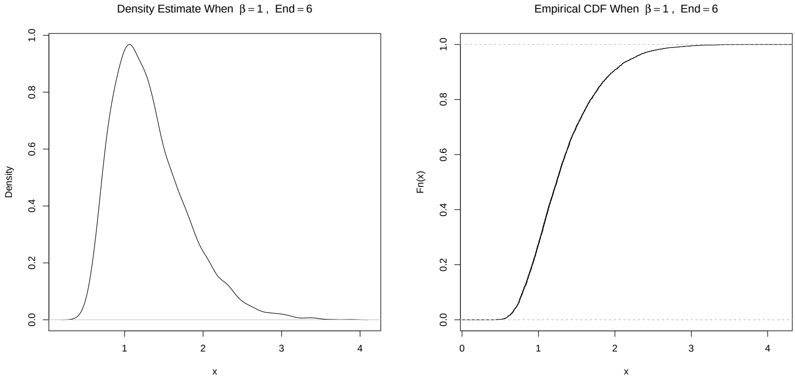
$\Theta(u)$ will change little after some u.

Upper Quantiles

Table: Asymptotic critical values chosen for Θ with different values of β

Critical Values			
β	$c_{90\%}$	$c_{95\%}$	$c_{99\%}$
1	1.961971	2.236345	2.725577
2	1.813968	2.105014	2.625778
3	1.756301	2.046612	2.586148
4	1.735013	2.010628	2.568760

Distribution of $\Theta(u)$



I.I.D. Simulation Example 1

Table: Empirical performance of test size, where $X_i \sim \text{Exp}(\lambda = 2)$

M	T	β	$\mathrm{P}(\Theta_{M,T} > c_{.90})$	$\mathrm{P}(\Theta_{M,T} > c_{.95})$	$\mathrm{P}(\Theta_{M,T} > c_{.99})$
100	100	1	0.1	0.0688	0.033
		2	0.0774	0.0474	0.0192
		3	0.0612	0.0342	0.0122
		4	0.043	0.0258	0.0088
1000	100	1	0.0854	0.053	0.022
		2	0.0648	0.037	0.013
		3	0.0494	0.026	0.007
		4	0.0346	0.018	0.0046
100	1000	1	0.1476	0.091	0.0422
		2	0.1276	0.0802	0.0314
		3	0.118	0.0734	0.0274
		4	0.107	0.0646	0.0236
1000	1000	1	0.0896	0.0512	0.0172
		2	0.0812	0.0438	0.014
		3	0.074	0.0388	0.0116
		4	0.0676	0.0358	0.0094

I.I.D. Simulation Example 2

Table: Empirical performance of test size, when $X_i \sim \text{Gamma}(k = 1, \theta = 1)$

M	T	β	$P(\Theta_{M,T} > c_{.90})$	$P(\Theta_{M,T} > c_{.95})$	$P(\Theta_{M,T} > c_{.99})$
100	100	1	0.1114	0.074	0.0376
		2	0.081	0.055	0.0248
		3	0.0652	0.0406	0.0142
		4	0.0508	0.0296	0.0082
1000	100	1	0.0758	0.045	0.0176
		2	0.0798	0.0412	0.0136
		3	0.0402	0.0198	0.0046
		4	0.0282	0.0142	0.0022
100	1000	1	0.1466	0.1002	0.016
		2	0.1258	0.0842	0.0378
		3	0.1168	0.0764	0.0318
		4	0.107	0.0718	0.0266
1000	1000	1	0.0908	0.05	0.016
		2	0.0796	0.0442	0.0116
		3	0.073	0.0384	0.0084
		4	0.0658	0.034	0.007

Long-Run Variance

We calculate the long-run variance, $\sigma^2_{x_t}$, by

$$\sigma^2_{x_t} = \mathrm{Cov}(X_1, X_1) + 2\sum_{j=2}^{\infty} \mathrm{Cov}(X_1, X_j),$$

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For dependent sequences we use the autocovariance function, γ

$$\sigma^2_{X_t} = \gamma_0 + 2\sum_{j=2}^{\infty} \gamma_j.$$

These preserve the assumptions we have established on the partial sums, by the Invariance Principle.

