Project 1 FYS-STK4155 Autumn 2017

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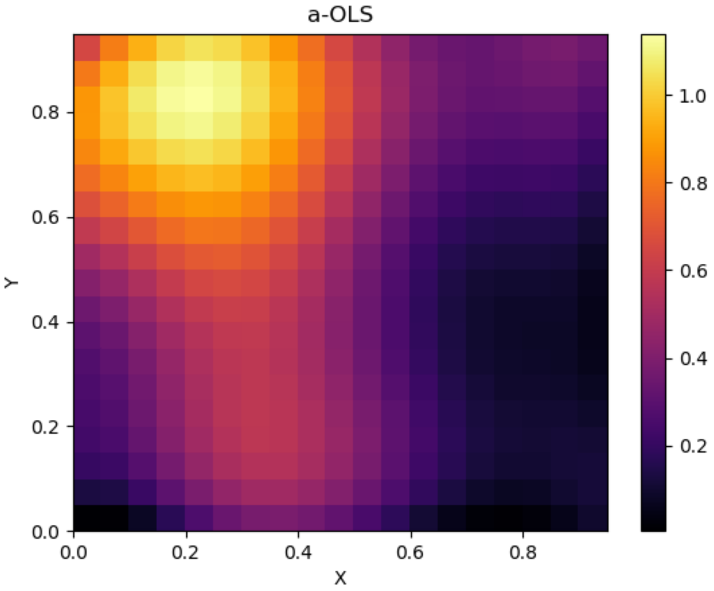
All of the plots of the models are done with degree equals 5.

Introduction

In this project we were to analyse different sets of data; test-data from the Franke-function and real data, using different linear regression models. The main goal of the project was to evaulate the different models, and come to a conclusion in terms of which models would fit best with which data.
Most of our code is in the GitHub repository. We have chosen to not use the code in our report.
We have, however, made a python file, project01.py, that provides most of our code in sequence, so that it is possible to follow the code that produce our tests and plots in order.

Ordinary least square on test data

First we have generated some test data, with the noise being very little, which we have plotted to get a more intuitive feel for how it looks. We have done this for all the models.
Ordinary Least square of the Frankefunction(the same as Ridge with $\lambda = 0$):



As you can tell from our code, we haven't made an own function for ordinary least square, since it is the same as Ridge, just with the λ set to 0. This will also show later that when the λ gets low it is very similar to OLS.

Here is the values for calculating the five first degrees and their MSE and R2-score:

OLS Test Data We can tell that our model is fitting our test data better and better with a higher degree. Now we have to check with bootstrap as well to see if our model fits the data good.

k	MSE	R2
1	0.020836568820240136	0.7159598591230791
2	0.015631136506458913	0.7869193218104034
3	0.007175355204257789	0.9021869233537347
4	0.003960217232650187	0.9460150723293508
5	0.0018602479352178107	0.9746414541595732

Before the resampling we have also calculated the betas of $k = 5$:

Var of Beta, degree 5

$$\begin{array}{llll}
[9.47238055e-03 & 8.01217226e-01 & 4.12183842e-01 & 9.89887798e+00 \\
\ 8.39455751\text{e}+00 & 4.14211294e+00 & 2.71771303e+01 & 3.28888083e+01 \\
\ 1.78285914\text{e}+01 & 1.31093669e+01 & 1.91368860e+01 & 2.67004422e+01 \\
\ 1.91391193\text{e}+01 & 9.34737951e+00 & 1.13215581\text{e}+01 & 2.55450522e+00 \\
\ 4.40599813\text{e}+00 & 3.60735967e+00 & 1.80107762e+00 & 1.00979408e+00 \\
 & & & 1.50021893e+00]
\end{array}$$

