Math Notes: Properties and Conventions of the Number Line

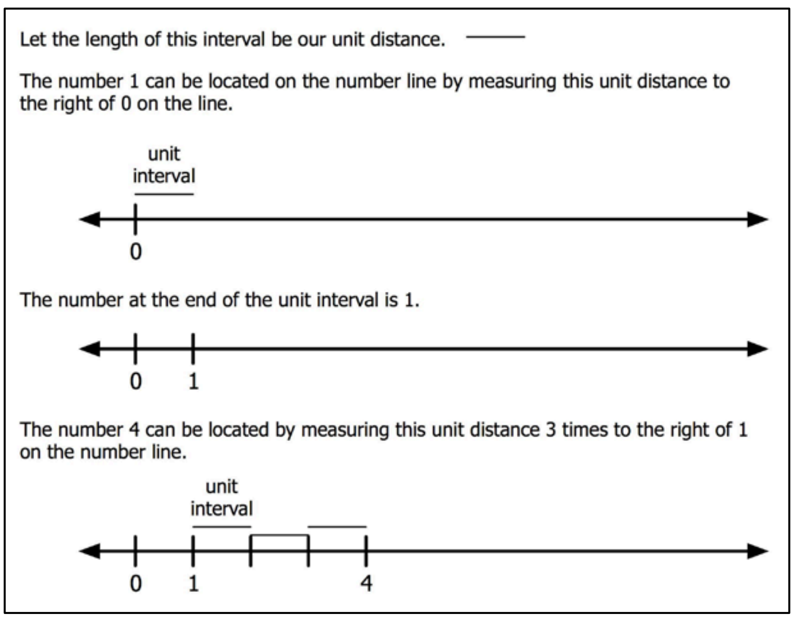
The number line, a model of the real numbers, is a representation that is introduced in the primary grades and continues to be used in the intermediate grades and beyond. For example, in the primary grades, students frequently are introduced to number lines for counting and addition/subtraction of whole numbers. In the intermediate grades, students are formally introduced to negative integers and fractions on the number line. In the middle and secondary grades, students use number lines to represent the rational numbers (integers and positive and negative fractions) and irrational numbers such as $\sqrt{2}$ and π. This is why the number line is often referred to as the *real line*. Students also see a pair of number lines in the context of the coordinate plane.

The number line is a mathematical object with specific properties and conventions. This document discusses several of these properties and conventions as well as various properties that result from them.

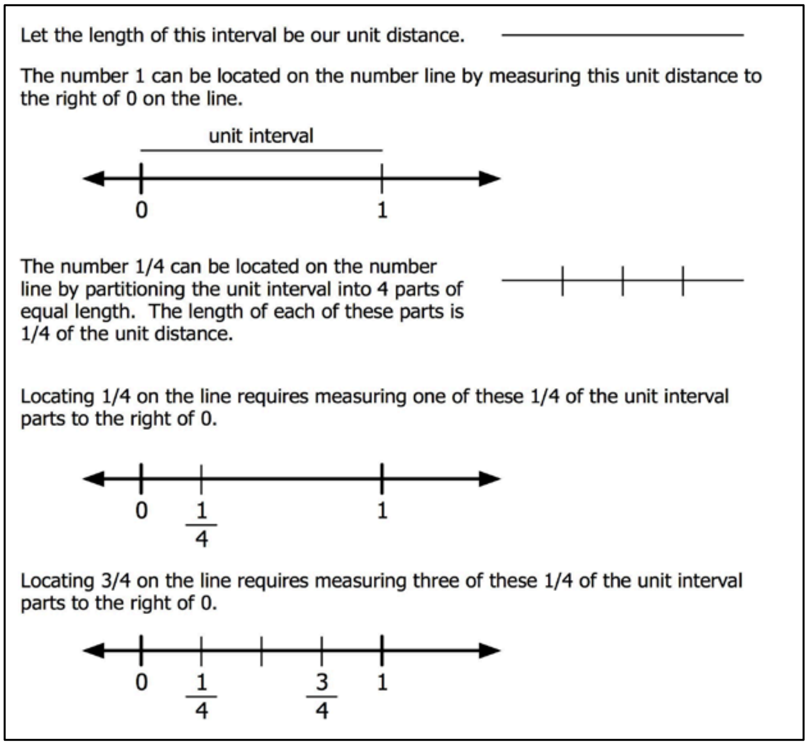
Properties and Conventions of the Number Line

The number line has seven fundamental properties/conventions, each of which are described below in the context of a horizontally displayed number line. These properties/conventions can be modified to think about a vertically displayed number line (such as a thermometer or the *y*-axis of the coordinate plane).