Parametric Long-Run Variance

Every stationary process can be represented by Wold decomposition. In Wold form it can be shown that

$$\sigma_{x_t}^2 = \sigma_\varepsilon^2 \psi(1)^2.$$

Lag-Operator Notation

With ARMA(p, q) models we use AR and MA polynomials to approximate the Wold polynomial,

$$\psi(L) = \frac{\theta(L)}{\delta(L)}. \quad \theta(L) = 1 + \theta_1 L + ... + \theta_k L^k$$

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$$\psi(L) = \frac{\theta(L)}{\delta(L)}. \quad \theta(L) = 1 + \theta_1 L + ... + \theta_k L^k$$

For GARCH models we use the form of the unconditional variance, or the ratio of the mean to the ARCH and GARCH component polynomials $\alpha(L)$ and $\beta(L)$ respectively

$$\psi(L)^2 = \begin{cases} \dfrac{\omega}{\alpha(L)} & \text{ARCH}(p), \\ \dfrac{\omega}{\alpha(L) + \beta(L) - 1} & \text{GARCH}(p, q). \end{cases}$$

Parametric Long-Run Variance Example

Take a simple AR(1) model,

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The AR(1) lag polynomial, $\delta(L) = 1 - \delta_1 L$, along with $\sigma_\varepsilon^2 = 1$ give the long-run variance

$$\sigma_{X_t}^2 = \frac{1}{(1-\delta_1)^2}.$$

Autoregressive Simulation Results

Using the parametric formula for $\sigma^2_{x_t}$, under H_0 we obtained the following simulation results.

Table: Size of the test for AR(1) sequences with $\delta_1 = 0.2$

M	T	β	$\mathrm{P}(\Theta_{M,T} > c_{.90})$	$\mathrm{P}(\Theta_{M,T} > c_{.95})$	$\mathrm{P}(\Theta_{M,T} > c_{.99})$
100	100	1	0.0588	0.029	0.008
		2	0.0496	0.202	0.0038
1000	100	1	0.0544	0.022	0.002
		2	0.0406	0.0158	0.002
100	1000	1	0.1176	0.057	0.02
		2	0.1062	0.0582	0 .0158
1000	1000	1	0.0734	0.039	0.0075
		2	0.0696	0.0328	0.0054

Non-Parametric Long-Run Variance

For practical applications, we need a non-parametric estimator. We use weights on the estimated acf, $w_{j,M}$, made popular by Newey and West (1987)

$$\hat{\sigma}^2_{x_t} = \hat{\gamma}_0 + 2\sum_{j=1}^{R_M} w_{j,M}\hat{\gamma}_j,$$

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$$\hat{\sigma}^2_{x_t} = \hat{\gamma}_0 + 2\sum_{j=1}^{R_M} w_{j,M}\hat{\gamma}_j,$$

where $w_{j,M}$ sum to unity and R_M is a truncation lag parameter that satisfies $R_M = O(M^{1/3})$. The argument

$$j/(f(M)+1), \quad f(M) = \left\lfloor 4(M/100)^{2/9} \right\rfloor$$

is passed to the kernel function to derive the weight for index j.

