Assignment 4

Name:_____

Write or type solutions on a separate paper. If written, write legibly.

1. Prove or disprove that 859 is prime.

2. Prove using induction that

$$\sum_{i=1}^{n} (i+1)^3 - i^3 = n^3 + 3n^2 + 3n$$

3. Find $\gcd(403, 91)$ by using the Euclidean method.

4. Rewrite the program below and define the function `Phi()` which returns the count of the integers between 1 (included) and n (excluded) that are relatively prime to n where n is the parameter. If n is $n < 1$, it returns 0. Furthermore, you are allowed to make additional functions, but you cannot include additional libraries.

```
#include <iostream>
#include <fstream>
#include <string>
#include <iomanip>
using namespace std;

int Phi(int);

int main()
{
 fstream out;
 out.open("data.dat",fstream::out);

 for(int i = 1;i <= 100;i += 1)
 {
  out << setw(3) << setfill('0');
  out << "phi(" << i << ") = " << Phi(i) << "\n";
 }
 out.close();
 return 0;
}
```

5. Given

$$T(n) = \begin{cases} 3 & \text{if } n = 1 \\ 4T(n-1) + 2n - 1 & \text{if } n > 1 \end{cases}$$

Find $T(2)$, $T(4)$, $T(6)$ and $T(9)$.

Extra Credit Prove using induction that

$$\sum_{i=1}^{n} \sum_{j=1}^{i} j = \frac{n(n+1)(n+2)}{6}$$