Antikythera Mechanism: Eclipse Seasons and the 63 Tooth Gear in Fragment D.

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Purpose

This paper proposes a gear train and dial presenting the Eclipse Season, employing the apparently superfluous 63 tooth gear in the Antikythera Mechanism Fragment D, in addition to a 45 tooth gear as possibly suggested by "ME" (which represents "45" in the ancient Greek letters-for-numbers system) engraved on the 63 tooth gear.[1] This paper is a natural follow up to the recently proposed[2] hypothetical dial that correlates the Egyptian regnal year with the solar year, as the premise in both these papers is that the side of the Antikythera Mechanism potentially contained dials.

This paper is primarily focused on the mathematics behind the proposed gearing, which reveals a very strong correlation to the Eclipse Seasons. The paper falls short, though, in exploring the concept from other facets which will require other subject matter experts along with review of the available evidence. Note also that this paper uses modern estimates of the Eclipse Seasons, and does not explore whether ancient astronomical knowledge of the Eclipse Seasons existed.

The Eclipse Season

A large portion of the Antikythera Mechanism provides lunisolar information, including the Metonic, Saros, and Exeligmos cycles, with the latter two supporting eclipse prediction.[3] One lunisolar cycle that does not appear to be mentioned in literature surrounding the Antikythera Mechanism is the Eclipse Season. (There are various other terms that refer to this cycle, specifically the draconic year, draconitic year, eclipse year, or ecliptic year[4], but for the purposes of this paper, the term "eclipse season" will be used.) The Eclipse Season is described as follows:

> The Sun makes one complete circuit of the ecliptic in 365.24 days, so its average angular velocity is 0.99° per day. At this rate, it takes 34.5 days for the Sun to cross the 34° wide eclipse zone centered on each node. Because the Moon's orbit with respect to the Sun has a mean duration of 29.53 days, there will always be one and possibly two solar eclipses during each 34.5 day interval when the Sun passes through the nodal eclipse zones. These time periods are called eclipse seasons.

[1] The Cosmos in the Antikythera Mechanism, Tony Freeth and Alexander Jones, Feb 2012.
[2] Antikythera Mechanism: A hypothetical A1 dial to provide in absolute terms both the Egyptian Regnal Year and the offset of the Egyptian Calendar Ring, Trent, Jan 2020.
[3] The Cosmos in the Antikythera Mechanism, Tony Freeth and Alexander Jones, Feb 2012.
[4] http://www.year.lu.rs/draconic/

> The mid-point of each eclipse season is separated by 173.3 days which is the mean time for the Sun to travel from one node to the next. The period is a little less than half a calendar year because the lunar nodes slowly regress westward by 19.3° per year.[5]

Additionally, the Eclipse Season cycle assists in using the Saros to predict eclipses.

> A solar eclipse requires more than a new Moon. It also requires the Moon to be aligned with Sun. This doesn't happen every month, because the Moon's orbit is tilted. The period of time needed for the apparent path of the Sun to carry it from the Moon's ascending node to its descending node and back again is called an eclipse year lasting 346.6201 days. (The nodes are the points where the moon crosses the plane of the earth's orbit.) Nineteen (19) eclipse years is equivalent to 6,585.78 days, which is almost — but not quite — a perfect match with 223 synodic months (the Saros cycle).[6]

For the purposes of this paper, the semi Eclipse Season of 173.31 days will be used in the mathematical analysis, and it will be assumed that B1 represents the Julian year of 365.25 days, although referred to as the "solar year" throughout this paper in line with other literature.[7]

The Critical Base Gearing Ratio

The A1 crown gear, being perpendicular in rotation to every other gear of the Antikythera Mechanism, offers the possibility of dials on the side of the mechanism. Bearing this premise in mind, there is the possibility of extending the B1 / A1 gear train by adding the 63 tooth gear to the A axis, which will then share the same rotation rate with A1. As the evidence of Fragment D indicates that the 63 tooth gear is marked with "ME" or 45 in Greek,[8] this gear train will further be extended by having the 63 tooth gear engage with a gear of 45 teeth. This does not necessarily have to be the arrangement as noted later, but for this analysis, the evidence is taken at face value. Thus, the gear train, organized by axis, appears as follows:

$$223\ (B1) \rightarrow \frac{48\ (A1)}{63\ (A2)} \rightarrow \frac{45\ (X1)}{}$$

That is, the B1 gear engages with the A1 gear, which is collocated with the 63 tooth gear on the same A axis, which in turn engages with a gear of 45 teeth. The following table lays out the mathematics.

Table 1: Gearing ratio of Solar year thru the 63 and 45 teeth gears.

