

Pathology Segmentation with Group Equivariant Networks

Jon Middleton

Supervisors: Akshay Pai, Stefan Sommer,
Mads Nielsen

Innovationsfonden

UNIVERSITY OF COPENHAGEN



Takeaways

Takeaways

- There's a need for sample efficient networks in medical imaging

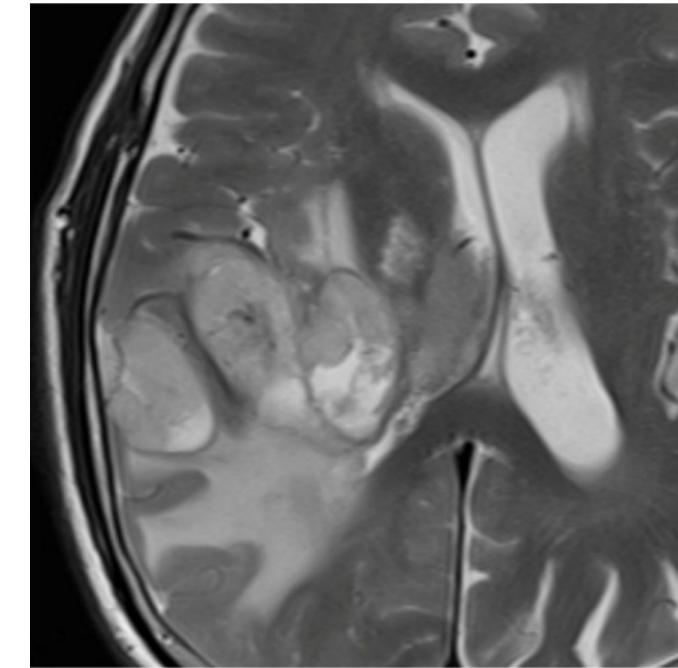
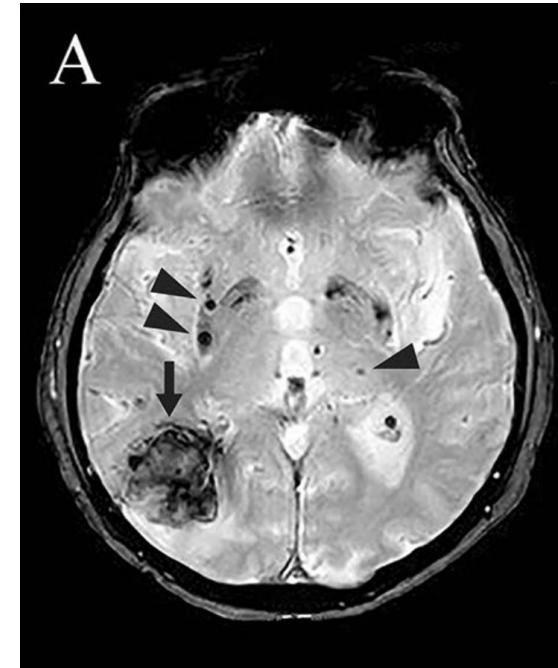
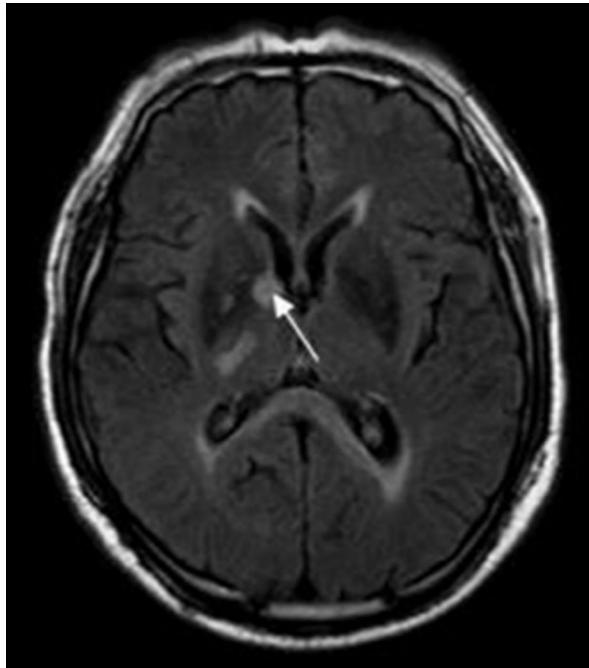
Takeaways

- There's a need for sample efficient networks in medical imaging
- Group equivariant networks are sample efficient

Takeaways

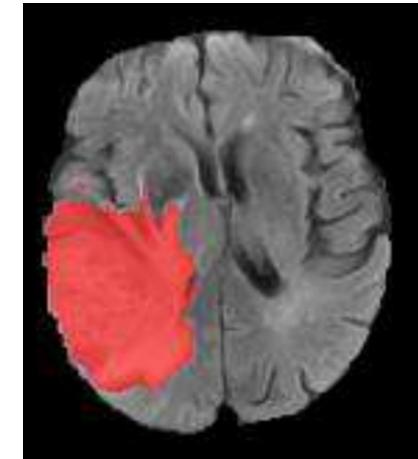
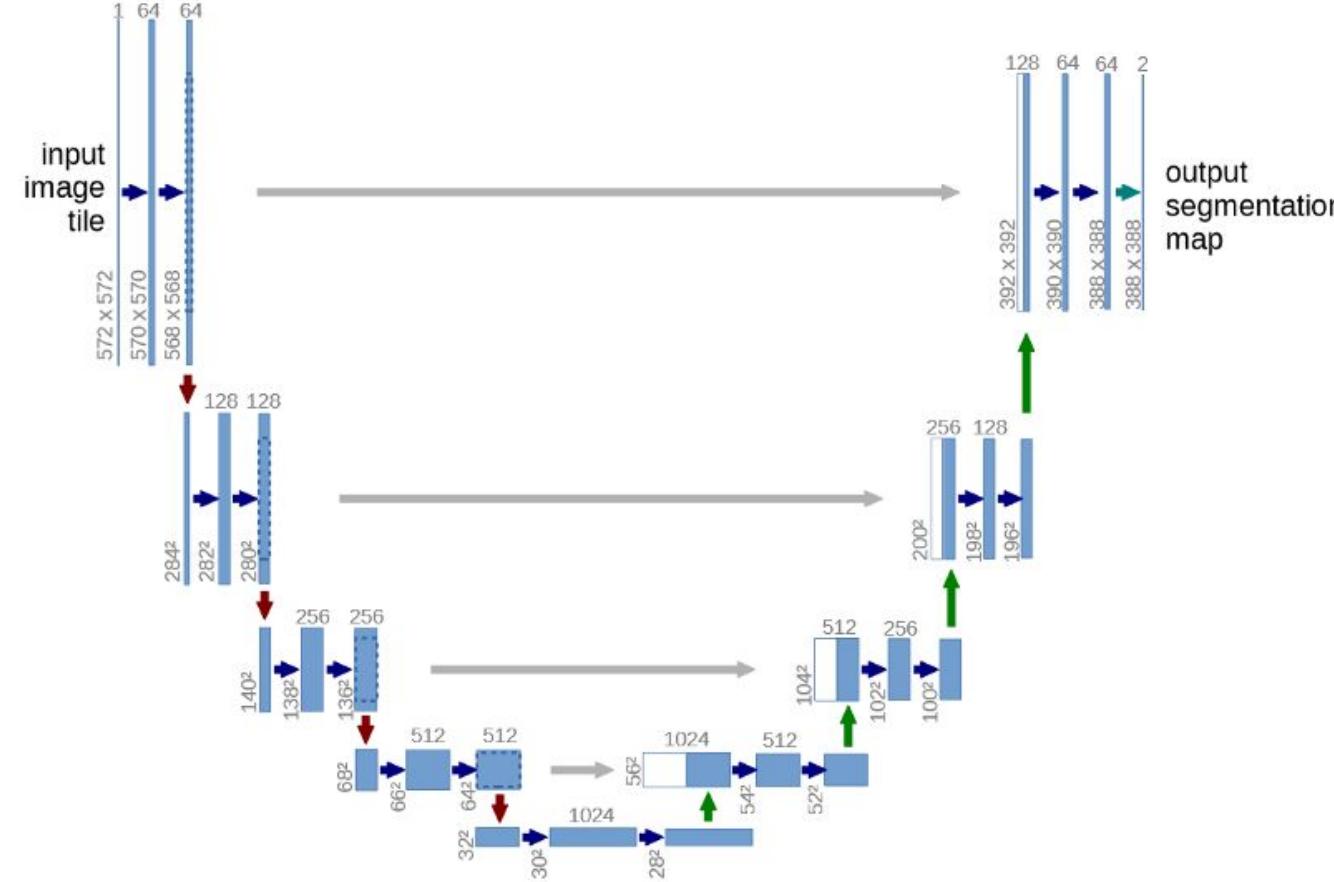
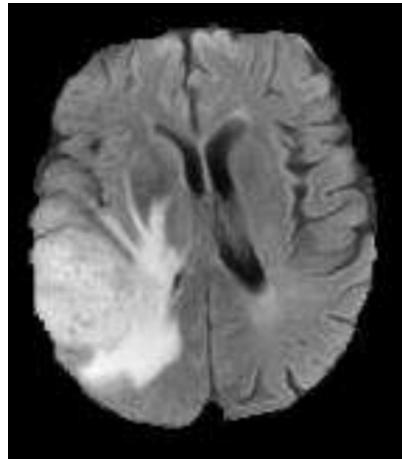
- There's a need for sample efficient networks in medical imaging
- Group equivariant networks are sample efficient
- Applications of group equivariant networks into detection tasks in brain MRIs are worth investigating

Cerebriu highlights pathologies in brain MRIs



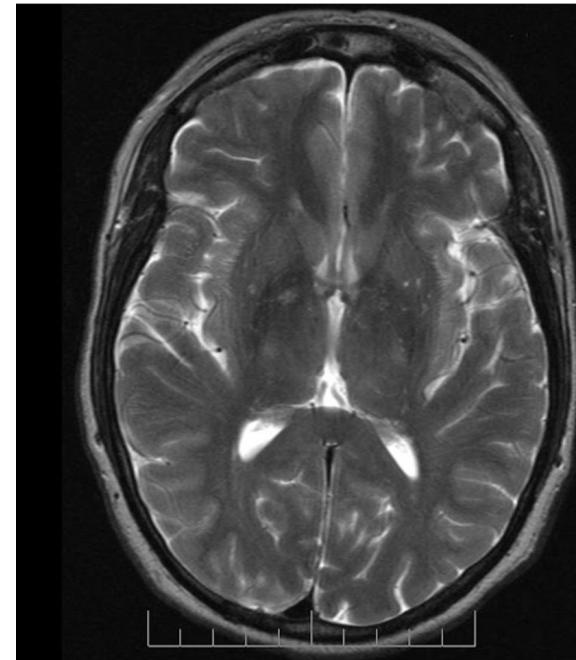
Sources: [Luo et al. 2020](#); [Renard et al. 2018](#); [Maekawa et al. 2020](#)

Cerebriu highlights pathologies with deep neural networks



Source: [Ronneberger et al. 2015](#)

Deep neural networks are data-hungry

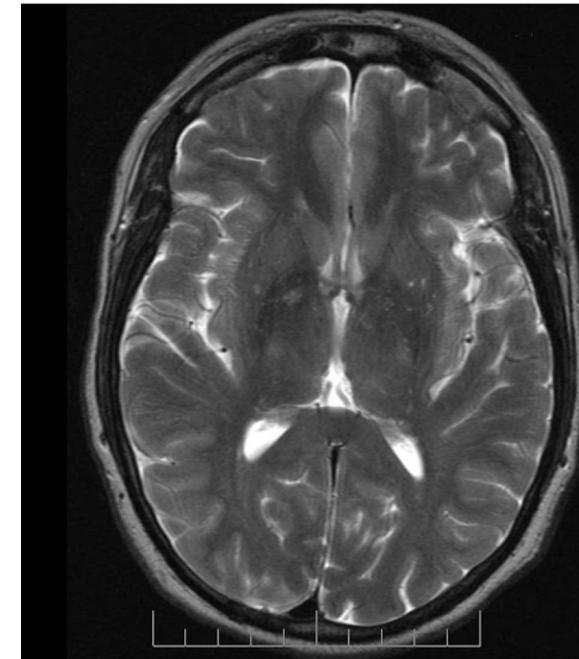


Source: https://commons.wikimedia.org/wiki/File:MRI_T2_Brain_axial_image.jpg

Deep neural networks are data-hungry

Solutions:

- *Explicit* data augmentation (e.g. data transformations)

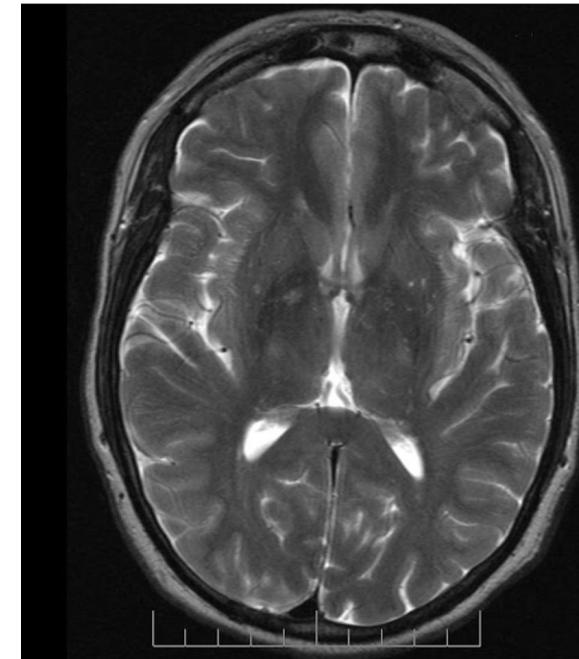
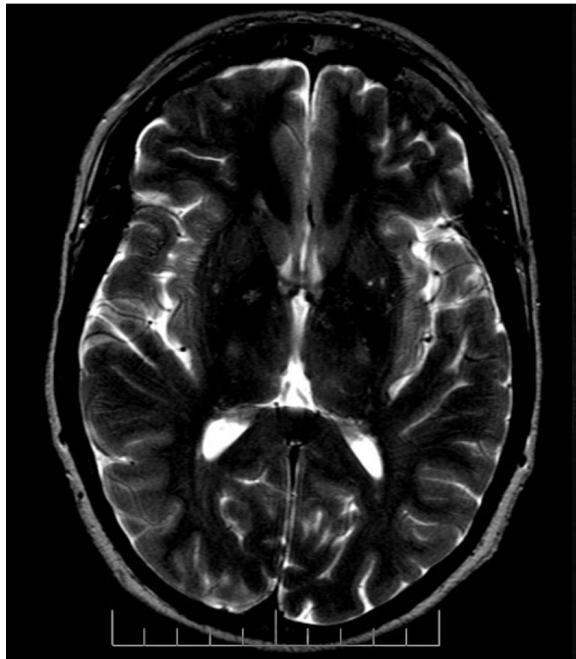


Source: https://commons.wikimedia.org/wiki/File:MRI_T2_Brain_axial_image.jpg

Deep neural networks are data-hungry

Solutions:

- *Explicit* data augmentation (e.g. data transformations)