Kernels Used

Kernel Name	Abbreviation	Kernel, $K(z)$, where $0 \leq z \leq 1$
Bartlett	B.	$K(z) = 1 - z$
Parzen	P.	$K(z) = \begin{cases} 1 - 6z^2 + 6z^3, & 0 \leq z \leq 0.5, \\ 2(1-z)^3, & 0.5 < z \leq 1, \end{cases}$
Flat-Top	F.T.	$K(z) = \begin{cases} 1, & 0 \leq z \leq 0.5, \\ 2(1-z), & 0.5 \leq z \leq 1, \end{cases}$
Quadratic Spectral	Q.S.	$K(z) = \frac{25}{12\pi^2 z^2}\left(\frac{\sin(6\pi z/5)}{6\pi z/5} - \cos(6\pi z)\right)$
Tukey-Hanning	T.H.	$K(z) = \frac{1+\cos(\pi z)}{2}$

ARMA Simulation Results

Table: Size of the test for ARMA(1,1) sequences with $\delta_1 = 0.2, \theta_1 = 0.3$

Kernel	β	$\mathrm{P}(\Theta_{M,T} > c_{.90})$	$\mathrm{P}(\Theta_{M,T} > c_{.95})$	$\mathrm{P}(\Theta_{M,T} > c_{.99})$
B.	1	0.1096	0.0602	0.0226
	2	0.08	0.0446	0.014
P.	1	0.1362	0.0794	0.0294
	2	0.097	0.0552	0.0192
F.T.	1	0.0736	0.0384	0.0132
	2	0.0542	0.0288	0.0094
Q.S.	1	0.0912	0.0502	0.0182
	2	0.0668	0.0358	0.0116
T.H.	1	0.1036	0.0556	0.021
	2	0.0754	0.0418	0.0134

ARCH/GARCH Simulation Results

Table: Size of the test for ARCH(1) sequences with $\omega = 0.3, \alpha_1 = 0.25$

β	$\mathrm{P}(\Theta_{M,T} > c_{.90})$	$\mathrm{P}(\Theta_{M,T} > c_{.95})$	$\mathrm{P}(\Theta_{M,T} > c_{.99})$
1	0.0832	0.041	0.0134
2	0.0548	0.0258	0.007

Table: Size of the test for GARCH(1,1) sequences with $\omega = 0.3, \alpha_1 = 0.25, \beta_1 = 0.1$

β	$\mathrm{P}(\Theta_{M,T} > c_{.90})$	$\mathrm{P}(\Theta_{M,T} > c_{.95})$	$\mathrm{P}(\Theta_{M,T} > c_{.99})$
1	0.0842	0.0426	0.0138
2	0.0558	0.0272	0.0076

GARCH Simulated Stopping Times

In the simulations, we change parameters of the test examples to evaluate results of sequential test under H_A.

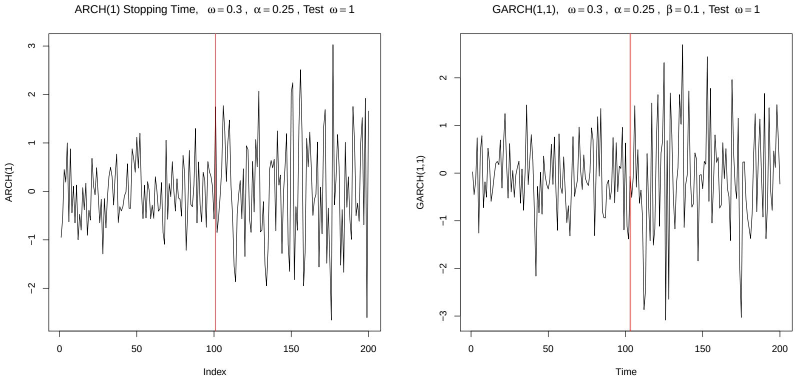

Figure: Simulated stopping time of ARCH(1), GARCH(1,1) processes

Stock Returns

Need to convert monthly price to log return r_t.

