Definitions and Formula Sheet

Connectives The truth table below lists the truth values of each connective.

A	B	$A \wedge B$	$A \vee B$	$A \to B$	$A \leftrightarrow B$	A'
T	T	T	T	T	T	F
T	F	F	T	F	F	
F	T	F	T	T	F	T
F	F	F	F	T	T	

Note: prime (′) is an unary connective. It operates on a single statement; hence, it only has two possible truth values.

Order of Precedence of Connectives The order of precedence of connectives from highest to lowest priority is

1. (), Parentheses
2. ′, Negation
3. $\wedge$, $\vee$, Conjunction and Disjunction
4. $\to$, Implication
5. $\leftrightarrow$, Equivalence

Note: connectives are evaluated from left to right.

Equivalence Rules		
Expression	**Equivalent to**	**Name - Abbreviation of Rule**
$A \vee B$ $A \wedge B$	$B \vee A$ $B \wedge A$	Commutative - comm
$(A \vee B) \vee C$ $(A \wedge B) \wedge C$ $(A \to B) \to C$	$A \vee (B \vee C)$ $A \wedge (B \wedge C)$ $A \to (B \to C)$	Associative - assoc
$A \vee (B \wedge C)$ $A \wedge (B \vee C)$	$(A \vee B) \wedge (A \vee C)$ $(A \wedge B) \vee (A \wedge C)$	Distributive - dis
$A \vee 0$ $A \wedge 1$	A A	Identity - iden
$A \vee 1$ $A \wedge 0$	1 0	Domination - dom
$A \vee A'$ $A \wedge A'$	1 0	Complement - neg
$(A \vee B)'$ $(A \wedge B)'$	$A' \wedge B'$ $A' \vee B'$	De Morgan's Law - deMor
$A \to B$	$A' \vee B$	Implication - imp
$A \to B$	$B' \to A'$	Contrapositive - cp
A	A''	Double Negation - dn
$A \wedge A$ $A \vee A$	A A	Idempotent - self

$A \leftrightarrow B$	$(A \to B) \wedge (B \to A)$	Equivalence - equ
$(A \to B) \wedge A$	$A \wedge B$	Reduction - red
$(A \wedge B) \to C$ $(A \vee B) \to C$	$(A \to C) \vee (B \to C)$ $(A \to C) \wedge (B \to C)$	Expansion - exp
$A \vee (A \wedge B)$ $A \wedge (A \vee B)$	A A	Absorption - abs
$A \to (B \to C)$	$A \wedge B \to C$	Deductive - ded
$A \to B$	$A \to (A \wedge B)$	Redundancy - rep
$[(\forall x)P(x)]'$ $[(\exists x)P(x)]'$	$(\exists x)[P(x)]'$ $(\forall x)[P(x)]'$	Negation - ng
$P(x) \to (\forall y)Q(y)$ $P(x) \to (\exists y)Q(y)$	$(\forall y)[P(x) \to Q(y)]$ $(\exists y)[P(x) \to Q(y)]$	Shifting - sh
Inference Rules		
Expression	**Derives**	**Name - Abbreviation of Rule**
$A \wedge B$	$A \vee B$	Weakening - wk
$(A \to B) \wedge A$	B	Modus ponens - mp
$(A \to B) \wedge B'$	A'	Modus tollens - mt
$(A \wedge B)$	A B	Simplification - sim
$A \wedge B$	$(A \wedge B)$	Conjunction - con
A	$A \vee B$	Addition - add
$(A \vee B) \wedge A'$ $(A \vee B) \wedge B'$	B A	Disjunctive syllogism - ds
$(A \to B) \wedge (B \to C)$	$A \to C$	Hypothetical Syllogism - hs
$A \to B$ $A \to B$	$A \wedge C \to B \wedge C$ $A \vee C \to B \vee C$	Monotonic - mono
$(\forall x)P(x)$	$P(c)$	Universal Instantiation - ui$_{\dagger}$
$(\exists x)P(x)$	$P(c)$	Existential Instantiation - ei$_{\dagger\dagger}$
$P(c)$	$(\forall x)P(x)$	Universal Generalization - ug$_{\ddagger}$
$P(c)$	$(\exists x)P(x)$	Existential Generalization - eg$_{\ddagger\ddagger}$
$(\forall x)(\forall y)P(x,y)$ $(\exists x)(\exists y)P(x,y)$	$(\forall y)(\forall x)P(x,y)$ $(\exists y)(\exists x)P(x,y)$	Reordering - ord

† **If c is a variable, then it must not have already be quantified in $P(x)$.**

†† **c must be a new constant.**

‡ **c cannot be a free variable or derived from ei.**

‡‡ **x cannot be an existing variable or constant in $P(c)$.**

Set A set is an unordered collection of distinct elements.