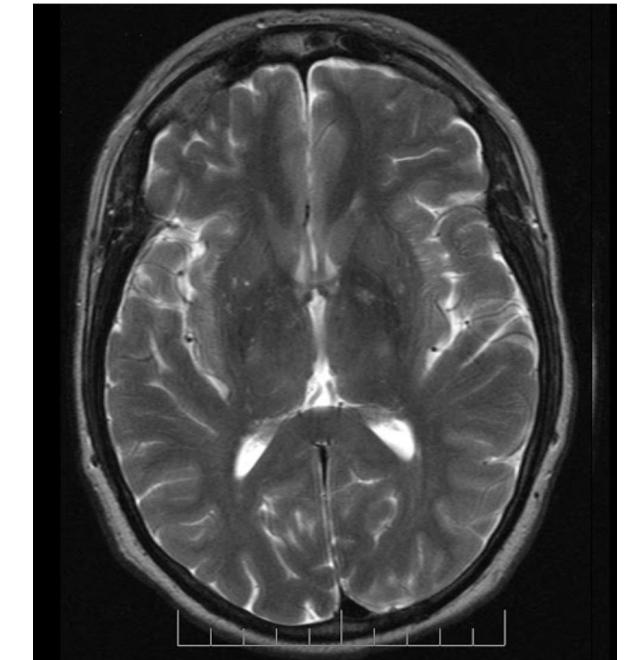
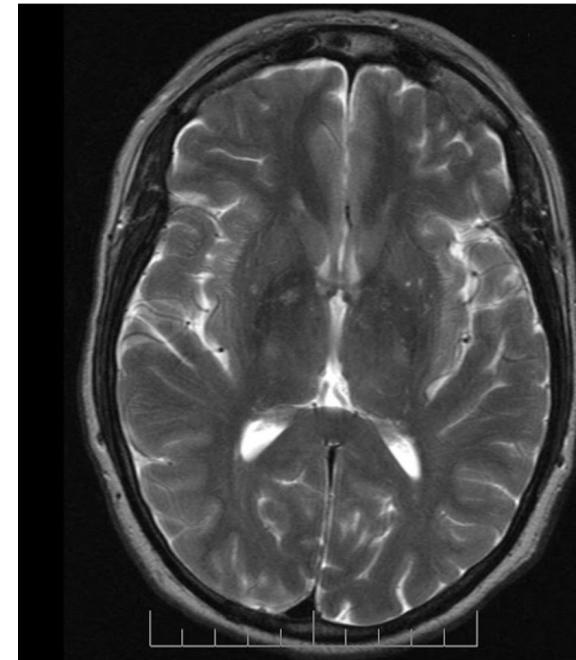
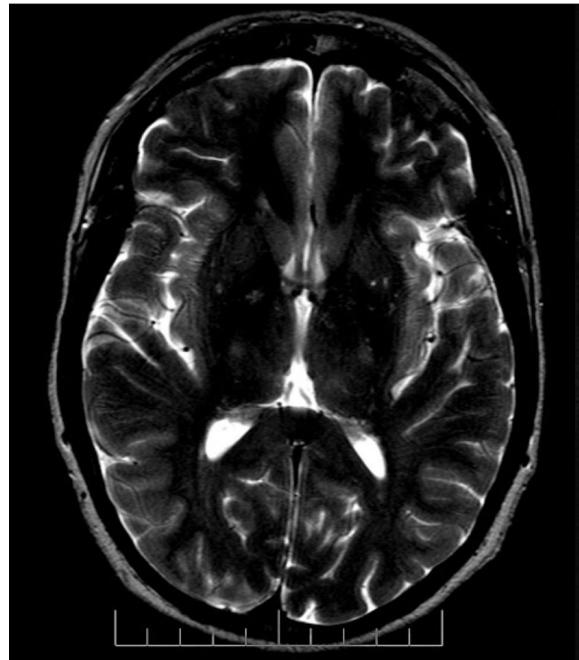


Source: https://commons.wikimedia.org/wiki/File:MRI_T2_Brain_axial_image.jpg

Deep neural networks are data-hungry

Solutions:

- *Explicit* data augmentation (e.g. data transformations)



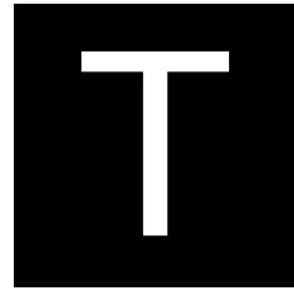
Source: https://commons.wikimedia.org/wiki/File:MRI_T2_Brain_axial_image.jpg

Deep neural networks are data-hungry

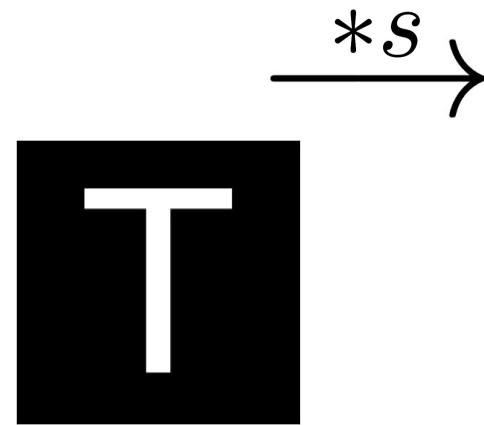
Solutions:

- *Explicit* data augmentation (e.g. data transformations)
- *Implicit* data augmentation (e.g. group equivariant networks)

