Definitions and Formula Sheet

Connectives The truth table below lists the truth values of each connective.

A	B	$A \wedge B$	$A \vee B$	$A \to B$	$A \leftrightarrow B$	A'
T	T	T	T	T	T	F
T	F	F	T	F	F	
F	T	F	T	T	F	T
F	F	F	F	T	T	

Note: prime ($'$) is an unary connective. It operates on a single statement; hence, it only has two possible truth values.

Order of Precedence of Connectives The order of precedence of connectives from highest to lowest priority is

1. (), Parentheses
2. $'$, Negation
3. $\wedge$, $\vee$, Conjunction and Disjunction
4. $\to$, Implication
5. $\leftrightarrow$, Equivalence

Note: connectives are evaluated from left to right.

Equivalence Rules		
Expression	**Equivalent to**	**Name - Abbreviation of Rule**
$A \vee B$ $A \wedge B$	$B \vee A$ $B \wedge A$	Commutative - comm
$(A \vee B) \vee C$ $(A \wedge B) \wedge C$	$A \vee (B \vee C)$ $A \wedge (B \wedge C)$	Associative - assoc
$A \vee (B \wedge C)$ $A \wedge (B \vee C)$	$(A \vee B) \wedge (A \vee C)$ $(A \wedge B) \vee (A \wedge C)$	Distributive - dis
$A \vee 0$ $A \wedge 1$	A A	Identity - iden
$A \vee 1$ $A \wedge 0$	1 0	Domination - dom
$A \vee A'$ $A \wedge A'$	1 0	Complement - neg
$(A \vee B)'$ $(A \wedge B)'$	$A' \wedge B'$ $A' \vee B'$	De Morgan's Law - deMor
$A \to B$	$A' \vee B$	Implication - imp
$A \to B$	$B' \to A'$	Contrapositive - cp
A	A''	Double Negation - dn
$A \wedge A$ $A \vee A$	A A	Idempotent - self
$A \leftrightarrow B$	$(A \to B) \wedge (B \to A)$	Equivalence - equ

$A \vee (A \wedge B)$ $A \wedge (A \vee B)$	A A	Absorption - abs
$A \to (B \to C)$	$A \wedge B \to C$	Deductive - ded
$[(\forall x)P(x)]'$ $[(\exists x)P(x)]'$	$(\exists x)[P(x)]'$ $(\forall x)[P(x)]'$	Negation - ng
Inference Rules		
Expression	**Derives**	**Name - Abbreviation of Rule**
$(A \to B) \wedge A$	B	Modus ponens - mp
$(A \to B) \wedge B'$	A'	Modus tollens - mt
$(A \wedge B)$	A B	Simplification - sim
$A \wedge B$	$(A \wedge B)$	Conjunction - con
A	$A \vee B$	Addition - add
$(A \vee B) \wedge A'$ $(A \vee B) \wedge B'$	B A	Disjunctive syllogism - ds
$(A \to B) \wedge (B \to C)$	$A \to C$	Hypothetical Syllogism - hs
$(\forall x)P(x)$	$P(c)$	Universal Instantiation - ui$_{\dagger}$
$(\exists x)P(x)$	$P(c)$	Existential Instantiation - ei$_{\dagger\dagger}$
$P(c)$	$(\forall x)P(x)$	Universal Generalization - ug$_{\ddagger}$
$P(c)$	$(\exists x)P(x)$	Existential Generalization - eg$_{\ddagger\ddagger}$
$(\forall x)(\forall y)P(x,y)$ $(\exists x)(\exists y)P(x,y)$	$(\forall y)(\forall x)P(x,y)$ $(\exists y)(\exists x)P(x,y)$	Reordering - ord

† **If c is a variable, then it must not have already be quantified in $P(x)$.**

†† **c must be a new constant.**

‡ **c cannot be a free variable or derived from ei.**

‡‡ **x cannot be an existing variable or constant in $P(c)$.**

Set A set is an unordered collection of distinct elements.