	B axis	A axis	New "X" axis
Gearing by Name	B1	$\frac{A1}{A2}$	$\frac{X1}{}$
Gearing by Teeth	223	$\frac{48}{63}$	$\frac{45}{}$
Rotation Ratio Computation	1 (365.25 days)	223 / 48	223 / 48 * 63 / 45
Rotation Ratio	1	4.645833	6.504167

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[5] Periodicity of Solar Eclipses, https://eclipse.gsfc.nasa.gov/SEsaros/SEperiodicity.html, Fred Espenak, Jan 2012.
[6] What is the Saros Cycle and How Does It Foretell Eclipses?, https://www.skyandtelescope.com/observing/saros-cycle-solar-eclipse-lunar-eclipse/, Graham Jones, 24 Jan 2018.
[7] The Cosmos in the Antikythera Mechanism, Tony Freeth and Alexander Jones, Feb 2012.
[8] The Cosmos in the Antikythera Mechanism, Tony Freeth and Alexander Jones, Feb 2012.

Thus for every solar year represented by the complete revolution of the B axis, the X axis rotates 6.504167 times. Table 2 depicts the outcome when the B axis only rotates the duration of the semi Eclipse Season of 173.31 days.

Table 2: Gearing ratio of Eclipse Season

	B axis (Solar Year)	A axis	New "X" axis
Gearing by Name	B1	$\frac{A1}{A2}$	$\frac{X1}{}$
Gearing by Teeth	223	$\frac{48}{63}$	$\frac{45}{}$
Semi Eclipse Season Rotation Ratio Computation	173.31 / 365.25	(173.31 days of B axis) * 223 / 48	(173.31 days of B axis) * 223 / 48 * 63 / 45
Rotation	0.4744969	2.204434	3.086207

That is, if the B axis rotates 173.31 / 365.25 or 0.4744969 times, then the resulting X axis rotation will be 3.086207 times. In order for this relationship between the B axis and X axis to be useful for calculating a lunisolar cycle via gearing, the X axis must present a repeating whole number cycle with a relatively low number of turns, so that reasonable extended gearing can be implemented to drive a dial with minimal error.

Table 3 shows the angle of rotation of the X axis over the course of 58 semi Eclipse Seasons, incrementing the rotation of the B1 solar gear by the equivalent of 173.31 days. Thus, after 1 semi Eclipse Season, representing 173.31 days, the B axis will have rotated 0.474497 times (ie, 173.31 / 365.25) which in turn will drive the X axis 3.086207 times (ie, 0.474497 * 6.504167). After 2 semi Eclipse Seasons, with a total of 346.62 days, the B axis will have rotated 0.948994 times and the X axis a corresponding 6.172414 times, and so on. This is extrapolated through all 58 semi seasons.

Table 3: Eclipse Season gearing cycle

Semi Eclipse Season	Days	B axis Rotation	X axis Rotation	X axis Degrees
0	0	0.000000	0.000000	0.000000
1	173.31	0.474497	3.086207	31.034538
2	346.62	0.948994	6.172414	62.069076
3	519.93	1.423491	9.258621	93.103614
4	693.24	1.897988	12.344828	124.138152
5	866.55	2.372485	15.431035	155.172690
…	…	…	…	…
53	9185.43	25.148337	163.568974	204.830513
54	9358.74	25.622834	166.655181	235.865051
55	9532.05	26.097331	169.741388	266.899589
56	9705.36	26.571828	172.827595	297.934127
57	9878.67	27.046324	175.913802	328.968665
58	10051.98	27.520821	179.000009	0.003203

The critical result is that after only 58 semi Eclipse Seasons, the X axis is back to its original orientation with an error of only 9 parts in a million. This relatively small repeat cycle lends itself to gearing of reasonable number of teeth, made even more critical in the tight confines of the mechanism's side.

This is an appropriate place to pause, and ask, what of other potential cycles for any other given number of days? In other words, what are the odds that 173.31 days, which of course is the semi Eclipse Season, rather than any other daily cycle driven by the B axis provides a relatively short repeating cycle whereby the X axis returns to its original position, presenting a reasonable gearing ratio with minimal error?

Appendix C explores this question, and the analysis concludes that the base gear train strongly supports the Eclipse Season as the purpose behind the design. That is, there are no other apparent cosmological daily cycles that occur both within a reasonable number of whole number daily cycles and whole number X axis rotations with a very small margin of error necessary for a base set of gearing, an error which must in turn be absorbed by any downstream gearing which itself will likely introduce more error…

Thus, the combination of…

- the physical evidence of the superfluous 63 tooth gear,
- the possibility that the "ME" refers to a 45 tooth gear,
- the possibility that the 63 and 45 tooth gears can extend existing gears B1 and A1, and
- the high correlation that the gearing ratio represents the Eclipse Season

…arguably lends credence to a gear train of "223 → 48 / 63 → 45" in support of a dial representing the Eclipse Seasons. Furthermore, this cycle is a lunisolar measure that is in accordance with the Antikythera Mechanism's feature of eclipse prediction.

