Elementary Mathematics Laboratory for incoming fifth graders
Park City Mathematics Institute
Tuesday, July 11, 2006

Seating Arrangement

Jessica								Maddie
Ally								Cozy
Sabrina								Holly
Paige								Luke
Tori								Arthur
Brianna								Britney
David	Vinnie	Rebecca	Sarah	Ben	Trevor	Michael	Sean	Autumn

July 11, 2006.

Problem:

Teacher: _… With the number line and the group of circles, I want you to show what one-eighth means in each of those._
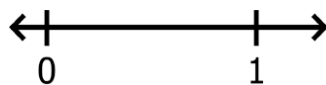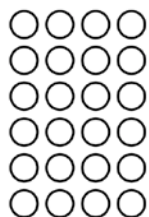
| 1 | Teacher: | So now the other one is the twenty-four circles. Would |
2		somebody like to explain the way that they decided
3		what one-eighth would look like in that drawing? Let's
4		see, who's not had a chance to talk? Sean, you wanna
5		go up and do it? Are you doing okay?
6	Ally:	Yeah
7	Teacher:	Okay.
8	Sean:	Okay, so the- There's twenty-four squares and-
9	Teacher:	Circles, I think. Circles.
10	Sean:	Circles. Oh, circles and eight goes into twenty-four
11		three times, so you make it into three groups. Into
12		groups of three. (_Draws:_)
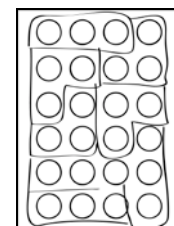
13		And if you count them, you'll have eight. One, two,
14		three, four, five, six, seven, eight.
15	Teacher:	So what are those- What's the eight?
16	Sean:	Three.
17	Teacher:	Okay, but why- What- In each eight is- The eight-
18		You made eight groups, and how much is in each one?
19	Sean:	Three.
20	Teacher:	Three. And why did you make it into eight groups?
21	Sean:	Because- Eight- You need to make it into eight groups-
22		for an eighth.
23	Teacher:	Eight- What kind of groups?

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24	Sean:	Into groups of three to make eight.
25	Teacher:	Equal groups, right? Okay-
26	Sean:	Equal groups.
27	Teacher:	And so what are you calling one-eighth in this drawing?
28	Sean:	Three circles.
29 30	Teacher:	Three circles. Okay, do you understand what he did? Let's get some comments now. Rebecca?
31 32	Rebecca:	Well, I'm wondering maybe there's a different way to do it.
33 34 35 36	Teacher:	Okay, but first let's comment on this and then we can see a different way. What- Can someone comment on what David did? He took the twenty-four and what did he do then?
37	Vinnie:	David? Sean.
38 39	Teacher:	I'm sorry, Sean. What did he do then? He took the twenty-four and did what? Yes?
40	Brianna:	He divided it into three.
41	Teacher:	He divided it into groups of three.
42	Brianna:	Into groups of three.
43	Teacher:	And how many groups of three did he get?
44	Brianna:	Eight.
45 46 47	Teacher:	Eight. So let's check it against- Can someone check it against the poster now? What's the whole in this drawing? Art?
48	Art:	All the circles.
49	Teacher:	How many?
50	Art:	The twenty-four circles.
51	Teacher:	Twenty-four circles. Did Sean divide it into equal parts?
52	Art:	Yes.
53	Teacher:	Yes. How many equal parts did he make? Maddie?
54	Maddie:	Eight.
55 56 57 58 59	Teacher:	Eight. Now, the name of the kind of part comes from the number of equal parts which was eight equal parts, so it's one-eighth. And the answer was- How many was in each eighth? Three. Okay? Do- So you want to show a different thing? Okay, Rebecca, go ahead.
60	Autumn:	After Rebecca if she doesn't use…
61	Teacher:	Thank you, Sean. What?
62	Autumn:	After Rebecca can I go show this way?
63	Teacher:	Yes, in a minute.
64	Autumn:	Okay.
65	Teacher:	Oh, let me give you a new drawing. I'm sorry.
66	Rebecca:	Okay.
67	Teacher:	There you go. Can people see Rebecca?
68 69 70 71	Rebecca:	Well, you have twenty-four as your whole and then- well, twenty- Eight goes into twenty-four three times, and so what I did is I just made eight groups- Well, three groups of eight, (*draws:*)
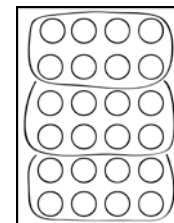
| 72 | | and so- and then to get my one-eighth, (*draws:*) |
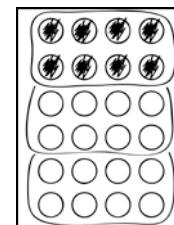
73		I just shaded in one of those groups.
74	Teacher:	Okay, so that's very interesting. So Rebecca knows that
75		eight times three is twenty-four and- How is her picture
76		different from Sean's? Can someone explain? I should
77		see more hands than that because that's a pretty easy
78		question. How is Sean's picture- How is Rebecca's
79		picture different from Sean's? They're both on the
80		board. You should be able to see them both. What's
81		different? Ben?
82	Ben:	On Rebecca's there are three groups and on Sean's
83		there are eight.
84	Teacher:	Good. They have the same whole, they both have
85		twenty-four circles and they both made equal groups,
86		but their equal groups are different. So Rebecca made
87		three equal groups and Sean made eight equal groups.
88		So what does Rebecca's picture show? Can somebody
89		use what we're developing about fractions to say what
90		her picture shows? What kind of fraction did Rebecca
91		show? Vinnie?
92	Vinnie:	One-third.
93	Teacher:	Okay, why one-third, Vinnie?
94	Vinnie:	Because she's- she did three groups and she colored
95		one of them.
96	Teacher:	Okay, good. She made thirds because she divided into
97		three equal groups and made one-third. And how many
98		is in one- How many circles in one-third, Vinnie?
99	Vinnie:	Eight.
100	Teacher:	Eight. So that's very interesting, Rebecca. So, you
101		made a different fraction, but you can see why you
102		were thinking it. Why did you end up thinking about
103		that one? You had a good reason. Why did you end up
104		doing that? You made one-third, but why did you end
105		up with thirds when you were thinking about eighths?
106		Why did that happen?
107	Rebecca:	Because I thought if you just make three groups of
108		eight and shaded in one of them, it would equal to one-
109		eight.