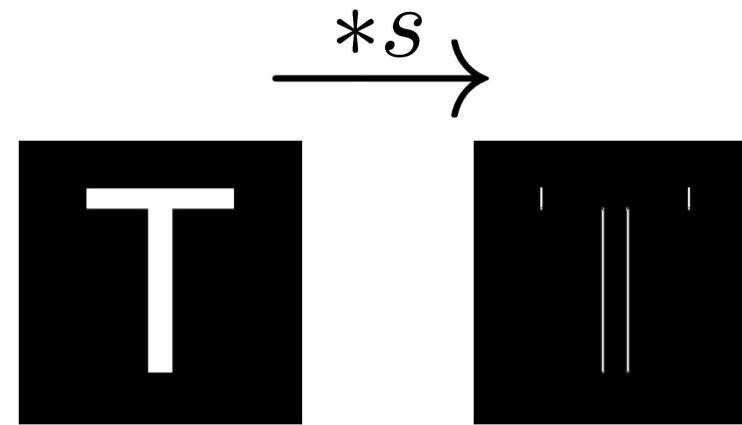
Convolution is translation-equivariant



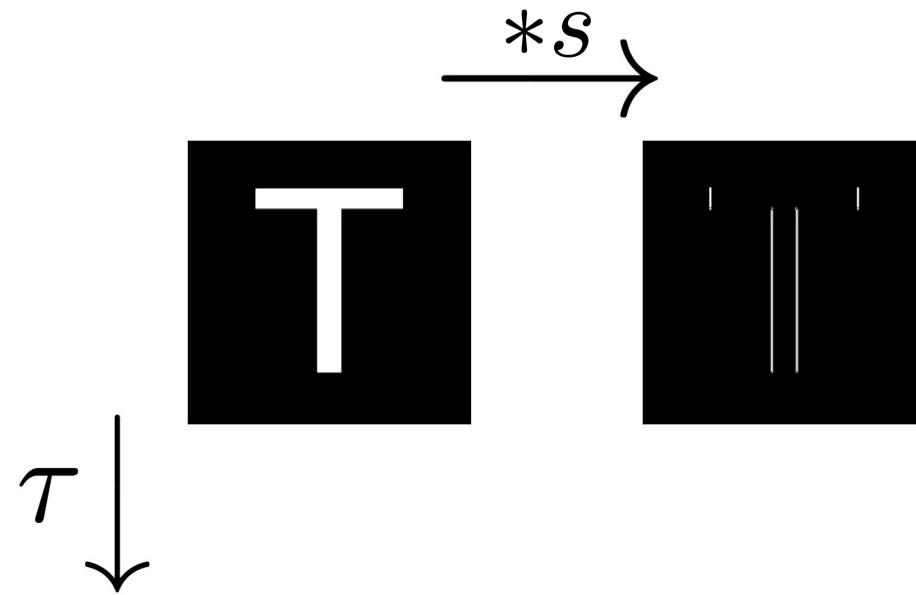
Convolution is translation-equivariant



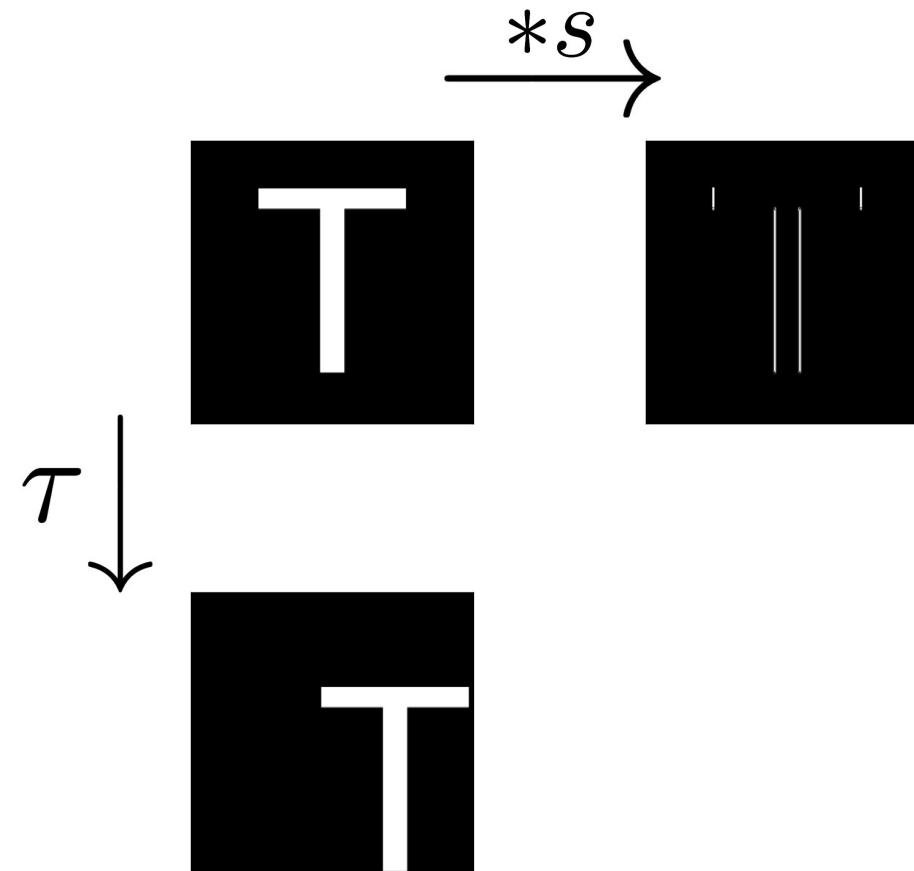
Convolution is translation-equivariant



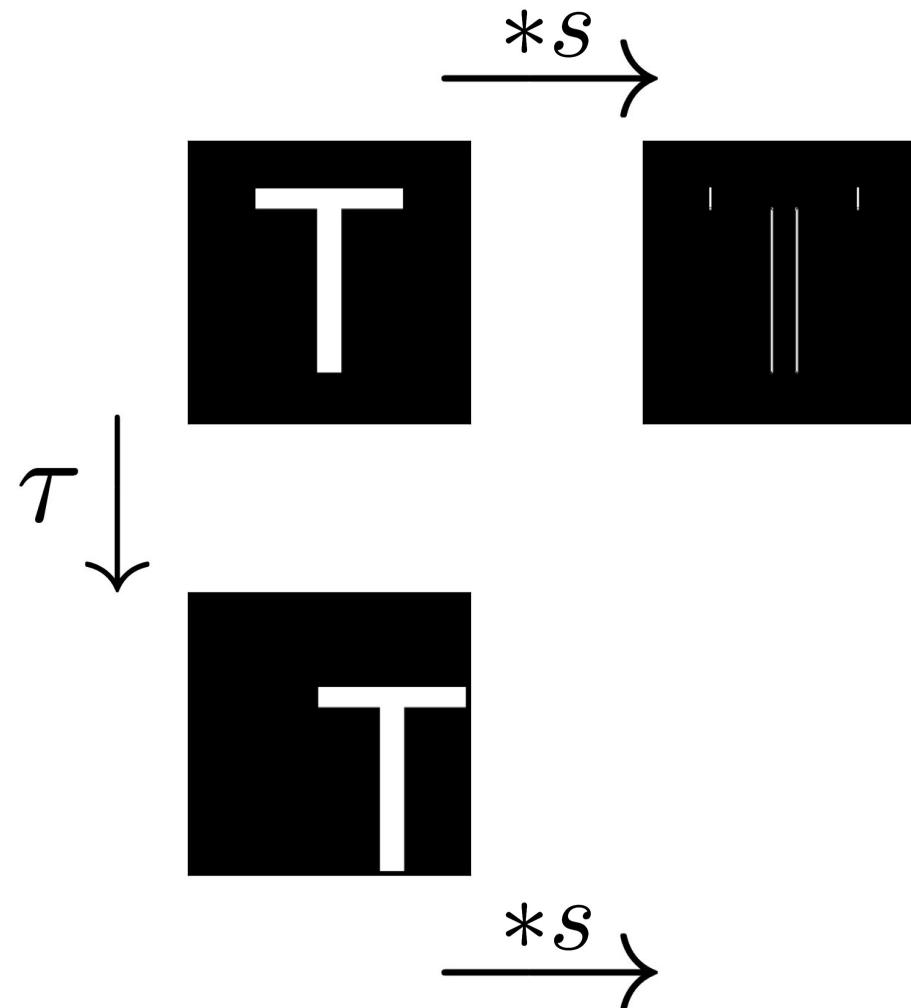
Convolution is translation-equivariant



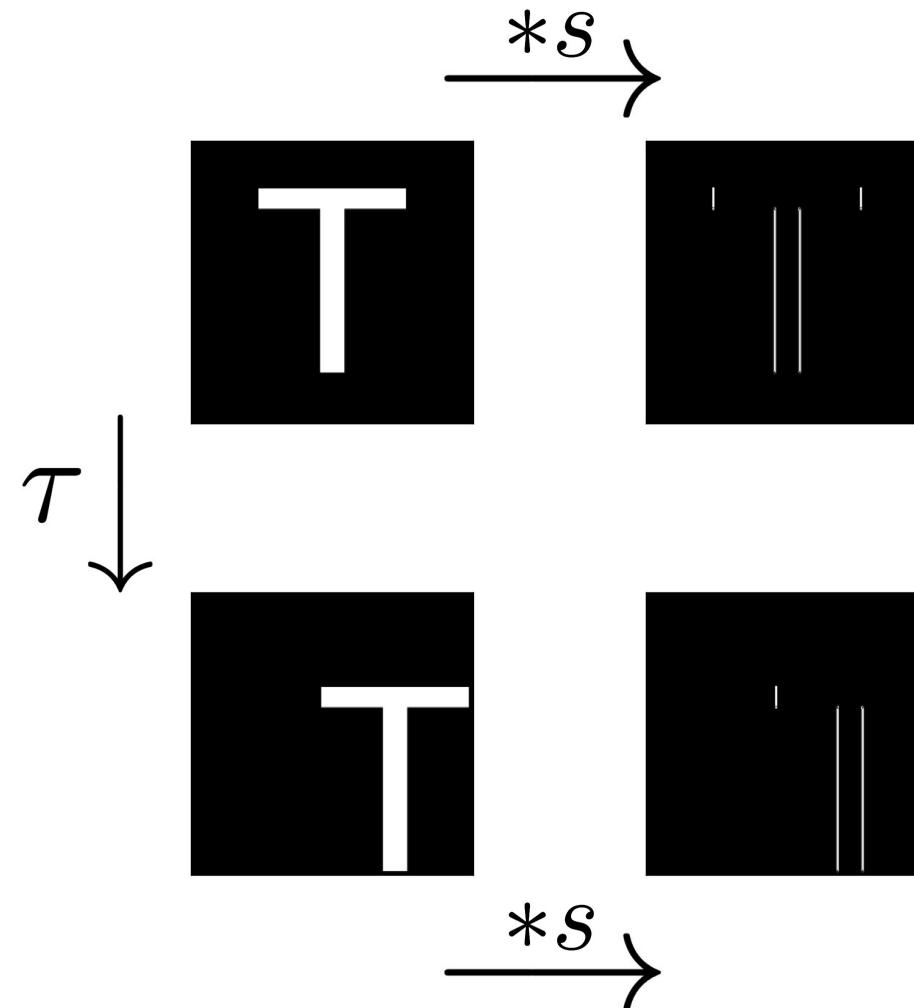
Convolution is translation-equivariant



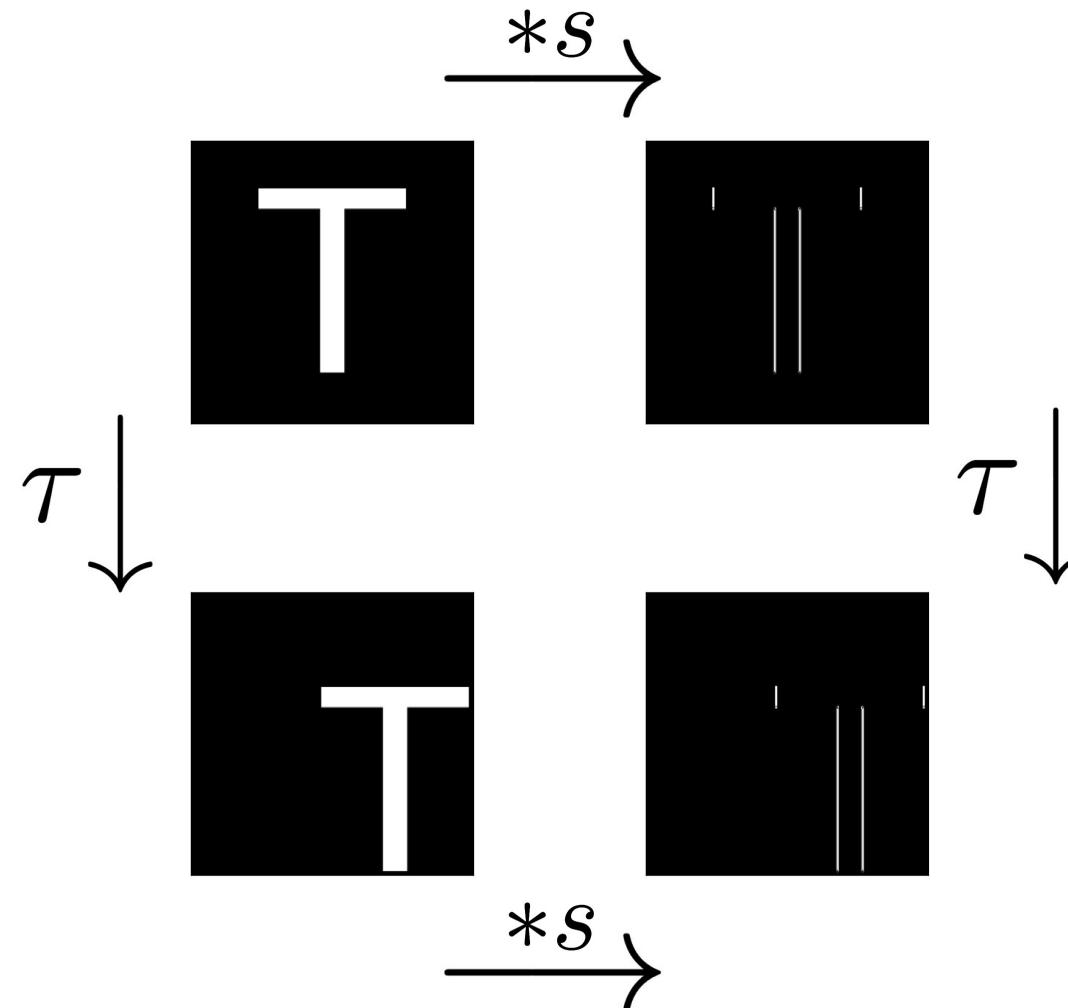
Convolution is translation-equivariant



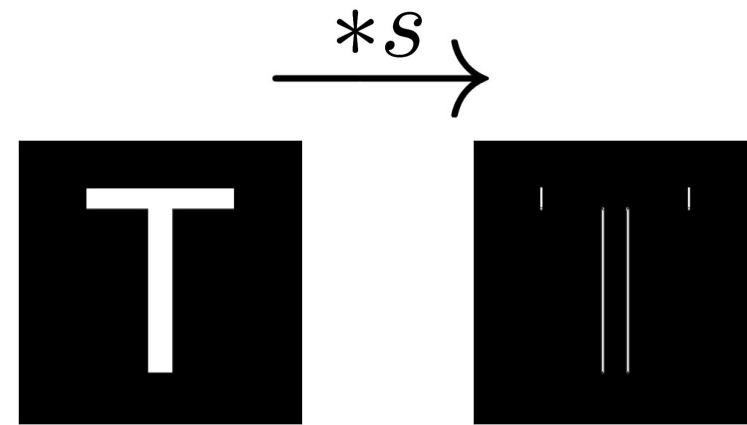
Convolution is translation-equivariant



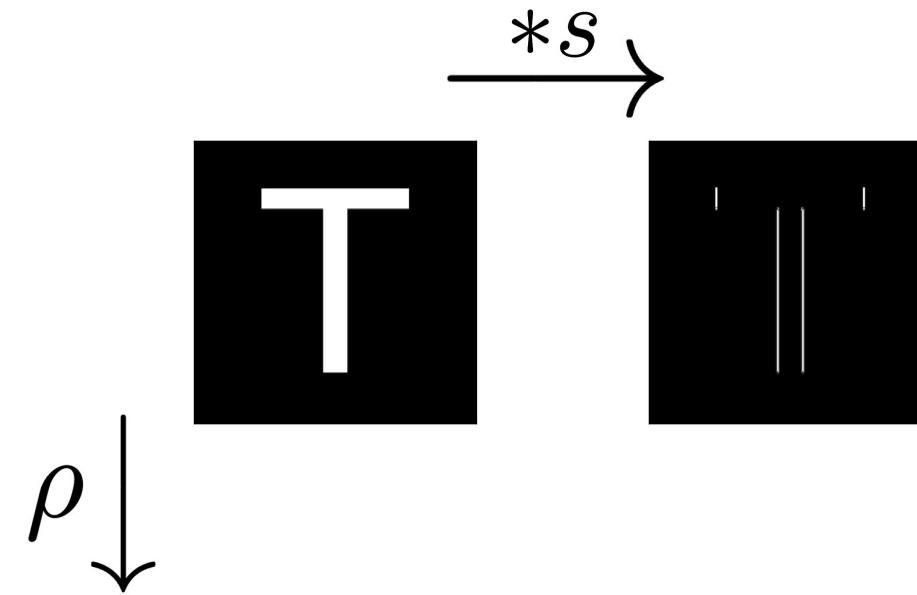
Convolution is translation-equivariant



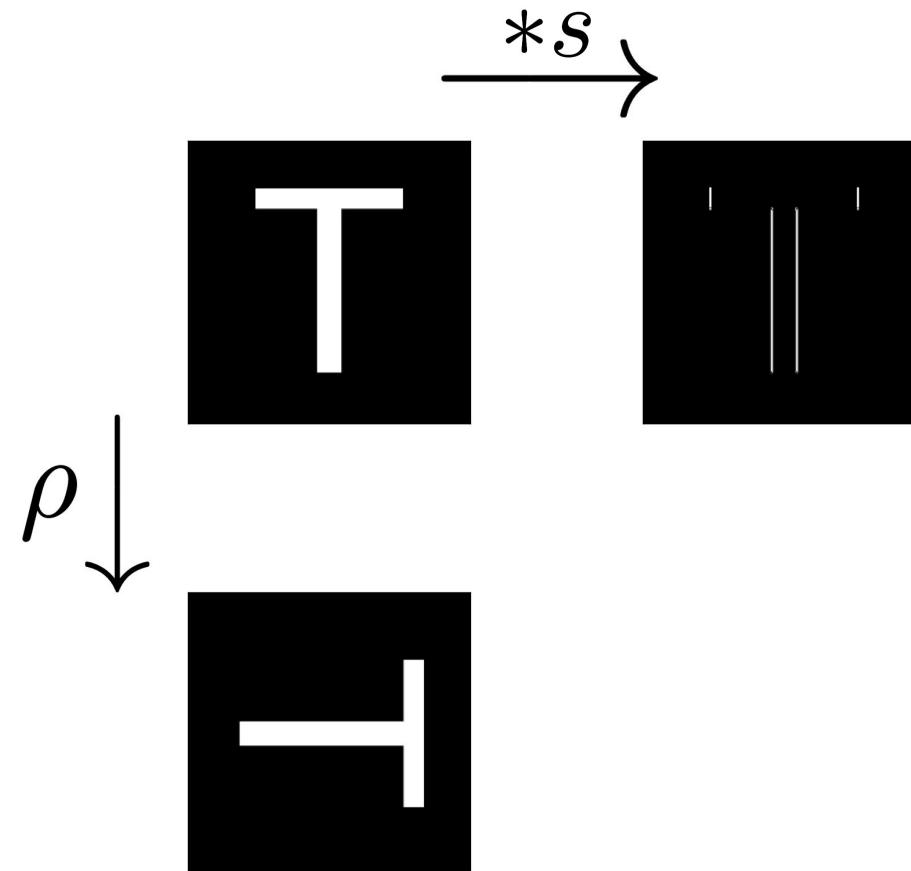
Convolution is *not* rotation-equivariant



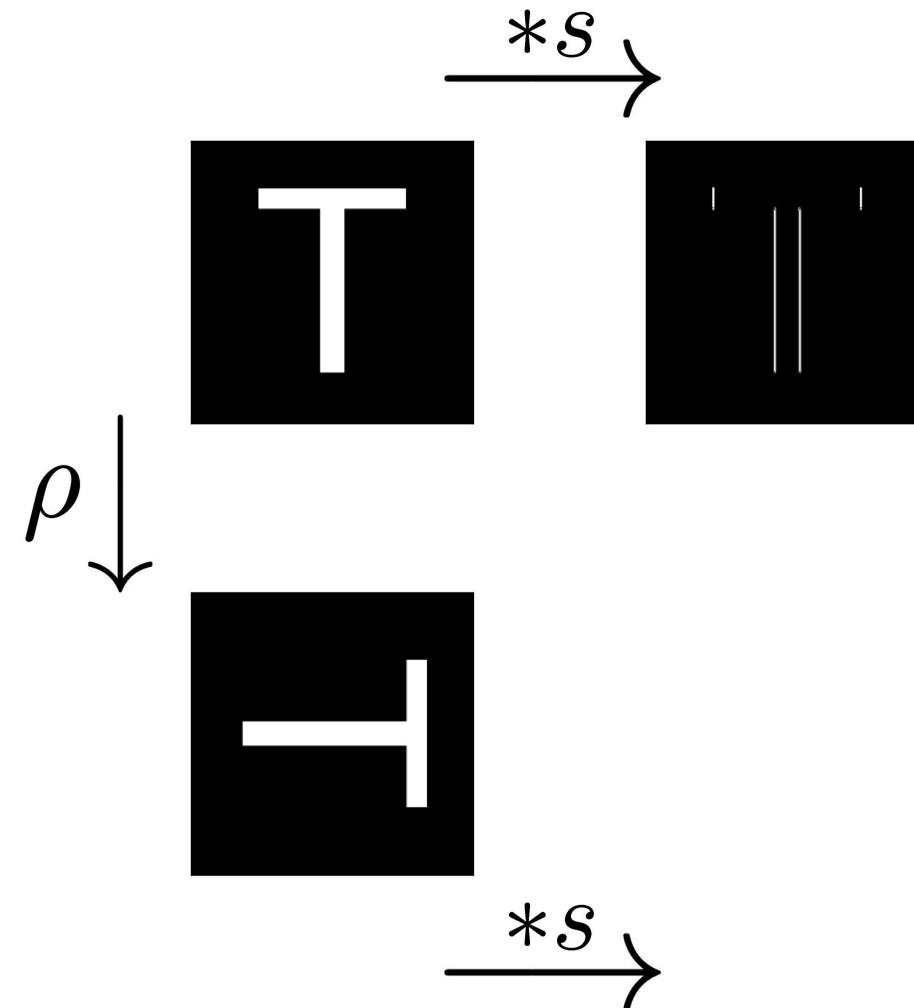
Convolution is *not* rotation-equivariant



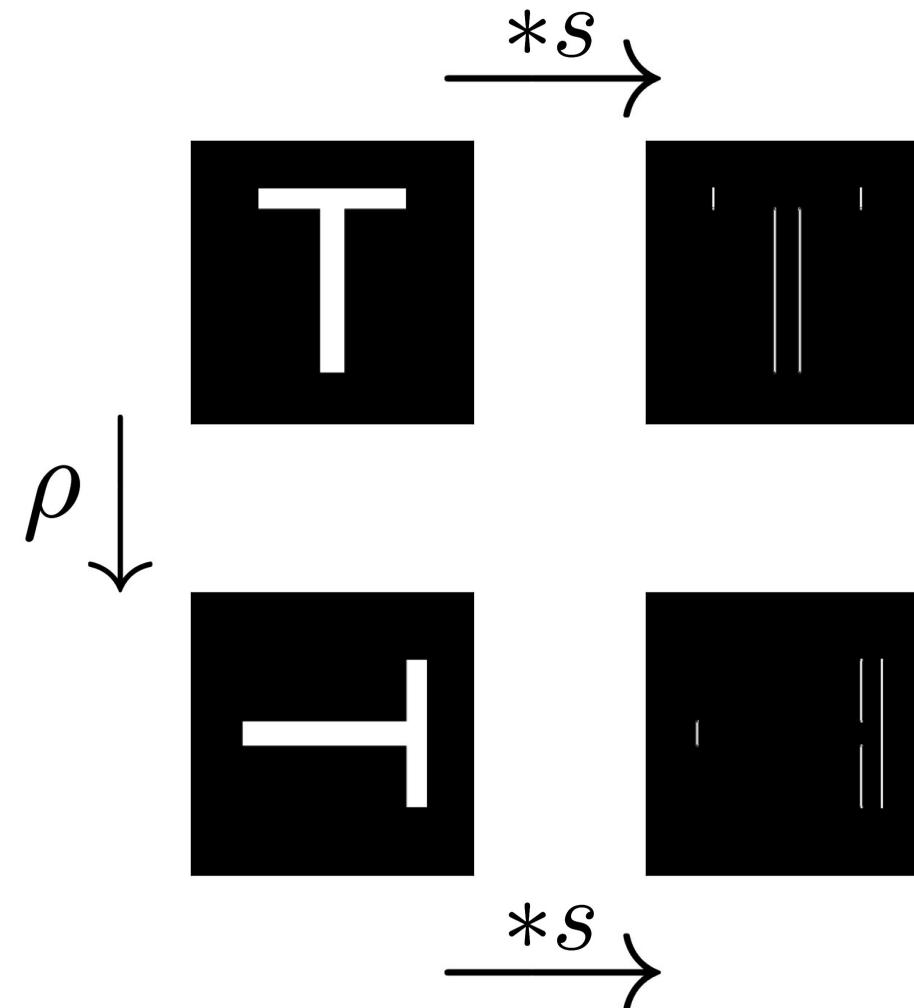
Convolution is *not* rotation-equivariant