Extended Gear Train

First and foremost, the rotation of 3.086207 revolutions of the new proposed X axis relative to the B axis per Eclipse Season appears of little value to present lunisolar information via a dial, and like other dials of the mechanism, needs to be normalized via supplemental gear train. This involves converting the fraction 1 / 3.086207 (or 0.3240223) into a ratio of whole numbers and then boiling this down to reasonable primes to form a useful gear train. Of course, the greater the precision of the ratio, the less likely the result will contain a workable set of primes to support a practical gearing ratio.

Appendix B shows the analysis of the possible gearing ratios, and the prime factorization of those ratios. The analysis concludes that a gearing ratio of 81 / 250 is optimal, as 1) it is composed of small prime numbers, 2) contains a balance in the quantity of prime numbers, and 3) minimizes the additional error to the gear train. Appending gears of 81 teeth and 250 teeth to the current hypothesized Eclipse Season gear train results in a rotation ratio of the final output gear to very nearly 1 rotation. The gear train will then be…

$$223\ (B1) \rightarrow \frac{48\ (A1)}{63\ (A2)} \rightarrow \frac{45\ (X1)}{81\ (X2')} \rightarrow 250\ (Z1')$$

…with the resulting ratio of the B axis to the Z axis being…

$$\frac{223}{48} * \frac{63}{45} * \frac{81}{250} = 2.10735$$

This appears to be a useless ratio, but if the B axis, rather than turning 365.25 days (1 full revolution), only turns 173.31 days (the semi Eclipse Season), the resulting rotation of the Z axis is…

$$\frac{173.31}{365.25} * 2.10735 = 0.999931$$

That is, the Z axis will rotate once every 173.31 days of rotation of the B axis, with an error of 0.0069% or 1 part in ~14500. Now a complete gear train exists to present useful information about the Eclipse Season. Of course, gears of 81 and 250 teeth are impractical, so the gear train needs to be adjusted with more reasonable gearing, using teeth of 9 & 9 and 25 & 10 as follows…

$$223\ (B1) \rightarrow \frac{48\ (A1)}{63\ (A2)} \rightarrow \frac{45\ (X1)}{9\ (X2)} \rightarrow \frac{25\ (Y1)}{9\ (Y2)} \rightarrow 10\ (Z1)$$

Note too that the option exists to shuffle this a bit (including the underlying prime factors) and still retain the same rotation rate output at the Z axis. That is, there is nothing preventing the 45 tooth gear being at the end of the train, and say, moving the 63 tooth gear elsewhere, and still obtain the same ratio of rotation of the Z axis relative to the B axis…

$$223\ (B1) \rightarrow \frac{48\ (A1)}{9} \rightarrow \frac{10}{63} \rightarrow \frac{25}{9} \rightarrow 45$$

This gear train also produces the Z axis desired rotation rate of 2.10735 relative to the rotation rate of the B axis. Again, though, other considerations must come into play, such as the size of the gears and the reach required to mesh with its partner, whether the gears or axes might have auxiliary purposes in driving other trains, etc.

Hypothetical Eclipse Season Dial

The following diagram depicts the side of the Antikythera Mechanism, albeit crudely, with the Eclipse Season dial and the previously hypothesized Egyptian regnal year dial. This diagram is based on the discussion above, whereby the Eclipse Season dial rotates one full turn every semi Eclipse Season (ie, 173.31 days), and due to the stretch of the gear train connected to the A axis, will be either above or below the position of the A1 crown gear. Obviously, as with other gear trains on the Antikythera Mechanism, adjusting the module of the gears expands or contracts the size of the gears while retaining the same number of teeth, permitting optimal placement of the dial.

The small arc of blue, labelled "Eclipse Season", represents the period during which eclipses will occur. Per references, this period is a mean of 34.5 days, or a range of 31 to 37 days.[9,10] Thus, the arc will sweep 37 / 173.31 or ~21% of a full circle.

[9] Periodicity of Solar Eclipses, https://eclipse.gsfc.nasa.gov/SEsaros/SEperiodicity.html, Fred Espenak, Jan 2012.
[10] The Science: Eclipse Cycles, https://moonblink.info/Eclipse/why/cycles, Ian Cameron Smith, Oct 2019.