Also the 95-percentage confidence interval of the betas: 95-percentage CI of betas, degree 5

$$\begin{array}{rr}
[[7.25873511e-02 & 4.54098870e-01] \\
[7.23335269e+00 & 1.07421092e+01] \\
[3.44378188e+00 & 5.96043621e+00] \\
[-4.41211552e+01 & -3.17880887e+01] \\
[-2.42828028e+01 & -1.29254533e+01] \\
[-1.57022805e+01 & -7.72437196e+00] \\
[4.22571363e+01 & 6.26923830e+01] \\
[4.18269196e+01 & 6.43072224e+01] \\
[1.67482166e+01 & 3.32996879e+01] \\
[-1.02120404e+01 & 3.98078784e+00] \\
[-3.30453851e+01 & -1.58973753e+01] \\
[-7.33667324e+01 & -5.31114961e+01] \\
[-1.94596236e+01 & -2.31061330e+00] \\
[-3.98768180e+01 & -2.78922323e+01] \\
[1.90552365e+01 & 3.22448231e+01] \\
[-2.43921072e+00 & 3.82593943e+00] \\
[1.88142647e+01 & 2.70423776e+01] \\
[8.68290825e+00 & 1.61280472e+01] \\
[-7.84195698e+00 & -2.58124771e+00] \\
[1.66275865e+01 & 2.05666637e+01] \\
[-1.80168037e+01 & -1.32155417e+01]]
\end{array}$$

The code and commenting for the calculations is to be found in python-file project01.py

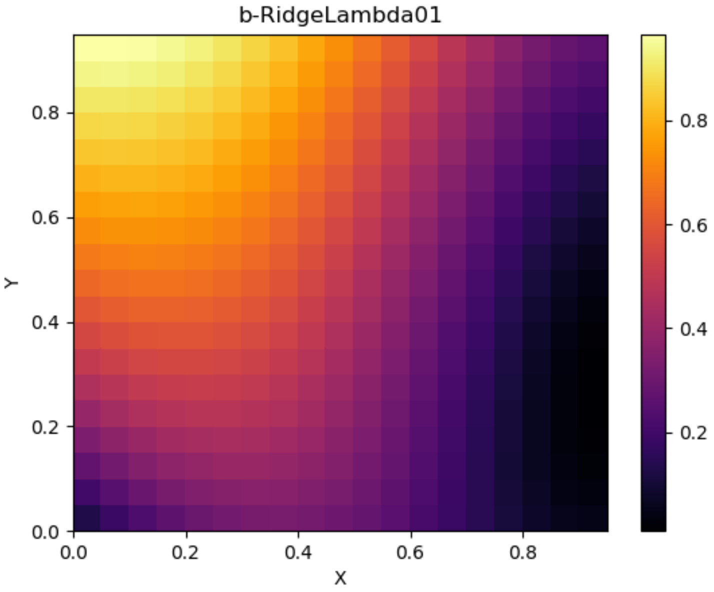
Resampling

Using our bootstrapping algorithm with a resampling of 100, degree of five, we get these values:
VAR: 0.000052
BIAS: 0.001933
Bootstrap mean of MSE: 0.0020
Bootstrap mean of r2Score: 0.9757

The bootsrap values aligns pretty well with our original ones.

Ridge regression

Ridge Regression with $\lambda = 0.1$
Graphic plot of how it looks:



Ridge Test Data

k	MSE	R2
1	0.025965982389446095	0.6906471643146342
2	0.018247163430545398	0.7826074259086047
3	0.010258352759015437	0.8777843076427536
4	0.009382588378732645	0.8882179662029949
5	0.009143926633340667	0.8910613282063765

Compared to OLS, we can tell that Ridge does significantly worse then OLS.
Var of Beta, degree 5