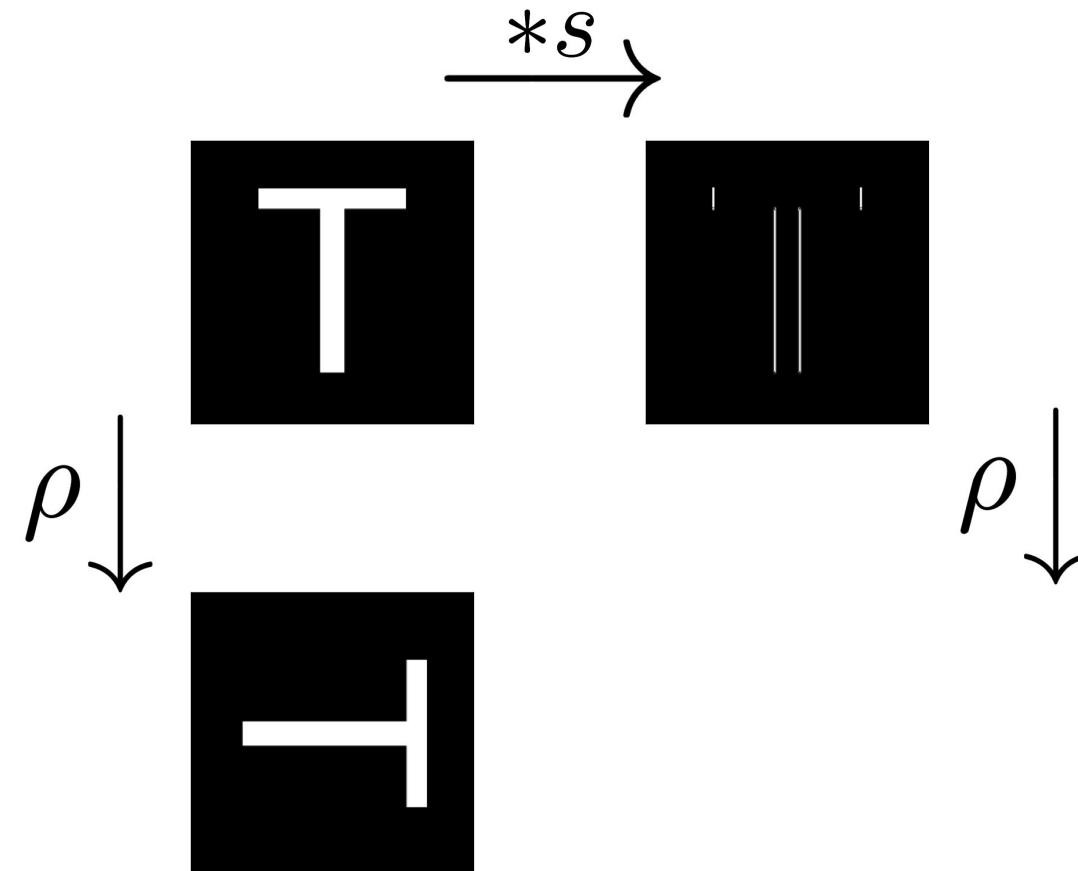
Convolution is *not* rotation-equivariant



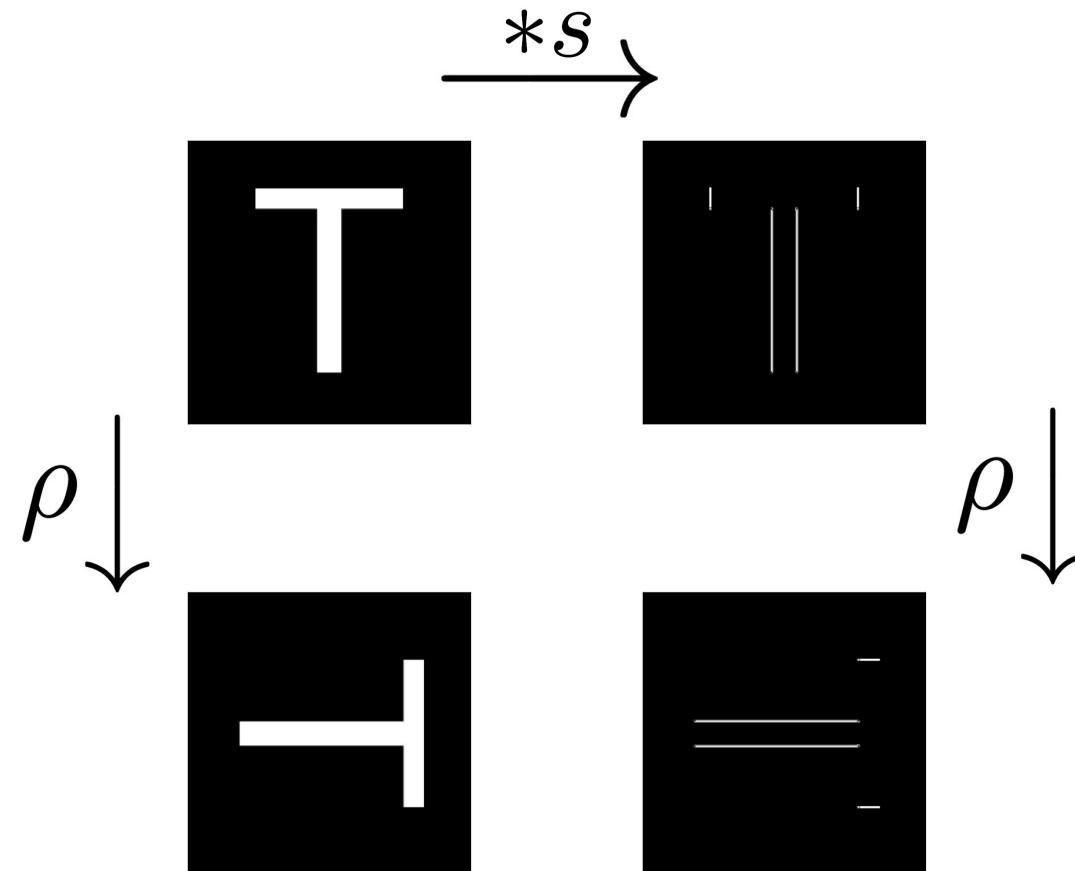
Convolution is *not* rotation-equivariant



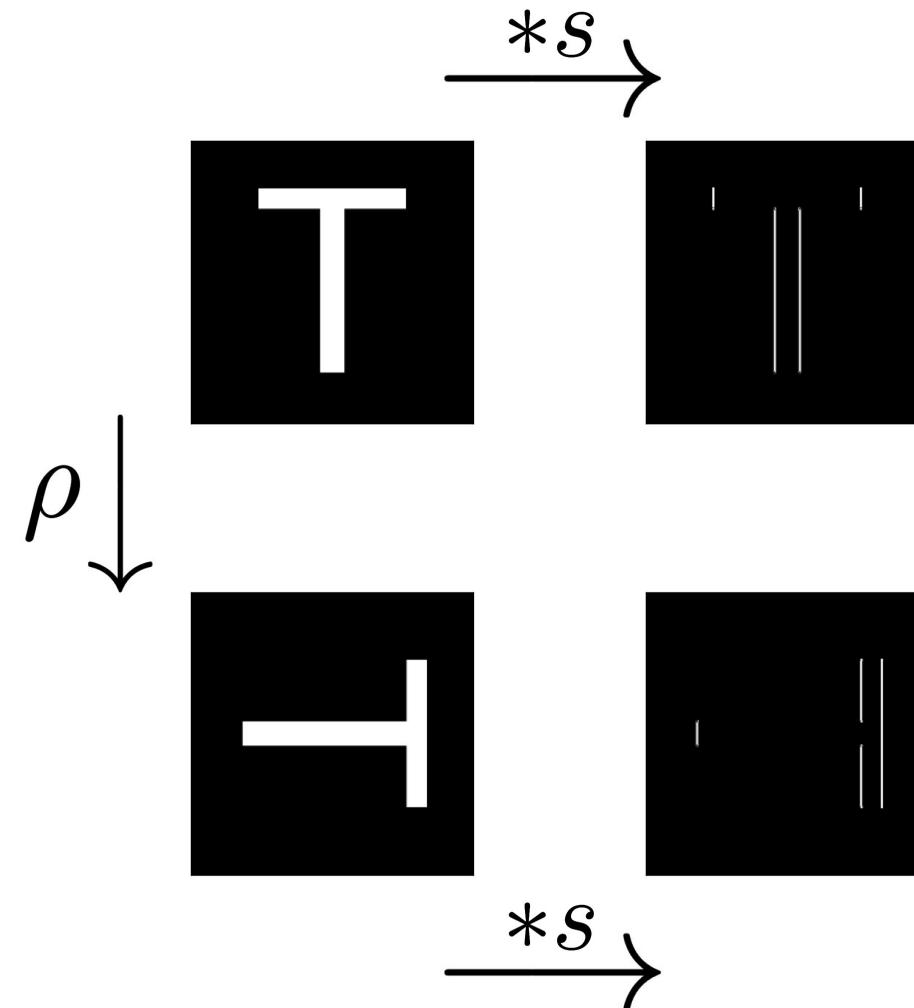
Convolution is *not* rotation-equivariant



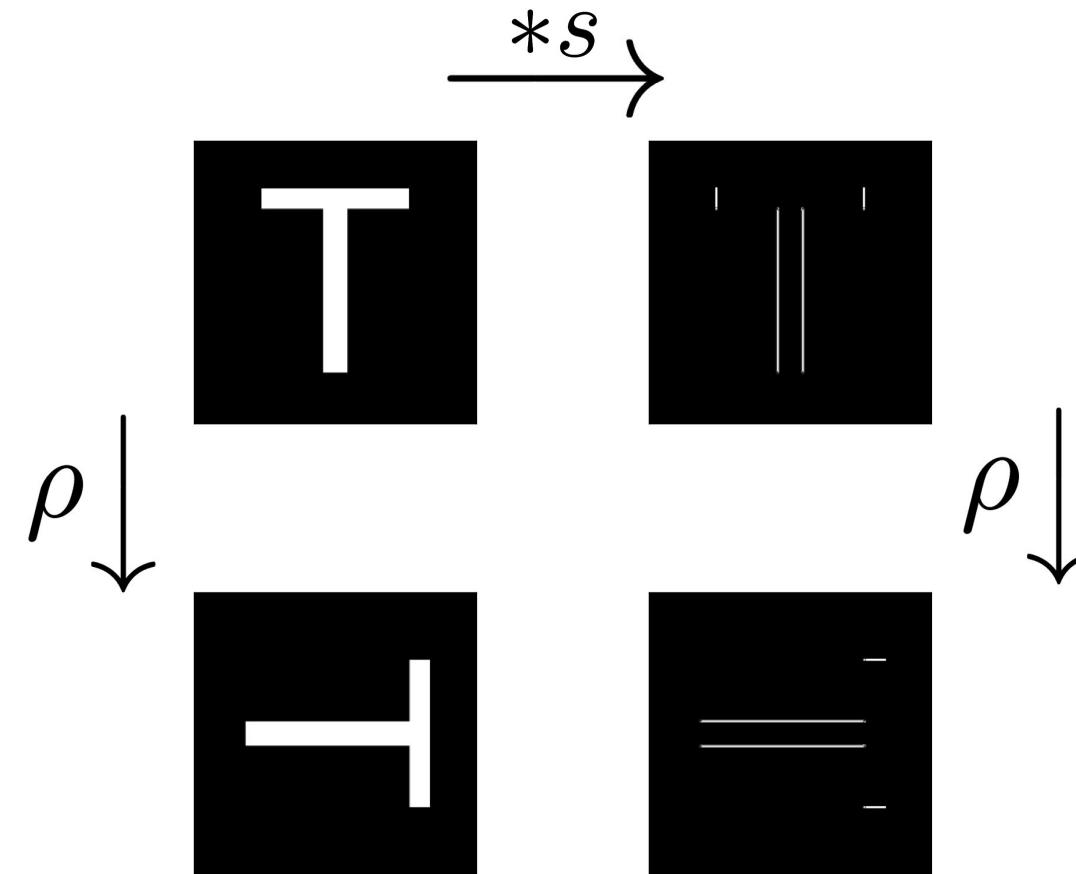
Convolution is *not* rotation-equivariant