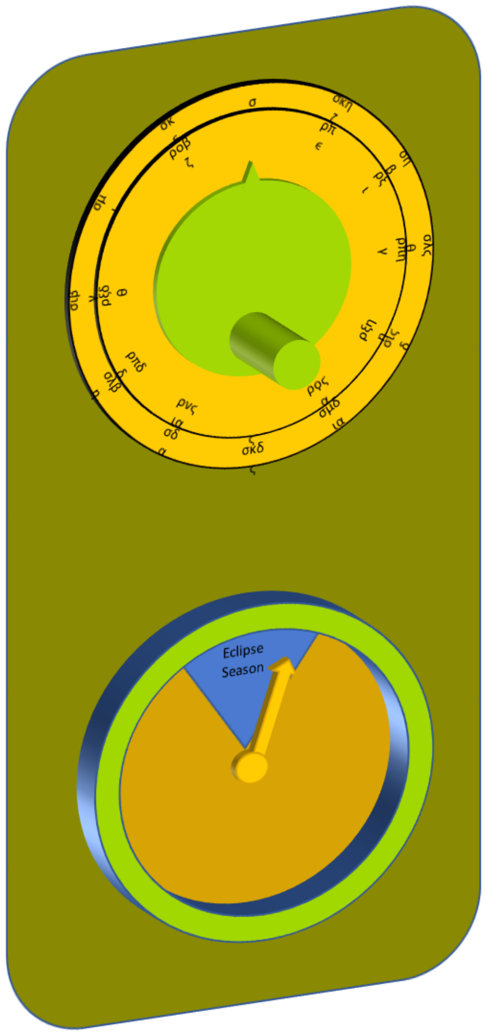

Figure 1: Hypothetical side of Antikythera Mechanism with Eclipse Season Dial

Such a dial permits the operator to identify whether the mechanism is aligned to an Eclipse Season period, before referring to the Saros and Exeligmos dials for more eclipse details.

Concluding Remarks

To reiterate, the hypothesis of this paper was born from the premise that the side of Antikythera Mechanism might possibly contain dials, originating of course from the A1 crown gear. Although this paper shows a strong mathematical correlation of applying both the 63 tooth gear in Fragment D and a 45 tooth gear to the B1 / A1 gear train to support an Eclipse Season dial, it falls short in many critical areas. Questions and issues that readily come to mind:

- Were ancient astronomers aware of the Eclipse Season cycle of 346.62 days and the mean 34.5 day window during each semi Eclipse Season?[11] And is there evidence of this knowledge?

[11] Periodicity of Solar Eclipses, https://eclipse.gsfc.nasa.gov/SEsaros/SEperiodicity.html, Fred Espenak, Jan 2012.

- Is there evidence that the A1 gear and the 63 tooth gear in Fragment D shared an axis? (Note that it is not a requirement of the gear train that the 63 tooth gear reside on the A axis, but the incidental notes in Appendix A hint at this configuration to support other opportunities.)
- If the 63 tooth gear is on the A axis, what might be the purpose behind the holes and rivets as evidenced by the X-ray CT of Fragment D[12]?
- Knowing that the 63 tooth gear might now be part of an Eclipse Season gear train, is the "ME" engraved on the 63 tooth gear actually a reference to a subsequent 45 tooth gear?
- In light of the latest planetary models[13] and available physical evidence, is there sufficient physical space to contain the Eclipse Season gear train?
- Is the final rotation rate of only 1 full turn of the Z axis for every turn of 173.31 days of the B1 gear depriving the opportunity of additional information that can potentially correlate the Eclipse Seasons with other aspects of the Saros, Exeligmos, and Metonic calendars?
- Is it possible to also show the 19 year Eclipse Season cycle, which equates to just about exactly one Saros cycle, with minor additional gearing? Will such an extension of the gear train provide any value to the operation of the mechanism?
- Is there the possibility that the X axis (45 tooth gear) was also used for Egyptian Regnal calendar alignment, bolstering the premise of dials on the side? (See appendix A Incidental Subjects.)

[12] ISAW Papers 4: The Cosmos in the Antikythera Mechanism
[13] ISAW Papers 4: The Cosmos in the Antikythera Mechanism

Appendix A: Incidental Subjects

During the course of writing this paper, incidental questions began to present themselves, and this appendix simply captures the musings…

Possible Egyptian regnal year down gearing

The author's previous paper, suggesting a dial associated directly with the A axis to provide in absolute terms the Egyptian regnal year and leap years,[14] potentially needs reevaluation because during the course of performing this analysis, it was discovered that if table 3 (Eclipse Season base gearing cycle) is taken just one more iteration, the result is:

Semi Eclipse Season	Days	B axis Rotation	X axis Rotation	X axis Degrees
59	10225.29	27.995318	…	…

Note that the B axis rotation is 0.5% from being exactly 28 solar years. This relationship is very convenient, as 28 years is exactly 7 occurrences of 4 solar years, which introduces the possibility that another gear train emanates from the X axis (the axis of the 45 tooth gear) offering a potentially better means of tracking Egyptian Regnal years and leap years via a dial in a more continuous fashion. Plus, although the error is relatively large, the fact that it is spread over 4 rotations of the B axis (1461 days) reduces the effects of the error.

