Assignment 3

Name: _____

Write or type solutions on a separate paper. If written, write legibly.

1. Describe each of the following sets as a list

   a) $\{x \in \mathbb{N} \mid |x| < 6\}$

   b) $\{x \in \mathbb{N} \mid x < 30 \wedge (\forall y \in \mathbb{N})(x \neq 2y)\}$

   c) $\{x \in \mathbb{R} \mid 6x^4 + 43x^3 - 78x^2 + 5x - 12 = 0\}$

   d) $\{x \in \mathbb{N} \mid (\forall y \in \mathbb{N})(y \leq 10 \wedge x = 3y + 2)\}$

2. Given $\mathbf{A}, \mathbf{B}, \mathbf{C} \subseteq \mathbf{S}$ where

$$\mathbf{S} = \{x \in \mathbb{N} \mid x \leq 20\} \qquad \mathbf{A} = \{x \in \mathbf{S} \mid (\exists y \in \mathbb{Z})(\exists z \in \mathbb{Z})(20y + xz = 1)\}$$
$$\mathbf{B} = \{x \in \mathbf{S} \mid (\exists y \in \mathbb{Z})(x = 2y + 1)\} \qquad \mathbf{C} = \{x \in \mathbf{S} \mid (\exists y \in \mathbb{N})(x = 5y)\}$$

   Find

   (a) $(\mathbf{A} \cap \mathbf{B}' \cap \mathbf{C})'$

   (b) $\mathbf{C} \cup (\mathbf{A}' - (\mathbf{S} - \mathbf{B}))$

3. Given $\mathbf{A}, \mathbf{B}, \mathbf{C} \subseteq \mathbf{S}$ where

$$\mathbf{S} = \{x \in \mathbb{N} \mid x \leq 50\} \qquad \mathbf{A} = \{x \in \mathbf{S} \mid (\exists y \in \mathbb{Z})(x = 4y)\}$$
$$\mathbf{B} = \{x \in \mathbf{S} \mid (\exists y \in \mathbb{Z})(\exists z \in \mathbb{Z})(5y + xz = 3)\} \qquad \mathbf{C} = \{x \in \mathbf{S} \mid (\exists y \in \mathbb{N})(x = 3y + 2)\}$$

   Find

   (a) $|\mathbf{A} \cup \mathbf{B} \cup \mathbf{C}|$

   (b) $|\wp(\mathbf{A} \cup \mathbf{B}) \times \mathbf{C}'|$

4. Rewrite the program below and define the function `IsSet()`. The function `IsSet()` should return true if the file it opens represents a set; otherwise, it should return false. The files will have no more than 500 numbers. Furthermore, you are allowed to make additional functions, but you cannot include additional libraries.

```cpp
#include <iostream>
#include <fstream>
#include <string>
using namespace std;

bool IsSet(string filename);

int main()
{
string data[2] = {"data1.txt","data2.txt"};

for(int i = 0;i < 2;i += 1)
{
cout << data[i] << ((IsSet(data[i])?(" does "):(" does not ")) << " represent a set\n";
}

return 0;
}
```

Extra Credit Given $\mathbf{A}, \mathbf{B}, \mathbf{C} \subseteq \mathbf{S}$, prove
$$[\mathbf{A} \cap (\mathbf{B} \cup \mathbf{C})]' = \mathbf{A}' \cup (\mathbf{B}' \cap \mathbf{C}')$$
by using set identities and a direct proof.