Convolution is *not* rotation-equivariant

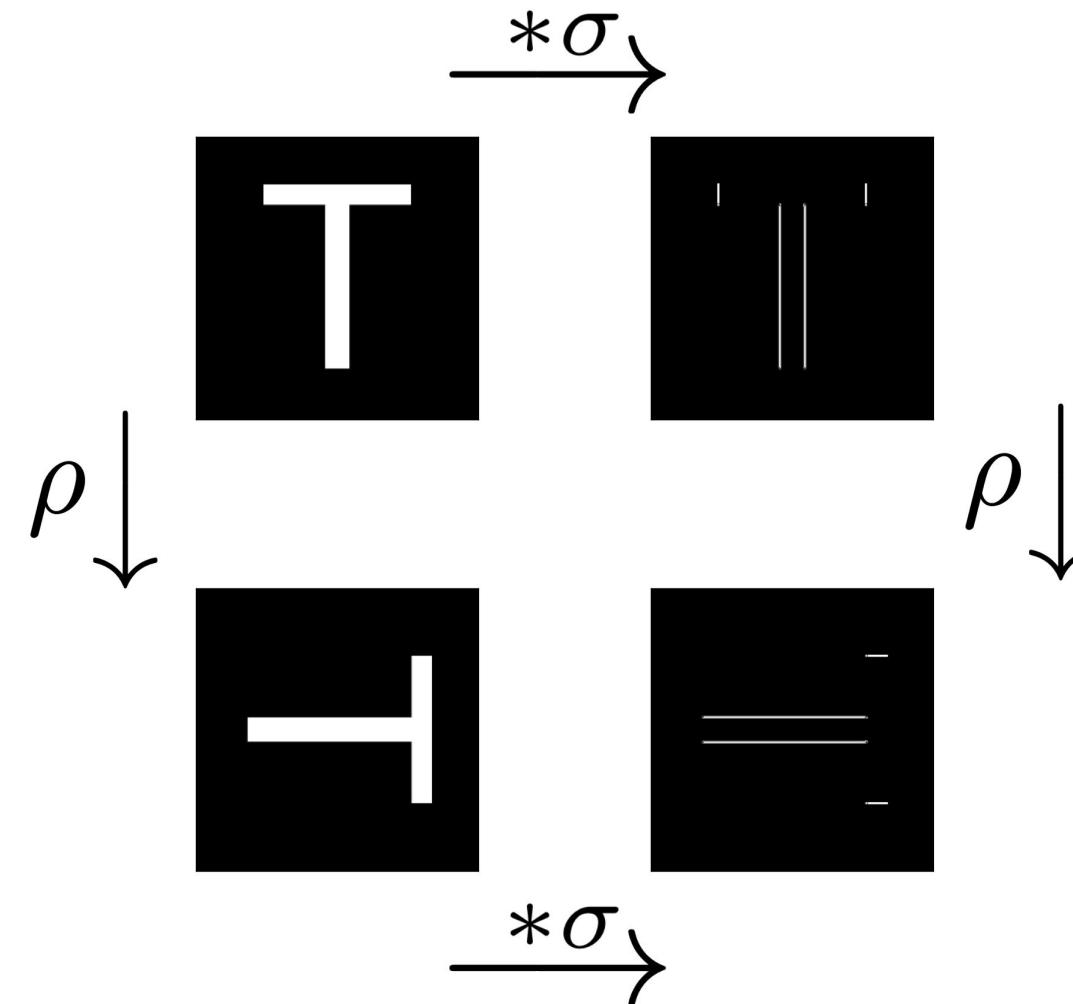


Convolution is *not* rotation-equivariant

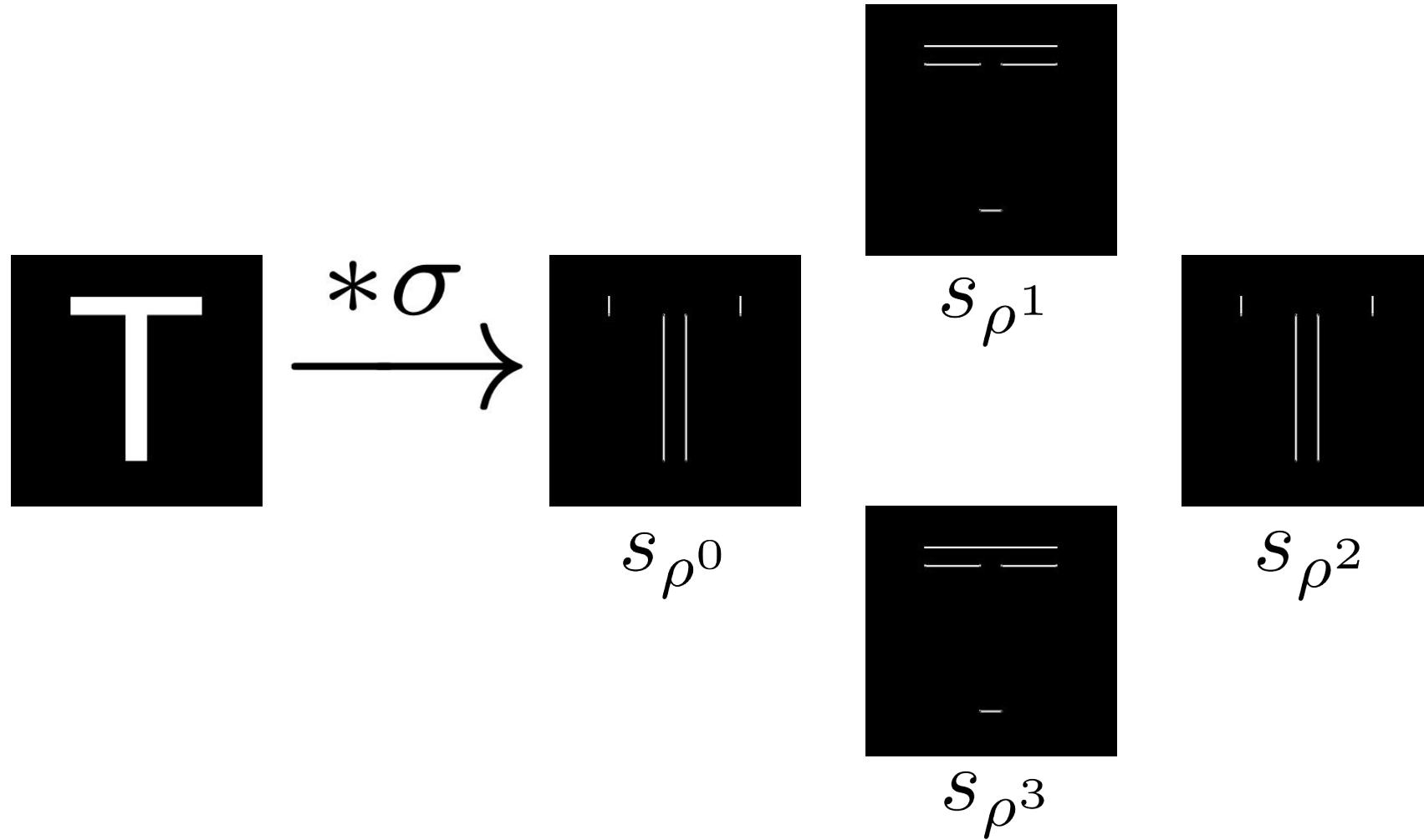


Problem: convolution and rotation are incompatible, in general

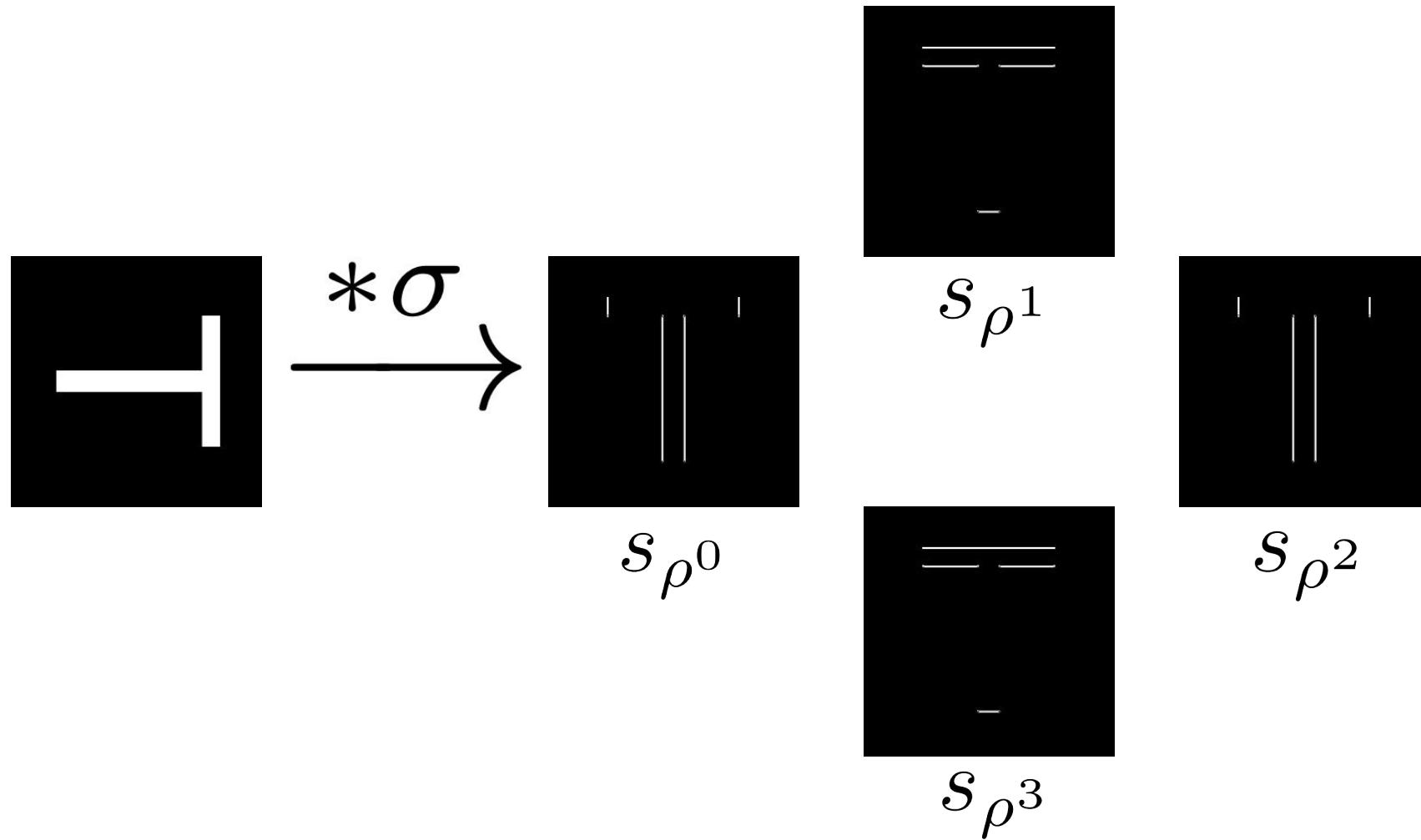
Rotation-equivariance is achieved with restructured filters



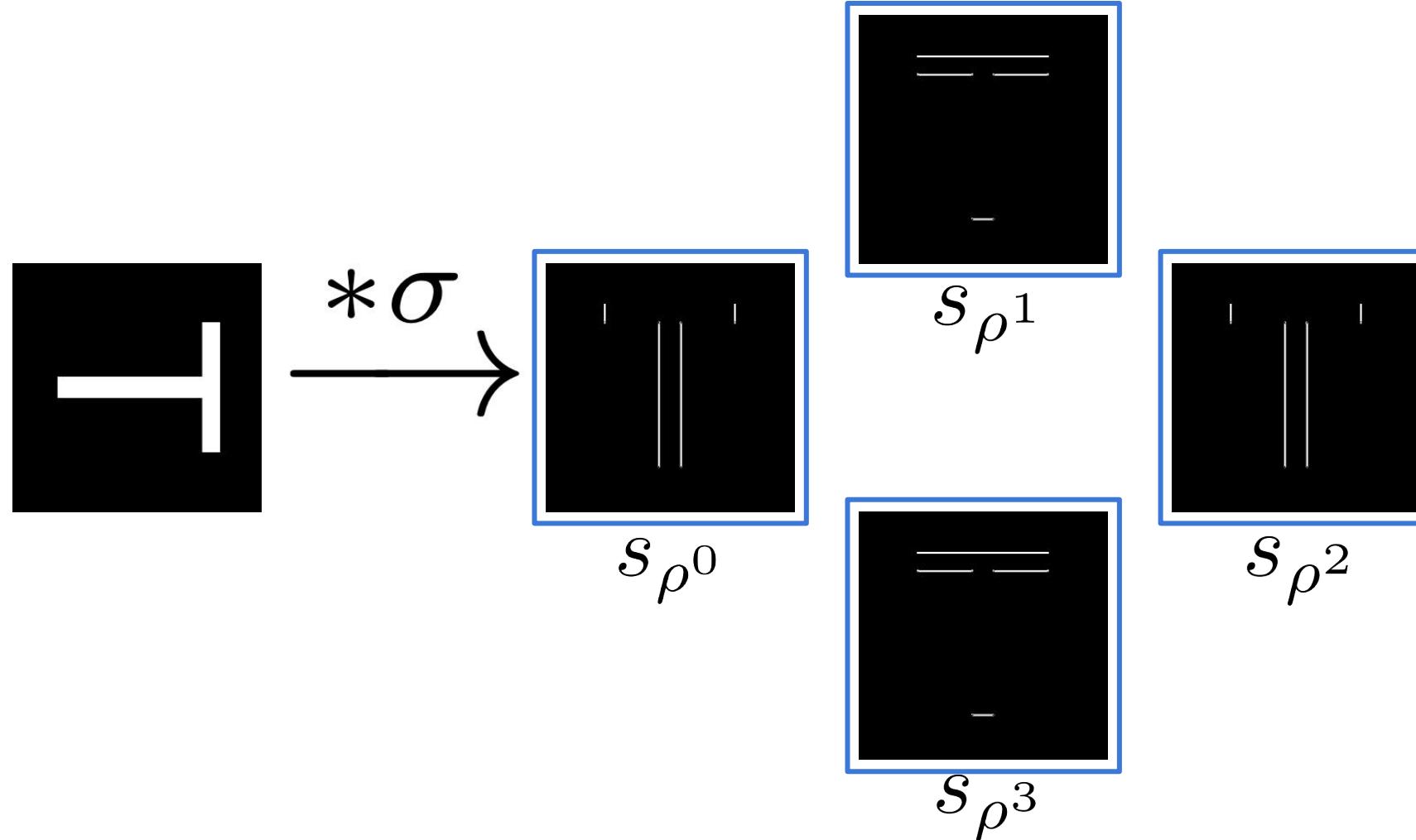
Convolution with a finite-group equivariant filter