If this pans out, this will further bolster the base gear train of "223 (B1) → 48 (A1) / 63 (A2) → 45 (X1)" as it will now serves dual purposes, akin to the N axis which drives both the Olympiad / Games and Callippic via two different gear trains.

Input knob vector

The author only formed the premise that the side had dials, as various modern attempts in various forms at reconstructing the function of the Antikythera Mechanism seem to avoid the use of A1 as the input gear due to the large gearing up required on the D axis leading into the E axis, and the subsequent internal stresses. If the concept of dials on the side of the mechanism holds water, then this opens consideration for input from other vectors of the gearing, despite some, including the author, likely finding this mildly displeasing as it detracts from the design appeal of the input knob on the side. Per the latest comprehensive research, "at the right-hand side of the Mechanism there was an input, and *we assume that this was turned by hand with some sort of handle or crank, though only the keyway for the input remains*."[15] A mechanical analysis, and possibly only a physical build authentic to the latest research, will likely be required to vet this…

[14] Antikythera Mechanism: A hypothetical A1 dial to provide in absolute terms both the Egyptian Regnal Year and the offset of the Egyptian Calendar Ring, Trent, Jan 2020.
[15] ISAW Papers 4: The Cosmos in the Antikythera Mechanism

Appendix B: Extended Gear Train

When the B1 solar gear turns 173.31 days of the 365.25 days represented by a full rotation, the gear train of "223 (B1) → 48 (A1) / 63 (A2) → 45 (X1)" will see the X axis rotate 3.086207 times. If the desire is to represent a 173.31 day turn of B1 as a single full turn on a dial, this involves gearing down the X axis to a single rotation by determining the possible gearing that corresponds to the reciprocal of the X axis rotation rate, which is 0.3240223 (ie, 1 / 3.086207).

The technique for converting a fraction to a whole number ratio usually involves Farey or Stern-Brocot techniques, but in the case of this analysis a brute force combination of every numerator and denominator less than 500 was tested. Any ratio in error with 0.3240223 to less than 0.0001 is then considered a candidate. The results, sorted by smallest error:

Num	Den	Error	Error: 1 in ...	Numerator Factored	Denominator Factored	Small Primes	Balanced Quantities of Primes
58	179	0.0000000	21566265	2.29	179		
116	358	0.0000000	21566265	2.2.29	2.179		
151	466	0.0000120	83093	151	2.233		
139	429	0.0000130	77065	139	3.11.13		
93	287	0.0000195	51251	3.31	7.41	Yes	
81	250	0.0000223	44843	3.3.3.3	2.5.5.5	Yes	Yes
162	500	0.0000223	44843	2.3.3.3.3	2.2.5.5.5	Yes	Yes
128	395	0.0000283	35295	2.2.2.2.2.2.2	5.79		
104	321	0.0000348	28768	2.2.2.13	3.107		
127	392	0.0000427	23415	127	2.2.2.7.7		
150	463	0.0000482	20739	2.3.5.5	463		
35	108	0.0000518	19315	5.7	2.2.3.3.3	Yes	Yes
70	216	0.0000518	19315	2.5.7	2.2.2.3.3.3	Yes	Yes
105	324	0.0000518	19315	3.5.7	2.2.3.3.3.3	Yes	Yes
140	432	0.0000518	19315	2.2.5.7	2.2.2.2.3.3.3	Yes	Yes
152	469	0.0000715	13983	2.2.2.19	7.67		
117	361	0.0000774	12916	3.3.13	19.19	Yes	Yes
23	71	0.0000786	12716	23	71		
46	142	0.0000786	12716	2.23	2.71		
69	213	0.0000786	12716	3.23	3.71		
92	284	0.0000786	12716	2.2.23	2.2.71		
115	355	0.0000786	12716	5.23	5.71		
138	426	0.0000786	12716	2.3.23	2.3.71		
161	497	0.0000786	12716	7.23	7.71		
82	253	0.0000884	11316	2.41	11.23	Yes	
129	398	0.0000983	10173	3.43	2.199		

Of course, notice that the ratio of 58 / 179 is the top candidate, as derived by Table 3. The optimal ratio for a gear train, though, is typically one involving small primes (otherwise the gear train will require gears with a large number of teeth) and a balance between the numerator and denominator in the number of primes, to assist in flexibly configuring the gearing ratios in the gear train.