Set Membership Given a set, $\mathbf{S}$, we denote that an element, a, is a member of $\mathbf{S}$ as $a \in \mathbf{S}$; otherwise, if a is not a member of $\mathbf{S}$, it is denoted $a \notin \mathbf{S}$.

Subsets Given sets $\mathbf{A}$ and $\mathbf{B}$, $\mathbf{A}$ is a subset of $\mathbf{B}$, denoted $\mathbf{A} \subseteq \mathbf{B}$, implies $\{x \mid x \in \mathbf{A} \to x \in \mathbf{B}\}$.

Proper Subsets Given sets $\mathbf{A}$ and $\mathbf{B}$, $\mathbf{A}$ is a proper subset of $\mathbf{B}$, denoted $\mathbf{A} \subset \mathbf{B}$, implies

$\{x \mid x \in \mathbf{A} \rightarrow x \in \mathbf{B} \land (\exists y)(y \in \mathbf{B} \land y \notin \mathbf{A})\}$.

Empty Set The empty set or null set is the set that contains no elements. It is denoted $\emptyset$ or $\{\}$. The empty set is a subset of every set.

Equality of Sets Given sets $\mathbf{A}$ and $\mathbf{B}$, $\mathbf{A}$ is equals $\mathbf{B}$, denoted $\mathbf{A} = \mathbf{B}$, implies $\{\mathbf{A} \subseteq \mathbf{B} \land \mathbf{B} \subseteq \mathbf{A}\}$.

Powerset Given set $\mathbf{S}$, $\wp(\mathbf{S}) = \{x \mid x \subseteq \mathbf{S}\}$. The function $\wp(\mathbf{S})$ is called the power set of $\mathbf{S}$.

Cardinality If $\mathbf{S}$ is a finite set, $|\mathbf{S}|$ is the number of elements in $\mathbf{S}$. It is called the cardinality of $\mathbf{S}$.

Number Sets Below is a list of numerical sets:

Whole Numbers $\mathbb{W} = \{0, 1, 2, 3, 4, 5, \ldots\}$

Natural Numbers $\mathbb{N} = \mathbb{W}^*$

Note: the $*$ exponent means that 0 is excluded from the set.

Integers $\mathbb{Z} = \{\ldots, -2, -1, 0, 1, 2, \ldots\}$

Rational Numbers $\mathbb{Q} = \{\frac{a}{b} \mid a \in \mathbb{Z} \land b \in \mathbb{Z}^*\}$ or $\{$decimals that either repeats or terminates$\}$

Irrational Numbers $\mathbb{I} = \{i \mid (\forall a \in \mathbb{Z})(\forall b \in \mathbb{Z}^*)(i \neq \frac{a}{b})\}$ or $\{$decimals that neither repeats nor terminates$\}$

Real Numbers $\mathbb{R} = \{r \mid r \in \mathbb{Q} \lor r \in \mathbb{I}\}$

Complex Numbers $\mathbb{C} = \{a + ib \mid a \in \mathbb{R} \land b \in \mathbb{R} \land i = \sqrt{-1}\}$

Set Operations Given sets $\mathbf{A}, \mathbf{B} \subseteq \mathbf{S}$, the following operations can be performed

Union

$$\mathbf{A} \cup \mathbf{B} = \{x \mid x \in \mathbf{A} \lor x \in \mathbf{B}\}.$$

Intersection

$$\mathbf{A} \cap \mathbf{B} = \{x \mid x \in \mathbf{A} \land x \in \mathbf{B}\}.$$

Difference

$$\mathbf{A} - \mathbf{B} = \{x \mid x \in \mathbf{A} \land x \notin \mathbf{B}\}.$$

Complement

$$\overline{\mathbf{A}} = \{x \mid x \in \mathbf{S} \land x \notin \mathbf{A}\}.$$

Cartesian Product

$$\mathbf{A} \times \mathbf{B} = \{(x, y) \mid x \in \mathbf{A} \land y \in \mathbf{B}\}.$$

Disjoint Sets The sets $\mathbf{A}$ and $\mathbf{B}$ are disjoint if $\mathbf{A} \cap \mathbf{B} = \emptyset$.