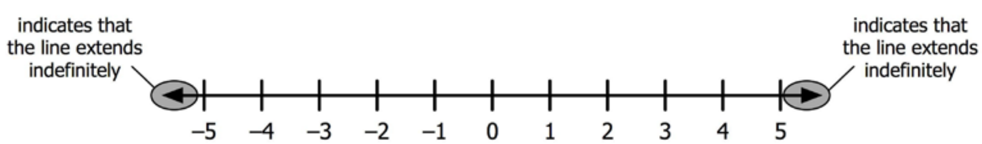
Locating Numbers*.* The whole numbers are located on the number line by starting at 0 and repeatedly measuring the unit distance to the right. The figure below illustrates the location of 1 and 4 on the number line.



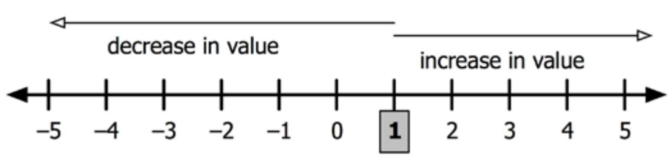
A fraction is similarly located on the number line by partitioning the unit interval into parts of equal length where the number of parts corresponds to the denominator. A fraction can be located by repeatedly measuring the length of one of these parts. The figure below illustrates the location of $\frac{1}{4}$ and $\frac{3}{4}$ on the line.



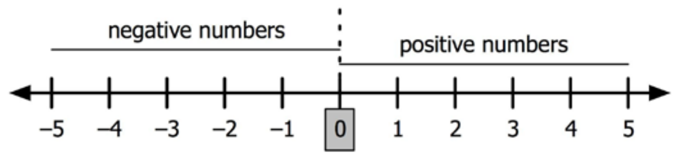
Two infinite directions: The number line has two directions and extends infinitely in both. As shown below, we use arrowheads on either end of the line to indicate that the line extends indefinitely.



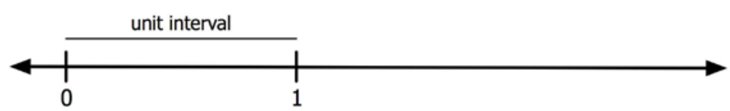
Increase to the right: Numbers are represented by points on the line. As illustrated below, numbers increase in value as you move to the right (and decrease in value as you move to the left). So, for example, 3 is greater than 1 (denoted, 3 > 1) because 3 is to the right of 1 on the number line. Similarly, –1 is greater than –3 (–1 > –3).



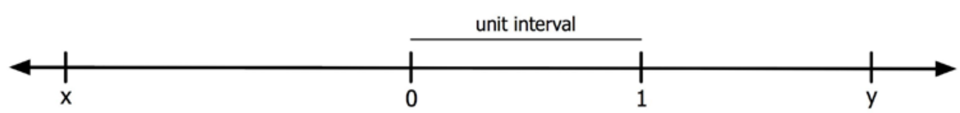
Positive and negative: Once the number 0 is located, points to the right of 0 are "positive" numbers (> 0) and points to the left of 0 are "negative" numbers (< 0). Zero (0) is neither positive or negative.



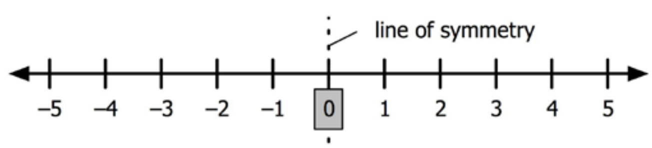
Unit interval: The interval from 0 to 1 is defined to be the *unit interval* and its length is the *unit distance*.



Labeling Points: The number corresponding to a point is determined by its distance from 0 in relation to the unit distance and its direction from 0. In the figure below, the point y represents the number 2 because y is located to the right of 0 where positive numbers are located and the distance from 0 to y on the line is equal to twice the unit distance. The point x represents the number $-1\frac{1}{2}$ because x is located to the left of 0 where the negative numbers are located and the distance from 0 to x on the line is one unit distance plus one-half the unit distance.