In this case, 81 / 250 is the optimal solution to extending the gear train from the X axis, given the small error, the representation via small primes, and a balance in the number of primes between the numerator and denominator.

Appendix C: Analysis of "223 (B1) ➔ 48 (A1) / 63 ➔ 45" Daily Cycles

This appendix poses the question as to what other daily cycles, driven by the base gear train from the B to X axis, potentially represent a relevant cosmological cycle. In other words, this paper asserts that 173.31 days of rotation of B1 repeated 58 times yielding 179 turns of the X axis is the purpose behind the gearing. So, this appendix answers whether there are other daily cycles that produce a similar relationship between the B and X axes, and hence whether the design purpose of such a gear train is to support the Eclipse Season rather than some other cosmological daily cycle.

To explore this, a brute force method of iterating through daily cycles (to the 1/100th of each day) was applied to the B axis upwards to 250 times, until the X axis rotated a whole number of times to within a tolerance of 0.00005 rotations. (That is, a day of 0.01 days was tested, then 0.02 days, then 0.03 days, etc until 365.25 days, for a total of 36,525 test sequences.) ***The fundamental logic being that such a daily cycle will precipitate from the analysis due to the original designer's intent of the gear train. That is, the gear train was designed to minimize the error through repeated cycles of whatever cosmological cycle it represents.***

The margin of error might seem unnecessarily tight, but note that any error of a base gear train is degraded with any further down gearing required to present a calculation via a dial, so the base gearing margin of error at the X axis must be minimal, hence the tight tolerance of 0.00005 rotations to narrow the field of candidates. Again, this analysis is to find the design intent, and hence the table below is sorted by smallest margin of error first.

The interpretation of the table below is thus, best explained via example. Take the second row, which represents 52.77 days of the B1 gear. This equates to 52.77 / 365.25 * 6.50416666667 rotations of the X axis, and then this cycle of 52.77 days is incremented (for up to 250 times) until the X axis is back to its original orientation to within 0.00005 rotations. In this case, the 199th time of incrementing the B axis by 52.77 days results in the X axis rotating 187.0000003 times, nearly back to its original position by 1 part in 2,922,000. So, if there was some need to represent a single cycle of 52.77 days via a dial, the base gearing ratio of the X axis will need to be extended with gearing having a ratio of 199 / 187, and in this case the representation will be very accurate relative to all other daily cycle candidates.

Then, since 52.77 is a candidate daily cycle, the author searched the internet for any cosmological daily cycles involving 52.77 days. Five asterisks ('*****') represents a direct correlation with a cosmological daily cycle, and one asterisk represents some internet search hits but mostly involving tidal data, eclipse tables, IP addresses, etc. Again, the table is sorted by the smallest error first, with the logic being that the gear ratio of "223 (B1) ➔ 48 (A1) / 63 ➔ 45" was by design, and therefore thoughtfully crafted to minimize error.

Days represented on B axis (Step: 0.01)	Cycles of Days (<= 250)	Rotations of X axis (45 tooth gear) (Error < 0.00005)	Error: 1 part in…	Reference Hits
365.25	240	1561	Infinity	*****
52.77	199	187.0000003	2922000	*
105.54	199	374.0000007	1461000	*
158.31	199	561.000001	974000	
82.63	70	103.0000011	876600	
165.26	35	103.0000011	876600	
211.08	199	748.0000014	730500	
99.91	163	290.0000015	674308	*