110	Teacher:	It makes sense because eight times three is twenty-four
111		but in fact, what you ended up doing is showing us
112		what it looks like when you make three equal groups.
113		Very nice job. But that's not one-eighth, that's one-
114		third. But you did a good job of thinking about that.
115		Okay, so now I think Autumn has one more to show.
116		Do you want to show yours, Autumn, or do you not
117		want to show it now?
118	Autumn:	I don't care.
119	Teacher:	What?
120	Autumn:	I don't care. But I'll go.
121	Teacher:	Okay. Watch carefully, because this is again a different
122		one, and you need to decide: Is it just a different way
123		of showing it? Or, is Autumn thinking about something
124		else? Just like with what Rebecca just did. Here's a
125		clean one. This is the last one we're gonna look at, and
126		then you're gonna do a little bit of independent work
127		with fractions. Here's a blank one for you.
128	Autumn:	Okay.
129	Teacher:	I'll move this up. Okay, there you go.
130	Autumn:	Okay. I'm not sure if it's right, but I-
131	Teacher:	Did you say you're not sure it's right?
132	Autumn:	Yeah.
133	Teacher:	Okay, so watch carefully. She's saying even while she
134		starts to talk about it that she's not sure it's right. So
135		watch with her and think about whether it's right or not.
136	Autumn:	So, I did like what Rebecca and Sean did. I put them in
137		groups of eight. (*Draws:*)
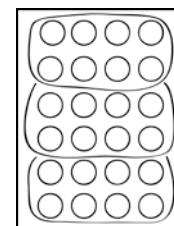
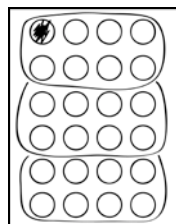|		
138		And then I- And then-
139 140	Teacher:	So, first of all, wait one second. Let's ask a question. Whose picture is that more like right now? Maddie?
141	Maddie:	Rebecca's.
142 143	Teacher:	It's more like Rebecca's. Now, what are you gonna do next?
144 145	Autumn:	And then, because each group is eight, and it's one-eighth, I just colored in one. (*Draws:*)

