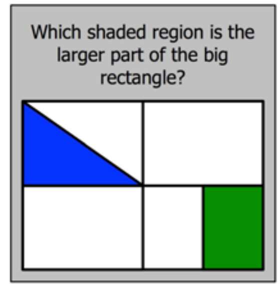
Math Notes: Analysis of the Fractions-of-an-Area Task

Description of the task:

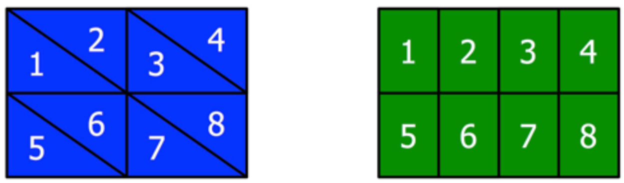


The big rectangle is partitioned into six regions of varying sizes and shapes. An area interpretation of fractions is used to compare the fractions of the big rectangle covered by the blue triangle and by the green rectangle. While the question asks which region is a larger part of the big rectangle, in this case, the answer is "neither" as they are equal in area (i.e., they both cover $\frac{1}{8}$ of the big rectangle).

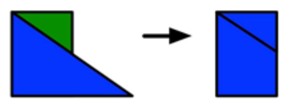
What approaches could be used to compare the sizes of the shaded regions?

Four common ways of showing the areas are equal:

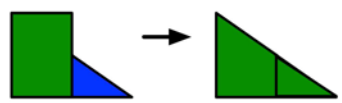
1. It takes 8 copies of either the blue triangle or the green rectangle to cover the whole. So the blue triangle and the green rectangle each cover $\frac{1}{8}$ of the big rectangle.



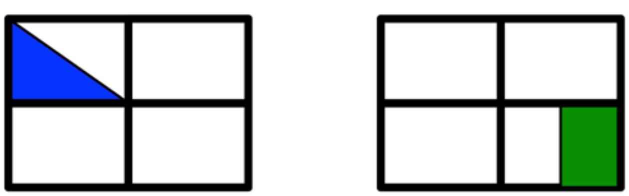
2. If you overlay the blue triangle on the green rectangle, a small piece of the triangle that doesn't overlap the green rectangle can be cut and moved to cover the remaining portion of the green rectangle. This shows directly that the blue triangle and the green rectangle have equal areas.



3. If you overlay the green rectangle on the blue triangle, a small piece of the rectangle that doesn't overlap the blue triangle can be cut and moved to cover the remaining portion of the blue triangle. This again shows that the green rectangle and the blue triangle have equal areas.

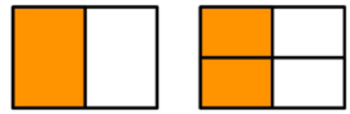


4. Each shaded region is one-half of one-fourth of the big rectangle. The big rectangle can be shown to be divided into four equal-sized rectangles (i.e., each of these white rectangles is one-fourth of the big rectangle). It takes two of the blue regions to cover one of these small white rectangles. Thus, one blue region is one-half of one-fourth of the big rectangle. A similar argument can be made for the green region.



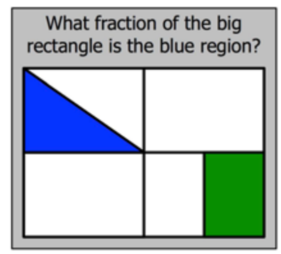
What do equivalent fractions mean in the context of an area model?

In the context of an area model, fractions refer to the area of a region in relation to the area of the whole. Thus, two regions of *the same whole* represent equivalent fractions if and only if they have the same area. For example, the figures to the right show orange regions of the same area, and so they show the equivalence of the fractions $\frac{1}{2}$ and $\frac{2}{4}$. In the figure on the left, the whole is divided into two equal parts and one part is orange. In the figure on the right, the whole is divided into four equal parts and two parts are orange. In both cases, the total area covered by the orange regions is identical so we can say that $\frac{1}{2}$ and $\frac{2}{4}$ are equivalent fractions.

