Math Notes: Analysis of the Fractions-of-a-Length Task

Description of the task:

This task uses the linear interpretation of fractions to compare two lengths that are fractions of a linear unit, the inch. The task might be represented with rulers, fraction strips, or number lines.

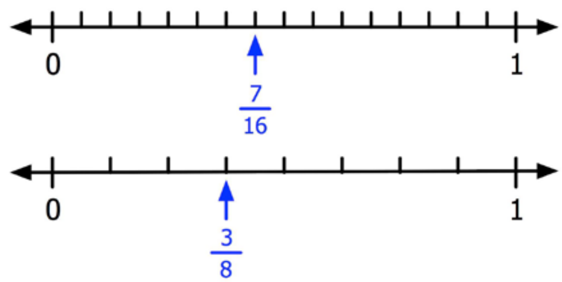
Which is longer $\frac{3}{8}$ of an inch or $\frac{7}{16}$ of an inch?

What approaches could be used to compare the two fractions?

There are three common strategies that can be used to compare two fractions. Other strategies are possible as well.

1. *Distance from a benchmark.* One strategy involves determining the (directed) distance of each fraction from a benchmark. For example, $\frac{3}{8}$ is $\frac{1}{8}$ less than $\frac{4}{8}$ (or $\frac{1}{2}$) and $\frac{7}{16}$ is $\frac{1}{16}$ less than $\frac{8}{16}$ (or $\frac{1}{2}$). Since $\frac{1}{16}$ is less than $\frac{1}{8}$, it follows that $\frac{3}{8}$ is farther to the left of $\frac{1}{2}$, making $\frac{3}{8}$ the lesser fraction.

2. *Comparing using a number line (or fraction strips).* Another strategy involves showing the two fractions using a fraction paper strip or a number line. Each fraction is located (in the case of the number line) or constructed (in the case of the fraction strips) and the fraction that is the farthest to the right of 0 is greater. In the case of $\frac{3}{8}$ and $\frac{7}{16}$, $\frac{7}{16}$ is the greater fraction because it is further to the right of 0.



3. *Common denominator.* A third strategy involves finding a common denominator and then comparing the numerators of the two fractions. In the case of $\frac{3}{8}$ and $\frac{7}{16}$, $\frac{3}{8}$ can also be written as $\frac{6}{16}$. Then, $\frac{6}{16}$ and $\frac{7}{16}$ can be compared. $\frac{7}{16}$ is the greater fraction because it consists of 7 one-sixteenth parts while $\frac{6}{16}$ only consists of 6 one-sixteenth parts.

What are common ways that students interpret this task?

In addition to the three approaches just described, that yield the correct comparison, there are also strategies that students might use that indicate either overgeneralization of fraction comparison strategies or partial understandings of the meanings of fractions.

1. *Focusing on the number of parts (numerator) without considering the size of the parts (denominator).* Students may argue that $\frac{7}{16}$ is greater than $\frac{3}{8}$ because 7 is greater than 3. In the case of these two fractions ($\frac{3}{8}$ and $\frac{7}{16}$), focusing solely on the number of parts without considering their sizes does yield the correct comparison; however, this strategy leads to incorrect conclusions in other cases (e.g., a fraction comparison like $\frac{5}{8}$ and $\frac{7}{16}$).

2. *Focusing on the size of the parts (denominator) without considering the number of those parts (numerator).* When students initially compare fractions, they are often comparing unit fractions (e.g., $\frac{1}{3}$ and $\frac{1}{5}$) and so the focus is on the size of the parts. When comparing non-unit fraction (e.g., $\frac{3}{8}$ and $\frac{7}{16}$), students may overgeneralize the strategy that worked with unit fractions (or other comparisons in which the numerators of the fractions being compared is the same). For example, a student might indicate that $\frac{3}{8}$ is greater than $\frac{7}{16}$ because $\frac{1}{8}$ is greater than $\frac{1}{16}$.

3. *Focusing on the numerators and denominators separately.* Students may separately compare numerators and denominators. In the case of comparing $\frac{3}{8}$ and $\frac{7}{16}$, a student might indicate that $3 < 7$ and $8 < 16$, so $\frac{3}{8}$ is less than $\frac{7}{16}$. This separate comparison of numerators and denominators yields correct comparisons in some cases, but not in others (e.g., comparing $\frac{5}{8}$ and $\frac{7}{16}$ using this strategy leads to an incorrect comparison).

What do equivalent fractions mean in the context of linear models?

In the context of a linear model, a fraction represents a directed distance of a point from 0 in relation to a unit distance. Equivalence of fractions has a simple meaning in this context: Two fractions are equivalent if and only if they name the same point on the number line. The figure to the right illustrates geometrically the equivalence of $\frac{3}{8}$ and $\frac{6}{16}$.