146	Teacher:	Okay, so is that one-eighth of what?
147 148	Autumn:	One-eighth of this (*gestures to the top group of eight circles*).
149 150 151 152 153 154 155 156 157 158	Teacher:	Okay, so can someone say what question Autumn is answering? Lots of times when people get a different answer in math, they're actually answering a different question which is a useful thing for you to think about because that helps you to learn more math if you can figure out what question someone is actually answering. She doesn't get the same answer as Sean. Sean got the answer three for one-eighth of twenty-four, and we agreed that was right. But what is Autumn doing? Can someone explain what question she is answering? Yes.
159 160 161 162	Paige:	She's answering- If you do all of them, it's actually- Well, since there's eight circles in it, she colored one in, which would be one of the eight, and that's what she was thinking, but if you- There's still the two parts left.
163 164 165	Teacher:	Well, don't go to that yet. So what is that one-eighth of? It's one-eighth of something. What is it one-eighth of? Her drawing. Look at her drawing.
166	Paige:	It's one-eighth of a third.

167	Teacher:	One-eighth of what? Say it again.
168	Paige:	One-eighth of a third.
169 170 171 172 173	Teacher:	One-eighth of a third of the twenty-four. And why is Paige- Sorry. Why is Autumn calling that one-eighth? There's a good reason. Because there's something about that that's right that she's doing. What is it? Sean?
174	Sean:	Because there's eight circles in the rectangle-
175	Teacher:	Right.
176	Sean:	-that she shaded.
177 178	Teacher:	So, in that group, it's one-eighth of those eight circles, right? Now go on to say something was missing, Paige?
179 180	Paige:	They're- The two ha- The two of the thirds are still left.
181	Teacher:	Okay. So-
182 183	Paige:	And, if you put them all together, it would actually be three-eighths instead.
184 185 186	Teacher:	Can you go up and do that? So there's a way of working with Autumn's drawing to see the- to see the one-eighth of twenty-four. Can you work with it?
187 188 189	Paige:	Well, if you- Because these two are still left and- So, you need to color in two more. (*Draws on Autumn's diagram:*)
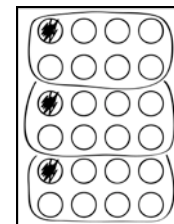
190 191 192 193		But when you do that, then that would equal three-eighths instead of one-eighth and it would be three-eighths of a third. She was right on the first part, but there's still those two.

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195	Teacher:	Okay. So let me just clear this up a little and then we're
196		going to do some work together. That's really very
197		interesting what Autumn did and what Paige did with it,
198		so see if you can follow this. Can everyone look up
199		here for a minute? Everybody's eyes should be up here.
200		Okay, ready? So what Autumn did is she took the
201		whole twenty-four circles and made three groups.
202		That's a lot like what who did? Rebecca. And then she
203		colored in one out of the eight. And we just said, well,
204		one out of eight would be one-eighth, right? So, she
205		was only looking- It's like this part didn't exist (*folds the*
206		*paper so only the top group of eight circles is showing*).
207		Okay. Is that one-eighth? Yeah. Because it's one out
208		of eight parts. One equal part out of eight of the parts.
209		So then, Paige did something else interesting. She said,
210		well, there are two thirds left and she took one-eighth
211		of this one, and what did she do here?
212	Student:	She took one-eighth.
213	Teacher:	One-eighth of that one. So, she took one eighth of this
214		one, one-eighth of this one, and one-eighth of this one.
215		Now she's actually taken one-eighth of the whole thing
216		because she's just done it in steps. One-eighth of this
217		group, then one-eighth of this group, then one-eighth of
218		this group. How many circles are shaded when Paige is
219		done?
220	Student:	Three.
221	Teacher:	Ben?
222	Ben:	Three.
223	Teacher:	Three, which is the same answer that who got? Who
224		else got three for an answer? Sarah?
225	Sarah:	Sean.
226	Teacher:	Who?
227	Sarah:	Sean.
228	Teacher:	Sean. Now, where's Sean's drawing?
229	Sarah:	Down there.
230	Teacher:	Right there. So, they both arrived at the answer of
231		three but in a different way. And what this- Between
232		Paige and Autumn together is a little bit harder to see.
233		But, if you take one-eighth of one equal part and then
234		one-eighth of the next equal part and one-eighth of the
235		next equal part, you've taken one-eighth of the whole
236		thing. This (*Sean's drawing*) is probably the easiest way
237		to see it because he took the twenty-four and made
238		eight equal parts out of it. But that's very interesting,
239		these different things that people tried to do.