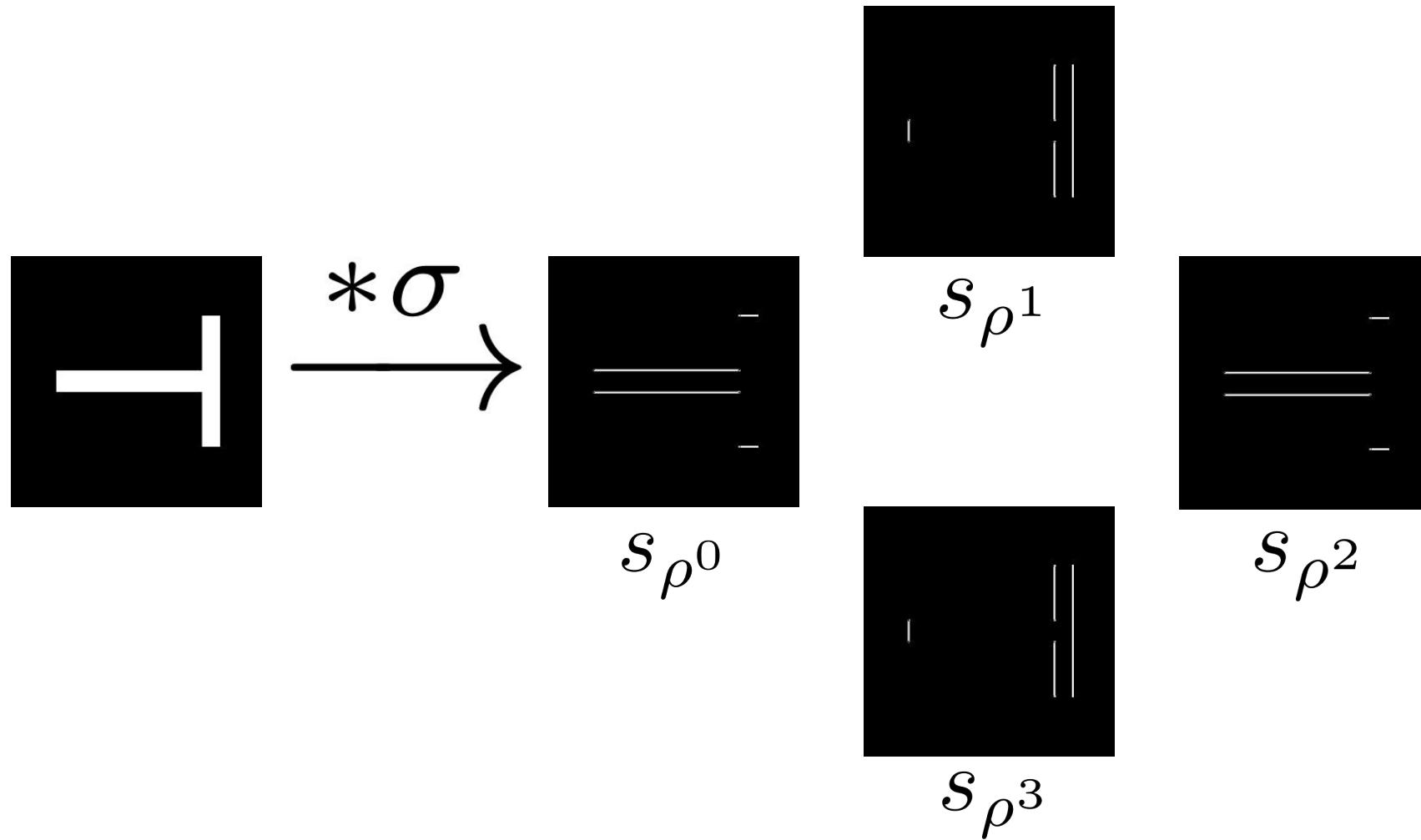
Convolution with an equivariant filter



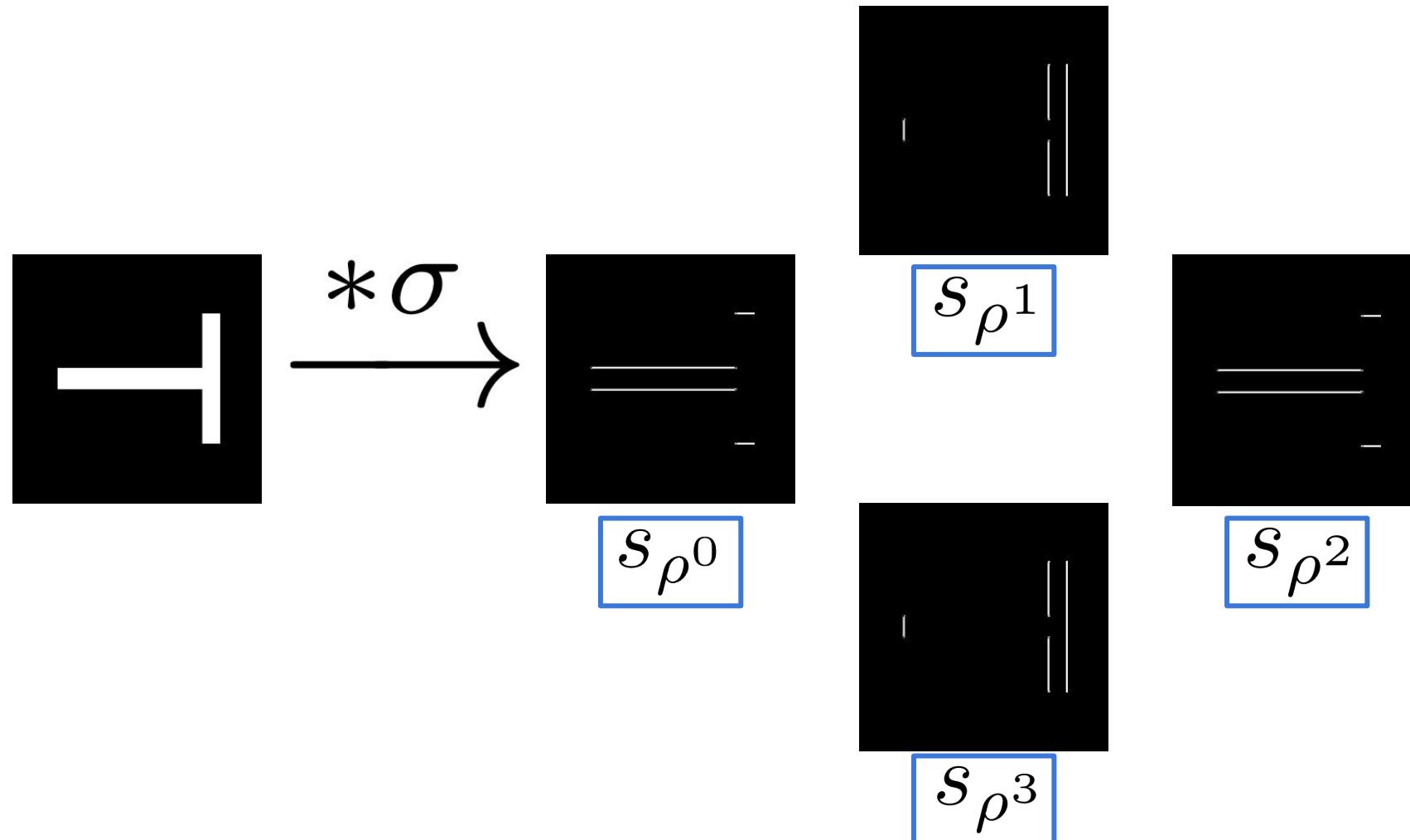
Convolution with an equivariant filter



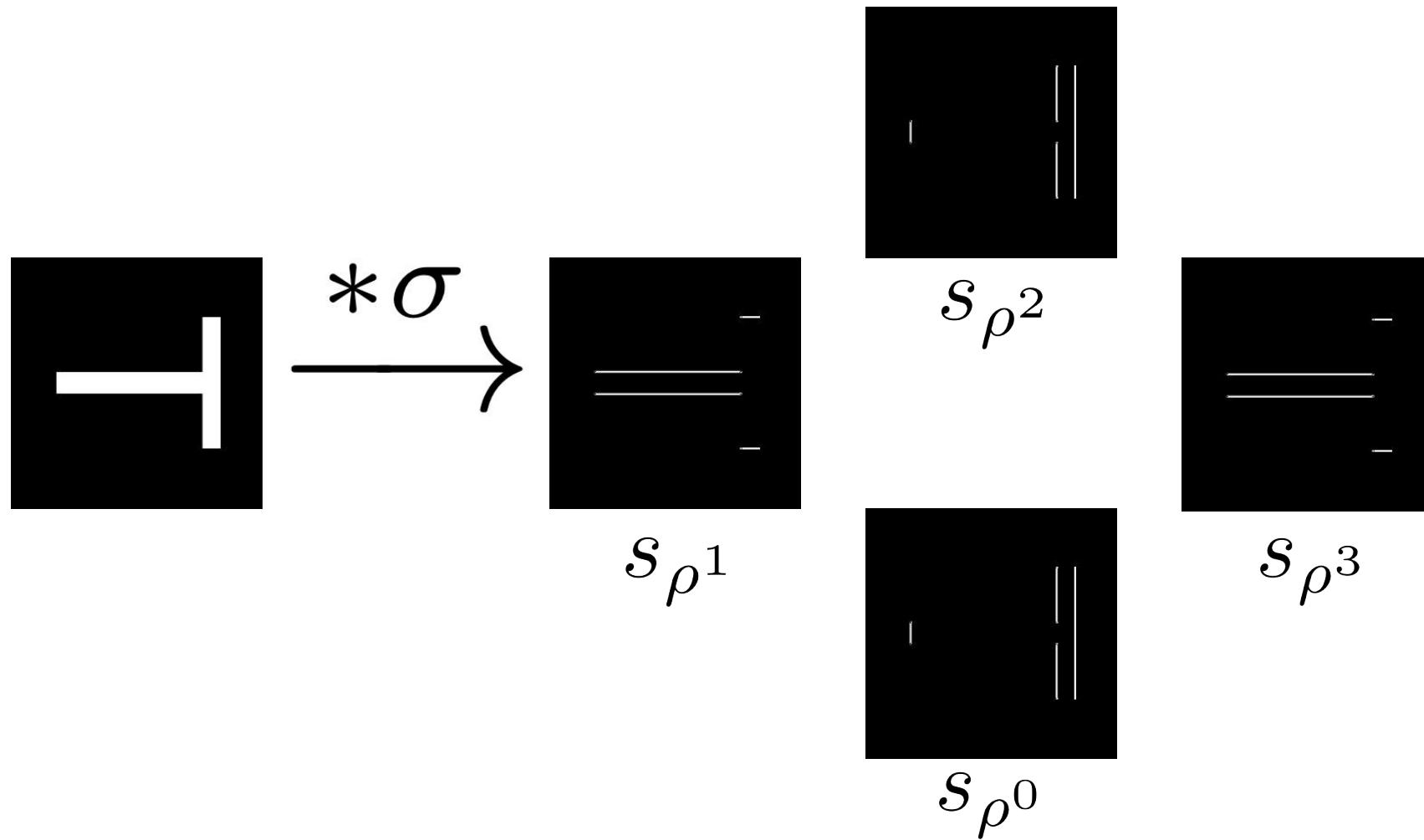
Convolution with an equivariant filter



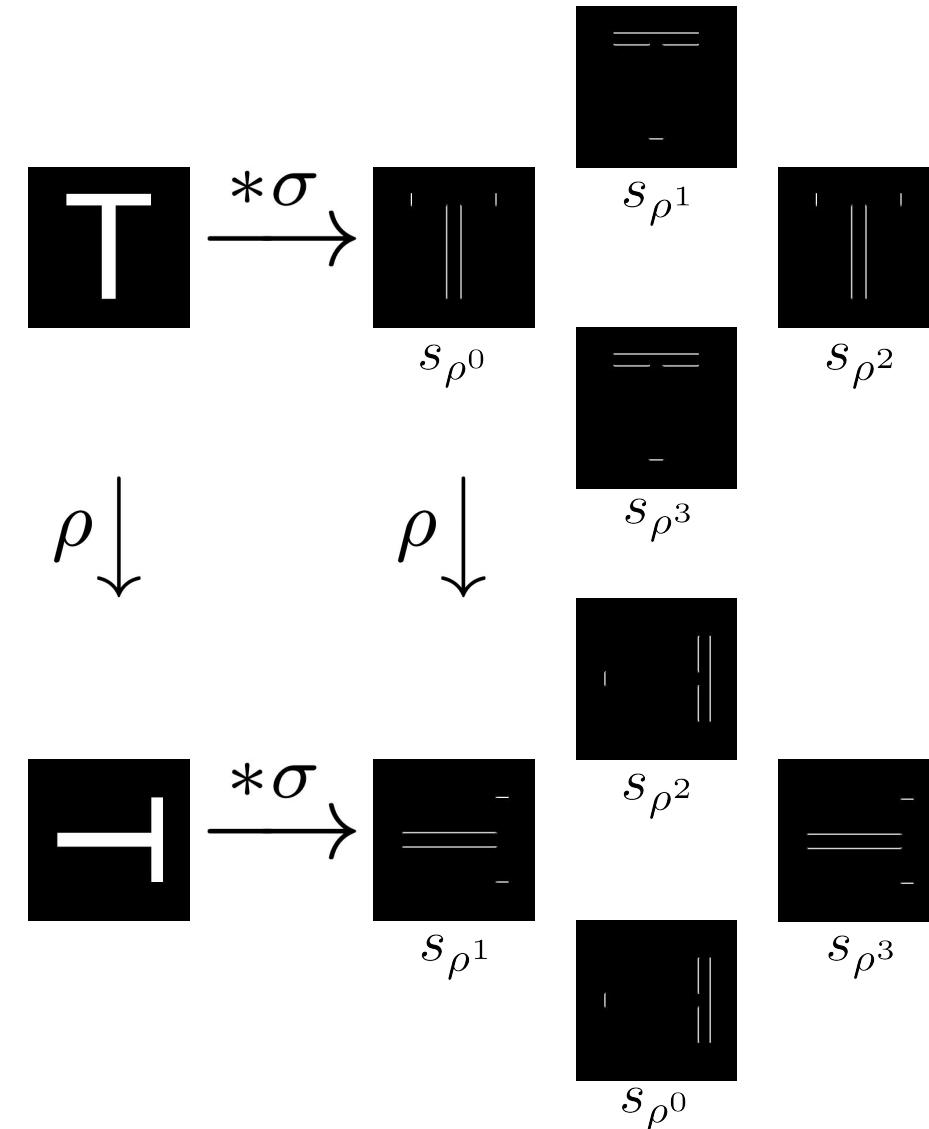
Convolution with an equivariant filter



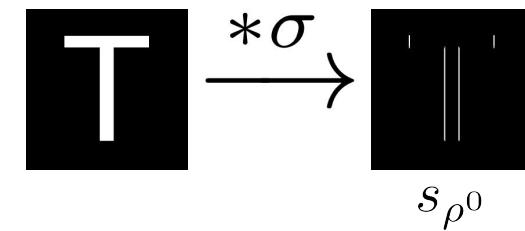
Convolution with an equivariant filter



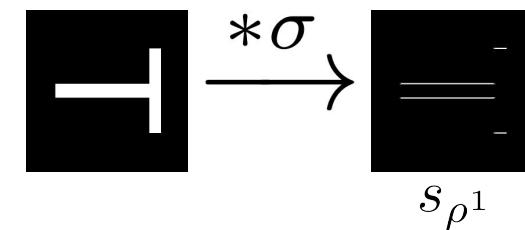
Convolution with an equivariant filter



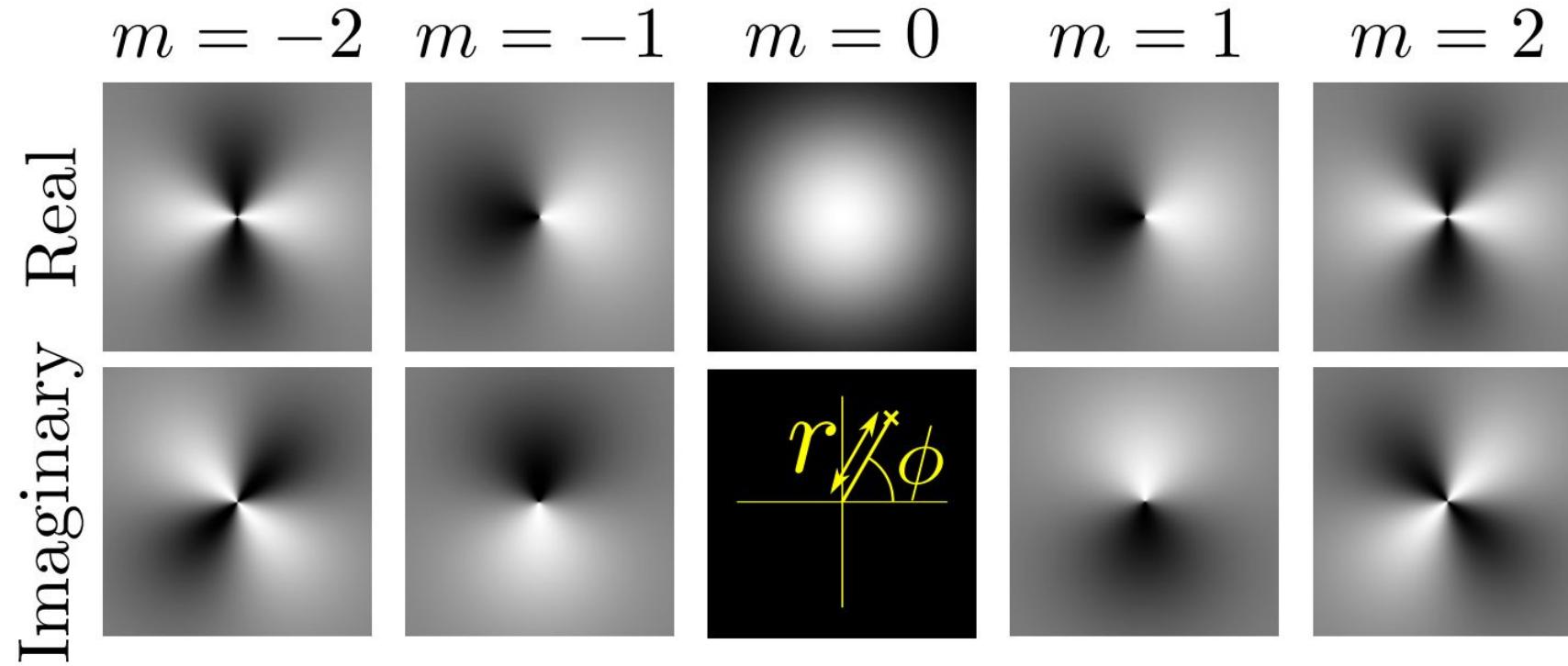
Convolution with an equivariant filter



$$\rho \downarrow \qquad \qquad \rho \downarrow$$

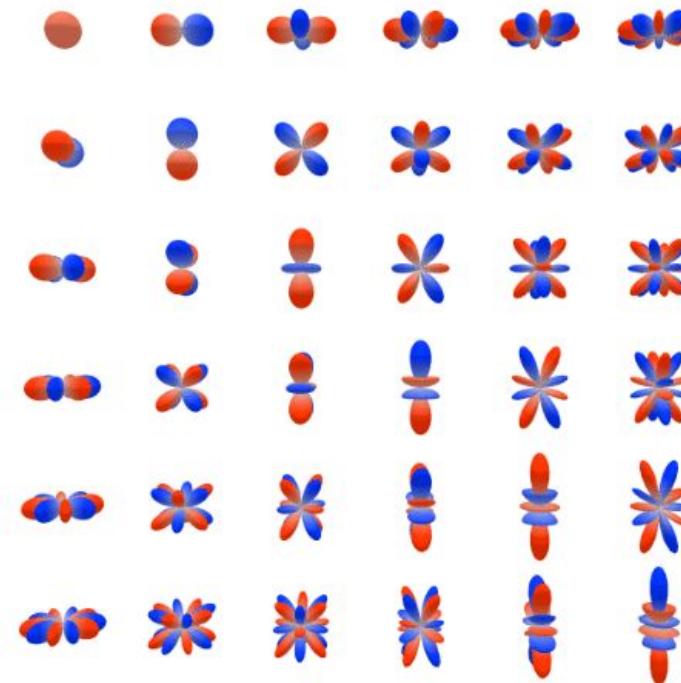


Convolution with a 2D harmonic filter



$$\mathbf{W}_m(r, \phi'; e^{-r^2}, 0) = e^{-r^2} e^{im\phi}$$

Convolution with a 3D harmonic filter

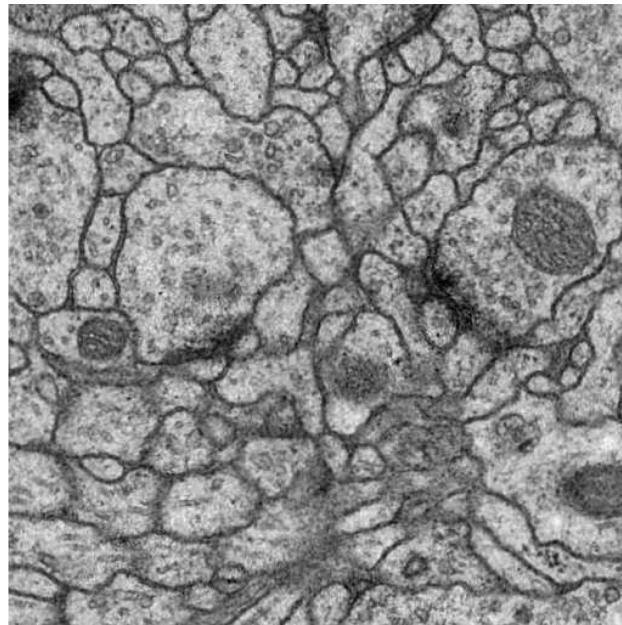