Days represented on B axis (Step: 0.01)	Cycles of Days (<= 250)	Rotations of X axis (45 tooth gear) (Error < 0.00005)	Error: 1 part in...	Reference Hits
158.11	103	290.0000015	674308	
167.89	97	290.0000015	674308	
263.85	199	935.0000017	584400	
196.96	136	477.0000018	547875	*
209.27	128	477.0000018	547875	*
46.39	23	19.00000194	515647	*
316.62	199	1122.000002	487000	
330.52	35	206.0000023	438300	
199.82	163	580.000003	337154	*
316.22	103	580.000003	337154	
335.78	97	580.000003	337154	
247.89	70	309.0000034	292200	
145.83	191	496.0000038	265636	*
92.78	23	38.00000388	257824	
299.73	163	870.0000044	224769	
152.27	52	141.000005	199227	
304.54	26	141.000005	199227	
297.27	235	1244.000005	194800	
139.17	23	57.00000582	171882	
291.66	191	992.0000075	132818	
185.56	23	76.00000776	128912	
46.11	218	179.0000089	112385	*
57.77	174	179.0000089	112385	*
63.22	159	179.0000089	112385	*
92.22	109	179.0000089	112385	*
94.83	106	179.0000089	112385	
115.54	87	179.0000089	112385	*
173.31	58	179.0000089	112385	*****
189.66	53	179.0000089	112385	
346.62	29	179.0000089	112385	*****
231.95	23	95.0000097	103129	
327.21	127	740.0000099	100759	
145.51	181	469.0000104	96330	
154.43	4	11.0000105	95283	
308.86	2	11.0000105	95283	
361.68	202	1301.000011	91312	
143.18	151	385.0000112	89449	*
278.34	23	114.0000116	85941	
41.09	41	30.00001243	80422	*
240.67	7	30.00001243	80422	*
53.92	226	217.0000128	78268	*
107.84	113	217.0000128	78268	*
117.55	193	404.0000131	76226	*
199.93	166	591.0000135	74288	*
324.73	23	133.0000136	73664	
342.98	158	965.0000141	70694	
227.77	125	507.0000143	70128	
152.87	18	49.00001437	69571	
305.74	9	49.00001437	69571	
59.43	223	236.0000147	67953	*
345.97	143	881.0000149	66916	
239.34	187	797.0000157	63522	
52.31	73	68.00001631	61301	*
278.64	236	1171.000016	60875	

Days represented on B axis (Step: 0.01)	Cycles of Days (<= 250)	Rotations of X axis (45 tooth gear) (Error < 0.00005)	Error: 1 part in...	Reference Hits
304.44	216	1171.000016	60875	
186.23	215	713.0000165	60455	*
166.51	86	255.0000167	60041	*
333.02	43	255.0000167	60041	*
145.96	242	629.0000173	57671	*
198.44	178	629.0000173	57671	*
215.38	164	629.0000173	57671	*
291.92	121	629.0000173	57671	*
126.44	159	358.0000178	56192	*
184.44	109	358.0000178	56192	
231.08	87	358.0000178	56192	
132.49	231	545.0000181	55132	
19.7	248	87.00001825	54788	*
24.8	197	87.00001825	54788	*
31.52	155	87.00001825	54788	*
39.4	124	87.00001825	54788	*
61.07	80	87.00001825	54788	*
78.8	62	87.00001825	54788	*
122.14	40	87.00001825	54788	*
157.6	31	87.00001825	54788	*
244.28	20	87.00001825	54788	
305.35	16	87.00001825	54788	
194.77	79	274.0000186	53779	*
9.91	17	3.000019051	52491	*
168.47	1	3.000019051	52491	
360.29	101	648.0000193	51870	
152.63	39	106.0000202	49525	*
74.79	220	293.0000205	48700	*
83.1	198	293.0000205	48700	*
91.41	180	293.0000205	48700	
99.72	165	293.0000205	48700	*
121.88	135	293.0000205	48700	*
124.65	132	293.0000205	48700	*
149.58	110	293.0000205	48700	
152.35	108	293.0000205	48700	
166.2	99	293.0000205	48700	*
182.82	90	293.0000205	48700	
249.3	66	293.0000205	48700	*
274.23	60	293.0000205	48700	
299.16	55	293.0000205	48700	
304.7	54	293.0000205	48700	*
291.02	181	938.0000208	48165	
319.07	22	125.0000221	45186	
336.39	205	1228.000022	44954	
286.36	151	770.0000224	44724	
140.11	200	499.0000228	43830	*
280.22	100	499.0000228	43830	
69.77	33	41.00002293	43612	
209.31	11	41.00002293	43612	*
250.59	93	415.0000236	42348	
293.23	203	1060.000024	41943	
82.18	41	60.00002487	40211	
360.37	139	892.0000254	39309	
215.68	113	434.0000256	39134	*

