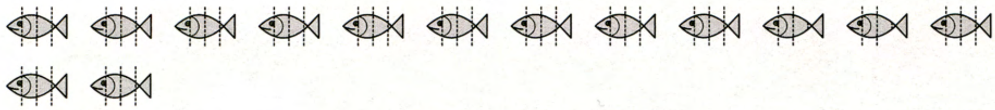
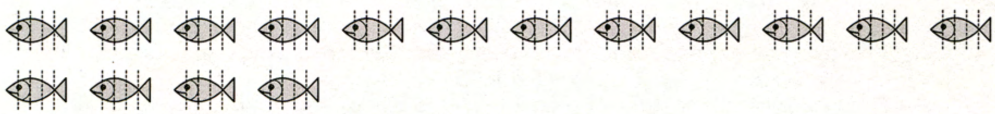
The operator interpretation of fraction can also be used to solve this problem. If we divide all 10 fish into thirds, all 14 fish into fourths, and all 16 fish into fifths, each unit fraction can operate on the total number of fish. For example, multiplying $\frac{1}{3}$ by 10 gives a mixed number, $\frac{10}{3}$, or $3\frac{1}{3}$; multiplying $\frac{1}{4}$ by 14 gives $\frac{14}{4}$, or $3\frac{2}{4}$; and multiplying 16 by $\frac{1}{5}$ results in $\frac{16}{5}$, or $3\frac{1}{5}$ fish per person.

10 fish, $\frac{1}{3}$ per person



14 fish, $\frac{1}{4}$ per person



16 fish, $\frac{1}{5}$ per person

▲

2. Equivalence and Ordering

Equivalence is one of the most important mathematical ideas for students to understand, particularly with regard to fractions. Equivalence is used when comparing fractions, ordering fractions, and adding and subtracting fractions. Equivalent fractions are fractions that represent equal value; they are numerals that name the same fractional number. When represented using a number line, equivalent fractions represent the same distance. Equivalent fractions are obtained when both the numerator and the denominator of a fraction are either multiplied or divided by the same number:

$$\frac{a}{b} = \frac{a \times c}{b \times c} = \frac{ac}{bc} \quad \text{or} \quad \frac{ac}{bc} = \frac{ac \div c}{bc \div c} = \frac{a}{b}$$

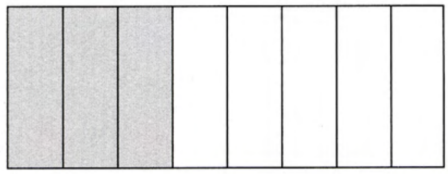
These relationships are illustrated in the diagrams on page 115, using the parts-of-a-whole interpretation of fraction. The shaded portion of Figure 1 shows $\frac{3}{8}$. Figure 2 was created by dividing each of the eighths in half—we can also say that the number of shaded regions is doubled, as is the total number of regions:

$$\frac{3}{8} = \frac{3 \times 2}{8 \times 2} = \frac{6}{16}$$

If each eighth was instead cut into four pieces, another equivalent fraction would be obtained (see Figure 3):

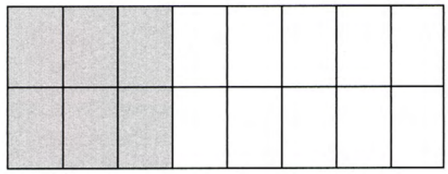
$$\frac{3}{8} = \frac{3 \times 4}{8 \times 4} = \frac{12}{32}$$

Figure 1



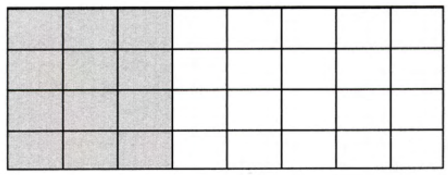
$\frac{3}{8}$

Figure 2



$\frac{6}{16}$

Figure 3



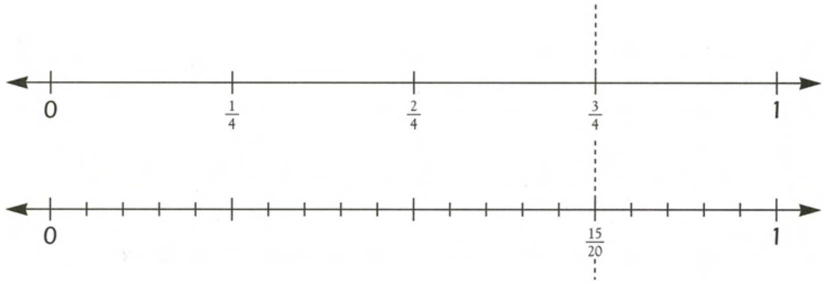
$\frac{12}{32}$

Children often use a doubling strategy to form equivalent fractions. This can be an effective strategy in some situations, as shown in Figures 1 through 3 ($\frac{3}{8} = \frac{6}{16} = \frac{12}{32}$), but notice that doubling does not produce all equivalent fractions ($\frac{9}{24}$ and $\frac{15}{40}$ to name a few).

One underlying assumption regarding equivalent fractions is that when we state that two fractions are equivalent, this implies the "wholes" are the same size. However, students do not always recognize this important fact. Furthermore, students sometimes do not focus on the areas covered by equivalent fractional amounts but instead count the number of pieces, incorrectly assuming that $\frac{3}{8}$ is not equivalent to $\frac{6}{16}$ because $3 \neq 6$. Instructional tasks that focus on equivalence need to direct students' attention to whether or not the "wholes" are the same size and whether the fractional amounts, distances on a number line, or areas in each of the wholes are the same.