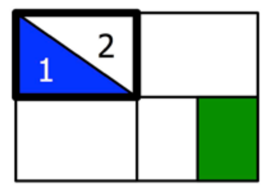


What are common ways that students interpret area model tasks?

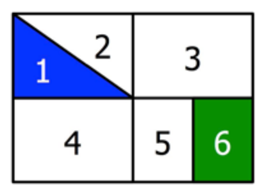
When this task is used with elementary school students, students may initially be asked to consider the fraction of the rectangle that is shaded blue and the fraction of the rectangle that is shaded green. There are several patterns of reasoning that students typically use to interpret area model tasks. These patterns are described in the context of considering the fraction of the big rectangle that is the blue region.



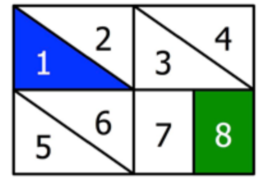
1. *Redefining the unit.* When students are presented with area model tasks in which parts vary in size, students may redefine the whole. For example, as shown to the right, a student might treat the smaller rectangular region outlined in black as the whole and then indicate that the blue region is $\frac{1}{2}$ of the "new" whole because the "new" whole is divided into two equal parts and one of those parts is blue. A similar argument would be made for the green rectangle.



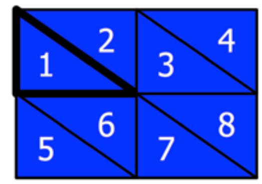
2. *Discrete count strategy.* When students interpret area models, students may not consider the size of the part in relation to the size of the whole; rather, students might consider the number of shaded parts in relation to the number of total parts. When a region is partitioned into parts that vary in size, the fraction produced by the student will not represent the relationship of the area of the shaded part in relation to the area of the whole. For instance, a student using this strategy would label the blue region shown to the right as $\frac{1}{6}$ of the whole because the whole is divided into 6 parts and one of those parts is blue. Similarly, the green rectangle also would be called $\frac{1}{6}$ of the big rectangle. Notably, in 2005, Geoffrey Saxe and colleagues at the University of Californa, Berkeley found that when students produce correct answers on area model tasks that are partitioned into equal parts, they may be using a discrete count strategy. The use of unequally partitioned area model tasks has the potential to reveal instances where students are using this strategy, and can be used to prompt discussion about the importance of equal-sized parts.



3. *Part-whole strategy.* A part-whole strategy involves partitioning the whole into parts that are equal in size (they may or may not be the same shape) and then considering the number of shaded parts in relation to the total number of parts. In the figure to the right, $\frac{1}{8}$ of the big rectangle is blue because the whole is divided into 8 parts of equal size and one of those parts is shaded blue.



4. *Measurement strategy.* A measurement strategy involves reasoning about the number of copies of a given part needed to cover the whole. The fraction of the rectangle that is shaded is the number of parts of the given size (the blue triangle) that are shaded in relation to the total number of copies of that part (blue triangles) needed to cover the whole. In the figure to the right, the one blue part is $\frac{1}{8}$ of the rectangle because it takes eight copies of the triangular region to cover the whole area within the rectangle.



What are the common ways that students compare the area of the blue and the area of the green regions?

Even when students are able to determine that both the blue and green regions are $\frac{1}{8}$ of the big rectangle, they may not correctly compare the two areas. When asked to compare regions of a whole that are not the same shape, some students will compare the regions based on perceptual features. For example, when upper elementary school students are asked to compare the areas of the blue and green regions, a common response is that students indicate that the blue triangle is larger because it "looks bigger" than the green rectangle even though they have already labeled the blue region and the green region as both representing $\frac{1}{8}$ of the big rectangle. It is also noteworthy that other students may correctly conclude that the blue and the green regions represent the same area, without being able to articulate that the two regions each represent $\frac{1}{8}$ of the rectangle.

References

Saxe, Geoffrey B., Edward V. Taylor, Clifton McIntosh, and Maryl Gearhart. 2005. Representing Fractions with Standard Notation: A Developmental Analysis. *Journal for Research in Mathematics Education 36*(2): 137-57.