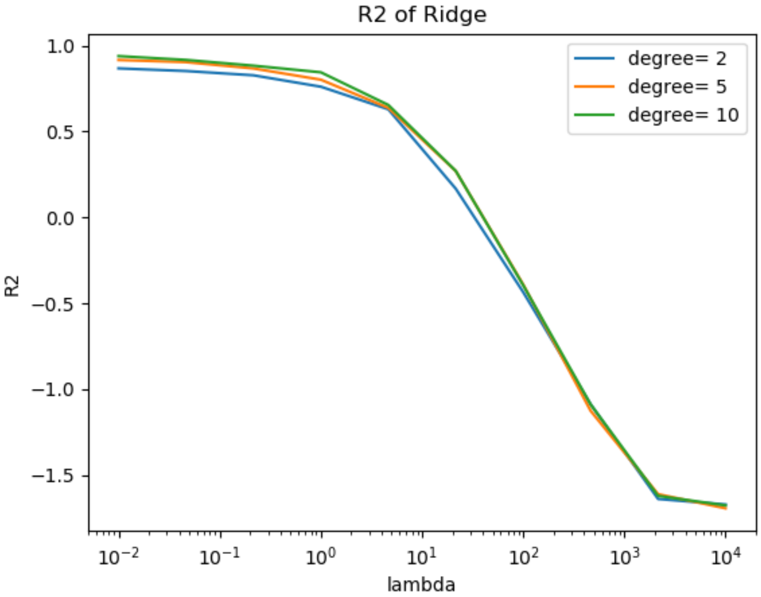
```
[0.00077004  0.01729839  0.01384995  0.06816008  0.06826285  0.05696662
  0.0417508  0.02034709  0.06671475  0.03500783  0.02294549  0.01073849
 0.03470761  0.01261125  0.02127092  0.03349824  0.01062464  0.03548007
                                    0.03422875  0.04204926  0.02520149]
```

Also the 95-percentage confidence interval of the betas: 95-percentage CI of betas, degree 5

$$\begin{array}{r}
[[8.39518095e-01 \quad 9.45603016e-01]\\
[6.00189901e-01 \quad 1.10406786e+00]\\
[9.22637923e-01 \quad 1.52633094e+00]\\
[-6.51421032e+00 \quad -5.30387095e+00]\\
[5.84279933e-01 \quad 1.67511842e+00]\\
[-5.82384066e+00 \quad -4.85326698e+00]\\
[3.42317950e+00 \quad 4.16050815e+00]\\
[1.31502300e+00 \quad 2.10193829e+00]\\
[-2.00335018e+00 \quad -1.07460992e+00]\\
[1.38884378e+00 \quad 1.92131292e+00]\\
[3.21253289e+00 \quad 3.98989176e+00]\\
[5.79118852e-01 \quad 1.17557901e+00]\\
[-3.41043372e-02 \quad 7.77207310e-01]\\
[-8.02285903e-01 \quad -2.23570539e-01]\\
[3.31692208e+00 \quad 3.80174824e+00]\\
[-3.69457810e+00 \quad -2.99905374e+00]\\
[-2.10226995e+00 \quad -1.54360569e+00]\\
[-5.64130918e-03 \quad 8.94344017e-01]\\
[-1.07674065e+00 \quad -1.87377295e-01]\\
[3.14047856e-01 \quad 8.79022520e-01]\\
[-1.87908328e+00 \quad -1.35609332e+00]]
\end{array}$$

Resampling

We can take a look at how different lambdas and different degrees of the polynomial makes a change in the R2-score and the MSE.
Here is a plot to show how they develop as a function of lambda.



We can tell pretty easily that the degree of the predictions doesn't matter much compared to how much the choice of lambda do. We can still tell that a lower degree function does worse then the other functions.
Some interesting values from bootstrap:
Bootstrap-values from degree of 5, lmb = 0.1 and 100 bootstrap-samples
VAR: 0.000067
BIAS: 0.008640
Bootstrap mean of MSE: 0.0087
Bootstrap mean of r2Score: 0.8980