$$f(\rho, \theta, \phi) = \sum_{n=0}^N h_n(\rho) \sum_{m=-n}^n C_n[m] Y_{n,m}(\theta, \phi)$$

Sources: <https://github.com/e3nn/e3nn>, <https://doi.org/10.1016/j.media.2020.101756>

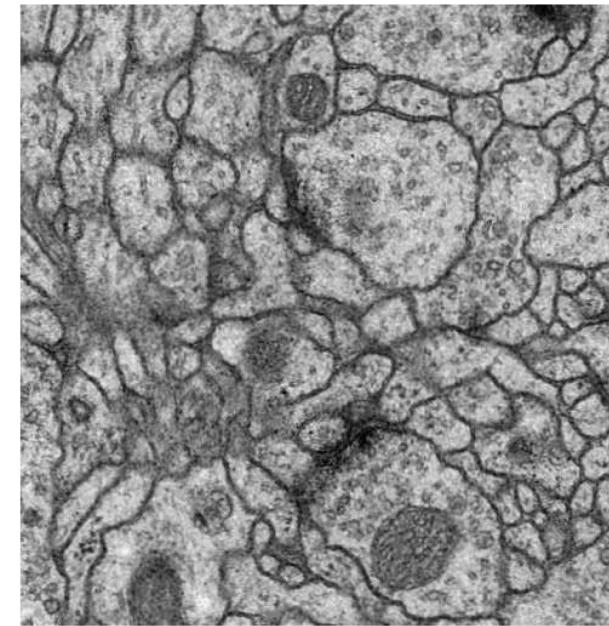
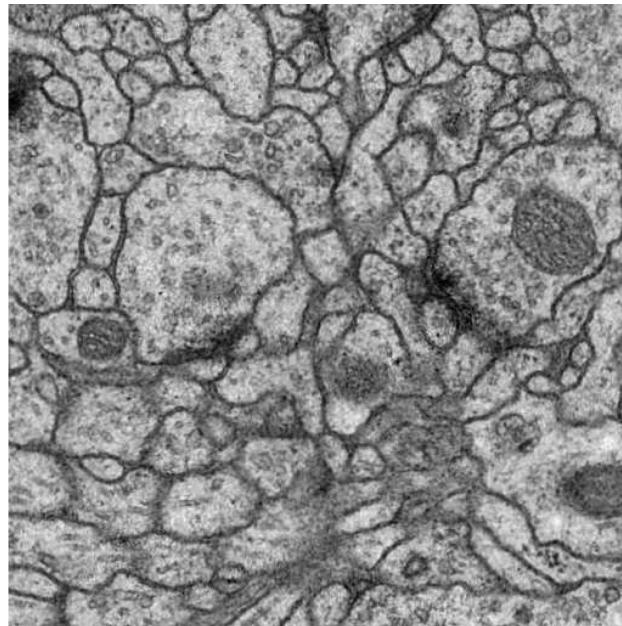
When should equivariant networks be applied?

When should equivariant networks be applied?



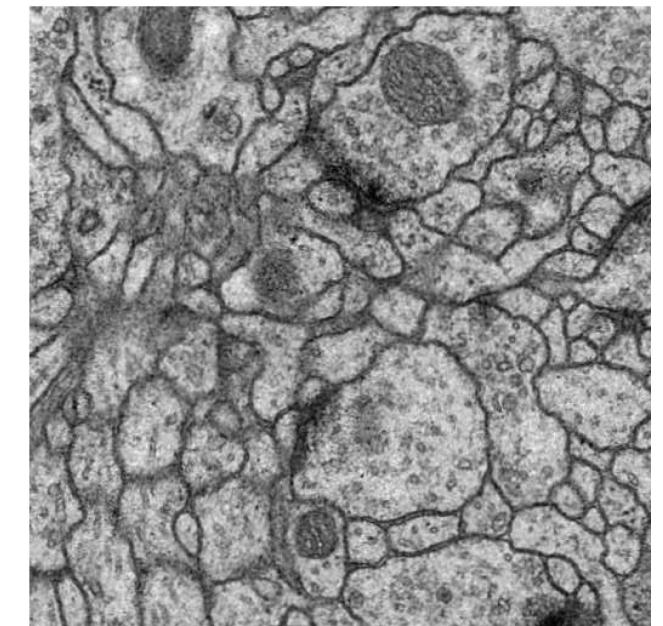
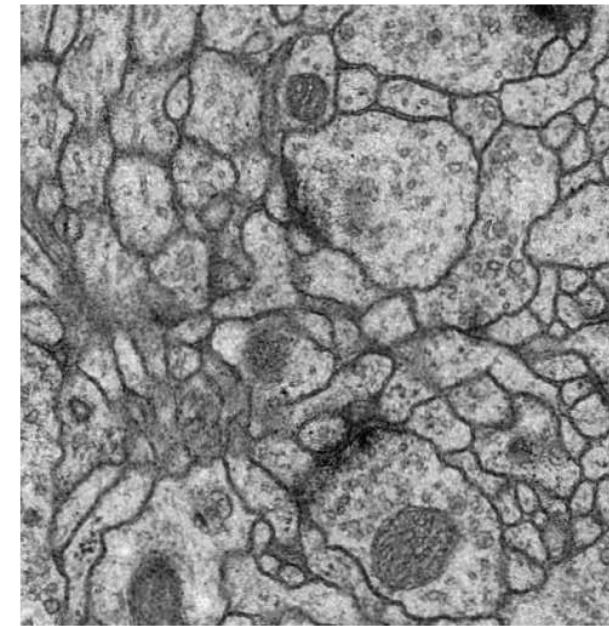
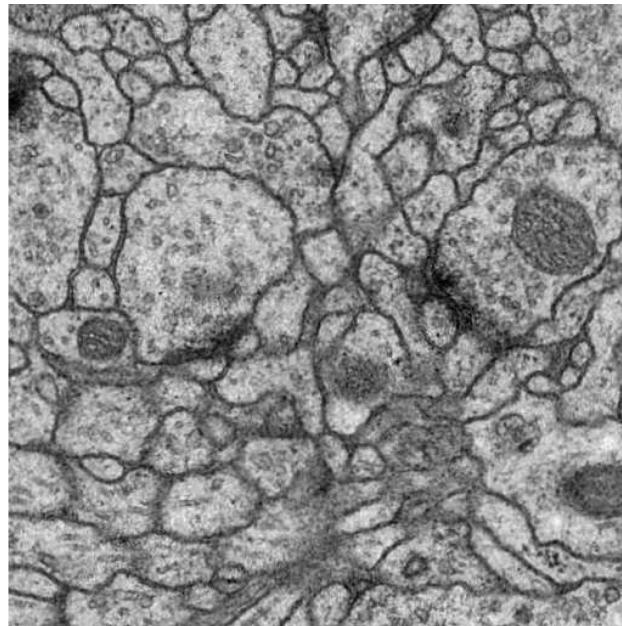
Source: https://openaccess.thecvf.com/content_ECCV_2018/html/Daniel_Worrall_CubeNet_Equivariance_to_ECCV_2018_paper.html

When should equivariant networks be applied?



Source: https://openaccess.thecvf.com/content_ECCV_2018/html/Daniel_Worrall_CubeNet_Equivariance_to_ECCV_2018_paper.html

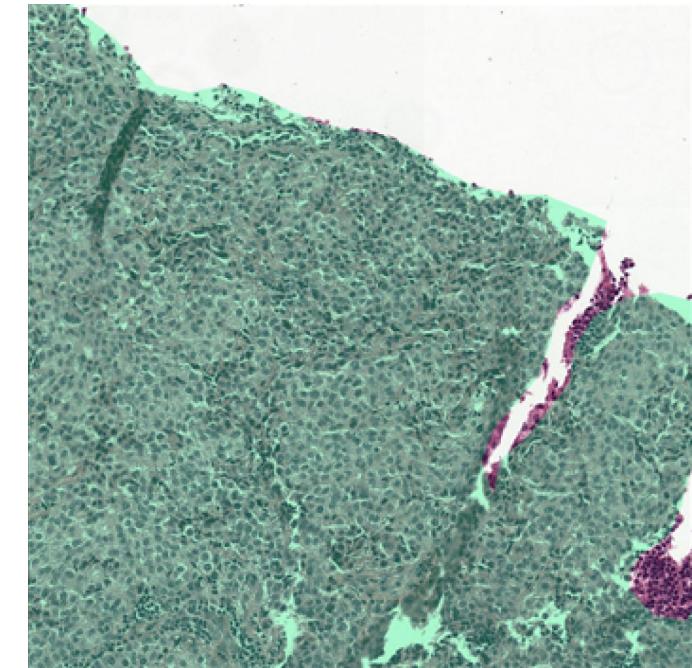
When should equivariant networks be applied?