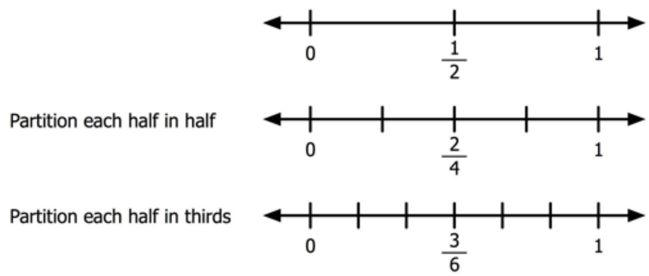
Symmetry: The number line can be reflected through 0. This reflection maps each number x to its additive inverse (opposite), $-x$. This means that each number x is the same distance from 0 as its additive inverse (opposite). In other words, 2 and –2 are both located the same distance from 0 but in opposite directions.



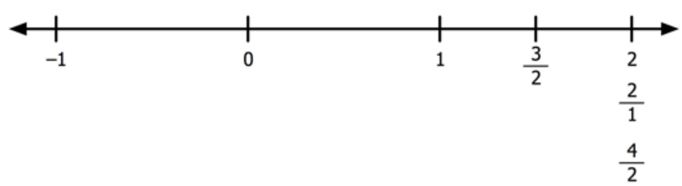
Other Observations about the Number Line

Using these properties and conventions, we can make several observations about the number line.

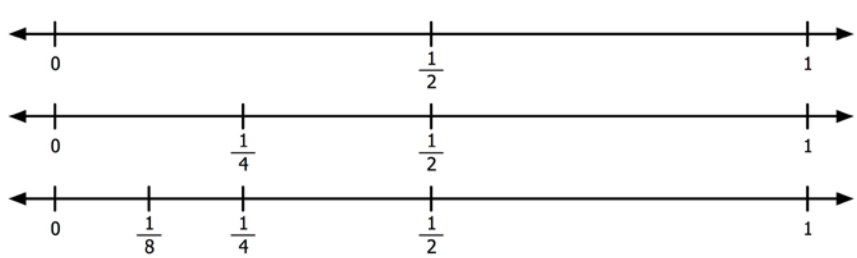
1. Every number can be written in infinitely many equivalent forms. For example, the number $\frac{1}{2}$ can be rewritten as $\frac{2}{4}$ and $\frac{3}{6}$ as shown in the figure below. Other names are possible too. There are infinitely many names for this point on the line. In fact, two fractions are equivalent if and only if they represent the same point on the number line.



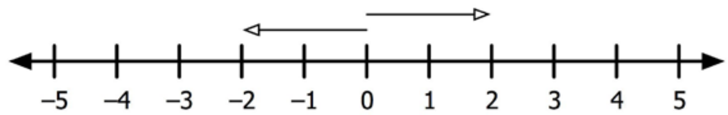
2. All integer points can be written as fractions, but not all fractions represent integers. For example, as shown below, the integer 2 can be rewritten as $\frac{2}{1}$ and $\frac{4}{2}$. On the other hand, the fraction $\frac{3}{2}$ cannot be rewritten as an integer as its distance from 0 is not a multiple of the unit distance.



3. Between any two points on the number line there is always a rational number (a number that can be written as a fraction). This is what we mean when we say that the rational numbers are *dense* in the line. As shown in the figure below, using a repeated halving approach, we see that the fraction $\frac{1}{2}$ is between 0 and 1, the fraction $\frac{1}{4}$ is between 0 and $\frac{1}{2}$, the fraction $\frac{1}{8}$ is between 0 and $\frac{1}{4}$, and so on. While it might appear that space will eventually run out, one can think about "zooming in" on the interval to show additional numbers.



4. Any number, x, and its additive inverse (opposite), $-x$, are the same distance from 0. For example, as shown in the figure below, –2 and 2 are both at distance 2 from 0. The same is true for –3 and 3 and so on.



5. The numbers represented by points on the number line are called the real numbers and so the number line is also called the "real line". The real numbers include the rational numbers, but not all real numbers are rational; those that are not are called *irrational numbers* (e.g., $\sqrt{2}$ and π can be shown to be irrational).