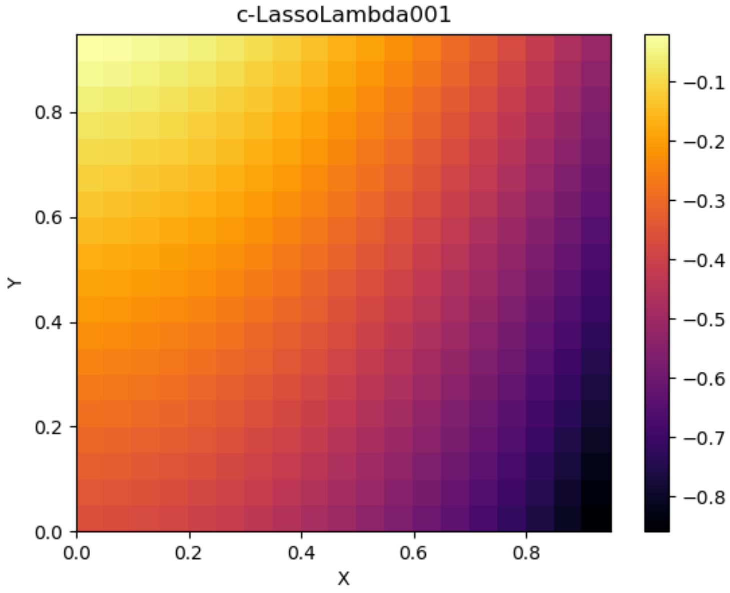
Bootstrap-values from degree of 5, lmb = 1 and 100 bootstrap-samples
VAR: 0.000059
BIAS: 0.011915
Bootstrap mean of MSE: 0.0120
Bootstrap mean of r2Score: 0.8597

Bootstrap-values from degree of 5, lmb = 10 and 100 bootstrap-samples
VAR: 0.000066
BIAS: 0.020274
Bootstrap mean of MSE: 0.0203
Bootstrap mean of r2Score: 0.7617

Bootstrap-values from degree of 2, lmb = 10 and 100 bootstrap-samples
VAR: 0.000057
BIAS: 0.022754
Bootstrap mean of MSE: 0.0228
Bootstrap mean of r2Score: 0.7327

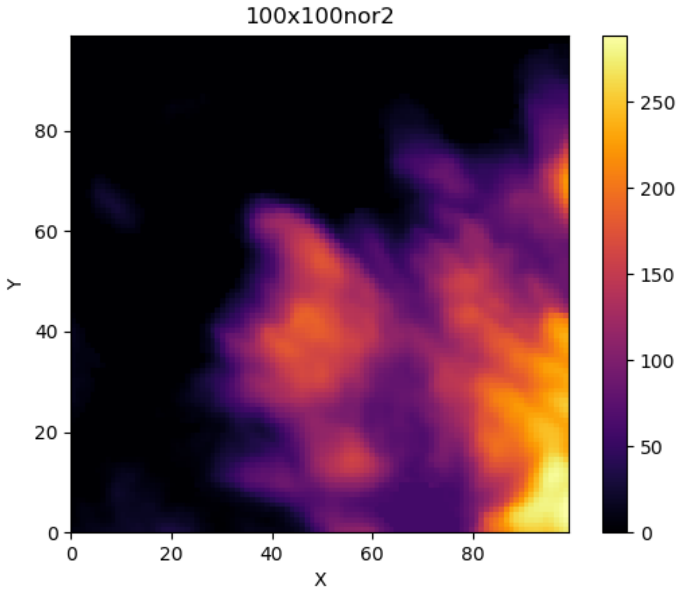
Part c)