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Take ratio of prices at time t, S_t, and time $t-1$, S_{t-1}, to get

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Take ratio of prices at time t, S_t, and time $t-1$, S_{t-1}, to get

$$\frac{S_t}{S_{t-1}}.$$

Then take log to get return series,

$$r_t = \log\left(\frac{S_t}{S_{t-1}}\right) = \log(S_t) - \log(S_{t-1}).$$

DJIA Test Setup

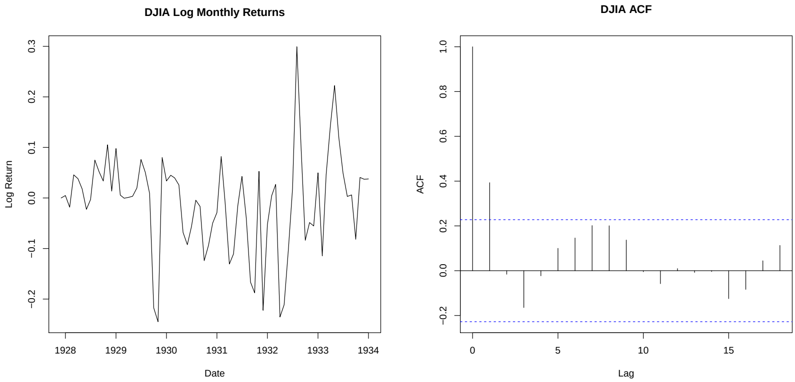

Figure: Dow-Jones Industrial log monthly stock returns and acf

DJIA Test Results

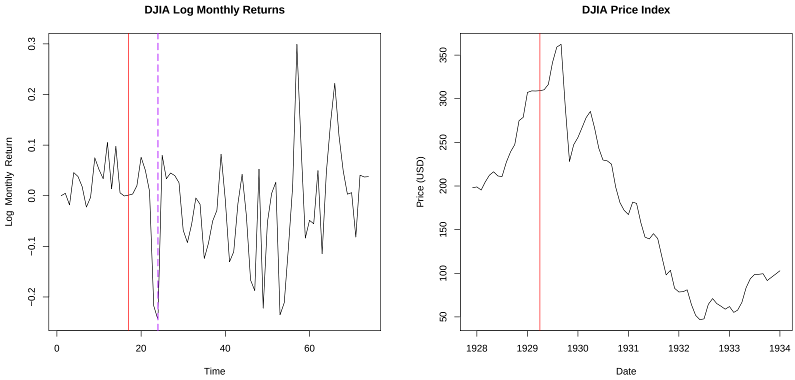

Figure: Dow-Jones Industrial log monthly stock returns and stop time

Stocks Used

Group Name	Tickers
Loss	AIG, AXP, BAC, C, GE, GS, JPM, ZION
Neutral	AZO, CLX, DLTR, JNJ, WMT
Gain (Bubble)	BTC, CSCO, GPRO, MSFT, N225, QCOM

Other Stop Times

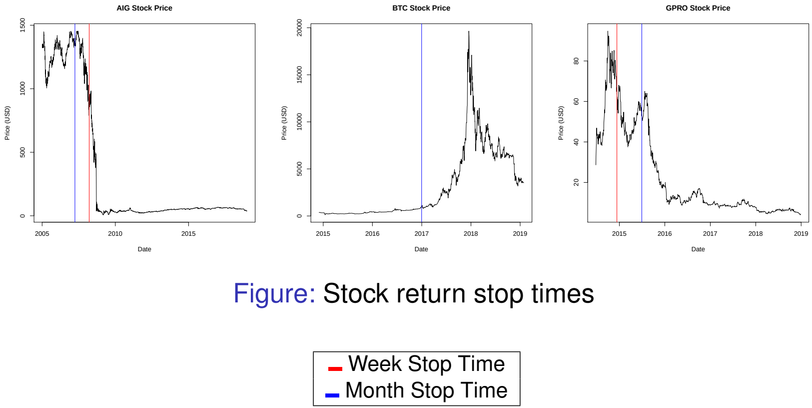

Figure: Stock return stop times

Further Research

Power function of sequential test.

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How powerful is it for different random variables?

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Develop R package with implementation interface.

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