Set Identities Given $\mathbf{A}, \mathbf{B}, \mathbf{C} \in \wp(\mathbf{S})$

Expression	**Equivalent to**	**Name - Abbreviation of Rule**
$\mathbf{A} \cup \mathbf{B}$ $\mathbf{A} \cap \mathbf{B}$	$\mathbf{B} \cup \mathbf{A}$ $\mathbf{B} \cap \mathbf{A}$	Commutative - comm
$(\mathbf{A} \cup \mathbf{B}) \cup \mathbf{C}$ $(\mathbf{A} \cap \mathbf{B}) \cap \mathbf{C}$	$\mathbf{A} \cup (\mathbf{B} \cup \mathbf{C})$ $\mathbf{A} \cap (\mathbf{B} \cap \mathbf{C})$	Associative - ass
$\mathbf{A} \cup (\mathbf{B} \cap \mathbf{C})$ $\mathbf{A} \cap (\mathbf{B} \cup \mathbf{C})$	$(\mathbf{A} \cup \mathbf{B}) \cap (\mathbf{A} \cup \mathbf{C})$ $(\mathbf{A} \cap \mathbf{B}) \cup (\mathbf{A} \cap \mathbf{C})$	Distributive - dis
$\overline{\overline{\mathbf{A}}}$	$\mathbf{A}$	Double Negation - dn
$\mathbf{A} \cup \emptyset$ $\mathbf{A} \cap \mathbf{S}$	$\mathbf{A}$ $\mathbf{A}$	Identity - iden

$\mathbf{A} \cup \overline{\mathbf{A}}$ $\mathbf{A} \cap \overline{\mathbf{A}}$	$\mathbf{S}$ $\emptyset$	Complement - unit
$\mathbf{A} \cup \mathbf{S}$ $\mathbf{A} \cap \emptyset$	$\mathbf{S}$ $\emptyset$	Domination - dom
$\overline{(\mathbf{A} \cup \mathbf{B})}$ $\overline{(\mathbf{A} \cap \mathbf{B})}$	$\overline{\mathbf{A}} \cap \overline{\mathbf{B}}$ $\overline{\mathbf{A}} \cup \overline{\mathbf{B}}$	De Morgan's Law - deMor
$\mathbf{A} \cap \mathbf{A}$ $\mathbf{A} \cup \mathbf{A}$	$\mathbf{A}$ $\mathbf{A}$	Idempotent - self
$\mathbf{A} - \mathbf{B}$	$\mathbf{A} \cap \overline{\mathbf{B}}$	Difference - diff

N-ary Union Given $\mathbf{S} = \{\mathbf{A}_1, \mathbf{A}_2, \ldots, \mathbf{A}_n\}$ where $\mathbf{A}_i$s are sets,

$$\bigcup_{x \in \mathbf{S}} x = \mathbf{A}_1 \cup \mathbf{A}_2 \cup \ldots \cup \mathbf{A}_n$$

If $n = 1$,

$$\bigcup_{x \in \mathbf{S}} x = \mathbf{A}_1$$

N-ary Intersection Given $\mathbf{S} = \{\mathbf{A}_1, \mathbf{A}_2, \ldots, \mathbf{A}_n\}$ where $\mathbf{A}_i$s are sets,

$$\bigcap_{x \in \mathbf{S}} x = \mathbf{A}_1 \cap \mathbf{A}_2 \cap \ldots \cap \mathbf{A}_n$$

If $n = 1$,

$$\bigcap_{x \in \mathbf{S}} x = \mathbf{A}_1$$

Partition Given a set, $\mathbf{A}$, $P_{\mathbf{A}}$ is a partition of $\mathbf{A}$ implies $P_{\mathbf{A}} \subset \wp(\mathbf{A})$ such that

- $\forall x \in P_{\mathbf{A}}, |x| \neq 0$
- $\forall x_i, x_j \in P_{\mathbf{A}}, x_i \cap x_j = \emptyset$
- $\bigcup_{x \in P_{\mathbf{A}}} x = \mathbf{A}$

Summation

$$\sum_{i=a}^{b} f(i) = f(a) + f(a+1) + f(a+2) + \ldots + f(b)$$

where $a, b \in \mathbb{Z}$ and $a \leq b$

Summation Properties

Sum Rule

$$\sum_{i=a}^{b} f(i) + g(i) = \sum_{i=a}^{b} f(i) + \sum_{i=a}^{b} g(i)$$

Difference Rule

$$\sum_{i=a}^{b} f(i) - g(i) = \sum_{i=a}^{b} f(i) - \sum_{i=a}^{b} g(i)$$

Scalar Product Rule

$$\sum_{i=a}^{b} cf(i) = c\sum_{i=a}^{b} f(i)$$

where $c \in \mathbb{R}$

Separation Rule

$$\sum_{i=a}^{b} f(i) = \sum_{i=a}^{c} f(i) + \sum_{i=c+1}^{b} f(i)$$

where $a \leq c < b$

Inclusion and Exclusion Rule Let $\mathbf{S} = \left\{\mathbf{A}_1, \mathbf{A}_2, \ldots, \mathbf{A}_n\right\}$ where $\mathbf{A}_i$s are sets, and $\mathbf{p}_i = \left\{x \in \wp(\mathbf{S}) \mid \left|x\right| = i\right\}$ for $1 \leq i \leq n$,

$$\left|\bigcup_{x\in\mathbf{S}} x\right| = \sum_{i=1}^{n} (-1)^{i+1} \sum_{j\in p_i} \left|\bigcap_{x\in j} x\right|$$