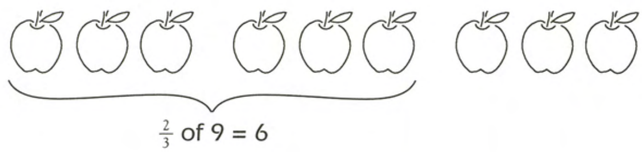
Encountering a variety of instructional models and applications may help students generalize some of the key ideas about equivalent fractions. In particular, the idea that multiplication and division can be used to form equivalent fractions needs to be examined in different situations with the different interpretations of fractions. For example, number lines can be used to show that $\frac{3}{4}$ is equivalent to $\frac{15}{20}$; both fractions represent the same distance though on the second number line each fourth is divided into five sections in order to create twentieths.

Chapin, S. H., & Johnson, A. (2006). Fractions. In T. Gordon (Ed.), Math matters: Understanding the math you teach grades K-8 (2nd ed.) (pp. 114-117). Sausalito, CA: Math Solutions Publications.

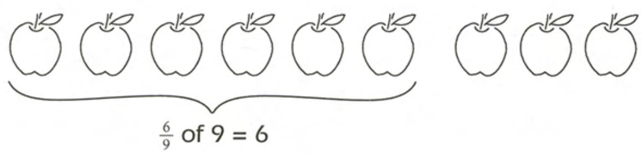


Dividing each fourth into five equal parts is equivalent to multiplying the numerator and denominator of $\frac{3}{4}$ by 5.

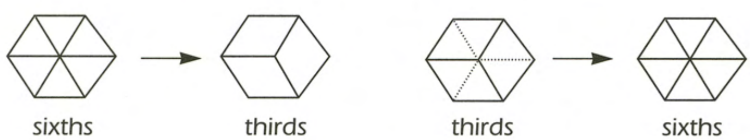
Using the operator interpretation, students might explore how $\frac{2}{3}$ of 9 is equivalent to $\frac{6}{9}$ of 9 using objects or pictures. One way to think of $\frac{2}{3}$ of 9 is to divide the 9 apples below into 3 groups, with 3 apples in each. Two groups of 3 is 6.



Likewise, $\frac{6}{9}$ of 9 also equals 6. Since $\frac{2}{3} = \frac{6}{9}$, operating (in fact, multiplying 9 by the fraction) with either of these fractions results in the same product, 6.



Pattern blocks can also be used to explore the multiplicative relationships in equivalence. If the yellow hexagon from a set of pattern blocks represents one "whole," then a blue rhombus represents one third and two green triangles represent two sixths. Students can construct models of these fractions and then use the models to describe how sixths can be grouped to form thirds or how thirds can be divided to form sixths.



Children deal with equivalence informally when counting money, using measuring cups, telling time, folding paper, sharing snacks, and eating pizza. Thus, many children are familiar with the general idea of equivalency long before the concept is introduced in school. Why then do children have such difficulty with this important idea? In part it is because we sometimes forget to design instruction around children's prior knowledge—we jump into a topic as if they know nothing or know everything! In addition, some textbooks devote very little time to exploring the meaning of equivalent fractions and instead focus on symbolic manipulation (having students practice finding common denominators, for example). Instructional activities that use models and drawings and ask students to reflect on why two fractions are or are not equivalent are necessary. The activities in this section highlight important ideas related to equivalence and order.

Activity

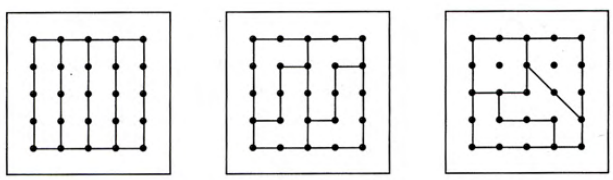
6

Exploring Fourths on a Geoboard

Objective: explore the idea that equal fractions, using a part-whole model, do not have to have the same shape but must have equal area.

Materials: a geoboard or geoboard dot paper.

On a geoboard (or geoboard dot paper) make the largest square possible. Now divide the square to show fourths. Each fourth must be an area that if cut out of paper would remain in one piece. Make another square and show fourths that are irregularly shaped. Next make fourths that are not congruent but have equal area. Take each of your previous drawings of fourths and divide them further to show eighths.



Things to Think About

What strategies did you use to make fourths? Since there are 16 square units in the square, each fourth must cover exactly 4 square units. Textbooks often unwittingly create misconceptions about fractions by presenting pictures of "wholes" partitioned into identical fractional parts. In this case, where there are 16 square units in total, any configuration that uses 4 square units is one fourth of the total area. The idea that equal fractional pieces don't have to be identical in shape but simply must cover the same area or space is an important one: it's part of the foundation on which other equivalence relationships are built. For example, a rectangular-shaped $\frac{1}{4}$ of the geoboard covers the same area as an irregularly shaped $\frac{1}{4}$ of the geoboard (4 square units), but the $\frac{1}{4}$s do not look identical. Students must overcome their tendency to rely on visualization for verification of equality and instead consider the relationships established by dividing a whole into *n* number of parts.

Chapin, S. H., & Johnson, A. (2006). Fractions. In T. Gordon (Ed.), Math matters: Understanding the math you teach grades K-8 (2nd ed.) (pp. 114-117). Sausalito, CA: Math Solutions Publications.