Days represented on B axis (Step: 0.01)	Cycles of Days (<= 250)	Rotations of X axis (45 tooth gear) (Error < 0.00005)	Error: 1 part in…	Reference Hits
38.46	238	163.000026	38447	*
44.87	204	163.000026	38447	
76.92	119	163.000026	38447	
89.74	102	163.000026	38447	
108.97	84	163.000026	38447	*
134.61	68	163.000026	38447	
179.48	51	163.000026	38447	
217.94	42	163.000026	38447	*
269.22	34	163.000026	38447	
326.91	28	163.000026	38447	
235.1	193	808.0000262	38113	*
138.33	218	537.0000267	37462	
276.66	109	537.0000267	37462	
284.49	106	537.0000267	37462	
53.45	83	79.00002681	37302	
213.16	240	911.0000274	36525	*
233.6	219	911.0000274	36525	*
266.45	192	911.0000274	36525	*
319.74	160	911.0000274	36525	
350.4	146	911.0000274	36525	*
143.95	71	182.0000279	35780	
241.18	173	743.000029	34512	
118.86	223	472.0000294	33977	*
27.11	29	14.00002955	33846	*
330.14	190	1117.00003	33715	
300.87	123	659.0000298	33586	
335.21	65	388.0000302	33079	
136.86	167	407.0000322	31085	
104.62	73	136.0000326	30650	*
288.54	152	781.0000329	30438	
304.57	144	781.0000329	30438	
329.76	133	781.0000329	30438	
348.08	126	781.0000329	30438	
191.52	229	781.0000329	30437	*
192.36	228	781.0000329	30437	*
256.48	171	781.0000329	30437	
261.06	168	781.0000329	30437	*
67.91	43	52.00003342	29918	
105.68	127	239.0000338	29615	*
219.26	157	613.0000345	29026	*
120.27	247	529.0000352	28369	*
140.79	211	529.0000352	28369	
30.67	130	71.00003536	28277	*
61.34	65	71.00003536	28277	*
153.35	26	71.00003536	28277	
306.7	13	71.00003536	28277	*
252.88	159	716.0000356	28096	
190.76	131	445.000036	27741	
328.81	76	445.000036	27741	*
264.98	231	1090.000036	27566	*
49.6	197	174.0000365	27394	*
63.04	155	174.0000365	27394	*
315.2	31	174.0000365	27394	*
123.27	41	90.0000373	26807	*

Days represented on B axis (Step: 0.01)	Cycles of Days (<= 250)	Rotations of X axis (45 tooth gear) (Error < 0.00005)	Error: 1 part in…	Reference Hits
81.87	190	277.0000376	26564	
136.45	114	277.0000376	26564	
163.74	95	277.0000376	26564	*
272.9	57	277.0000376	26564	*
292.77	89	464.000038	26324	
19.82	17	6.000038102	26246	*
336.94	1	6.000038102	26246	
265.01	235	1109.000038	26167	*
161.76	226	651.0000383	26089	
323.52	113	651.0000383	26089	
53.39	203	193.0000384	26012	
81.49	133	193.0000384	26012	*
265.87	177	838.0000387	25858	
355.31	162	1025.000039	25632	*
40.27	152	109.0000392	25483	
80.54	76	109.0000392	25483	
161.08	38	109.0000392	25483	
322.16	19	109.0000392	25483	*
352.65	193	1212.000039	25409	
339.23	49	296.0000396	25262	
362.04	217	1399.00004	25190	
159.55	170	483.0000399	25046	
319.1	85	483.0000399	25046	*
8.21	171	25.00004004	24974	*
24.63	57	25.00004004	24974	*
73.89	19	25.00004004	24974	*
155.99	9	25.00004004	24974	
263.11	143	670.0000403	24833	*
305.26	39	212.0000404	24763	
199.44	165	586.0000411	24350	*
243.76	135	586.0000411	24350	
332.4	99	586.0000411	24350	*
323.03	214	1231.000041	24215	
82.66	214	315.0000415	24082	
165.32	107	315.0000415	24082	
129.91	217	502.0000419	23886	*
188.74	205	689.0000422	23692	
155.01	121	334.0000435	23008	*
20.45	173	63.00004392	22769	*
178.29	223	708.0000441	22651	
230.55	218	895.0000445	22477	
288.85	174	895.0000445	22477	
316.1	159	895.0000445	22477	*
49.85	187	166.0000451	22192	*
109.67	85	166.0000451	22192	*
169.49	55	166.0000451	22192	
139.54	33	82.00004586	21806	
172.99	236	727.0000461	21698	
345.98	118	727.0000461	21698	
335.69	45	269.0000462	21644	*
357.51	101	643.0000469	21328	
144.07	145	372.0000473	21123	
22.73	42	17.0000486	20577	*
45.46	21	17.0000486	20577	*

Days represented on B axis (Step: 0.01)	Cycles of Days (<= 250)	Rotations of X axis (45 tooth gear) (Error < 0.00005)	Error: 1 part in…	Reference Hits
68.19	14	17.0000486	20577	
136.38	7	17.0000486	20577	
159.11	6	17.0000486	20577	
318.22	3	17.0000486	20577	
156.93	73	204.0000489	20434	
114.36	192	391.0000493	20292	*
152.48	144	391.0000493	20292	
171.54	128	391.0000493	20292	
228.72	96	391.0000493	20292	
304.96	72	391.0000493	20292	*
343.08	64	391.0000493	20292	
164.36	41	120.0000497	20106	