Source: https://openaccess.thecvf.com/content_ECCV_2018/html/Daniel_Worrall_CubeNet_Equivariance_to_ECCV_2018_paper.html

Recent work

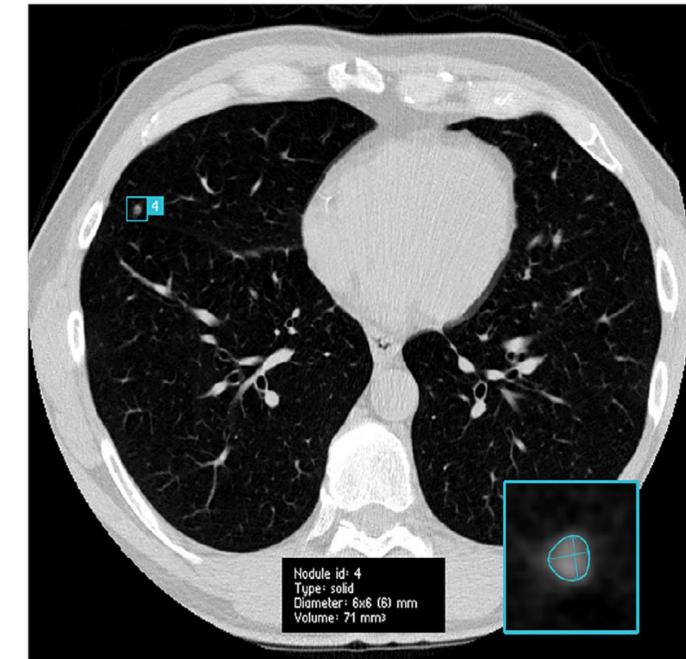
- Veeling, et al., 2018
 - 2D G-CNN for tumor segmentation



Source: <https://arxiv.org/abs/1806.03962>

Recent work

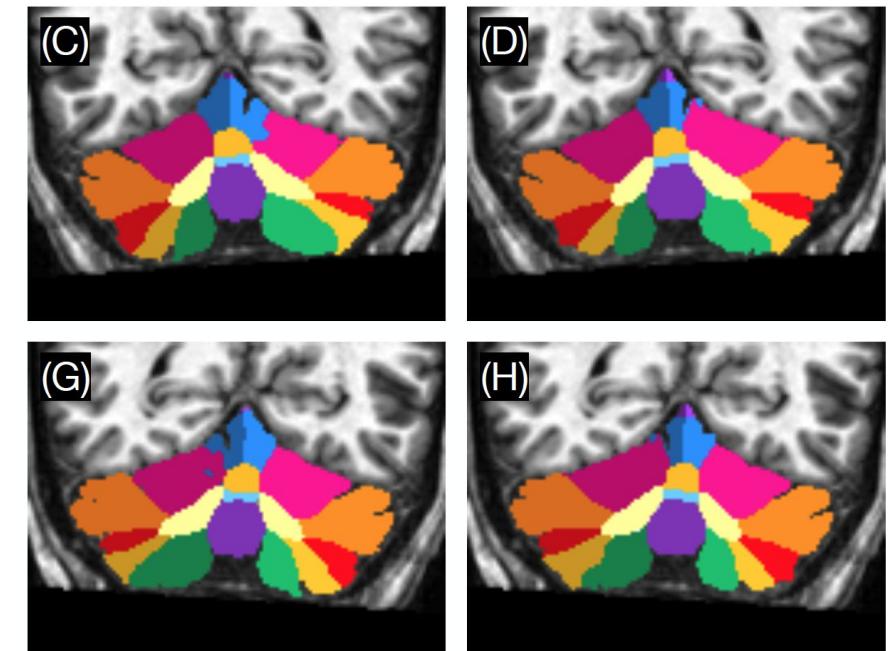
- Veeling, et al., 2018
 - 2D G-CNN for tumor segmentation
- Winkels and Cohen, 2019
 - 3D G-CNN for lung nodule detection



Source: <https://pubmed.ncbi.nlm.nih.gov/31003034/>

Recent work

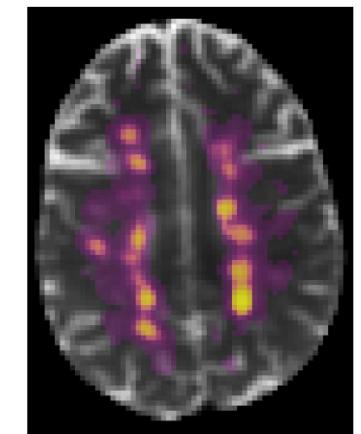
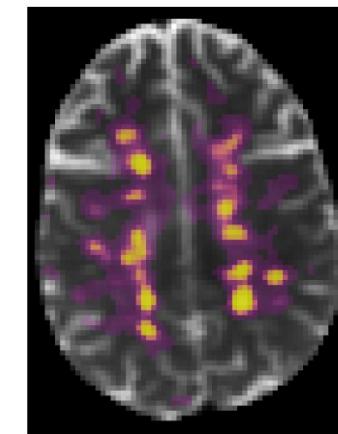
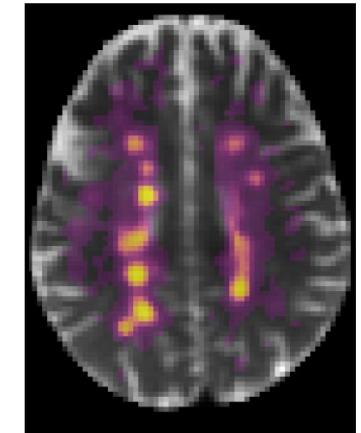
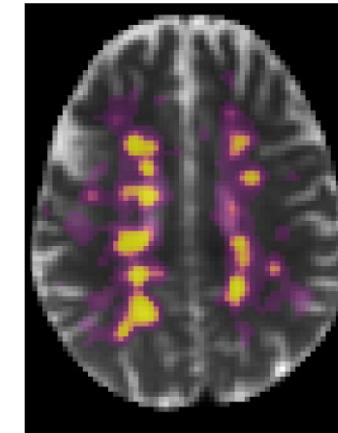
- Veeling, et al., 2018
 - 2D G-CNN for tumor segmentation
- Winkels and Cohen, 2019
 - 3D G-CNN for lung nodule detection
- Han et al., 2020
 - 3D G-CNN for subcortical segmentation and cerebellum parcellation



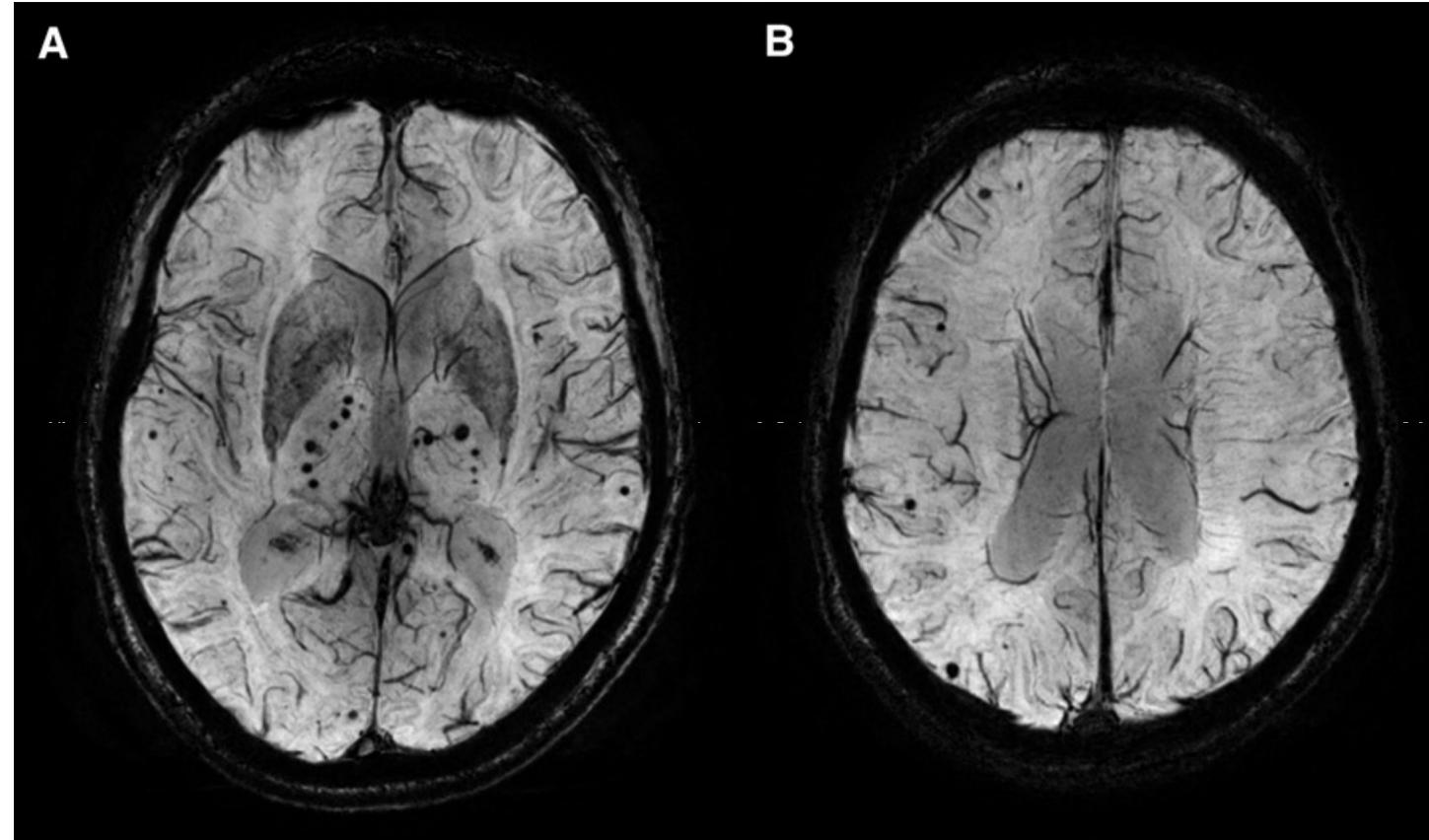
Recent work

- Veeling, et al., 2018
 - 2D G-CNN for tumor segmentation
- Winkels and Cohen, 2019
 - 3D G-CNN for lung nodule detection
- Han et al., 2020
 - 3D G-CNN for subcortical segmentation and cerebellum parcellation
- Müller et al., 2021 (pre-print)
 - 6D G-CNN for multiple sclerosis lesion segmentation

Equivariant 2 Non-equivariant



Application: cerebral microbleed quantification



Source: [Haller et al. - Cerebral Microbleeds: Imaging and Clinical Significance](#)

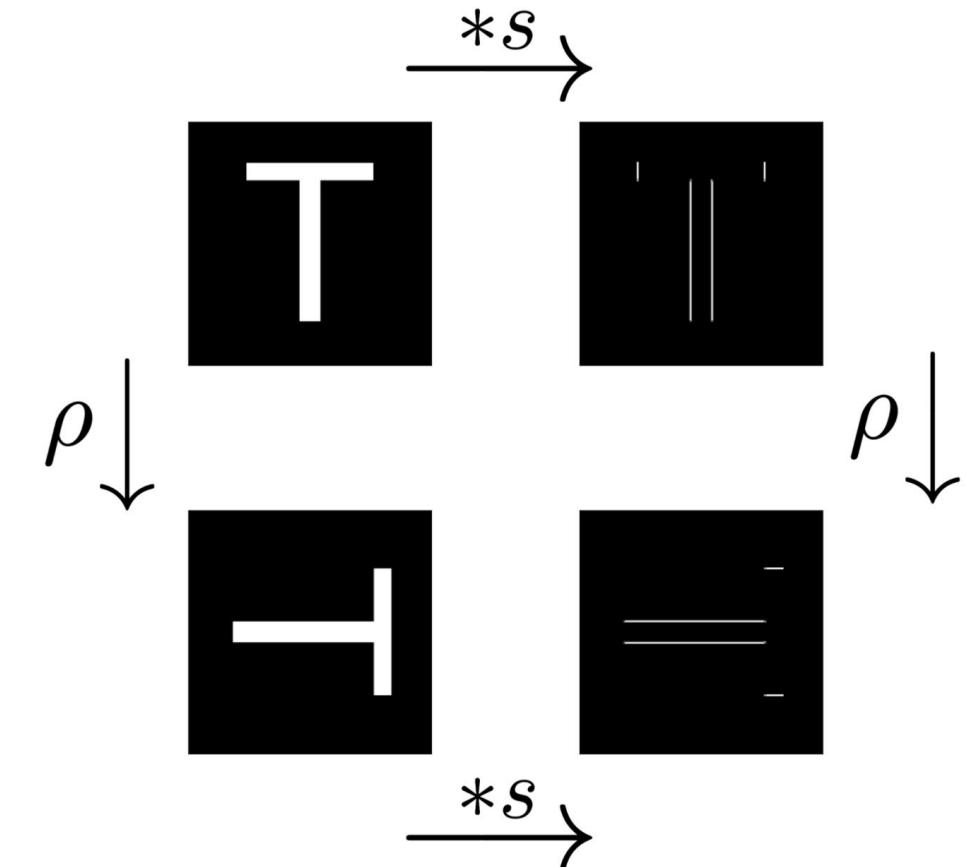
How do equivariant networks interact with brain MRIs?

Hypotheses:

- Higher sample efficiency
- Better segmentation performance
- Fewer false positives

Challenges:

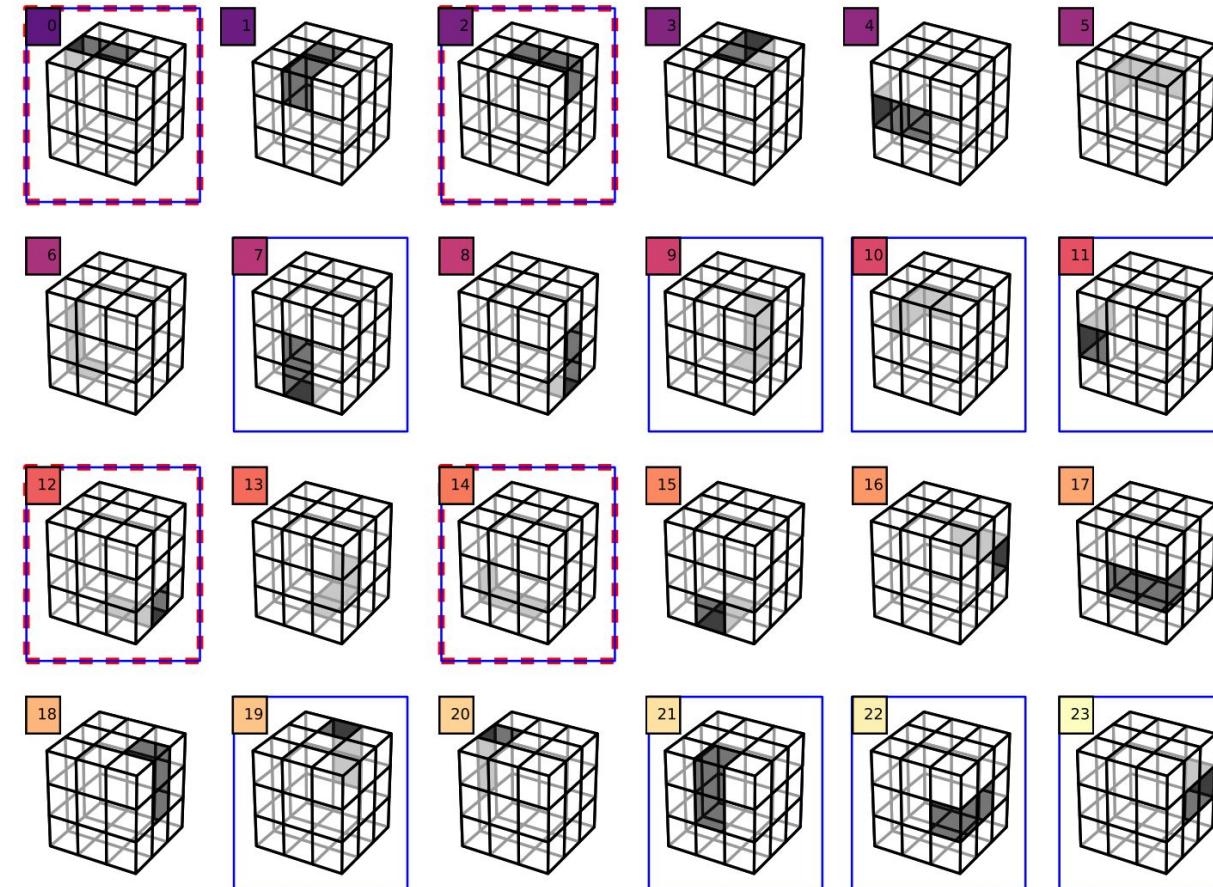
- Influence of surrounding tissue
- Non-trivial implementation



Thank you!

jami@di.ku.dk

Orbit of a filter under the octahedral group



Source: https://openaccess.thecvf.com/content_ECCV_2018/papers/Daniel_Worrall_CubeNet_Equivariance_to_ECCV_2018_paper.pdf