Lasso Regression with $\lambda = 0.01$



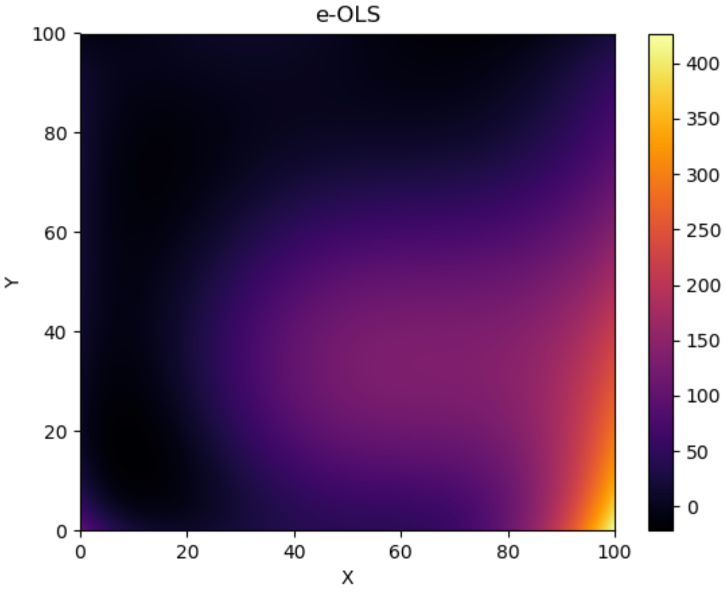
Part d)

Imports 100x100 chunk of real data from top left corner of dataset nr.1.
Plot of real data

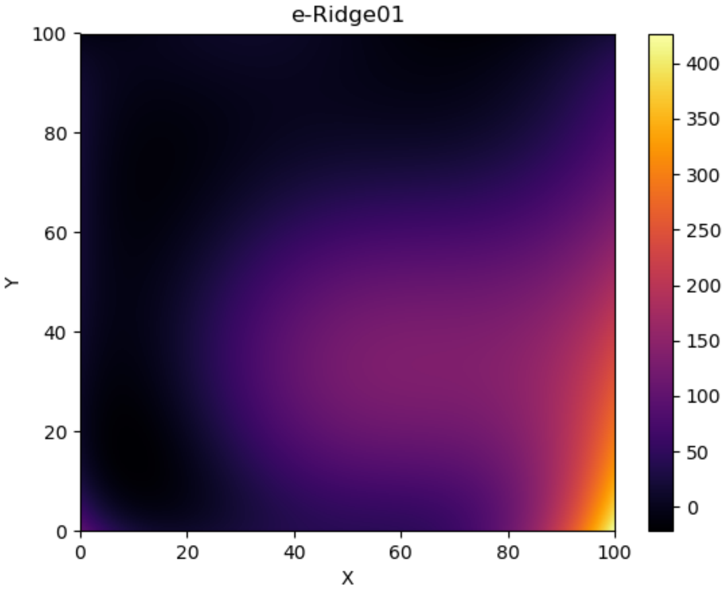


Part e)

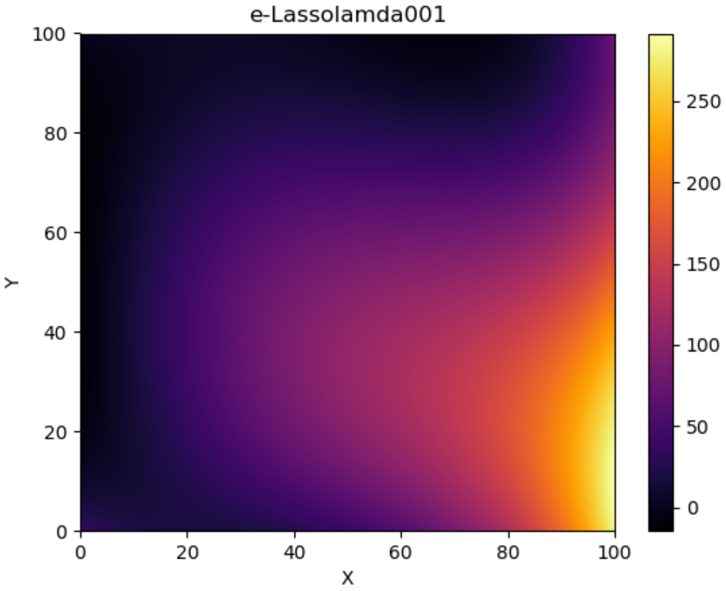
Repeat of a-c, but with real data from d)
OLS:



Ridge:



Lasso:


e-Lassolamda001
Y
X
0
20
40
60
80
100
0
20
40
60
80
100
250
200
150
100
50
0