Constructing a Truth Table of a Compound Well-Formed Formula

To construct a truth table of a compound wff, you need to perform four (4) steps. We will illustrate the steps by constructing the wff

$$\big(A \to B\big) \wedge \big(C \to D\big) \wedge \big(B' \vee D'\big) \to A' \vee C'$$

Step 1. **Start constructing the header row by listing the unique statement letters of the wff**. Our wff has four (4) statement letters, namely A, B, C, and D. The truth table so far wil look like the following:

A	B	C	D

Step 2. **Populate the remainder of the header row with the wffs that builds the compound wff using the order of precedence rules.** Based on order of precedence (review the formula sheet), the first wff to evaluate is $A \to B$. So we will have

A	B	C	D	$A \to B$

What remains is $P_1 \wedge \big(C \to D\big) \wedge \big(B' \vee D'\big) \to \big(A \vee C\big)$ where $P_1 = A \to B$. Like evaluating an algebraic expression, once you perform an operation, the result of the operation is used in the remainder of the evaluation of the expression. The next wff is $C \to D$. After that it would be B', D', $B' \vee D'$, A', C', $A' \vee C'$, $\big(A \to B\big) \wedge \big(C \to D\big)$, $\big(A \to B\big) \wedge \big(C \to D\big) \wedge \big(B' \vee D'\big)$, and finally, $\big(A \to B\big) \wedge \big(C \to D\big) \wedge \big(B' \vee D'\big) \to \big(A' \vee C'\big)$. The complete header row will look as follows:

A	B	C	D	$A \to B$	$C \to D$	B'	D'	$B' \vee D'$	A'	C'	$A' \vee C'$	$(A \to B) \wedge (C \to D)$	$(A \to B) \wedge (C \to D) \wedge (B' \vee D')$	$(A \to B) \wedge (C \to D) \wedge (B' \vee D') \to A' \vee C'$

Step 3. **Generate the truth values for the statement letters**. There will be 2^n truth values for each wff where n is the number of statement letters. The truth values for each statement letter are $\big(T^{2^{n-i}}F^{2^{n-i}}\big)^{2^{i-1}}$ where i is the position of the statement letter in the table. For example, the truth values of A is

$$\begin{aligned}\big(T^{2^{4-1}}F^{2^{4-1}}\big)^{2^{1-1}} &= \big(T^{2^3}F^{2^3}\big)^{2^0}\\ &= \big(T^8F^8\big)^1\\ &= \big(T^8F^8\big)\\ &= TTTTTTTTFFFFFFFF\end{aligned}$$

The truth values for the remaining statement letters are

A	B	C	D
T	T	T	T
T	T	T	F
T	T	F	T
T	T	F	F
T	F	T	T
T	F	T	F
T	F	F	T
T	F	F	F
F	T	T	T
F	T	T	F
F	T	F	T
F	T	F	F
F	F	T	T
F	F	T	F
F	F	F	T
F	F	F	F

Step 4. **Generate the truth values for the remaining wffs by using the truth values of the previously evaluated wffs and the connective rule of the wff**. For instance, to evaluate $A \to B$ use the truth values of A and B, and the implication rule, which will yield $TTTTFFFFTTTTTTTT$. Continue the process until all wffs are populated.

A	B	C	D	$A \to B$	$C \to D$	B'	D'	$B' \vee D'$	A'	C'	$A' \vee C'$	$(A \to B) \wedge (C \to D)$	$(A \to B) \wedge (C \to D) \wedge (B' \vee D')$	$(A \to B) \wedge (C \to D) \wedge (B' \vee D') \to A' \vee C'$
T	T	T	T	T	T	F	F	F	F	F	F	T	F	T
T	T	T	F	T	F	F	T	T	F	F	F	F	F	T
T	T	F	T	T	T	F	F	F	F	T	T	T	F	T
T	T	F	F	T	T	F	T	T	F	T	T	T	T	T
T	F	T	T	F	T	T	F	T	F	F	F	F	F	T
T	F	T	F	F	F	T	T	T	F	F	F	F	F	T
T	F	F	T	F	T	T	F	T	F	T	T	F	F	T
T	F	F	F	F	T	T	T	T	F	T	T	F	F	T
F	T	T	T	T	T	F	F	F	T	F	T	T	F	T
F	T	T	F	T	F	F	T	T	T	F	T	F	F	T
F	T	F	T	T	T	F	F	F	T	T	T	T	F	T
F	T	F	F	T	T	F	T	T	T	T	T	T	T	T
F	F	T	T	T	T	T	F	T	T	F	T	T	T	T
F	F	T	F	T	F	T	T	T	T	F	T	F	F	T
F	F	F	T	T	T	T	F	T	T	T	T	T	T	T
F	F	F	F	T	T	T	T	T	T	T	T	T	T	T